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Cooperative UAV Trajectory Planning with Multiple Dynamic Targets

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The paper addresses the problem of multiple UAV trajectory planning with dynamic targets. The problem is studied under the MILP framework, where how to express the nonlinear time-dependent cost function between two targets in a linear form makes the key difficulties. To solve the problem, the cost function between two nodes is determined using propotional guidance law to achieve shortest chasing time, then it is linearized with non-uniform segmented time intervals to keep the problem solvable with MILP. To process the problem with obstacle avoidance, additional time intervals corresponding to blocked obstacle regions are introduced into the cost function. Target leaving time decision variable values fallen in the intervals are treated as infeasible by introducing new logic decision variables. Various simulation examples verify the proposed method.

Nomenclature

B_1	=	a large enough number used in MILP constraints
c_{ij}	=	flying time from target i to target j; in the dynamic target case it is time-dependent
K	=	number of UAVs
n	=	number of targets
N	=	total number of obstacle polygon vertices
t_j	=	the time the UAV leaving target j
x_{ij}	=	0-1 decision variable
$\mathbf{v}_i$	=	target velocity
v_p	=	UAV speed

I. Introduction

Although much effort has been made regarding the problem of cooperative UAV trajectory planning with static targets, research aiming at dynamic targets seems rare. The mixed integer linear programming (MILP) based approaches constitute an important class of solutions,[1-5] due to the efficient software implementation and global convergence features of MILP algorithms. Among these, a general design scheme is to minimize the cost function containing performance (usually minimum time) and destination reaching decision variables, and the constraint part includes aircraft dynamics, obstacle avoidance and other constraints. These methods have been so mature that they can work well in complex conditions where multiple UAVs cooperation and miscellaneous task constraints are considered. The problem scale in these methods depends heavily on the aircraft dynamic state equation constraints, and a long planning horizon will generate too much decision variables to be implemented in nowadays linear programming software. To deal with the problem, the rolling optimization techniques are usually adopted, such as receding horizon control and predictive control.[1,2] However, the selection of proper time horizon arouses the new

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problem. Too short horizon will produce less optimized result, and long horizon will increase the problem scale. Obviously, the underlying complexity of this problem arises from the inherent coupling between the task assignment and the trajectory generation. Therefore, Ref. 3 proposed to decompose these two problems to gain much faster computation. Following the idea, task assignment becomes the key problem,[4-6] for the trajectory planning after definite task assignment is a common problem and can be solved easily using MILP. However, most existing results only consider static targets, and the time-varying features of the targets prevent these methods to be extended to dynamic target situation.

Compared with the situation where all targets are static, the dynamic version is fundamentally a time-dependent TSP problem and difficult to solve. Good approximation results have been achieved only in very limited cases.[7-8] In recent years some researchers try to address the problem using intelligent optimization methods. Ref. 9 solves the problem with genetic algorithm, under the assumption that relative to the UAVs the targets are approximately fixed in position. Ref. 10 considers a more complex case with more constraints using PSO, however all vehicles in the problem are assumed to travel from one destination to another with the unit speed. Besides, no global convergence to the minimum is guaranteed in these methods.

In the paper we address the problem of single and cooperative UAV trajectory planning with multiple dynamic targets, especially the task assignment problem, under MILP framework. The key idea is the non-uniformly piecewise linearization of the time cost between targets, and choosing proper constraint formulation to fit the problem in the MILP framework. The reminder is organized as follows: Section II studies the cases of single and cooperative UAVs for multiple dynamic targets without obstacle avoidance. Section III extends section II's result to the case with obstacle avoidance. Conclusions are given in Section IV.

II. Trajectory Planning without Obstacle Avoidance

In this section, we address the problem of dynamic target assignment problem with single or cooperative UAVs when no obstacle avoidance is considered.

A. Problem Formulation

Consider a single or a team of UAVs executing searching and reconnaissance tasks against multiple dynamic targets, as shown in Fig. 1. The underlying problem is a moving target TSP and can be described as follows:

Given a set of targets $G=\{g_1, \ \cdots \ g_n\}$, each target g_i moving at constant velocity $\mathbf{v}_i=[u_i, v_i]$, and a UAV starting from the same origin at constant speed v_p, find the shortest tour starting and ending at the origin, such that the vehicles visits all targets.

