Inverse Scattering Investigation

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ABSTRACT

The approach to the inverse scattering problem based upon the representation of the scattered field in terms of plane waves is investigated. This technique is shown to have several advantages. If the scattered field is thought of as arising from a set of discrete sources, the field can be obtained everywhere outside and between each individual source, i.e. it is not restricted to the region outside the minimum convex shape enclosing the sources. This could have practical uses for investigating cavities or antennas mounted on the surface of the body. In addition, if the scattered field (phase and amplitude) is known only over some angular bistatic sector, the near field (in the high frequency case) can be still obtained in certain regions. Thus, if it is assumed apriori that the body was a perfect conductor, then those portions of the scattering body giving rise to the observed portions of the scattered field can be found.

For non-magnetic and non-perfectly conducting bodies, it is shown that the exact total field inside the body could be represented in terms of a plane wave expansion involving the far field quantities. This representation involves an appropriate split up of the far field data, and a fundamental problem still exists to uniquely determine the split up from the knowledge of the far field data alone. It is possible that additional information will be needed; perhaps knowledge of the complete scattering matrix for all frequencies. This is an important problem since its solution will yield both the shape and material of the body.

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INTRODUCTION

The approach to obtaining the near field from knowledge of the complete scattered field, based upon the plane wave representation is investigated for a fixed frequency. Such a technique has been used for particular direct scattering problems, and modified versions of it appropriate to the high frequency case, have been employed in geometric optics (Kline and Kay, 1965).

I

GENERAL THEOREM

1.1 Determination of the Near Field from the Far Field.

A problem of fundamental significance in inverse scattering theory is that of determining the electromagnetic field at all points in space from a knowledge of the field in the far zone. In this regard, an important representation of the electromagnetic field in free space may be obtained as a combination of infinite plane waves whose amplitude factors are given by the far-field and whose directions of propagation are, in general, complex. Although the representation discussed in this section is valid only at points outside the sources of the field, the extension to points within a source region is under investigation and will be discussed in Section 4.

Consider the electromagnetic field produced by a given volume distribution of electric currents $\underline{j}$ varying harmonically with time ($e^{-i\omega t}$) and located in some finite volume V of free space. The field everywhere in space may be expressed in terms of the vector potential $\underline{A}$ given by

$$\underline{A}(\underline{x}) = \frac{\mu_o}{4\pi} \int_V \underline{j}(\underline{x}') \frac{e^{ikR}}{R} d\underline{x}' \tag{1.1}$$

where $R = |\underline{x} - \underline{x}'|$, and the far-field distribution has the form

$$\underline{A}(r, \theta, \phi) \underset{r \to \infty}{\sim} \frac{e^{ikr}}{r} \underline{A}_o(\theta, \phi) \tag{1.2}$$

with

$$\underline{A}_o(\theta, \phi) = \frac{\mu_o}{4\pi} \int_V \underline{j}(\underline{x}') \, e^{-i\underline{k} \cdot \underline{x}'} d\underline{x}' \ , \qquad (1.3)$$

$$\underline{k} = k(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \ .$$

The currents $\underline{j}$ may be thought of as equivalent sources for some scattered field or as real sources for some radiation field.

For points exterior to V the Green's function e^{ikR}/R can be expanded into plane waves. We shall employ the well-known integral representation due to Weyl (1919) (see also Stratton, 1941, p. 578)

$$\frac{e^{ikR}}{R} = \frac{ik}{2\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2} - i\infty} e^{i\underline{k} \cdot (\underline{x} - \underline{x}')} \sin\alpha \, d\alpha \, d\beta \quad (z \geqslant z') \quad (1.4)$$

where now $\underline{k} = k(\sin\alpha\cos\beta, \sin\alpha\sin\beta, \cos\alpha)$ is a function of the variables of integration α and β running from 0 to $\frac{1}{2}\pi - i\infty$ and 0 to 2π, respectively. It is seen that in this expansion of the spherical wave e^{ikR}/R all possible plane-wave directions within the limits $0 \leqslant \beta \leqslant 2\pi$, $0 \leqslant \alpha \leqslant (\pi/2)$ are included; values of α lying in $(\pi/2) \leqslant \alpha \leqslant \pi$ correspond to plane waves travelling in from infinity in the half-space $z \geqslant z'$, and are, therefore, excluded. In addition, however, inhomogeneous plane waves with an exponentially decreasing amplitude in the z-direction (for $z > z'$) are included in order to yield the necessary singularity at $R \to 0$. These waves correspond to that part of the integration path running from $\alpha = (\pi/2)$ to $\alpha = (\pi/2) - i\infty$. An alternative representation valid in the half-space $z \leqslant z'$ may be obtained by selecting a different path of integration in the α-plane; thus, for example, we may write

$$\frac{e^{ikR}}{R} = -\frac{ik}{2\pi}\int_0^{2\pi}\int_{\pi}^{\frac{\pi}{2}+i\infty} e^{i\underline{k}\cdot(\underline{x}-\underline{x}')}\sin\alpha\, d\alpha\, d\beta \qquad (z \lessgtr z') \tag{1.5}$$

When (1.4) is introduced into (1.1) and the orders of integration interchanged, one obtains for the vector potential $\underline{A}$ the following result

$$\underline{A}(\underline{x}) = \frac{ik}{2\pi}\frac{\mu_o}{4\pi}\int_V \underline{j}(\underline{x}')\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot(\underline{x}-\underline{x}')}\sin\alpha\, d\alpha\, d\beta\, d\underline{x}'$$

$$= \frac{ik}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot\underline{x}}\left\{\frac{\mu_o}{4\pi}\int_V \underline{j}(\underline{x}')\, e^{-i\underline{k}\cdot\underline{x}'}\, d\underline{x}'\right\}\sin\alpha\, d\alpha\, d\beta\,, \tag{1.6}$$

and, upon recognizing in view of (1.3) that the quantity contained in $\{\ \}$ immediately above is merely $\underline{A}_o(\alpha,\beta)$, one finds

$$\underline{A}(\underline{x}) = \frac{ik}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot\underline{x}}\,\underline{A}_o(\alpha,\beta)\sin\alpha\, d\alpha\, d\beta \tag{1.7}$$

provided $\underline{x}$ lies in the half-space formed by the portion of the z-axis above the source volume V, that is, $z > z'_{max}$. In this upper half-space, then, Eq. (1.7) provides a representation of the near field in terms of the far-field data. For $\underline{x}$ lying in the lower half-space below the source region, $z < z'_{min}$, we have by virtue of (1.5)

$$\underline{A}(\underline{x}) = -\frac{ik}{2\pi} \int_0^{2\pi} \int_{\pi}^{\frac{\pi}{2}+i\infty} e^{i\underline{k}\cdot\underline{x}} \underline{A}_o(\alpha,\beta) \sin\alpha \, d\alpha \, d\beta \ . \tag{1.8}$$

The integrals (1.7) and (1.8) together give the field everywhere in space except in the region $z'_{min} \leqslant z \leqslant z'_{max}$ which sandwiches the sources. It is clear, however, that other paths of integration in the α-plane depending on the observation angle θ may be selected to yield results even within $z'_{min} \leqslant z \leqslant z'_{max}$, although the source region must still be excluded. Choosing other paths of integration is tantamount to rotating the reference axes and will be discussed shortly.

