THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING
Department of Atmospheric and Oceanic Sciences

Technical Report

A STUDY OF THE ANNUAL VARIATION OF CERTAIN ASPECTS
OF THE ATMOSPHERIC GENERAL CIRCULATION

Aksel C. Wiin-Nielsen

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ABSTRACT

Values of the mean zonal wind, the meridional transport of momentum, and sensible heat for each month of a year have been used to calculate the annual average and the first few Fourier components of the annual variation in order to supplement earlier studies of the annual variation of atmospheric energy. It is found that the annual average and the first component of the annual variation give by far the largest contribution to the observed variance.

The data mentioned above have been used to calculate the annual average and the first component of the annual variation of the surface stress, the mean meridional wind component and the zonally averaged vertical velocity using a quasi-geostrophic formulation. These values are compared with values based directly on the observed winds with generally fair agreement.
1. INTRODUCTION

During the last few years there has been a renewed interest in the climatology of a number of parameters of direct importance for the description of the atmospheric general circulation. A few examples of these types of studies are those by Kung (1966a,b, 1967, 1969, 1970) of kinetic energy, its generation and dissipation, by Starr et al. (1970), of angular momentum and the generation of zonal kinetic energy, and by Van Loon et al. (1968), of the zonal mean temperature and wind in the Southern Hemisphere.

The present study is a continuation of the author's (1967) investigation of the annual variation of available potential energy and kinetic energy in both zonal and eddy form. It was found that most of the annual variation of the quantities could be expressed by the annual average and the first Fourier component of the annual variation. In the same study it was found that typical time lags existed between the generation of available potential energy, conversion from this form of energy to kinetic energy, and the dissipation of kinetic energy. It is relatively simple to describe the annual variation of all these quantities because they are obtained as integrals over as large a portion of the Northern Hemisphere as possible. They are therefore to be considered as functions of time only. However, it is conceivable that one may find annual variations of the more basic parameters which may be described in an equally simple manner using the same basic technique of Fourier analysis in time. Such a technique has already been used by Van Loon et al. (1968), to describe the major aspects of the annual variation of the zonal mean temperature and wind in the Southern Hemisphere. In this study we shall describe an analysis of the mean zonal wind, the meridional transport of momentum, and sensible heat. Since these quantities depend upon time, latitude, and pressure, we must describe them using mean meridional cross sections.

It is possible to investigate the annual variation of a number of other parameters in the atmosphere if we adopt a specific model. We shall here be interested in the zonal average of the surface stress, the meridional velocity, and the vertical velocity, which can be calculated if we assume a quasi-geostrophic model, and when we know the annual variation of the mean zonal wind and the momentum transport. The details of the computational procedure will be given in a following section.

There has recently been considerable interest in computations of the mean meridional circulation. The studies by Starr et al. (1970), and Oort and Rasmusson (1970) are both based on the same data sample and on the use of observed winds. The method is the most direct way to obtain the mean meridional circulation, but it is also the procedure which is most sensitive to the data analysis as demonstrated in the first of these studies. Indirect calculations have been made by Mintz and Lang (1955), Kuo (1956), Holopainen
(1965), Lorenz (1967), and Verneker (1967) for the major portions of the Northern Hemisphere, while Gilman (1965) has made calculations for the Southern Hemisphere using an indirect method. Several other calculations dealing with separate regions such as the tropics and the lower stratosphere have been made. A review of these studies can be found in Lorenz (1967) and Oort and Rasmusson (1970).

The present study will be made using the indirect method based on a quasi-geostrophic model, which makes the calculations simpler than Gilman's (loc. cit.). We shall on the other hand include internal as well as boundary layer friction.

It is particularly desirable to use the indirect methods in the middle and high latitudes in view of the comments made by Oort and Rasmusson (1970): "Both the increased variance in time and the 'observed' mass imbalance, which probably indicates systematic sampling errors in space, support our conclusion that the details of the mean meridional circulation as presented here must be taken with reservation at middle and high latitudes." These authors as well as Starr et al. (1970), find that the vertical and time average, of $v_z$, the mean meridional wind, may be as large as 0.4 msec$^{-1}$, while it should be zero in order to assure a vanishing mass transport across a latitude circle.

The indirect calculations may therefore help to settle the questions concerning the middle- and high-latitude mean meridional circulation. The present calculation will, on the other hand, give no contribution to the knowledge of the Hadley circulation in low latitudes because our data do not extend below 20°N.
2. COMPUTATIONAL PROCEDURE

The data available for this study are monthly mean values of \( u_z \), the zonally averaged wind component, \( M = (u_E v_E)_z \), the transport of relative momentum, and \( N = c_p (T_E v_E)_z \), the transport of sensible heat by the eddies. The period for which these data are available is February 1963 to January 1964, inclusive. It is, needless to say, important to expand the data sample to cover a longer time period at a later time. The results obtained here should be considered as an example, but cannot be regarded to be representative in the same sense as a climatological average.