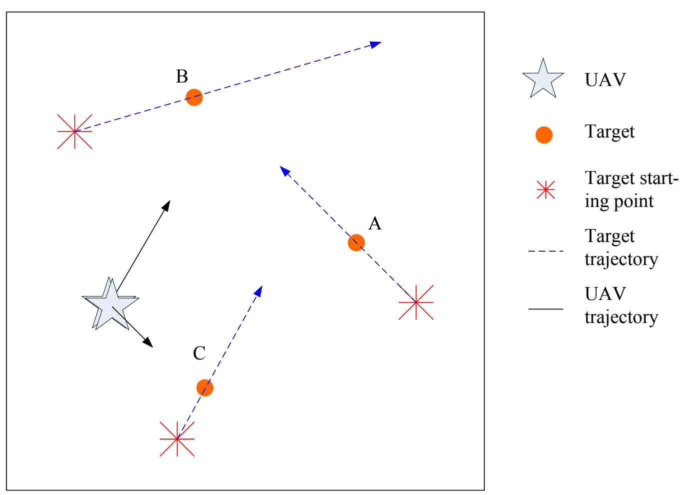


Figure 1. Demonstration of trajectory planning problem with dynamic targets

To establish solvable MILP formulation, we augment the target space as follows. Let target 1 be the starting point of all UAVs, and introduce nodes n+1, n+2, $\cdots$, n+K corresponding to returning points for each of the K vehicles respectively. Choosing decision variable

$$x_{ij} = \begin{cases} 1 & \text{if UAV fly from node } i \text{ to } j \\ 0 & \text{others} \end{cases}$$

and t_j is the time when a UAV leaving node j, using the problem formulation techniques in Ref. 11, the problem can be formulated in a quasi-MILP form:

$$\min \sum_{k=1}^{K} t_{n+k} \tag{1}$$

s.t.

$$\sum_{\substack{i=1 \\ i \neq j}}^{n} x_{ij} = 1 \qquad (j = 2, \dots, n+K) \tag{2}$$

$$\sum_{\substack{j=2 \\ j \neq i}}^{n+K} x_{ij} = 1 \qquad (i = 2, \dots, n) \tag{3}$$

$$\sum_{j=2}^{n} x_{1j} = K \tag{4}$$

$$t_1 = 0 \tag{5}$$

$$t_j - t_i - B_1 x_{ij} \geq c_{ij} - B_1 \quad (i = 1, \dots, n; j = 2, \dots, n+K, i \neq j) \tag{6}$$

$$\sum_{j \in Tr} \sum_{i \in Tr} x_{ij} \leq \|Tr\| - 1 \quad Tr \subset \{1, \dots, n\} \tag{7}$$

$$x_{ij} = \{0, 1\} \qquad (i = 1, \dots, n; j = 2, \dots, n+K) \tag{8}$$

$$t_i \geq 0 \qquad (i = 1, \dots, n+K) \tag{9}$$

The objective function (1) minimizes the total flying time of all UAVs. Beside this, we can also consider minimizing the maximal flying time of all UAVs as the objective function, as shown in (11):

$$\min \max_{j \in \{1, 2, \dots, K\}} t_{n+j} \tag{10}$$

Constraints (2) and (3) ensure that each target is reached once and only once. Constraints (2) and (3) ensure that each target is reached once and only once. Constraint (4) ensures that exactly K UAVs are used. Constraint (5) sets the starting time of all UAVs as the reference time. Constraints (6) compute the leaving time at node j, where c_{ij} is the chasing time cost and will be explained in detail below. Constraint (6) and (7) work together to eliminate possible subloops.

B. Linearization of Time-Dependent Flying Cost

If the flying cost c_{ij} in (6) is constant, the problem degrades to the common TSP problem and can be solved easily using MILP. However, for the dynamic targets the cost is obviously time dependent. Our task is (1) decide the expression of the time cost relative to current time decision variable; and (2) linearize the expression to make the problem solvable in the MILP framework.

