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INTERIM REPORT

DETAILED STUDY OF ICE ON AN INLAND LAKE

By

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PREFACE

This interim report contains three chapters summarizing work carried on during the Winter of 1951-52 under Contract DA-11-190-ENG-2, 11/1/51, with the Snow, Ice and Permafrost Research Establishment, Corps of Engineers, U.S. Army, 1215 Washington Street, Wilmette, Illinois.

Chapter I reports quantitative studies on thermal expansion and contraction of the ice on Wamplers Lake in Jackson and Lenawee Counties in southeastern Michigan. Presented here are the first quantitative data on the amount of push resulting from the thermal expansion of ice under changing weather conditions. The measurements are correlated with records of air temperature and other meteorological factors.

Chapter II presents a theory of thermal ice push that takes into account the vertical temperature gradient in a floating ice sheet. While the theory is extremely rough, it accounts quite satisfactorily for the observations.

Chapter III presents considerable data on the texture of lake ice, with particular reference to Wamplers Lake during the Winter of 1951-52. Information is presented in quantitative form. A comparison is made with ice textures observed in Lake Michigan ice during previous winters.

CHAPTER I

QUANTITATIVE STUDIES ON THERMAL EXPANSION AND CONTRACTION OF LAKE ICE

by

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CHAPTER I

QUANTITATIVE STUDIES ON THERMAL EXPANSION AND CONTRACTION OF LAKE ICE

Abstract*

Field observations during the winter of 1951-52 on Wamplers Lake in southeastern Michigan resulted in measurements on the movement of the sheet of lake ice in response to air-temperature fluctuations.

The following generalities are made as a result of the study:

(1) A temperature rise of 1°F per hour prolonged over a period of 12 hours on an 8-inch sheet is sufficient to cause thrust on a shore composed of unconsolidated glacial outwash containing some boulders.

(2) The direction of ice thrust against the shore is not everywhere orthogonal to the trend of the shore line on an elongate lake, but may be oblique at certain points.

(3) Tensional fracturing of the ice due to rapid cooling results in one set of cracks that radiate from the central part of the lake, and another set roughly concentric with the shore line.

Wamplers Lake is about 2 miles long and 1 mile wide.

Introduction

Expansion and contraction of a sheet of lake ice due to temperature changes has long been known to be responsible for the formation of ice ramparts along shore areas, or pressure ridges in the ice itself. Geologists have confined their investigations to the formation of the ice ramparts, while engineers have been more interested in the effects of the expanding ice sheet on shore structures such as reservoir dams and bridge abutments.

Geologists who have reported on the origin and development of ice ramparts include Buckley (1900), Fenneman (1902), Gilbert (1908), J. B. Tyrrell (1910), Hobbs (1911), Scott (1926), and Zumberge (1952). The emphasis in all these studies has been on the results of the expansion and contraction process rather than on the actual process itself.

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Engineering studies, on the other hand, have attempted to evaluate the magnitude of the forces involved during expansion, and their conclusions have been based on either mathematical computations in connection with laboratory experiments (Sawyer 1911, Barnes et al. 1914, Brown 1932, Brown et al. 1946, Rose 1946,) or by direct measurement (Hill 1935).

The underlying principles involved in the formation of ice ramparts by thermal expansion of lake ice were established by Buckley (1900) and later re-emphasized by Hobbs (1911), so only a brief resume is pertinent here. Ice reacts to temperature changes like any other solid; that is, it contracts on cooling and expands when its temperature is raised. A rapid fall in air temperature causes the ice on a lake to contract, so that tension cracks are developed in the ice sheet. These cracks immediately fill with water that quickly freezes so that the total mass of the ice cover is increased. A subsequent rise in air temperature will result in expansion of the ice, so that the total surface area will be greater than it was before contraction. This increase in area of the ice cover will result in compressive forces against the shore, or, in other words, the ice exerts a "push" on the shore so that shore debris is forced into ridges called ice ramparts.

Neither the geological investigations nor the engineering studies have resulted in detailed information concerning the direction of the expansive force of the ice on a natural lake and the correlation of the amount of expansion with temperature changes. For this reason the writers attempted to measure the factors involved in the ice push process during the winter of 1951-52.

Since the north shore of Wamplers Lake, Michigan (Fig. 1) showed evidence of past ice activity in the form of old ice ramparts (Pl. I,B), it was felt that such information could be obtained by close observation on the north shore of that lake during the ice year. Accordingly, reference points on the shore area were established so that the movement of the ice sheet could be followed quantitatively throughout the winter, and such movements could be correlated with air temperature changes recorded on a continuous recording thermograph placed near the lake. The procedure for making the measurements and the results are discussed in the pages that follow.

Field Procedure

Ice Expansion. Two stations on the north shore were established, W_n and E_n in Fig. 1, by driving 4-foot lengths of 1-inch iron pipe into the ground. The stations were so placed that they would not be disturbed by ice action during the winter, although they were put as close to the shore as conditions would permit. Markers were also set on the ice out from shore between W_n and E_n. Periodically during the course of the winter months, after the ice cover on the lake was well established, a transit was set over W_n and E_n, and the angles determined by the base line and a line of sights to the various markers were read to the nearest minute of arc. In this way any small change in the position of the markers could be determined.

The markers were made of 3-foot lengths of 1 by 1-1/2 inch pine. Each was nailed to the bottom of a 1-pound coffee can which was then filled with concrete. The markers were frozen into the lake surface in such a way that the top of the marker base was level with the top of the ice surface. Early in the investigation it was found that melting around the bases of the markers on warm days caused them to loosen and be blown over by moderate winds. This difficulty was remidied by reducing the height of the markers to about 18 inches (Pl. II,A). Thereafter they remained in place except for a slight tilting due to occasional thawing and refreezing around the bases.

The markers were set out in the form of a cross, one axis of which was perpendicular to the shore (Fig. 2). Between January 23, 1952, and March 1, 1952, six observations were made on each marker from W_n and E_n. The original and subsequent positions of each marker were plotted on a large scale. Figure 3 is a composite diagram showing the movement of each of the markers between successive observations as well as the net movement for the entire period of observation, January 23, 1952, to March 1, 1952 (1 to 7 on Fig. 3).

Observations and measurements were also made on the edge of the lake ice as it was pushed up on the shore at other points around the lake. Figure 4 shows, diagrammatically, the manner in which the measurements of ice push on the east shore of the lake were made. Plate I,A is a photograph of the upturned ice on which measurements were made on the east shore.

Another manifestation of ice expansion developed in the form of a buckled zone of ice extending from a point just west of station E_n, along a line having a southerly direction that trended toward the east shore of the lake (Fig. 2). This compression ridge did not come into existence until February 23. The axis of this ridge was at right angles to the direction of push shown in the movement of the markers on the north shore.

Ice Contraction. On January 10 the atmosphere temperature at Wamplers Lake dropped from 30°F to 8°F in 12 hours, a decrease of 1.7°F per hour. Previous to this time the thermograph records showed a three-day period during which the temperature remained nearly constant around 30°F with only minor fluctuations to 23°F and 34°F. During this three-day period of nearly constant temperature, the ice sheet had sufficient time to reach equilibrium conditions with both the top and bottom of the ice layer remaining at almost the same temperature. The abrupt temperature drop of the 10th, therefore, caused the ice layer to contract rapidly, so that cracks developed. On January 12 these cracks were very apparent and were easily followed and mapped for distances of almost 2,000 feet out into the central part of the lake (Pl. II, B; Fig. 5). Some of the cracks intersected each other, with offsetting of one crack by another.

The cracks ranged in width from a few tenths of an inch to 1.5 inches (Pl. II,C). Most of the larger cracks were open to the water. These were the largest cracks produced by tension observed during the entire season.

Discussion of Results

Relationship of Temperature Changes to Ice Expansion as Indicated by Movement of Markers. From the thermograph records at the temporary weather station established near the southeast shore of the lake, air temperature gradients in terms of °F rise per hour were determined. All gradients of any significance appear in Table. 1. Gradients ranged in value from 1.3°F per hour to 5.3°F per hour. Duration of the rises varied from 3 to 13 hours.*

The cumulative °F in Table 1 shows a rough correlation to the magnitude of the push of each interval between observations on the north shore markers. Thus, periods 1 to 2, 3 to 4, and 6 to 7 had the largest number of cumulative °F during which expansion of the ice was possible, and these same periods were also times of greatest movement of the markers on the ice near the north shore (Fig. 3). Exact agreement is not to be expected since the factors of solar radiation during those periods cannot be evaluated precisely. Furthermore, it should be emphasized that the temperatures on which the gradients are based were atmospheric temperatures and not ice temperatures. Snow cover on the ice of various thicknesses and densities would tend to insulate the ice against rapid transmission of atmospheric temperature changes.

Besides the fact that the net movement of all the markers was of the same order of magnitude, there also seems to be some consistency in the pattern of movement for all markers between one observation and the next. Deviations

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[*] The upper limit of any gradient was taken as 32°F. It is realized that rises above this temperature would continue to cause the ice to become warmer, since the temperature gradient in the ice would lag behind the temperature gradient of the air, but because the lag is not the same for each gradient the limit of 32°F of the air temperature was used for the sake of consistency.

of certain markers from the general pattern of movement of all other markers may be accounted for by errors in reading the vernier of the transit or minor tilting of the marker itself due to differential thawing and refreezing around the base. Some error also may be due to the inability of the instrument man to set the transit over the exact point on the iron stake at each time when angles were measured from stations W_n and E_n.

TABLE 1

TEMPERATURE GRADIENT DATA

Observation No. (Date of measurement on Fig. 3)	Interval Between measurements on north shore markers (see Fig. 3)	Date of gradient recorded on thermograph	Temperature Range (°F) From	To	Total Rise, °F	Duration of Rise hours	Gradient, °F/hr
1.	Jan. 23	Jan. 24	15	32	17	13	1.3
	to	Jan. 29	0	14	14	8	1.7
2.	Feb. 2	Jan. 30	4	20	16	8	2.0
		Jan. 31	20	32	12	3	4.0
			Cumulative °F		59		
2.	Feb. 2 to	Feb. 7	14	24	10	6	1.7
3.	Feb. 8		Cumulative °F		10		
3.	Feb. 8	Feb. 9*	18	31	13	10	1.3
	to		16	32	16	3	5.3
4.	Feb. 16		10	25	15	9	1.7
			11	25	14	7	2.0
			Cumulative °F		58		
4.	Feb. 16	Feb. 16	22	30	8	4	2.0
	to	Feb. 17	17	32	15	6	2.5
5.	Feb. 19		Cumulative °F		23		
5.	Feb. 19 to	Feb. 22	13	32	19	4	4.8
	Feb. 23		Cumulative °F		19		
6.	Feb. 23	Feb. 25	20	30	10	4	2.5
	to	Feb. 26	23	32	9	6	1.5
7.	Mar. 1	Feb. 27	20	32	12	7	1.7
			Cumulative °F		31		

*Indicates more than 2 inches of continuous snow cover.

If the movement of the markers is an index of both the magnitude and direction of ice push, then the significant result appears to be the fact that the expansive force of the ice is not directed orthogonal to the shore in the area of observation. The pressure ridge that formed at right angles to this push is also corroborating evidence that the markers did indicate a true direction of movement. Figure 2 illustrates graphically the net movement and direc- of movement of each marker.

