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FACILITY PLANNING - STATE OF THE ART

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I. INTRODUCTION

Facility Planning has been practiced by man since prehistoric times. The search for a strategic location for shelter, concern for craftsman-like construction, the insistence on proper orientation, and attention to reasonable maintenance - all these facility planning problems have been part of man's struggle to control his environment. The degree of his success is exemplified by the historian's acclaim for some of man's accomplishments: Solomon's Temple; The Hanging Gardens of Babylon; The Great Pyramid; The Gothic Cathedrals and The Taj Mahal. Each of these facilities is directed toward a spiritual need of the time. More recently, the growth of concentrated industrial activities has emphasized a different objective of facility planning: the achievement of optimum productive or economic efficiency. A comparison of today's economic facilities on esthetic and economic grounds with those of one hundred years ago is striking. The textile mills built in 1860 are unable to compete with the more efficient mills of 1960. The foundries of today no longer resemble their Stygian counterparts of a hundred years ago. Similar changes have taken place in offices, farms, warehouses, banks; the art of Facility Planning continues to develop and adjust to the changing needs of man.

In this monograph, we focus our attention on Facility Planning for economic activities. Although our emphasis is on manufacturing facilities, many of the ideas examined have equal validity in service

activities (hospitals, cafeterias, libraries, offices, etc.). Indeed, it is hoped that this monograph will dramatize the fact that many of the basic principles of manufacturing Facility Planning are common to all Facility Planning activities, and that a unified body of Facility Planning knowledge does exist, or can be uncovered. These principles should be independent of the physical hardware characteristic of the particular system or process under consideration. We emphasize in our development therefore, those principles and ideas which can be considered in the abstract and applied to non-economic facilities, such as homes, churches, and schools, as well as to manufacturing facilities.

For this purpose, we define Facility Planning as the synthesis of physical equipment, human elements, and structures into an integrated system for efficiently furnishing designated services or for manufacturing specified products. The scope of such a system is necessarily broad. Simultaneously, the esthetic impact, safety, organizational features, supervision, employee or customer comfort, lighting, local customs, and information gathering, must be considered for an integrated design. In an effort to limit the content of this monograph, considerations of methods and work place layout, applied psychology, data and information systems, production and inventory control systems, organization and processing problems will be omitted. The implication that these can be overlooked in the practice of Facility Planning is not intended. These aspects of industrial engineering affecting Facility Planning will be mentioned when needed, but the reader is referred to the other writings for details of their considerations. Emphasis in this monograph is placed

on problems of planning related to location, process capability, material flow characteristics, layout, and maintenance of systems of physical facilities and equipment.

The literature of plant location, plant layout, material handling, maintenance, and construction is voluminous and spans many years. Familiarity with the classical (or descriptive) literature in these fields will be assumed and only minimal reference made to it in this monograph. The few books and articles of this descriptive type referenced have been selected because of their recent publication or relevance, and their superiority or definitiveness is not thereby implied. The reader however, will find these works extremely useful in helping him appraise the present state of the art. If references to literature before 1960 are desired the books and articles mentioned will furnish additional direction.

Concepts of usefulness to the Facility Planner are appearing in many diverse places at an increasing rate. Some of these concepts, the use of simulation for example, have already established themselves as practical aids when applied to parts of the Facility Planning activities. Others, such as servo-theory, for example, up to this writing hold promise for future application or for furnishing needed insight into the behavior of the physical systems under consideration. The writer admits to human fallibility in his selection of concepts with potential use in Facility Planning. Nevertheless, further consideration of many of these concepts cannot help but improve the validity and increase the number of principles available to Facility Planners twenty

years hence. With this expectation therefore we examine in this monograph those concepts which seem potentially useful to the Facility Planner, even though their immediate relevance may be tenuous. We will frequently lean toward theoretical rather than toward applied problems, confident that development of, and insight into, a theory of Facility Planning is the best way to improve the application of Facility Planning in future decades.

In presenting the developments of proven or implicit value to Facility Planners, this monograph is intended to be suggestive, rather than comprehensive. The references cited are frequently only representative of their field, and the interested reader is encouraged to investigate more exhaustively on his own. The particular references have been selected primarily because of their implied relevance to Facility Planning and do not necessarily constitute the best reference for the individual needing a basic introduction to their topic. For these individuals we have appended a bibliography which might serve as a basis for study.

A sampling of reports from industry yields the impression that the majority of Facility Planning is deeply rooted in traditional and empirical methods. It may well be that competitive or proprietary motivations dissuade many firms from reporting work of significance which contributes to Facility Planning as an engineering technology. It is hoped that this monograph may serve to encourage more open communication among Facility Planning engineers. Possibly many of the concepts suggested in this monograph have been applied with marked success or

failure. Perhaps others will be tried and also found useful or deficient. Exchange of this knowledge will certainly accelerate the expansion of the basic techniques of facility planners and lead to more economic planning in the future. Comments regarding the topics of this monograph are therefore solicited. If interest is sufficient, a supplemental issue will be compiled from these comments.

The presentation will draw upon many of the writings from areas unfamiliar to the practicing industrial engineer and may occasionally assume a forbidding mathematical appearance. The writer hopes that his readers will agree that the use of mathematical notation at certain points does clarify the monograph. Certainly the precise specification of reported work avoids ambiguity. The reader, it is hoped, can more fully appreciate the state of the art and needed extensions if these are precisely stated. Detailed proofs or solution procedures are avoided; these may be found in the original article listed in the bibliography at the end of the monograph.

The monograph is divided into topics each of which bears some relation to the others. For example, the problem of plant location is similar in many respects to the problem of locating machines in a manufacturing plant. The transportation of materials between plant locations shares some of the same routing and scheduling difficulties which characterize intra-plant material handling. The interrelation between facility location and material handling planning is well known, yet we find it necessary to look at these topics separately before considering ways of synthesizing an overall system design. The reader is aware

that progress in Facility Planning has yet to approach the point where all factors can be considered explicitly and simultaneously. As we shall see, however, the number of factors which can be considered is increasing. The sub-division of topics in this monograph therefore is an expository convenience, not desirable in practice. In fact, many benefits may derive from astute application of ideas from several topics to the same Facility Planning problem.

To be more explicit about the nature of the Facility Planning problems, we attempt an operational classification of the variables which are relevant to the field. There are several reasons for doing this. First, the extremely large number of forms which these variables can assume, or can be assumed to possess, will become apparent. Second, our later presentations of existing quantitative techniques will be more meaningful when they can be viewed in relation to a comprehensive classification. Thus the scope of the techniques, as well as ways of extending them to include additional or different forms of the variables, can be readily inferred by the reader.

Our structure or classification system treats facility planning as an abstract operational system, deriving from the concept of a network of service facilities and queues. This classification results in emphasizing the operational or dynamic characteristics of the facility rather than the physical or engineering features. This does not in any sense imply that the physical engineering aspects of facility planning are to be ignored. It does suggest, however, that they are but one set variables (albeit important) to be considered in

synthesizing a complete facility design. Our classification system conceives of all facilities as composed of networks of four broad categories, (1) the fixed facilities such as building and process equipment, (2) the product flow network, (3) a network of feasible paths for materials handling equipment, including humans; and (4) networks for communication of messages. Within each category the usual facility has multiple networks. For example there may be many products, or alternative material handling means, or alternative ways of communicating orders. Furthermore, when visualizing the net some nets may be partitioned or severely truncated if for example the product undergoes a complete metamorphosis, and then later returns to its original state. If we consider each network individually we can think of it as a queueing network, with homogeneous units of products, messages, or people moving through it. Each network would exhibit specific characteristics of inputs to each of its service facilities; methods of coping with waiting lines or arrival inputs to each service facility are classified and the characteristics of the service facility itself must be described. Hence we see that each stage of each network of each category will have particular input, line, and service characteristics. We outline a few factors which must be considered in completely describing such a facility planning system.

I. Service System Characteristics

A. Fixed (Processing) Nodes

1. Service space characteristics
 - a. Input capabilities
 - 1) Physical
 - 2) Chemical
 - 3) Location of inputs
 - b. Transformation capabilities
 - c. Output capabilities
 - 1) Physical
 - 2) Chemical
 - 3) Location of outputs
 - d. Node polarity weights (location relationships)
2. Equivalence among nodes (parallel nodes)
3. Service time characteristics
 - a. Constant or stochastic
 - b. Batch, discrete, or continuous
 - c. Linear or non-linear service time
 - d. Transient or stationary
4. Breakdown characteristics

B. Communication Nodes

1. Space characteristics
 - a. Flow direction
 - b. Parallel nodes
 - c. Input-output links to and from other nodes

- 1) Locations
 - 2) Physical state of message
 - 3) Information content
 2. Service time characteristics (see C 2 a)
 3. Breakdown characteristics
- C. Mobile (Material Handling) Nodes
1. Mobile node space characteristics
 - a. Direction
 - 1) Uniformly distributed over area
 - 2) Uni-directional
 - 3) Bi-directional
 - 4) Cyclic
 - b. Number of parallel nodes, transient or stationary
 - c. Transition links to and from other nodes
 - 1) Locations in space
 - 2) Physical and chemical capabilities
 - d. Node cost characteristics
 - e. Space coordinates of path
 2. Mobile node time characteristics
 - a. Service time distributions
 - 1) Constant or stochastic
 - 2) Batch, discrete or continuous
 - 3) Linear or non-linear service time
 - 4) Transient or stationary
 - 5) Function of location or space
 - b. Independent or cooperative service function
 3. Breakdown characteristics

II. Characteristics of Inputs to Fixed, Mobile, and Communications Nodes

A. Number of Different Inputs to Each Node Class

B. Flow Path Characteristics of Each Input

1. Sequence of transformations required for each input

a. Physical and chemical characteristics

2. Channel fusions (assemblies)

3. Channel splits (disassemblies)

C. Arrival Time Characteristics of Each Input

1. Deterministic or stochastic

2. Transient or stationary

3. Batch, discrete or continuous

III. Characteristics of Lines Before Fixed, Mobile, and Communications Nodes

A. Space Characteristics

1. Infinite inventory space

2. Finite inventory space

3. No inventory with blocking

4. Unpaced belt sequencing

5. Parallel inventory or alternative lines

B. Discipline

1. Scheduled

2. Priorities

3. First come, first served

4. Random

GENERAL BIBLIOGRAPHY

1. Ackoff, R. L.(Ed.), Progress in Operations Research, Vol. I, John Wiley & Sons, (1961) p. 106-210.
2. Apple, J. M., "Lesson Guide Outline On Material Handling Education", Material Handling Institute, Pittsburgh 22, Pennsylvania (1961).
3. Buffa, E. S., Modern Production Management, John Wiley & Sons, New York, (1961).
4. Charnes, A., and Cooper, W., Management Models and Linear Programming, Vol. I & II, John Wiley & Sons (1961).
5. Dantzig, G., Linear Programming and Extensions, Princeton (1963).
6. Demyanyuck, F. S., Principles of Flow Line Automated Production, Macmillan (1963).
7. Gass, S. I., Introduction to Linear Programming, John Wiley & Sons.
8. Ireson, W. G., Factory Planning & Layout, Prentice Hall, New York, (1952) p. 69-73.
9. Jernstedt, G. W., "Prepare Now for 1965", Automation, Vol. 9, No. 1, (Jan. 1962) p. 48-53.
10. Moore, James M., Plant Layout and Designs, Macmillan & Co., New York, (1962).
11. Morris, Wm., Analysis for Material Handling Mgt., Irwin & Co., Homewood, Illinois, (1962).
12. Morse, P., Queues, Inventories, and Maintenance, John Wiley & Sons.
13. Muther, Richard, Practical Plant Layout, McGraw Hill, New York, (1955).
14. Muther, Richard, Systematic Layout Planning, Industrial Education Institute, Boston 16, Massachusetts, (1961).
15. Reed, Ruddell I., Plant Layout, Richard D. Irwin Inc., Homewood, Illinois, (1961).
16. Reinfield, N. V., Vogel, W. R., Mathematical Programming, Prentice Hall, (1958).
17. Saaty, T., Elements of Queueing Theory, McGraw Hill Co., New York (1962).
18. Vajda, S., Mathematical Programming, Addison Wesley, (1961).
19. Woollard, F. G., Principles of Mass and Flow Production, Iliffe (1954).

II. PLANT LOCATION

The subject of plant location continues to be studied on what may be classified as three levels: theoretical, descriptive or applied, and analytical. In this monograph we will briefly review the available source materials of theoretical work (Section 2.1), descriptions of practice or recommended methodology (Section 2.2) and then investigate in greater detail examples of analytic (or mathematical) efforts to precisely define and solve some types of plant location problems (Section 2.3). The material is more suggestive than complete, the objective being to achieve brevity and yet hopefully sufficient detail for the reader to acquire a "feel" for the current state of theory and application, supplemented with sufficient references to permit the interested to continue their investigations.

2.1. Plant Location Theory

A comprehensive theory of plant location has been under development by economists for many years. There are two reasons for examining this theory: 1) it should serve as a conceptual framework for plant location studies in practice and 2) the pattern of development utilized may be useful for suggesting the manner in which a theory of plant layout should develop. The economic theories rest on the principle of substitution, that is the particular resources inherent in an available plant site are to be substituted, or balanced, against the resources available in alternative or competing sites in order to find the maximum cost advantages. An extensive development of the following ideas will be found in Greenhut.⁽³⁷⁾

In 1875, Von Thünen formulated a model of a fixed market point which supplied a surrounding agricultural area with manufactured goods in exchange for agricultural produce. The profit obtained for the produce is reduced by the transportation costs to the market, a function of distance from the market. The land rent decreases as the distance from market increases. A difference in labor costs for different locations appears as variations in rent. Von Thünen thus substitutes land cost for transportation cost to obtain the optimum product for a given site.

In contrast, Weber's location theory of 1909 treats three factors of locations: transportation cost, labor cost, and agglomerating forces. If fuel and raw material costs are different among locations they are included in transportation costs. Other things being equal, the location with overall lowest transportation cost is preferred. The labor cost factor however may exert a stronger location pull than transportation. And thirdly, agglomerating forces, e.g., industrial services, market outlets, or economics of size may counteract transportation or labor factors. More explicit consideration of costs and the influence of taxes and climate or labor factors was developed by Hoover in 1948.⁽⁴⁰⁾

The location theories implicitly assume an unlimited market for each firm, and hence explicitly avoid references to the spatial separation which may affect the demand curves of markets. Nevertheless, as Greenhut points out, the assumptions about interdependence of markets and location fit the framework if a firm selects sites on the basis of profit potential rather than minimum cost. The secondary effects of demand created by the firm's presence in a given location are not included.

Greenhut comprehensively develops the factors relevant to plant location under four broad headings 1) transportation, 2) processing costs, 3) demand, and 4) cost reducing factors. Their importance in plant location decision of eight firms is reported in detail.

A theory for plant location should incorporate not only immediate cost or profit considerations of the firm, but also the interdependency of these considerations with competitor's location and marketing behavior as well as the dynamic changes which occur in regions, nations, or internationally, insofar as they affect the firm. In this sense, population movements, money flows, regional resources, and restrictions, are relevant to plant location in both theory and practice. A comprehensive development of regional science theory and methodology is found in Isard.^(42,43) A chapter of perceived research needs in the field is also included.

2.2. Description of Plant Location Practice

The traditional literature on plant location is in close agreement with the economists' theory of plant location in its selection of factors for consideration in a location study. The factors can be readily categorized into the four headings used by Greenhut for example. In this sense most such articles add little to the state of the knowledge in the field. They do however serve to translate the language of the economist into specific recommendation couched in industrial terminology, and often in the context of a particular industry. Characteristic of some of the briefer articles is that of Smith⁽⁵⁵⁾ who advises consideration of the following factors:

1. Raw materials supply
2. Market location
3. Power and fuel supply
4. Climate
5. Water Supply
6. Waste disposal
7. Transportation facilities
8. Taxes
9. Regulatory Laws and Practices
10. Labor availability and rates
11. Proximity to other industries
12. General surroundings
13. Site characteristics
14. Living conditions

He suggests the following sequence of steps in a complete location study:

1. Determine those four factors which most clearly distinguish regions.
2. On the basis of those factors select the best region.
3. Within the region find the best location based on secondary factors.
4. Get outside aid (Area development groups, Chamber of Commerce etc.)
5. Appraise available sites.
6. Review and select final site.

Crouch⁽³⁰⁾ is more detailed in his development of considerations for selecting the specific site. These include:

1. Site size
2. Transportation
3. Utilities
4. Highways
5. Zoning
6. Taxes
7. Title check

In brief, there are a large number of sources where the practitioner can obtain instructions on good procedure. A typical group is listed in the bibliography. ^(31,35,39,41,44,53,56,57,58,64,65) In addition, articles specific to special location problems are common: warehouses, ⁽⁶³⁾ food industry, ⁽³³⁾ chemical plants⁽⁶⁰⁾, environmental effects, ⁽²⁰⁾ transportation, ^(21,28) and taxation. ⁽²⁷⁾ The Survey Research Center ⁽⁵¹⁾ reports the results of an attitude survey of Michigan and Ohio manufacturers. When asked to choose from a list of 20 the five most important factors in locating plants, the most frequently selected were labor costs, proximity to customers and markets, availability of skilled labor, industrial climate, the tax bill, and proximity to materials. Thus general agreement on the significant factors in plant location seems implied by both economists and industry. The measurement or analysis of such factors however is still a challenge to both groups.

2.3. Analytic Plant Location

In this section we first look at some of the abstract representations of location problems and the proposed methods of solution, given that needed data were available. Reference to many of the analytic tools such as "gravity weighting", factor analysis, and input-output analysis which are described by Isard⁽⁴³⁾ will be omitted. Second, we examine a selected set of location studies which incorporate ideas attempting to extend beyond a straight forward classical cost analysis of alternative locations.

Analytic location models are generally directed to determining a location of one or more activities in order to optimize some spatially dependent function. Repeated application of the transportation algorithm for example, permits a comparison of the minimum transportation costs for a finite number of known alternative facility locations relative to some given fixed locations. Section 4.11 develops this and related models in more detail. If a large number of alternative locations are to be considered, the straightforward approach may be prohibitive without some rules for deciding which locations are more desirable than others, and thus proceeding in a rational fashion to try only those alternatives which are at least as good as the current locations. Thus, as the number of available alternatives approaches infinity the problem generalizes to finding the optimal points on a map for one or more sources which supply destinations at specified fixed locations. The simplest form of the problem assumes that one source must supply known quantities to m destinations, where the costs of

shipment per unit distance are a linear function of the volume on that route. Miehle⁽⁸⁰⁾ describes three methods of solution: 1) mechanical, 2) numeric, 3) soap film for the more general case of n sources.

Another mechanical analog is described by Brown⁽²⁶⁾ with more explicit consideration to plant location problems. McHose⁽⁴⁸⁾ points out the useful expedient of a quadratic formulation of the cost-distance function since it can be solved explicitly rather than by analog or approximation. A more general statement of the problem is examined by Wester and Kantner⁽⁶¹⁾ and Brink and de Cani.⁽²⁵⁾ Wester and Kantner give the following statement: "given:

- a) a set of n sinks (destinations) with coordinates (x_j, y_j) , $j = 1, \dots, n$ and
 - b) the requirements (r_j) of these sinks $j = 1, \dots, n$
- find
- 1) the locations (X_i, Y_i) of m sources $(i = 1, \dots, m)$
 - 2) the flows (f_{ij}) from the m sources to the n sinks, $i = 1, \dots, m$ $j = 1, \dots, n$
 - 3) the capacities (k_i) of m sources $i = 1, \dots, m$
- such that the cost function

$$C = \sum_i \sum_j f_{ij} C_{ij}$$

is minimized, subject to the restrictions

$$\begin{aligned} \sum_i f_{ij} &= r_j \\ \sum_j f_{ij} &= k_i \\ f_{ij} &\geq 0 \end{aligned}$$

A usual cost formula could be the two dimensional distance formula

$$c_{ij} = \sqrt{(X_i - x_j)^2 + (Y_i - y_j)^2}$$

The solution is to be obtained on digital computer by iterative application of the transportation algorithm, an application of concepts of static equilibrium of forces acting on a point to determine the location, and finally an allocation of needed capacities to achieve this optimal location. Brink and de Cani describe an analog computer for quickly solving such problems. In subsequent work⁽⁶²⁾, Wester defines a location-allocation model in which the requirements r_j at the m destinations are a function of time.

A number of interesting application of analysis to plant location studies are reported. An early paper by Bowman and Stewart⁽²⁴⁾ examined the territory size and number of warehouses needed by a firm in a specific region. Using linear regression, a model predicting total cost as a function of area served, volume handled, fixed costs of operation, variable costs of distribution, and area-dependent costs of distribution was obtained. Baumol and Wolfe⁽²³⁾ consider the following problem: given m factories, n warehouses, and q retailer customers, cost of shipment from factory to warehouse and also from warehouse to retailer, and the number of product units available at each factory and required by each retailer. The warehouses are rented and hence costs of warehousing depend upon the particular set and number of warehouses used. A suggested iterative procedure using the transportation algorithm permits finding locally optimal solutions. Barache⁽²²⁾ reports the results of an extensive study of the distribution system for food products. The distribution network, and placement of depots for minimum cost were evaluated on the basis of (1) a fixed

plus variable cost of transport from the factory to the depots; (2) a linear function relating total tonnage through a depot to cost of handling, and (3) costs of delivery from the depot to the customer which is a function of the two variables: distance travelled inside and outside a city. The study technique considered iteratively the effect on the total delivery costs of addition or removal of a depot in the network. A related study of warehousing for petroleum distribution by Wanty⁽⁵⁹⁾ investigates the number of warehouses, the location of each, the products to be stored in each, the capacity required and the area to be covered. A step by step description is given of the approach used during the more than ten men - months of effort. Clapham⁽²⁹⁾ makes a plea for simpler models, such a Bowman's, for the initial location studies and for small businesses. He treats three case studies 1) economic feasibility and location of an amalgamated fruit packing house to serve a region, 2) location of a single center to distribute a product to a regular group of customers on a regular schedule and 3) simulation of truck delivery scheduling with alternative numbers of dispatch points to minimize cost in the face of stochastic demands. A study specific to location of classification yards for railroads' is presented by Mansfield and Wein,⁽⁴⁶⁾ but the quadratic cost criteria used are proposed for possible application to other transportation and location problems. The following is an abstract of a paper by Goldman and Bender⁽³⁶⁾ relating to Post Office Department problems:

"The optimal locations of sorting centers are determined, and the degree of centralization investigated for the following mathematical model of a distribution network. The area served by the network is a square which is a uniform source and sink of the distributed items. It is divided by lines parallel to its sides into subregions, each with a single sorting center. Transportation costs are subject to economics of scale, and sorting costs are also taken into account."

