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COLLEGE OF ENGINEERING
Department of Mechanical Engineering

Progress Report

DIFFUSION OF WATER VAPOR INTO ARTIFICIAL SOILS

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LIST OF FIGURES

LIST OF SYMBOLS

$\frac{x}{L}$	depth ratio
L	depth of artificial soil, inches
$\frac{P_{\frac{x}{L}} - P_{\infty}}{P_o - P_{\infty}}$	dimensionless vapor-pressure ratio
P_{∞}	vapor pressure of medium surrounding soil
P_o	intrinsic vapor pressure of soil at the bottom of the soil bed
$\frac{Q}{Q_o}$	$\frac{\text{\% gain of water/time at time } \theta}{\text{\% gain of water/time at the equilibrium pt. of soil}}$
B_i	Biot modulus
F_o	Fourier modulus
W	fractional water in clay-glycol-water mixture
K_w	$\frac{\text{fractional water in mixture}}{\text{vapor pressure of water}}$
θ	time in days

INTRODUCTION

This report is a first step in attempting to reach a solution to the problem of whether a certain soil will support a given vehicle under specific weather conditions. A set of tables and analyzing equipment to determine if the ground soil has the necessary properties to support a given vehicle, and what action, if any is needed to correct it, would be the ideal solution. Hence this problem compels us to start with the soil itself; i.e., to study the diffusion of water vapor into the soil.

PRELIMINARY DIFFUSION-RATE STUDIES IN ARTIFICIAL SOILS

The purpose of these tests was to determine, both analytically and experimentally, the rates at which artificial soils reach an equilibrium point. Consideration must be given to such factors as temperature, vapor pressure, relative humidity, soil composition, experimental conditions, boundary and surface conditions of soil, etc. Some of the factors concerning the experiment must be considered to be constant to allow an analytic correlation between idealized and experimental data. It is hoped that later these constant factors may be considered as variables under both experimental and analytical treatment.

Vapor pressure and temperature will be considered as constants in the desiccators where the experiments took place. This is a feasible assumption since the laboratory temperature was 77°F ± 3° and the relative humidities in the desiccators, controlled by chemical solution, are believed to be reasonably accurate.

Soil composition was assumed to be homogeneous throughout, since clay (Bentonite) was the only substance used in these initial diffusion-rate studies.

Another assumption made was that the petrie dishes in which the soil was placed had perfectly insulated boundaries except, of course, at the open surface.

EXPERIMENTAL PROCEDURES

The initial weight of the soil samples was determined by weighing the petrie dishes first, then weighing the combined weight of the dish and the soil sample, and subtracting the difference between the dish weight and combined weight.

The following relative humidities were used in the experiment: 100, 85, 75, 62, and 32%. The average laboratory temperature was 77°F.

The samples were weighed daily, when possible, and their changes in weight were noted on a data sheet, a sample of which is attached. Care was taken in the weighing process to keep the daily weight error to a minimum.

The surface of the petrie dish is 9.28 sq in., approx. The depth of the petrie dish is approx. 1/2 in.

Date	% R. H.					Temp., °F
	100	85	75	62	32	
	Dish Weight					
	33.35	28.91	32.10	28.87	28.49	
	Total Weight, gm					
	100% Bentonite—Large Samples For Diffusion-Rate Studies					
10-12-60	113.65	119.10	116.80	110.46	100.70	82
10-13-60	117.56	120.92	118.15	111.14	100.30	82
10-14-60	119.57	121.91	118.82	111.49	100.05	80
10-18-60	123.22	123.42	119.74	111.91	99.69	78.5
10-19-60	123.77	123.53	119.78	111.89	99.67	78
10-20-60	124.20	123.61	119.81	111.88	99.63	78
10-21-60	124.63	123.69	119.81	111.85	99.61	78
10-25-60	126.04	124.00	119.93	111.93	99.59	77.5
10-27-60	126.52	124.07	119.93	111.89	99.585	79
10-28-60	127.56	124.07	119.93	111.89	99.56	80
11-1 -60	127.61	124.13	119.95	111.85	99.54	78.5
11-3 -60	127.89	124.27	120.04	111.81	99.56	76
11-15-60	129.15	124.45	120.10	111.91	99.60	78
11-17-60	129.28	124.38	120.07	111.86	99.58	77
11-22-60	129.67	124.42	120.10	111.92	99.58	77
11-29-60	130.03	124.42	120.09	111.85	99.58	77

Date	% R. H.					Temp., °F
	100	85	75	62	32	
	Dish Weight, gm					
	28.88	37.76	33.30	28.86	29.32	
	Total Weight, gm					
	10% Glycol-Bentonite, Large Samples					
May 10	107.43	109.56	106.09	99.22	93.34	
May 11	113.77	112.10	109.63	100.24	94.40	Average
May 12	116.70	115.94	112.36	102.84	95.37	Temp.
May 13	Sample	117.77	113.92	104.65	95.77	75
May 15	was	118.16	114.22	104.81	95.81	± 4
May 17	Accidentally	119.48	115.02	106.01	96.48	
May 19	Saturated,	120.06	115.23	106.17	96.62	
May 24	Could	120.41	115.40	106.41	96.72	
May 26	Not	120.77	115.38	106.37	96.72	
June 8	Be	120.77				
June 9	Restored					
June 10						

