T H E   U N I V E R S I T Y   O F   M I C H I G A N

College of Engineering
Department of Mechanical Engineering
Heat Transfer Laboratory

Technical Report No. 5

INFLUENCE OF FLOW AND ROTATIONAL OSCILLATIONS ON THE MECHANICS OF TWO-DIMENSIONAL LAMINAR BOUNDARY-LAYER FLOW PAST CYLINDERS, INCLUDING UNIFORM SUCTION OR BLOWING

Wen-Jei Yang
John A. Clark

ORA Project 05065

under contract with:

AEROSPACE RESEARCH LABORATORIES, OAR
AERONAUTICAL SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO
CONTRACT NO. AF 33(657)-8368

administered through:

OFFICE OF RESEARCH ADMINISTRATION   ANN ARBOR

September 1965

ACKNOWLEDGMENTS

The authors wish to express their appreciation to Dr. Max. Scherborg of the Aeronautical Research Laboratories for his assistance and encouragement and Dr. Eiichi Hori of the Central Research Laboratory of the Hitachi Ltd., Japan for his cooperation in providing information on the programming for a digital computer. Their gratitude is also extended to Messrs Tomokita Izumi, R. Mutyara and Hsu-Chieh Yen, all graduate students of Mechanical Engineering, who analyzed the problems of fluctuating circulation and rotational oscillation as their graduate research works in ME 600.

The financial support of the Aeronautical Research Laboratories is gratefully acknowledged.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES (Continued)

LIST OF FIGURES (Concluded)

NOMENCLATURE

Symbols

a: Coefficient depending on the geometrical configuration of the body, dimensionless.

b: Coefficient depending on the nature of flow oscillation, dimensionless.

C: Dimensionless concentration, $= \dfrac{C_w^* - C^*}{C_w^* - C_\infty^*}$.

C^*: Concentration, lbm - mole; C_w^* at the wall; C_∞^* of the free stream.

F: Functional coefficient or universal distribution function of temperature (or concentration), dimensionless; F_{0k} for the zeroth-order approximation; $F_{1\ell k}$ for the first-order approximation; $F_{2\ell jk}$ for the second-order approximation.

f: Functional coefficient or universal distribution function of velocity, dimensionless; f_{0k} for the zeroth-order approximation; $f_{1\ell k}$ for the first-order approximation; $f_{2\ell jk}$ for the second-order approximation.

i: $(-1)^{1/2}$.

k: Integer, dimensionless.

L: Characteristic length, ft, = 2R for a circular cylinder.

ℓ: Integer, dimensionless.

M,m: Constant, dimensionless.

Nu: Nusselt number, dimensionless, $= \left(\dfrac{\partial\theta}{\partial y}\right)_{y=0}$.

n: Constant, dimensionless.

Pr: Prandtl number, dimensionless.

q: Rate of heat transfer, BTU/hr-ft^2.

R: Radius of a circular cylinder, ft.

Symbols

Re: Reynolds number, dimensionless.

Sc: Schmidt number, dimensionless.

Sh: Sherwood number, dimensionless, $= \left(\frac{\partial c}{\partial y}\right)_{y=0}$.

T: Dimensionless temperature, $= \frac{T_w^* - T^*}{T_w^* - T_\infty^*}$.

T*: Temperature, $^{\circ}F$; T_w^*, of the wall; T_∞^*, of the free stream.

t: Dimensionless time, $= \frac{t^* u_\infty}{L}$.

t*: Physical time, hr.

U(x,t): Velocity of potential flow in dimensionless form, $= \frac{U^*(x,t)}{U_\infty}$.

$U_o(x)$: Time-average or steady velocity of potential flow in dimensionless form, $= \frac{U_o^*(x)}{U_\infty}$.

$U_1(x)$: Oscillation amplitude of potential flow or cylinder surface in dimensionless form, $= \frac{U_1^*(x)}{U_\infty}$.

U_∞: Velocity of potential flow at infinity, ft/hr.

$U^*(x,t)$: Velocity of potential flow, $= U_o^*(x) + \epsilon U_1^*(x) \cos \omega t$ for parts A and B, $= U_o^*(x)$ for part C, ft/hr.

$U_o^*(x)$: Time-average or steady velocity of potential flow, ft/hr.

$U_1^*(x)$: Oscillation amplitude of potential flow or cylinder surface, ft/hr.

u: Dimensionless velocity in x-direction, $= \frac{u^*}{U_\infty}$; u_o for the zeroth-order perturbation, $= \frac{\partial \psi_o}{\partial y}$; u_1 and $u_{1\ell}$ for the first-order perturbation, $u_{1\ell} = \frac{\partial \psi_{1\ell}}{\partial y}$; u_2 and $u_{2\ell j}$ for the second-order perturbation, $u_{2\ell j} = \frac{\partial \psi_{2\ell j}}{\partial y}$.

Symbols

u^*: Velocity component in x-direction, ft/hr.

V: Velocity of uniform suction in dimensionless form, $= \frac{V^*}{U_\infty}$.

V^*: Velocity of uniform suction, ft/hr.

v: Dimensionless velocity in y-direction, $= \frac{v^*}{U_\infty}$; v_o for the zeroth-order perturbation, $= -\frac{\partial \psi_o}{\partial x}$; v_1 and $v_{1\ell}$ for the first-order perturbation, $v_{1\ell} = -\frac{\partial \psi_{1\ell}}{\partial x}$; v_2 and $v_{2\ell j}$ for the second-order perturbation, $v_{2\ell j} = -\frac{\partial \psi_{2\ell j}}{\partial x}$.

v^*: Velocity component in y-direction, ft/hr.

x: Distance measured along the wall in dimensionless form, $= \frac{x^*}{L}$.

x^*: Distance measured along the wall, ft.

y: Dimensionless distance measured in the direction perpendicular to the wall, $= \frac{y^*}{L}$.

y^*: Distance measured in the direction perpendicular to the wall, ft.

γ: Integer, dimensionless.

ϵ: Small constant parameter, dimensionless.

η: Dimensionless distance measured in the direction perpendicular to the wall; $= y(a_o Re)^{1/2} x^{m-1/2}$ for generalized Blasius series solutions; $= y(a_1 Re)^{1/2}$ for ordinary Blasius series solutions.

Θ: Dimensionless temperature; Θ_o for the zeroth-order perturbation, $= T_o$; $\Theta_{1\ell}$ for the first-order perturbation, $\Theta_{2\ell j}$; for the second-order perturbation.

μ: Absolute viscosity, lbm/hr-ft.

Symbols

τ: Wall shear stress in dimensionless form, $= \frac{\tau^* L}{\mu U_\infty}$ or $= \left(\frac{\partial u}{\partial y}\right)_{y=0}$.

τ^*: Wall shear stress, $= -\mu \left(\frac{\partial u^*}{\partial y^*}\right)_{y=0}$, lbf/ft^2.