The method of analysis is an application of Fourier analysis of a time series. Let \( b_z(\phi, p, t) \) be the zonal average of \( b \). We write \( b_z \) in the form

\[
b_z = B_o + \sum_{n=1}^{N} \left( B_n \cos(nst) + C_n \sin(nst) \right) \tag{2.1}
\]

where \( B_o \) is the annual average:

\[
B_o = \frac{1}{T} \int_0^T b_z \, dt \tag{2.2}
\]

where \( T = 1 \) year, and

\[
B_n = \frac{2}{T} \int_0^T b_z \cos(nst) \, dt \tag{2.3}
\]

and

\[
C_n = \frac{2}{T} \int_0^T b_z \sin(nst) \, dt \tag{2.4}
\]

In the expressions above \( s = 2\pi/T \) is the frequency corresponding to the period of 1 year, while \( n \) is a positive integer, and \( N = n_{\text{max}} \).

The coefficients \( B_n \) and \( C_n \) are computed from the finite sum equivalence of (2.3) and (2.4). It is often desirable to express a wave component in (2.1) in terms of an amplitude and a phase angle. The amplitude of the \( n \)'th component is \( (B_n^2 + C_n^2)^{1/2} \) while the phase angle may be computed from the formula

\[
\gamma_n = \frac{1}{n} \arctan \left( \frac{C_n}{B_n} \right) \tag{2.5}
\]
With only 12 data points in the total period we must limit the number of components which can be computed. Theoretically, we may have \( N = 5 \), and this value was used in the preliminary calculations in order to investigate how rapidly the amplitude decrease with increasing \( n \). The expression (2.1) was used for all quantities calculated in this investigation.

In addition to the basic data available for this study, it is possible to calculate a number of additional quantities as long as we adopt a reasonable model. We shall use the data of \( u_z \) and \( M \) to calculate the mean meridional circulation, \( v_z \) and \( \omega_z \), i.e., the zonal averages of the meridional and vertical wind components. It turns out that the annual mean and the first component describe most of the variance as will be demonstrated later for \( u_z \) and \( M \). We shall therefore limit the calculation of \( v_z \) and \( \omega_z \) to the same two components. Let us therefore assume that \( u_z \) and \( M \) are written in the form

\[
\begin{align*}
    u_z &= U_o + A(u_z) \cos st + B(u_z) \sin st \\
    M &= M_o + A(M) \cos st + B(M) \sin st
\end{align*}
\] (2.6) (2.7)

The basis for the calculation of the mean meridional circulation will be the zonal average of the first equation of motion in the form

\[
\frac{\partial u_z}{\partial t} + \frac{1}{a \cos \varphi} \frac{\partial M \cos \varphi}{\partial p} = f v_z + g \frac{\partial \tau_z}{\partial \varphi}
\] (2.8)

where \( \tau_z \) is the zonal average of the zonal component of the stress, \( g \) gravity, \( f = 2\Omega \sin \varphi \) the Coriolis parameter, \( \Omega \) the angular velocity of the earth, \( \varphi \) latitude, \( p \) pressure, and \( a \) the radius of the earth. It should be mentioned that (2.8) corresponds to the quasi-geostrophic approximation because we have neglected the vertical transport of momentum, and because \( M \) is calculated from geostrophic winds. The terms on the left side of (2.8) are known in principle according to the specifications (2.6) and (2.7). We shall make use of (2.8) to calculate the meridional component, \( v_z \), and the surface stress, \( \tau_s \). Let us assume that

\[
\begin{align*}
    v_z &= V_o + A(v_z) \cos st + B(v_z) \sin st \\
    \omega_z &= \omega_o + A(\omega_z) \cos st + B(\omega_z) \sin st \\
    \tau_s &= \tau_o + A(\tau_s) \cos st + B(\tau_s) \sin st
\end{align*}
\] (2.9) (2.10) (2.11)
while

\[ \tau_z = T_o + A(\tau_z) \cos st + B(\tau_z) \sin st \]  \hspace{1cm} (2.12) 

When (2.6), (2.7), (2.9), and (2.12) are introduced in (2.8), and when we equate time-independent terms, terms containing \( \cos st \) and \( \sin st \), respectively, we obtain

\[ - \frac{1}{a \cos^2 \varphi} \frac{\partial M}{\partial \varphi} \cos^2 \varphi + fV_o + g \frac{\partial T_o}{\partial \varphi} = 0 \]  \hspace{1cm} (2.13) 

\[ -sB(u_z) - \frac{1}{a \cos^2 \varphi} \frac{\partial A}{\partial \varphi} \cos^2 \varphi + fA(v_z) + g \frac{\partial A(\tau_z)}{\partial \varphi} = 0 \]  \hspace{1cm} (2.14) 

\[ +sA(u_z) - \frac{1}{a \cos^2 \varphi} \frac{\partial B}{\partial \varphi} \cos^2 \varphi + fB(v_z) + g \frac{\partial B(\tau_z)}{\partial \varphi} = 0 \]  \hspace{1cm} (2.15) 