1.2 Additional Comments.

As we have seen, the integral representation of the near field in terms of the far field requires integration over a surface element $d\Omega = \sin\alpha \, d\alpha \, d\beta$ of the complex unit sphere Ω. It is interesting to note that integration over the real portion of the unit sphere yields a result which contains both incoming and outgoing waves. Thus, in view of the representation (Stratton, 1941, p. 410)

$$\frac{\sin kR}{R} = \frac{k}{4\pi} \int_0^{2\pi} \int_0^{\pi} e^{i\underline{k}\cdot(\underline{x}-\underline{x}')} \sin\alpha \, d\alpha \, d\beta \ , \tag{1.9}$$

one finds

$$\frac{k}{4\pi}\int_0^{2\pi}\int_0^{\pi} e^{i\underline{k}\cdot\underline{x}}\,\underline{A}_o(\alpha,\beta)\sin\alpha\, d\alpha\, d\beta = \frac{\mu_o}{4\pi}\int_V \underline{j}(\underline{x}')\,\frac{\sin kR}{R}\,d\underline{x}' \ . \qquad (1.10)$$

On the other hand, complex values of β as well as α may be included since, by a straightforward modification of the Weyl formula (1.4), we have

$$\frac{e^{ikR}}{R} = \frac{ik}{2\pi}\int_{-\frac{\pi}{2}+i\infty}^{\frac{\pi}{2}-i\infty}\int_{-\frac{\pi}{2}+i\infty}^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot(\underline{x}-\underline{x}')}\sin\alpha\, d\alpha\, d\beta \ , \qquad (1.11)$$

and thus

$$\underline{A}(\underline{x}) = \frac{ik}{2\pi}\iint_{-\frac{\pi}{2}+i\infty}^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot\underline{x}}\,\underline{A}_o(\alpha,\beta)\sin\alpha\, d\alpha\, d\beta \ . \qquad (1.12)$$

1.3 Rotations of the Reference Axes.

The integral representations (1.7) and (1.8) taken together exclude certain portions of free space, namely the free-space points lying within the region $z'_{min} \leqslant z \leqslant z'_{max}$ sandwiching the source volume V. These points may be included by means of a rotation of the reference axes. Consider the integral (1.4)

$$\frac{e^{ikR}}{R} = \frac{ik}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot(\underline{x}-\underline{x}')}\sin\alpha\, d\alpha\, d\beta \qquad (z \geqslant z')$$

where $\underline{k} = k(\sin\alpha\cos\beta,\ \sin\alpha\sin\beta,\ \cos\alpha)$. This integral is invariant to a rotation of the reference axes, thus change to new variables α', β' defined by direction cosine relations

$$\begin{aligned}
\sin\alpha'\cos\beta' &= \sin\alpha\cos\theta_o\cos(\beta-\phi_o) - \cos\alpha\sin\theta_o\ , \\
\sin\alpha'\sin\beta' &= \sin\alpha\sin(\beta-\phi_o)\ , \qquad (1.13)\\
\cos\alpha' &= \sin\alpha\sin\theta_o\cos(\beta-\phi_o) + \cos\alpha\cos\theta_o\ ,
\end{aligned}$$

where θ_o, ϕ_o are arbitrary (real) angles. The inverse transformation, defining the old variables (α,β) in terms of the new variables (α',β'), is

$$\begin{aligned}
\sin\alpha\cos\beta &= \cos\alpha'\sin\theta_o\cos\phi_o + \sin\alpha'\left[\cos\theta_o\cos\beta'\cos\phi_o - \sin\beta'\sin\phi_o\right]\ , \\
\sin\alpha\sin\beta &= \cos\alpha'\sin\theta_o\sin\phi_o + \sin\alpha'\left[\cos\theta_o\cos\beta'\sin\phi_o + \sin\beta'\cos\phi_o\right]\ , \\
\cos\alpha &= \cos\alpha'\cos\theta_o - \sin\alpha'\sin\theta_o\cos\beta'\ . \qquad (1.14)
\end{aligned}$$

Further

$$d\Omega = \sin\alpha\, d\alpha\, d\beta = \sin\alpha'\, d\alpha'\, d\beta' = d\Omega' \qquad (1.15)$$

and the limits of integration may remain the same. The integral representation is essentially unchanged in form:

$$\frac{e^{ikR}}{R} = \frac{ik}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot(\underline{x}-\underline{x}')}\sin\alpha'\, d\alpha'\, d\beta'\ , \qquad (1.16)$$

where the directional cosines of $\underline{k}$ are to be obtained from the inverse transformation relations given in (1.14). The integral, however, now converges at $\alpha' = (\pi/2) - i\infty$

provided

$$(x-x')\sin\theta_o\cos\phi_o + (y-y')\sin\theta_o\sin\phi_o + (z-z')\cos\theta_o > 0 \,, \qquad (1.17)$$

that is, provided

$$\underline{x}\cdot\hat{x}_o > \underline{x}'\cdot\hat{x}_o \qquad (1.18)$$

where $\hat{x}_o$ is the unit vector

$$\hat{x}_o = (\sin\theta_o\cos\phi_o,\ \sin\theta_o\sin\phi_o,\ \cos\theta_o)\ . \qquad (1.19)$$

We have, therfore,

$$\underline{A}(\underline{x}) = \frac{ik}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot\underline{x}}\,\underline{A}_o(\alpha,\beta)\sin\alpha'\,d\alpha'\,d\beta' \qquad (1.20)$$

provided

$$\underline{x}\cdot\hat{x}_o > \max\,(\underline{x}'\cdot\hat{x}_o)\ . \qquad (1.21)$$

This representation is thus valid for all $\underline{x}$ lying in the half-space formed by the portion of the $\hat{x}_o$ axis not containing the sources. Since $\hat{x}_o$ is an arbitrarily directed vector, it is clear that portions of free space within $z'_{min} \leqslant z \leqslant z'_{max}$, which were previously excluded, may now be included. In particular, by rotating the $\hat{x}_o$ vector we may generate the field everywhere in the space outside some minimum convex shape surrounding the sources.