Finally, we quote from McGillivray⁽⁴⁷⁾ on a warehouse location study:

"The objective was to develop a plan for storage facility expansion that would also be useable in the future to take advantage of changing conditions. The evaluation considered distribution systems utilizing:

1. Plant Consolidation Warehouses.
2. Consolidation Warehouses in producing area.
3. Sales warehouses adjacent to major consuming area.
4. Sales warehouses intermediate between producing and consuming areas.
5. Combination of above.

The locations considered away from producing areas were arbitrarily selected on the basis of geographical location in relation to consuming areas. The usual pertinent data for site selection was gathered for the 23 cities selected. Rail service, storage-in-transit privileges from packing areas to destinations (56 were used), and rail transit times from sites to destinations were of primary importance.

Specifically, the relative merits of the possible combinations were measured in terms of three criteria:

1. Cost.
2. Delivery time (from location to destination).
3. Product Mix.

The method of analysis developed for the study was based on the concept of the efficient surface.

To aid in the analysis of the large number of systems possible and to select points on the efficient surface, a computer program for the IBM 650 was written and used for:

1. Generating combinations.
2. Determining the least cost method of serving each destination.
3. Determining the average delivery time for serving all areas in each system.
4. Determining the average product mix of each combination.
5. Selecting the lowest cost combination of warehouses that would provide each particular level of service as measured by product mix and delivery time.
6. Providing detailed distribution plans for each of the lowest cost combinations.

Over 18,000 combinations were analyzed after certain assumptions were established to reduce the computational volume."

PLANT LOCATION BIBLIOGRAPHY

20. "Environment & Industry; Report on the Princeton Conference", Arch. Rec. Vol. 124, (July, 1958) p. 159-62.
21. Ball, C. and Teitz, M., "Expressway & Industrial Location", Traffic Quarterly, Vol. 12 (Oct., 1958) p. 589-601.
22. Barache, J., "A Study to Determine the Optimum Distribution Network For Food Products", Rev. Recherche Operationelle, Vol. 2, No. 6, (1958) p. 32-47; also Quality Control Abstracts, Vol. 5, No. 3, (1960) p. 267.
23. Boumol, W. J. and Wolfe, P., "A Warehouse Location Problem", O.R., Vol. 6, (1958) p. 252.
24. Bowman, E. H. and Stewart, J. B., "A Model for Scale of Operations", Journ. of Marketing, (Jan., 1956) p. 242-47; also Bowman and Fetter, Analysis for Production Management, R. D. Irwin & Co., Homewood, Ill., (1957) p. 248-255.
25. Brink, E. L. and de Cani, John S., "Analog Solution of the Generalized Transportation Problem", Proceedings of the First International Conference on Operations Research, Oxford (1957).
26. Brown, Harry, Solution of a Class of Transportation Problems, American Mach. & Foundry, 689 Hope St., Springdale, Conn., (Jan., 1961) Technical Report, CRL-T-37.
27. Burkhead, J. and Steele, D. C., "The Effect of State Taxation on the Migration of Industry", Journal of Business, Vol. 23, (Nov., 1949), p. 414-422.
28. Chintz, B., "Effect of Transportation on Regional Economic Growth", Traffic Q., Vol. 14, (April, 1960) p. 129-142.
29. Clapham, J. C., Number & Location of Plants to Serve a Region, Mimeo Paper Presented at Twentieth National Meeting of Op. Res. Soc., (Nov., 1961).
30. Crouch, C. W., "Plant Site Selection Demands Careful Study", S.A.E. Journal, Vol. 67, (June, 1959) p. 104-5.
31. "Site Selection", Factory, (May, 1960) p. 180-206.
32. Florence, P. Sargent, Investment Location and Size of Plant, National Institute of Economics & Social Research, Study VII, Cambridge; Cambridge University Press, (1949).

33. Forbes, J. H. and Riches, C. V., "New Technique Averts Site Selection Hazards", Food Eng., Vol. 30, (April, 1958) p. 59-62.
34. Fulton, M., "Plant Location, 1965", Harvard Bus. R., Vol. 33, (May, 1955) p. 40-50.
35. Fulton, M., "Where to Locate Your Plant", Am. Mach./Metalworking Mfg., Vol. 104, (Dec. 12, 1960) p. 121-7.
36. Goldman, A. J. and Bender, B. K., Optimization Models of Distribution Networks, Applied Mathematic Division National Bureau of Standards, Washington, D.C., Paper No. 413; 13th Nat'l. Meeting ORSA.
37. Greenhut, Melvin L., Plant Location In Theory and In Practice, Chapel Hill, University of North Carolina Press (1956).
38. Hammond, R. A., "Reducing Fixed Plus Variable Distribution Costs", Mgt. Tech., 3 (Dec. 1963).
39. Hay, R. F., "38 Check Points for Plant Locating", Sales Management, Vol. 79, (July 10, 1957) p. 115-16.
40. Hoover, E. M., Location of Economic Activity, McGraw Hill, New York, (1948).
41. "Site Selection Handbook", Ind. Dev. & Mfg. Rec., Conway Pub., Inc., Baltimore 2, Md.
42. Isard, W., Location & Space Economy, Wiley & Sons (1956) p. 350; Reviewed in Mgt. Sci., Vol. 4, 2 (Jan. 1958) p. 197.
43. Isard, W., Methods of Regional Analysis, An Introduction to Regional Analysis, Technology Press and Wiley & Son, New York (1960) p. 784.
44. Larke, A. G. and Blake, J. K., "Location Analysis; Business and Industrial Locations", Dun's R. & Mod. Ind., Vol. 67 (April 1956) p. 59.
45. Koopmans, T. C. and Beckmann, M., "Location of Economic Activities", Econometrica, 25 (1957).
46. Mansfield, Edwin and Wein, Harold H., "A Model for the Location of a Railroad Classification Yard", Mgt. Sci., Vol. 4, (1958) p. 292.
47. McGillivray, W. T., Private Correspondence (Feb. 1962) California Packing Co, San Francisco, Calif.
48. McHose, Andre, H., "A Quadratic Formulation of Activity Location Problem", Journ. I.E., Vol. 12, No. 5 (Sept. 1961) p. 334-337.

49. Morgenstern, O., Economic Activity Analysis, Wiley & Son, (1954).
50. Moses, L. N., "Location Theory, Input-Output, and Economic Development; An Appraisal", R. Econ & Stat., Vol. 37 (August 1955) p. 308-12.
51. Mueller, E., Wilken, A., and Wood, M., Location Decisions and Industrial Mobility in Michigan, Institute for Social Research, University of Michigan, Ann Arbor, Publication No. 22, (1961).
52. Root, G., "Symbolic Logic Applied to a Selection Problem", O.R. 12 (July 1964).
53. Roterus, V. and March, S. R., Industrial Development and Plant Location, Washington, D.C., U.S. Dept. of Commerce (1950).
54. Smidt, R. and Reis, I. L., "Symbolic Logic and Plant Location", Journ. I.E., 14 (Jan. 1963).
55. Smith, S. P., "Selections of a Plant Site", Chem. Eng. Prog., Vol. 51 (March 1955) p. 134-8.
56. Stewart, O., "Plant Site Selection Guide", Factory, Vol. 115 (May 1957) p. 180a-130f.
57. "Techniques of Plant Location", Studies in Business Policy No. 61, National Industrial Conference Board, (1953).
58. "Basic Industrial Location Factors", U.S. Dept. of Commerce, Ind. Series No. 74, Area Development Division, Washington D.C., U.S. Gov. Printing Office (1947).
59. Wanty, J. R. M., "Location of Depots in a Petroleum Co.", Op. Res. Qtrly., Vol. 9, No. 2 (June 1958) p. 149.
60. Wessel, H. E., "Locating Tomorrow's Chemical Plants", Oil and Gas Journ., Vol. 57, (July 13, 1959) p. 106-8.
61. Wester, Leon and Kantner, Harold H., Optimal Location-Allocation, Paper presented at 6th Annual Meeting of ORSA (May 16, 1958).
62. Wester, Leon, Location Allocation Models With Time Dependent Requirements at Destinations, Paper presented at 7th Annual Meeting of ORSA (May 14, 1959).
63. Willard, E. W., "How Many Warehouses & Where?", Material Handling Engineering, Vol. 16, No. 11 (Nov. 1961) p. 59-61.
64. Yaseen, Leonard C., Plant Location, New York American Research Council (1956) (Revised, 1960).
65. Yaseen, Leonard C., "Ten Biggest Pitfalls in Plant Locations", Dun's R. & Mod. Ind., Vol. 69 (March 1957) p. 49-50.

III. FACILITY PLANNING AND CAPACITY ESTIMATION

This section is sub-divided into the following four topics:

- 3.1. Cost and Service Time Estimation
- 3.2. Capacity Estimation
- 3.3. Process Selection
- 3.4. Line Balancing

All facility planning must necessarily rest upon estimates or forecasts of future demand for the services for which the facilities are provided. The study of methods for long range planning is rapidly broadening in scope but is outside the subject matter of this monograph. The closely related area of forecasting likewise has an extensive substantive content of its own which is not explored in this monograph. Instead, our attention is directed towards those considerations which may ordinarily fall to the planning engineer rather than to the entrepreneur or administrator. This by no means implies the lesser importance of the setting of strategic goals and entrepreneurial objectives of the firm, but is rather an attempt to restrict this monograph to reasonable scope. This section emphasizes reported methods for estimating cost and service time characteristics of fixed facilities, for analyzing the capacity and expansion timing of overall as well as individual equipment, and finally to point out the relationship between capacity and assembly line balancing. With reference to the classification characteristics established in Section 1 of this monograph, the subject matter of this section is centered on the characteristics of the fixed facilities service system. The capacity of the facility is clearly

a function of the service time characteristics, the number of parallel nodes, the breakdown characteristics, as well as the arrival time characteristics of the inputs and the queue discipline of the input lines. We assume the reader is familiar with traditional methods of measuring standard process times, cost estimating, and of calculating the number of machines required to meet a desired production rate. Our presentation, therefore, emphasizes the recent and unique developments reported in current periodicals which we believe have implications for future facility planning.

3.1. Cost and Service Time Estimation

The use of multi-variate statistical regression is a recognized method for predicting a dependent variable from a number of significant independent decision variables. Its application to estimating special equipment or operating costs should be rewarding, but reports of such applications are unknown to the writer. A non-statistical method of estimating equipment costs which is closely related to regression analysis but lacking in statistical consistency, is proposed by Dexter.⁽⁷²⁾ He suggests that variables such as machine speed, range of adjustment, size, nature of the product, etc., may be important in predicting equipment costs. Only empirical justification is given for selection of these factors. A more comprehensive investigation of a similar cost estimation problem is made by Watkins⁽⁸⁴⁾ in connection with central station boilers. Making use of engineering data covering a number of years, Watkins analyzes boiler types and designs trends in order to develop a predictive framework for boiler design parameters based on fuel cost and

boiler investment trends. He then uses this framework to predict the design of boilers for minimum cost operation under the future conditions extrapolated from his past data. The generality of his approach suggests the possibilities and limitations of the use of a similar method in cost analysis of other classes of equipment.

Obtaining the data necessary for statistical cost estimating requires careful planning, and frequently special data collection and analysis. Discussion⁽⁷⁶⁾ of some of the measurement problems encountered in determining the ultimate capacity of a slabbing mill further emphasizes the difficulties of data collection. We expect that the cost of obtaining data about plant facilities will decrease with the expanded use of data processing equipment. The IBM manual⁽⁷⁷⁾ on plant and equipment accounting indicates the ease of maintaining equipment inventory by location, by type of equipment, by year of acquisition, by department, as well as deriving maintenance and other operating expense data on punched cards. The extension of data processing to the specific problem of facility planning is clearly set forth by Monsell.⁽⁷⁹⁾ His planning information consists of four IBM punched-card decks as follows:

1. A complete list of parts needed to make each product model.
2. Operating and Facilities File, and listing by part number of the standard time for each operation on the machine or equipment used.
3. The master pricing or line-up file of job rates showing the number of pieces per hour required to meet time standards.
4. The facilities file giving the square footage of the production facilities together with in-process and raw material storage.

Additional data on idle time, set-up time, machine down-time, re-work time and spoilage are compiled. Outputs, which can be obtained from analysis at a particular production rate, are labor requirements, hours needed by male and female to do the job, a list of the unused facilities, the number of hours on each machine needed, and the square footage required to meet the scheduled rate.

In contrast to data collection and analysis which are based on single point estimates of performance times, Nelson⁽⁸⁰⁾ studies the arrival, service time, and waiting time distributions of a job shop production process. A hypothesis of Poisson arrival distributions to the total shop was not rejected and Poisson arrival distributions to individual machine centers were rejected at only one machine center. Service time distributions were best fitted by hyper-exponential distributions in some cases, and hyper-Erlang distributions in others. Theoretical waiting time properties using hypothesized distributions were compared with simulation results using observed data. Errors ranged from 3 to 10 percent of the simulated values. In view of the increased recognition of the presence of uncertainty in production processes, we believe that the future will see more data collection and analysis of the type presented by Nelson. Hopefully, the study of such analyses will lead to discovery of standard distributions for particular classes or kinds of processing equipment.

3.2. Plant Capacity

The estimation of facility capacity is fraught with difficulties, arising in part from the absence of a standard definition of "capacity". Typical of a plant wide measure of capacity is the suggestion

by Gravely to use "percent of facilities operated" based on standard data.⁽²⁵³⁾

It is well recognized, however, that the ultimate capacity of a facility can be affected by the scheduling procedures, the methods utilized, the in-process inventory permitted, the frequency of equipment failure, and even worker motivation. We may, for example, be able to define the capacity of each individual component of the facility in terms of some physical or engineering constraint. When such individual equipments are assembled into a production network, however, we know that the overall facility capacity is, in a deterministic sense, restricted by the maximal flow through the network of all connected paths. This problem is discussed in more detail in Section 4.3.1. Hunt⁽¹⁶³⁾ has effectively demonstrated the loss in capacity resulting from restricting inventories between queueing service facilities in series. Our attention in this section therefore will be directed toward estimation of system facility capacity as opposed to individual unit capacity which we assume has been determined from engineering analysis or work measurement.

The use of linear programming to determine an optimal allocation or production to fixed facilities has received wide attention in the literature.^(4,5,7,18) Fabian⁽⁷⁴⁾ presents a special application of linear programming as a model of iron and steel production which offers the possibilities of determining the least cost rate of input of materials within the constraint of available production capacity. From

the standpoint of capacity planning, the parametric dual programming formulation by Gass⁽⁷⁾ suggests a procedure for finding the range of values of capacity for which a given allocation of production is optimal. Mc Eachron⁽⁷⁸⁾ reports the inclusion of a capital charge for new facilities of variable size in a linear programming model. Thus the optimum size of the facility considering the desired corporate rate of return on investment, as well as the optimum mode of operation of the entire system represented by the model is obtained. A similar approach is described by Bertolotti et al.⁽⁷¹⁾ Mc Eachron also reports on experimental applications of "logical set" programming. The approach first computes the best linear programming solution. It then "backs down" from this optimum solution in a stepwise fashion, yielding as little as possible as it moves toward complete compliance with the logical requirements.

The use of queueing theory for capacity planning can be a useful procedure. Stover⁽⁸²⁾ reports a case study of the application of queueing theory to chemical plant operation and expansion. Tests of the analytic queueing solution against the actual operating data gave an accurate forecast of waiting time and estimates of loss of production capacity due to alternative arrangements of facilities. Sespaniak⁽⁸¹⁾ also uses queueing theory but, in his case, for determining the number of inspectors required for an in-line assembly department. Given the average number of pieces arriving per mean inspection time, and the ratio of cost of delaying the assembly station compared to the cost of inspector idle time, Sepaniak is able to select the optimal number of inspectors for the line. Another example of queueing

theory in capacity planning is given by Ventura.⁽⁸³⁾ His problem requires evaluation of operating an ore-boat loader for additional hours, compared to an expenditure for equipment with a 50% higher loading rate. Queue equations are developed for the case of uninterrupted service, and also for the interrupted (single shift) service, and a total expected cost function developed based on the length of stay of a loading ore-boat. Costs are shown to be more sensitive to changes in working hours than in loading rate.

A more complex capacity study which did not yield to queueing theory analysis is reported by Banbury and Taylor.⁽⁶⁹⁾ The effect of delays or queues on the production capacity of a steel mill is studied by simulation. The study indicates that an increase in output could be obtained only by methods changes and scheduling priorities and that no significant increase in delay time would result.

Dille⁽⁷³⁾ describes the following problem with specific application to selecting electric power transformer sizes: "The principle problem in the efficient utilization of fixed-capacity equipment serving a growing demand is basically one of how much spare capacity to install initially over and above that required by the present load. Closely associated is the problem of what action to take when the initial installation is fully loaded." He reaches the following conclusions:

"1. The problem of capacity-limited equipment selection may logically be treated as an inventory problem, with units of capacity corresponding to stock items. 2. Equipment prices as well as other physical parameters may be expressed by continuous mathematical functions which are

useful in the model. 3. A generalized solution of an inventory model for a particular type of equipment will yield planning guide lines, that is: higher growth rates call for shorter re-order cycle time." A similar problem except described in terms of electrical generating facilities is formulated and solved under some strict assumptions by Arrow, Beckmann, and Karlin.⁽⁶⁷⁾ Determining the optimal facility capacity is pertinent in manufacturing industries as well as electric utilities, but meaningful forecasts of demand as well as useful measures of production capacity are both much more difficult to obtain in a manufacturing enterprise. Another factor, the selection of line or process type layouts, also affects the ultimate capacity of the system because of unbalanced utilization of equipment on line production. Further discussions of, and references to, this problem are given in Sections 3.4 and 5.2.

3.3. Process Selection

Although the study of processing methods is outside the main content of this monograph, it cannot be avoided by the facility planner. Amber⁽⁶⁶⁾ is outspoken in emphasizing the need for attention to the mathematical modeling of the operational characteristics of automation systems. A method for carrying out such a study is described by Banbury⁽⁷⁰⁾ in his attempt to provide a yardstick by which alternative forging techniques can be compared. Of particular interest is the fact that the study is conducted in a large number of different companies and reveals the difficulties of trying to find universal standards among firms. A number of descriptive studies of "principles" for performing operations analyses to determine the suitability of automation have been published.

Typical of these is a special report emanating from the General Electric Company.⁽⁶⁸⁾ Of more specific nature is a report by French et al.,⁽⁷⁵⁾ which gives an integer linear programming procedure for selecting optimal material handling systems when components have discontinuous cost functions. The suggested procedure also has application in the selection of alternative processing equipment which has minimum purchase plus operating cost at a pre-established operating level.

PLANNING BIBLIOGRAPHY

66. Amber, G. H. and Paul, S., "Operations Research for Computer Control", Operational Res. Qtrly., Vol. 10, No. 2 (June 1959) p. 105-109.
67. Arrow, K. J., Beckman, M. J. and Karlin, S., "Optimal Expansion of the Capacity of the Firm", Chapt. 7 in Studies in Mathematical Theory of Inventory Control, Stanford Univ. Press (1958).
68. "Automation for Small Lot Producers", Automation (March 1958).
69. Banbury, J. and Taylor, R. J., "Congestion in a Steelworks", Op. Res. Quart., Vol. 9, No. 2 (June 1958) p. 89.
70. Banbury, J. and Chelsom, J. V., "Evaluation of Possible Alternative Forging Techniques", Opr. Res. Qtrly., Vol. 10, No. 2 (June 1959).
71. Bertolotti, M. and Chapiro, J., and Rieznik, H., "Optimization of Investment - A Solution By Linear Programming", Management Technology, Monograph No. 1, (Jan. 1960) p. 64-74.
72. Dexter, R.G., "Estimating Special Equipment Costs", Automation, (May 1958).
73. Dille, E. K., Economic Selection of Capacity Limited Equipment, Mimeo Paper Presented at Joint ORSA-TIMS Meeting (Nov. 8, 1961) San Francisco, Calif.
74. Fabian, Tibor, "A Linear Programming Model of Integrated Iron & Steel Production", Mgt. Sci., Vol. 4 (1959) p. 415.
75. French, C. E., Isaacs, G. W. and Peart, R. M., "Optimizing Systems Having Components With Discontinuous Cost Functions", Op. Res., Vol. 9, No. 4 (July 1961) p. 468-78.
76. Jones, H. G., Steer, D. T. and Dickerson, P. D., "Ultimate Capacity of a Slabbing Mill", Operational Research Quarterly, Vol. 7, No. 1 (March 1956) p. 1.
77. Plant and Equipment, IBM Corp. Booklet 320-6245-0 (June 1959).
78. McEachron, W. D., Standard Oil Co. of Indiana; Private Communication, (Feb. 1962).
79. Monsell, Jack, Manufacturing Planning With Data Processing, General Electric Co. Mimeographed Paper, (June 1959).