This oblique movement of the ice on the north shore might well be ascribed to the shape of the lake itself. Obviously, more expansion will develop along the long axis than along the short axis of the lake, and therefore, it is reasonable to assume that because Wamplers Lake has an east-west elongation, there would be a pronounced eastward component of shove for the ice between stations E_n and W_n. The question does arise, however, as to what effect skin-friction action on the ice surface caused by strong westerly winds could have in adding an increment of movement of the entire ice sheet to the east. Because the ice was frozen tightly to the peripheral shore area from freeze-up to break-up, it seems highly unlikely that skin friction was responsible for any component movement of the ice as indicated by the movement of the markers. The amount of expansion on other parts of the ice sheet (Fig. 4 and Pl. I,A) was of the same order of magnitude as that which developed between E_n and W_n, indicating that that area was not atypical in comparison with other push zones.

The statement has often been made that expansion of the ice and, therefore, push on the shore can only result from "rapid" rises in temperature that are quickly communicated to the ice layer. Just what gradients should be classified as "rapid" has heretofore been unknown. Rose (1946, p. 574) calculated thrusts of expanding lake ice for gradients of 5°F, 10°F, and 15°F per hour. Hobbs (1911, p. 158) maintained that there must be "relatively sudden alteration of lower and higher air temperatures...", but gave no quantitative data for the needed gradients.

The Wamplers Lake study provided at least one example of push that resulted from a gradient of 1.3°F per hour. This rise in temperature took place on January 24, when the temperature changed from 15°F to 32°F in 12 hours, and continued to rise up to 37°F in 2 more hours, after which it remained nearly constant at 34-37°F for the next 50 hours. Visible push of considerable magnitude was noticed on January 25, and because there had been no push previous to this date, it was unequivocally attributable to the rise in temperature on January 24. Overcast skies on the 24th and 25th precluded any expansion due to solar radiation, so the push was due entirely to atmospheric temperature rise. The conclusion deduced from this situation is that a rise of about 1°F per hour prolonged over a period of a half day is sufficient to cause thrusting along the shore. This conclusion is in agreement with field data collected by Hill (1935) at the Hastings Lock and Dam on Lake Pepin, Minnesota, where he noted the development of considerable ice thrust against a test beam as the result of a temperature rise of 1°F per hour over a period of

13 hours, a gradient which compares favorably with that of January 24 on ~~Wamplers Lake.~~

Ice Buckling. The buckled zone on Wamplers Lake (Pl. I,B) developed between February 19 and February 23. The ice was less than 8 inches thick at the time of buckling. The axis of this pressure ridge lies at right angles to the direction of ice movement as recorded by the markers (Fig. 2). Prior to this time no buckling of the ice had taken place, but considerable thrust had developed on the east shore (Pl. I,A).

Rose (1946, p. 584) considers that arching and buckling rarely occurs in an ice sheet which is over 1 foot thick, and Hobbs (1911) also emphasized the competency of the ice sheet in transmitting stresses induced in it by temperature changes, but did not set a maximum thickness above which buckling was unlikely or rare in occurrence. He also pointed out the fact that the horizontal distance or the length and width of the ice sheet was also a controlling factor in determining whether the ice sheet would fail by buckling or would push on the shore.

Both the thickness of the ice and the position of the thrust zone bear out the statements of these two previous observers. The axis of the compression ridge was more or less at right angles to the long axis of the lake, and it is to be expected that expansion stresses could be less easily transmitted over the long axis of the ice sheet than over the short one, especially when the ice is only 8 inches thick. The problem lies in the time of formation. Just why the failure of the ice took place after considerable thrust had already developed all around the lake is a matter of speculation. The answer probably lies in the critical thickness of the ice for the length of the lake. Before the buckling developed, the ice was probably strong enough to transmit the stress to the east shore, but for some reason it had weakened sufficiently by late February so that it failed by buckling instead of exerting a thrust on the shore. Another possible explanation is that the resistance of the east shore against thrusting had increased, perhaps by an increase in ice thickness near shore so that the ice became more firmly anchored to the bottom by the time the thrust which produced the buckling was initiated.

Solar Radiation Effects. On a lake-ice surface that is free of snow, bright sunshine has a very noticeable effect. Heat absorbed by radiation causes the ice to expand very rapidly, so rapidly, in fact, that the formation of cracks due to compressive forces of expansion is manifested in a low rumbling noise over the entire lake. Once during the ice season this phenomenon was observed. On February 12 the lake ice was snow-free and bright sunshine brought about the necessary conditions for expansion and fracturing due to solar radiation. An observer walking on the ice during the period could actually see the fractures developing as expansion stresses were relieved in the ice sheet. The rumbling would cease whenever the sun was masked by a cloud, and then recommence when the cloud passed. Eventually, however, the rumbling noise ceased even though the sun continued to shine brightly; this cessation of noise is explained by the supposition that the ice had reached equilibrium condition, all the stresses having been relieved by this time because of push, fracture, and plastic flow. The total absence of snow is apparently mandatory for this condition to prevail, because even a 1-inch cover of light snow can prevent the solar radiation from being effectively absorbed by the ice surface so that rapid expansion will result.

It is noteworthy, also, that the temperature of the ice surface must be considerably below 32°F before effects of solar radiation can be of any significance. The lower surface of the ice is never at a temperature much below freezing, and if the upper surface is also near the freezing point, the total ice layer remains at equilibrium.

Ice Contraction. In contrast to ice expansion due to temperature rises, contraction results when the temperature is lowered fast enough so that the ice sheet cannot deform by plastic flow. The fracture pattern shown on the map (Fig. 5) of February 10 is in response to a drop in temperature of 22°F in 12 hours. The unusually large cracks that formed as a result of this temperature decline can be related to the fact that the ice layer was nearly uniform in temperature for the three-day period immediately preceding the day of the drop in temperature; that is, both the top and bottom of the ice sheet were very close to 32°F.

The fracture pattern is of considerable interest because it shows a tendency toward a radial pattern. Although not all the cracks are shown on the map the major trend of the pattern seems to be well established. Besides the radial cracks, unmapped tension cracks developed parallel to the east shore, so that the complete pattern, ideally, would show a series of cracks extending at right angles from the shore toward the central part of the lake, and another series concentric to the shore and crossing the first set at an angle approaching 90 degrees. It is hoped that future work will permit the mapping of a complete fracture system due to tension in order to establish the general pattern better. Of primary interest would be the difference in fracture pattern between an elongate lake and one that is more closely equidimensional.

Future Studies

There seem to be two elements in the study of ice push that need more detailed attention in any undertaking of future studies. One is more frequent observation, preferably in the form of continuous recording devices set on the lake shore. The other is a record of ice temperatures, so that the rapidity of communication of air temperatures to the ice and the effect of snow-cover insulation can be evaluated more precisely. A device whereby solar radiation could be measured at the lake site would also be advantageous.

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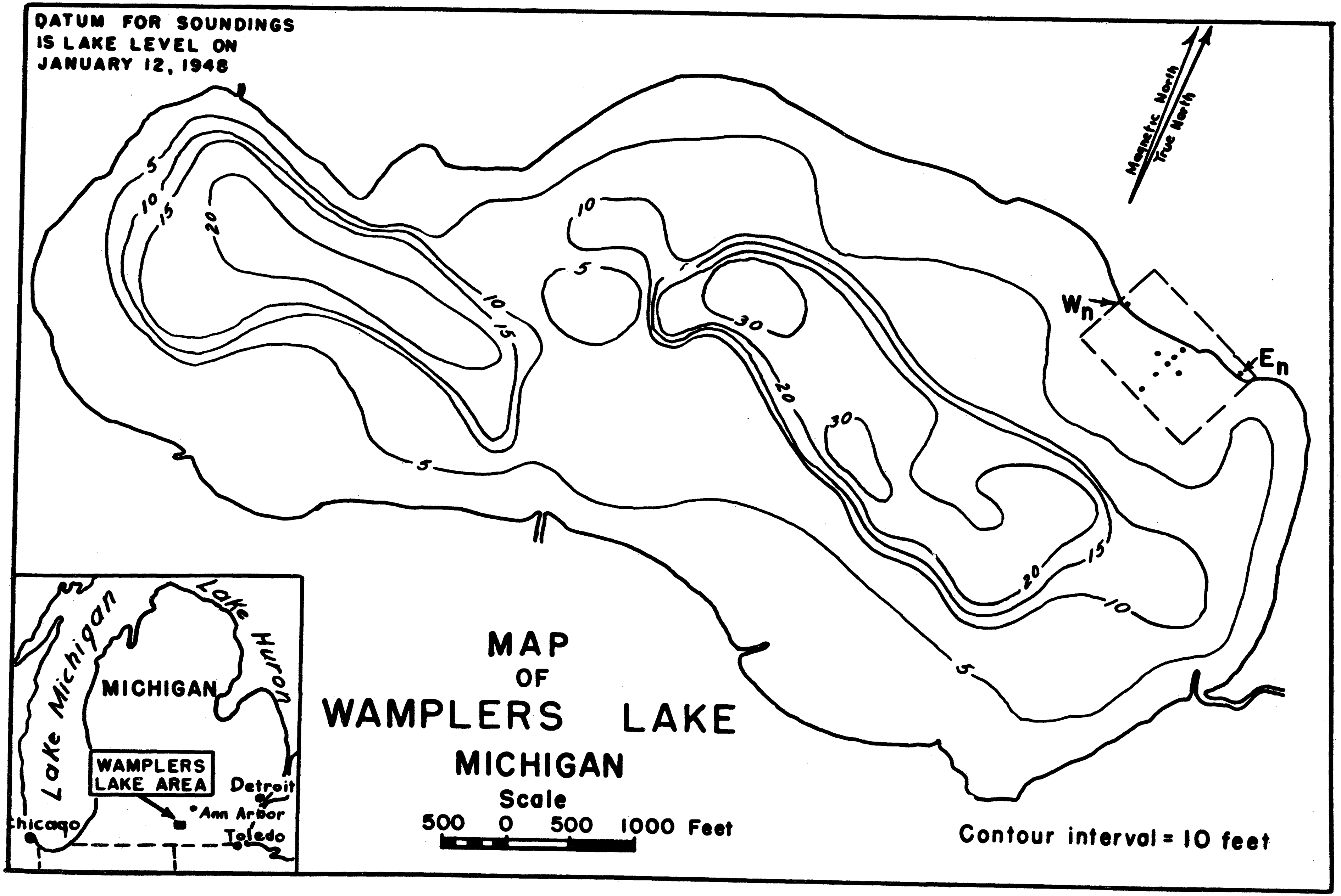

Fig. 1. Bottom contour map of Wamplers Lake. Area enclosed by dashed line on northeast side of lake is area of detailed study. W_n and E_n are ends of the base line in Fig. 2. Small circles between W_n and E_n off-shore show position of markers on the ice.