ψ: Stokes stream function in dimensionless form, $= \frac{\psi^*}{LU_\infty}$; ψ_o for the zeroth-order perturbation, $= \frac{\psi_o^*}{LU_\infty}$; $\psi_{1\ell}$ for the first-order perturbation, $= \psi_{1\ell}^* U_\infty^{\ell-1}/L^{\ell+1}$; $\psi_{2\ell j}$ for the second-order perturbation, $= \psi_{2\ell j}^* U_\infty^{\ell-1}/L^{\ell+1}$.

ψ^*: Stokes stream function, ft^2/hr; ψ_o^* for the zeroth-order perturbation; $\psi_{1\ell}^*$ for the first-order perturbation; $\psi_{2\ell jk}^*$ for the second-order perturbation.

ω: Frequency of oscillation in dimensionless form, $= \frac{\omega^* L}{U_\infty}$.

ω^*: Frequency of oscillation, rad/hr.

Superscripts

$'$, $''$, $'''$: First, second and third derivatives with respect to η respectively.

Subscripts for Functional Coefficients f and F

o, 1, 2: Zeroth, first, and second-order perturbation.

j: Refers to either steady- or transient-state in the second-order perturbation.

ℓ: Order of approximation in term of the frequency of oscillation, $(i\omega)^\ell$.

k: Order of function.

s,t: Steady- and transient-states in the second-order perturbation respectively.

For Example

Order of Perturbation	Subscript 1st	2nd	3rd	4th	5th
Zeroth	Order of ϵ	Exponent of x	Order of Function		
1st	Order of ϵ	Order of ω	Exponent of x	Order of Function	
2nd	Order of ϵ	Order of ω	Steady- or Transient-State	Exponent of x	Order of Function

ABSTRACT

A theoretical investigation of the influence of flow oscillation, fluctuating circulation and rotational oscillation upon the transfer of momentum, heat and mass in two-dimensional laminar boundary-layer flow past cylinders with or without uniform suction. The boundary-layer equations for flow, temperature and concentration are linearized by means of a perturbation procedure and the first three terms retained. The solutions of the velocity, temperature and concentration components are obtained in the forms of an ordinary Blasius series for a symmetrical blunt body and a generalized Blasius series for a sharp-edged body. Theoretical results include the frequency response of fluid velocity, temperature and concentration, the streamline patterns of the streaming, the distribution of the steady second-order temperature and concentration, and the alternations in the shear stress, rates of heat and mass transfer. For flow around a circular cylinder numerical results show that the permanent alterations in the skin friction and heat transfer rate induced by the flow oscillation, fluctuating circulation and rotational oscillation are very small in the range of small amplitude and low frequency.

I. INTRODUCTION

In recent years, considerable attention has been focussed on the problems of periodic boundary layers. Since fluctuations in a stream incident upon a body is known to occur, it is important to understand how the boundary layer responds to the oscillations of the stream. For instance, in the occurrence of flutter on aircraft, the boundary-layer effects may be considerable.

It is generally observed that a steady flow, which is known as secondary or streaming flow, exists in an oscillating fluid or is generated in a quiescent fluid where solid boundaries oscillate. This phenomenon is also known to occur when oscillating acoustic waves interact with a stationary object.

For periodic boundary layers in the absence of a mean flow, Rayleigh[1] has given a unified theory for the steady secondary motion, valid both inside and outside the boundary layer, in connection with certain acoustic phenomena of Kundt's dust tube. Schlichting[2] has applied his theory to the periodic flow generated by a circular cylinder oscillating along a diameter. The existence of the steady streaming flows which persists both inside and outside the oscillatory boundary layer was mathematically established. Photographic evidence of the streaming was observed in air by Andrade[3] and Holtsmark, et al.[4] and in water by Schlichting.[2] The Reynolds number is defined as $U_\infty^2/\omega\nu$, where $U_\infty \cos \omega t$ is the cylinder velocity. For large values of the Reynolds number, as has been pointed out by Stuart,[5] there exists a second, outer boundary layer. The nonlinear inertia terms are not negligible within the outer layer at the edge of which the steady velocity component along the surface tends to zero. Other related works include West,[6] Westervelt,[7] Andres and Ingard,[8] Nyborg[9,10] and Segal.[11]

For the periodic boundary layers in a fluctuating flow, Lighthill[12] has studied the response of skin friction and heat transfer rate to small oscillations in the main stream. Lin[13] and Lighthill[12] have independently treated high-frequency oscillating flows by means of the theory of differential equations. More general case in which the stream fluctuates both in magnitude and in direction has been investigated by Gibson.[14] The method of series expansion was used by Hori[15] to solve various oscillation problems. The exact solution for the fluctuating flow past an infinite flat plate with uniform suction is obtained by Stuart.[16]

Unsteady Blasius flow has been treated by Moore,[17] and Cheng and Elliott[18] for low frequency case and by Illingworth[19] for compressible flow of both high and low frequencies. For the flow near a stagnation point, Wuest[20] has investigated the response of velocity in the boundary layer when a flow impinges on a wall which is oscillating parallel to the stagnation line. Glauert[2] and Rott[22] studied the two-dimensional flow against an infinite flat plate making transverse oscillations in its own plane. Solutions are obtained for small and large values of the frequency as well as for the whole frequency range.

Later work pertaining to the effects of periodic boundary layer on both natural and forced convective transfer phenomena include Jackson, et al.,[23] Fand, et al.,[24] Bayley, et al.,[25] Kesten, et al.,[26-28] Clark, et al.,[29-32] Eshgy[33] and Na.[34]

The present work, which consists of two parts, is devoted to a study of problems pertaining to the effects produced on forced convection flows from harmonically fluctuating stream and from rotational oscillation in two-dimensional laminar boundary-layer flow past cylinders. The perturbing effects are respectively from a potential flow and rotational oscillation of small fluctuating amplitude and low frequency. Both induce fluctuations of velocity, temperature and concentration as well as the secondary or steady-state alternations in the forced convection boundary layer. The governing differential equations are linearized through the use of the perturbation technique with the first three terms retained. The first terms being the case of steady-state forced convection are the classical problem. The second terms are the frequency response of the fluid velocity, temperature and concentration. The third terms consist of two components, one which is harmonic with twice the frequency of flow oscillation and one which is time independent and gives rise to a net change in the steady-state values of the shear stress and the rates of heat and mass transfer at the wall.

To solve the transfer equations for low frequencies, Lighthill[12] used a Karman-Pohlhausen method and Hori[15] used the method of Blasius and Howarth. At high frequencies, since viscosity is only effective for oscillation within a very thin shear-wave boundary layer closed to the wall, the theory of differential equations with a large parameter for high frequency approximation was applied independently by Lighthill[12] and Lin.[13] For the present investigation, the method of Blasius and Howarth is used. By means of this method, the transfer equations may be solved by power-series development from the stagnation point, without any arbitrary assumptions regarding the velocity, temperature and concentration profiles. Furthermore, the solutions as expressed in terms of the coefficients representing both the geometrical configuration and the nature of the flow or rotational oscillations and the universal distribution functions may be applied to any two-dimensional flow.