New 100% R. H. Sample (Glycol was added to sample after the sample was dried)

June 19	128.72	July 20	143.23
June 21	129.92	July 21	143.57
June 23	131.01	July 25	145.10
June 27	133.01	July 30	145.87
June 30	134.51	Aug. 3	148.41
July 3	135.67	Aug. 9	150.78
July 7	137.80	Aug. 10	151.10
July 11	139.57	Aug. 11	151.50
July 13	140.43	Aug. 14	152.45
July 14	140.83		
July 18	142.49		

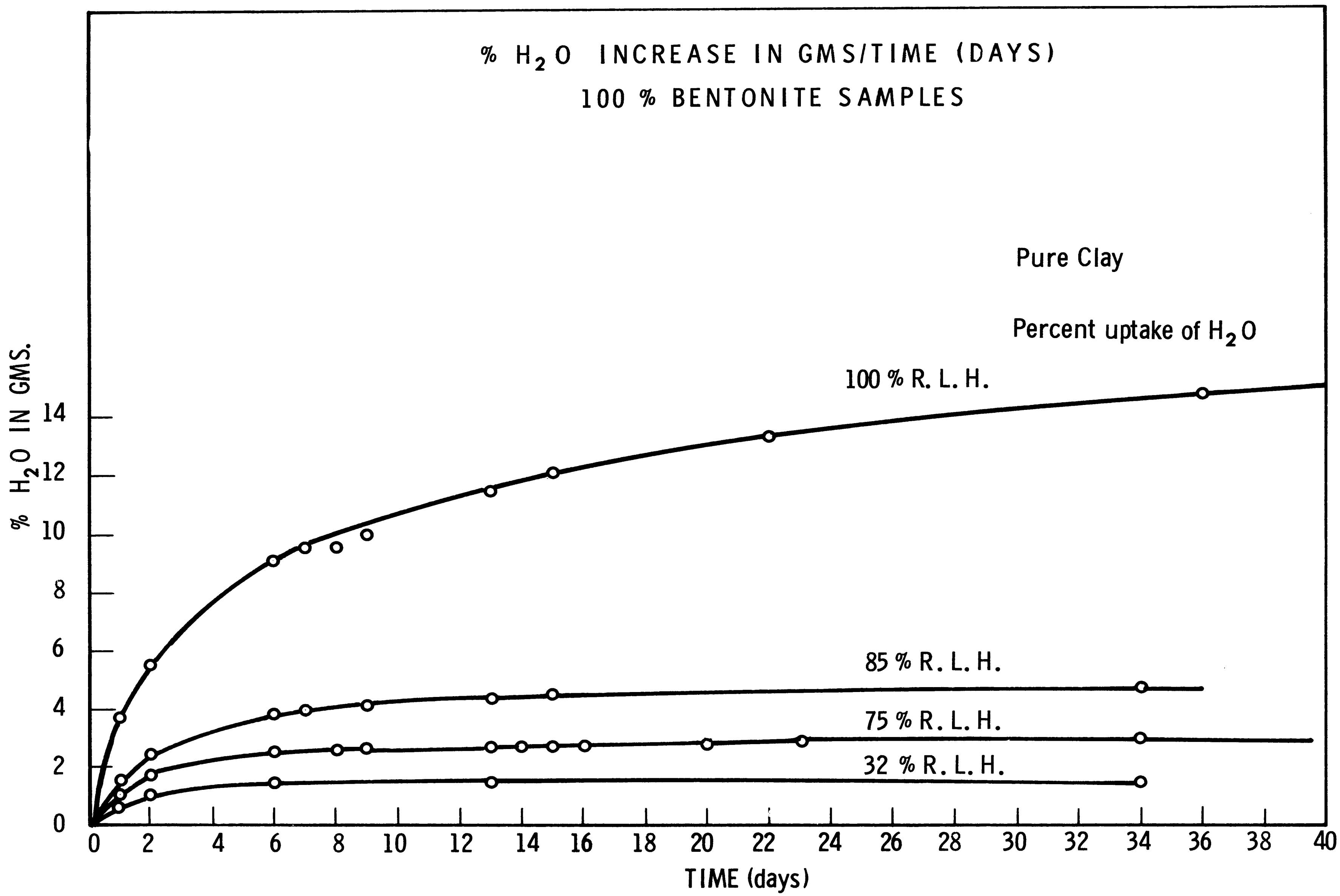

Fig. 1. Plot of the diffusion of water vapor into pure clay.