We introduce a vertical average with respect to pressure using the definition

\[ \langle \cdot \rangle = \frac{1}{p_2 - p_1} \int_{p_1}^{p_2} \langle \cdot \rangle dp \]  \hspace{1cm} (2.16) 

when, ideally \( p_1 = 0 \) and \( p_2 = P_s \), the surface pressure. We shall assume that the net mass transport across any latitude circle vanishes. This is equivalent to the statement that

\[ \tilde{v}_z = 0 \]  \hspace{1cm} (2.17) 

or, as can be seen from the zonally averaged continuity equation

\[ \frac{1}{a \cos \varphi} \frac{\partial \mathbf{v}_z \cos \varphi}{\partial \varphi} + \frac{\partial \mathbf{v}_z}{\partial p} = 0, \]  \hspace{1cm} (2.18) 

to the boundary conditions \( \omega_z = 0 \) at \( p = 0 \) and \( p = P_s \). The data available in this case do not permit us to extend the integration to the complete depth of the atmosphere. We shall use \( p_1 = 10 \text{ cb} \) and \( p_2 = 100 \text{ cb} \) covering approximately 90% of the mass of the atmosphere. We shall still assume that (2.17) holds, and according to (2.18) we are assuming that \( \omega_z = 0 \) at \( p = 10 \text{ cb} \) and \( p = 100 \text{ cb} \).
Applying (2.16) and (2.17) to (2.13) (2.14), and (2.15) we get after some rearrangement:

\[ T_{so} = \frac{P_2 - P_1}{g} \left( 1 - \frac{\hat{M}_o}{a \cos^2 \varphi} \right) \frac{\partial \hat{M}_o}{\partial \varphi} (2.19) \]

\[ A(T_s) = \frac{P_2 - P_1}{g} \left[ \frac{1}{a \cos^2 \varphi} \frac{\partial \hat{A}(z) \cos^2 \varphi}{\partial \varphi} + sB(u_z) \right] (2.20) \]

\[ B(T_s) = \frac{P_2 - P_1}{g} \left[ \frac{1}{a \cos^2 \varphi} \frac{\partial \hat{B}(z) \cos^2 \varphi}{\partial \varphi} - sA(u_z) \right] (2.21) \]

where it has been assumed that the stress component vanishes at the top of the layer, in this case at \( p_1 = 10 \) cb. The expressions (2.19) - (2.21) were used to calculate the annual mean and the first Fourier component of the surface stress.

In order to calculate \( u_z \) one must either neglect the internal stress in the atmosphere or make a reasonable assumption. We shall select the latter possibility, and assume that

\[ r_z = \nu \rho \frac{\partial u_z}{\partial z} = -v \nu \rho^2 \frac{\partial^2 u_z}{\partial p^2} = -\frac{v\nu}{R T^2} \rho^2 \frac{\partial u_z}{\partial p} \] (2.22)

The last term in (2.8) becomes, using (2.22):

\[ g \frac{\partial r}{\partial p} = -K \frac{\partial}{\partial p} \left( \rho^2 \frac{\partial u_z}{\partial p} \right) \] (2.23)

where it has been assumed that \( K \) is a constant and equal to:

\[ K = \frac{v \nu}{R T^2} \] (2.24)

Assumptions equivalent to (2.24) have been made by Roberts (1966) and Derome and Wiin-Nielsen (1971). The numerical value of \( K \) is obtained from the values \( v = 90 \text{ m}^2 \text{sec}^{-1} \), \( g = 9.8 \text{ msec}^{-2} \), \( R = 287 \text{ m}^2 \text{sec}^{-2} \text{deg}^{-1} \) and \( T = 250^\circ \text{K} \), and we find \( K = 1.7 \times 10^{-6} \text{sec}^{-1} \). Introducing (2.23) in (2.13), (2.14), and (2.15) we find after rearrangement that
\[ V_o = \frac{1}{r} \left[ \frac{1}{a \cos^2 \phi} \frac{\partial M_o}{\partial \varphi} \cos^2 \phi + K \frac{\partial}{\partial p} \left( p^2 \frac{\partial U_o}{\partial p} \right) \right] \]  
(2.25)

\[ A(v_z) = \frac{1}{r} \left[ \frac{1}{a \cos^2 \phi} \frac{\partial A(M \cos^2 \phi)}{\partial p} + K \frac{\partial}{\partial p} \left( p^2 \frac{\partial A(u_z)}{\partial p} \right) + sB(u_z) \right] \]  
(2.26)

\[ B(v_z) = \frac{1}{r} \left[ \frac{1}{a \cos^2 \phi} \frac{\partial B(M \cos^2 \phi)}{\partial p} + K \frac{\partial}{\partial p} \left( p^2 \frac{\partial B(u_z)}{\partial p} \right) - sA(u_z) \right] \]  
(2.27)