Also treated are the effects of uniform suction or flowing imposed on the cylinder surface. Theoretical analyses include the shear stress, and the rates of heat and mass transfer for a symmetrical blunt body and a sharp-edged body. Numerical results are obtained for flow around a circular cylinder with flow oscillation, fluctuating circulations and rotational oscillations.

II. THEORETICAL ANALYSIS

Part A. Oscillation of Free Stream

1. THE FUNDAMENTAL EQUATIONS

The physical system, consists of a heated cylindrical body around which flows fluctuate harmonically with time. A coordinate system x^*, y^* is fixed at the forward stagnation point, with x^* measured along the cylindrical surface and y^* in the direction perpendicular to the surface as illustrated in Fig. 1 for a sharp-edged body. The analysis is restricted to two-dimensional, incompressible flow in the x^*-y^* plane. The external potential flow is represented by $U^*(x^*t^*) = U_0^*(x^*)+\epsilon U_1^*(x^*) \cos \omega^* t^*$. Where $U_0^*(x)$ is the time-average velocity, $U_1^*(x)$ is the amplitude of oscillation, ω^* is the frequency of oscillation, and t^* is the physical time.

The cylinder with surface concentration of C_W^* is heated to a uniform temperature, T_W^*. It is maintained in contact with a fluid at temperature T_∞^*, and concentration C_∞^* which otherwise would flow with a constant velocity U_∞ at infinity.

The following assumptions are imposed on the analysis:

(a) The velocity components are small compared to sonic velocity so that the compressibility effects are negligible.

(b) In the temperature boundary-layer, the viscous dissipation and the heat generated by change in pressure may be neglected.

(c) The differences in temperature and concentration are not so large that the physical properties of the fluid vary from point to point.

(d) For problems involving suction or blowing, the flow through the surface is assumed to be wholly normal, since the pressure gradient through the surface is usually large.

With these assumptions, the boundary-layer equations for flow, temperature and concentration read (the fundamental equations for parts A, B and C are all presented here, although the statements of the problem for parts B and C will be given later).

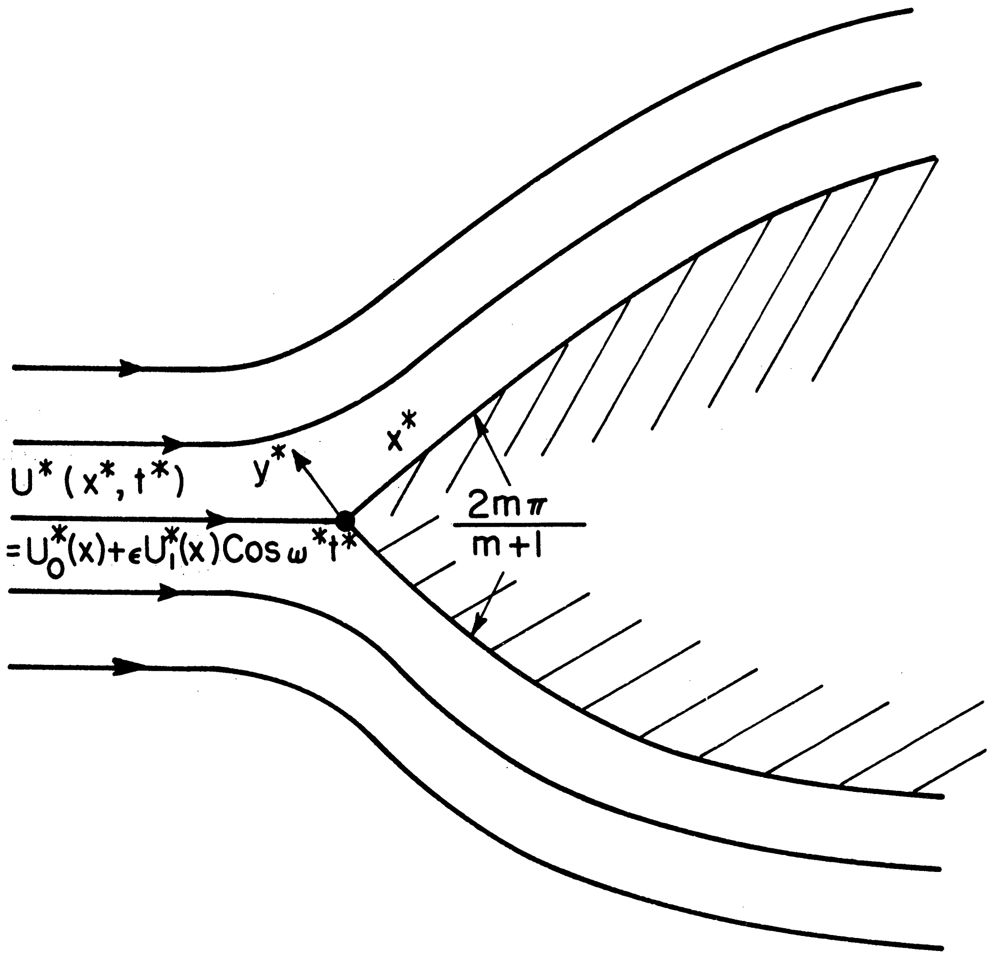


Fig. 1. Oscillating flow past a sharp-edged body.

$$
\begin{cases}
\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 \\[2ex]
\dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = \dfrac{1}{Re}\dfrac{\partial^2 u}{\partial y^2} + \begin{cases} \dfrac{\partial U}{\partial t} + U\dfrac{\partial U}{\partial y} & \text{for both flow oscillation (Part A) and fluctuating circulation (Part B) of free stream} \\[2ex] U_0\dfrac{dU_0}{dx} & \text{for rotational oscillation of cylinder surface (Part C)} \end{cases} \\[2ex]
\dfrac{\partial T}{\partial t} + u\dfrac{\partial T}{\partial x} + v\dfrac{\partial T}{\partial y} = \dfrac{1}{RePr}\dfrac{\partial^2 T}{\partial y^2} \\[2ex]
\dfrac{\partial c}{\partial t} + u\dfrac{\partial c}{\partial x} + v\dfrac{\partial c}{\partial y} = \dfrac{1}{ReSc}\dfrac{\partial^2 c}{\partial y^2}
\end{cases}
$$

The boundary conditions are

$$
\begin{cases}
y = 0: \quad u = \begin{cases} 0 & \text{for Parts A and B} \\ \epsilon U_1(x)\cos\omega t, & \text{for Part C} \end{cases} \quad T = C = 0, \quad v = \begin{cases} 0 & \text{without suction or blowing} \\ V & \text{positive for uniform blowing and negative for uniform suction} \end{cases} \\[3ex]
y = \infty: \quad u = \begin{cases} U & \text{for Parts A and B} \\ U_0(x) & \text{for Part C}, \end{cases} \quad T = C = 1,
\end{cases}
$$

where the external potential flow is $U(x,t) = U_0(x)+\epsilon U_1(x)\cos\omega t$. Because of the identical form of the equations as well as boundary conditions for temperature and concentration, both may be treated simultaneously. In the following analysis one may, therefore, interchange the quantities T and C, Pr and Sc, and Nu and Sh.