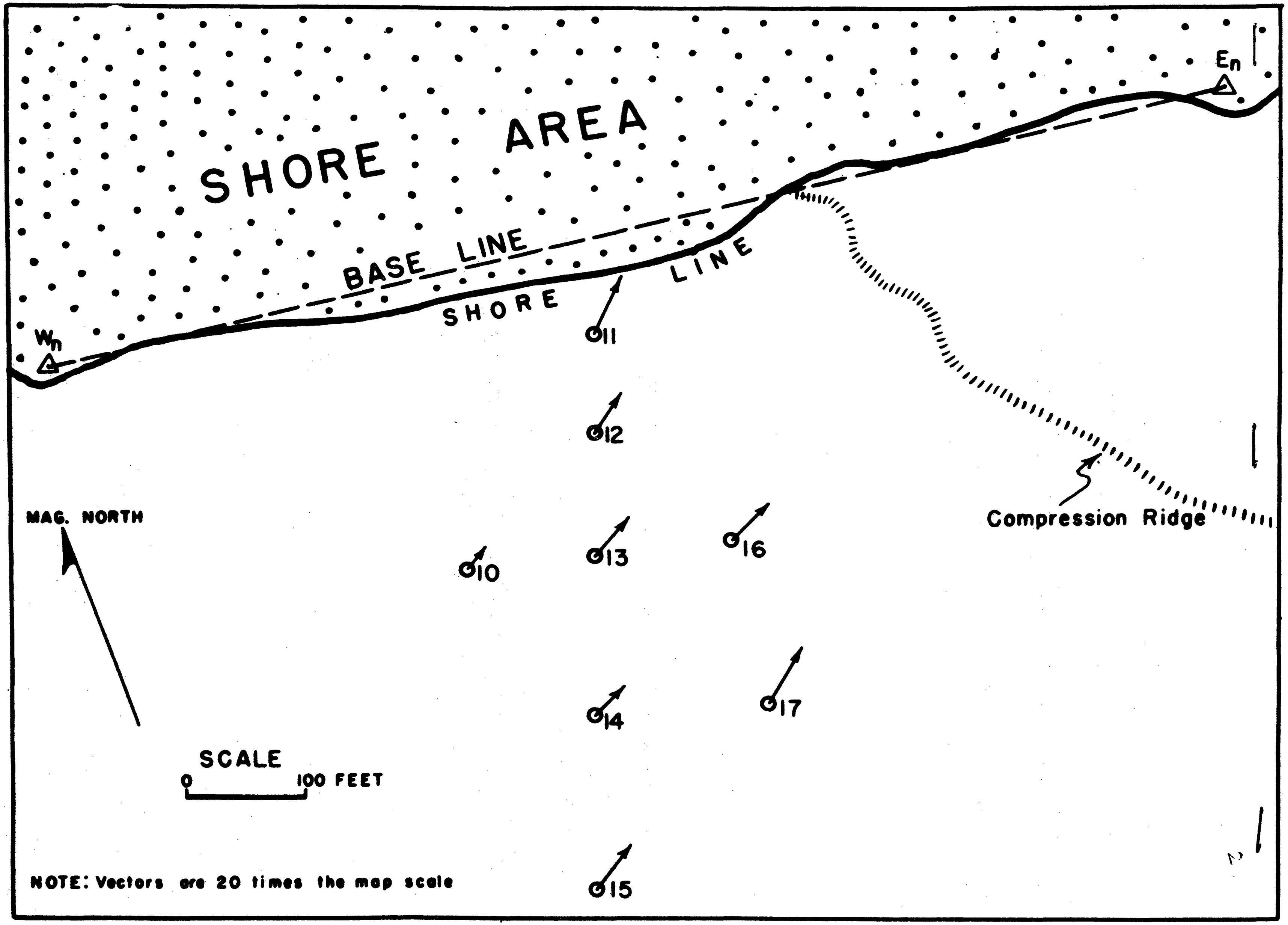

Fig. 2. Map of shore area on Wamplers Lake. Net movement of each marker is shown by a vector originating at the marker position on January 23, 1952 (position 1 on Fig. 3), and ending at the marker position on March 1, 1952 (position 7 on Fig. 3).

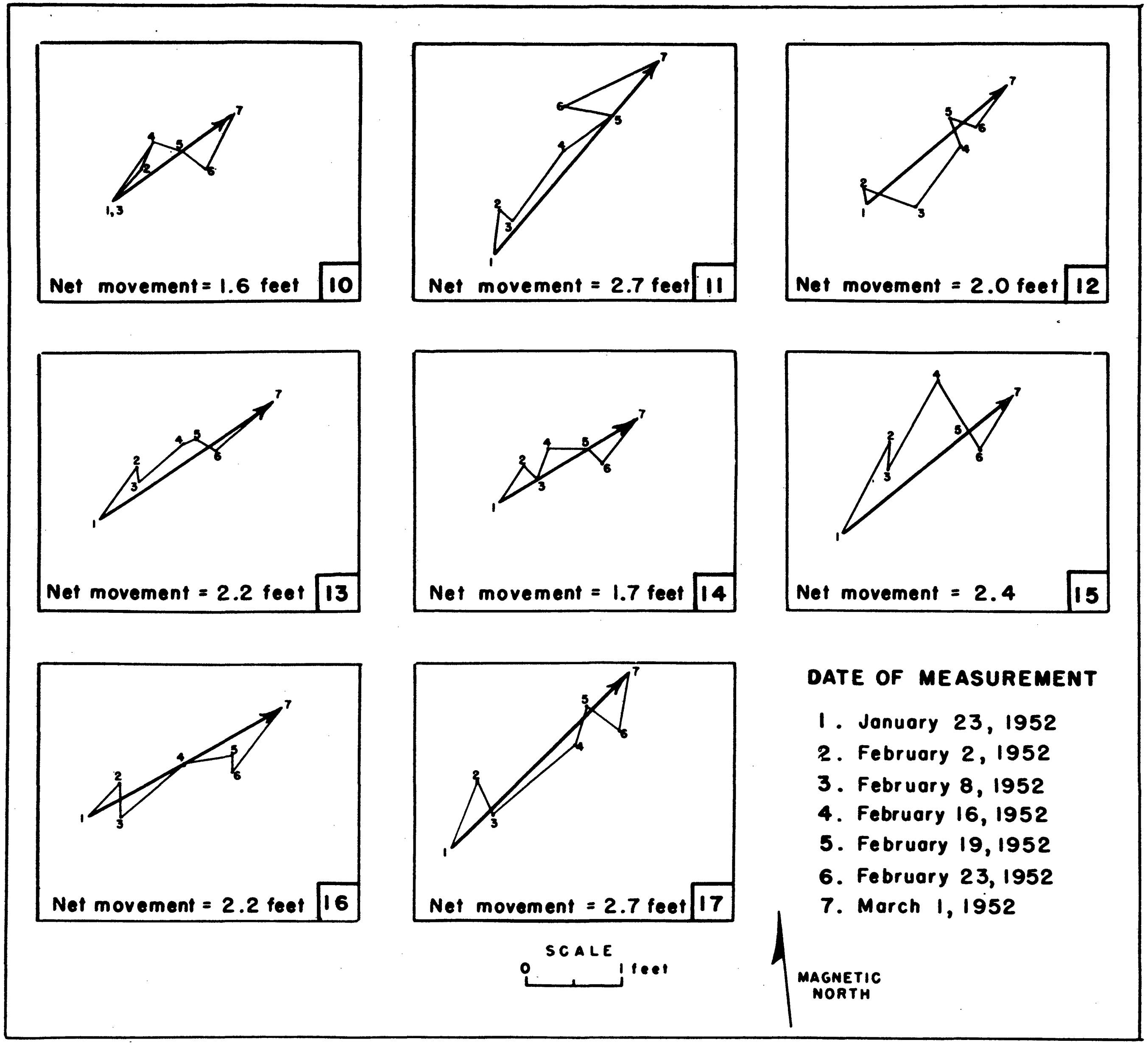

Fig. 3. Map diagrams showing movement of ice markers between successive dates of measurement and total net movement. Marker number is shown in the lower right-hand corner of each map. For position of each marker with respect to base line, see Fig. 2.

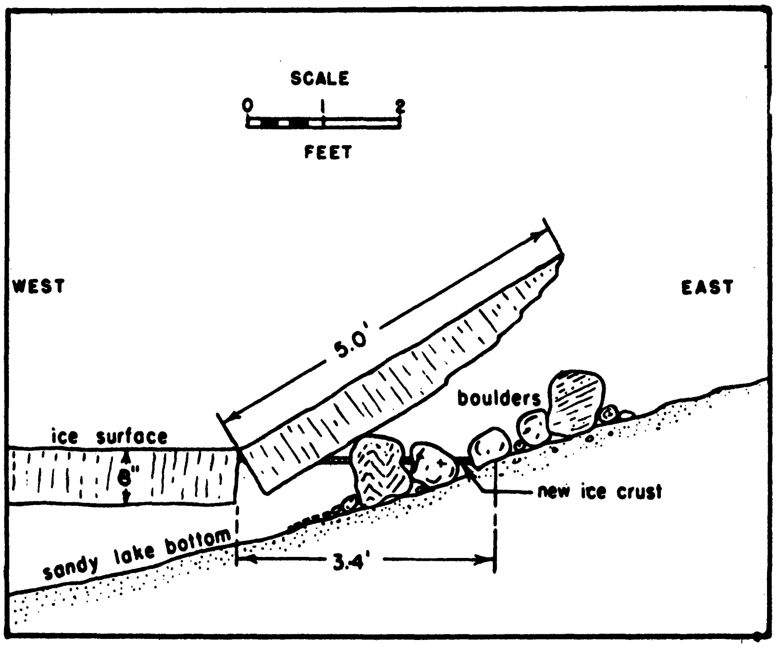

Fig. 4. Cross-sectional diagram of ice edge on east shore of Wamplers Lake. Total "push" is equal to 5.0 feet minus 3.4 feet, 1.6 feet or 19 inches.