1.4 Relationship to Spherical Harmonics.

Assume the far field is known as an expansion in spherical harmonics

$$\underline{A}_o(\theta, \phi) = \sum_n \sum_m \underline{a}_{nm} P_n^m(\cos\theta) e^{im\phi} . \tag{1.22}$$

Then, because of the integral representation derived by Erdélyi (1937):

$$i^n h_n^{(1)}(kr) P_n^m(\cos\theta) e^{\pm im\phi} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot\underline{x}} P_n^m(\cos\alpha) e^{\pm im\beta} \sin\alpha \, d\alpha d\beta , \tag{1.23}$$

we have immediately from (1.7)

$$\underline{A}(\underline{x}) = ik \sum_n \sum_m \underline{a}_{nm} i^n h_n^{(1)}(kr) P_n^m(\cos\theta) e^{im\phi} , \tag{1.24}$$

thereby giving the field as an expansion in spherical wave functions.

1.5 Relationship to Aperture Problems.

Assume the source currents $\underline{j}(\underline{x}')$ are confined to the plane $z'=0$ and denote the directional cosines of $\underline{k}$ by u, v, w where

$$\begin{aligned} u &= \sin\alpha \cos\beta, \\ v &= \sin\alpha \sin\beta, \\ w &= \cos\alpha . \end{aligned} \tag{1.25}$$

The far-field amplitude (1.3) may be written in the form

$$\underline{A}_o(u, v) = \frac{\mu_o}{4\pi} \int \underline{j}(x', y')\, e^{-ik(ux' + vy')}\, dx'\, dy' \tag{1.26}$$

and the spherical wave (1.4) may be expressed as

$$\frac{e^{ikR}}{R} = \frac{ik}{2\pi} \iint_{-\infty}^{\infty} e^{ik\left[u(x-x') + v(y-y') + w(z-z')\right]} \frac{dudv}{w} \tag{1.27}$$

since $\sin\alpha\, d\alpha\, d\beta = (1/w)dudv$. Thus, remembering $z'=0$, we have

$$\underline{A}(\underline{x}) = \frac{\mu_o}{4\pi} \int \underline{j}(x', y') \frac{e^{ikR}}{R} dx'dy' = \frac{ik}{2\pi} \iint_{-\infty}^{\infty} e^{ik(ux + vy + wz)} \underline{A}_o(u, v) \frac{dudv}{w} . \tag{1.28}$$

When $z=0$ this leads to the well-known result that polar diagrams and aperture distributions are related by two-dimensional Fourier transformations (see e.g. Bouwkamp, 1954).

1.6 The Field Quantities.

The electromagnetic field is derived from the vector potential by means of the relations

$$\underline{H} = \frac{1}{\mu_o} \text{curl}\, \underline{A},$$

$$\underline{E} = \frac{i}{\epsilon_o \mu_o \omega} \text{curl curl}\, \underline{A}. \tag{1.29}$$

In the far zone these equations give

$$\underline{H}(r,\theta,\phi) \sim \frac{e^{ikr}}{r}\,\underline{H}_o(\theta,\phi) = \frac{e^{ikr}}{r}\left(\frac{i}{\mu_o}\right)\underline{k}\wedge\underline{A}_o(\theta,\phi)\,,$$

$$\underline{E}(r,\theta,\phi) \sim \frac{e^{ikr}}{r}\,\underline{E}_o(\theta,\phi) = \frac{e^{ikr}}{r}\left(\frac{-i}{\epsilon_o\mu_o\omega}\right)\underline{k}\wedge\underline{k}\wedge\underline{A}_o(\theta,\phi)\,. \qquad (1.30)$$

When these relations are applied to the integral representation (1.7) of the vector potential, one finds

$$\underline{H}(\underline{x}) = \frac{ik}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot\underline{x}}\left(\frac{i}{\mu_o}\right)\underline{k}\wedge\underline{A}_o(\alpha,\beta)\sin\alpha\,d\alpha\,d\beta$$

$$= \frac{ik}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot\underline{x}}\,\underline{H}_o(\alpha,\beta)\sin\alpha\,d\alpha\,d\beta\,, \qquad (1.31)$$

and similarly for the electric field

$$\underline{E}(\underline{x}) = \frac{ik}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot\underline{x}}\,\underline{E}_o(\alpha,\beta)\sin\alpha\,d\alpha\,d\beta\,. \qquad (1.32)$$

The near fields may thus be represented directly in terms of the electromagnetic fields in the far zone.

1.7 Determination of Field in Free-Space Region between Distinct Sources.

If the original source region V can be separated into a number of disjointed volumes V_i in free space, then it may be possible to determine the field in the space between these distinct source regions. For example, let the currents $\underline{j}$ be located in two finite volumes V_1 and V_2, and further assume that V_1 lies within the range $z_1 < z < z_2$ while V_2 lies within the range $z_3 < z < z_4$ with $z_2 < z_3$. Also, let $\underline{H}_o^{[1]}(\theta, \phi)$ and $\underline{H}_o^{[2]}(\theta, \phi)$ denote the far-field amplitudes due to the sources in V_1 and V_2, respectively. Then the field in the free-space region $z_2 < z < z_3$ between the two volumes V_1, V_2 can be represented in the form

$$\underline{H}(\underline{x}) = \frac{ik}{2\pi}\left\{\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot\underline{x}}\,\underline{H}_o^{[1]}(\alpha,\beta)\sin\alpha\,d\alpha\,d\beta - \int_0^{2\pi}\int_{\pi}^{\frac{\pi}{2}+i\infty} e^{i\underline{k}\cdot\underline{x}}\,\underline{H}_o^{[2]}(\alpha,\beta)\sin\alpha\,d\alpha\,d\beta\right\} \qquad (1.33)$$

The first integral above converges for $z > z_2$ while the second converges for $z < z_3$; hence, this representation is valid in the desired region $z_2 < z < z_3$. This has an immediate application in providing a means of separating out distinct sources of the scattered field that may occur, such as an antenna or other protuberance mounted on a smooth surface.

II

ANALYTIC CONTINUATION

The field $\underline{H}_o(\theta, \phi)$ in the far zone is measurable only for real values of θ, ϕ in the ranges $0 \leqslant \theta \leqslant \pi$, $0 \leqslant \phi \leqslant 2\pi$. However, in order to obtain the near field by means of the integral representation discussed in the previous section, it is necessary to know $\underline{H}_o(\alpha, \phi)$ where $\alpha = \theta + i\xi$. Therefore, we need an extension into the complex α-plane based upon the measured quantity $\underline{H}_o(\theta, \phi)$.