80. Nelson, Ross T., An Empirical Study of Arrival, Service Time, and Waiting Time Distributions of a Job Shop Production Process, Univ. of Calif., Los Angeles, Management Sciences Research Report No. 60 (June 1959).
81. Sespaniak, C. D., "An Application of Queueing Theory for Determining Manpower Requirements for an In-Line Assembly Inspection Department", Journ. of I.E., Vol. 10, No. 4 (July-Aug. 1959).
82. Stover, A. M., Application of Queueing Theory to Operation and Expansion of a Chemical Plant, Maugatuck Chemical Co. Mimeo. Paper (Sept. 1956).
83. Ventura, E., "Operations Research & Port Investments", Rev. Recherche Operationelle, Vol. 2, No. 6 (1958) p. 45-58; and Quality Control Abstracts, Vol. V, No. 3 (1960) p. 273.
84. Watkins, G. E., Cost Determinants of Process Plant Design: Central Station Boilers, Unpublished Ph.D. Dissertation, Columbia Univ. (1957).
85. Westinghouse Electric Corp., Buyer's Guide-Freight Elevators Booklet B7702-B (Aug. 1956); Buyer's Guide-Passenger Elevators Booklet B4572-B (Jan. 1957).

3.4. Assembly Line Balancing

The production capacity of an assembly line can be improved by careful "balancing" of the assembly tasks among the individual workers along the line. Because balancing is dependent upon the desired production schedule, the problem typifies the importance of simultaneous consideration of scheduling and plant facility planning. The Assembly Line Balancing problem has received considerable attention in the literature which develops theoretical models as well as procedures for heuristic or non-optimal solutions to the problem. We first describe and then give an analytic statement of the assembly balancing problem. A review of procedures for solution follows.

The line balancing problem arises in progressive assembly typical of conveyORIZED production lines in the following ways. The specified product is to be assembled at a sequence of stations or by a sequence of operations, each station or operator to perform one or more assigned tasks. The tasks are technologically irreducible; that is, it is not practical or possible to devise tasks of shorter duration which could, if desired, be conveniently performed by different operators. A precedence, or order relation exists among the tasks. Some tasks must be completed before a subsequent task can be started; for example, a bolt must be positioned before it can be tightened. Others can be performed simultaneously; for example, two bolts could be inserted at the same time without technological difficulty. Because the number of stations or operators must be discrete integers, and because the task times are not usually integers, the tasks cannot generally be assigned to

stations or operators so that the total time is the same for each operator. Furthermore, as the desired rate of production changes, the number of stations or operators and/or the resulting idle time must also change. The line balancing problem then seeks to maximize the production output given a collection of assembly tasks with restrictions on the order of their performance.

Salveson's papers^(116,117) contain the first published analytic statement of the problem. The following terminology is used:

- $i = 1, \dots, I$ a set of elemental tasks
- $k = 1, \dots, K$ a set of assembly stations
- $t = 1, \dots, T$ number of units of time in the period
- a_i = number of units of time to perform task i
- q = number of units of output desired during the period
- I_k = a sub-set of tasks i , assigned to station k
- c = cycle time of the line, units of time per unit of output.

Then cycle time is:

$$c = \frac{T}{q} . \tag{1}$$

In order that balance can be achieved, it is required that all assembly task times be less than the cycle time:

$$a_i \leq c \quad (i = 1, \dots, I) . \tag{2}$$

The minimum possible number of integer stations for an assembly line is:

$$K_{\min} = \text{minimum integer} \geq \frac{\sum a_i}{c} . \tag{3}$$

The maximum number of stations results when one task is assigned to each station; therefore,

$$K_{\max} = I \quad (4)$$

One criterion (hereafter called Criterion I) of the assembly line problem is, for a given c , to minimize the total amount of idle time:

$$\sum_{k=1}^K (c - \sum_{i \in I_k} a_i)_k \quad (5)$$

subject to:

$$\text{adherence to the set up precedence requirements on the element tasks } i, \quad (5a)$$

and

$$\text{the total time units assigned to any station must not exceed the cycle time:} \quad (5b)$$

$$(c - \sum_{i \in I_k} a_i)_k \geq 0 \quad k = 1, \dots, K \quad (6)$$

A second criterion (Criterion II) is, for a given c , to minimize the total number of stations K subject to (5a) and (5b). Salvesson⁽⁴⁷⁾ shows that this is equivalent to Criterion I.

A third criterion (Criterion III) is, for a given total number of stations K , to minimize cycle time c (or equivalently to minimize (5)) subject to (5a) and (5b).

An alternative statement of Criterion I in linear programming form follows:

$$\text{If } X_{ik} = \begin{cases} 1 & \text{if the task } i \text{ is assigned to station } k, \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

then assignment of a task to one and only one station implies:

$$\sum_k^K X_{ik} = 1 \quad (i = 1, \dots, I) \quad (8)$$

And restriction (5b) implies:

$$\sum_{i=1}^I X_{ik} a_i \leq C \quad (k = 1, \dots, K) \quad (9)$$

The precedence requirements (5a) are set by restrictions on the precedence matrix. The objective is found from a reformulation of Equation (5):

Minimize: $\sum_{k=1}^K (C - \sum_{i=1}^I X_{ik} a_i)$
subject to (8), (9) and the precedence relations.

Salveson suggests an "abbreviated" procedure for finding a near optimal solution to the line balancing problem of Criterion I. The procedure requires enumeration of all possible combinations of tasks and has limited application in practical situations. An improved procedure, developed by Jackson,⁽¹⁰³⁾ using stepwise enumeration of possible assignments to work stations expands the size of computable problems.

Helgeson and Kwo⁽¹⁰⁰⁾ suggest an additional criterion for evaluation of solutions: minimizing the variation in task load among stations.

Another common constraint, zoning, is included in the IBM 650 program⁽¹¹⁵⁾ stemming from Jackson's algorithm for Criterion I.

Tonge^(119,120) defines zoning as the "division of the set of elemental tasks into (possible overlapping) subsets corresponding to physical constraints on the assembly operation. Zoning of an assembly line may

be determined by the position of the product on the conveyor, the layout of the production facility, or both. For example, certain tasks may be performed only from the back of the product, or only while it is lying on its side; likewise some elements may be carried out on a smaller sub-line joining the main conveyor. The constraint that all elemental tasks assigned to a work station must be in the same zone is added to the definition of a solution." The IBM program can handle up to 99 zones in the assembly line, with up to 50 different tasks performed per zone. A maximum of 24 tasks can be assigned each station with a normal maximum of 12 precedence tasks for each task. The number of tasks which can be optionally assigned to more than one zone is restricted to 95. Using the program, it is feasible to calculate the total number of stations and station assignments for a wide range of production rates. From a graph of the results, the highest production rate possible for a given number of operators may be determined. Hence, instead of finding one balance at a preselected production rate, the program shows how to assign the same number of stations to obtain the maximum production rate, within the optimality of the procedure.

A heuristic program for assembly line balancing using Criterion I is described by Tonge.^(119,120) His objective is to explore the use of heuristic programs for problem solving, rather than to study the line balancing problem as such. The general heuristic scheme calls for:

Phase I Repeated application of aggregative procedures, creating a hierarchy of simplified line balancing problems ranging in complexity from the initial problem to one containing a single compound element.

Phase II Recursive application to these simplified problems of a procedure for assigning men to tasks, down to the level of problems whose component tasks require one man each.

Phase III Smoothing the resulting balance by transferring tasks among work stations until the distribution of assigned time is as even as possible."

Computing effort to reach a balance depends on the number of tasks in the problem, the number of stations desired, and the per cent idle time available. On the same problem, a program using Jackson's algorithm required 2.5 times as many operations to reach a balance. A linear programming solution to the problem yielded groupings of fractional parts of elements, and was, therefore, unsatisfactory. Helgeson and Birnie,⁽⁹⁹⁾ and Hoffman⁽¹⁰¹⁾ describe two other computer programs for solution of the Criterion I problem. The Helgeson and Birnie procedure makes use of "positional weights". Positional weight is the sum of the time values for a specific task and all tasks which must follow it. A cycle time is selected and tasks assigned to stations starting with those whose positional weight is largest. After a satisfactory (but not necessarily optimal) balance is found an iterative procedure is used to find an alternative solution with minimum cycle

time for the same number of stations. Hoffman uses a "code number" row adjoined to the bottom row of the precedence matrix for his Criterion I procedure. Comparison of the efficiency of these methods is reported underway.

The practical importance of Criterion III solution is pointed out by Salveson,⁽¹¹⁰⁾ Helgeson and Birnie,⁽⁹⁹⁾ Hoffman,⁽¹⁰¹⁾ and Kilbridge and Wester.⁽¹⁰⁴⁾ Bryten⁽⁹⁰⁾ attacks the problem directly by a technique for including precedence considerations during appropriate solution of the assignment problem. Hu⁽¹⁰²⁾ describes a simplified version of the Criterion III problem in which each task is assumed to require the same time. With this assumption the problem is easily solved by the network flow algorithm presented, but has only limited practical use for line balancing.

Kilbridge and Wester^(104,105,107) outline a different heuristic procedure which aims to find those solutions where the "balance delay" is zero. Balance delay is defined as the amount of idle time on the line due to the imperfect division of tasks among the stations. A necessary condition for balance delay to be zero is that:

$$K = \frac{\sum_i a_i}{c} .$$

Kilbridge and Wester begin by finding those K's for which a perfect balance is possible, and then assigning tasks to stations in a heuristic fashion according to precedence constraints. Thus, those solutions which satisfy criterion I and III simultaneously with zero balance delay are found.

Bowman⁽⁸⁸⁾ presents two formulations of the assembly line balancing problem as linear programs using Criterion I. The algorithm required for integer solution by linear programming of a modest size line balancing problem is too large for more than theoretical interest at the present. Cord⁽⁹⁴⁾ suggests an interesting scheme for solving a sequence of small integer programs to balance sequentially. None of the described procedures guarantee optimal solutions in problems the size and complexity of those found in practice. In fact, "solutions" often serve as a base for further improvement by inspection. As Tonge⁽¹¹⁹⁾ says ".no satisfactory general scheme for resolving large combinatorial problems involving partial ordering relations has yet been devised."

Salveson⁽¹¹⁷⁾ suggests other important problems beyond the formal mathematical problems. Task times used are almost always assumed to be deterministic whereas in reality they are a distribution of times. Experience has shown that the time to perform a task also is related to the task grouping and sequence in which it is done. Salveson also suggests that the line balancing procedure using time distributions should establish a cycle time which balances the costs of finishing a per cent of parts in repair stations at the end of the line against the costs of idleness due to excessive cycle time. Counter proposals however indicate that the time to repair is usually many times the time to assemble the part originally and that bad parts tend to come in batches. One procedure installs repair stations at planned intervals on the line to which travelling repairmen are assigned as the need arises.

Schreiber⁽¹¹⁸⁾ details a large number of factors which should be included in overall design of a production line. One of these is line balance. Other relevant factors not considered in line balancing, but impinging on the design problem are:

1. Size or existance of float or bank
2. Downtime
3. Scrap
4. Learning
5. Absenteeism
6. Turnover
7. Aptitude of workers
8. Time of day efficiency
9. Worker pool.

In the absence of any analytic model capable of treating all factors simultaneously, Schreiber has written a production line simulation to study the importance of the factors listed. If it can be shown that some factors have no significant effect on the line performance, the design problem will thereby be simplified. Buffa⁽⁹¹⁾ reports on related experiments studying the time a part is available, belt travel rate, and productivity of a conveyor line.

Studies have been made at General Electric Co. on some of these factors. Two alternatives are possible with regard to learning: 1) bring the line up to normal speed following the learning curve with normal staff or 2) put additional staff in the few critical stations until regular operators are trained. Some investigation of the effect

of matching worker capabilities with station load by judgment was also attempted. A 5 to 10 per cent increase in capacity of lines over random assignments is reported. These investigations point to one of the major values of analytic methods of line balancing--as a simulator to uncover critical elements, precedence relationships, and restrictions which create inefficient balances. More explicit statements would be of help in efficiently dividing the effort between the "peak-seeking" search procedures of the analytic methods and the "hill-climbing" improvements which can be made by operating supervision of analytic line balances.

A number of analytic problems remain unexplored. How can the probable best cycle time for K stations be estimated? Perhaps data about average task time, range of task times, and flexibility of the line could be utilized. The ratio of zero's to one's in a precedence matrix or the "fatness" of the precedence graph (ratio of number of task chains to the number of tasks in the longest chain) might be useful indices. Further, none of the current analytic methods are able to treat cases where one or more task times a_i are greater than c the cycle time. In such cases two or more stations may be combined to meet the desired output rate. Another complex problem arises in automobile assembly lines, for example, when a balance is desired for a line assembling more than one end-product. Kilbridge and Wester discuss the implications of dispatching to such assembly lines. ⁽¹⁰⁶⁾

ASSEMBLY LINE BALANCING BIBLIOGRAPHY

86. Ambill, H. W., A Method for Line Balancing, Mimeo, Westinghouse Electric Corp., Pittsburgh 35 Pa., (1963).
87. Barankin, E. W., Precedence Matrices, Management Sciences Research Project, Univ. of Calif. at Los Angeles, (1954).
88. Bowman, E. H., "Assembly-Line Balancing by Linear Programming", OP. Res., Vol. 8, No. 3 (May-June 1960) p. 385.
89. Brennecke, Donald H., An Investigation of the Role of Element Time Variability in the Assembly Line Balancing Problem, unpublished DBA dissertation, Indiana Univ. (1962).
90. Bryton, Benjamin, Balancing of a Continuous Production Line, unpublished Master of Science thesis, Northwestern Univ., Evanston (1954) (see also Moore, Chap. 19).
91. Buffa, E., "Pacing Effects in Production Lines", Journal A.I.I.E., Vol. XII, No. 6 (Nov. 1961) p. 383.
92. Burgeson, J. W. and Daum, T. E., Production Line Balancing, File No. 10.3.002, IBM Corp., Akron, O. (1958).
93. Capretta, J. N., "Operations Analysis in Four Manufacturing Problems", Proceedings 13th Annual Conf A.I.I.E (May 1962) p. 257.
94. Cord, Joel, A Method of Balancing Assembly Lines Using L. P., Mimeo paper (Oct. 1963).
95. Foulkes, J. D., "Directed Graphs and Assembly Schedules", in Combinatorial Analysis, American Math. Society, Providence, R.I., (1960) p. 281.
96. Gutjahr, A. L. and Nemhauser, G. L., "An Algorithm for The Line Balancing Problem", Mgt. Science, 11, No. 2 (Nov. 1964) p. 308.
97. Held, M., Karp, R. M. and Shreshian, R., Assembly Line Balancing, Dynamic Programming with Precedence Constraints, IBM Corp. Research Paper RC-797 (Sept. 1962); Also O.R., 11, No. 3 (May 1963) p. 442.
98. Held, M. and Karp, R. M., "A Dynamic Programming Approach to Sequencing Problems," J.S.I.A.M., 10 (1962) p. 196-210.
99. Helgeson, W. B. and Birnie, D. P., "Assembly Line Balancing Using the Ranked Positional Weight Technique", Journal of Industrial Engineering, Vol. XII, No. 6 (Nov. 1961) p. 394.

100. Helegeson, W. B. and Kwo, T. T., "Letter to the Editor", Management Science, Vol. 3, No. 1 (Oct. 1956).
101. Hoffman, Thomas R., Optimization Criteria and Assembly Line Balancing by Digital Computer, Paper presented at 7th Annual International Meeting Institute of Management Sciences, (Oct. 1960) Univ. of Wis., School of Commerce Study, Vol. II, No. 3 (Feb. 1961).
102. Hu, T. C., "Parallel Sequencing and Assembly Line Problems", OP Res., Vol. 9, No. 6 (Nov. 1961) p. 841-8.
103. Jackson, James R., "A Computing Procedure for a Line Balancing Problem", Management Science, Vol. 2, No. 3 (April 1956).
104. Kilbridge, M. and Wester, L., "A Heuristic Method of Assembly Line Balancing", Journ. of I.E., Vol. XII, No. 4 (July 1961) p. 292.
105. Kilbridge, M., Wester, L. and Lewis, T., Analytic Systems of Line Balancing, Paper given at 20th Nat'l. Meeting of Oper. Res. Soc., San Francisco, Calif. (Nov. 1961).
106. Kilbridge, M. D. and Wester, Leon., Assembly Line Model-Mix Sequence Problem, Mimeo Paper, Univ. of Chicago (1962).
107. Kilbridge, M. D. and Wester, Leon., "Heuristic Line Balancing a Case", Journ. of I.E., Vol. XIII, No. 3 (May 1962) p. 139.
108. Kilbridge, M. and Wester, L., "The Balance Delay Problem", Management Science, Vol. 8, No. 1 (Oct. 1961) p. 69-84.
109. Klein, M., "On Assembly Line Balancing," OR, 11, No. 2 (March 1963) p. 274.
110. Mansoor, E. M., "Assembly Line Balancing - An Improvement on the Ranked Position Weight Technique", Journ. I.E., Vol. XV, No. 2 (March 1964) p. 73.
111. Mansoor, E. M., "Comments on Assembly Line Balancing", Journ. I.E., 15, No. 6 (Nov. 1964) p. 322.
112. Marimont, R. B., "Method for Checking the Consistency of Precedence Matrices", Journ. ACM, 6, No. 2 (April 1959) p. 164.
113. Moore, James M., "Line Balancing", Chapter 19 in Plant Layout and Design, Macmillan, (1962).
114. Moodie, C. L. and Young, H. H., "A Heuristic Method of Assembly Line Balancing", Journ. I.E., 16, No. 1 (Jan. 1956) p. 23.

115. Production Line Balancing at Westinghouse, IBM Corp. General Information Manuel, E20-4037, (1959).
116. Salveson, M. E., Smith, W. W. and Helgeson, W. B., How to Balance an Assembly Line, Center for Adv. Mgt., New Canaan, Conn.
117. Salveson, M. E., "The Assembly-Line Balancing Problem", Transactions of the ASME, Vol. 77, No. 6 (August 1955); also Journ. I.E., 6, (May 1955) p. 18.
118. Schreiber, Harry, Simulation of a Production Line, Proceedings AIIE National Meeting, Detroit, Michigan (1961).
119. Tonge, F. M., A Heuristic Program of Assembly Line Balancing, Prentice Hall Co. (1961).
120. Tonge, F. M., "A Summary of a Heuristic Line Balancing Procedure", Mgt. Sci., 7, No. 1 (Oct. 1960).
121. White, W. W., "Comments on a Paper by Bowman", Op. Res., 9, No. 2 (March 1961) p. 274.

IV MATERIAL FLOW SYSTEMS

This section is subdivided into the following topics:

- 4.1 Analysis of Materials
- 4.2 Classification and Selection of Material Handling Equipment
- 4.3 Material Handling Flow Systems Planning
 - 4.3.1 Deterministic Models
 - 4.3.2 Stochastic Models
 - 4.3.3 Dynamic Models
- 4.4 Material Handling Flow Systems Scheduling and Operation

A comprehensive list of reference books, technical associations, periodicals and visual aids describing equipment and empirical principles for material handling may be found in the Material Handling Institute's "Lesson Guide."⁽²⁾ Readers who desire an introduction to queueing theory in the context of material handling should refer to Morris.⁽¹¹⁾ A more extensive development of queueing theory may be found in Morse⁽¹²⁾ or Saaty.⁽¹⁷⁾ Among the many recent texts on linear programming are Charnes and Cooper,⁽⁴⁾ Vajda,⁽¹⁸⁾ Gass,⁽⁷⁾ and Dantzig.⁽⁵⁾

4.1 Analysis of Materials

The material handling engineer is faced with early consideration of the physical characteristics of the materials under movement and their effect on his selection of routes, equipment, and investment. A thorough knowledge of the characteristics of the materials implies not just facts about the density, weight, shape, value, and hazards of the material, but

also how to relate these characteristics to the selection of technically feasible material handling equipment. No engineering axiom exist which make it possible to measure the engineering appropriateness of the handling device with the material to be moved. Mis-application has probably occurred when excessive damage occurs in transit, when belts wear out prematurely, or when materials hang up in bins.

Some effort is underway to relate materials and material handling equipment. Research on the problem of classifying bulk materials according to their "convey-ability" is reported by Erisman.⁽¹⁴²⁾ He classifies bulk materials by size, flowability, abrasiveness, and other factors such as corrosiveness, explosiveness, etc. Erisman attempts to relate these characteristics to specific kinds of conveyors. He indicates that the full power of his approach will be realizable when it is possible to analyze material characteristics together with capacity and other requirements by a computer program which selects the most appropriate conveyor. Erisman also points out that "there is nothing available for measuring or defining... abrasion," and that the study of abrasion warrants research through-out the bulk handling industry.

The movement and handling of packaged or containerized materials represents a large portion of material flow activity, particularly when long distance or intra-plant movement is considered. The material handling engineer in this instance usually is required to select the package or container with the most desirable and economic physical characteristics. Specifying size, material, and durability of packaging or container must be carried out in concert with selection of the material handling equipment

or subject to the restrictions which the existing equipment imposes. The extensive engineering developments in package design are treated by Brown⁽¹³²⁾ and Friedman and Kipnees⁽¹⁵²⁾ and will not be pursued here.

The economic savings from high utilization of cargo space in long haul material transit has motivated a number of studies. The extensive charts in the classic Akrep and Stambler report⁽¹²²⁾ on pallet pattern selection criteria for 40 x 48 pallets fully exemplify the complex combinational problem of trying to fit small boxes into bigger ones with minimum void space. The problem has since been reduced to a computer program which, given a pallet size and carton size, will select the layer configuration which minimizes the void area in the layer (unpublished, University of Michigan, Industrial Engineering Department). As part of a larger study on ship cargo containerization, Carrabino⁽¹³⁴⁾ carried out a simulation to fit different size packages into different size containers. The study indicates the importance of considering not one dimension at a time but the relative shapes of packages and containers when attempting to minimize void space. The evaluation of containerization economics made by Carrabino is coincidentally supplemented by Weldon's study⁽²⁰⁵⁾ of the economics of "Cargo Containerization in the West Coast-Hawaiian Trade" and by Fleisher's cautions.⁽¹⁴⁴⁾ King and Goode⁽¹⁶⁷⁾ describes techniques used to determine the optimum number of packages per carton and the optimum number of cartons per case for packaging and shipping a product. The unit costs of a case were approximated from basic labor and material elements as a function of carton size, case size, and physical volume per package.