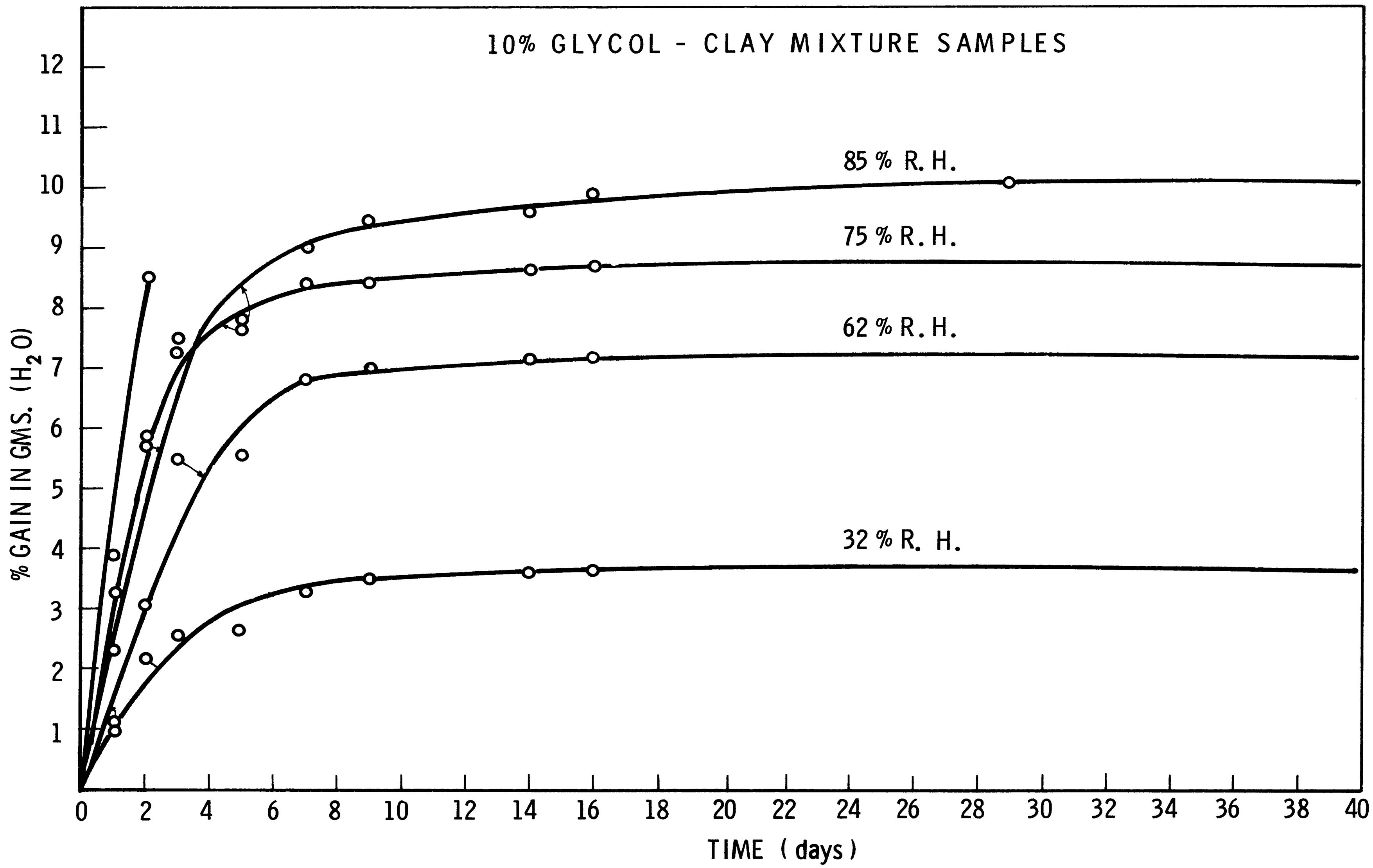

Fig. 2. Plot of the diffusion of water vapor into 10% Glycol-clay mixture.