Now consider the first problem. Assume target i and j move with a velocity of (v_{xi}, v_{yi}) and (v_{xj}, v_{yj}), repectively. At time t_i and t_j, the UAV reaches targets i and j in order at position P and T, as shown in Fig. 2. Let v_p be the maximum UAV speed, it is easy to prove that (1) reaches the minimum only when the UAV flys at maximum speed,[7] therefore we can assume it flys at constant speed v_p. According to the propotional guidance law, UAV chases the target at the shortest time only when its velocity component along the PQ direction is equal to target j's velocity component in that direction. Therefore, we have

$$t_j - t_i = \frac{\|PQ\|}{v_2 - v_1} \tag{11}$$

where $\|\bullet\|$ stands for the Euclidean distance, and v_2 and v_1 are shown in Fig. 2.

After simple geometric calculations we have [12]

$$c_{ij} = (t_j - t_i) \cdot v_p$$

$$= \frac{(\Delta x)^2 + (\Delta y)^2}{\sqrt{(v_p^2 - v_y^2)(\Delta x)^2 + (v_p^2 - v_x^2)(\Delta y)^2 - 2v_x v_y \Delta x \Delta y} - (v_x \Delta x + v_y \Delta y)} \tag{12}$$

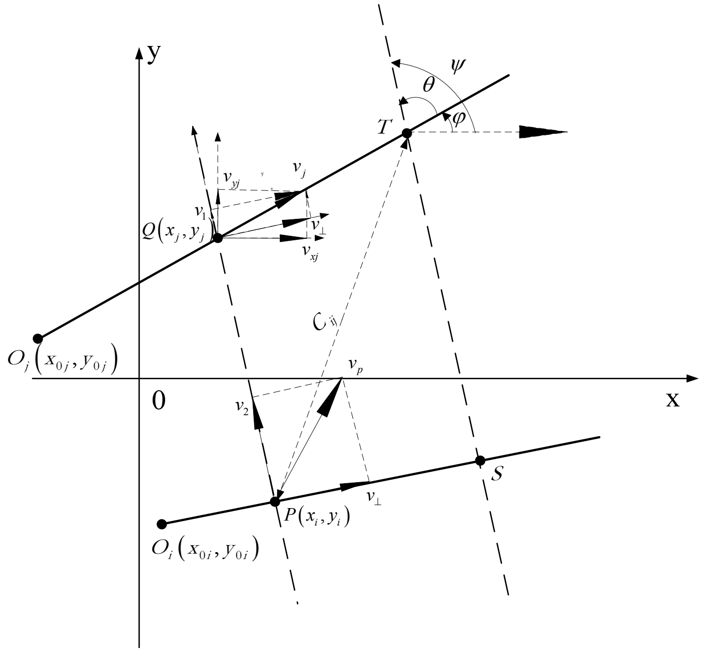


Figure 2. Geometric demonstration of dynamic flying cost calculation. *Two targets start at positions O_i and O_j, reach positions P and Q at time t_i, and positions S and T at time t_j. UAV leaves target i at time t_i and reaches target j at time t_j. v_2 and $v_\perp$ are UAV's velocity component along PQ and its perpendicular direction.*

where

$$\Delta x = x_j - x_i = (x_{j0} - x_{i0}) + (v_{xj} - v_{xi})t_i \tag{13}$$

and

$$\Delta y = y_j - y_i = (y_{j0} - y_{i0}) + (v_{yj} - v_{yi})t_i \tag{14}$$

From (12)-(14) we know c_{ij} depends only on decision variable t_i. Although the function is a complex nonlinear function, its shape like a quadratic function so much. To fit into the MILP framework, the function needs to be linearized. Consider its quadratic-like shape where the valley is curving and two sides are nearly linear, we use piecewise linearization method where the *t* axis is adaptively segmented into non-uniform intervals, as shown in Fig. 3. More turning points means more precise result, however increase the computational burden. In the following simulation the point count is set to 6, which proves to be reasonable in most computation.