Carton and package costs were approximated in a similar way. Total annual costs on a given product were calculated as the sum of unit cost of each type of container times its annual usage. A graph of total cost versus annual sales for different combinations of container sizes was used to find optimum combinations for pertinent ranges of annual usage.

4.2 Material Handling Equipment Classification and Selection

We next turn to classification of material handling equipment.

A large number of periodicals regularly carry descriptions and pictures of creative applications of material handling equipment. Haynes⁽¹⁵⁷⁾ has compressed descriptions of most standard material handling equipment into one volume; most books relating to plant layout or material handling also contain descriptive sections on material handling equipment. Zimmerman and Lavine⁽²⁰⁹⁾ prepared extensive empirical cost data on shovels, cranes and powered and non-powered industrial trucks. Some attempts have been made to develop practical universal standards for predicting equipment capability and performance. For example, the Power Crane and Shovel Association⁽¹⁹⁴⁾ publishes information on hourly yardage of drag-lines and of power shovels under varying operating conditions. The Yale and Towne-Warton School studies⁽¹⁹⁵⁾ on measurement of fork truck performance have been available since 1954. The extensive data on basic motions (straight-runs, turns and stacking) and on operating variables applies to 4000-6000 pound electric fork trucks. Wimmert⁽²³⁶⁾ develops deterministic relationships for these variables in deriving his "equivalent move distance" concept for fork left trucks. Statistical reliability of the performance standards are not included in these studies.

The overwhelming variety of material handling equipment and of material characteristics offers a large number of possible ways of combining material and equipment (some better than others) to accomplish a desired material flow. The application engineer needs extensive experience or exhaustive study to avoid the many pitfalls of incorrect application. Comprehensive check lists and tables are characteristic devices offered as a help in narrowing the search for the feasible combinations and to force consideration of unsuspected problems. For this purpose, "Mill and Factory"⁽¹⁶⁰⁾ presents a "Material Handling Equipment Selector Table." Ninety types of equipment are classified into three groups: (1) fixed path (conveyor, elevators); (2) limited area (cranes, hoists), and (3) wide area (tractors, trailers, trucks). Tabulation of the equipment is made against forty-six varying factors also classified into three broad classes: (1) limitations of the equipment (mobility, transfer characteristics, building factors); (2) properties of material which can be handled (nature of material, weight ranges) and (3) characteristics of movement (frequency, direction, distance). Typical of the abundance of available instructions on proper selection of material handling equipment are the Material Handling Institute's "Library of Know How," Sayre's "Selection and Use of Pneumatic Systems,"⁽¹⁹⁶⁾ Berg's "Gravity Flow"⁽¹²⁶⁾ and Materials Handling Engineering "Handling with Hoists." This statement by Winter⁽¹⁸⁸⁾ may be a just evaluation of available literature for most types of material handling equipment: "Very little research has been published with reference to the economics and comparative performance of industrial fork-lift trucks together with

accessories, and tractor operating conditions and in like environments." He then presents results for the handling of both palletized and unpalletized materials by this equipment under like conditions, based on data obtained by the U.S. Department of Agriculture in the cotton and apple industries.

Existing empirical knowledge of handling equipment suffers from the absence of a structure defining the equipment according to its operational characteristics. For example, the "Mill and Factory" classification system does not include estimated relative costs or adequate information on flow capacity of the ninety different equipments. In general, the capacity of the equipment will be a function of the installation, as both the Yale and Towne and Winter Studies indicate. Ideally then, effective equipment selection must include the additional factors of equipment operation characteristics, as well as the technological capability and cost factors. As we would expect, adequate descriptions of operation characteristics can be quite complex, and therefore usually require a mathematical or simulation model.

Before turning to models of material handling equipment, we should observe that some of the studies in traffic and transportation problems may offer substantial guidance. A significant study by Beckman et al. ⁽¹²⁴⁾ relating to highway traffic and railroad transportation appeared in 1955. The authors pioneered in drawing together concepts, methods, and models of potential usefulness in assessing capabilities and appraising efficiency of operation of these two transportation systems. Queueing theory is applied to such traffic situations as the flow

of cars through an intersection and the passing of slower cars by faster cars by using gaps which occur in the opposing traffic stream. The concepts of individual demand and congestion on the highway then are incorporated into the assumptions of a non-linear model of highway traffic on a road network. Fragmentary models of railroad "accumulation delay" problems, classification yard operation, and train scheduling between yards to minimize accumulation delay are discussed. A useful supplement to this study in traffic situations is the 1960 bibliography of the Committee on Theory of Traffic Flow.⁽¹⁸⁸⁾

Turning now to material handling equipment, we find the most comprehensive examination of material handling models in Morris' book.⁽¹¹⁾ Methods of approach to materials handling problems are applied to a fixed schedule fleet size problem and to waiting line (queueing theory) analysis of random flow systems. The efforts of Kwo⁽¹⁷⁰⁾ and Mayer⁽¹⁷⁶⁾ reported below toward development of a theory of conveyors of the hook storage-delivery type, are expanded by Morris in his book. He also illustrates the application of simulation to materials handling problems, by examples reported in the literature from the steel industry,⁽¹⁶⁵⁾ department store warehousing,⁽¹⁹⁸⁾ and bus terminal design.⁽¹⁶⁴⁾

Kwo's paper⁽¹⁷⁰⁾ studies the operation of a common overhead loop-conveyor linking two manufacturing process areas which have different operating rates. Three principles are stated: (1) The speed of the conveyor (under the assumed conditions), in terms of carriers per unit time, must be greater than the maximum of the loading or

unloading rate desired and less than the minimum of the maximum technological speed of the conveyor or of the human ability to load or unload parts; (2) the conveyor must have enough capacity to accommodate the intentional reserve stock, temporary transit requirements due to the geographic separation of the loading and unloading points and any accumulated items due to differences in loading and unloading schedules; and (3) the conveyor must be loaded and unloaded uniformly throughout its entire length. Using these principles, two methods of simulation are developed, and suggestions advanced for consideration of operating policies. In a related paper,⁽¹⁵⁸⁾ Helgeson presents a nomograph for use in finding candidates workable revolution times for consideration when operating an existing loop conveyor systems. The candidate times must then be checked by simulation or by actual operation of the conveyor to be certain that the required storage capacity is available. In a separate article⁽¹⁷¹⁾ Kwo extends these principles of operation to checking the possibility of design of a loop conveyor system.

Mayer⁽¹⁷⁶⁾ develops a mathematical model of an overhead hook carrier trolley conveyor which is loaded by a worker completing a work cycle. He assumes constant conveyor speeds, work stations all with constant and equal cycle times and only one hook available to the worker at any time. If the available hook is already loaded, the completed part is placed on the floor and removed from further consideration. He then develops an index of performance which is the ratio of the expected quantity of production placed on the floor to the total production. The probability of an attempt to load a particular hook

at a particular work station is one minus the number of hooks which pass the work station during the work cycle. With these assumptions, he gives an example of the evaluation of competing proposals for improvement of conveyor performance by doubling conveyor speed, or by doubling the capacity of each hook. Doubling hook capacity is shown to give a better (lower) index of performance.

Moving away from conveyor systems, Mayer⁽¹⁷⁷⁾ also develops a mathematical model for each of three job shop storage systems requiring intermittent storage of discrete unit loads of material. He classified a manual order-picker as one dimensional, a mechanical transporter as two dimensional and a fork-lift truck as three dimensional material handling devices. A single cycle of operation is defined as either depositing a unit load in storage, or retrieving a unit load from storage but not both on the same trip. A dual cycle both deposits and retrieves on the same trip in that order. Assignment of storage locations is by sampling from a uniform distribution.

The order picker is compared for two methods: material handled in (1) a sequence of single cycles or (2) a sequence of dual cycles.

If T_1 = expected time for single cycle operation,

T_2 = expected time for dual cycle operation,

W = width of storage cell (feet),

S = speed of order picker (feet/min.),

N = number of storage cells in a line,

Mayer shows that

$$T_1 = \frac{2W(N+1)}{S}$$

and

$$T_2 = \frac{4W(N+1)}{3S} .$$

In short, the dual cycle is one third faster than the single cycle.

The mechanical transporter is defined as a self-propelled machine which operates in a single aisle consisting of storage cells in R rows and C columns. Results are also obtained for both the single and dual cycles. Finally, the fork-lift truck case is examined for both cycles. Expressions are obtained for T_1 and T_2 , the expected cycle times for each method of operation. Mayer also comments on planning storage allocation.

In a still different area, Pollack⁽¹⁹⁰⁾ gives a mathematical description of shuttle and of assembly line flow processes. Actually, this article is one of a number emanating from the comprehensive engineering analysis of cargo handling at University of California at Los Angeles. The shuttle process consists of a transporting agent (lift-truck for example) which picks up a unit load at one point (or node), delivers it to a second node, and then returns to the first position to repeat the sequence. The shuttle is always confined to this link or "stage". Additional stages of shuttles, similar to the first, continue the sequence of shuttles, resulting in the N-stage shuttle process. More than one shuttle may operate in a particular stage, but passing is not permitted. Analytic solutions to two and

three link stochastic systems with zero-storage capacity at the nodes and one shuttle per stage are presented by Davis and Weinstock.⁽¹⁴⁰⁾ Solutions to the N-stage shuttle process have not been obtained, but distributions of delay time for various systems have been obtained by simulation.⁽¹²⁵⁾ The assembly line processes are described as a series of machines where each item is processed in order. After the processing is complete, the item proceeds to the next stage where it is accepted into the machine or waits if the stage is busy. This process is shown to be analogous to the shuttle process described earlier where the machines in each stage play the role of the transporting agents in the shuttle. In this way, both systems are seen to be waiting line processes. A later study⁽¹⁹¹⁾ examines the effect of communication on a N-stage shuttle process and for the assumptions made, finds no significant change in operation.

4.3. Material Handling Flow Systems

We have indicated the large number of variations of materials and material handling equipments which may be considered in seeking a desirable combination for a flow system. In trying to predict operational characteristics of material handling equipment, we have already seen some mathematical descriptions of the equipment and the system in which they operate.

A further separation of the subsequent work will be useful for our presentation although not so simple in practice. We will consider material handling system planning or design as a before-the-fact study of flow requirements and facilities. For existing systems, we

will consider separately the reported schema for routing, dispatching and control. In practice, of course, sound design requires simultaneous consideration of operational factors just as operational problems lead to re-design of existing systems. A material handling system (or for that matter any operational system) can in the long run be viewed as self-adapting organism which compensates and alters itself to best cope with its environment. The implications of this concept of an industrial complex as an adaptive system are just being recognized and will therefore not be explicitly pursued here. Some empirical evidence of adaptation can be inferred however from the following statement about planning manufacturing facilities by Jernstedt:⁽⁹⁾

"... we find that if we have a good planning organization we secure continual improvements year after year and can meet sales forecasts with existing plants when and if they occur. One new plant at the time of completion was able to produce 35,000 units a month. With continuous planning and normal year to year improvements, three years later it was able to produce 70,000 units a month."

Thus, although in practice planning and control are intimately intermixed, we will arbitrarily consider them separately wherever possible.

In considering planning and subsequently operational control problems of material handling flow systems, different authors have used different objective measures of effectiveness. In some cases it is possible to relate measures of effectiveness by a change in dimension

as when time is measured in terms of dollar cost. A large number of objectives may be used for material handling system design and operation. To list just a few:

1. Maximize flow capacity
2. Minimize trip time
3. Minimize number of handling units
4. Minimize capital investment
5. Minimize operating costs
6. Maximize system reliability
7. Maximize on-time deliveries
8. Minimize inventory hold-up
9. Minimize idle time.

Many writers utilize different objectives for justifying material handling systems and thus obtain different operating policies for otherwise identical facilities. Furthermore, several measures of effectiveness may be pertinent in a given application, but dimensionally uncomparable. We will examine this problem further later in this monograph.

4.3.1. Analysis and Design of Deterministic Material Flow Systems

In this section we examine developments in deterministic models of flow systems. Many of these models appear in the literature of linear programming, where they have been presented primarily on the basis of their intrinsic mathematical properties. Although the names of many of the models such as transportation or assignment, are suggestive of handling problems, reports on specific applications to

material flow systems are meager. In this section then, we describe the operational characteristics of the model types which have been developed, and to a lesser extent report on applications to specific material handling system problems. In addition to the previously cited books by Charnes and Cooper,⁽⁴⁾ Vajda,⁽¹⁸⁾ and Gass,⁽⁷⁾ a useful survey of the present state of linear programming can be found in Ackoff.⁽¹⁾ A complete development in the special area of network flows is given in Ford and Fulkerson.⁽¹⁵¹⁾

The conceptually simplest of the flow problems is that of finding the shortest route through a network (Figure 1). Demand on the network per period is irrelevant (assumed one unit, without loss of generality), one homogeneous material is considered, steady state, uni-directional, fixed-path flow, with linear, deterministic service time, no storage and with multiple stages. Since the measure of performance is minimum distance (or associated cost or time), time related variables such as scheduling policy or nature of the load are not considered. Flow is from one origin node to one specified destination node.

Even listing all possible paths in such a network may be a difficult and tedious process. In his paper on higher transitions in a Markov process, Miehle⁽¹⁸¹⁾ describes a matrix method of determining a list of alternative paths by manipulation of the matrix of connections between adjacent points.

A number of different procedures have been proposed for finding the shortest route through a network.^(139,184,192) A useful extension of the single path problem is to find and rank all possible

paths from the origin to the destination.^(128,159,193) Alternatively, Brown⁽¹³¹⁾ presents a computational scheme for generating shortest paths between all pairs of nodes of a directed network.

A second flow problem of considerable interest, the traveling salesman problem, differs from the first primarily in that the system is cyclic, that is, the flow-path destination is also the starting node or origin (Figure 2). The problem in this case is given the distances (or other cost measure) between any two nodes, to find the minimal travel distance necessary to visit each node once. Notice that travel is permitted between any two nodes and that the distance (or cost measure) need not be the same in both directions. This problem has not yielded to a general solution, although a number of procedures for finding near optimal paths are available.^(136,145,172) A theorem from Flood⁽¹⁴⁵⁾ useful in application to problems of this kind has been proven: the minimal path does not intersect itself. A further generalization of the problem has been proposed in which more than one tour is permitted (or required) in order to visit all nodes. Research on solutions to this problem is underway.⁽⁸²⁾

A third network flow problem may be stated as follows: given a network of connected paths each with maximal capacity per period, find the maximum number of units of goods per period which can be transported from a specified origin to a specified destination.^(137,146,147,148) It is useful to know that under certain conditions, this problem can be converted to its dual and solved by the same

methods as the shortest route through a network problem. The capacity problem differs from the shortest route problem only in that instead of minimizing distance travelled, it seeks to determine maximum capacity allocation. Physical analogy can be made to trying to allocate production of one product to a sequence of alternative facilities in order to maximize output per period. Alternatively, in the design process, given a sequence of constrained maximum flow capacities one may seek the maximum overall capacity of the system assuming optimal utilization is practiced. Computational procedures for this problem are simple and capable of treating networks of thousands of links very quickly. Boldyreff⁽¹³⁰⁾ describes an application to find the maximum flow capacity of a railroad network. Hu⁽¹⁶²⁾ gives a procedure for finding the maximum capacity routes between all pairs of nodes in a network. The work of Shapley⁽¹⁹⁹⁾ suggests some ideas which could be helpful in deciding on the placement of additional links when capacity increases are desired.

A useful and important practical extension of the maximum flow capacity problem would add the possibility of shipping n heterogeneous materials from several sources to several different destinations. To more rigorously define the problem, we use the notation in Ford and Fulkerson.⁽¹⁵⁰⁾ Let A be an $m \times n$ incidence matrix with elements

$$(a_{rs}) = \begin{cases} 1 & \text{if } C_s \text{ contains } A_r \\ 0 & \text{otherwise,} \end{cases}$$

where A_1, \dots, A_m is a list of arcs of the network and C_1, \dots, C_n is a list of all sequences of arcs for all commodities which join all the sources with all sinks for the same commodity. Let X_s ($s = 1, \dots, n$) denote the amount of material S which flows along sequence C_s and b_r the maximum capacity of path A_r . For example, if we have the network in Figure 4, with sources P_1, P_2 , sink P_3 for material 1, and source P_4 , sink P_1 for material 2, the A list of paths will be A_1, \dots, A_6 and the C list can be found by considering all paths from P_1 to P_3 , and from P_2 to P_3 for material 1 etc. Thus we can select C_1 to be made of A_3 from P_1 to P_3 only, C_3 to be A_1 from P_1 to P_2 and A_2 from P_2 to destination P_3 . The A matrix then is as shown:

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}		
A_1		1	1		1			1		1		1			1		
A_2			1	1		1								1	1		
A_3	1							1	1				1	1			
A_4		1		1					1	1	1						
A_5				1	1		1		1			1		1			
A_6		1				1	1			1			1		1		
				Material 1										Material 2			

The linear programming problem is then, to:

$$\begin{aligned}
 & \text{Maximize} && \sum_{s=1}^n x_s \\
 & \text{Subject to} && \sum_{s=1}^n a_{rs} x_s + x_{n+r} = b_r \\
 & && x_1, \dots, x_{n+r} \geq 0,
 \end{aligned}$$

that is, maximize the total flow of all materials through the network. Different flow values also can be attached to different products, but the procedure assumes unlimited supplies at any source.

Yet another version of the network flow capacity problem in a single commodity permits consideration of flow over more than one period. Ford and Fulkerson⁽¹⁴⁹⁾ state the problem thus:

"Suppose there is a network in which each path has associated with it two positive integers, one a material flow capacity and the other a material traverse time, and assume that material flow originates at some particular source node and is ultimately destined for some other sink node. If each remaining node of the network can either tranship the material immediately after receiving it or hold it for later shipment, what is the maximal amount that can be shipped from source to sink in any given number of time periods?"

We see that this model introduces storage into the network and also permits the study of dynamic effects. In solving for a given time period T_1 optimal solutions for all lesser time periods are a by-product. Paths which serve as bottlenecks for the flow are singled out, as well as the time periods in which they act as such. Since the procedure is a straight forward labelling process, the solution of networks with more than 1,000 branches is feasible by computer.

A question likely to arise in the design stages of a network flow system is: given a flow network with existing maximum flow capacity for each link, and a unit cost of additional capacity for each link, how should a fixed amount of funds be spent in order to maximize the resulting network flow capacity between a given source and sink? If the cost of increasing each link capacity is linear, Fulkerson's labelling procedure (154) permits solution of large networks. Practical material handling situations where this assumption is true may be scarce, however. Models for some kinds of non-linear cost problems will be mentioned later. Notice that if we are interested in flows from any of several sources to any of several sinks, the same procedure can be used.

Matthys and Ricard⁽¹⁷⁵⁾ extend the maximum network flow procedure in their study of a railway network. The study considers direct freight trains and priority trains in a section of double track railway and determines the theoretical maximum flow, the bottleneck areas and a freight train timetable corresponding to the maximum flow.

The "transportation" model extends the network flow problem to include time or cost estimates which are a function of the number of units of material shipped by each route, and imposes requirements and availabilities at each destination and origin. Figure 3 is a pictorial representation of this classic transportation problem. The structure of the problem and the many different procedures available for obtaining an optimal solution are well reported in the literature. Because of the simplicity of hand computation of small problems and the availability of rapid computer programs for solving large problems, the

transportation procedure has been used for many applications. A few which relate to materials handling flow are included here to illustrate the value and difficulties of applications.

Berrisford⁽¹²⁷⁾ describes the application in Britain of the transportation procedure to the selection of minimum cost routes for shipment of coal from mines to gas producers. He reports a saving of around 2-1/2 per cent (£65,000) over previous empirical methods against a computer time cost of £300. Erickson and Randolph⁽¹⁴¹⁾ used the transportation procedure for deciding which of 3 handling methods to use for moving twelve different products in a shipping area. Costs are assumed linear and capacities of the methods are unlimited or known. It is interesting to note that the solution found by this procedure could be improved by assigning some materials to one of the partially idle methods. The costs in this case are only approximated by linear functions, and the assumptions of the model lead to a non-optimal solution for this application. Metzger⁽¹⁷⁸⁾ applies the same procedure to minimize the total time required to move empty containers, which become available in m departments, by fork-lift-trucks to n destinations, where the number of containers required and available is known. He then considers the material delivery requirements also and by inspection combines deliveries and empty trips into cyclic trips. In this way, schedules can be prepared for truck dispatching at the start of each day.

Two other variations of the transportation model have been successfully developed. One of these, commonly called the transshipment problem, broadens the classical transportation problem to permit any

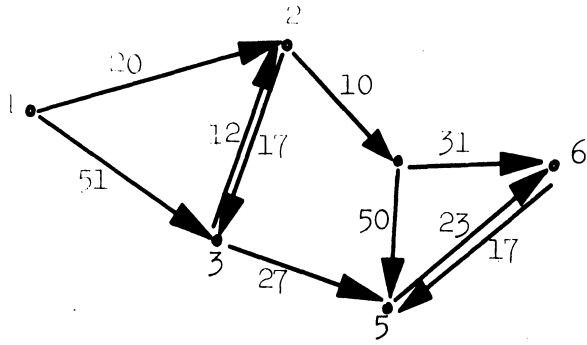


Figure 1. Network Flow Problem.
Find Shortest Path
from 1 to 5 .