The data for \( M \) are available at 15, 20, 30, 50, 70, and 85 cb, while \( u_z \) is available at these levels plus the levels of 10 and 100 cb. We may use (2.25) - (2.27) to compute the components of \( v_z \) at the same levels as those where \( M \) is available except at 85 cb, where we must use the computed values of the surface stress obtained from (2.19) - (2.21) at the level 100 cb, while the stress at 70 cb is calculated using (2.22). The values obtained from (2.25) - (2.27) do not necessarily satisfy (2.17). The vertical mean was therefore formed for \( V_o, A(v_z), \) and \( B(v_z) \), and \( V_o, \tilde{A}(v_z), \) and \( \tilde{B}(v_z) \) were subtracted from the individual values of the proper quantities. This procedure guarantees that the corrected quantities have a vanishing vertical average.

The values of \( \omega_o, A(\omega_z), \) and \( B(\omega_z) \) are obtained by integration of the continuity equation which takes the following form when (2.9) and (2.10) are substituted in (2.18)

\[ \frac{\partial \omega_o}{\partial p} + \frac{1}{a \cos \phi} \frac{\partial V_o \cos \phi}{\partial \varphi} = 0 \]  
(2.28)

\[ \frac{\partial A(\omega_z)}{\partial p} + \frac{1}{a \cos \phi} \frac{\partial A(v_z) \cos \phi}{\partial \varphi} = 0 \]  
(2.29)

\[ \frac{\partial B(\omega_z)}{\partial p} + \frac{1}{a \cos \phi} \frac{\partial B(v_z) \cos \phi}{\partial \varphi} = 0 \]  
(2.30)

The integration of (2.28) - (2.30) is straightforward using \( \omega_o = A(\omega_z) = B(\omega_z) = 0 \) at \( p = p_1 \) as a boundary condition. The equivalent condition at \( p = p_2 \) is automatically satisfied because the vertical averages of \( V_o, A(v_z), \) and \( B(v_z) \) vanish.

The horizontal derivatives which appear in the expressions are all of the same type. The following approximation was used

\[ \frac{1}{\cos^2 \phi} \frac{\partial Z(\phi) \cos^2 \phi}{\partial \varphi} = \frac{Z(\phi + \Delta \phi) \cos^2 (\phi + \Delta \phi) - Z(\phi - \Delta \phi) \cos^2 (\phi - \Delta \phi)}{2 \Delta \phi \cos^2 \phi} \]  
(2.31)
where $\Delta p = 2.5^\circ \text{ lat} = 0.0436 \text{ rad}$.

The vertical derivatives are also of the same form. Let us assume that we shall calculate the differential expression at level $p$, where $p(u) < p$ is the nearest upper level, while $p(t) > p$ is the nearest lower level. We have then

$$\frac{\partial}{\partial p} \left( \frac{2}{p} \frac{\partial Z}{\partial p} \right) = \frac{1}{p'' - p'} \left[ \left( \frac{2}{p} \frac{\partial Z}{\partial p} \right)_{p''} - \left( \frac{2}{p} \frac{\partial Z}{\partial p} \right)_{p'} \right] \quad (2.32)$$

where $p'' = 1/2 (p(t) + p)$ and $p' = 1/2 (p + p(u))$. We obtain further

$$\frac{\partial}{\partial p} \left( \frac{2}{p} \frac{\partial Z}{\partial p} \right) = \frac{1}{p'' - p'} \left[ \frac{p'^2}{p(t)} \frac{Z(t) - Z}{p(t) - p} - \frac{p'^2}{p} \frac{Z - Z(u)}{p - p(u)} \right] \quad (2.33)$$

where $Z(t) = Z$ at $p = p(t)$ and $Z(u) = Z$ at $p = p(u)$.
3. RESULTS FOR ZONAL WINDS, HEAT AND MOMENTUM TRANSPORTS

Figure 1 shows the annual average of the zonal wind $u_z$ as a function of latitude and pressure. We observe the single maximum of a little more than 25 m/sec$^{-1}$ in the subtropical jet stream, the very weak polar easterlies and the somewhat stronger easterlies in the low latitudes.

The amplitude of the first Fourier component of $u_z$ is shown in Figure 2 as a function of latitude and pressure. It is seen that the largest annual variation takes place in the upper part of the troposphere. The primary maximum is located at 25-30°N with a much weaker secondary maximum at 60°N. It is interesting to note that two maxima in the amplitude of the first harmonic were also found in the Southern Hemisphere by Van Loon et al. (1968), in the corresponding positions (25°S and 60°S). However, the major maximum in the Southern Hemisphere is found in connection with the high-latitude winds.