Now $\underline{H}_o(\theta, \phi)$ is immediately known for the range $-\pi \leqslant \theta \leqslant \pi$. This follows from the definition

$$\underline{H}_o(\theta, \phi) = \frac{i}{4\pi} \int_V \underline{k} \wedge \underline{j}(\underline{x}')\, e^{-i\underline{k}\cdot\underline{x}'}\, d\underline{x}' \tag{2.1}$$

and the fact that $\underline{k}$ as a function of θ and ϕ satisfies

$$\underline{k}(-\theta, \phi) = \underline{k}(\theta, \phi \pm \pi) ; \tag{2.2}$$

hence

$$\underline{H}_o(-\theta, \phi) = \underline{H}_o(\theta, \phi \pm \pi) . \tag{2.3}$$

In addition, $\underline{H}_o$ is periodic with period 2π in both θ and ϕ.

To obtain an expression for $\underline{H}_o(\alpha, \phi)$ in the complex α-plane, we observe from (2.1) that $\underline{H}_o(\alpha, \phi)$ is a harmonic function in the variables θ and ξ; that is

$$\left(\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial\xi^2}\right) \underline{H}_o(\theta + i\xi, \phi) = 0 . \tag{2.4}$$

As such, $\underline{H}_o(\alpha, \phi)$ may be expressed in the following form

$$\underline{H}_o(\alpha, \phi) = \sum_{n=-\infty}^{\infty} \underline{a}_n e^{in(\theta + i\xi)} , \tag{2.5}$$

where the coefficients $\underline{a}_n$ are derived by means of the relation

$$\underline{a}_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-in\theta'} \underline{H}_o(\theta', \phi) \, d\theta' . \tag{2.6}$$

This provides an extension into the complex α-plane. The series (2.5) may be partially summed and put into closed form as follows:

$$\underline{H}_o(\alpha, \phi) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{e^{i(\theta + i\xi)} \underline{H}_o(\theta', \phi)}{e^{i(\theta + i\xi)} - e^{i\theta'}} d\theta' + \frac{1}{2\pi} \sum_{n=1}^{\infty} e^{in(\theta + i\xi)} \int_{-\pi}^{\pi} \underline{H}_o(\theta', \phi) e^{-in\theta'} d\theta' \tag{2.7}$$

for $\xi < 0$, and

$$\underline{H}_o(\alpha, \phi) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{e^{i\theta'} \underline{H}_o(\theta', \phi)}{e^{i\theta'} - e^{i(\theta + i\xi)}} d\theta' + \frac{1}{2\pi} \sum_{n=1}^{\infty} e^{-in(\theta + i\xi)} \int_{-\pi}^{\pi} \underline{H}_o(\theta', \phi) e^{in\theta'} d\theta' \tag{2.8}$$

for $\xi > 0$.

To investigate the convergence of the series (2.5) we examine the behavior of $\underline{a}_n$ as $n \rightarrow \infty$. Now

$$\underline{a}_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-in\theta}\left\{\frac{i}{4\pi}\int_V \underline{k}\wedge\underline{j}(x')\, e^{-i\underline{k}\cdot\underline{x}'}\, d\underline{x}'\right\} d\theta = \frac{-i}{8\pi^2}\int_V \underline{j}(\underline{x}')\wedge\int_{-\pi}^{\pi} \underline{k}\, e^{-in\theta - i\underline{k}\cdot\underline{x}'}\, d\theta\, d\underline{x}' \tag{2.9}$$

where

$$\underline{k} = k(\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta) \tag{2.10}$$

Write $\underline{k}$ in the form

$$\underline{k} = \frac{k}{2}(e^{i\theta}\,\underline{t} + e^{-i\theta}\,\underline{t}^*) \tag{2.11}$$

with

$$\underline{t} = (-i\cos\phi,\ -i\sin\phi,\ 1)\ , \tag{2.12}$$

then

$$\underline{a}_n = \frac{-ik}{16\pi^2}\int_V \underline{j}(\underline{x}')\wedge\left\{\underline{t}\int_{-\pi}^{\pi} e^{-i(n-1)\theta - i\underline{k}\cdot\underline{x}'}\, d\theta + \underline{t}^*\int_{-\pi}^{\pi} e^{-i(n+1)\theta - i\underline{k}\cdot\underline{x}'}\, d\theta\right\} d\underline{x}'\ . \tag{2.13}$$

But $\underline{k}\cdot\underline{x}'$ may be written in the form

$$\begin{aligned}\underline{k}\cdot\underline{x}' &= kr'\left[\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi-\phi')\right]\\ &= kr'\rho\cos(\theta-\psi)\ ,\end{aligned} \tag{2.14}$$

where

$$\begin{aligned}\rho^2 &= \cos^2\theta' + \sin^2\theta'\cos^2(\phi-\phi')\ ,\\ \tan\psi &= \tan\theta'\cos(\phi-\phi')\ ,\end{aligned} \tag{2.15}$$

and, in view of the integral representation

$$e^{-im\psi} J_m(kr'\rho) = \frac{i^n}{2\pi} \int_{-\pi}^{\pi} e^{-im\theta - ikr'\rho\cos(\theta-\psi)}\, d\theta, \qquad (2.16)$$

we have the closed form expression

$$\underline{a}_n = \frac{(-i)^n k}{8\pi} \int_V \underline{j}(\underline{x}') \wedge \left\{ \underline{t}\, e^{i\psi} J_{n-1}(kr'\rho) - \underline{t}^* e^{-i\psi} J_{n+1}(kr'\rho) \right\} e^{-in\psi}\, d\underline{x}' \ . \qquad (2.17)$$

As n tends to infinity the dominant contribution is due to the first term within the braces in the integrand

$$\underline{a}_n \underset{n\to\infty}{\sim} \frac{(-i)^n k}{8\pi \Gamma(n)} \int_V \underline{j}(\underline{x}') \wedge \underline{t} \left(\frac{kr'\rho}{2} e^{-i\psi} \right)^{n-1} d\underline{x}' \ , \qquad (2.18)$$

hence

$$\left| \underline{a}_n \right|_{n\to\infty} \leqslant \frac{\text{Constant}}{\Gamma(n)} \left(\frac{kR}{2} \right)^{n-1} \qquad (2.19)$$

where R represents the maximum value of r' . A similar result holds for $\left| \underline{a}_{-n} \right|$. The convergence of the series (2.5) is therefore secured for all $\alpha = \theta + i\xi$ because of the gamma function in the denominator of (2.19).