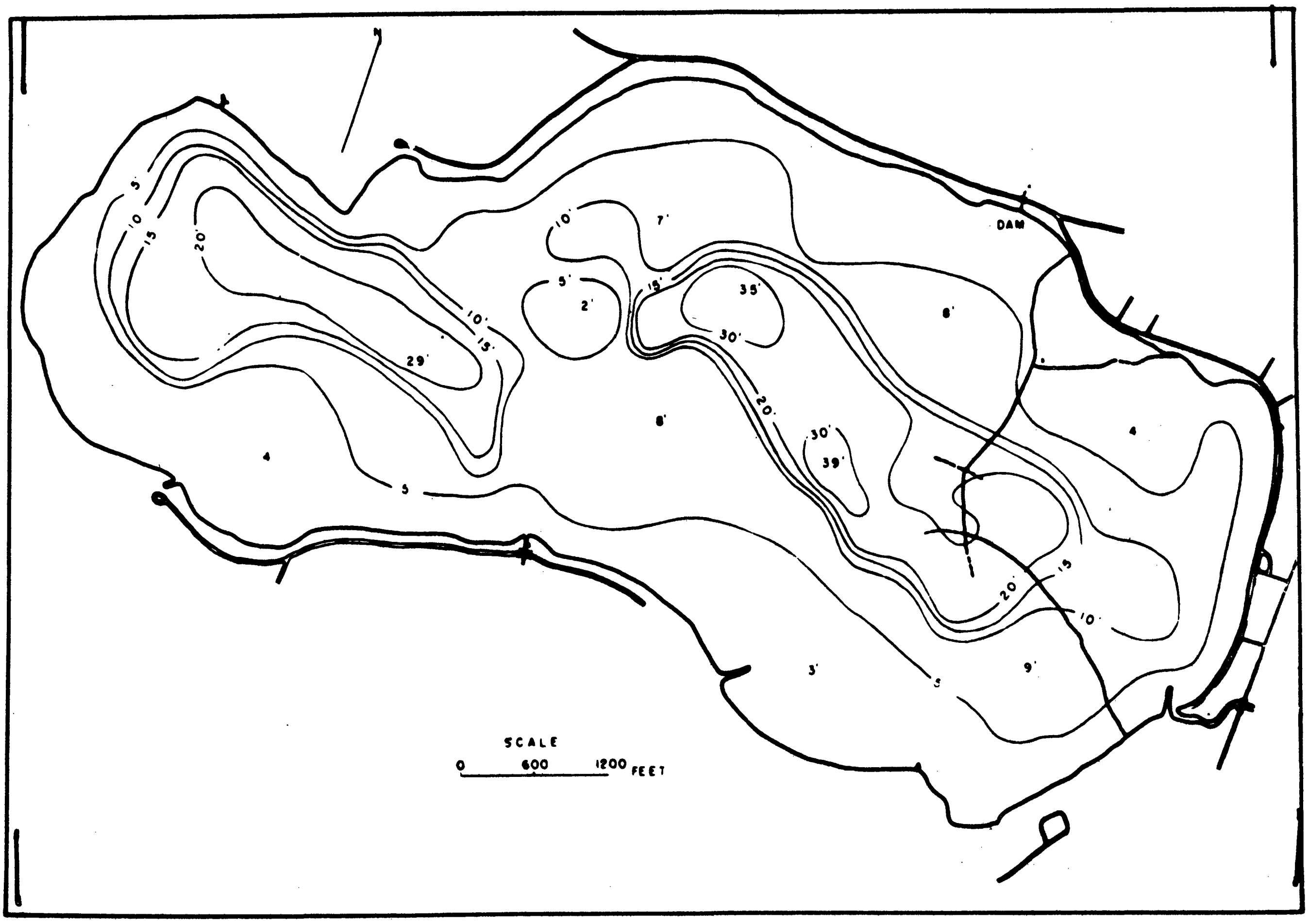

Fig. 5. Map of Wamplers Lake showing tensional fracture pattern (heavy lines near east end of lake) in ice cover, developed because of rapid temperature decline on February 10, 1952.

CHAPTER II

THEORY OF THERMAL ICE PUSH

by

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December, 1952

CHAPTER II. THEORY OF THERMAL ICE PUSH

1. Introduction

The push of lake ice as the result of thermal expansion cannot be likened to the simple expansion of a uniformly warmed plate except in its most general aspects, because the lower surface of the ice is held at its freezing point regardless of temperature changes at the upper surface.

Consider an ice sheet in equilibrium with the lower surface at the freezing point and the upper surface at a lower temperature. If the upper surface is warmed, the sheet will expand differentially and tend to become curved convexly upward. The weight of the sheet and the buoyant forces on it will tend to constrain the sheet to a plane parallel to the water surface. If we assume the sheet large in comparison with its thickness, we can, to a first approximation, neglect the confining effects of the shore. As the sheet is weak in transverse loading, it will break into segments. Each segment will have a curvature appropriate to the temperature difference and the boundaries between the segments will be marked by cracks which open downward. The area of the upper surface will have increased and the ice will have pushed on shore. Figure 1 illustrates this diagrammatically. The shoreward push will be less than the total expansion of the upper surface, as some of the expansion is absorbed in the curvature.

Ice behaves plastically, particularly at temperatures near the melting point, and consequently the stress in the distorted sheet will relax with time. This will lead to flattening of the segments, some thickening of them, and perhaps some further push. However, it will take but little shore resistance to inhibit any push due to the plastic settling of the arched segments.

2. Calculation of the Temperature Rise

Let us now make a rough calculation of the temperature rise necessary to break the ice sheet. In this we will follow to a certain extent Timoshenko's treatment of a differentially heated plate with clamped edges.[1]

If the upper surface is warmed an amount T, the "effective" temperature gradient through the sheet will be T/h, where h is the ice thickness. If the sheet were completely free, it would deform to a spherical shell of radius.

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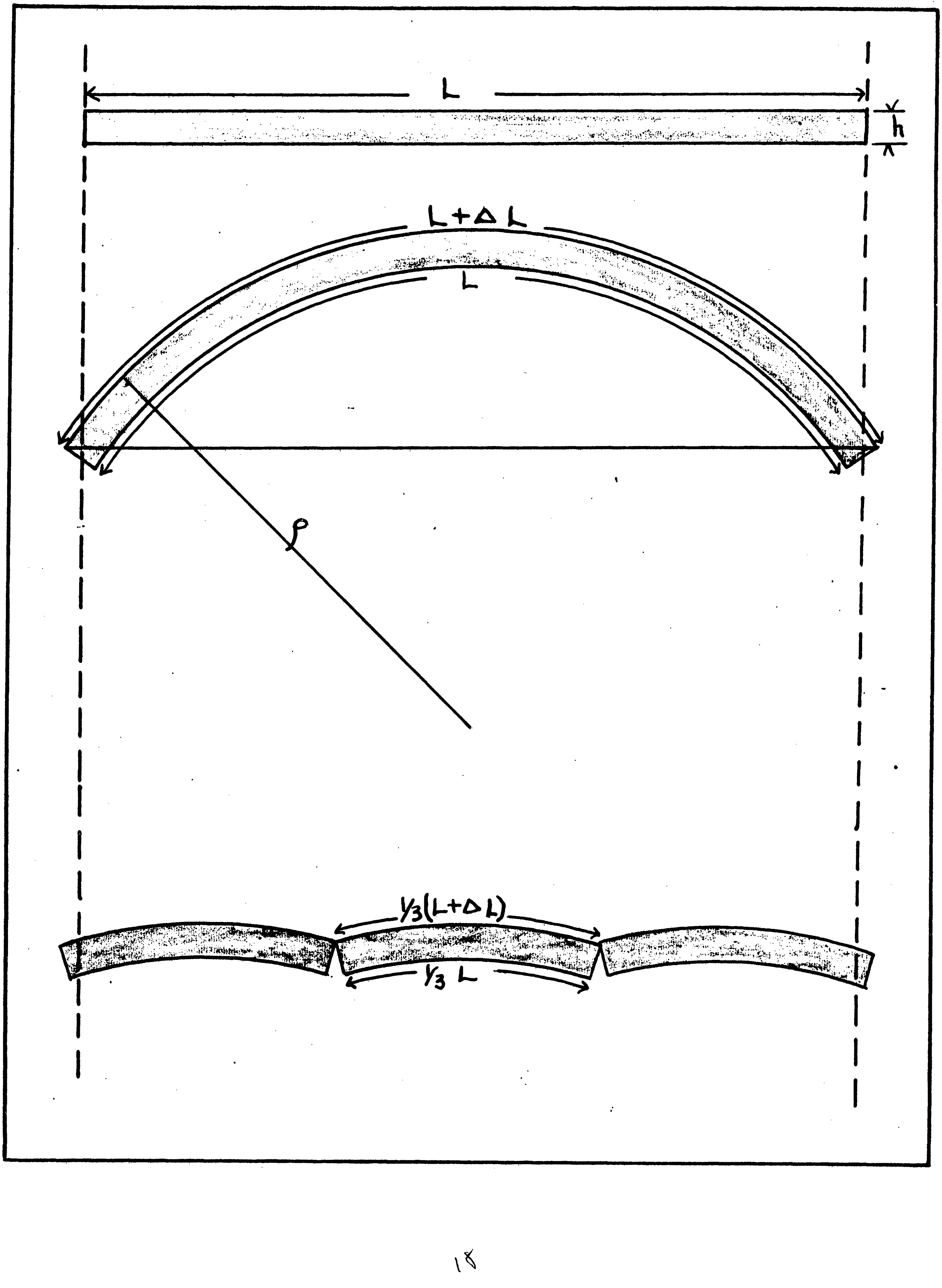
[1] Timoshenko, S., Theory of Plates and Shells, First Ed., McGraw-Hill Book Co., 1940, p. 53 ff.


L
h
L+ΔL
L
ρ
1/3(L+ΔL)
1/3 L

$$\rho = \frac{h}{\alpha T} ,$$

where α is the coefficient of linear expansion.

Now the condition that the ice remain plane is more or less equivalent to considering bending moments uniformly distributed around the edges and of such a magnitude as to nullify the bending due to differential heating. The heating may then be considered to produce a bending moment M per unit length of the edge, given by

$$M = \frac{D(1 + \nu)}{\rho} ,$$

where $D = Eh^3/12(1 - \nu^2)$, E = Young's Modulus, and ν = Poisson's Ratio. Then

$$\frac{M}{D(1 + \nu)} = \frac{1}{\rho} = \frac{\alpha T}{h} .$$

The maximum stress due to the bending moment is then

$$\sigma m = \frac{6 M}{h^2} ,$$

which becomes

$$\sigma m = \frac{\alpha T E}{2(1 - \nu)} .$$

The ice will be expected to fail in tension. Then for

$$\begin{aligned} \sigma m &= 12 \times 10^6 \text{ dynes/cm}^2 \\ E &= 9 \times 10^6 \text{ dynes/cm}^2 \\ \nu &= 1/3 \\ \alpha &= 50 \times 10^{-6}/^\circ C \\ T &= 3.^\circ 6C = 6.^\circ 5F \end{aligned}$$

This rough calculation will apply equally well to cooling of the upper surface and contraction.

It is to be noted that the necessary temperature rise is proportional to the strength and independent of the thickness. A thick sheet, however, would be expected to break into larger segments.

3. Calculation of Segment Size

Let us now estimate the size of the segments into which the sheet will break. To do this we will calculate the maximum size of a floating

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circular segment of a spherical shell of ice that will not be broken by the unbalanced buoyant forces.