The phase angle, i.e., the time of the maximum in the first harmonic, for $u_z$ is shown in Figure 3 in the unit: days. The zero line at about 40°N shows that a maximum of $u_z$ at this latitude occurs on the first of January. At latitudes to the south of 40°N we find a positive phase angle indicating a maximum wind toward the latter part of January in the regions where the amplitude is large, i.e., in the upper part of the troposphere. The maximum winds north of 40°N occur generally in the later part of the year, but it may be noted the maximum winds at 50-55°N occur as early as the latter part of September. The rather rapid changes in the phase angle found in the lower right and left parts of Figure 3 are of minor significance because the wind speeds in these regions are small. A comparison between Figure 3 and the corresponding figure in Van Loon et al. (1968), shows a general agreement between the Northern and Southern Hemispheres with respect to the phase angle of the first harmonic. The maximum winds occur in the region 25-30°S during the latter part of July, or roughly one month after the equinox for the hemisphere.

The results displayed in Figures 1, 2, and 3 show clearly why the annual variation of the zonal kinetic energy shows a maximum around the latter part of January. The reason is that the largest annual mean and the large amplitude of the first harmonic are found in the region 25-35°N where the phase of $u_z$ is approximately January 20.

We are next turning our attention to the annual variation of the meridional transport of sensible heat by the eddies. Figure 4 shows the annual mean value of this quantity as a function of latitude and pressure. The unit is $10^8$ kj sec$^{-1}$ cm$^{-1}$. The distribution in Figure 4 is similar to earlier calculations of the heat transport such as those displayed by Wijn-Nielsen et al. (1964). We find two maxima in middle latitudes, the primary one at lower elevations and the secondary one at the tropopause level. The configuration of
Figure 1. Annual mean of the zonally averaged u-component of the velocity as a function of latitude and pressure. Unit: m sec$^{-1}$. 
Figure 2. Amplitude of the first Fourier component of the zonally averaged u-component as a function of latitude and pressure. Unit: m sec⁻¹.
Figure 3. Phase of the first Fourier component of the zonally averaged u-component as a function of latitude and pressure. Unit: days relative to January 1.
Figure 4. Annual mean of the meridional transport of sensible heat as a function of latitude and pressure. Unit: $10^8$ kJ sec$^{-1}$ cb$^{-1}$.
the amplitude of the first harmonic, shown in Figure 5, is quite similar to the averaged transport in Figure 4. The change in the heat transport through the year is mainly a change in intensity and a smaller change in the shape of the curve as seen from Figure 6 which shows the phase of the first harmonic. In a broad zone (30°N - 60°N) we find a winter maximum in the heat transport. In the higher latitudes the maximum occurs in December, while it takes place in January south of approximately 40°N. The large gradients in the phase in the lower left and upper right part of Figure 6 are again connected with relatively small values of the amplitude.

Figure 7 shows the annual average of the meridional transport of relative momentum by the eddies. The quantity mapped in Figure 7 is $(u_E v_E)_z$ in the unit m$^2$sec$^{-2}$. The distribution pictured in Figure 7 is very similar to the earlier results by Win-Nielsen et al. (1964), and Starr et al. (1970). However, the numerical values in the maximum located approximately at 35°N and 20 cb are considerably larger than the annual mean values shown by other authors as seen from the comparisons made by Lorenz (1967). This discrepancy between the present data and other results has also been discussed by Holfapainen (1967). However, the variation of the eddy momentum transport with respect to latitude and pressure is in good agreement with other calculation in a qualitative sense, and no great harm will be made by using the present data it is remembered that such a discrepancy exists between the various data samples.

The amplitude of the first harmonic of the momentum transport is shown in Figure 8. The greatest annual variability is found in the upper troposphere in connection with the maximum northward transport in the lower latitudes, and the maximum southward transport in the higher latitudes of the region. A comparison between Figure 7 and Figure 8 shows, however, that the region south of approximately 55°N is dominated by a northward transport throughout the year just as the region north of the same latitude is characterized by a southward transport. These results are in agreement with those shown by Starr et al. (1970), indicating again that the present data sample is not biased with respect to the qualitative aspects of the eddy momentum transport.

The phase of the first harmonic of the momentum transport is shown in Figure 9. In contrast to the previous figures we have here divided the region in two parts corresponding to the northward transport (south of 55°N, approximately) and the southward transport (north of 55°N). From the left part of the figure it is seen that the maximum southward transport occurs in January and February. The phase of the northward transport shows a considerable variation as seen from the right part of Figure 9. However, in the region of the largest annual mean value and the largest variation, we find a phase which varies between December and January.

The results displayed in Figure 4 through Figure 9 of the annual variation of the transports of sensible heat and relative momentum are consistent with
Figure 5. Amplitude of the first Fourier component of the meridional transport of sensible heat as a function of latitude and pressure. Unit: $10^8$ kj sec$^{-1}$ cb$^{-1}$. 
Figure 6. Phase of the first Fourier component of the meridional transport of sensible heat as a function of latitude and pressure. Unit: days relative to January 1.
Figure 7. Annual mean of the meridional transport of relative momentum as a function of latitude and pressure. Unit: m$^2$sec$^{-2}$. 
Figure 8. Amplitude of the first Fourier component of the meridional transport of relative momentum as a function of latitude and pressure. Unit: m^2·sec^{-2}.
Figure 9. Phase of the first Fourier component of the meridional transport of relative momentum as a function of latitude and pressure. The left part gives the phase of the southward transport in the high latitude, while the right part is the phase of the northward transport. Unit: days relative to January 1.
the analysis of the annual variation of the energy conversions from zonal to eddy available potential energy and from eddy to zonal kinetic energy made by the author (1967).