C. Simulation Results

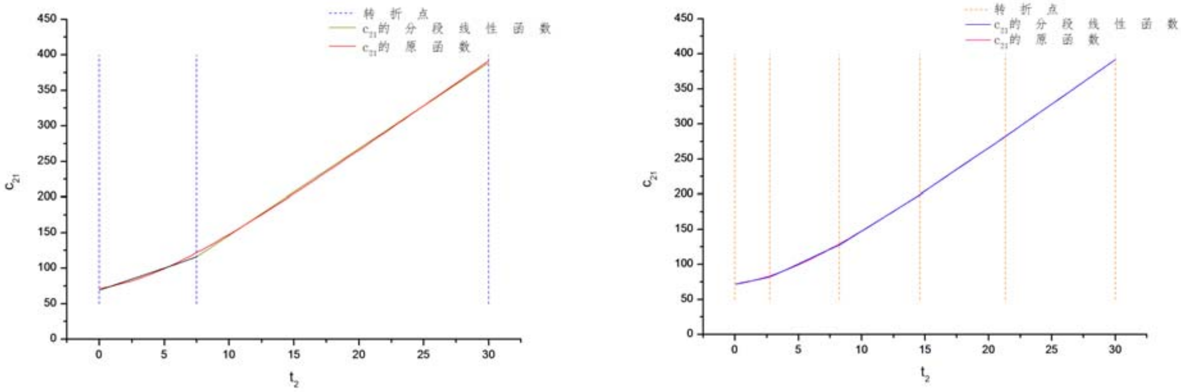
The simulation was performed on PC platform with Intel Core2 CPU and 2G memory. The MILP problem is processed using and IBM OPL CPLEX.[13] UAVs' starting point is set to be the origin. All targets' initial position and their velocity are randomized at the beginning of simulation. The time-dependent cost term is linearized using SLMTools in Matlab. [14]

1. Single UAV

First a simple case was considered: a single UAV with three dynamic targets. Initial simulation values are shown in Table 1, and UAV speed is set to 10.

In OPL IDE, the problem was solved and the following decision variables are given:

$$\mathbf{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad time = \begin{bmatrix} 0 & 16.697 & 3.9103 & 12.289 & 23.375 \end{bmatrix}$$



a) Linearization with 3 turning points. Error can be easily observed at the second point.

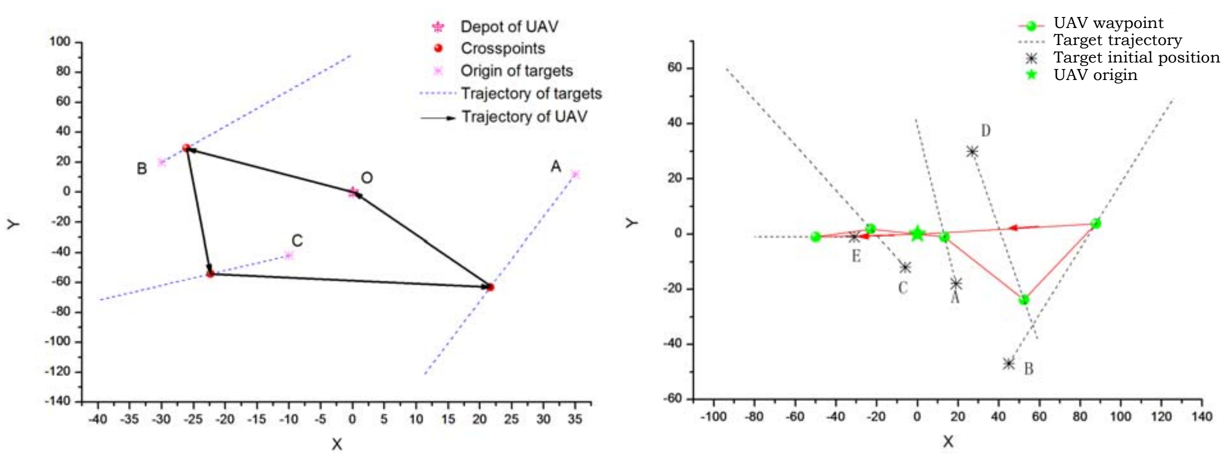
b) Linearization with 6 turning points. No obvious error can be observed.