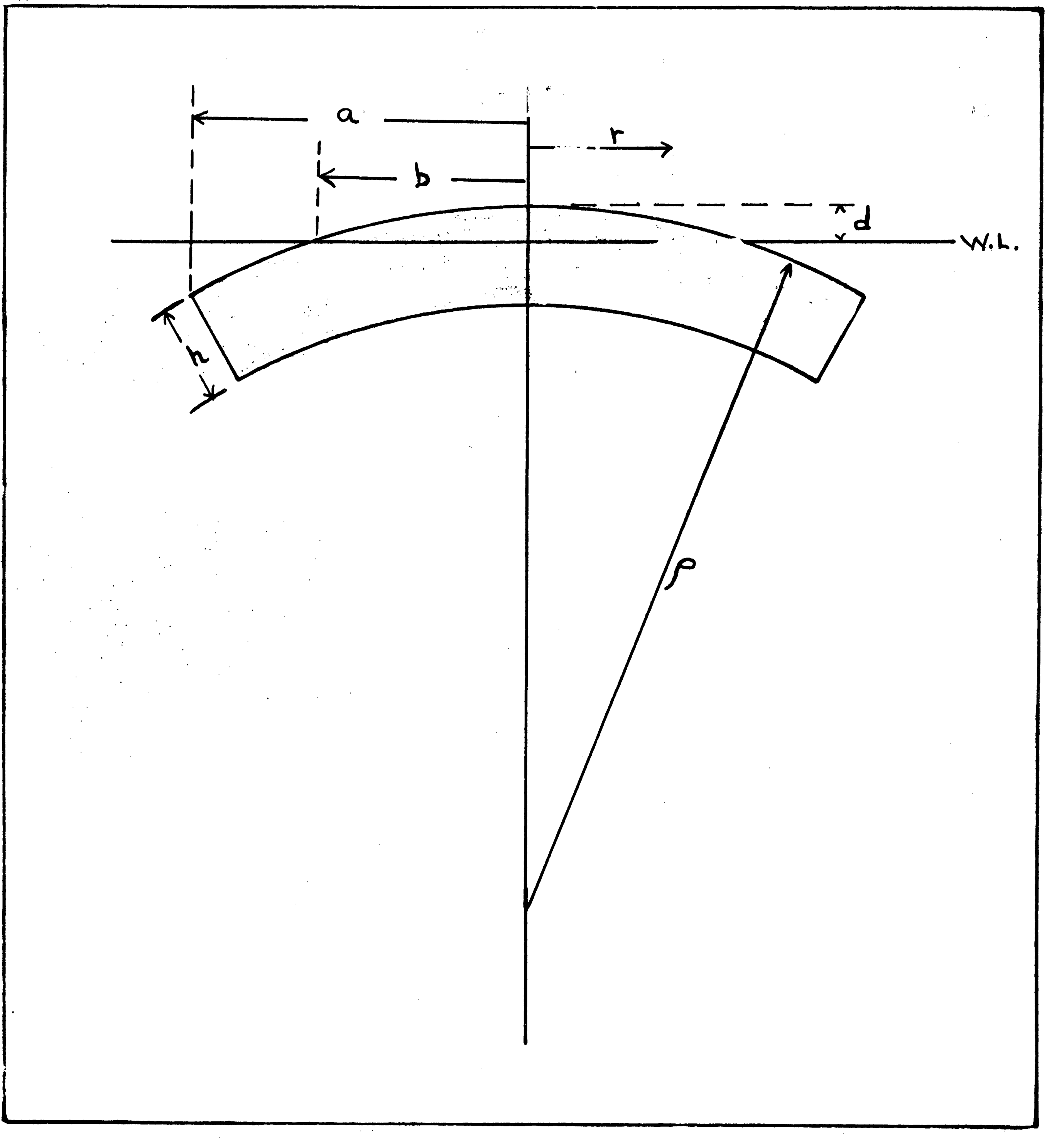

Fig. 2

Referring to the cross section of such a sheet shown in Fig. 2, the exposed volume V is

$$V = \frac{1}{3}\pi d^2 (3\rho - d) = \frac{1}{6}\pi d (d^2 + 3b^2) \approx \frac{\pi b^4}{4\rho}$$

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If the shell were flat, the exposed volume would be

$$V' = \pi d^2 \gamma h,$$

where $\gamma = 1 - \delta$ and δ = Specific gravity of ice. As the shell is floating

$$V = V'$$

or

$$b^2 = 2a \sqrt{\gamma h \rho}$$

The "excess" buoyancy over the ring $a \geqq r \geqq b$

is

$$W' = \pi(a^2 - b^2) \gamma h \delta_w,$$

where δ_w = density of water.

The "excess" load, W, over the circle of radius b must equal the excess buoyancy. Therefore

$$W = \pi(a^2 - b^2) \gamma h \delta_w,$$

or

$$W = \pi(a^2 - 2d \sqrt{rhp}) \gamma h \delta_w.$$

To a crude approximation, we may now consider the shell as a flat plate simply supported around the circle $r = b$ and loaded uniformly with a load q per unit area over $r \leqq b$. Then

$$q = \frac{W}{\pi b^2}$$

For a plate so loaded the maximum stress, σm, is the tensional stress in the bottom surface at the center and[2]

$$\sigma m = \frac{3(3 + \nu)}{8h} q b^2$$

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[2] Timoshenko, S., Loc. Cit., p. 62.

or

$$\sigma m = \frac{3(3+\nu)}{8h} \left(a^2 - 2d\sqrt{\gamma hp}\right)\gamma\delta w.$$

Now

$$\rho = \frac{h}{\alpha T}$$

and

$$a = h\sqrt{\gamma/\alpha T} + \sqrt{h^2\frac{\gamma}{\alpha T} + \frac{8\,h\,\sigma m}{3(3+\nu)\gamma\,\delta w}}$$

Taking

$$\begin{aligned} \sigma m &= 12 \times 10^3 \text{ gm/cm}^2 \\ \nu &= 1/3 \\ \delta w &= 1 \text{ gm/cm}^3 \\ \gamma &= 0.09 \\ \alpha &= 50 \times 10^{-6}/^\circ\text{C}, \end{aligned}$$

we find the following values

h. cm	a. meters, at T = 3.6°C	a. meters, at T = 9°C
10	13	12
30	25	22
100	62	50
300	155	114

4. Calculation of Maximum Push

Referring back to Fig. 1,

$$L + \Delta L = L(1 + \alpha T)$$

and the push on the shore is

$$P = \frac{1}{2}(C - L),$$

where C is the chord length.

$$C = 2\rho \sin\frac{L(1+\alpha T)}{2\rho}$$

or

$$C \approx L(1 + \alpha T) - \frac{L^3}{24\rho^2}$$

and

$$P \approx \frac{1}{2} (L\alpha T - \frac{L^3}{24\rho^2}).$$

Substituting for ρ,

$$P = \frac{1}{2} L\alpha T (1 - \frac{L^2\alpha T}{24h^2})$$

or when there are n segments,

$$P \approx \frac{1}{2} L\alpha T (1 - \frac{L^2\alpha T}{24n^2h^2}).$$

It is to be noted that for

$$T = T_o = \frac{24n^2h^2}{\alpha L^2} \approx \frac{n^2h^2}{2L^2} \times 10^6 \text{ °C}$$

$$P = 0.$$

Of more interest is T_m, the temperature for maximum push

$$T_m = \frac{12\ n^2h^2}{\alpha L^2} \approx \frac{n^2h^2}{4L^2} \times 10^6 \text{ °C},$$

for which the push is

$$P_m = \frac{6n^2L^2}{L} (1 - \frac{1}{24}) \approx \frac{3\ nh^2}{a}$$

Applying these theoretical results to Wamplers Lake in the Winter of 1951-52, we had

$$L \approx 2 \times 10^5 \text{ cm}$$
$$h \approx 30 \text{ cm}$$

Then

$$\alpha \approx 25 \text{ m}$$
$$n \approx 40,$$

and

$$T_m = 9\text{°C} = 16\text{°F}$$
$$P_m = 43\text{cm} = 14 \text{ in}$$

The agreement with observation is amazing and perhaps coincidental considering the crudeness of the theory, but it seems to bear out the basic principle.