2. SOLUTIONS

Considering the nature of the velocity of the potential flow $U(x,t)$, which is imposed as perturbations, the following forms may be assumed for the functions involved:

$$u(x,y,t) = u_0(x,y) + \epsilon u_1(x,y,t) + \epsilon^2 u_2(x,y,t) + \ldots$$

$$v(x,y,t) = v_0(x,y) + \epsilon v_1(x,y,t) + \epsilon^2 v_2(x,y,t) + \ldots$$

$$T(x,y,t) = T_0(x,y) + \epsilon T_1(x,y,t) + \epsilon^2 T_2(x,y,t) + \ldots$$

By substituting these expansions into the governing equations and boundary conditions, and by separating terms according to the powers of ϵ^n, a set of simultaneous, linear differential equations and boundary conditions are found as follows:

Zeroth-order perturbation, ϵ^0

$$\left.\begin{aligned}
&\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0 \\
&u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} = \frac{1}{Re}\frac{\partial^2 u_0}{\partial y^2} + U_0 \frac{dU_0}{dx} \\
&u_0 \frac{\partial T_0}{\partial x} + v_0 \frac{\partial T_0}{\partial y} = \frac{1}{RePr}\frac{\partial^2 T_0}{\partial y^2} \\
&\text{with the boundary conditions} \\
&y = 0;\quad u_0 = T_0 = 0 \\
&\qquad\text{and}\quad v_0 = \begin{cases} 0 & \text{without suction or blowing} \\ V & \text{positive for uniform blowing and negative for uniform suction} \end{cases} \\
&y = \infty;\quad u_0 = U_0(x),\quad T_0 = 1
\end{aligned}\right\} \quad (1)$$

First-order perturbation, ϵ^1

$$\left.\begin{aligned}
&\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \\
&\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_0}{\partial x} + u_0 \frac{\partial u_1}{\partial x} + v_0 \frac{\partial u_1}{\partial y} + v_1 \frac{\partial u_0}{\partial y} = \left(U_1 \frac{dU_0}{dx} + U_0 \frac{dU_1}{dx}\right) \cos \omega t - \omega U_1 \sin \omega t \\
&\frac{\partial T_1}{\partial t} + u_0 \frac{\partial T_1}{\partial x} + u_1 \frac{\partial T_0}{\partial x} + v_0 \frac{\partial T_1}{\partial y} + v_1 \frac{\partial T_0}{\partial y} = \frac{1}{RePr} \frac{\partial^2 T_1}{\partial y^2}
\end{aligned}\right\} \quad (2)$$

with the boundary conditions

$$y = 0; \quad u_1 = v_1 = T_1 = 0$$

$$y = \infty; \quad u_1 = U_1(x) \cos \omega t, \quad T_1 = 0$$

Second-order perturbation, ϵ^2

$$\left.\begin{aligned}
&\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0 \\
&\frac{\partial u_2}{\partial t} + u_2 \frac{\partial u_0}{\partial x} + u_0 \frac{\partial u_2}{\partial x} + u_1 \frac{\partial u_1}{\partial x} + v_2 \frac{\partial u_0}{\partial y} + v_0 \frac{\partial u_2}{\partial y} + v_1 \frac{\partial u_1}{\partial y} \\
&\quad = \frac{U_1}{2} \frac{dU_1}{dx} (1 + \cos 2\omega t) + \frac{1}{Re} \frac{\partial^2 u_2}{\partial y^2} \\
&\frac{\partial T_2}{\partial t} + u_0 \frac{\partial T_2}{\partial x} + u_2 \frac{\partial T_0}{\partial x} + u_1 \frac{\partial T_1}{\partial x} + v_0 \frac{\partial T_2}{\partial y} + v_2 \frac{\partial T_0}{\partial y} + v_1 \frac{\partial T_1}{\partial y} = \frac{1}{RePr} \frac{\partial^2 T_2}{\partial y^2}
\end{aligned}\right\} \quad (3)$$

with the boundary conditions

$$y = 0; \quad u_2 = v_2 = T_2 = 0$$

$$y = \infty; \quad u_2 = T_2 = 0$$

Here the value of ϵ is chosen small such that the first three terms of the expansion will approximate the physical problem.

The best analytical method available at present for the solution of the nonlinear partial differential equations of the boundary layer such as Eqs. (1), (2), and (3) is that of the power series. Frössling (1938) has discussed the series solutions of the steady boundary-layer Eq. (1) for several types of outer velocity distribution $U_o(x)$. For a flow over a symmetrical blunt body, Blasius (1939) has introduced a symmetrical velocity distribution for $U_o(x)$ as

$$U_o(x) = \sum_{k=0}^{\infty} a_{2k+1} x^{2k+1}. \tag{4}$$

Later Howarth (1940) has extended the Blasius series solution to an unsymmetrical velocity distribution.

$$U_o(x) = \sum_{k=0}^{\infty} a_k x^k \tag{5}$$

For a sharp-edged body, Görtler (1941) has adopted

$$U_o(x) = x^m \sum_{k=0}^{\infty} a_k x^{k(m+1)} \tag{6}$$

as the external flow. The corresponding solution of the heat transfer problem with the Görtler series development has been given by Sparrow (1942) and Wrage (1943).

The Blasius technique was further generalized by Frössling (1938) using the very general velocity distribution

$$U_o(x) = x^m \sum_{k=0}^{\infty} a_k x^{Mk} \tag{7}$$

for a two-dimensional velocity field, where m and M are arbitrary numbers. The velocity distributions described by Eqs. (4), (5) and (6) are special cases of this general form. The outer velocity distribution of Eq. (7) represents approximately a wedge flow $U_o(x) = a_o x^m$ in the vicinity of $x = 0$ when the wedge angle is $2m\pi/m+1$.

In the following the solutions of the boundary-layer Eqs. (1), (2) and (3) are obtained in two forms: an ordinary Blasius series for a symmetrical blunt body and a generalized Blasius series for a sharp-edged body.

a. Ordinary Blasius Series Solutions for a Symmetrical Blunt Body

1. Solution to the Zeroth-Order Perturbation

The zeroth-order perturbation is the case of steady-state forced convection, Frössling (1937). The dimensionless distance from the wall, defined as $\eta = (a_1 Re)^{1/2} y$, is selected as the similarity variable. The equation of continuity is satisfied by the introduction of the stream function defined by $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. This definition of the stream function is employed for all other orders of perturbation. Using Eq. (4) as an expression for the external velocity distribution around a symmetrical blunt body, the solutions for the stream function and temperature are obtained as:

$$\left.\begin{aligned} \psi_0(x,\eta) &= \left(\frac{a_1}{Re}\right)^{1/2}\left(f_{01}x + 4\,\frac{a_3}{a_1}\,f_{03}\,x^3 + 6\,\frac{a_5}{a_1}\,f_{05}\,x^5 + \ldots\right), \\ \theta_0(x,\eta) &= F_{00} + 4\,\frac{a_3}{a_1}\,F_{02}\,x^2 + 6\,\frac{a_5}{a_1}\,F_{04}\,x^4 + \ldots , \end{aligned}\right\} \quad (8)$$

where

$$f_{05} = f_{051} + \frac{a_3^2}{a_1 a_5}\,f_{052}$$

$$f_{07} = f_{071} + \frac{a_3 a_5}{a_1 a_7}\,f_{072} + \frac{a_3^2}{a_1^2 a_7}\,f_{073}$$

. . . . .

and

$$F_{04} = F_{041} + \frac{a_3^2}{a_1 a_5}\,F_{042}$$

$$F_{07} = F_{071} + \frac{a_3 a_5}{a_1 a_7}\,F_{072} + \frac{a_3^2}{a_1^2 a_7}\,F_{073} .$$

. . . . .