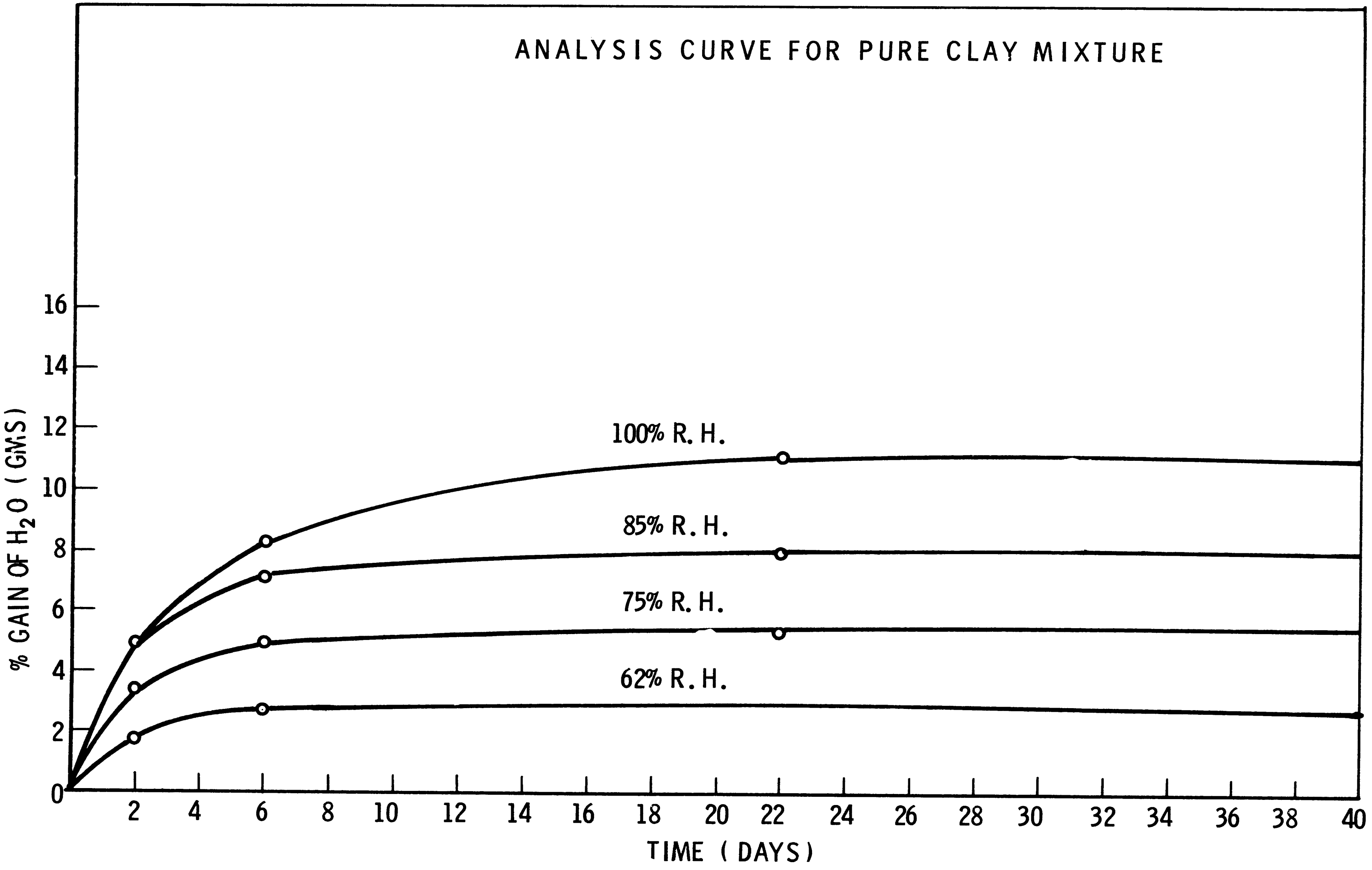

Fig. 3. Plot of Fig. 1 from analytical work.

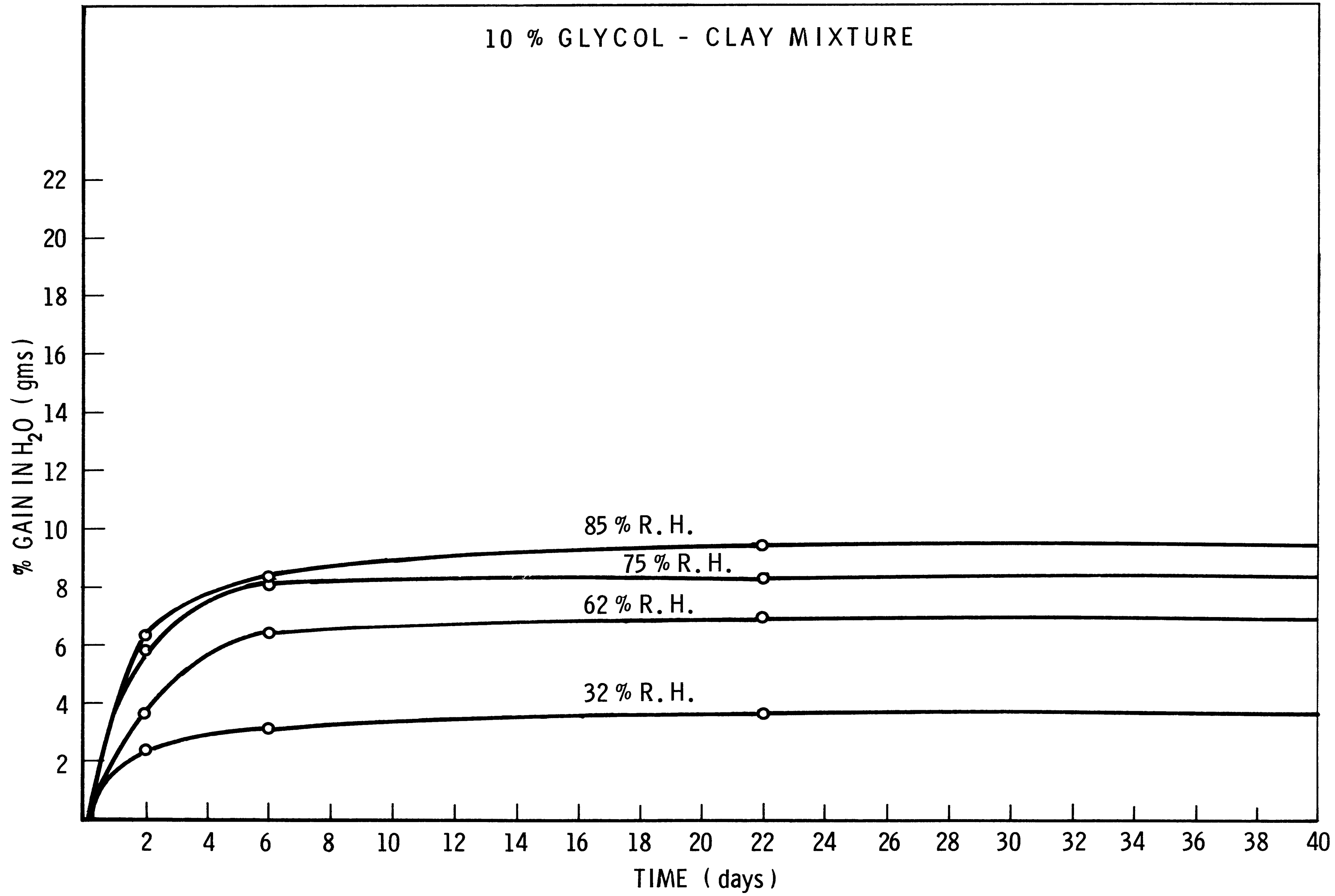

Fig. 4. Plot of Fig. 2 from analytical work.