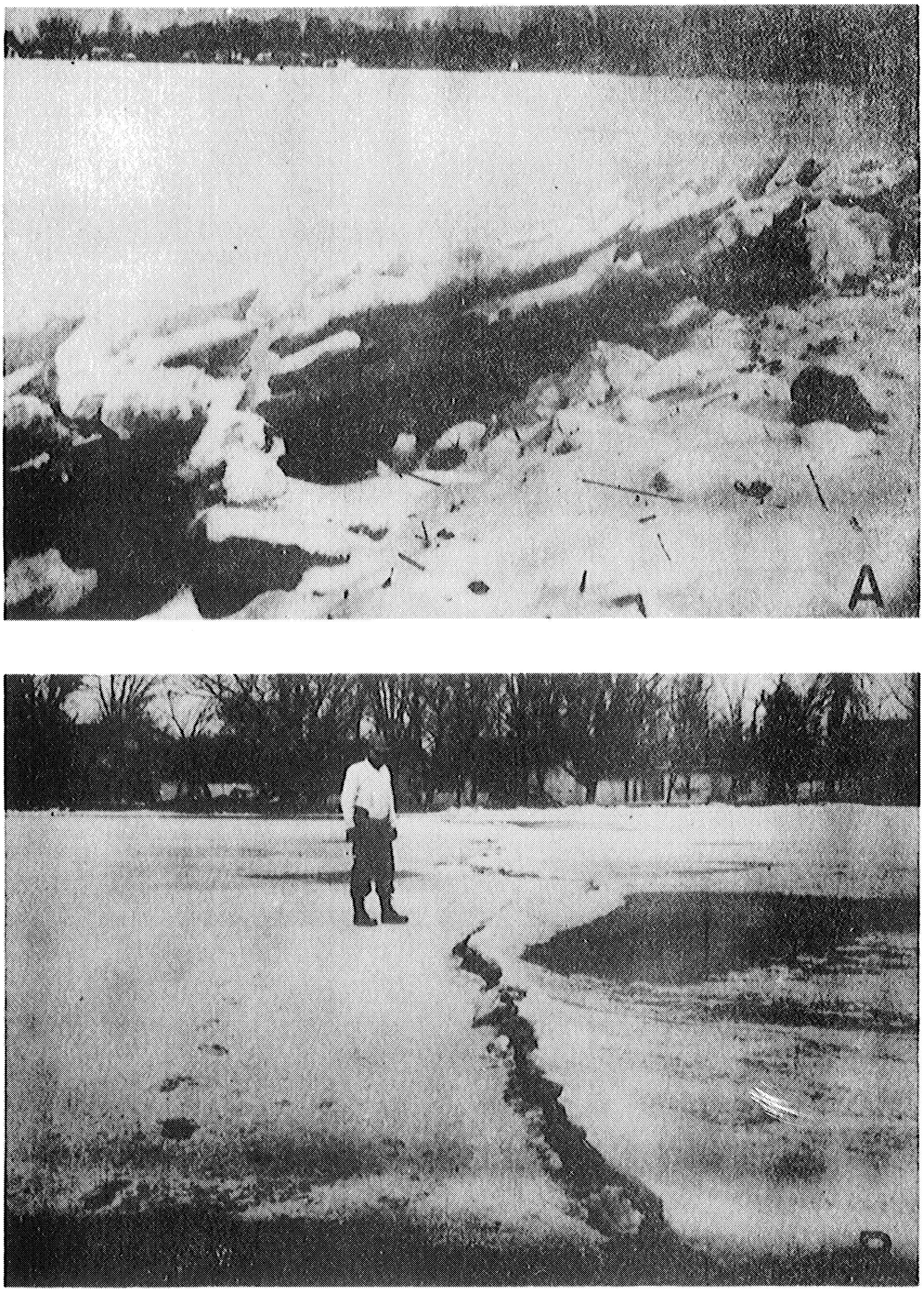

PLATE I

A. Edge of ice sheet on east shore of Wamplers Lake after about 19 inches of "push" had developed. Compare with Fig. 5.

B. Looking northwest along compression ridge shown on map in Fig. 2.

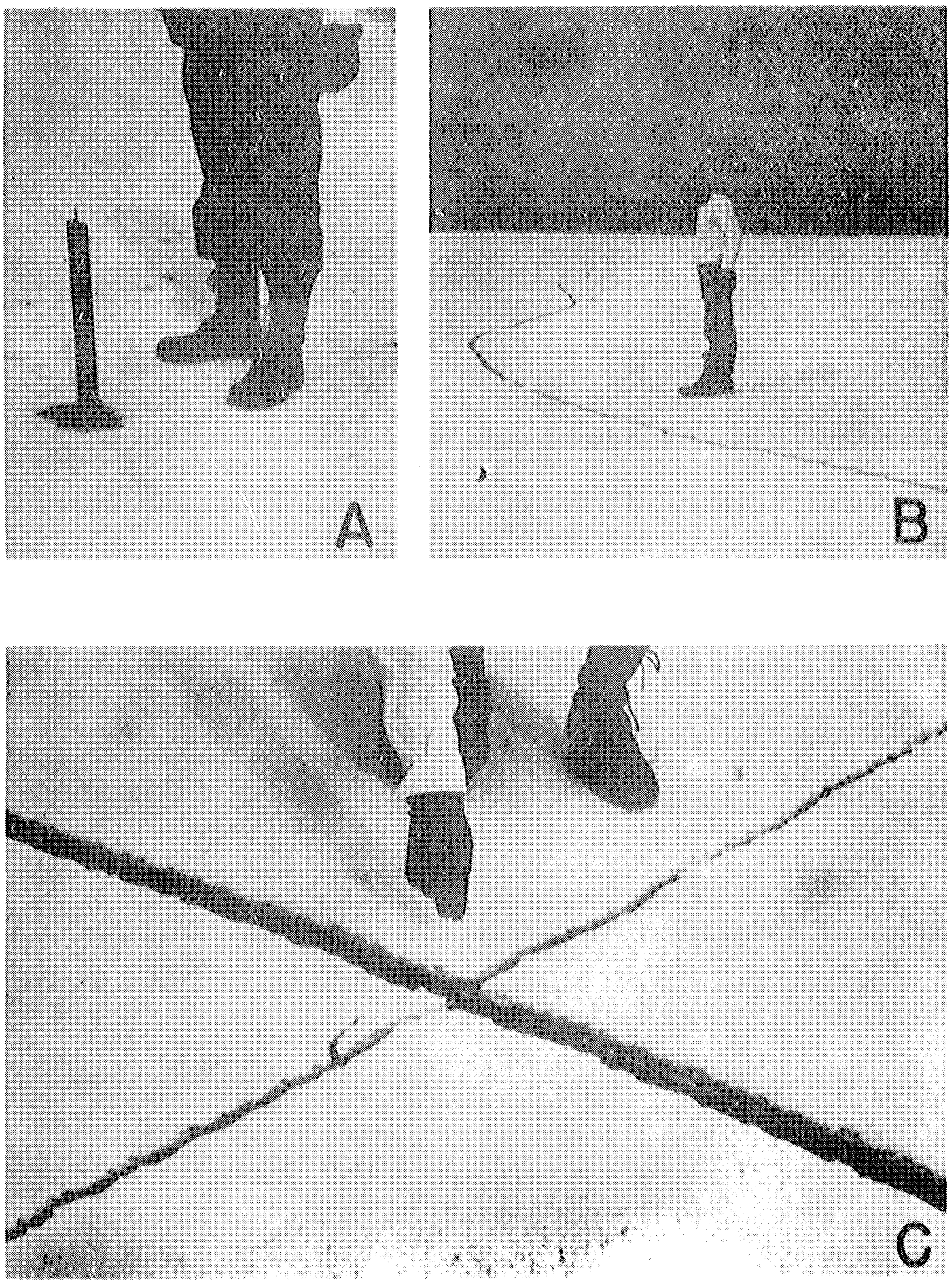

PLATE II

A. Marker frozen into ice.

B. Tension crack radiating toward center of Wamplers Lake from the east shore.

C. Intersecting tension cracks in Wamplers Lake ice. The larger fracture radiates toward the center of the lake, and the smaller one is parallel to the shore.

CHAPTER III

TEXTURE OF LAKE ICE

(With particular respect to the ice on
Wamplers Lake, Michigan, winter of
1951-52)

by

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CHAPTER III. TEXTURE OF LAKE ICE

Introduction: Previous work

Observations by Wilson and Horeth in the Winter of 1947-48, during research on the strength of natural lake ice, supported previous observations on the texture of lake ice as being a columnar aggregate of macroscopic crystals, with the long direction of the crystals at right angles to the refrigerating surface. The desirability of more detailed studies of the history and texture of ice sheets was recognized at that time.

During the Winter of 1950-51, studies were made on Lake Huron along the northeast shoreline of the Lower Peninsula of Michigan and on some of the adjacent inland lakes. These observations were reported in detail in a preliminary report by Wilson and Marshall, Research on Mechanics and Crystallinity of Natural and Artificial Ice, Engineering Research Institute, University of Michigan, May, 1952.

A summary of the observations made in this report is as follows: Two basic textures were observed in lake ice: (1) a coarsely crystalline columnar aggregate (Fig. 1), of which there were found to be several variations, and (2) a finely crystalline granular aggregate. Normal ice sheet accretion gives rise to a columnar aggregate of varying texture. The crystalline granular aggregate is produced by the effects of heavy snowfall at the time of freeze-up or by snowfall on the ice surface which, by processes of wet and dry snow metamorphism, changes into the granular aggregate forming a minor and in many cases a major portion of the ice sheet.

The most common coarsely crystalline texture is the ordinary columnar aggregate. In ice sheets of this texture, the crystals are of macroscopic size, a fraction of an inch to several inches in diameter and with lengths comparable to the thickness of the ice sheet. The larger crystals which extend through the thickness of the ice sheet usually enlarge downward (Fig. 2), and where the ice is a foot or more in thickness the crystals have been observed to be as much as two inches in diameter at the bottom. In the upper part of the aggregate the crystals are smaller and many of them pinch out with depth. Two modifications of the columnar aggregate have been observed: (1) a porphyritic type, in which there are large phenocrysts scattered through an ordinary columnar aggregate (Fig. 3), and (2) a columnar aggregate of crystals so large in the horizontal dimension that they are tabular rather than columnar (Fig.4).

Crystal Texture: Wamplers Lake

During the Winter of 1951-52, a project was conducted on Wamplers Lake, Lenawee and Jackson Counties, Michigan (see map in chapter II). As one phase of the project, observations were made on the crystal texture of the ice sheet photographically and by means of rubbings of selected specimens.

Most significant is the wide variation of crystal size within a single ice sheet. Observations were made and recorded of areas in which the average crystal cross section (on the basis of a count of several hundred crystals) averaged from a fraction of a centimeter to several centimeters, with scattered crystals to 100 centimeters. In the field, crystal boundaries can be seen clearly under certain favorable conditions. Areas protected from snowfall and drifting snow (as under bridges) showed the crystal boundaries by the fact that the boundaries were slightly depressed and caught windblown silt, while the crystal surface was swept clean. In late Winter and early Spring before break-up, areas on which there is no snow ice show the crystal boundaries due to the fact that increased solar radiation melts them out. Figures 5 and 6 show the effect of this phenomenon. In the laboratory crystal texture can be observed by means of rubbings, which provides a means by which detailed crystal size distribution studies can be made. Thin ice sections can also be cut from the ice sheet and observed between polaroids. The varying orientation of the crystals refracts the light differentially, thus revealing the limits of individual crystals.

Crystal size distribution data are presented here in histogram form. These have been compiled from rubbings taken in some cases at the top and bottom of a particular sample and in others at specific intervals through the thickness of the sample. The histograms have been prepared from measurements made on rubbings of ice surfaces. The dimensions of each crystal cross section shown in a particular rubbing were measured and have been compiled according to the logarithmic scale shown on the histograms. For example, for Histogram I, A, 3.9 per cent of the crystals were less than 0.25 centimeter across, 12.3 per cent greater than 0.4 and less than 0.6 centimeter across, 21.3 per cent greater than 0.6 and less than 1.0 centimeter across, etc.

In the majority of cases the crystal size shows a normal distribution pattern at both top and bottom, with a shift toward larger and fewer crystals with depth. The wide distribution of surface crystal size can be attributed to varying degrees of supercooling which occur on various parts of the water surface at the time of the formation of the initial ice skim. The irregular distribution in sections taken at intervals through an ice sheet may be explained by the thermal conditions that existed at the time of accretion due to snow cover, lack of it, or to its removal during the accretion process.

A study made on Evans Lake (in the vicinity of Wamplers Lake) and one from Lake Huron have been included for comparison.

Description of Histograms

Histograms I,A and I,B were compiled from rubbings from the top and bottom ice surfaces respectively of a sample from Wamplers Lake. The ice thickness was 2-3/4 inches. In all, 179 crystal cross sections were measured on the top and 67 on the bottom of the specimen.

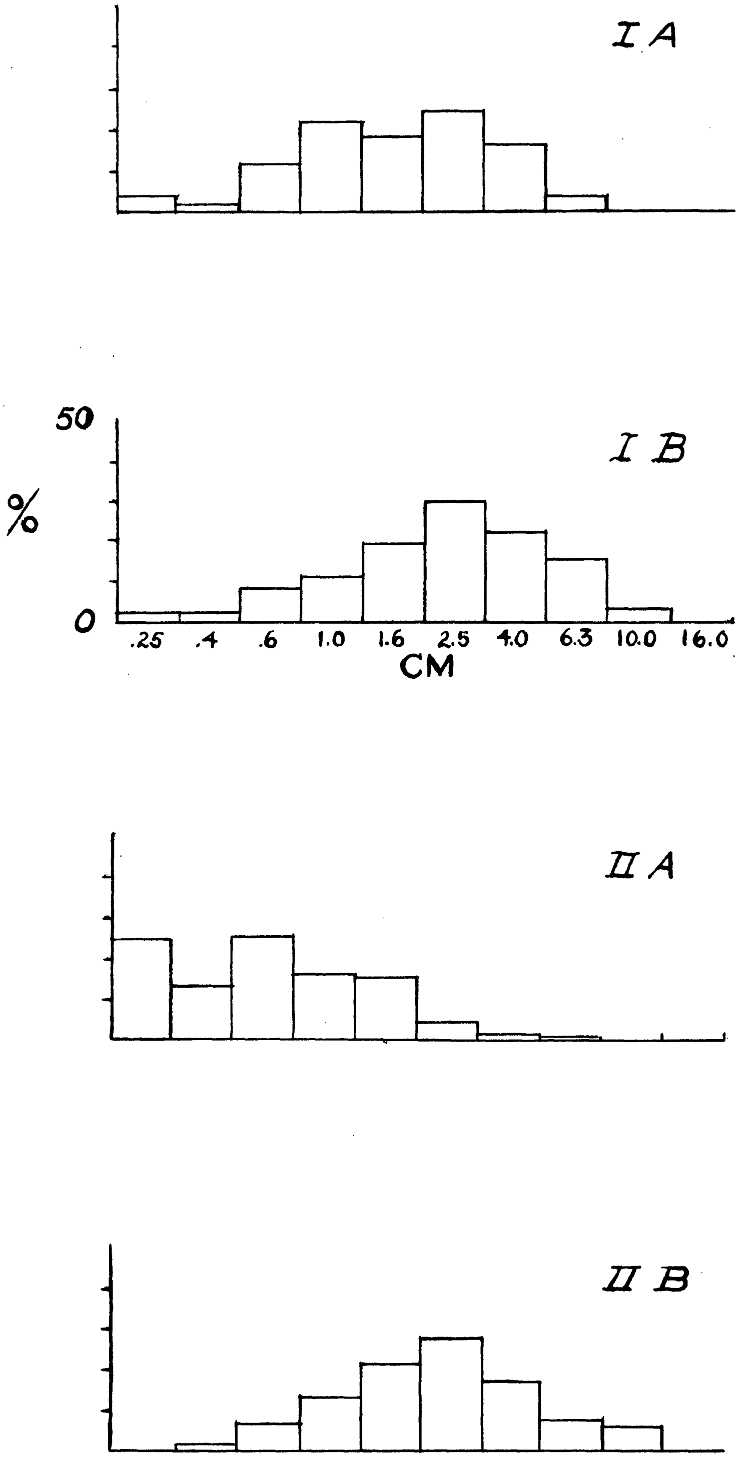
Histograms II,A and II,B were prepared from the top and bottom respectively of a specimen of ice from Wamplers Lake. This specimen was approximately 4-1/2 inches thick. From it, 433 crystal sections were counted on the top and 76 on the bottom.