The discussion in this section has been limited to the annual mean value and the first Fourier component of the annual variation. The reason is naturally that these components in general are the most important in explaining the annual variation. However, we do not imply, as will be seen later, that the first harmonic explains most of the variance at all locations. We stress in this connection that we are concerned with the large time scales because our analysis is based on monthly mean data. However, it is possible to compute several harmonics from the twelve data points represented by the monthly mean data and this was done for all quantities treated in this section. The results of this analysis will not be reproduced here in detail, but we shall be satisfied by giving selected examples. Table 1 shows the amplitudes of the harmonics of $u_z$ at various latitudes at 20 cb which is close to the maximum zonal winds as seen from Figure 1 and Figure 2. An inspection of Table 1 shows that $n = 1$ explains most of the variance in the low latitudes, but plays a less dominant role as latitude increases. For example, at 20°N $n = 1$ explains 98.5% of the annual variance while $n = 2$ describes only 0.4%. The corresponding numbers are 57.4% and 25.8%, respectively, at 70°N.

TABLE 1

AMPLITUDES OF THE HARMONICS OF $u_z$ AT 20 cb

<table>
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<td>0.97</td>
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In order to illustrate the role of the higher harmonics in the annual variation of the eddy transport of sensible heat, we have selected the layer between 65 and 70 cb close to the level of the primary maximum in the heat transport. The data are presented in Table 2, which shows that the first harmonic is dominant in middle latitudes where the transport has its maximum, but also that the higher harmonics play an increasing role as we move away from the maximum toward higher and lower latitudes. For example, at 40°N we find
that $n = 1$ explains 96.7% of the variance, while the corresponding number for $n = 2$ is 1.2%. However, the analogous percentages are 68.9% ($n = 1$) and 18.0% ($n = 2$) at 20°N, and 51.8% ($n = 1$) and 14.1% at 70°N.

### TABLE 2

AMPLITUDE OF THE HARMONICS OF THE EDDY TRANSPORT
OF SENSIBLE HEAT IN THE LAYER 85-70 cb

Unit: $10^3$ kJ sec$^{-1}$ cb$^{-1}$

<table>
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<td>23</td>
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<td>18</td>
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</table>

We shall finally illustrate the role of the higher harmonics in the annual variation of the eddy transport of momentum. The data in Table 3 has been obtained from 20 cb which is close to the maximum transport of relative momentum. It is seen from Table 3 that the higher harmonic play a larger role in the momentum transport than in the heat transport and the zonal wind. The percentages of the total annual variance explained by the first and second harmonics are 71.8% and 6.3% at 20°N, 71.4% and 6.9% at 40°N, and 43.8% and 7.5% at 70°N.

### TABLE 3

AMPLITUDE OF THE HARMONICS OF THE EDDY TRANSPORT
OF ANGULAR MOMENTUM AT 20 cb

Unit: m$^2$sec$^{-2}$

<table>
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</tr>
<tr>
<td>70</td>
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<td>7.56</td>
<td>3.13</td>
<td>5.49</td>
<td>5.36</td>
<td>2.17</td>
</tr>
</tbody>
</table>
4. RESULTS FOR THE SURFACE STRESS AND THE MEAN MERIDIONAL CIRCULATION

The surface stress and the mean meridional circulation were calculated following the procedures outlined in section 2. For display purposes we have made the following calculation. From the annual mean and the first Fourier component, we have computed the distribution of the various quantities on the days: January 1, April 1, July 1 and October 1 by using expressions of the type (2.1) for each of the quantities with $N = 1$.

Figure 10 shows the surface stress as a function of latitude for the annual mean and the four selected days during the year. Note, that we have defined the stress in such a way that a negative value means a decrease in westerly relative momentum. We observe that the magnitude of the stress is somewhat larger than the estimates made by other authors due to the magnitude of the transport of relative momentum indicated by our data (Holopainen, 1967). The annual variation of the maximum stress during the year is apparent from the figure. It is furthermore noted that the maximum negative stress in the middle latitudes move northward by about $10^\circ$ of latitude from winter to summer.