III

HIGH-FREQUENCY SCATTERING

It will often happen at high frequencies that the far scattered field from a body can be characterized either by a rapidly varying phase function (e.g. specular scattering) or by a rapidly varying amplitude function (e.g. scattering by a flat plate). Under these conditions the contour integral representation of the near field in terms of the far field is particularly convenient because the powerful methods of asymptotic analysis are then at one's disposal. As an application of the integral representation at high frequencies we shall here consider the geometrical optics field produced by a plane wave incident on a perfectly conducting sphere.

The incident field is taken in the form

$$\underline{E}^i = \hat{x}\, e^{-ikz} ,$$
$$\underline{H}^i = -\sqrt{\frac{\epsilon_o}{\mu_o}}\, \hat{y}\, e^{-ikz} , \tag{3.1}$$

and the scattered field is given by

$$\underline{E}^s(\underline{x}) = \frac{ik}{2\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot\underline{x}}\, \underline{E}_o^s(\alpha,\beta) \sin\alpha \, d\alpha \, d\beta ,$$

$$\underline{H}^s(\underline{x}) = \frac{ik}{2\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot\underline{x}}\, \underline{H}_o^s(\alpha,\beta) \sin\alpha \, d\alpha \, d\beta , \tag{3.2}$$

The geometrical optics field in the far zone may be written as

$$\underline{E}_o^s(\alpha,\beta) = -\frac{a}{2}\hat{e}(\alpha,\beta)\, e^{-2ika\cos(\alpha/2)} ,$$

$$\underline{H}_o^s(\alpha,\beta) = -\sqrt{\frac{\epsilon_o}{\mu_o}}\,\frac{a}{2}\hat{h}(\alpha,\beta)\, e^{-2ika\cos(\alpha/2)} , \tag{3.3}$$

where $\hat{e}(\alpha,\beta)$ and $\hat{h}(\alpha,\beta)$ are the unit vectors

$$\hat{e}(\alpha,\beta) = \hat{x}(\cos\alpha\cos^2\beta + \sin^2\beta) - \hat{y}(1-\cos\alpha)\sin\beta\cos\beta - \hat{z}\sin\alpha\cos\beta ,$$

$$\hat{h}(\alpha,\beta) = -\hat{x}(1-\cos\alpha)\sin\beta\cos\beta + \hat{y}(\cos\alpha\sin^2\beta + \cos^2\beta) - \hat{z}\sin\alpha\sin\beta . \tag{3.4}$$

The exponential behavior of the integrands in (3.2) is thus governed by the factor

$$e^{ikf(\alpha,\beta)} \tag{3.5}$$

where

$$f(\alpha,\beta) = \hat{k}\cdot\underline{x} - 2a\cos\frac{\alpha}{2} = r\left[\sin\theta\sin\alpha\cos(\phi-\beta) + \cos\theta\cos\alpha\right] - 2a\cos\frac{\alpha}{2} . \tag{3.6}$$

Upon examining the convergence of the integral as $\alpha \to (\pi/2) - i\infty$, one finds that the integrand decays exponentially so long as $r\cos\theta > 0$, that is, $z > 0$. When $z = 0$, however, the integrand grows exponentially as $\alpha \to (\pi/2) - i\infty$ and therefore diverges. Hence the representation (3.2) with (3.3) is valid for all $z > 0$, or what is the same, $0 \leqslant \theta < (\pi/2)$.

As $k \to \infty$ the dominant contribution to the integral arises from the vicinity of the stationary phase point ($\beta = \phi$, $\alpha = \alpha_o$) where α_o satisfies the equation

$$r \sin(\alpha_o - \theta) = a \sin(\alpha_o / 2) \quad . \tag{3.7}$$

The physical interpretation of this equation is shown in Fig. (3-1). The quantity $p = a \sin(\alpha_o/2)$ may be interpreted as the impact parameter associated with an incident ray, and this is precisely the incident ray that reaches the observation point P after being reflected at the surface according to the laws of geometrical optics. The angle α_o is twice the angle of incidence.

By means of a first order stationary phase evaluation we obtain immediately

$$\underline{E}^s(\underline{x}) \sim -\frac{a}{2} \sqrt{\frac{\sin \alpha_o}{\left[r \cos(\alpha_o - \theta) - \frac{a}{2} \cos\frac{\alpha_o}{2}\right] r \sin\theta}} \; \hat{e}(\alpha_o, \phi)\, e^{ikf(\alpha_o, \phi)} \, , \tag{3.8}$$

and similarly for the magnetic field. If we let s denote the distance along the reflected ray from the point of reflection

$$\begin{aligned} s &= r \cos(\alpha_o - \theta) - a \cos(\alpha_o/2) \, , \\ r \sin\theta &= s \sin\alpha_o + a \sin(\alpha_o/2) \, , \end{aligned} \tag{3.9}$$

then the result (3.8) may be written in the form

$$\underline{E}^s(\underline{x}) \sim -\left[\frac{D(0)}{D(s)}\right]^{1/2} \hat{e}(\alpha_o, \phi)\, e^{iks - ika\cos(\alpha_o/2)} \, , \tag{3.10}$$