Figure 3. Time cost function linearization

Table 1. Initial values for the simulation of a single UAV with three dynamic targets

	Target A	Target B	Target C
Initial position	(35, 12)	(-30, 20)	(-10, -42)
Velocity	(-0.8, -4.5)	(1, 2.4)	(-1, -1)

The corresponding trajectory is $O \rightarrow B \rightarrow C \rightarrow A \rightarrow O$, as shown in Fig. 4a). The final minimum value is $f^* = 23.375$. Since only three targets are considered, the result can be easily verified using enumerating all feasible paths. Fig. 4b) shows the result for five dynamic targets, where the corresponding trajectory is $O \rightarrow E \rightarrow C \rightarrow A \rightarrow D \rightarrow B \rightarrow O$. Readers may refer to Ref. 12 for detailed parameters and verification process.



a) Three dynamic targets

b) Five dynamic targets

Figure 4. Trajectory planning results for single UAV case

2. Cooperative UAVs

To extend results above, multiple cooperative UAVs are now considered. In this case the objective function (10) is selected to better fit the requirement (minimum completion time). We first considered the case with 2 UAVs and 3 targets. Initial simulation values and UAV speed settings are the same as in 1. The solved decision variables are

$$\mathbf{X}=\begin{bmatrix}0&1&1&0&0\\0&0&0&1&0\\0&0&0&0&1\\1&0&0&0&0\end{bmatrix},\quad time=\begin{bmatrix}0 & 9.4412 & 3.8620 & 4.9227 & 7.7140 & 13.527\end{bmatrix}$$

The final minimum value is $f^*=13.527$. Corresponding trajectories are $O\rightarrow C\rightarrow A\rightarrow O$ and $O\rightarrow B\rightarrow O$, as shown in Fig. 5. The result has been verified to be the correct solution. Simulations containing more UAVs and targets (up to 10) were also performed and tested. [12]

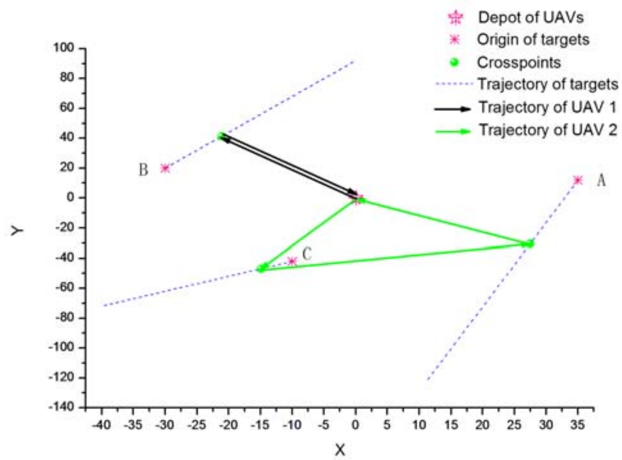


Figure 5. Trajectory planning results for 2 UAVs and 3 dynamic targets case

III. Trajectory Planning with Obstacle Avoidance

Now consider the planning problem with obstacle avoidance, as shown in Fig. 6. Here the "obstacle" means polygonal zones in the workspace which the UAVs cannot traverse whereas the targets can. Therefore, it represents not only obstacles but also hazerous zones where the defence units exist.

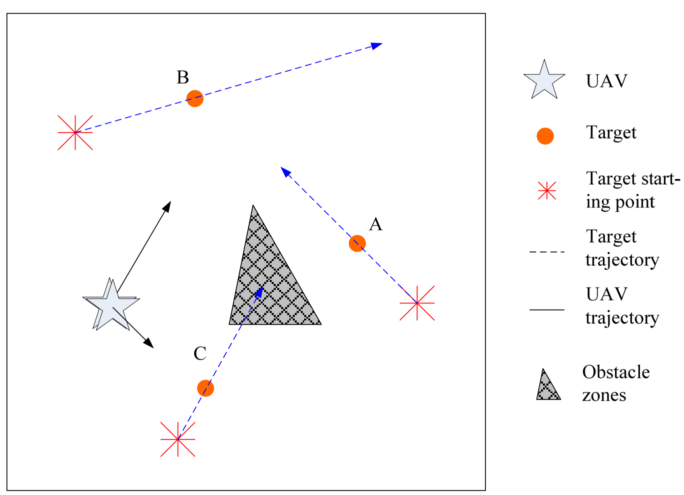


Figure 6. Demonstration of trajectory planning problem with obstacle avoidance

A. MILP Formulation

Compared with section II, the only difference is the introduction of obstacles. Observe that if a path connecting two targets is blocked by an obstacle, the feasible path between the two targets must contain one or more vertices of the obstacle polygon. Therefore, by still choosing decision variables x_{ij} and t_j, the problem can be formulation in the following MILP form:

$$min \max_{k\in\{1,2,\dots,K\}} t_{n+N+k} \tag{15}$$

s.t.

$$\sum_{j=1}^{n+N} x_{0j} = K \tag{16}$$

$$\sum_{j=n+N}^{n+N+K} x_{0j} = 0 \tag{17}$$

$$\sum_{i=0}^{n+N} x_{ij} = 1 \quad \left(j = 1,\dots,n,n+N+1,\dots,n+N+K\right) \tag{18}$$

$$\sum_{i=0}^{n+N} x_{ij} \le 1 \quad \left(j = n+1,\dots,n+N\right) \tag{19}$$

$$\sum_{j=1}^{n+K+N} x_{ij} = 1 \quad \left(i = 1,\dots,n\right) \tag{20}$$

$$\sum_{i=1}^{n+N} x_{ii} = 0 \quad \left(j = n+1,\dots,n+N\right) \tag{21}$$

$$\sum_{j=1}^{n+K+N} x_{ij} \le 1 \quad \left(i = n+1,\dots,n+N\right) \tag{22}$$

$$\sum_{i=0}^{n+N} x_{ij} = \sum_{i=1}^{n+N+K} x_{ji} \quad \left(j = n+1,\dots,n+N\right) \tag{23}$$

$$t_j - t_i - B_1 x_{ij} \ge c_{ij} - B_1 \quad \left(i = 0,\dots n+N; j = 1,\dots,n+K+N; i \ne j\right) \tag{24}$$

$$\sum_{i\in Tr}\sum_{j\in Tr} x_{ij} \le \left|Tr\right| - 1 \quad \forall Tr \subset \left\{1,\dots,n+N\right\} \tag{25}$$

$$t_0 = 0 \tag{26}$$

$$x_{ij} = \left\{0,1\right\} \qquad \left(i = 0,\dots,n+N; j = 1,\dots,n+K+N\right) \tag{27}$$

$$t_j \ge 0 \qquad \left(j = 1,\dots,n+K+N\right) \tag{28}$$

Similar to section II, constraints (16) and (17) ensure that exactly K UAVs are used. Constraints (18) ensure that each target is reached once. Constraints (19) ensure that each obstacle polygon vertex is reached once at most. Constraints (20) ensure the next destination after reaching a target should be an unreached one. Constraints (21) ensure UAVs can't leaving a vertex and return to that vertex. Constraints (22) ensure each UAV reach any vertex at most only once and leave that vertex. Constraints (23) ensure that if a vertex is reached by a UAV, a path leaving from that vertex must exist, and vice versa.

B. Revised Cost Function for Obstacle Avoidance

Although constraints (24) are the same as (6), we notice the former exert hidden constraints that any feasible path doesn't cross the obstacle. One way to meet the constraints is to add additional segment intersection detection constraints. Unfortunately, by now we can't find a linear algorithm to fit into the above MILP formulation. Another way is to process the cost function c_{ij} beforehand with additional intervals corresponding to blocked time segments. This method can not only meet the obstacle constraint, but save the computation by not adding additional constraints in the online MILP optimization process.