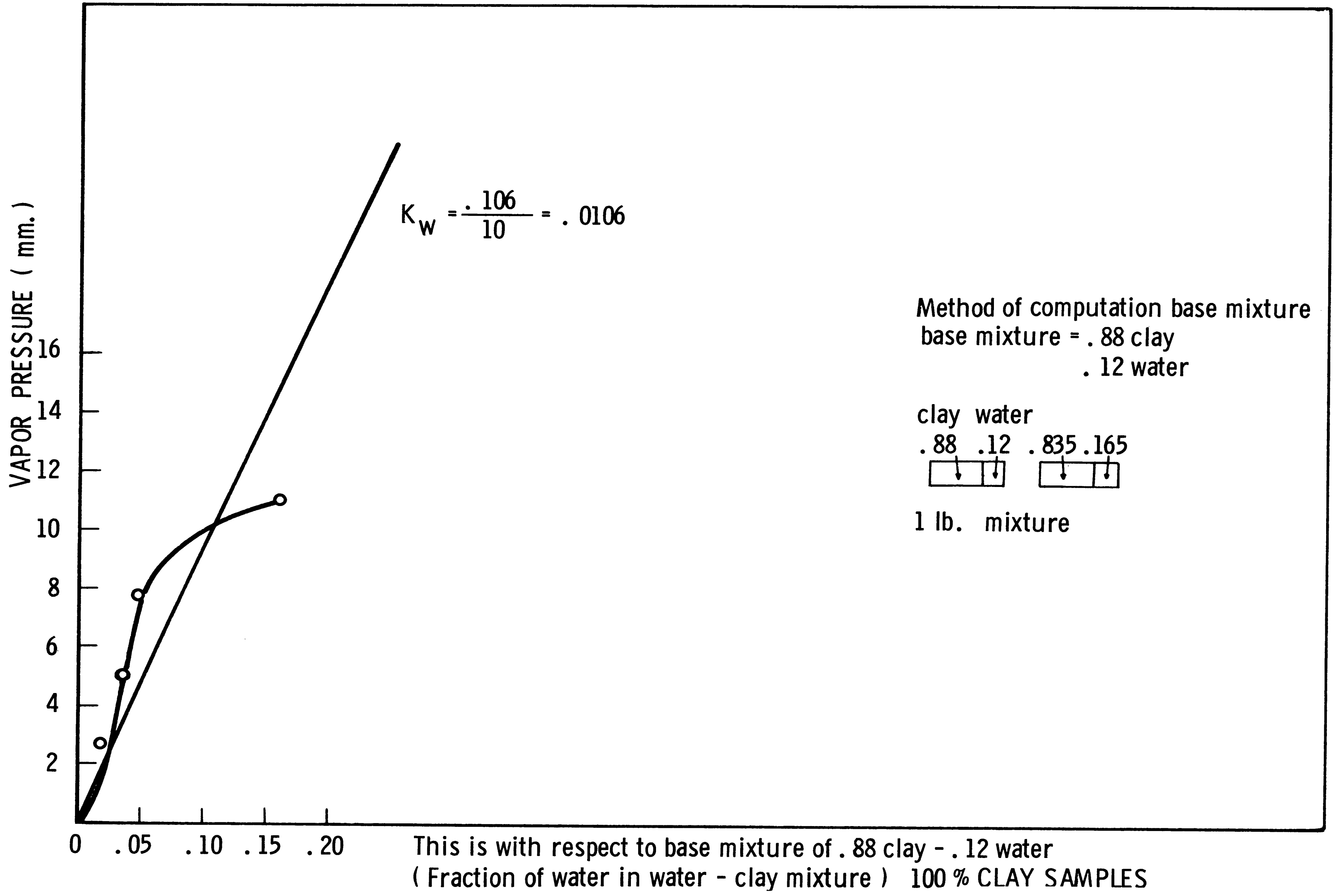

Fig. 5. Plot of the determination of K_w for Fig. 1.