A system of simultaneous ordinary differential equations and appropriate boundary conditions for the functions f and F described presented in Tables I and II.

TABLE I

EQUATIONS AND BOUNDARY CONDITIONS WHICH DEFINE AN ORDINARY BLASIUS SERIES SOLUTION FOR VELOCITY IN OSCILLATING FLOW PAST A SYMMETRICAL BLUNT BODY

For	f	Equation for f	Boundary conditions for f: f(0)	f(0)	f'(∞)	Equation Number Used in the Computer Program
ψ_0	f_{01}	$f_{01}'''+f_{01}f_{01}''-f_{01}'^2 = -1$	0 or ± V	0	1	1
	f_{03}	$f_{03}'''+f_{01}f_{03}''-4f_{01}'f_{03}'+3f_{01}''f_{03} = -1$	0	0	1/4	2
	f_{051}	$f_{051}'''+f_{01}f_{051}''-6f_{01}'f_{051}'+5f_{01}''f_{051} = -1$	0	0	1/6	
	f_{052}	$f_{052}'''+f_{01}f_{052}''-6f_{01}'f_{051}'+5f_{01}''f_{051} = -\frac{1}{2}+8(f_{03}'^2-f_{03}f_{03}'')$	0	0	0	
ψ_{11}	f_{111}	$f_{111}'''+f_{01}f_{111}''-2f_{01}'f_{111}'+f_{01}''f_{111} = f_{01}'+\frac{1}{2}\eta f_{01}''-1$	0	0	0	3
	f_{113}	$f_{113}'''+f_{01}f_{113}''-4f_{01}'f_{113}'+3f_{01}''f_{113} = \frac{\eta}{2}f_{03}''+f_{03}'-(3f_{03}f_{111}''-4f_{03}'f_{111}'+f_{03}''f_{111})-\frac{1}{4}$	0	0	0	4
ψ_{12}	f_{121}	$f_{121}'''+f_{01}f_{121}''-2f_{01}'f_{121}'+f_{01}''f_{121} = f_{111}'$	0	0	0	5
	f_{123}	$f_{123}'''+f_{01}f_{123}''-4f_{01}'f_{123}'+3f_{01}''f_{123} = f_{113}'-(3f_{03}f_{111}''-4f_{03}'f_{111}'+f_{111}f_{03}'')$	0	0	0	6
ψ_{20s}	f_{20s1}	$f_{20s1}'''+f_{01}f_{20s1}''-2f_{01}'f_{20s1}'+f_{01}''f_{20s1} = -1+(f_{01}'+\frac{\eta}{2}f_{01}'')^2-\frac{1}{4}(f_{01}+\eta f_{01}')(3f_{01}''+\eta f_{01}''')$	0	0	0	7
	f_{20s3}	$f_{20s3}'''+f_{01}f_{20s3}''-4f_{01}'f_{20s3}'+3f_{01}''f_{20s3} = 4f_{03}'f_{20s1}'-1+4(f_{01}'+\frac{\eta}{2}f_{01}'')(f_{01}'+\frac{\eta}{2}f_{01}'')$ $-\frac{3}{4}(3f_{01}''+\eta f_{01}''')(f_{03}+\eta f_{03}')-\frac{1}{4}(f_{01}+\eta f_{01}')(3f_{03}''+\eta f_{03}''')-f_{03}''f_{20s1}$	0	0	0	8
ψ_{21t}	f_{21t1}	$f_{21t1}'''+f_{01}f_{21t1}''-2f_{01}'f_{21t1}'+f_{01}''f_{21t1} = 2f_{20s1}'+f_{111}'(f_{01}'+\frac{\eta}{2}f_{01}'')-\frac{1}{2}(f_{01}+\eta f_{01}')f_{111}''$	0	0	0	9
	f_{21t3}	$f_{21t3}'''+f_{01}f_{21t3}''-4f_{01}'f_{21t3}'+3f_{01}''f_{21t3} = 2f_{20s3}'+4f_{03}'f_{21t1}'+f_{111}'(f_{03}'+\frac{\eta}{2}f_{03}'')+3f_{113}'(f_{01}'+\frac{\eta}{2}f_{01}'')$ $-3f_{03}f_{21t1}''-\frac{1}{3}f_{113}''(f_{01}+\eta f_{01}')-\frac{3}{2}f_{111}''(f_{03}+\eta f_{03}')-f_{03}''f_{21t1}$	0	0	0	10
ψ_{22s}	f_{22s1}	$f_{22s1}'''+f_{01}f_{22s1}''-2f_{01}'f_{22s1}'+f_{01}''f_{22s1} = 2f_{121}'(f_{01}'+\frac{\eta}{2}f_{01}'')-f_{111}'^2-\frac{f_{121}''}{2}(f_{01}+\eta f_{01}')-\frac{f_{121}}{2}(3f_{01}''$ $+\eta f_{01}''')+f_{121}f_{121}''$	0	0	0	11
	f_{22s3}	$f_{22s3}'''+f_{01}f_{22s3}''-4f_{01}'f_{22s3}'+3f_{01}f_{22s3} = 4f_{03}'f_{22s1}'-3f_{03}f_{22s1}''-f_{03}''f_{22s1}+4f_{121}'(f_{03}'+\frac{\eta}{2}f_{03}'')$ $+4f_{123}'(f_{01}'+\frac{\eta}{2}f_{01}'')-4f_{111}'f_{113}'-\frac{3}{2}f_{121}''(f_{03}+\eta f_{03}')-\frac{1}{2}f_{123}''(f_{01}+\eta f_{01}')-\frac{3}{2}f_{123}(3f_{01}''+\eta f_{01}''')$ $-\frac{f_{01}}{2}(3f_{03}''+\eta f_{03}''')+3f_{121}''f_{123}+f_{121}f_{123}''$	0	0	0	12
ψ_{22t}	f_{22t1}	$f_{22t1}'''+f_{01}f_{22t1}''-2f_{01}'f_{22t1}'+f_{01}''f_{22t1} = 2f_{121}'(f_{01}'+\frac{\eta}{2}f_{01}'')+f_{111}'^2-\frac{f_{121}''}{2}(f_{01}+\eta f_{01}')-f_{111}f_{111}''$ $+2f_{21t1}'-\frac{f_{121}}{2}(3f_{01}''+\eta f_{01}''')$	0	0	0	13
	f_{22t3}	$f_{22t3}'''+f_{01}f_{22t3}''-4f_{01}'f_{22t3}'+3f_{01}''f_{22t3}-4f_{03}'f_{22t1}'-3f_{03}f_{22t1}''-f_{03}''f_{22t1}+4f_{121}'(f_{03}'+\frac{\eta}{2}f_{03}'')$ $+4f_{123}'(f_{01}'+\frac{\eta}{2}f_{01}'')+4f_{111}'f_{113}'-\frac{3}{2}f_{121}''(f_{03}+\eta f_{03}')-\frac{1}{2}f_{123}''(f_{01}+\eta f_{01}')-\frac{3}{2}f_{123}(3f_{01}''+\eta f_{01}''')$ $-\frac{f_{01}}{2}(3f_{03}''+\eta f_{03}''')-3f_{121}''f_{123}-f_{121}f_{123}''-2f_{21t3}'$	0	0	0	14