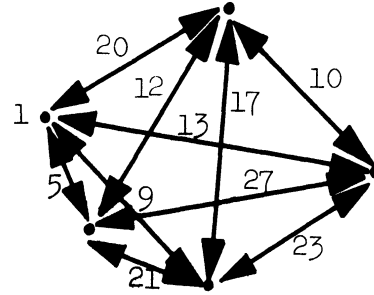


Figure 2. Travelling Salesman Problem.
Find Shortest Route from 1
and Return which Visits Each
Node Once.

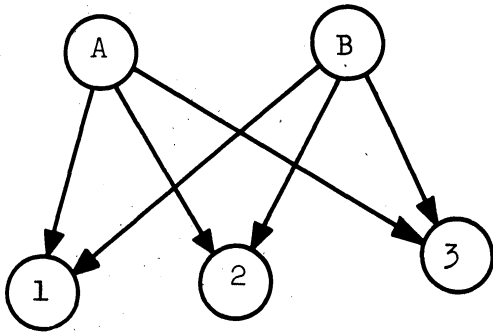


Figure 3. Transportation Problem.

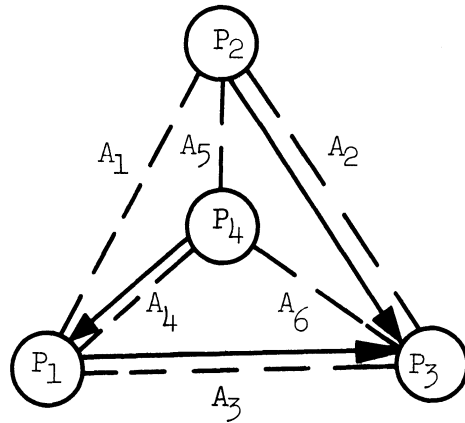


Figure 4. Multi-Commodity Network.
Capacity Problem.

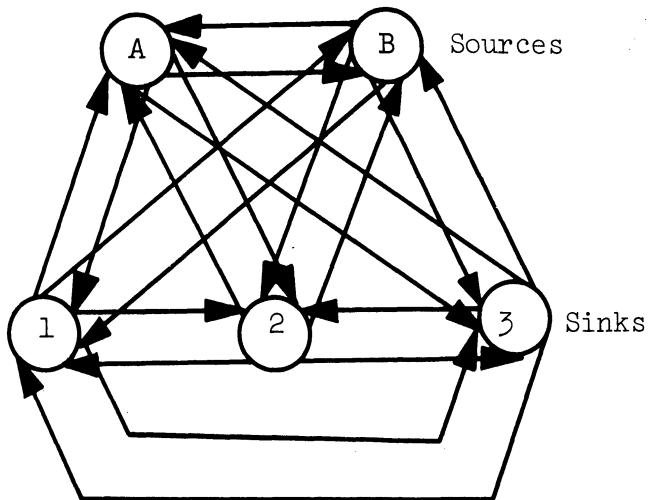


Figure 5. Transshipment Problem.

origin or destination to also act as an intermediate point during the allocation or shipment. Williams and Haley⁽²⁰⁷⁾ report on a practical application of the transshipment procedure in the coal mining industry. By interpretation of the requirements and availabilities at the source and destination as being time dependent rather than spatially dependent, it is also possible to interpret the standard transportation model as a dynamic problem. In this sense it is possible to study optimal routing under time dependent demand requirements.

A major stumbling block in the practical application of the transportation procedure to many industrial problems is the requirement that costs be linearly related to the number of units allocated to each route. The following application reported by Gould⁽¹⁵⁶⁾ is typical:

"An investigation was carried out to establish a method of determining the allocation between certain factories of a known production requirement so as to minimize the total expense of production. Each factory was made up of a number of department or services whose total expense varied non-linearly with the production level. The method was to divide the production range of each department in each factory into regions of approximate linear expense, and consider the associated total company expense connected with the optimum allocation for each possible group of regions (one region in each department).

In theory the Simplex technique could be applied to each group to find its optimum allocation, but as the number of groups is prodigious for quite simple problems, this is not practical. A procedure was devised using the transportation technique together with various lemmas proved in the article and feasibility considerations, by which it was possible to reduce rapidly the number of groups needed to be considered in detail to just a few. Other problems should be amenable to this method provided they are similar to the one above in two respects: 1. the departmental variation of expense with production from one region to the next is in general continuous with decreasing rate of change of expense i.e., the variation is concave; 2. it should be possible to put the requirements into one of the units of capacity usage in which there are important physical or maximum capacity restrictions."

Shetty⁽²⁰⁰⁾ also makes a contribution to application of transportation algorithm to the many situations with non-linear action costs. He describes his model as follows:

"We are given a set of m sources for production of a material and a set of n destinations with known requirements, the cost of transportation for a unit of material from each source to each destination, and a set of functions defining the total cost of production at each source as a function of the volume of resource produced at the source with marginal cost of production continuous and non-decreasing. Find the amount of material to be shipped from each source to each market to minimize the total transportation and production costs."

Mathematically the problem is stated as follows:

Let c_{ij} = cost of transportation of a unit of material
from source i to destination j

x_{ij} = number of units shipped from i to j

a_i = number of units available at source i

b_j = number of units required at destination j

$P_i(a_i)$ = cost function of production at source i .

Then, minimize

$$\sum_{i=1}^m \sum_{j=1}^n c_{ij} X_{ij} + \sum_{i=1}^m P_i(a_i)$$

subject to

$$\sum_{j=1}^n X_{ij} = a_i$$

$$\sum_{i=1}^m X_{ij} = b_j \quad \text{and} \quad X_{ij} \geq 0$$

O'Rourke and Blitz⁽¹⁸⁷⁾ summarize the nature of a transportation problem in which there is a fixed charge for every route which appears in the final solution in addition to the usual variable charge per unit shipped. The fixed charge might be thought of as analogous to a set-up cost. The fixed charge transportation problem is described as:

$$\text{minimize:} \quad \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} + f_{ij} \delta(x_{ij})$$

where

$$\delta(x_{ij}) = \begin{cases} 0 & \text{for } x_{ij}=0 \\ 1 & \text{for } x_{ij}>0 \end{cases}$$

$$\text{Subject to } \sum_{i=1}^m x_{ij} = b_j \quad (j=1, \dots, n)$$

$$\sum_{j=1}^n x_{ij} = a_i \quad (i=1, \dots, m)$$

$$\sum_{j=1}^n b_j = \sum_{i=1}^m a_i$$

The problem may be solved through use of a simplex procedure for integer solutions to general linear programming problems. However, this procedure is expensive and severely restricts the size of the problem which can be handled in practice. Balinski⁽¹²³⁾ suggests an approximation procedure and illustrates several examples using this technique. The transportation problem has been extended to multiproduct problems, although, as would be expected, the array is considerably enlarged. Knödel⁽¹⁶⁹⁾ uses this example:

"Supply 230 consumption points with four kinds of sugar and six sorts of salt from seven sugar plants and six salt mines. Find the optimal distribution plan and prescribe the quantities of the different kinds of goods to produce at each origin if maximum amounts are stipulated."

More completely the problem is formulated thus:

"There is a set $k = 1, 2, \dots, l$ of kinds of commodities which can be substituted in production or consumption. The plant i of total capacity a_i has a capacity a_{ki} for commodity k such that:

$$\sum_{k=1}^l a_{ki} \geq a_i \quad (i = 1, 2, \dots, m).$$

The problem is to find x_{kij} , the amounts of product k produced at plant i and shipped to destination j to minimize

$$\sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^{\ell} c_{kij} x_{kij}$$

where the c 's are unit shipping costs, subject to:

$$\sum_{j=1}^n x_{kij} \leq a_{ki} \quad \begin{array}{l} (k = 1, \dots, \ell) \\ (i = 1, \dots, m) \end{array}$$

(for a given product and plant, total amount shipped to all destination \leq capacity) and, for a given plant i , the total quantity of all production to all destinations \leq total capacity:

$$\sum_{k=1}^{\ell} \sum_{j=1}^n x_{kij} \leq a_i \quad (i = 1, \dots, m)$$

and, for a given product k and a given destination j , total shipments = requirements:

$$\sum_{i=1}^m x_{kij} = b_{jk} \quad \begin{array}{l} (j = 1, \dots, n) \\ (k = 1, \dots, \ell) \end{array}$$

and

$$x_{kij} \geq 0$$

where b_{jk} represents the requirements for product k at destination j . Schell (197) has examined other variations of the three dimensional transportation problem, with various degrees of success.

In passing, it should be pointed out that transportation problems with capacity restrictions as well as requirements and availability constraints have been solved. The capacitated transshipment problem has also been studied⁽²⁰³⁾ as have transportation problems in which some variables are stochastic. On the other hand, some fairly simple extensions of these problems have failed to yield any suitable solutions thus far. The multiproduct transshipment problem is a case in point.⁽¹⁵⁵⁾

4.3.2. Analysis of Stochastic Material Flow Systems

In this section we look at models of material handling flow systems, one or more of whose operational characteristics are best described by distributions of random variables. As an example, the assemblyline processes developed by Pollock⁽¹⁹⁰⁾ (described earlier) are stochastic models if the process time for each stage is a random variable. Other specific discussions of applications of waiting line or queueing theory models in material handling are frequent in the literature.^(143,189,202,206) Extensive description here of the underlying operational models of queueing systems which have been developed, does not seem warranted in view of the excellent books in this area. Morris⁽¹¹⁾ devotes a major portion of his book to development of waiting line analysis in the specific context of material handling problems. Both Morse,⁽¹²⁾ and especially Saaty⁽¹⁷⁾ in his recent book, cover the extremely large number of different models which have been developed thus far. In addition, Saaty gives a comprehensive bibliography of queueing theory, containing over 900 entries. For those interested in

applications a particularly interesting pair of studies in airport service problems is reported by Friend.⁽¹⁵³⁾ The first of these studies is a good example of the practical problems in attempts to apply classical queueing theory, whereas the second study, superficially similar to the first, had to be solved by a crude scheduling approach. The two studies and reasons for the different approaches are contrasted by Friend in an attempt to justify the divergence in method.

4.3.3. Material Handling Flow System Planning - Dynamic Models

In our development so far we have emphasized steady state models, that is, flow problems in which the demand on the system is constant. Only brief mention has been made of dynamic network, transportation, or queueing models. The importance of considering dynamic characteristics of material flow systems is perhaps too obvious to need mention, but methods for analyzing the effect of changing rates of operation are not widely reported. A good description of the need for such consideration is given by Webb:⁽²⁰⁴⁾

"Synchronization of conveyor drives is necessary to permit automatic or manual transfer of products or product-supporting carriers between separate conveyors. An overall conveyor system is usually made up of separate conveyors for one or more of several reasons:

- 1) In a conveyor system it is sometimes necessary to operate conveyors independently for short periods of time.

- 2) Different types of conveyors are often included in the same system.
- 3) Different load spacings are often required on different conveyors within the same system.
- 4) Production requires variable speed to maintain schedules.
- 5) Most conveyors would be too long for mechanical ties driven by one unit.

"In most production lines where individual units are handled on a given clearance or work spacing, proper positional relationships take on a great deal of importance. When units have a fixed position on a given conveyor and are to be transferred to a relative fixed position on another conveyor, ... position synchronization requires a great deal of consideration.

Certain basic requirements must be met:

- 1) All load spaces on the conveyor must be equal.
- 2) Drives require a means for remote control speed changing and brakes for uniform stopping.
- 3) Controls must provide interlocking so all conveyors must run and change speed together.
- 4) Controls must set the speed of individual conveyors to maintain positional relationships, permit manual emergency operation, and provide signals to show malfunction or local stoppage.
- 5) The conveyor path and control should provide for positive transfer of carriers across gaps between independent conveyors."

Webb describes the kind of control hardware needed to implement synchronized conveyor systems, but he says nothing about how to analyze desirable speed and spacing relationships under dynamic operating conditions. In fact, the literature of Industrial Engineering is almost barren of procedures other than simulation for analyzing dynamic flow patterns. The literature of Chemical Engineering has some relevance, however; for example, analysis of the flow dynamics of both discrete and continuous processes is introduced by Campbell.⁽¹³³⁾ His presentation of the kinematics of material handling, and of fluids in motion is relevant for stability analysis of deterministic process systems with varying inputs. Some transient queueing models have also been solved, but applications to material flow problems have not been reported.

4.4. Material Handling Flow Systems: Scheduling and Operation

Ideally, when planning a material flow system the designer should also consider the scheduling and operation procedure which will be utilized. Effective operation can increase capacity and minimize costs compared to haphazard operation. The designer's responsibility in planning material flow systems includes consideration of control procedures and provision for facilitating control. The scheduling and dispatching problem exists, however, even if the flow system has already been established. Given a particular system and the current demands on the system, how can one most effectively schedule and dispatch material or material handling equipment? This problem is only one of many kinds of activity scheduling and dispatching problems. A fuller discussion of the scheduling art is appropriate to production and inventory control.

Our attention centers therefore on scheduling problems with specific material handling or transportation applications. In fact, we are concerned primarily with the scheduling of handling equipment of the discrete load type; trucks, trailers, railroads, barges, etc. In these cases, poor scheduling is evidenced by inability to meet shipment demands, together with excessive dead-heading or idle equipment because it is not in the needed location. The problem of scheduling continuous handling devices will not be discussed, partly because the author feels this is primarily a production control problem except in the case of dynamic synchronization considerations mentioned earlier by Webb and also Kwo. More important, however, is the absence of known reports of more than descriptive nature of the interaction of continuous flow systems and scheduling. A notable exception is Mandel's discussion (174) of a materials handling system designed to be an integral part of the job-shop production control system.

Morris⁽¹¹⁾ provides a basic introduction to the problem of designing fixed schedules which will optimize some criterion such as the fleet size required or the level of service provided. The approach to scheduling may be described as an inventory problem in which random demands for transportation from various locations to other destinations are to be carried out at a scheduled time. The optimum time is selected so that "costs" of waiting for the departure in a period are balanced by costs of missing departure in a period and having to wait for departure in the next period.

One version of the problem may be described as a shuttle service between two plants. Units for shipment arrive at each plant from outside the system according to (independent) time-dependent known probability distributions. The shuttle is restricted to one departure from each plant during the total time period under consideration. If travel-time between plants is assumed known and constant, what are the best scheduled departure times in the period in order to minimize total expected unit-periods of waiting for arrival units? Rules for a simpler case are derived, but solutions to this and more complex systems are not given.

Meyer and Wolfe⁽¹⁷⁹⁾ start with the same physical problem of two (plants) sources of shipping demand served by one shuttle running between the two. Instead of attempting to design a fixed schedule, however, they concentrate on the comparison of operating policies of the following kind:

- A. A free shuttle always remains at the place where it becomes free.
- B. A free shuttle always returns to the busier of the two points.

Communications is available between the points of demand and the shuttle so that it can be recalled to pick up a load.

If we define utilization as the mean aggregate demands per hour times the travel time between sources, or equivalently, the fraction of time that loads are being transported, it is shown that policy B leads to less inventory waiting time when utilization is less than 50%. If the

shuttle operates at a level of utilization greater than 50%, the additional dead-heading under policy B becomes uneconomic. It should be noted that Morris' study⁽¹¹⁾ and this approach use different criteria but otherwise attack the same problem. Meyer and Wolfe extend their study to a geographic region with uniform demand distribution. The following dispatching strategy for a fleet of shuttles is studied:

- a) If free shuttles are available, the dispatcher sends the free shuttle which can reach the call first.
- b) If no free shuttles are available, he assigns the shuttles which can reach the loads first in the order in which they placed their calls.

Expressions are derived for the distribution of load waiting times and number of shuttles required to provide a given level of service under conditions of exponential holding times and median waiting times small compared to trip times. Simulation is used to study load waiting times and deadheading for cases of several point sources with random demands, and different fleet sizes with results confirming the analytic work. Howard⁽¹⁶¹⁾ uses an example of taxi cruising to illustrate the concepts of policy optimization in Markov chains.

Dispatching, or the assignment of vehicles to routes in an optimal fashion, has been attacked in a number of different ways. Reinfeld and Vogel⁽¹⁶⁾ describe the use of approximate transportation procedures to the routing of tractor trailers at Joliet Arsenal. In a procedure similar to Metzger's⁽¹⁷⁸⁾, they show methods of dispatching tractor-trailers daily which minimizes the travel distance and determines

the number of vehicles required. The procedure considers movement between any of four sources and six destinations. The method with minor variations, is also applied to the railroad yard movements at the arsenal.

It will be useful to be explicit about the assumptions of Reinfield and Vogel's procedure, in order to contrast it to assumptions of other dispatching procedures. Given =

c_{ij} as the distance (or time) to return an empty trailer from move destination i to a move origin j

a_i is the number of empties becoming available at i

b_j is the number of empties needed at j

x_{ij} is the number of trailers returned from i to j

minimize

$$\sum_i \sum_j c_{ij} x_{ij} \quad \text{subject to} \quad \sum_i x_{ij} = b_j; \quad \sum_j x_{ij} = a_i, \quad \text{and} \\ x_{ij} \geq 0$$

This is, of course, the standard transportation formulation which can be used to minimize returntrips. The optimal empty return routing is then combined with the required loaded move-routings in order to develop complete round trips for the trailers. The procedure for doing this is logical, but the result is not unique. Notice in this approach that only full loads are moved to a destination i and only empties are carried to the next origin j . Transshipment of partial loads is precluded and i to j routings must be used for loaded moves.

A problem related to the travelling salesman procedures is called the "Truck Dispatching Problem" by Dantzig and Ramser.⁽¹³⁸⁾

The problem is described as trying to find "the optimum routing for a fleet of gasoline delivery trucks between a bulk terminal and large number of service stations supplied by the terminal. The shortest routes between any two points in the system are given and a demand for one or several products is specified for a number of stations within the distribution system. It is desired to find a way to assign stations to trucks in such a manner that station demands are satisfied and total mileage covered by the fleet is a minimum." If the capacity of a truck is at least as large as the total demand of all stations within the system, the problem is a travelling salesman type. If however, the truck capacities are not as large as the system demands, several sub-tours, or returns to the terminal may be required. The problem is formally stated thus:

- (1) Given a set of n station points P_i ($i = 1, \dots, n$) to which deliveries are made from a point P_0 called the terminal point.
- (2) A distance matrix C is given whose entries $C_{ij} = C_{ji}$ specify the distance between any two points ($i, j = 0, 1, \dots, n$).
- (3) A delivery vector B is given whose entries b_i specify the amount to be delivered to every point P_i ($i = 1, 2, \dots, n$).
- (4) Truck capacity is C (initially assumed identical for all trucks) where $C > \max b_i$.

- (5) If $X_{ij} = X_{ji} = 1$ is interpreted to mean that points P_i and P_j are paired, and $X_{ij} = X_{ji} = 0$ ($i, j=0, 1, \dots, n$) means that the points are not paired, $X_{ij} = 0$, and

$$\sum_{j=0}^n X_{ij} = 1 \quad (i = 1, 2, \dots, n)$$

since every point P_i is connected with P_0 or a most one point P_j

- (6) The problem is to find those values of X_{ij} which minimize

$$\sum_i \sum_j C_{ij} X_{ij}$$

The paper presents a procedure for obtaining an approximate solution to the problem, suitable for hand or computer computation.

A somewhat related problem with dynamic characteristics is described by Minas and Mitten.⁽¹⁸³⁾ In this particular problem, travel between points P_i ($i = 1, 2, \dots, m$) is prohibited, hence all trucks must operate through the terminal, or hub, P_0 . Unlike the previous problem, loads may originate at any P_i ($i = 0, 1, \dots, n$) with changing or dynamic demand over time. At scheduling time the immediate demands are known, but demands for the following period are known only as probability distributions. Trips, in either direction, empty or loaded, between hub terminal and points require one day. For this reason, allocations of trucks to particular loads commits the dispatcher to a pattern of available trucks at specific points to meet the next day's probabilistic demand. A two day planning horizon is used to develop a balance of shipment delay costs against the cost of running empty trucks between points and terminal. Changes in the size of the truck fleet are also considered. If

V_i = the number of trucks available at start of period at terminal i

D_i = demand in loads at terminal i

v_i = number of trucks arriving from other terminals to terminal i

A_i = expected net addition of trucks to the system at terminal i

e_i = empty trucks dispatched from terminal i

and u_i = the uncommitted balance of trucks at the end of the period at terminal i ,

then

$$u_i = V_i - D_i + v_i + A_i - e_i$$

The total costs to be minimized are the sum of:

- (1) the cost of moving empty trucks in the current period
- (2) the total cost of holding-over loads bound from the hub to outlying terminals in the current period, and
- (3) the total expected future cost of holding-over loads from the next period to the following period, which is a function of u_i .

Computational procedures for rapid calculation of schedules is given.

A number of other unrelated analytic approaches to material handling scheduling problems are reported. For example, suppose it is desired to find the minimum cost allocation of loads to shipping method. This was described earlier as a transportation problem. Shepard⁽²⁰¹⁾ applies dynamic programming to a similar problem: the daily scheduling of deliveries, where loads can be carried on first or second shift via company-owned trucks or by common carrier and wages on the second shift are higher in direct proportion to delivery time on that shift.

Liebel⁽¹⁷³⁾ surveys the (german) literature on the special problem of optimal use of locomotives and trucks in mines. Graphical representations of simple quantity-time and distance-time relationships are recommended with simulation suggested for complex cases.

Difficulties in analytic formulation of scheduling problems have resulted in many simulation studies. For example Joyner⁽¹⁴⁶⁾ reports a study on the scheduling and operation of tractors and lift trucks at Robins Air Force Base. The study revealed that radio dispatch of the material handling equipment would only add to the cost of the present system. A new system which centrally controls some floating equipment and permanently assigns others on the basis of work load in the area is recommended. It is claimed that the new system would reduce equipment idle time, reduce customer waiting time and yield annual savings in excess of \$500,000. A simulation study of tugs and barge operation on the Ohio Mississippi River System is described by O'Brien and Crane.⁽¹⁸⁵⁾ The objective was two-fold: to determine a schedule of tugs which could move the maximum number of expected barge loads arising at various parts, and second to determine the proper balance between tugs and barges.