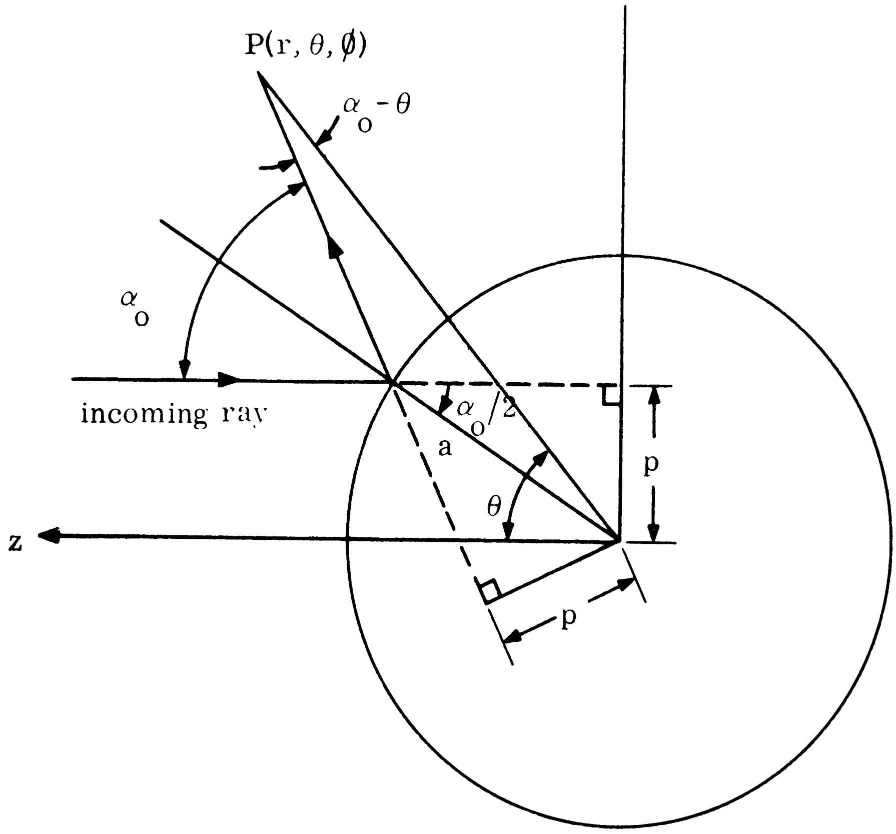

FIG. 3-1: PHYSICAL INTERPRETATION OF STATIONARY PHASE POINT α_0 FOR HIGH-FREQUENCY (GEOMETRICAL OPTICS) SCATTERING BY CONDUCTING SPHERE.

where

$$D(s) = \left(\cos\frac{\alpha_o}{2} + \frac{2s}{a}\right)\left(1 + \frac{2s}{a}\cos\frac{\alpha_o}{2}\right) \quad . \tag{3.11}$$

The magnetic field takes the analogous form

$$\underline{H}^s(\underline{x}) \sim -\sqrt{\frac{\epsilon_o}{\mu_o}}\left[\frac{D(0)}{D(s)}\right]^{1/2} \hat{h}(\alpha_o, \phi)\, e^{iks - ika\cos(\alpha_o/2)} \quad . \tag{3.12}$$

The amplitude factor $\left[D(0)/D(s)\right]^{1/2}$ in the equations above accounts for the divergence of the rays after reflection at the surface. This factor has been derived on the basis of geometrical optics for reflection from an arbitrary convex body by Fock (1948), and it is easy to verify that the quantity (3.11) agrees with the expression given by Fock in the case of the sphere. The phase of the field is also in agreement with standard geometrical optics considerations.

The ordinary stationary phase evaluation fails in the vicinity of the caustic given by the equation

$$r\cos(\alpha_o - \theta) - \frac{a}{2}\cos\frac{\alpha_o}{2} = 0 \; ; \tag{3.13}$$

however, it must be emphasized that the behavior of the field near the caustic may still be examined by applying a modified asymptotic analysis to the integral representation in (3.2). The elegance and simplicity of this representation for application to high-frequency scattering is evident.

IV

REPRESENTATION OF THE FIELD INSIDE THE SCATTERING BODY

In the previous sections the scattered field was represented in terms of a vector potential involving currents that were physical or otherwise; i.e. the fact that the scattered field arose from induced sources was not prescribed, only that it arose from some current distribution. Outside the source region the scattered field was then expressed in terms of an integral operator acting on the far-zone scattered field components. In this section the possibility of obtaining an expression for the total field inside the scattering body, in terms of the far-zone scattered field is examined.

As a preliminary, the derivation of the total field in terms of a vector potential relating to the actual induced currents (conduction and polarization) is reviewed. It will be assumed that the scattering body is contained in a finite volume V_s. The material of the body will be taken to be non-magnetic (i.e. $\mu = \mu_o$), and characterized by the relative permittivity ϵ' which may be complex allowing for conductivity. For present purposes the conductivity will be taken to be finite (but can be extremely large) thus ruling out the mathematical concept of a perfect conductor. Let the incident field be generated by a current source $\underline{J}_o$ outside the body. The source will first be taken a finite distance from the body, then later allowed to go to infinity, to account for plane wave incidence. Maxwell's equations become

$$\underline{\nabla} \cdot \underline{H} = 0 \tag{4.1}$$

$$\omega \epsilon_o \underline{\nabla} \cdot \epsilon' \underline{E} = i \underline{\nabla} \cdot \underline{J}_o \tag{4.2}$$

$$\underline{\nabla} \wedge \underline{E} = i \omega \mu_o \underline{H} \tag{4.3}$$

$$\underline{\nabla} \wedge \underline{H} = i \omega \epsilon_o \epsilon' \underline{E} + \underline{J}_o \tag{4.4}$$

The field quantities $\underline{H}$ and $\underline{E}$ will be represented in terms of a vector potential $\underline{A}$ and a scalar potential ϕ as follows

$$\mu_0 \underline{H} = \underline{\nabla} \wedge \underline{A} \qquad (4.5)$$

$$\underline{E} = i\omega \underline{A} + \underline{\nabla}\, \phi \qquad (4.6)$$

Equations (4.1) and (4.3) are automatically satisfied by the potential representation. Equations (4.2) and (4.5) become

$$i\omega\, \underline{\nabla} \cdot \epsilon' \underline{A} + \underline{\nabla} \cdot \epsilon' \underline{\nabla}\phi = \frac{i\omega\mu_0}{k_0^2}\, \underline{\nabla} \cdot \underline{J}_0 \qquad (4.7)$$

$$\nabla^2 \underline{A} - \underline{\nabla}(\underline{\nabla} \cdot \underline{A}) + k_0^2\, \epsilon' \underline{A} - i\epsilon' \nabla(\omega\mu_0 \epsilon_0 \phi) = \mu_0 \underline{J}_0 \qquad (4.8)$$

Since in place of $\underline{A}$ one could have used $\underline{A}+\underline{\nabla}\psi$ where ψ is arbitrary, still automatically satisfying Eqs. (4.1) and (4.3), one can impose an additional condition on the potentials in terms of a gauge transformation. The particular choice will be taken as follows

$$\omega\mu_0 \epsilon_0 \phi = i\, \underline{\nabla} \cdot \underline{A} \qquad (4.9)$$

Equation (3.8) reduces to

$$\nabla^2 \underline{A} - (1-\epsilon')\underline{\nabla}(\underline{\nabla} \cdot \underline{A}) + k_0^2\, \epsilon' \underline{A} = \mu_0 \underline{J}_0 \qquad (4.10)$$

Taking the divergence of this equation, one obtains Eq. (4.7) automatically.

Thus, it is seen that with condition (4.9), the vector potential $\underline{A}$ must satisfy Eq. (4.10). Outside the body $\epsilon' = 1$, and this reduces to the free space Helmholtz equation operating on the components of $\underline{A}$.