TABLE II

EQUATIONS AND BOUNDARY CONDITIONS WHICH DEFINE AN ORDINARY BLASIUS SERIES SOLUTION FOR TEMPERATURE OR CONCENTRATION IN OSCILLATING FLOW PAST A SYMMETRICAL BLUNT BODY

For	F	Equation for F	Boundary conditions for F: F(0)	Boundary conditions for F: F(∞)	Equation Number Used in the Computer Program
θ_0	F_{00}	$-f_{01}F'_{00} = \frac{F''_{00}}{Pr}$	0	1	15
	F_{02}	$2f'_{01}F_{02} - 3f_{03}F'_{00} - f_{01}F'_{02} = \frac{F''_{02}}{Pr}$	0	0	16
	F_{041}	$-f_{01}F'_{041} + 4f'_{01}F_{041} - 5f_{041}F_{00} = \frac{F_{041}}{Pr}$	0	0	
	F_{042}	$-f_{01}F'_{042} + 4f'_{01}F_{042} - 5f_{042}F'_{00} + \frac{8}{3}(2f'_{03}F_{02} - 3f_{03}F'_{02}) = \frac{F''_{042}}{Pr}$	0	0	
θ_{11}	F_{110}	$-f_{01}F'_{110} - \left(f_{111} - \frac{\eta}{2}\right)F'_{00} = \frac{F''_{110}}{Pr}$	0	0	17
	F_{112}	$2f'_{01}F_{112} + 2f'_{111}F_{02} - 3f_{03}F'_{110} - f_{01}F'_{02} - 3f_{113}F'_{00} - f_{111}F'_{02} + \frac{F'_{02}}{2}\eta = \frac{F''_{112}}{Pr}$	0	0	18
θ_{12}	F_{120}	$-f_{121}F'_{00} - f_{01}F'_{120} + F_{110} = \frac{F''_{120}}{Pr}$	0	0	19
	F_{122}	$-3f_{03}F'_{120} - 3f_{123}F'_{00} - f_{121}F'_{02} + 2f'_{01}F_{122} + 2f'_{121}F_{02} - f_{01}F'_{122} + F_{112} = \frac{F''_{122}}{Pr}$	0	0	20
θ_{20s}	F_{20s0}	$-f_{01}F'_{20s0} - \frac{1}{4}(F'_{00}+\eta F''_{00})(f_{01}+\eta f'_{01}) - f_{20s1}F'_{00} = \frac{F_{20s0}}{Pr}$	0	0	21
	F_{20s2}	$2f'_{01}F_{20s2} + F'_{02}\left(f'_{01} + \frac{\eta}{2}f''_{01}\right) + 2f'_{20s1}F_{02} - 3f_{03}F'_{20s0} - f_{01}F'_{20s2} - \frac{3}{4}(F'_{00}+\eta F''_{00})(f_{03}+\eta f'_{03})$	0	0	22
		$- \frac{1}{4}(F'_{02}+\eta F''_{02})(f_{01}+\eta f'_{01}) - 3f_{20s3}F'_{00} - f_{20s1}F'_{02} = \frac{F''_{20s2}}{Pr}$			
θ_{21t}	F_{21t0}	$-\frac{f_{111}}{2}(F'_{00}+\eta F''_{00}) - \frac{F'_{110}}{2}(f_{01}+\eta f'_{01}) + \eta F'_{00} - f_{21t1}F'_{00} - f_{01}F'_{21t0} = \frac{F_{21t0}}{Pr}$	0	0	23
	F_{21t2}	$2f'_{01}F_{21t2} + 2f'_{21t1}F_{02} - 3f_{03}F'_{21t0} - f_{01}F'_{21t2} - 3f_{21t3}F'_{00} - f_{21t1}F'_{02} + \eta f'_{01}F'_{02} + 2\left(f'_{01} + \frac{\eta}{2}f''_{01}\right)F_{112}$	0	0	24
		$- \frac{3}{2}f_{113}(F'_{00}+\eta F''_{00}) - \frac{1}{2}f_{111}(F'_{02}+\eta F''_{02}) - \frac{3}{2}(f_{03}+\eta f'_{03})F'_{110} - \frac{1}{2}(f_{01}+\eta f'_{01})F'_{02} + \eta F'_{02} = \frac{F''_{21t2}}{Pr}$			
θ_{22s}	F_{22s0}	$-f_{01}F'_{22s0} - f_{22s1}F'_{00} - \frac{1}{2}f_{121}(F'_{00}+\eta F''_{00}) - \frac{1}{2}(f_{01}+\eta f'_{01})F'_{120} + f_{111}F'_{110} = \frac{F''_{22s0}}{Pr}$	0	0	25
	F_{22s2}	$2f'_{22s1}F_{02} - 3f_{03}F'_{22s0} - f_{01}F'_{22s2} - 3f_{22s3}F'_{00} - f_{22s1}F'_{02} + 2f'_{01}F_{22s2} + 2\ f'_{01} + \frac{\eta}{2}f''_{01}\ F'_{112} - 2f'_{111}F'_{112}$	0	0	26
		$- \frac{3}{2}f_{113}(F'_{00}+\eta F''_{00}) - \frac{f_{121}}{2}(F'_{02}+\eta F''_{02}) - 2(f_{03}+\eta f'_{03})F'_{00} - \frac{1}{2}(f_{01}+\eta f'_{01})F'_{122} + 3f_{113}F'_{110} + f_{111}F'_{112} = \frac{F''_{22s2}}{Pr}$			
θ_{22t}	F_{22t0}	$-f_{01}F'_{22t0} - f_{22t1}F'_{00} - \frac{1}{2}f_{121}(F'_{00}+\eta F''_{00}) - \frac{1}{2}(f_{01}+\eta f'_{01})F'_{120} - f_{111}F'_{110} + \frac{1}{2}F_{21t0} = \frac{F_{22t0}}{Pr}$	0	0	27
	F_{22t2}	$2f'_{22t1}F_{02} - 3f_{03}F'_{22t0} - f_{01}F'_{22t2} - 3f_{22t3}F'_{00} - f_{22t1}F'_{02} + 2f'_{01}F_{22t2} + \eta f'_{121}F'_{02} + 2\left(f'_{01} + \frac{\eta}{2}f''_{01}\right)$	0	0	28
		$F'_{122} + 2f'_{111}F'_{112} - \frac{3}{2}f_{03}(F'_{00}+\eta F''_{00}) - \frac{f_{121}}{2}(F'_{02}+\eta F''_{02}) - 2(f_{03}+\eta f'_{03})F'_{120} - \frac{1}{2}(f_{01}+\eta f'_{01})F'_{122}$			
		$- 3f_{113}F'_{110} - f_{111}f'_{112} + \frac{1}{2}F_{21t2} = \frac{F''_{22t2}}{Pr}$			