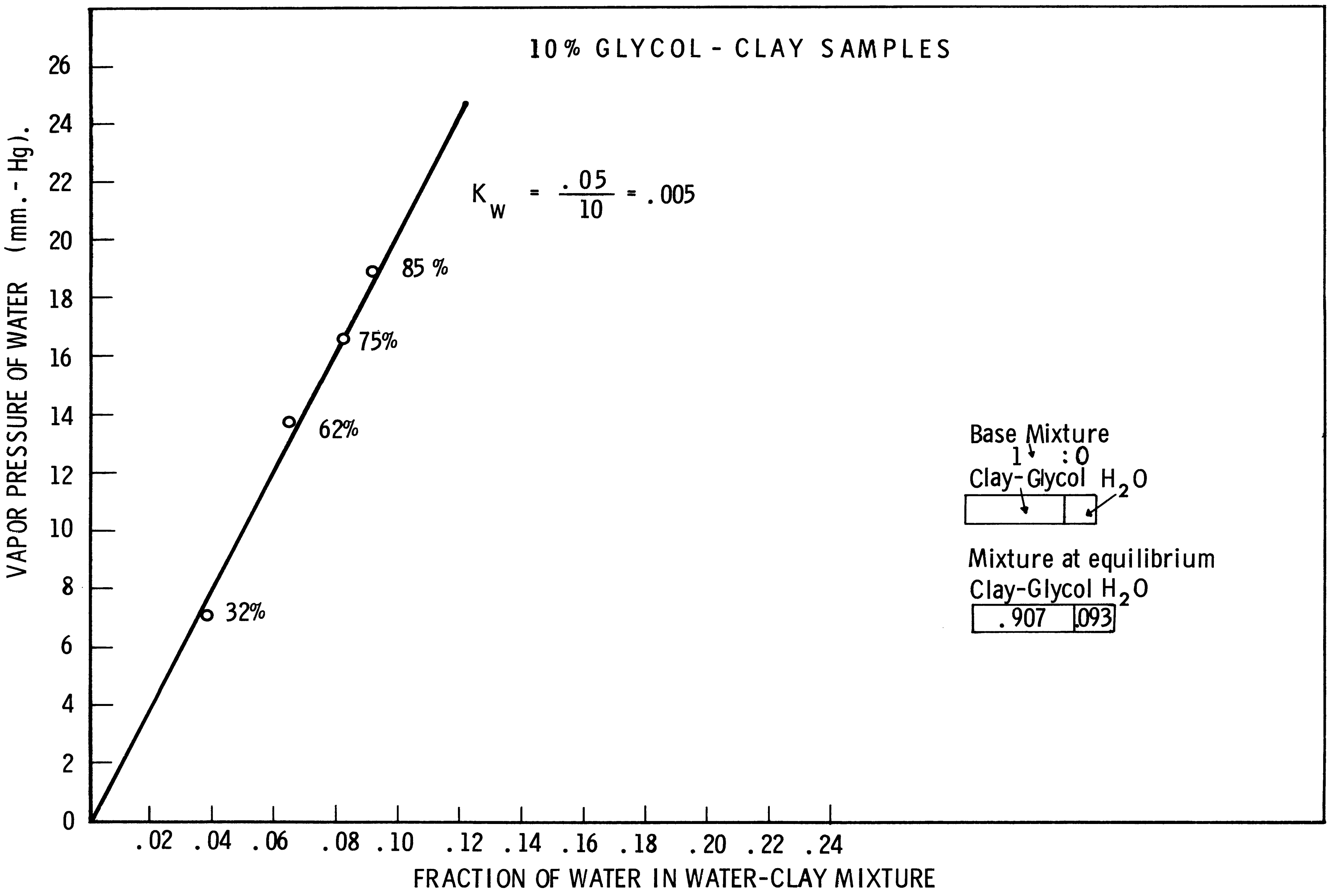

Fig. 6. Plot of the determination of K_W for Fig. 2.

Pure Clay Samples - Analysis Data

$\frac{x}{L}$	$F_o = \frac{a\theta}{L^2}$	$\frac{P_{\frac{x}{L}} - P_\infty}{P_o - P_\infty}$	Water = W % H_2O/day	Relative Humidity	Misc.
.00	.1	.92	.019	100%	temp. 75°F
.20	.1	.88	.024	"	$P_\infty = 11.1$, $P_o = 1$
.40	.1	.80	.032	"	$\theta = 2$ days
.60	.1	.62	.051	"	$\Sigma W/6 \approx .050$
.80	.1	.35	.080	"	
1.00	.1	.20	.096	"	$(Q/Q_o) = .35$
.00	.35	.54	.06	100%	$P_\infty = 11.1$, $P_o = 1$
.20	.35	.50	.064	"	$\theta = 6$ days
.40	.35	.45	.07	"	$\Sigma W/6 \approx .083$
.60	.35	.32	.083	"	
.80	.35	.17	.099	"	$(Q/Q_o) = .56$
1.00	.35	.00	.12	"	
.00	.9	.134	.103	100%	$P_\infty = 11.1$, $P_o = 1$
.20	.9	.13	.103	"	$\theta = 22$ days
.40	.9	.12	.105	"	$\Sigma W/6 \approx .111$
.60	.9	0	.118	"	
.80	.9	0	.118	"	
1.00	.9	0	.118	"	$(Q/Q_o) = .894$
.00	.32	.60	.033	85%	$P_\infty = 7.8$
.20	.32	.58	.034	"	$P_o = 0$
.40	.32	.50	.041	"	$\Sigma W/6 \approx .051$
.60	.32	.35	.054	"	$\theta = 2$ days
.80	.32	.18	.067	"	
1.00	.32	0	.080	"	$(Q/Q_o) = .52$
.00	.70	.225	.064	85%	$P_\infty = 7.8$
.20	.70	.22	.064	"	$P_{\theta=o} = 0$
.40	.70	.185	.067	"	$\theta = 6$ days
.60	.70	.135	.071	"	$\Sigma W/6 \approx .072$
.80	.70	0	.080	"	
1.00	.70	0	.080	"	$(Q/Q_o) = .84$

Pure Clay Samples (Continued)