The most convenient way to display the mean meridional circulation is to define a "streamfunction," which satisfies the zonally averaged continuity equation. We have selected to define the streamfunction by the relations

$$v_z = \frac{a}{\cos \varphi} \frac{\partial \psi}{\partial p}, \quad \omega_z = -\frac{1}{\cos \varphi} \frac{\partial \psi}{\partial \varphi}$$

(4.1)

where $\psi$ has the unit cb sec$^{-1}$ in the MTS-system. The mean meridional component $v_z$ was computed as described in section 2. $\psi = \psi(\varphi, p)$ was then computed by a numerical integration of the first of the relations in (4.1), setting $\psi = 0$ at $p = 0$. The results of these calculations are shown in Figures 11a-e. These figures are dominated by the indirect Ferrel cell during the whole year. The position of the center of the Ferrel cell varies little during the year. In the annual mean the position is approximately $40^\circ N$ and 72.5 cb. There is a slight northward shift during summer and fall. The intensity of the mean meridional Ferrel circulation varies by about a factor 2 from winter to summer. During winter and spring it is hardly possible to see any indications of the Hadley circulation in the low latitudes, but the northern branch can be observed in the summer and fall due to the well-known annual variation of the Hadley circulation around the equator. In all the figures except Figure 11d for July 1 we find a very weak direct polar cell in the high latitudes.
Figure 10. The surface stress as a function of latitude for the annual mean, January 1, April 1, July 1, and October 1. Unit: dyn cm⁻².
Figure 11a. Streamfunction for the mean meridional circulation as a function of latitude and pressure. Unit: $10^{-7}$ cb sec$^{-1}$. A 1 average.
Figure IIId.  As Figure IIa.  Time: July 1.
Figure 1.1e. As Figure 1.1a. Time: October 1.
It is of interest to mention that the mean meridional velocities which form the basis for Figures 1la-e are generally of a magnitude less than 1 m/sec⁻¹. The only place where 1 m/sec⁻¹ is exceeded is the lowest layer in the middle latitudes where the maximum values of v_z are about 1.5 m/sec⁻¹.

Supplementing the mean meridional streamfunction in Figures 1la-e, we have prepared the meridional cross-sections of the vertical velocity v_z shown in Figures 12a-e. They show naturally essentially the same features as Figures 11a-e. The maximum vertical velocity of 35x10⁻⁶ cb sec⁻¹ corresponds to about 0.4 cm sec⁻¹. The weak sinking motion in the very high latitudes is apparent through the whole year. The upward branch of the Hadley circulation can be seen in the summer only, when the downward motion at 30°N is the strongest through the whole year.

The mean meridional circulation derived here by the indirect methods is in good agreement with similar calculations using the same method as summarized by Lorenz (1967) as far as the general patterns are concerned. As mentioned earlier we find that the mean meridional winds v_z are somewhat larger than those obtained by other investigators using different and, on occasion, larger data samples. The differences in v_z are naturally reflected in the streamfunctions Ψ derived from them. In the comparison between these results and those obtained by indirect and direct methods (Lorenz, 1967; Starr et al., 1970), it must be remembered that the time period used in this study includes the highly abnormal winter months in early 1963. For this period it has been shown (Wiin-Nielsen, 1964, 1966) that the atmospheric energy cycle behaved in a rather unusual manner with a much larger contribution from the divergent part of the flow which in turn is reflected in the greater intensity of the mean meridional circulation. From this point of view we should consider the results presented here as somewhat abnormal. On the other hand, it was also demonstrated by Starr et al. (1970), that the intensity of the mean meridional circulation is rather sensitive to the data analysis and the data density. The five different calculations of the mean meridional streamfunction presented by Starr et al. (loc. cit.), show variations by at least a factor 2 between the extremes. Our results are also influenced by the analysis procedures and must have a similar uncertainty. In any case, our results were derived mainly to investigate the annual variation of the mean meridional circulation which is shown in Figures 11a-e and 12a-e.
Figure 12a: The zonal mean of the vertical velocity $\alpha z$ as a function of latitude and pressure. Units: 10$^{-6}$ cm sec$^{-1}$. Annual average.
Figure 12b. As Figure 12a. Time: January 1.
Figure 12c. As Figure 12a. Time: April 1.
Figure 12a. As Figure 12a. Time: July 1.
Figure 12e. As Figure 12a. Time: October 1.
5. CONCLUDING REMARKS

The main purpose of this paper has been to describe the annual variation of the zonally averaged wind field, the meridional transport of relative momentum and sensible heat, the surface stress, and the mean meridional circulation. The technique of Fourier analysis in time was used to describe the first three quantities with the result that the major part of the annual variation is described by the annual mean and the first Fourier component at most latitudes. The amplitude and the phase angle of the first Fourier component are given in detail.

The surface stress and the mean meridional circulation were computed indirectly, and the annual variation is displayed by calculating the distributions at specific times through the year from the annual mean and the first Fourier component.

The calculations reported here are based entirely on a quasi-geostrophic formulation. The transports of momentum and sensible heat are made using the geostrophic assumption. The indirect determination of the surface stress and the mean meridional circulation are furthermore made using a quasi-geostrophic model. Our results should therefore be considered as a supplement to the very extensive calculations based on observed winds made by Oort and Rasmusson (1970) and Starr et al. (1970). Such a supplement is needed in view of the fact that a determination of the mean meridional circulation directly from observed winds presents difficult problems because of the sparsity of observations and the errors in wind observations. The authors mentioned above have furthermore demonstrated that the computed mean meridional circulation is quite sensitive to the procedures employed in the data analysis.