Histograms III,A through III,G were prepared from rubbings made at approximately uniform intervals through a 7 inch thick specimen of ice from Wamplers Lake. III,A is from the upper surface and III,G from the bottom.

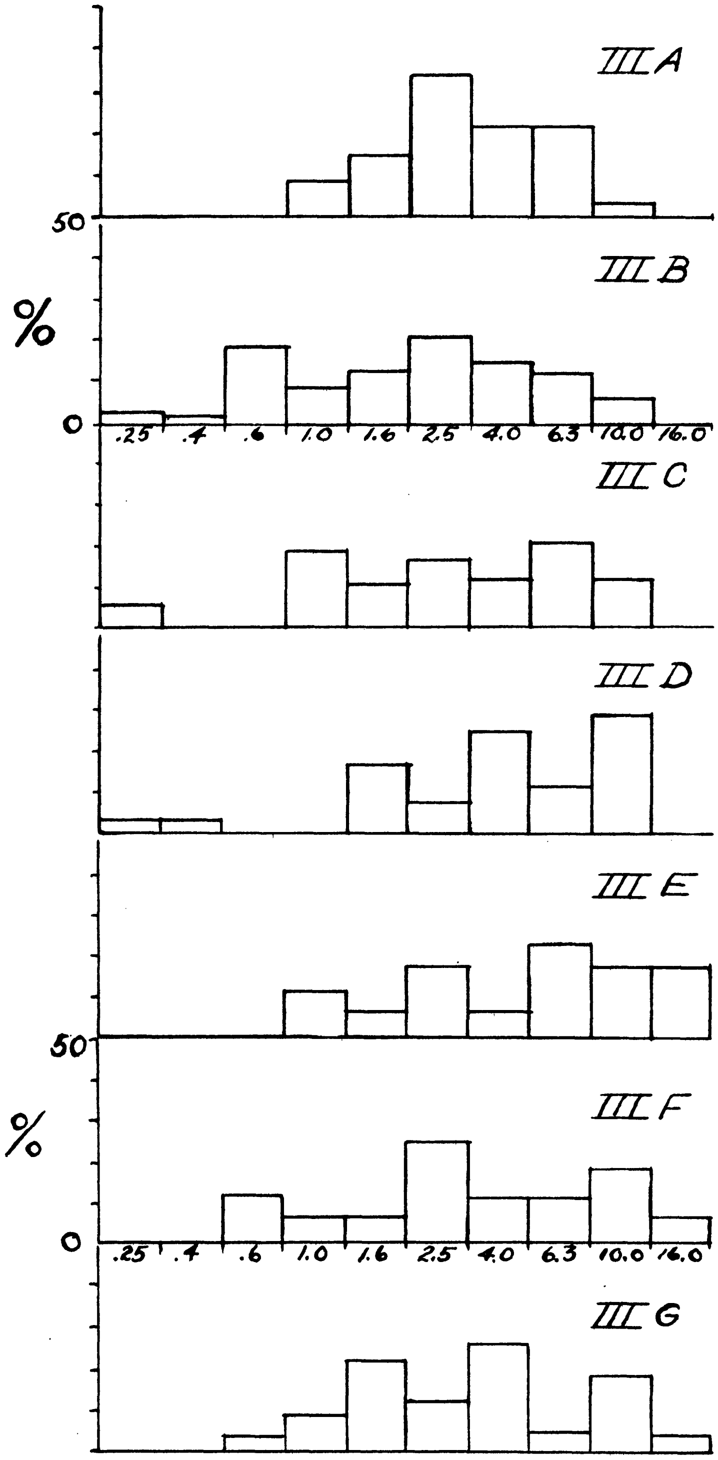
Histograms IV,A and IV,B were prepared from rubbings from the top and bottom of a 7-1/2 inch thick specimen from Evans Lake (Evans Lake is a somewhat smaller lake near Wamplers). In this specimen 159 crystals were counted in the top section and 36 in the bottom.

Histogram V was prepared from a rubbing taken from a large specimen of ice from Evans Lake. The rubbing was made on the upper surface of the ice, and 2061 crystals were counted. Figure 7 is a photograph of this rubbing.

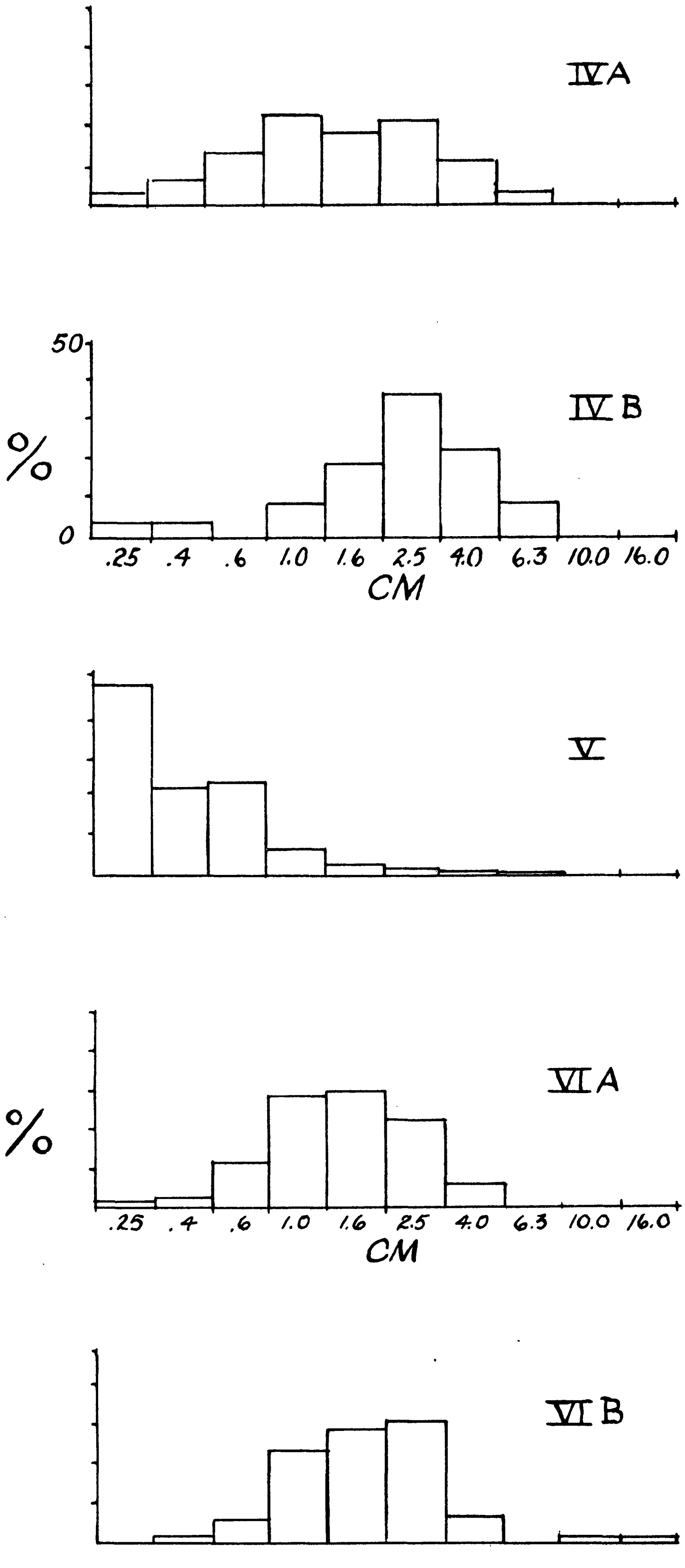
Histograms VI,A and VI,B have been prepared from rubbings made on two specimens of ice from the Lake Huron shoreline near Rogers City, Michigan. Both rubbings were from the bottom of the ice. The surface from which the rubbing furnishing the data for Histogram VI,B was obtained is shown in Fig. 3.



Histograms I and II



Histogram III



Histograms IV, V, and VI

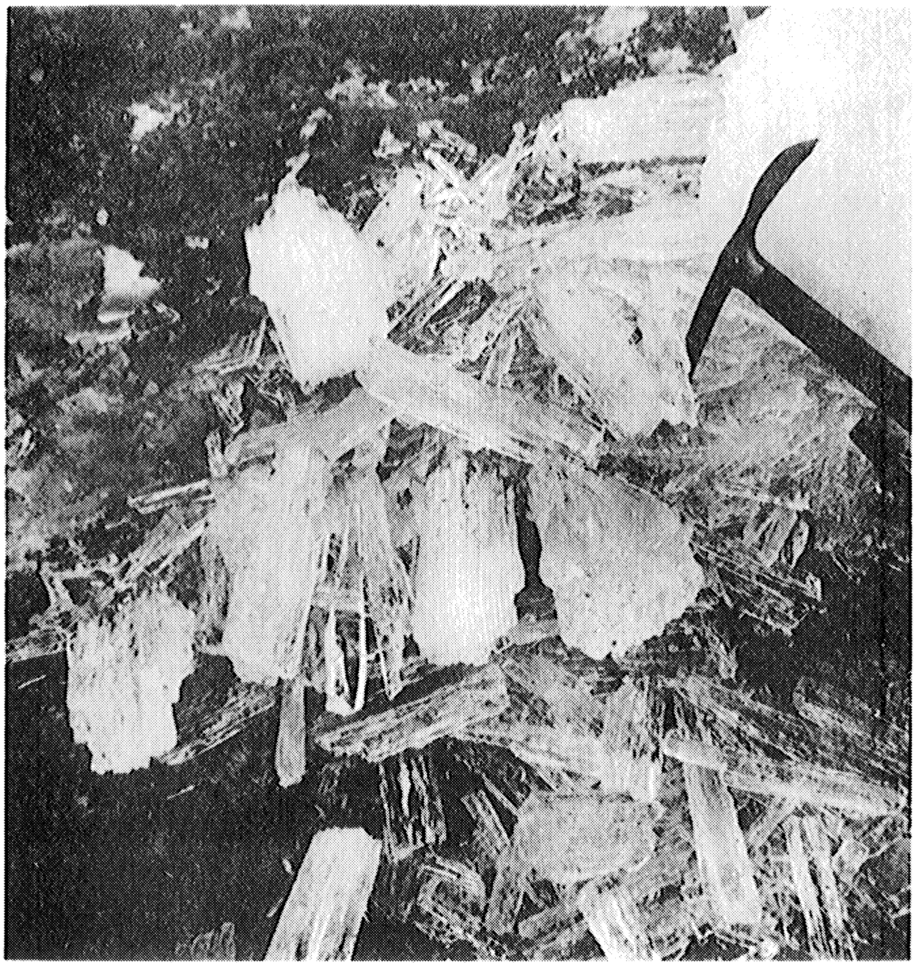
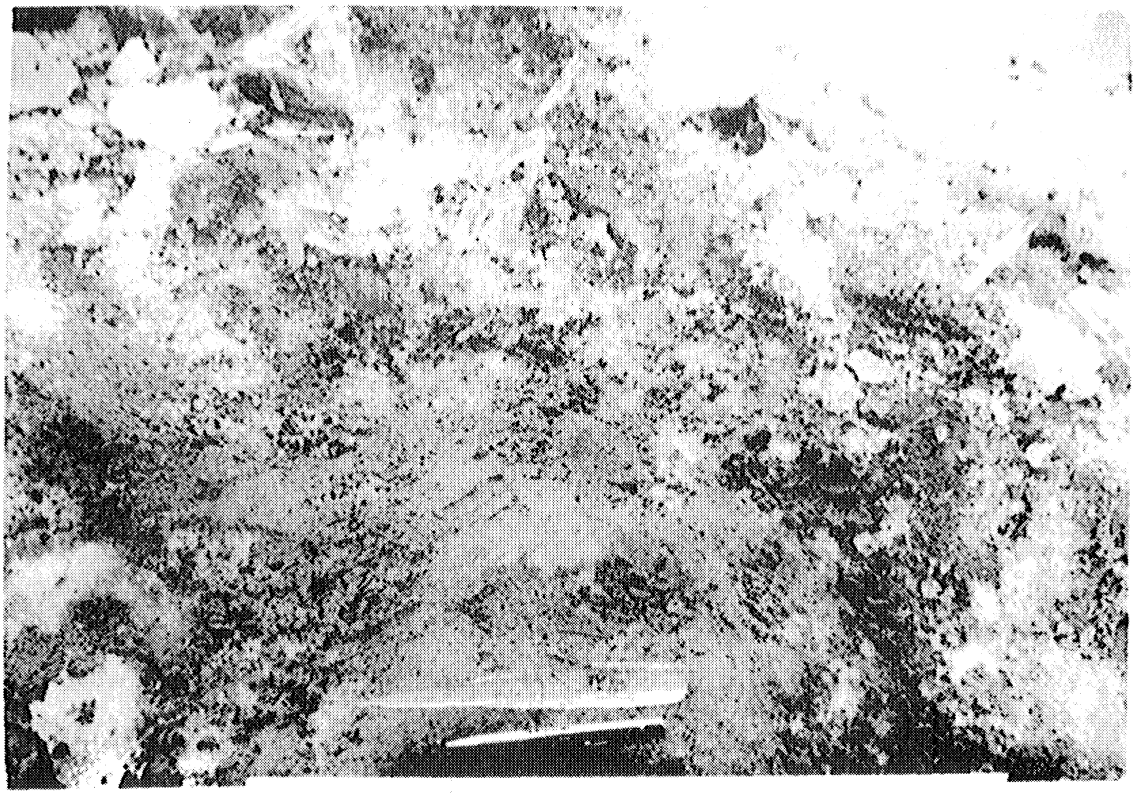