2. Solution to the First-Order Perturbation

In the first- and second-order perturbations, most terms depend upon the functions of solutions of the zero-order perturbation. For low frequencies, the velocity and temperature components are expanded in terms of frequency as follows. It is convenient to adopt complex notations and to write

$$\left.\begin{aligned} u_1(x,y,t) &= \text{real}\ \{[(u_{10}(x,y)+(i\omega)u_{11}(x,y)+(i\omega)^2u_{12}(x,y)+\ldots]e^{i\omega t}\} \\ v_1(x,y,t) &= \text{real}\ \{[(v_{10}(x,y)+(i\omega)v_{11}(x,y)+(i\omega)^2v_{12}(x,y)+\ldots]e^{i\omega t}\} \\ T_1(x,y,t) &= \text{real}\ \{[(\theta_{10}(x,y)+(i\omega)\theta_{11}(x,y)+(i\omega)^2\theta_{12}(x,y)+\ldots]e^{i\omega t}\} \end{aligned}\right\} \quad (9)$$

The new functions u_{10}, u_{10}, θ_{10},... are now complex quantities and independent of time. Substituting Eq. (9) into Eq. (2) and observing that the symbol "real" appears in front of every term, it is disclosed that since "real" is a linear operator, it can be dropped out of the equations. All terms are now linear in $e^{i\omega t}$ which can also be dropped out. In this manner, time dependency is omitted from the differential equations. Arranging the time-independent equations according to the powers (ℓ = 0,1,2,...etc.) or $i\omega$, one obtains:

ℓth order approximation, $(i\omega)^\ell$

$$\left.\begin{aligned} &\frac{\partial u_{1\ell}}{\partial x} + \frac{\partial v_{1\ell}}{\partial y} = 0 \\ &u_0 \frac{\partial u_{1\ell}}{\partial x} + u_{1\ell} \frac{\partial u_{1\ell}}{\partial x} + v_0 \frac{\partial u_{1\ell}}{\partial y} + v_{1\ell} \frac{\partial u_0}{\partial y} = \frac{1}{Re} \frac{\partial^2 u_{1\ell}}{\partial y^2} + \begin{cases} U_0 \dfrac{dU_1}{dx} + U_1 \dfrac{dU_0}{dx} & \text{for } \ell = 0 \\ U_1 - u_{10} & \text{for } \ell = 1 \\ -u_{1(\ell-1)} & \text{for } \ell > 1 \end{cases} \\ &u_0 \frac{\partial \theta_{1\ell}}{\partial x} + u_{1\ell} \frac{\partial \theta_0}{\partial x} + v_0 \frac{\partial \theta_{1\ell}}{\partial y} + v_{1\ell} \frac{\partial \theta_0}{\partial y} = \frac{1}{RePr} \cdot \frac{\partial^2 \theta_{1\ell}}{\partial y^2} + \begin{cases} 0 & \text{, for } \ell = 0 \\ -\theta_{1(\ell-1)} & \text{, for } \ell > 0 \end{cases} \end{aligned}\right\} (10)$$

with the boundary conditions

$$\eta = 0; \quad u_{1\ell} = v_{1\ell} = \theta_{1\ell} = 0$$

$$\eta = \infty; \quad u_{1\ell} = \begin{cases} U_1(x) & \text{for } \ell = 0 \quad \theta_{1\ell} = 0 \\ 0 & \text{for } \ell > 0 \end{cases}$$

where ℓ is the order of approximation, $(i\omega)$.

Let us consider a particular case of two-dimensional flow about a fixed symmetrical blunt body, when the fluctuations in the external flow are produced by fluctuations in the magnitude but not the direction of the velocity U_∞ of the oncoming stream relative to the body. The latter, which is called the fluctuating circulation,[15] will be treated later. If the fluctuation of the oncoming flow velocity is $U_\infty(1+\epsilon e^{i\omega t})$, then the external flow about the cylinder will fluctuate by the same factor and may be expressed as

$$U(x,t) = U_0(x)\,(1+\epsilon e^{i\omega t}). \tag{11}$$

This suggests that one is concerned with the case $U_0(x) = U_1(x)$ in this section. The solutions to the first-order perturbation are obtained:

$$\left.\begin{aligned} \psi_{1\ell} &= \begin{cases} \dfrac{1}{2}\left(\psi_0 + y\,\dfrac{\partial\psi_0}{\partial y}\right) & \text{for } \ell = 0 \\[2ex] \dfrac{1}{(a_1 Re)^{1/2} a_1{}^{\ell}}\left(f_{1\ell 1}\, x + 4\,\dfrac{a_3}{a_1}\, f_{1\ell 3}\, x^3 + 6\,\dfrac{a_5}{a_1}\, f_{1\ell 5}\, x^5 + \ldots\right), & \end{cases} \\[3ex] \theta_{1\ell} &= \begin{cases} \dfrac{1}{2}\, y\,\dfrac{\partial\theta_0}{\partial y} & \text{for } \ell = 0 \\[2ex] \dfrac{1}{a_1{}^{\ell}}\left(F_{1\ell 0} + 4\,\dfrac{a_3}{a_1}\, F_{1\ell 2}\, x^2 + 6\,\dfrac{a_5}{a_1}\, F_{1\ell 4}\, x^4 + \ldots\right), & \end{cases} \end{aligned}\right\} \tag{12}$$

where $f_{1\ell}$ and $F_{1\ell}$ are universal functions as in Tables I and II, respectively.