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$\frac{x}{L}$	$F_o = \frac{a\theta}{L^2}$	$\frac{P_{\frac{x}{L}} - P_\infty}{P_o - P_\infty}$	Water = W % H_2O/day	Relative Humidity	Misc.
.00	1.15	0	.080	85%	$P_\infty = 7.8$
.20	1.15	0	.080	"	$P_{\theta=o} = 0$
.40	1.15	0	.080	"	$\theta = 22$ days
.60	1.15	0	.080	"	$\Sigma W/6 = .080$
.80	1.15	0	.080	"	
1.00	1.15	0	.080	"	$(Q/Q_o) = .95$
.00	.37	.55	.024	75%	$P_\infty = 5.1$
.20	.37	.51	.027	"	$P_o = 0$
.40	.37	.46	.029	"	$\theta = 2$ days
.60	.37	.33	.036	"	$\Sigma W/6 \approx .035$
.80	.37	.18	.043	"	temp. 75°F
1.00	.37	0	.054	"	$(Q/Q_o) = .60$
.00	.80	.1 x 7	.044	75%	$P_\infty = 5.1$
.20	.80	.1 x 6	.045	"	$P_o = 0$
.40	.80	.14	.047	"	$\theta = 6$ days
.60	.80	.11	.048	"	$\Sigma W/6 \approx .049$
.80	.80	0	.054	"	$(Q/Q_o) = .89$
1.00	.80	0	.054	"	
.00	1.2	0	.054	75%	$P_\infty = 5.1$
.20	1.2	0	.054	"	$P_o = 0$
.40	1.2	0	.054	"	$\theta = 22$ days
.60	1.2	0	.054	"	$\Sigma W/6 = .054$
.80	1.2	0	.054	"	
1.00	1.2	0	.054	"	$(Q/Q_o) = .96$
.00	.31	.60	.012	62%	$P_\infty = 2.7$
.20	.31	.58	.012	"	$P_{\theta=o} = 0$
.40	.31	.50	.014	"	$\theta = 2$ days
.60	.31	.35	.018	"	$\Sigma W/6 \approx .018$
.80	.31	.18	.023	"	temp. 75°F
1.00	.31	0	.028	"	$(Q/Q_o) = .50$

Pure Clay Samples (Concluded)

$\frac{x}{L}$	$F_o = \frac{a\Theta}{L^2}$	$\frac{P_{\frac{x}{L}} - P_\infty}{P_o - P_\infty}$	Water = W % H_2O/day	Relative Humidity	Misc.
.00	2.0	0	.028	62%	$P_\infty = 2.7$
.20	2.0	0	.028	"	$P_o = 0$
.40	2.0	0	.028	"	$\Theta = 6$ days
.60	2.0	0	.028	"	$\Sigma W/6 \approx .028$
.80	2.0	0	.028	"	
1.00	2.0	0	.028	"	$(Q/Q_o) = 1$

10% Glycol-Clay Samples

$\frac{x}{L}$	$F_o = \frac{a\theta}{L^2}$	$\frac{P_{\frac{x}{L}} - P_\infty}{P_{\theta=o} = P_\infty}$	Water = W	Relative Humidity	Misc.
.00	.35	.54	.043	85%	$P_\infty = 18.9$
.20	.35	.50	.047	"	$P_o = 0$
.40	.35	.45	.052	"	$\theta = 2$ days
.60	.35	.32	.064	"	temp. 75°F
.80	.35	.17	.078	"	$\Sigma W/6 \approx .063$
1.00	.35	0	.094	"	$(Q/Q_o) = .56$
.00	.68	.21	.074	85%	$P_\infty = 18.9$
.20	.68	.19	.078	"	$P_o = 0$
.40	.68	.16	.079	"	$\theta = 6$ days
.60	.68	.11	.084	"	$\Sigma W/6 \approx .084$
.80	.68	0	.094	"	
1.00	.68	0	.094	"	$(Q/Q_o) = .81$
.00	2.0	0	.094	85%	$P_\infty = 18.9$
.20	2.0	0	.094	"	$P_o = 0$
.40	2.0	0	.094	"	$\theta = 22$ days
.60	2.0	0	.094	"	$\Sigma W/6 \approx .094$
.80	2.0	0	.094	"	
1.00	2.0	0	.094	"	$(Q/Q_o) = .99$
.00	.42	.48	.043	75%	$P_\infty = 16.6$
.20	.42	.43	.047	"	$P_o = 0$
.40	.42	.38	.051	"	temp. 75°F
.60	.42	.28	.060	"	$\Sigma W/6 \approx .059$
.80	.42	.15	.071	"	$\theta = 2$ days
1.00	.42	0	.083	"	$(Q/Q_o) = .68$
.00	1.15	0	.083	75%	$P_\infty = 16.6$
.20	1.15	0	.083	"	$P_o = 0$
.40	1.15	0	.083	"	temp. 75°F
.60	1.15	0	.083	"	$\Sigma W/6 \approx .083$
.80	1.15	0	.083	"	$\theta = 6$ days
1.00	1.15	0	.083	"	$(Q/Q_o) = .94$

10% Glycol-Clay Samples (Continued)