If one considers the dynamics of road system operation, the shortest path through a network may not be a usable measure of effectiveness for scheduling the operations of a centrally controlled transportation or communications system. It is obvious for example, that if all traffic between a specific origin node and destination node were routed over the shortest path, the congestion on this path would make utilization

of some longer path more efficient. The same kind of phenomena may be observed in the behavior of highway transportation systems where the individual user is free to choose the road. Bock and Cameron⁽¹²⁹⁾ describe an electrical analog having the properties that costs are functions of traffic volume, permitting instantaneous balancing of traffic on the network in view of the predicted travel time for each route. Charnes and Cooper⁽¹³⁵⁾ also examine this problem in a network flow context.

MATERIAL HANDLING BIBLIOGRAPHY

122. Akrep, J. P. and Stambler, S., Container Size & Pallet Pattern Selection Criteria for Use on 40" X 48" Pallets, U.S. Dept. of Commerce, Office of Tech. Serv., Wash. 25, D.C., Proj. No. NT003-016.
123. Balinski, M. L., "Fixed Cost Transportation Problems", NRLQ, Vol. 8, No. 1 (March 1961) p. 41-54.
124. Beckmann, M., McGuire, C. B. and Winsten, C. B., Studies in the Economics of Transportation, Yale Univ. Press, New Haven, Conn. (1956) p. 232.
125. Bellman, R., Fukuda, Y. and Pollack, M., Technical Studies in Cargo Handling-II Computation of Delays in Multi-Stage Shuttle Process, Rept. 57-13 (April 1957) Dept. of Engr., Univ. of Calif., Los Angeles.
126. Berg, Paul O., "Gravity Flow: Biggest Problem of Bulk Material Users", Modern Materials Handling, (Sept.1959) p. 117.
127. Berrisford, H., "The Economic Dist. of Coal Supplies in the Gas Industry: An Application of the Linear Programming Transport Problem", Op. Res. Qu., Vol. 11, No. 3 (Sept. 1960).
128. Bock, F., Kantner, H. and Haynes, J., An Algorithm for Finding & Ranking Paths Through a Network, Mimeo paper presented at Nat'l. Meeting Oper. Res. Soc. of Am., (Nov. 1957).
129. Bock, Frederick and Cameron, Scott, Allocation of Network Traffic Demand By Instant Determination of Optimum Paths, Paper presented at 6th Annual Meeting of ORSA (May 16, 1958).
130. Boldyreff, A., "Determination of Maximal Steady State Flow of Traffic Through a Railroad Network", Op. Res., 3, (1955) p. 443.
131. Brown, Edward A., Generation of All Shortest Paths of a Directed Network, IBM Res. Center, Yorktown Hts., N.Y., 416 - 18th Nat'l. Meeting, ORSA.
132. Brown, K., Package Design Engineering, J. Wiley & Sons, N.Y.
133. Campbell, D., Process Dynamics, J. Wiley & Sons, (1958).
134. Carrabino, Joseph D., Containerization, Univ. of Calif., Los Angeles, Engr. Dept. Report 57-56 (July 1957).

135. Charnes, A. and Cooper, W. W., "Extremal Principles for Simulating Flows in a Network of City Streets", Proceedings-National Academy of Sciences, 44 (Feb. 1958) p. 201; Also Vol. 2 of Management Models and Linear Programming, John Wiley & Sons, N.Y., (1961).
136. Croes, G. A., "A Method for Solving Travelling Salesman Problems", Opns, Res., 6, (1958) p. 791-812.
137. Dantzig, G. B. and Fulkerson, D. R., "Computation of Maximal Flows in Networks", NRLQ, 2, No. 4 (Dec. 1955).
138. Dantzig, G. B. and Ramser, J. M., "The Truck Dispatching Problem", Mgt. Sci., 6, No. 1 (Oct. 1959) p. 53-72.
139. Dantzig, G. B., "On the Shortest Route Through a Network", Mgt. Sci., 6, (1960) p. 187.
140. Davis, H. and Weinstock, Jack K., Analysis of Stochastic Model of Cargo Handling, Department of Engineering Report 56-34, Univ. of Calif., Los Angeles (1956).
141. Erickson and Randolph, P., "Application of L.P. to Assignment of Material Handling Equipment", Journ. of I.E., 8, No. 6 (Nov.-Dec. 1957) p. 386.
142. Erisman, M. J., "Research in Bulk Material Handling", Mechanical Engineering, Paper No. 60-WA-226 (March 1961).
143. Fetter, R. B. and Galliher, H. P., "Waiting Line Models in Material Handling", Journ. of Ind. Eng., 2, No. 3 (May-June 1958).
144. Fleisher, Gerald A., "Marine Containerization, Caution!", Journ. of I.E., 11, No. 4 (July 1960) p. 338-341.
145. Flood, M. M., "Traveling Salesman Problem", Op. Res., 4 (1956) p. 61.
146. Ford, L. R., Network Flow Theory, Rand Paper P-923 (1956).
147. Ford, L. R. and Fulkerson, D. R., "Maximal Flow Through a Network", Can. Journ. of Math., 8 (1956) p. 399-404.
148. Ford, L. R. and Fulkerson, D. R., "A Simple Algorithm for Finding Maximal Network Flows and an Application to the Hitchcock Problem", Can. Journ. of Math., 9 (1957) p. 710-718.
149. Ford, L. R. and Fulkerson, D. R., "Constructing Maximal Dynamic Flows From Static Flows", Op. Res., 6 (1958) p. 419-433.

150. Ford, L. R. and Fulkerson, D. R., "Computation for Maximal Multi-Commodity Network Flows", Mgt. Sci., 5, No. 1 (Oct. 1958) p. 97.
151. Ford, L. R. Fulkerson, D. R., Flow in Networks, Princeton University Press (1962).
152. Friedman, W. G. and Kipnees, J. J., Industrial Packaging, J. Wiley & Sons, N. Y.
153. Friend, J. K., "Two Studies in Airport Congestion", Op. Res. Qtrly., 9, No. 3 (Sept 1958).
154. Fulkerson, D. R., "Increasing the Capacity of a Network: The Parametric Budget Problem ", Mgt. Sci., 5, (1959) p. 472.
155. Gale, D., "Multi-Product Transshipment Problem", Theory of Linear Economic Models, McGraw Hill Co., (1960) p. 173.
156. Gould, S., "A Method of Dealing With Certain Non-Linear Allocation Problems Using Transportation Technique", Oper. Res. Quart., 10, No. 3 (Sept. 1959).
157. Haynes, D. Oliphant, Material Handling Equipment, Chilton Co., Phila., Pa. (1957).
158. Helgeson, W., "Planning For the Use of Overhead Monorail Loop Type Conveyors", Journal Ind. Eng., 11, No. 6 (Nov. 1960) p. 488-492.
159. Hoffman and Pavley, "Method for Solution of Nth Best Path", Journ. ACM, 6, No. 4 (Oct. 1959) p. 506-514.
160. Holtzman, A. O., "Selection of Material Hand. Equip.", Eng. Library Washington Univ., St. Louis S, Mo., (1955); Also Mill & Factory, 58, No. 6.
161. Howard, R., Markov Chains and Dynamic Programming, MIT Press (1961).
162. Hu, T. C., "Maximum Capacity Route Problem", Op. Res., 9, No. 6, (Nov. 1961) p. 898-900.
163. Hunt, G. C., "Sequential Arrays of Waiting Lines", Oper. Res., 4, (1956) p. 675.
164. Jennings, N. H. and Dickens, J. H., "Computer Simulation of Peak Hour Operations in a Bus Terminal", Mgt. Sci., 5, No. 1 (Oct. 1958) p. 106.
165. Jones, H. G., and Lee, "Monte Carlo Methods in Heavy Industry", Op. Res. Qtrly., 6, No. 3 (Sept. 1955) p. 108.

166. Joyner, E. L., Materials Handling Equipment Management, Robins Air Force Base, O. R. Division Mimeo paper (Aug. 1959).
167. King, E. P. and Goode, R. J., "Application of O. R. Techniques to a Packaging Problem", Am. Soc. Qual. Cont. Proc., 14th Mid. West Conf. (1959) p. 177-82; Also in Q. Con. Abstracts, Vol. 5, No. 3.
168. Klein, M. and Milberg, S. H., "Application of L.P. to Materials Hdlg.", Modern Materials Handling, (Feb. 1955); Also Modern Materials Handling Manual, Vol. 4 (1959) p. 28-32.
169. Knödel, W. "Solution of the Multi-Commodity Transportation Problem", Unternehmensforsch., 4, No. 1 (1960) p. 1-12; Also Abstract in Quality Control Abstracts, 6, No. 2, p. 147.
170. Kwo, T. T., "Theory of Conveyors", Management Science, 5, No. 1 (Oct. 1, 1958) p. 51-72.
171. Kwo, T. T., "A Method for Designing Irreversible Overhead Loop Conveyors", Journ. I.E., 11, No. 6 (Nov.-Dec. 1960) p. 459-466.
172. Lambert, F., "Traveling Salesman Problem", Cahiers Centre d'Etudes Recherche Operationelle, 2, No. 2 (1960) p. 180-91; Also Quality Control Abstracts, 6, No. 2, p. 187.
173. Liebel, E., "Optimal Use of Locomotives & Trucks in Mines", Qualitätskontrolle + Op. Res., 5, (May 1960) p. 68-71; Also Quality Control Abstracts, 6, No. 2, p. 143.
174. Mandel, J. R., Materials Handling System for Job-Shop Production Control, A.S.M.E. paper 57-F-27 (1957).
175. Matthys, G. and Ricard, M., "Study of Maximum Flow of Ordinary Direct Freight Trains", Rev. Franc. Recherche Operationelle 4, No. 15 (1960) p. 117-46; Also Quality Control Abstracts, Interscience Vol. 6, Issue 1.
176. Mayer, H. E., Jr., "Conveyor Theory", Western Electric Engineer, Vol. 4, No. 1 (1960) p. 42.
177. Mayer, H. E., Jr., "Storage and Retrieval of Material", The Western Electric Engineer, 5, No. 1 (Jan. 1961) p. 42-48.
178. Metzger, R. W., "Linear Programming Can Cut Deadheading", Flow Mag., (May 1957).
179. Meyer, R. F., and Wolfe, H. B., "Organization & Operation of a Taxi Fleet", NRLQ, 8, No. 2 (June 1961) p. 137-150.

180. Miehle, W., "Line Length Minimization in Networks", Opr. Res., 6, (1958) p. 232.
181. Miehle, W., "Calculation of Higher Transitions in a Markov Process", Opns. Res., 6, (1958) p. 693-698.
182. Miller, C. E., Tucker, A. W., and Zemlin, R. A., "Integer Programming Formulation of Traveling Salesman Problems", Journ. ACM, 7, No. 4 (Oct. 1960) p. 326.
183. Minas, J. S. and Mitten, L. G., "Hub Scheduling Problem", Opns. Res., 6, (1958) p. 329.
184. Minty, George, "A Variant of the Shortest Route Problem", Opns. Res., 6, (1958) p. 882.
185. O'Brien, G. and Crane, R., "Scheduling of a Barge Line", Opns. Res., 7, No. 5 (1957) p. 561.
186. O'Neill, R. R., "Analysis & Simulation of Cargo Handling", NRLQ, 4, No. 3 (Sept. 1957).
187. O'Rourke, G. and Blitz, M., On the Solution to the Fixed Charge Problem, Applied Research Lab., Sylvania Elec. Sys., Memorandum 192 (Dec. 1959).
188. "Bibliography on Theory of Traffic Flow & Related Subjects", Opns. Res., 9, No. 4 (Oct. 1960; July, 1961) p. 568-574.
189. Polentz, L. M. and O'Neill, R. R., "The Monte Carlo Method for Planning a Handling System", Mat. Hand. Eng., 15, No. 1 (Oct 1959).
190. Pollack, M., "Some Studies on Shuttle and Assembly Line Processes", Naval Research Logistics Quarterly, 5, No. 2 (1958) p. 125.
191. Pollack, M., Technical Studies in Cargo Handling-V-Effect of Communication on a N-Stage Shuttle Process, Dept. of Engr. Report, Univ. of Calif., Los Angeles, (May 1958) p. 58-43
192. Pollack, M. and Wiebenson, W., "Solution of the Shortest Route Problem--A Review", Op. Res., 8, No. 2 (March 1960) p. 224-230.
193. Pollack, M., "The K-th Best Route Through a Network", Op. Res., 9, No. 4 (July 1961) p. 578-780.
194. Power Crane and Shovel Association, Functional Design, Job Analysis of Power Cranes & Shovels, Tech. Bul. No. 1.

195. Rice, Robert S., "How to Measure Fork Truck Performance", Factory, 112, No. 4, (April 1954) p. 84-93.
196. Sayre, H. S., "Selection and Use of Pneumatic Conveyors", Materials Handling Engineering, (Sept. 1959) p. 66.
197. Schell, E. D., "Distribution of a Product of Several Properties", Proceedings of 2nd Symposium on Linear Programming, Directorate of Management Analysis, Bureau of Standards, Washington, D.C. (1955).
198. Schiller, Donald and Lavin, M. M., "Determination of Requirements for Warehouse Dock Facilities", Op. Res., 4, (April 1956).
199. Shapley, L. S., "On Network Flow Functions", NRLQ, 8, No. 2, (June 1961) p. 151-158.
200. Shetty, C. M., "A Solution to the Transportation Problem with Non-Linear Costs", Op. Res., 7, (Sept.-Oct 1959) p. 571.
201. Shephard, R. W., "Daily Minimum Cost Planning of Local Deliveries", Unternehmungs Forsh., 4, No. 2 (1960) p. 50-67; Also Quality Control Abstracts, 6, No. 2, p. 175.
202. Smith, S. B., "Waiting Line Theory; How to Put it to Work for You", Mod. Materials Handling, 14 (Feb. 1959) p. 94-96.
203. Wagner, H. M., "On a Class of Capacitated Transportation Problems", Mgt. Sci., 5, No. 3 (April 1959) p. 304-318.
204. Webb, Jervis, B., "Synchronization of Conveyor Drives", ASME Paper 60-WA-223 (Sept. 1960).
205. Weldon, Foster, L., "Cargo Containerization in the West Coast Hawaiian Trade", Op. Res., 6 (1958) p. 649.
206. Wilkinson, R. I., "Queueing Theory and Some of its Industrial Uses", Nat'l. Convention Trans., Am. Soc. Qual. Cont., (1958) p. 313-330.
207. Williams, K. B. and Haley, K. B., "A Practical Application of Linear Programming in the Mining Industry", Op. Res. Qtrly., 10, No. 3 (Sept. 1959) p. 131.
208. Winter, F. C., Materials Handling Transportation Fork Truck vs Trailer Train, A.S.M.E. Paper 60-WA-101 (1960) Winter Meeting.
209. Zimmerman, O. T. and Lavine, T., "Cost Data On; 1) Power Shovels, Hoes, Cranes, Draglines, 2) Power Industrial Trucks, 3) Non-Powered Industrial Trucks", Cost Engineering, 3, No. 3 (July 1958); 3, No. 4 (Oct. 1958); 4, No. 1 (Jan. 1959).

V PLANT LAYOUT

Assuming that initial studies of equipment requirements have been completed (see Section III) and that methods for evaluating material flow problems are at hand, the designer must consider how to most effectively use the space available. Again we point out that while we may choose to discuss this phase of the facility planning effort as an independent aspect, this is clearly an artificial stratagem. The location of equipment affects the material flow paths through the facilities just as altering the flow paths may affect the material handling equipment capacity, and perhaps the utilization of the fixed equipment as well. Our attention therefore is directed to describing the current procedures for assigning fixed equipment to available locations in order to achieve (perhaps multiple) layout objectives. We attempt to contrast the many alternative approaches, in order to aid the reader in understanding the assumptions, and hence applicability of the procedures. Furthermore, as we pointed out in Section II in Plant Location, there exists a mutual similarity between certain classes of Plant Location problems and at least a restricted class of plant layout problems. Because both problems are concerned with problems involving Euclidean space, the reader with particular interest in either area is advised to carefully consider ideas from the other.

The problems of plant layout have received ample attention from many authors. Some of the recent books in the field are those by Moore,⁽¹⁰⁾ Muther,⁽¹⁴⁾ and Reed.⁽¹⁵⁾ Buffa⁽³⁾ gives a concise presentation of the

area in one chapter of his text. For readers seeking a complete knowledge of the applied problems, reference to these texts is suggested. The broad interpretation of plant layout implies a synthesis of all elements of the production system into an economic facility, with capacity and services necessary to produce a desired end-product. Investigation of this broad integrating aspect of plant layout, will be deferred until Section VI.

Our orientation in this section is toward an examination of the (conceptually) fixed networks of processing facilities. Our concern is specialized to examination of those aspects of plant layout design which are related to locating equipment and services in a desirable manner. We are interested in the topology of the relationship among the location of machines, the spatial flow paths between the machines considered as nodes of the fixed network. We may think of processing layouts as consisting of a number of feasible points for transition from a fixed node to other nodes, or other networks (e.g. material handling system). There may be positive or negative "weights" assigned to links between fixed nodes, which are functions of the importance of locating facilities close together, or separated. There are specified physical and chemical restrictions assigned to each node restricting the set of inputs which it will accept and also the output transitions to other nodes or networks. Our plant layout problem is descriptively that of locating the nodes of the fixed facility network in three dimensional Euclidean (bounded) space, in such a way as to optimize the link "weights" within the constraints of the physical and chemical capabilities. In the terminology of the layout designer, poor

layouts are typified by inadequate equipment, poor flow patterns (back-tracking), congestion and crowding, scattered buildings, crooked aisles and poor "flexibility". We turn therefore to these specific problems, with emphasis first on determination of space requirements, and then on "optimal" location of fixed facilities.

5.1 Space Requirements

Muther⁽¹⁴⁾ devotes a chapter to a description of five basic ways in which to determine space requirements for a layout. The methods are described as 1) aggregation of individual space requirements to obtain a project total; 2) adding to or decreasing an existing layout to obtain the new project total; 3) use of space standards; 4) rough estimates; 5) projection of trends. The reader is referred to Muther for details on these practical methods. Space standards for offices are specified by General Electric.⁽²¹⁷⁾ Greene⁽²¹⁹⁾ describes an approach to office layout by a method using projections, and Wilcox⁽²³⁴⁾ reports on procedure for estimating area requirements for maintenance shop which is based on the man-power factor. Procedures for incorporating a shift factor, or for two shift, seven day operation are also described. Studies using methods not included in one of the five categories given by Muther are unknown to the writer.

A discussion of total space requirements is contingent upon our earlier investigations into capacity planning. The number of individual equipments needed is obviously a function of the capacity required and hence the total floor space required is also. The optimal number of stations obtained in balancing an assembly line is largely dependent on

cycle time, the inverse of the production rate. Clearly, the space requirements increase with the number of operators or stations necessary to meet the desired production rate. Correspondingly, there is a similar relationship between space required for in-process inventories and the ultimate capacity to be achieved. For example, the dramatic reduction in the maximum attainable throughput which results from limiting inventories between queueing service facilities in sequence is demonstrated by Hunt.⁽¹⁶³⁾ Koenigsberg⁽²²¹⁾ reviews the problem of in-process inventories on production line performance and presents an application to the design of transfer equipment. Studies on the optimal location of in-process inventories have been reported by Ackoff,⁽²¹⁰⁾ Simpson,⁽²³²⁾ and Hanssmann.⁽²²⁰⁾ Although this problem is usually treated as an inventory control problem, the layout planner must consider the implications of such studies on his spatial requirements.

5.2 Location of Facilities

The establishment of the spatial location of facilities which is most "desireable" in a particular layout problem is the distinguishing characteristic of plant layout work. An attack on the overall gross or departmental relationships and locations is usually recommended before consideration of individual machine location or detailed studies within each department. Muther suggests a preliminary consideration of the flow of materials, information or people in order to determine the flow need for juxtaposition of facilities. Considerations of physical characteristics of processes may dictate specific activity relationships, or

relative link weights in our classification scheme. These weights may conceptually be attractive (+) or repulsive (-) depending on whether two facilities should be located contiguous (+) on the basis of the nature of the activities (e.g., supervision and operations) or be separated (-) (forging and cleaning departments). Some facilities may be shared (+) whereas others must be isolated for safety (-). After overall considerations, adjustments are made to accommodate specific area requirements, and finally, the specific departmental locations can be established. The procedure is of course repeated in greater detail within each department to then locate individual machines with their more rigidly defined area configurations.

Procedures for assimilating the essential data, and for displaying the key activity relationships for analysis are many. The majority of them draw heavily on the human ability to quickly recognize qualitative spatial considerations by using schematic diagrams of great ingenuity. If the number of relationships under simultaneous consideration are not excessive, it is difficult to improve on the results of these schemes by any known quantitative procedures. When the number of facilities and products under consideration becomes large, however, considerations of more than some arbitrarily selected sample is precluded by schematic means. As a result, a complex layout is often a compromise between analysis and judgment, seasoned with the hope that all important factors have been considered. In the final stages of design, visual aids, including models have been resorted to (see Mallick⁽²²⁵⁾ and Paton⁽²³⁰⁾).

The methods developed for optimally locating facilities all have used in general the minimization of some measure of material handling cost as the objective. If we defer examination of layout criteria until Section V, we can quickly summarize the existing developments.

One broad class of methods attempts to elaborate on process charting concepts to optimally relate a number of products and facilities into sequential line flow. The idea was first proposed by Ireson,⁽⁸⁾ and expanded by Young⁽²³⁷⁾ to include consideration of the capacity required for cyclic scheduling. Noy⁽²²⁹⁾ also begins with a process chart tableau but arranges the facilities in line sequence with the help of weighting factors. An approach similar to both Noy's and Young's was apparently developed independently by Michel.⁽²²⁶⁾ We present this example to illustrate the process chart approach. A shop with nine types of machines produces three articles. Two new items are to be added to the line, which would load some of the present machines beyond their capacity. The problem is to re-layout the shop to minimize distance between successive machines, to minimize back-tracing, and to avoid acquiring new machines. Two hundred hours of operation is assumed per month.