Let's demonstrate the method by a simple example. In Fig. 7a), the original path between target A and C crosses the obstacle triangle. Represent coordinates of the two terminals of the path segment as (x_1, y_1) and (x_3, y_3), then the corresponding line equation is:

$$\frac{x - x_1}{x_3 - x_1} = \frac{y - y_1}{y_3 - y_1} \tag{29}$$

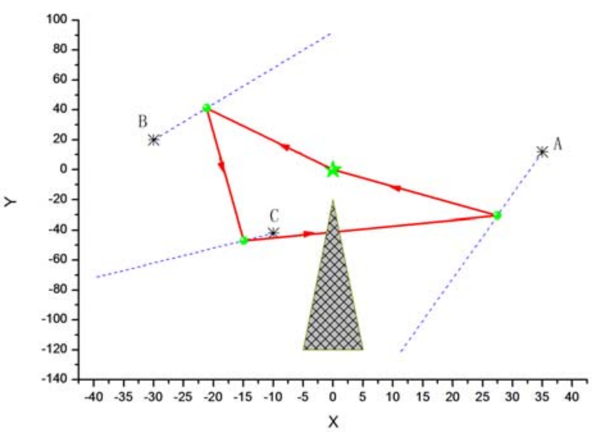
The time that the path between target A and C reaches each vertex of the triangle can be calculated by substituting coordinates of the vertices to (29). Therefore, the constraints can be defined as:

$$\sum_{a=1}^{m_a} \left(t_i \geq start_a \;\&\&\; t_i \leq end_a \right) \leq 1 - x_{ij} \tag{30}$$

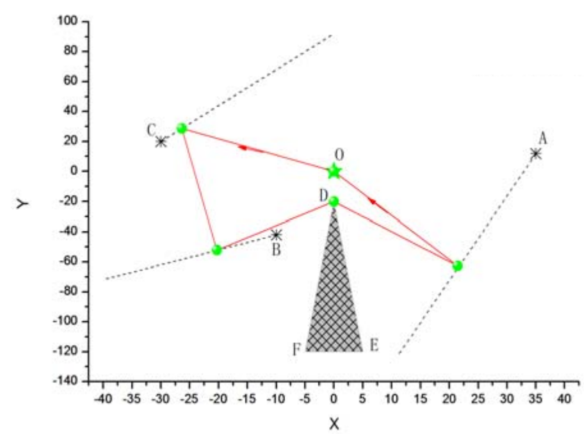
where $start_i$ and end_i are starting and ending time of vertex *i*'s corresponding region is crossed by the path segment, and m_a is the number of times the segment is blocked by the obstacle polygons. In this example m_a =1. The constraints means if $x_{ij} = 1$, then the value of t_i is not allowed to take within the designated interval.

C. Simulation Results

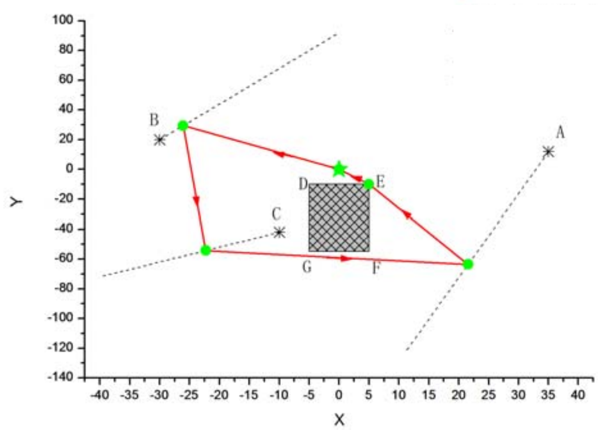
Still take the example used in section II with a triangle obstacle. Using proposed method we obtain the trajectory $O \to C \to B \to D \to A \to O$, as shown in Fig. 7b). For the same problem with a rectangle obstacle, the trajectory is $O \to B \to C \to A \to E \to O$, as shown in Fig. 7c). It is shown by exhaustive evaluation that these solutions are the optimal solution.



a) The solution is unfeasible after adding obstacle



b) Solution trajectory using proposed method



c) Solution trajectory using proposed method (rectangle obstacle)

Figure 7. Trajectory planning results for 1 UAV and 3 dynamic targets with obstacle avoidance

IV. Conclusion

Under the MILP framework, the paper proposes a solution to the cooperative UAV trajectory planning problem with dynamic targets. The approach minimizes the mission completion time or total path length, under the consideration of multiple UAVs, multiple dynamic targets, and obstacle avoidance. Simulation results demonstrate the feasibility of the proposed method in various conditions: single or multiple UAVs, with or without obstacle. Exhaustive calculation for simple case verifies the correctness of the result.

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