$\frac{x}{L}$	$F_o = \frac{a\theta}{L^2}$	$\frac{P_{\frac{x}{L}} - P_\infty}{P_{\theta=o} = P_\infty}$	Water = W	Relative Humidity	Misc.
.00	2.0	0	.083	75%	$P_\infty = 16.6$
.20	2.0	0	.083	"	$P_o = 0$
.40	2.0	0	.083	"	$\Sigma W/6 \approx .083$
.60	2.0	0	.083	"	
.80	2.0	0	.083	"	$\theta = 22$ days
1.00	2.0	0	.083	"	$(Q/Q_o) = 1.0$
.00	.22	.77	.016	62%	$P_\infty = 13.8$
.20	.22	.71	.020	"	$P_o = 0$
.40	.22	.64	.025	"	temp. 75°F
.60	.22	.47	.037	"	$\Sigma W/6 \approx .037$
.80	.22	.24	.053	"	$\theta = 2$ days
1.00	.22	0	.069	"	$(Q/Q_o) = .43$
.00	.80	.18	.057	62%	$P_\infty = 13.8$
.20	.80	.17	.057	"	$P_o = 0$
.40	.80	.14	.059	"	$\Sigma W/6 \approx .064$
.60	.80	.10	.062	"	
.80	.80	0	.069	"	$\theta = 6$ days
1.00	.80	0	.069	"	$(Q/Q_o) = .90$
.00	2.0	0	.069	62%	$P_\infty = 13.8$
.20	2.0	0	.069	"	$P_o = 0$
.40	2.0	0	.069	"	$\Sigma W/6 \approx .069$
.60	2.0	0	.069	"	
.80	2.0	0	.069	"	$\theta = 22$ days
1.00	2.0	0	.069	"	$(Q/Q_o) = 1$
.00	.38	.56	.016	32%	$P_\infty = 7.10$
.20	.38	.52	.017	"	$P_o = 0$
.40	.38	.47	.019	"	temp. 75°F
.60	.38	.33	.024	"	$\Sigma W/6 \approx .024$
.80	.38	.19	.029	"	$\theta = 2$ days
1.00	.38	0	.036	"	$(Q/Q_o) = .61$

10% Glycol-Clay Samples (Concluded)

$\frac{x}{L}$	$F_o = \frac{a\theta}{L^2}$	$\frac{P_{\frac{x}{L}} - P_\infty}{P_{\theta=o} = P_\infty}$	Water $\doteq$ W	Relative Humidity	Misc.
.00	.70	.23	.027	32%	$P_\infty = 7.10$
.20	.70	.21	.028	"	$P_o = 0$
.40	.70	.18	.029	"	temp. 75°F
.60	.70	.13	.031	"	$\Sigma W/6 \approx .031$
.80	.70	0	.036	"	$\theta = 6$ days
1.00	.70	0	.036	"	$(Q/Q_o) = .86$
.00	2.0	0	.036	32%	$P_\infty = 7.10$
.20	2.0	0	.036	"	$P_o = 0$
.40	2.0	0	.036	"	temp. 75°F
.60	2.0	0	.036	"	$\Sigma W/6 \approx .036$
.80	2.0	0	.036	"	$\theta = 22$ days
1.00	2.0	0	.036	"	$(Q/Q_o) = 1$

The theory behind the analysis curves to the experimental data lies in an analogy between a heat-conduction problem through a flat plate of certain thickness and diffusion of water vapor into an artificial soil of certain thickness. Of course in such an analogy certain terms must be re-defined and assumptions made to make the theory possible.

ASSUMPTIONS AND DEFINITIONS

(a) The unit surface conductance between the soil and surrounding medium was considered to be infinite.

(b) The soils were considered to be homogeneous throughout.

(c) The bottom of the petrie dish was analogous to the plane $x = L$ in the flat plate, since there was no diffusion taking place here.

(d) $\dfrac{T_{x/L} - T_\infty}{T_{\theta=0} - T_\infty}$ = dimensionless temp. ratio = $\dfrac{P_{x/L} - P_\infty}{P_{\theta=0} - P_\infty}$ = dimensionless vapor-pressure ratio

Where $P_{x/L} = K_w = K_w = \dfrac{\text{fractional } H_2O \text{ in mixture}}{\text{vapor pressure of } H_2O}$

(e) Vapor pressures of H_2O at 75°F

100% R.H.	22.2 mm Hg
85% R.H.	18.9 " "
75% R.H.	16.6 " "
62% R.H.	13.8 " "
32% R.H.	7.10 " "

(f) The solutions of this type of problem involves trigonometric relations. The analysis curves used in this report were resolved from graphical solutions used by Frank Kreith in his book, Principles of Heat Transfer. The graphs used, on pages 138 and 139, represent numerical data parametrized to non-dimensionalize the functional relationships.

CONCLUSION

In view of the many assumptions involved, the analogy between heat transfer and diffusion appears to be quite close as shown by the comparison of the experimental and calculated diagrams.

Additional work along the lines outlined appears to be justified, using clay and expanding to natural soils.

RECOMMENDATIONS

While the analysis curves fit the experimental data very closely, particularly in the samples which were dried initially, the results can hardly be generalized to include diffusion of water vapor into all possible combinations of artificial soils. A great deal more experimental data are needed, including many combinations of soils and humidities. Under existing conditions, the total time to run an experiment (approximately 2 months) is a severe handicap in collecting data. Expenditures in this area are necessary to resolve the diffusion-rate problem completely.

REFERENCES

Frank Kreith, Principles of Heat Transfer, 1958.

L. M. K. Boelter, V. H. Cherry, H. A. Johnson, and R. C. Martinelli, Heat Transfer Notes, 1946.