TABLE I

MACHINE TYPE REQUIRED FOR OPERATION NUMBER

Articles	1	2	3	4	5	6	7	8
a	A	B	C	D	A	E	F	G
b	A	B	C	M	D	A	E	D
c	A	C	D	A	E	D	G	-
d	B	C	M	A	F	D	G	H
e	B	C	D	M	A	E	F	D

Step 2 Calculate Table II:

TABLE II

MONTHLY WORK LOAD BY TYPE OF MACHINE

	Operation Number								Total Load	Number Machines
	1	2	3	4	5	6	7	8		
A	340			100	50	40			530	3
B	80	120							200	1
C		210	150						360	2
D			100	80	50	290		160	680	4
E					170	240	90		500	3
F					40		50		90	1
G							300	30	330	2
H								170	170	1
M			80	50					130	1

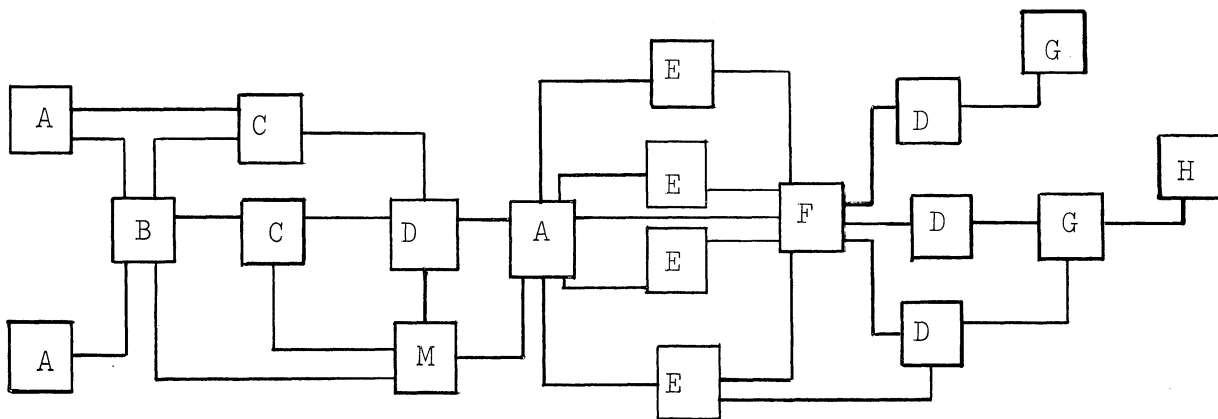
If pieces move regularly as if on production line, with the machines in order, then the figures in columns 1-8 of Table II will lie nearly in the diagonal. Table III is such an arrangement.

TABLE III

OPERATION NUMBER

	1	2	3	4	5	6	7	8	Total Monthly	
A	340								340	2
B	80	120							200	1
C		210	150						360	2
M			80	50					130	1
D			100	80					180	1
A				100	50	40			190	1
E					170	240			410	3
D					50	290			340	2
F					40		50		90	1
E							90		90	1
G							300	30	330	2
D								160	160	1
H								170	170	1

This analysis leads to the following layout:



Muther's Multiple Product Flow Chart gives the following Table IV, showing the backtracking:

TABLE IV

Product No.

		a	b	c	d	e
1	A	○	○	○		
2	B	○	○	○	○	○
3	C	○	○	○	○	○
4	M	○	○	○	○	○
5	D	○	○	○	○	○
6	A	○	○	○	○	○
7	E	○	○	○		○
8	D		○	○		○
9	F	○		○	○	○
10	E	○		○		○
11	G	○		○	○	○
12	D				○	○
13	H				○	

The limitations of the multiple product flow chart for analysis when the number of products is large, and when therefore a process type layout might be more usual, were circumvented by Cameron's travel chart technique⁽²¹⁴⁾ for analyzing facility flow relationships. The technique subsequently has been elaborated on by Smith,⁽²³³⁾ Lundy,⁽²²⁴⁾ Llewellyn,⁽²²³⁾ Anderson and Reis,⁽²¹¹⁾ and a punched card procedure by Schneider.⁽²³¹⁾ Although the travel chart is helpful in analyzing the importance of flows between a number of facilities for multiple products, it is no help in actually proscribing the spatial locations for the facilities or departments, nor does it include the location-department move-distance as an element of the optimal location criterion. Wimmert^(235,236) emphasized the value of including this location dependent distance criterion and then presented a method for locating n facilities in n available locations so that the product of the move-volume \times distance among the assigned facilities is minimized. In his dissertation he pointed out that the method does not guarantee optimality, as did Conway and Maxwell;⁽²¹⁵⁾ direct application of the method is very tedious by hand, and in application to a relatively small practical problem, would quickly stretch the capacity of current computers. The current status of the facility assignment problem posed by Wimmert and formalized by Conway and Maxwell may be summarized as follows: if X is a matrix of values (x_{ij}) equal to the move-volume \times distance between the i^{th} pair of facilities when assigned to the j^{th} pair of locations, find the set of independent elements of X corresponding to an assignment of facilities to locations and whose sum is a minimum.

Further if $(x_{ij}) = A_i \times B_j$ where A_1, A_2, \dots, A_n is the monotonic decreasing sequence of length of paths between pairs of locations, and B_1, B_2, \dots, B_n is monotonic increasing sequence of move-volumes between pairs of facilities, the minimum sum set of n elements of X lies along the diagonal. If this set is not a feasible assignment, an optimal assignment can be found only by search.

An alternative statement of the facility assignment problem in quadratic programming form is given by Armour⁽²¹²⁾ as follows:

N = number of facilities (and locations)

V_{ij} = number of unit loads from facility i to facility j

U_{ij} = cost per unit load per unit distance from facility i to facility j

I_{ij} = distance from location i to location j

$Y_{ij} = U_{ij}V_{ij}$ = total cost of moves from facility i to facility j per unit distance

Assuming all departments are represented as points and all available locations are known:

Let $x_{ik} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ if department i $\begin{pmatrix} \text{is} \\ \text{is not} \end{pmatrix}$ in location k .

Then, with the volume between facility i_1 and i_2 , $v_{i_1i_2} \geq 0$,

and the distance between location k_1 and k_2 , $I_{k_1k_2} \geq 0$

minimize $\sum_{k_2} \sum_{k_1} \sum_{i_2} \sum_{i_1} x_{i_1i_1} x_{i_2k_2} v_{i_1i_2} I_{k_1k_2}$

subject to $\sum_i x_{ik} = 1$ (all departments assigned)

and $\sum_k x_{ik} = 1$ (all locations occupied)

The only formal solution methods known for the problem in this form are given by Gilmore⁽²¹⁸⁾ and Lawler.⁽²²²⁾

Instead of considering the problem of locating n facilities in x locations in order to minimize the total move-volume distance among the n facilities, Moore^(227,228) has developed an alternative formulation. In his approach, n new facilities are to be assigned to n locations in such a way that the move-volume distance between each of the n new facilities and m existing facilities is to be minimized. Flows between the new facilities are not considered, and only point locations, not areas are considered. A plant location problem seems more readily defined in this way than plant layout. In this form, an optimal solution is readily obtained by means of the assignment algorithm.

All the procedures so far described have assumed that facilities or departments can be represented satisfactorily by points in space. Buffa⁽²¹³⁾ systematized the steps necessary to then consider the additional complexity involved when departmental areas, rather than just points, must be optimally integrated. Using move-distance measured along rectangular coordinates as a criterion, Armour⁽²¹²⁾ develops a heuristic algorithm and computer program for relative location of facilities. Because of the potential usefulness of his contribution, we present his approach in considerable detail. The following is assumed known:

n = number of departments (max 20)

$L_{k,l}$ = an initial layout matrix, arbitrarily chosen, which assigns a department identification key to each coordinate point

(k, l) of a two dimensional map of the layout. The number of points assigned each department is a measure of the area of that department.

$V = (v_{ij})$ the number of unit loads moving between departments i, j
 $U = (u_{ij})$ the cost to move a unit load a unit distance between departments i, j

An IBM 7090 computer program performs the following steps:

- 1 The Y matrix can be calculated, where:

$Y = (y_{ij}) = (u_{ij}v_{ij})$ the total move cost per unit distance between departments i, j in the period.

- 2 Next, the L matrix is calculated for the current layout, where:

$L = (l_{ij})$ the Euclidean rectangular distance between centers of each department.

- 3 Calculate the cost of transportation contributed by any two departments e and f where:

$$C = \sum_{j=1}^n l_{ej}y_{ej} + \sum_{j=1}^n l_{fj}y_{fj} .$$

- 4 Exchange locations of departments e and f . (If the two departments are of unequal area, configuration of contiguous departments are adjusted to accommodate the exchange). Determine the "best" new L

- 5 Calculate the new $C^1 = \sum_{j=1}^n l_{ej}y_{ej} + \sum_{j=1}^n l_{fj}y_{fj} + l_{ef}y_{ef}$ and then $(C^1 - C)_{ef}$

- 6 Repeat steps 3 through 5 for every department ($f = 1, \dots, n; f \neq e$) and y until the maximum positive $(C^1 - C)_{ef}$ is found.

If none, the search is completed. If one is found exchange e and that f and return to step 2.

The program is a logical procedure for obtaining the trial layouts which the engineer would make himself if time and cost were not prohibitive. Armour also emphasizes the importance of timely as well as efficient layout work. The use of this program makes the study of alternative layout proposals fast as well as exhaustive.

Although classic in its descriptive formulation, one plant layout problem has received relatively little attention of a quantitative nature. Methods for selecting the proper products to produce on a continuous line remain primitive in view of the publicized advantages of line production. Muther⁽¹⁴⁾ suggests a Pareto-type curve of product activity as one basis for analysis. Deming⁽²¹⁶⁾ proposes switching to line production whenever it maximizes the intensity of use of capital resources.

We conclude with a location problem of the following kind: given a number of fixed locations, design a system of connecting links which will be of minimum total length. Such a problem in more complex form, arises in the design of communication systems, power and service utilities and fixed path material handling systems, where the length of the system is a useful measure of effectiveness. Miehle⁽¹⁸⁰⁾ presents three alternative approaches to solutions: a mechanical analog, a numeric-analytic method, and a soap-film method.

PLANT LAYOUT BIBLIOGRAPHY

210. Ackoff, R. L., "Production and Inventory Control in a Chemical Process", Op. Res., 3, No. 3 (Aug. 1955) p. 319-338.
211. Anderson, G. E. and Reis, I. L., "Relative Importance Factors in Layout Analysis", Journ. I.E., 11, No. 4 (July 1960) p. 312-315.
212. Armour, G., A Heuristic Algorithm and Simulation Approach to Relative Location of Facilities, Unpublished Ph.D. Dissertation University of California at Los Angeles (1961).
213. Buffa, Elwood S., "Sequence Analysis for Functional Layouts", Journ. I.E., 6, No. 2 (March 1955) p. 12-13, 25.
214. Cameron, D. C., Travel Charts, Mod. Mat'l. Handling, 3, No. 1 (Jan. 1952) p. 37-40.
215. Conway, R. W. and Maxwell, W. L., "A Note on the Assignment of Facility Location", Journ. I.E., 12, No. 1 (Jan. 1961) p. 34-36.
216. Deming, D. D., "When to Shift to Straight Line Production", Harvard Business Review, 37, No. 6 (1959) p. 62-68.
217. General Electric Co., Manufacturing Services, An Approach to Office Planning, Schenectady, N.Y. (1957) p. 102.
218. Gilmore, P. C., "Quadratic Assignment Problem", Journ. SIAM, 10 (June 1962).
219. Greene, Richard M., Space Analysis for Management, System Development Corp., Santa Monica, Calif. (May 1959) Booklet SP-76, p. 135.
220. Hannsmann, F., "Optimal Inventory Location & Control in Production & Distribution Networks", Opn. Res., 7, No. 4 (July 1959) p. 483-498.
221. Koenigsberg, Ernest, "Production Lines and Internal Storage--A Review", Management Science, 4, No. 4 (1959) p. 413-33.
222. Lawler, E. L., "Quadratic Assignment Problem", Mgt. Sci., 9 (July 1963)
223. Llewellyn, Robert W., "Travel Charting With Realistic Criteria", Journ. I.E., 9, No. 3 (May 1958) p. 217-220.
224. Lundy, J. L., "A Reply to W. P. Smith", Journ. I.E., 6, No. 3 (May 1955) p. 9.

225. Mallick, R. W. and Sansonetti, J. H., "Templet or Model for Plant Layout", American Machinist, 90, No. 17 (Aug. 1946) p. 101-104.
226. Michel, Pierre, "Method of Laying Out a Production Line", (in French) Gestion, 3 (Dec. 1960) p. 441-44; See Statistical Abstracts, 6, No. 7, p. 759.
227. Moore, James M. and Bindshedler, A. E., "Optimal Location of New Machines in Existing Plant Layouts", Journ. I.E., 12, No. 1, (Jan. 1961) p. 41-48.
228. Moore, James M., "Optimal Location for Multiple Machines", Journ. I.E., 12, No. 5 (Sept. 1961) p. 307-311.
229. Noy, Peter C., "Make the Right Plant Layout Mathematically", American Machinist, 101, No. 6 (March 1957) p. 121-125.
230. Paton, Bruce L., "Models as a Design Tool", Mechanical Engineer, (Nov. 1956) p. 1019-1024.
231. Schneider, Marshall, "Cross Charting Techniques as a Basis for Plant Layout", Journ. I.E., 11, No. 6 (Nov. 1960) p. 478-483.
232. Simpson, K. F., "In Process Inventories", Opns. Res., 6 (1958) p. 863-873.
233. Smith, W. P., "Travel Charting-First Aid for Plant Layout", Journ. I.E., 6, No. 1 (Jan. 1955) p. 13-15.
234. Wilcox, W. W., Method of Estimating for a Shop Whose Area is Sensitive to Man Power Factor, United Airlines, Facilities Planning Mgr., San Francisco, Calif., Private Correspondence (Feb. 1962).
235. Wimmert, Robert J., Quantitative Approach to Equipment Location in Intermittent Manufacturing, Unpublished Ph.D. Thesis, Purdue Univ. (1957).
236. Wimmert, R. J., "Mathematical Method of Equipment Location", Journ. I.E., 9, No. 6 (Nov. 1958) p. 498.
237. Young, H. H., Functional Layout Analysis, Purdue Univ. Mimeo Paper (March 1960)

VI FACILITY DESIGN SYNTHESIS AND EVALUATION

Our presentation thus far has attempted to isolate independent aspects of facility planning such as material handling and location, in order to treat each of them in depth. The essential aspect of the design activity however is the creation of a unified facility which relates these independent aspects to obtain the most effective overall system. In this section therefore, we indicate some of the measures of effectiveness proposed for guiding and evaluating the design process, and then examine the methods reported in current use. The reader will recognize that many ideas about measures of effectiveness have already been explicitly developed in the context of special portions of earlier sections. For completeness we call attention to those in Section 4.3 on material handling systems and in Section 5.2 on facility location. The selection of one or more objective criteria was implicit in each of the techniques described and in some, linear programming models for example, explicit formulation and a single valued function was required. The simplicity of using single values as measures of effectiveness for any engineering design is appealing but certainly impossible in the present state of the facility planning art. In fact, the number of criteria which are in use or proposed is so large that the facility designer may be using many criteria of no real importance in order to guard against omission of the significant. As a result, the final evaluation of all factors in practice is usually made by judgment or group opinion.

We begin by examining some of these measures of effectiveness while simultaneously attempting to relate the previous sections to this. The objectives of good facilities planning have been succinctly stated by Muther⁽¹⁴⁾ as: "an arrangement of work areas and equipment which will be the most economical to operate and yet safe and satisfying for employees." A comprehensive list of 20 major factors is also offered as an aid in selecting a most suitable plan, with general descriptions serving to more precisely identify each of the 20 factors. A different set of about 75 factors recommended for evaluating the effectiveness and taking steps for improvement of existing layouts is proposed by Apple.⁽²³⁸⁾ Many others have also been proposed, but the completeness of such check lists has been achieved by adding many qualitative and often imprecise measures of effectiveness. Words which appear frequently are "satisfying," "flexible," "safe", "expandable," "compact," and "congestion." The necessity for consideration of these intangible and qualitative factors of designs has served to emphasize the difficulty of the design evaluation and also to resist reduction in the number of factors to be evaluated for fear that key aspects will be overlooked. Nevertheless, some schemes for quantifying layout evaluation are reported, such as weighted factor analysis similar to job evaluation techniques, ranking methods, or paired comparisons. Gantz and Pettit⁽²⁵¹⁾ propose ten measurements which are quantifiable as a basis of evaluating layouts. The following data is necessary:

a = the total distance a part moves without manual handling

b = the total distance a part moves

d = the sum of vertical movements by gravity

e = the sum of vertical distances a part moves

f = total percent automatic machine idle time where individual
machine idleness < 50%

g = total number of operators on automatic machines

h = same as f except individual machine downtime > 50%

j_1 = number of machines which can be moved to a new location on the
same part line in one shift

j_2 = same as j_1 except can be moved to any other location

k_1 = total number of machines working on part in its line

k_2 = total number of machines

m = extreme machine length

n = extreme machine width

p = total work area required by operator

q = total layout floor area

r = total aisle area

u = total area occupied by temporary or controlled storage

v = max-volume occupied by material storage

w = total volume available for storage

The measures of effectiveness suggested are:

- 1) Maximize a/b
- 2) Minimize b
- 3) Maximize d/e
- 4) Maximize $f/100g$
- 5) Maximize $h/100g$

- 6) Maximize j_i/k_i ($i = 1,2$) [given as a measure of "flexibility"]
- 7) Maximize $\frac{(m+2)(n+2)+p}{q-(r+u)}$
- 8) Minimize r/q
- 9) Maximize $\frac{q-u}{q}$ [called "index of storage space"]
- 10) Maximize v/w

Methods of designing a facility to achieve an optimal balance between these criteria, or a method for weighting the importance of the criteria, are not reported in this paper or in the effort to quantify measures of effectiveness by Bright.⁽²⁴³⁾ Bright proposes using the following:

- c = payroll of personnel assigned to material handling
- t = total operating workforce payroll
- lt = payroll lost due to time of direct labor spent on material handling
- tm = total number of moves
- tp = total production operation cycle time
- tt = actual production output per period
- ua = actual production output per period
- ut = theoretical production capacity per period
- rt = theoretical optimum total aisle space

Bright's suggested measures of effectiveness are:

- 1) Minimize c/t
- 2) Minimize $lt/(t-c)$
- 3) Minimize tm/tp
- 4) Maximize tc/tt
- 5) Maximize v/w
- 6) Maximize ua/ut
- 7) Minimize $\frac{r-rt}{r}$

Criterion 1 is also recommended by Beardslee.⁽²⁴²⁾ Other cost criteria specific to computer installations are proposed by Larsen⁽²⁶²⁾ in an article which then suggests ways of combining such costs into a single valued function. The importance of cost or economic criteria is emphasized by Muther, Bright, and is also implicit in the criteria suggested by Gantz and Pettit above. Explicit approaches to the measurement of costs must derive from work measurement and accounting costs of existing facilities, or from estimates of future costs. The measurement of materials handling work and associated costs has consistently eluded most firms, if published reports are correct. In a survey, Factory⁽²⁴⁸⁾ reports that few plants can pin-point material handling costs which are usually buried in the accounting system. A few of the work measurement problems in materials handling are outlined by Brown.⁽²⁴⁴⁾ Some yardsticks for new plant construction costs can be obtained from construction cost figures experienced by other plants.⁽²⁴⁷⁾ When one recognizes the difficulty of obtaining useful measurements for evaluating performance and costs of existing facility designs, the difficulty of economic evaluation (such as Weldons' "Cargo Containerization Study"⁽²⁰⁵⁾) of designs not yet in being is apparent.

The process of facility design when properly executed utilizes a systems engineering approach. Sims⁽²⁶⁴⁾ defines this as:

"The consideration of material handling, plant layout, and packaging problems as a part of the total system of moving material, product, and paperwork, from source to consumer into, through, and out of the manufacturing and distribution system."

Harvey⁽²⁵⁵⁾ describes systems engineering as "... an activity which investigates, analyzes, and designs, with a view to the relationship of each function to the total business. As applied to materials handling for example, it recognizes no artificial barriers between the warehouse and the rest of the business, between materials handling and paperwork handling, or between any of these operations and the types of information that management needs in the control of its business. It presents an appeal for ideas before hardware. Application of the systems concept promises these benefits:

1. Increased efficiency of existing operations without additional mechanization.
2. Economic justification of additional mechanization by employing equipment more efficiently.
3. Permits greater use of automatic equipment and controls.
4. Broadens the scope and effectiveness of management planning."

It seems safe to predict however that the explicit rules for carrying out the creative steps required to originate and integrate complex handling and processing equipments with humans are still years away. The systems approach therefore is a conceptual framework for meaningful analysis of portions of the facility planning problem. The final design synthesis can then be carried out using objective evaluation for these measures of effectiveness which can be efficiently quantified and subjective evaluation as a check or supplement on others. The mass of engineering detail essential to the final sound operation of a facility precludes objective evaluation of all aspects of a facility during design. Instead, the

judgment and experience of many people, both operating and engineering, can effectively contribute new ideas and also evaluate suggested designs. The experience at Westinghouse⁽²⁵⁰⁾ is testimony to the value of this approach. Furthermore, models^(225,230) can be of substantial value as visual aids in this "jury-of-informed-opinion" approach.