The above equation can be placed in a different form useful for deriving an integral expression for $\underline{A}$. Eliminating the term $(1-\epsilon')\underline{\nabla}(\nabla \cdot \underline{A})$ from Eq. (4.10), with the help of relations (4.6) and (4.9), one obtains

$$\nabla^2 \underline{A} + k_o^2 \underline{A} = \mu_o \underline{J}_o - i\omega\mu_o\epsilon_o(1-\epsilon')\,\underline{E} \qquad (4.11)$$

It follows that $\underline{A}$ can be expressed in the form

$$\underline{A}(\underline{x}) = \frac{\mu_o}{4\pi}\int_V \left[\underline{J}_o + \underline{J}\right]\frac{e^{ikR}}{R}\,d\underline{x}' \qquad (4.12)$$

where $R = |\underline{x} - \underline{x}'|$,

and

$$\underline{J} = i\omega\epsilon_o(1-\epsilon')\,\underline{E} \;. \qquad (4.13)$$

This can be represented in the form

$$\underline{A}(\underline{x}) = \underline{A}^i(\underline{x}) + \frac{\mu_o}{4\pi}\int_{V_s} \underline{J}(\underline{x}')\frac{e^{ikR}}{R}\,d\underline{x}' \qquad (4.14)$$

where $\underline{A}^i(\underline{x})$ is the vector potential of the incident field.

The magnetic field is thus given by

$$\underline{H}(\underline{x}) = \underline{H}^i(\underline{x}) + \frac{1}{4\pi}\int_{V_s} \underline{J}(\underline{x}') \wedge \underline{\nabla}'\left(\frac{e^{ikR}}{R}\right) d\underline{x}' \;. \qquad (4.15)$$

The source $\underline{J}_o$ giving rise to the incident field $\underline{H}^i$ can now be taken to infinity, in which case $\underline{H}^i$ will represent an incident plane wave. The current $\underline{J}(\underline{x}) = i\omega\epsilon_o(1-\epsilon')\underline{E}(\underline{x})$ is the current induced in the scattering body, being composed of conduction and polarization currents. Both the vector $\underline{H}$ and $\underline{A}$ will be continuous everywhere, due to the assumption that $\mu = \mu_o$ everywhere, and that ϵ' is finite. In the limiting case when the body is a perfect conductor, $\underline{H}$ is then discontinuous. It can be shown that in the limiting case when $\mathrm{Im}\,\epsilon' \longrightarrow \infty$, (i.e. a perfect conductor), the volume integral in (3.15) reduces to a surface integral, and expression (3.15) reduces to

$$\underline{H}(\underline{x}) = \underline{H}^i(\underline{x}) + \frac{1}{4\pi}\int_S (\underline{n}\wedge\underline{H})\wedge\underline{\nabla}'\,\frac{e^{ikR}}{R}\,ds \tag{4.16}$$

where S is the surface of the conductor and $\underline{n}$ is the unit outward normal.

Having considered the above preliminary work, we are now in a position to discuss the possible representation of the field inside the body in terms of the far scattered field. The notation $\underline{A}^s$ will be used to represent that part of the vector potential which results from the induced currents, i.e.:

$$\underline{A}^s(\underline{x}) = \frac{\mu_o}{4\pi}\int_{V_s} \underline{J}(\underline{x})\frac{e^{ikR}}{R}\,d\underline{x}' \quad . \tag{4.15}$$

The scattering body designated by the volume V_s will be split up into the following parts $V_+(\zeta)$, $V_-(\zeta)$ and $V_\delta(\zeta)$, where $V_+(\zeta)$ is the intersection of V_s and the half-space $z \leqslant \zeta$, $V_-(\zeta)$ is the intersection of V_s and the half-space $z \geqslant \zeta$, and $V_\delta(\zeta)$ the intersection of V_s and the slab $\zeta - \delta \leqslant z \leqslant \zeta + \delta$. The decomposition is displayed in Fig. (4-1).

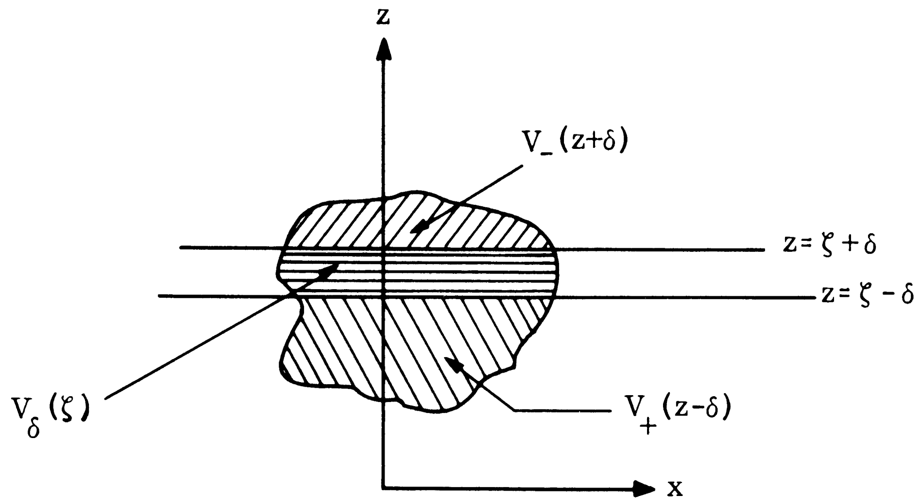

FIG. 4-1: DECOMPOSITION OF THE SCATTERING BODY

Associated with the above, the following vector potentials will be considered,

$$\underline{A}_+^s(\underline{x}) = \frac{\mu_o}{4\pi} \int_{V_+(z-\delta)} \underline{J}(\underline{x}') \frac{e^{ikR}}{R} \, d\underline{x}' \tag{4.16}$$

$$\underline{A}_-^s(\underline{x}) = \frac{\mu_o}{4\pi} \int_{V_-(z-\delta)} \underline{J}(\underline{x}') \frac{e^{ikR}}{R} \, d\underline{x}' \tag{4.17}$$

$$\underline{A}_\delta^s(\underline{x}) = \frac{\mu_o}{4\pi} \int_{V_\delta(z)} \underline{J}(\underline{x}') \frac{e^{ikR}}{R} \, d\underline{x}' \quad . \tag{4.18}$$