Fig. 1. Coarsely crystalline columnar aggregate from Lake Huron (Rogers City, Michigan). Drift ice blown up onto the beach had candled due to solar radiation; a slight blow disintegrated the ice sheet into individual crystals and bundles of crystals.



Lake Huron, Fig. 2. A single columnar crystals from enlarging with depth.

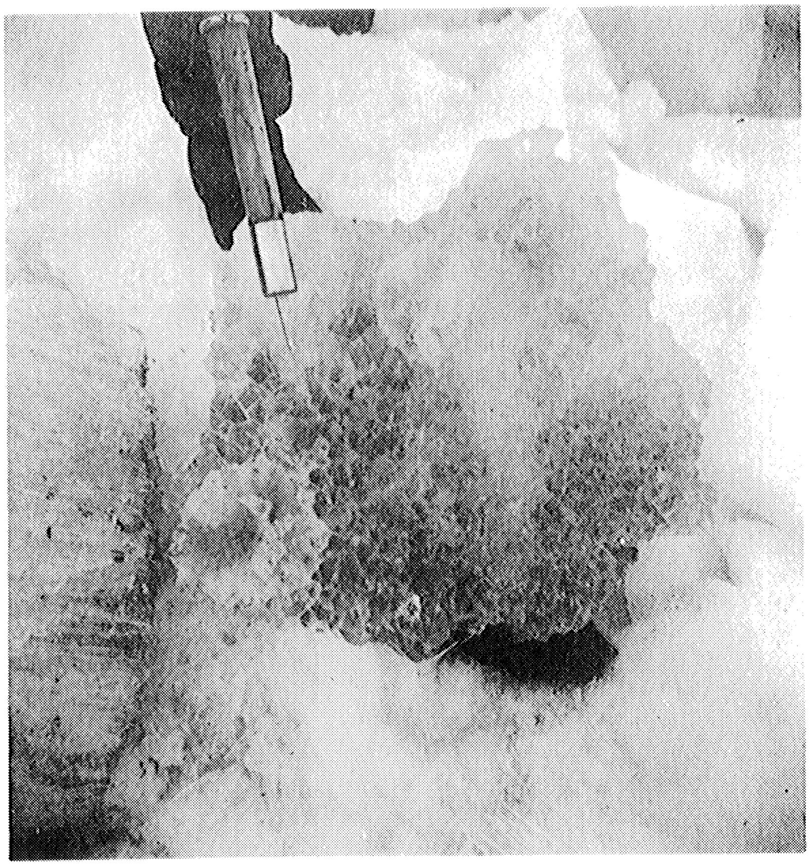


Fig. 3. Porphyritic texture, from Lake Michigan near Rogers City.

Fig. 4. Tabular ice crystals, from Wamplers Lake. The light, linear lines delimit crystal boundaries. Ice thickness 2-1/2 inches.

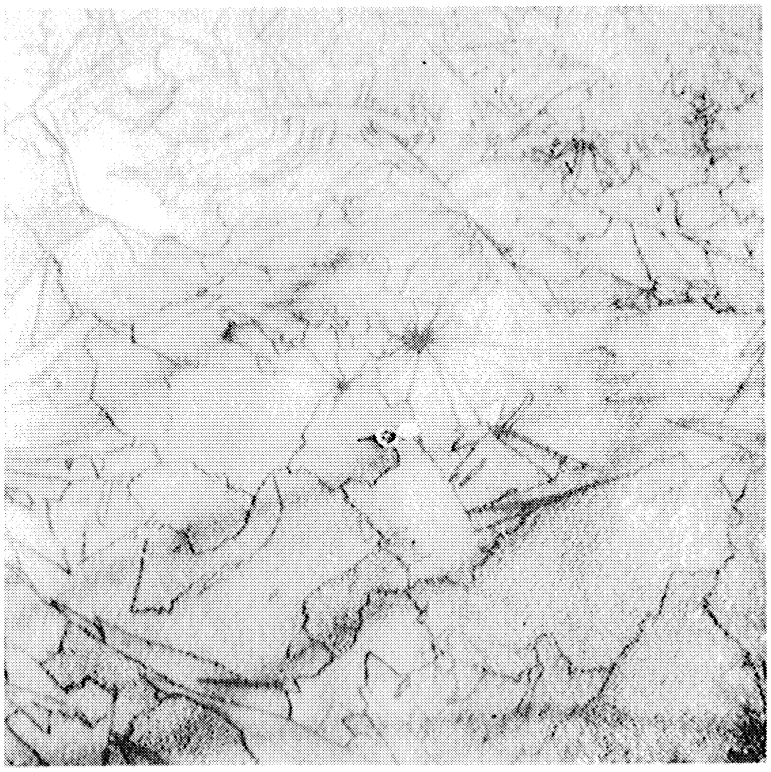


Fig. 5. Photograph of ice surface showing crystal boundaries, from Wamplers Lake. Note Brunton Compass for scale.

Fig. 6. A vertical photograph of the ice surface of Wamplers Lake showing large tabular crystals.

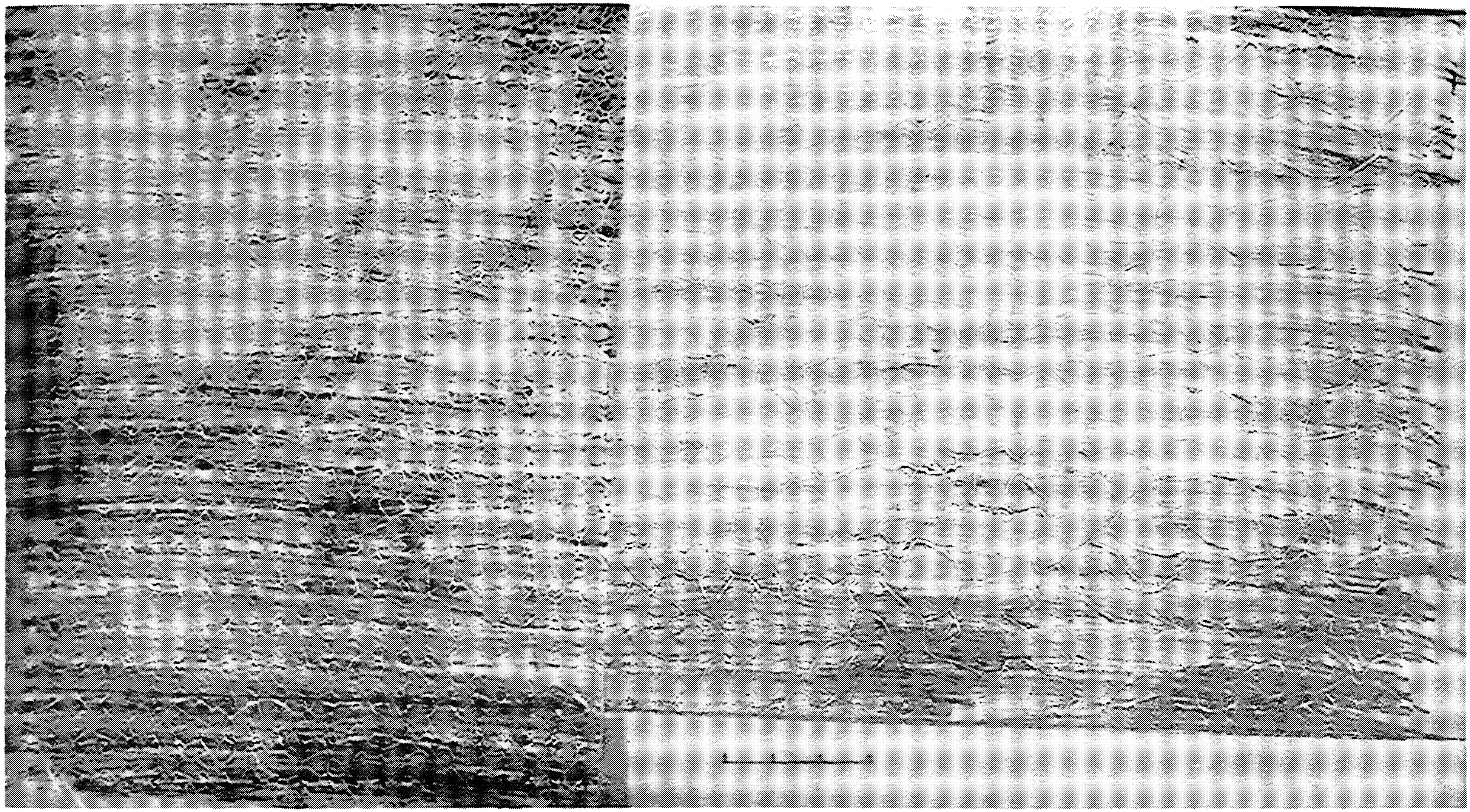


Fig. 7. A photograph of a rubbing of the upper surface of a specimen from Evans Lake. The divisions of the scales are centimeters. This is the specimen for which the size data are given in Histogram V.