The indirect method rests, on the other hand, on a different set of assumptions. The major steps are the adoption of a quasi-geostrophic model and the necessity to formulate empirical laws for the surface and internal stress.

In spite of the different approaches in the two methods, we obtain essentially the same qualitative features of the circulations although some differences exist in the intensity. These differences are at least in part due to the shorter time period used in this study, and the somewhat abnormal circulation which existed during the winter 1962-63. An additional factor is that the indirect method just as the other method is dependent on the analysis procedure which in this case is the objective analysis performed by the National Meteorological Center during the year 1963.
The original calculations of the zonally averaged winds and the meridional transports of heat and momentum were made by Miss Margaret Drake of the National Center for Atmospheric Research which is supported by the National Science Foundation. The Fourier analysis and the calculation of the surface stress and the mean meridional circulation was made while the author was a visiting professor at the Institute for Theoretical Meteorology, University of Copenhagen, in cooperation with Mr. Erik Rasmussen. Their help is gratefully acknowledged.
APPENDIX

The units used in this paper differ on occasion from those of other investigations. In order to ease the comparison with other results it is convenient to gather a few conversions between the more commonly used units.

We shall first consider the meridional transport of sensible heat which in this paper is measured in the unit: \(10^8\) kJ sec\(^{-1}\) cb\(^{-1}\). The unit used by Lorenz (1967) is \(10^{14}\) Watts (100 mb)\(^{-1}\). It is seen that

\[
10^{14}\text{ W(100 mb)}^{-1} = 10^{13}\text{ W cb}^{-1} = 10^{10}\text{ kJ sec}^{-1}\text{ cb}^{-1} = 100 \times (10^8\text{ kJ sec}^{-1}\text{ cb}^{-1})
\]

For the meridional transport of momentum we have used \(M = \left(\frac{\text{u}_w}{\text{v}}\right)_z\) measured in \(m^2\) sec\(^{-2}\), while Lorenz (1967) calculates the quantity

\[
T = \frac{2\pi a^2 \cos^2 \phi}{g} \Delta p M
\]

in which \(a\) is the radius of the earth, \(g\) gravity, and \(\Delta p\) the pressure difference over the layer. The dimension of \(T\) is \(M L^2 T^{-2}\), and the cgs unit used is \(10^{25}\) g cm\(^2\) sec\(^{-2}\) (100 mb)\(^{-1}\). We note first that

\[
10^{25}\text{ g cm}^2\text{ sec}^{-2}\text{ (100 mb)}^{-1} = 10^{15}\text{ tm}^2\text{ sec}^{-2}\text{ (10 cb)}^{-1}
\]

and therefore

\[
10^{15}\text{ tm}^2\text{ sec}^{-2}\text{ (10 cb)}^{-1} = 0.26 (\cos^2 \phi M) 10^{15}\text{ tm}^2\text{ sec}^{-2}\text{ (10 cb)}^{-1}
\]

where \(M\) is measured in \(m^2\) sec\(^{-2}\). This formula is used to find that the maximum value of \(M\) in the annual mean (60 m\(^2\) sec\(^{-2}\)) corresponds to approximately \(10^5 \times 10^{25}\) g cm\(^2\) sec\(^{-2}\) which is somewhat larger than the maximum values in any of the other annual averages, Holopainen (1967).

The streamfunction for the mean meridional flow is defined by (4.1) and has the unit cb sec\(^{-1}\). Most other investigators defined the streamfunction in such a way that it has the dimension of a mass flux according to the equations
\[ v_z = \frac{g}{2\pi a^2} \frac{s}{\cos \varphi} \frac{\partial \psi_*}{\partial p}, \quad w_z = -\frac{g}{2\pi a^2} \frac{1}{\cos \varphi} \frac{\partial \psi_*}{\partial \varphi} \]

We have therefore

\[ \frac{g}{2\pi a^2} \psi_* = \psi \]

Our unit for \( \psi \) is \( 10^{-7} \) cb sec\(^{-1} \) = \( 10^{-7} \) tm\(^{-1} \) sec\(^{-3} \) = \( 10^{-3} \) g cm\(^{-1} \) sec\(^{-3} \). Corresponding to this value we have

\[ \psi_* = \frac{2\pi a^2}{g} \times 10^{-3} \text{ g sec}^{-1} = 2.6 \times (10^{12} \text{ g sec}^{-1}) \]

The unit \( 10^{12} \text{ g sec}^{-1} \) for \( \psi_* \) is used by Lorenz (1967), Star et al. (1970) used the unit \( 2 \times 10^{11} \text{ g sec}^{-1} \), and Oort and Rasmusson (1970) \( 10^{13} \) g sec\(^{-1} \).
REFERENCES


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