The efficient division of work between such juries and quantitative methods is therefore constantly open to question, particularly as new methods such as simulation become available. Measures of effectiveness which, prior to simulation, were evaluated by subjective opinion or not at all, can now, in part at least, be evaluated with quantitative means. The use of simulation in certain industries for investigating a broad range of operating behavior particularly in scheduling, is probably extensive. The use of simulation for study of industrial plant layout-materials handling problems, however, has not been widely reported. The following is a representative list of applications in facilities planning:

1. Study of open hearth furnace productive capacity using different numbers of charges and the effect of routines for "unbunching" the time at which furnaces need charging.⁽¹⁶⁵⁾
2. Effect of loading platform length on waiting times of busses and passengers⁽¹⁶⁴⁾ (N.Y. Port authority).
3. Evaluation of lost production time of two machines and one operator attendant with variable set-up times⁽²⁵³⁾ (Lukens Steel).
4. Effect of times of pickup, transport loaded, release, and return empty, together with number and arrangement of links, number of

- transporting agents per link, storage permitted, and unit load size, on the capacity of a cargo handling system⁽²⁶³⁾ (UCLA).
5. Analysis of the effect of scheduling and dispatch rules on cycle time, work-in-process inventory, and delivery reliability on job-ship operations^(241,254,255) (GE Co.).
 6. A detailed description of the IBM 704 GEST simulation program can be found in References 258 and 259, and an application to the evaluation of nine different facility designs is given by Knight⁽²⁶¹⁾ (GE Co.).
 7. A capacity study of a conveyORIZED flow shop to determine needed additional equipment, storage space, and manpower⁽²⁵⁷⁾ (GE Co.).
 8. Warehouse design to determine building size and rack spacing to determine bottlenecks, number of trucks needed, and the effects of peak loads⁽²⁴⁹⁾ (IBM Corp.).
 9. Compute the most economic packaging and containerization for materials going into bin storage and to assign an appropriate storage area for each package⁽²⁶⁰⁾ (IBM Corp.).

The writer is aware of simulations which have been prepared and/or used by a number of other companies in the process of facility planning:

Westinghouse Electric Corporation, Eastman Kodak Company, Hughes Aircraft, United Airlines (see References 239 and 240 for examples). General purpose simulation compilers are also available (e.g. IBM) and the electrical utility industry has simulations for generating-station design and system design.⁽²⁴⁶⁾ Each of these simulations, however, is created primarily to

consider the time dependent (short-run) behavior of the system under study. The space-dependent measures of effectiveness which characterize the facility design problem are absent in almost all general simulations. Some work toward making use of the computer for the special nature of facility design is however, beginning to appear. For example, Armour's work⁽²¹²⁾ in using the computer for achieving heuristically a good area layout has been noted (Section 5.2). Huffman⁽²⁵⁶⁾ uses analysis and simulation to evaluate the cost of operating an idealized warehouse sub-system considered as a storage block and an overflow area. Two space-use policies are considered: 1) space is allocated to each commodity and 2) any commodity is stored in any space. Equations are derived for estimating the necessary sub-system capacity using criteria of: a) average system inventory, b) flow/space unit/unit time and c) the probability that the system inventory will exceed space allowed. Criterion c) is found to be ineffective as a design criterion. The effect of variation of space and block configuration on material handling effort is studied, and procedures for planning storage blocks which minimize total costs of space and handling are given. The row capacity of a block was found to have a minor cost effect under the homogeneity assumptions on the commodities. Regardless of the significance of the conclusions drawn, the study is a good example of the kind of investigations needed to enlarge our basic knowledge of space dependent system behavior. A somewhat different attack is used by Wilson⁽²⁶⁷⁾ in an attempt to devise rules for computer checking the engineering feasibility of a complex facility design. An exciting design tool for the layout

designer is the cathode ray display equipment used by Souder et al. (265) in the study of hospital planning. In spite of the still untapped potential which computers offer to facility designers, some factors will of necessity continue to be left open to judgment or subjective evaluation, if only for economic reasons. The explicit inclusion of all possible engineering details needed for facility design in a series of computer analyses defies the imagination. The role of creative design is not among the immediate accomplishments expected of this generation of computers, nor is the quantitative modelling of all behavioral characteristics of the "man" element in the system an accomplished fact. The power of computer analysis will continue to improve, but the ability to synthesize large systems most effectively and efficiently still belongs to the human.

MEASURES OF EFFECTIVENESS BIBLIOGRAPHY

238. Apple, James M., "How to Spot Plant Layout Problems", Factory (Jan. 1959) p. 212-216.
239. A.I.I.E., Report of System Simulation Symposium, (1958).
240. A.I.I.E., Report of Second System Simulation Symposium, (1960).
241. Barnes, W. E., Medium A-C Motor Simulation Denomstration, Mimeo Paper General Electric Co. (undated).
242. Beardslee, W. W., "Manufacturing Cycle Analysis", Materials Handling Manual, No. 4 (1959).
243. Bright, James R., Management Guide to Productivity, Yale & Towne Mfg. Co. (1961) 25 pages.
244. Brown, J. A., "Measuring Materials Handling Work", Mechanical Engineering (Dec. 1961) p. 66-69, Paper No. 61--Prod. 17.
245. Dickie, H. F. and Thronsen, E. C., "Manufacturing Systems Simulation", Factory, 118, No. 10 (1960) p. 114.
246. Dillard, J. K. and Everett, J. L., "Simulation of the Steam Power Plant", Westinghouse Engineer, (Nov. 1961) p. 174-179.
247. Factory, "How to Divy Up New-Plant Money", (Dec. 1958) p. 50-57.
248. Factory, "Material Handling in Atlanta, City of Contrasts", (Nov. 1960)
249. Factory, EDP Turns Architect, (Dec. 1961) p. 127.
250. Gagliardi, E. J. and Webber, W. W., "A Foreman Joins The Layout Planning Team", Factory (Oct. 1960) p. 106-107.
251. Gantz, S. P. and Pettit, R. B., "Plant Layout Efficiency", Modern Materials Handling, 8, No. 1 (Jan. 1953) p. 65-67.
252. Graveley, O. W., "New Way to Measure Plant Operations", Factory, (Dec. 1961) p. 69-73.
253. Green, P., Calhoun, S. R. and Haines, I. L., "Solving Your Plant Problems by Simulation", Factory, 117, No. 2 (Feb. 1959) p. 80.
254. Guernsey, P. N., Simulation in the Medium AC Motor & Generation Dep't, Mimeo G. E. Co. Paper (undated).

255. Harvey, A., "Systems Engineering Approach to Material Handling; Abstract", Automation, 5 (Dec. 1958) p. 121-122.
256. Huffman, John R., A Storage System for Discrete Entities, Ph.D. Dissertation, Univ. of Calif., Los Angeles (June 1960).
257. Hurley, O. R., Application of Computer Simulation at the Hickory Plant, General Electric Company, Mimeo Paper, (1959).
258. IBM Corp. General Information Manual, Improved Job Shop Management Through Data Processing, (1960)
259. IBM Corp. 704 Program Write-Up M & A-1, Job Shop Simulation-Application Revised (1960).
260. IBM Corp., Industrial Engineering Dept., Private Communication (Nov. 1961).
261. Knight, D. O., Test of Alternative Facility Designs & Operations by Simulations, G. E. Co., Tech. Info. Series, R61CD4.
262. Larsen, Robert P., "Objective Aspects of Computer Facility Designs", Journ. I.E., 11, No. 4 (July 1960) p. 328-331.
263. O'Neill, R. R., Simulation of Cargo Handling Systems, Univ. of Calif., Los Angeles, Dept. of Engineering Report 56-37 (Sept. 1956).
264. Sims, E. Ralph, "How To Make Use of the System Approach", Mat. Hand. Eng., 16, No. 4 (Aug. 1961) p. 68.
265. Souder, J. J., Clark, W. E., Elkind, J. and Brown, M., Planning for Hospitals, American Hospital Assoc., Chicago, Ill. (1964).
266. Westervelt, Frank H., An Automatic Compiler for System Simulation, Unpublished Ph.D. Dissertation, The Univ. of Mich. (1960) Ann Arbor, Mich.
267. Wilson, R. C., Evaluation of Spatial Relations and Empirical Plant Layout Criteria by Computer, Unpublished Ph.D. Dissertation, The Univ. of Mich. (May 1961) Ann Arbor, Mich.

VII MISCELLANY

A number of important topics, directly or tangentially related to facilities planning have been omitted from this monograph. In order to indicate the seriousness of this omission we list a few of the most obvious, together with several references which may whet the reader's curiosity enough to cause him to pursue the topic in the appropriate depth required.

The subjects of maintenance and reliability need the kind of consolidation attempted in this monograph on other topics. Their relationship to facility planning is important, and yet only preliminary efforts to quantitatively establish this relationship are apparent. Measurements of maintenance labor time are reported with increasing frequency and the Universal Maintenance Standards are one example of efforts to develop basic elemental time standards.^(274,287) Many articles describe maintenance control procedures⁽²⁸⁶⁾ and thereby also relate maintenance to handling and layout problems.^(270,290) Data processing has already demonstrated its potential in maintenance control.^(273,284) Further savings will follow the resulting increase in understanding of the characteristics of reliability and the effectiveness of preventive, replacement, and repair maintenance. The literature on statistical and analytical studies of reliability and failure is voluminous^(271,279,280,283,289 for example) and Buffa⁽³⁾ gives a concise one chapter summary of queueing theory contributions to maintenance policies and crew size determination.

An interesting class of problems is suggested by Koopman⁽²⁸¹⁾ and developed by Finch⁽²⁷⁸⁾ and Marshall.⁽²⁸²⁾ We quote freely from Koopman: "In the statistical study of failures of electronic equipment, we may have to consider an electrical network, composed of the various elements linked in an intricate fashion into a complex circuit, very possibly having some degree of redundancy for the sake of safety. Its probability of failure as a system as a result of random failures of individual elements is required. Further, in certain problems of providing servicing stations S for installations U , where each U requires at least one S , questions of a similar nature occur. If, to increase safety, each U can be serviced by a certain number of S (but only a minority), while on the other hand, for economy, a service station S may be called on to serve several U , a set of questions arise as to the chance of various numbers of U using all service facilities (at a given time) by virtue of some of the service stations S being taken out by chance events. A more complicated instance is that in which the U are assembly plants, each requiring the simultaneous supply of several different components." The importance of this problem to both maintenance and design of automated plants is obvious, and yet the difficulties of the combinational analysis have limited the scope of results.

The impact of architectural and construction concepts on industrial and commercial facilities is also omitted. The use of critical path methods for scheduling and control of both maintenance and construction is well publicized. A cursory view of ideas on these topics will be found in References 268, 276, and 277. Nicolson⁽²⁸⁵⁾ also points up another

topic not developed here: "The morale and output of the labor force can be much effected by working conditions, and much work has been carried out recently to measure their true effect. It has been found that drawing office output under the best artificial light is still 11% less than under good Northern natural light. Elimination of noise in a test in a weaving shed resulted in 12% more output. In summer, the difference between good and poor ventilation may account for some three to 10% more production. But no one has yet measured the psychological effect of poor working conditions and the resulting loss in cooperation, moral, and efficiency. All these factors naturally effect layout. So do communications, and care must be taken that the most important employee, the supervisor, can in fact have a small enough section to enable him to act as a true leader, and get around it."

Many aspects of automatic warehousing and physical distribution management are readily included among the topics of material handling and plant layout described in earlier sections. Those aspects of automatic warehousing which may be considered peculiar to the field are not discussed in this monograph. Descriptions of the impact of automatic order picking, sorting, and loading on warehouse operations may be found in a large number of articles such as References 269, 272, 275, and 288.

MISCELLANEOUS BIBLIOGRAPHY

268. Architectural Forum, "The Bold Look of Industry", 116, No. 4 (April 1962) p. 76
269. Business Week, "Warehouse Man Not Obsolete", (March 24, 1962) p. 93-96.
270. Calenoff, Raymond, "A Stitch of Design Can Save Nine at Maintenance Time", Factory (Jan. 1962) p. 67-74.
271. Chemical Week, "Statistics Show Human Side of Maintenance", 85, No. 9, p. 125-29, WADC Tech. Report 58-543.
272. Christian, R. W., "Automatic Warehousing - Where and How Soon?", Factory, (Sept. 1961) p. 98.
273. Factory, "Punch Card PM Days Off Fast", (Sept. 1958) p. 161-62.
274. Factory, "Work Sampling In Maintenance", (Dec. 1959) p. 72-76.
275. Factory, "This is Admos", (June 1961) p. 104-8.
276. Factory, "Building Benefits at a Glance", (Sept. 1961) p. 110-111.
277. Factory, "Trends, Problems and Opportunities in 1962", (Jan. 1962) p. 42-43.
278. Finch, P. D., Notes on a Combinatorial Problem of B.O. Koopman, Op. Res. Qtrly., 9, No. 3 (Sept. 1958) p. 169.
279. Hosford, J. E., "Measures of Dependability", Op. Res., 8, No. 1 (Jan. 1960) p. 53-64.
280. Hunter, L. and Proschan, F., "Replacement When Constant Failure Rate Preceeds Wearout", NRLQ, 8, No. 2 (June 1961) p. 127-136.
281. Koopman, B. O., Combinatorial Analysis of Operations, Proc. First International Conference on Operations Research (1957)
282. Marshall, C. W., "Full Service Probability for Regular Arrangements", Op. Res., 9, No. 2 (March 1961) p. 186-200.
283. Mayne, A. J., "Some Reliability Models of Production Lines", Op. Res. Qtrly., 2, No. 12 (March 1960) p. 16-30.
284. Mobil Oil Co., Planned Lubrication and Preventive Maintenance Through Data Processing, 150 E. 42nd. St., N. Y. 17, N. Y.

285. Nicolson, D. L., "The Modern Factory-A World Wide View", Mechanical Eng. (Dec. 1961) p. 46-49.
286. Price, Ralph N., "One-Page Score Sheet for Maintenance", Factory, (Sept. 1961) p. 86-90.
287. Robinson, A. M., "How to Gage Maintenance Output", Factory (Feb. 1959) p. 220-224.
288. Spinanger, Arthur, "Mechanized Warehousing of Case Goods", Mechanical Engineering, (Feb. 1961) p. 41-46.
289. Trans. Brit. Ceramic Soc., "Statistical Investigation Into Facts Affecting Life on Ladel Linings", 10, No 53 (Oct. 1954) p. 609-620.
290. Wray, J. D., "How You Can Hit the High Cost of Handling in Maintenance", Factory (June 1959) p. 94-100.

VIII SUMMARY AND CONCLUSIONS

We have now completed our review of the quantitatively oriented literature with either proven or implied potential for facilities planning as seen at the middle of 1962. Consideration of the monograph together with the literature reviewed will reveal many areas in which significant contributions have been made by 1965 and many others which remain to be made. Continued development of new engineering concepts together with quantitative tools for system analysis will continue to expand the capabilities of the facility designer. At the risk, however, of emphasizing insignificant problems and omitting the important, the writer feels it may be helpful to summarize some of the main areas which seems ripe for research effort. In doing so, we are assuming the same point of view as Michaelis⁽²⁹³⁾ in his stimulating paper on the creative engineering challenges to be found in the canning industry:

"The production processes are such that the product is always physically under control. In short, production in the canning industry is dealt with and looked upon by production supervisory personnel, and even by many old-time engineers, in essentially physical terms. This way of thinking is to be contrasted with conceiving of production operations in abstract terms. To illustrate this elusive but highly significant point, consider the operations of an oil refinery. The product is never seen, and the only way in which production can be supervised is through the interpretation of the abstract information generated by flow meters, temperature gauges, pressure gauges, etc. Thus, it is possible to gain complete control of an oil refinery's

operations without ever leaving a central control panel, where all operating information is displayed and the necessary remote controls are centered. This is what is meant by 'dealing with production in abstract terms'. It is entirely possible to conceive of fruit flowing through a cannery under the same control conditions as product flows in an oil refinery. All flow rates would be monitored and centrally reported; and distribution would be under centralized automatic control. Such a cannery would have instantaneous, rather than daily, control over operating results."

We believe that this statement applies in principle to a large number of manufacturing industries which are accustomed to controlling their production through physical means. These same industries will

gradually evolve toward control by abstract means, thus requiring statistical data and mathematical models of the plant system being controlled. With this need in mind, we suggest the following areas of broad research:

1. Development of more precise methods for predicting and controlling material handling costs. The evolutionary development of data processing systems together with the concurrent revisions in organization and accounting procedures may ultimately achieve this goal.
2. In the words of Mr. W. B. McClellan;⁽²⁹²⁾ "What the material handling industry needs is some basic thinking concerning the physical and chemical characteristics of materials which influence packaging and handling." We have already described the

traditional classifications attempted for material handling equipment⁽¹⁶⁰⁾ and the basic research instituted by Erisman⁽¹⁴²⁾ on the characteristics of materials in bulk material handling. McClellan suggests the following nine categories to describe all the materials purchased and received by any manufacturer:

- a. "Those which will be self unitizing
- b. Those requiring nothing but a base such as a palet
- c. Those requiring the base and four corner posts
- d. Those requiring the base, corner posts and liners suitable for metal items
- e. Those requiring the base, corner posts and leaves
- f. Those requiring the base, corner posts, and shelves
- g. The base, corner posts, and liners suitable for granular materials
- h. Base, corner posts, and liners suitable for liquids
- i. Base, corner posts, and liners suitable for frozen or heated items."

Similar effort is needed to describe the basic capabilities and characteristics of processing equipment, material handling equipment, and the linkage between the two as related to the material handled. Nelson has attempted to empirically determine the service time distributions for various classes of machine tools, and Wilson⁽²⁶⁷⁾ has formulated some concepts for testing the engineering feasibility of mechanical handling transfers.

4. Continued studies of the impact of digital computers on the facility planning process are indicated. Thus far, no individual or firm has expended the man-months of effort needed to develop what might be called "a general purpose facility design program" for digital computer use. Bits and pieces of the kinds of thinking which might well be incorporated into such a program have appeared: 1) in programs for checking the circuit logic, and locating circuit packages during design of digital computers; 2) the heuristic program concepts of Tonge⁽¹¹⁹⁾ to overcome the staggering combinatorial aspects of facility planning; 3) the information retrieval concepts proposed by Wortman and Weller⁽²⁹⁶⁾ for computer selection of components in the design of assembled metal-clad switchgear; 4) the heuristic procedure for locating layout areas optimally used by Armour;⁽²¹²⁾ and the computer scheme for evaluating feasibility of layout designs proposed by Wilson.⁽²⁶⁷⁾ A package of computer programs for facility design might best take the form of a compiler language by which the designer could quickly and precisely describe his particular physical facility to the machine. The computer could analyze the static engineering feasibility of the design and compile a program for use in dynamic evaluation of the facility design under simulated operating loads. The work of Souder et al.⁽²⁶⁵⁾ is certainly the most comprehensive approach to facility planning yet attempted.

5. Simulation programs designed to include space dependent variables in addition to the time-dependent variables included in system simulations such as those of Westervelt.⁽²⁶⁶⁾
6. The classical layout problem of equipment location remains theoretically unsolved for even the deterministic case. The author is aware of no work except that of Koopman⁽²⁸¹⁾ which could be construed as extending the work to examples where loads between points are described by frequency distributions. Analytic guidelines on which to base decision for arranging facilities in production lines rather than process groupings are also unavailable.
7. Considerable study is underway toward a better understanding of scheduling and dispatching rules for production control. To the writer's knowledge, consideration of the relationship between scheduling and dispatching rules and the material handling system or plant facility design has not entered into these production control studies. For example, if one accepts in principle that design efficiency is achieved by simultaneously considering the materials handling and production control system, as argued in the application reported by Mandel,⁽¹⁷⁴⁾ an omission in scheduling and dispatching research needs attention.
8. A concise, but satisfactory set of criteria for evaluating facility designs still eludes the designer. Statistical studies evaluating even the simple indices proposed by Gantz and Pettit⁽²⁹⁴⁾ are unreported, and more advanced quantitative techniques such as Shakan⁽²⁹⁴⁾ proposed seem to have had no affect on practice.

9. A broad theoretical framework is needed to precisely describe the major functional relationships and objectives of plant layout-- materials handling facility design. Conceptually, such a model could be a multi-dimensional network queueing model with spatial factors as outlined in Section I of this monograph. In any event, careful identification and definition of variables and relationships of layout is needed in order to arrive at more precise classifications of facility characteristics: assembly line, job-shop, etc.
10. Methods for predicting qualitative characteristics such as "safety", worker satisfaction, possible effectiveness of supervision, maintainability, flexibility, etc. of alternate facility designs would constitute a "breakthrough" in design.
11. The Post Office Department has described the problems in their field.⁽²⁹⁵⁾ A similar appraisal on the part of other industries could have a powerful impact on the effort currently being directed toward research in facilities planning. If materials handling, plant layout, and plant location problems are of significant importance to industry, a free interchange of ideas in this area would serve to define needed research and stimulate useful contributions by qualified individuals who may be unaware of the challenges in facility planning, perhaps only hinted at in this monograph.

SUMMARY BIBLIOGRAPHY

291. Forrester, J. W., Industrial Dynamics, Wiley & Sons, New York (1961).
292. McClelland, W. B., Organizing & Accounting for Material Handling in Manufacturing, Mimeo Paper, American Materials Handling Society Inc., (undated); Also Private Communication Dated 1/20/62.
293. Michaelis, George H., The Canning Industry An Outlet for Creative Engineering, Mimeo Paper (Hunt Foods & Industries Inc.) Fullerton, Calif. (Oct. 1961).
294. Shakun, M. G., Quantitative Method for Decision Making in Plant Layout, Unpublished Ph.D. Dissertation, Columbia, U. (1953).
295. U.S. Government Post Office Dept. publication 48, The Post Office Challenge to Industry, (Feb. 1962) 48 pages.
296. Wortman, H. B. and Waller, M. H., "Designing Assembled Switchgear by Computer", Westinghouse Engineer, 22, No. 2 (March 1962) p. 56-61.