Using the relations

$$\frac{e^{ikR}}{R} = \frac{ik}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot(\underline{x}-\underline{x}')}\sin\alpha\, d\alpha\, d\beta \qquad z \geqslant z'$$

$$= -\frac{ik}{2\pi}\int_0^{2\pi}\int_\pi^{\frac{\pi}{2}+i\infty} e^{i\underline{k}\cdot(\underline{x}-\underline{x}')}\sin\alpha\, d\alpha\, d\beta \qquad z \leqslant z'$$

$\underline{A}_+^s(x)$ and $\underline{A}_-^s(x)$ become

$$\underline{A}_+^s(\underline{x}) = \frac{ik}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} e^{i\underline{k}\cdot\underline{x}} \int_{V_+(z-\delta)} \frac{\mu_o}{4\pi}\underline{J}(\underline{x}')\, e^{-i\underline{k}\cdot\underline{x}'}\, d\underline{x}' \quad \sin\alpha\, d\alpha\, d\beta \tag{4.19}$$

$$\underline{A}_-^s(\underline{x}) = -\frac{ik}{2\pi}\int_0^{2\pi}\int_\pi^{\frac{\pi}{2}+i\infty} e^{i\underline{k}\cdot\underline{x}} \int_{V_-(z+\delta)} \frac{\mu_o}{4\pi}\underline{J}(\underline{x}')\, e^{-i\underline{k}\cdot\underline{x}'}\, d\underline{x}' \quad \sin\alpha\, d\alpha\, d\beta \tag{4.20}$$

provided that $\underline{J}(\underline{x}')$ is absolutely integrable, allowing the order of integration to be interchanged.

If $\underline{J}$ is bounded, it follows that each component of $\underline{A}_\delta^s(\underline{x})$ is bounded

$$\left|A_\delta^s(\underline{x})\right| \leqslant \frac{\mu_o}{4\pi} M \int_0^{2\pi}\int_0^{a}\int_{-\delta}^{\delta} \frac{\rho\, d\rho\, d\phi\, d\zeta}{\sqrt{\rho^2+\zeta^2}} = \mu_o M \int_0^{\delta} \sqrt{a^2+\zeta^2}\, d\zeta$$

where $\rho^2 = (x-x')^2 + (y-y')^2$, and a is the maximum value of ρ such that the cylinder $\rho = a$ encloses V_δ. It is easily seen then, that when $\delta \to 0$

$$\underline{A}^s_\delta(\underline{x}) \to 0 \ . \tag{4.21}$$

The above condition that $\underline{J}$ be bounded may be weakened, by allowing certain types of integrable singularities. However, these cases will not be considered at the present time.

Letting $\delta \to 0$, the vector potential $\underline{A}$ can be expressed as follows

$$\underline{A} = \underline{A}^i + \underline{A}^s_+(\underline{x}) + \underline{A}^s_-(\underline{x}) \tag{4.22}$$

where

$$\underline{A}^s_+(\underline{x}) = \frac{ik}{2\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2} - i\infty} e^{i\underline{k}\cdot\underline{x}} \, \underline{A}^+_o(\alpha,\beta) \sin\alpha \, d\alpha \, d\beta \tag{4.23}$$

and

$$\underline{A}^s_-(\underline{x}) = -\frac{ik}{2\pi} \int_0^{2\pi} \int_{\pi}^{\frac{\pi}{2} + i\infty} e^{i\underline{k}\cdot\underline{x}} \, \underline{A}^-_o(\alpha,\beta) \sin\alpha \, d\alpha \, d\beta \tag{4.24}$$

with the vector $\underline{k}(\alpha,\beta) = k(\sin\alpha\cos\beta, \sin\alpha\sin\beta, \cos\alpha)$. The quantities $\underline{A}^+_o(\alpha,\beta)$ and $\underline{A}^-_o(\alpha,\beta)$ are the far field components

$$\underline{A}^+_o(\alpha,\beta) = \frac{\mu_o}{4\pi} \int_{V_+(z-0)} \underline{J}(\underline{x}')\, e^{-i\underline{k}\cdot\underline{x}'} \, d\underline{x}' \tag{4.25}$$

$$\underline{A}_o^-(\alpha,\beta)=\frac{\mu_o}{4\pi}\int_{V_-(z+0)}\underline{J}(\underline{x}')\,e^{-i\underline{k}\cdot\underline{x}'}\,d\underline{x}' \tag{4.26}$$

arising from an appropriate decomposition of the quantity

$$\underline{A}_o(\alpha,\beta)=\frac{\mu_o}{4\pi}\int_{V_s}\underline{J}(\underline{x}')\,e^{-i\underline{k}\cdot\underline{x}'}\,d\underline{x}' \tag{4.27}$$

defined previously, i.e.

$$\underline{A}_o(\alpha,\beta)=\underline{A}_o^+(\alpha,\beta)+\underline{A}_o^-(\alpha,\beta)\quad . \tag{4.28}$$

It can be shown that the same results hold for the magnetic field, in which case

$$\underline{H}=\underline{H}^i+\underline{H}_+^s+\underline{H}_-^s \tag{4.29}$$

where

$$\underline{H}_+^s(\underline{x})=\frac{-k}{2\pi\mu_o}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty}e^{i\underline{k}\cdot\underline{x}}\,\underline{k}\wedge\underline{A}_o^+(\alpha,\beta)\,\sin\alpha\,d\alpha\,d\beta \tag{4.30}$$

$$\underline{H}_-^s(\underline{x})=\frac{+k}{2\pi\mu_o}\int_0^{2\pi}\int_{\pi}^{\frac{\pi}{2}+i\infty}e^{i\underline{k}\cdot\underline{x}}\,\underline{k}\wedge\underline{A}_o^-(\alpha,\beta)\,\sin\alpha\,d\alpha\,d\beta\,. \tag{4.31}$$

From the above it is seen that it is possible to obtain the magnetic field inside the body (composed of non-magnetic material with finite conductivity), from a knowledge of the far field data. This follows from the results in Section 2, which indicate how $\underline{k}(\alpha, \beta) \wedge \underline{A}_o(\alpha, \beta)$ may be determined for complex values of α where $\alpha = \theta + it$, from the knowledge of the far field quantities $A^o_\theta(\theta, \phi)$ and $A^o_\phi(\theta, \phi)$, measured in the range $0 \leqslant \theta \leqslant \frac{\pi}{2}$, $0 \leqslant \phi \leqslant 2\pi$. However, if the body is inside the slab $z_2 \leqslant z \leqslant z_1$, the appropriate split up of $\underline{k} \wedge \underline{A}$ must be sought. The key problem remains of determining a method of uniquely performing this decomposition from knowledge of the far field data alone. Additional knowledge will most likely be required, such as, knowledge of the scattered field for all frequencies, or all angles of incidence.

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