3. Solution to the Second-Order Perturbation

In the second-order perturbation for $u_2(x,y,t)$, $v_2(x,y,t)$, and $T_2(x,y,t)$, the convective terms of the governing equations will contribute terms with

$\cos^2 \omega t$. These, in turn, can be reduced to terms with $\cos 2\omega t$, $\sin 2\omega t$, and steady-state, i.e., time-independent terms, Schlichting (1935). Therefore, u_2, v_2, and T_2 for low frequencies may be expanded as follows:

$$\left.\begin{aligned}
u_2(x,y,t) &= \frac{1}{2}\,\text{real}\,\{u_{20s}(x,y) + (i\omega)u_{21s}(x,y) + (i\omega)^2 u_{22s}(x,y) + \ldots \\
&\quad + [u_{20t}(x,y) + (i\omega)u_{21t}(x,y) + (i\omega)^2 u_{22t}(x,y) + \ldots]e^{i2\omega t}\} \\
v_2(x,y,t) &= \frac{1}{2}\,\text{real}\,\{v_{20s}(x,y) + (i\omega)v_{21s}(x,y) + (i\omega)^2 v_{22s}(x,y) + \ldots \\
&\quad + [v_{20t}(x,y) + (i\omega)v_{21t}(x,y) + (i\omega)^2 v_{22t}(x,y) + \ldots]e^{i2\omega t}\} \\
T_2(x,y,t) &= \frac{1}{2}\,\text{real}\,\{\theta_{20s}(x,y) + (i\omega)\theta_{21s}(x,y) + (i\omega)^2 \theta_{22s}(x,y) + \ldots \\
&\quad + [\theta_{20t}(x,y) + (i\omega)\theta_{21t}(x,y) + (i\omega)^2 \theta_{22t}(x,y) + \ldots]e^{i2\omega t}\}
\end{aligned}\right\} \quad (13)$$

The equations of continuity, momentum and energy, which are obtained by substituting these relationships into Eq. (3) and by separating according to frequency- and time-dependency, are as follows. Although the analysis has been extended to include the first- and second-order approximations, only two resulting time-dependent (t) and time-independent (s) equations corresponding to the zeroth-order approximation, $(i\omega)^0$ are presented here. They are: (j = t or s)

$$\left.\begin{aligned}
&\frac{\partial u_{20j}}{\partial x} + \frac{\partial v_{20j}}{\partial y} = 0 \\
&u_0 \frac{\partial u_{20j}}{\partial x} + u_{10}\frac{\partial u_{10}}{\partial x} + u_{20j}\frac{\partial u_0}{\partial x} + v_0\frac{\partial u_{20j}}{\partial y} + v_{10}\frac{\partial u_{10}}{\partial y} + v_{20j}\frac{\partial u_0}{\partial y} \\
&\qquad = \frac{1}{Re}\frac{\partial^2 u_{20j}}{\partial y^2} + U_1\frac{dU_1}{dy} \\
&u_0 \frac{\partial \theta_{20j}}{\partial x} + u_{10}\frac{\partial \theta_{10}}{\partial x} + u_{20j}\frac{\partial \theta_0}{\partial x} + v_0\frac{\partial \theta_{20j}}{\partial y} + v_{10}\frac{\partial \theta_{10}}{\partial y} + v_{20j}\frac{\partial \theta_0}{\partial y} \\
&\qquad = \frac{1}{RePr}\frac{\partial^2 \theta_{20j}}{\partial y^2}
\end{aligned}\right\} \quad (14)$$

with the boundary conditions

$$\eta = 0: \quad u_{20j} = v_{20j} = \theta_{20j} = 0$$

$$\eta = \infty: \quad u_{20j} = \theta_{20j} = 0$$

where j refers to either the steady or transient state.

The solutions to the second perturbation are as follows:

$$\left.\begin{aligned} \psi_{2\ell j} &= \frac{1}{(a_1 Re)^{1/2} a_1^{\ell}} \left(f_{2\ell j1}\, x + 4\, \frac{a_3}{a_1}\, f_{2\ell j3}\, x^3 + 6\, \frac{a_5}{a_1}\, f_{2\ell j5}\, x^5 + \ldots \right) \\ \theta_{2\ell j} &= \frac{1}{a_1^{\ell}} \left(F_{2\ell jo} + \frac{4a_3}{a_1}\, F_{2\ell j2}\, x^2 + \frac{6a_5}{a_1}\, F_{2\ell j4}\, x^4 + \ldots \right), \end{aligned}\right\} \qquad (15)$$

where the functions $f_{2\ell j}$ and $F_{2\ell j}$ are defined in Tables I and II respectively.

b. Generalized Blasius Series Solutions for a Sharp-Edged Body

The Blasius technique, as demonstrated above, may be applied to obtain the solution of oscillating boundary-layer flow problems for symmetrical bodies with blunt noses. It is now desired to produce a more general solution from which several special cases of interest could be obtained. For example, the body has a sharp-edged nose or the fluctuations in the velocity of oncoming stream is both magnitude- and direction-dependent such that $U_o(x)$ is different from $U_1(x)$. In a paper by Hori[15] concerning unsteady boundary layers, an external flow

$$U_o(x) = x^m \sum_{k=0}^{\infty} a_k\, x^k$$

superimposed by an oscillating component with amplitude

$$U_1(x) = x^n \sum_{k=0}^{\infty} b_k\, x^k$$

has been studied for a two-dimensional velocity field by a Blasius technique and several universal functions have been defined. In this section a more general velocity distribution as expressed by Eq. (7) superimposed by a fluctuating component with amplitude

$$U_1(x) = x^n \sum_{k=0}^{\infty} b_k x^{Mk} \tag{16}$$

will be considered, wherefore the coefficients b_k depend upon the type of flow oscillation. The solutions thus obtained for fluctuating velocity and temperature in the unsteady boundary layers will be in the form of a generalized Blasius series. Since the velocity distribution, Eq. (7), represents approximately a wedge flow in the neighborhood of the forward stagnation point when the wedge angle is $2m\pi/m+1$, the solutions may be applied to fluctuating flow problems about a sharp-edged body.

Solutions for the velocity and temperature components are obtained following the same procedure as presented in the preceding section for a symmetrical blunt body. The zeroth-order perturbation is the case of steady-state forced convection, Frössling.[38] The dimensionless parameter, defined as $\eta = y(a_oRe)^{1/2} x^{m-1/2}$, is selected as the similarity variable. The solutions of equation are obtained as

$$\left.\begin{aligned} \psi_o(x,\eta) &= \frac{1}{(a_oRe)^{1/2}} x^{m+1/2} \sum_{k=0}^{\infty} a_k f_{ok}(\eta) x^{kM}, \\ T_o = \theta_o &= \sum_{k=0}^{\infty} x^{Mk} F_{ok}, \end{aligned}\right\} \tag{17}$$

The functions f_{ok} and F_{ok}, $f_{1\ell k}$ and $F_{1\ell k}$, and $f_{2\ell jk}$ and $F_{2\ell jk}$ as defined respectively by Eqs. (17), (18) and (19) must satisfy the equations and appropriate boundary conditions given in Tables III and IV.

The solutions to the first-order perturbation for $U_1(x)$ defined as Eq. (16), are obtained as

$$\left.\begin{aligned} \psi_{1\ell} &= \frac{1}{(a_oRe)^{1/2} a_o^{\ell}} x^{\frac{2n+(2\ell+1)(1-m)}{2}} \sum_{k=0}^{\infty} b_k f_{1\ell k} x^{Mk} \\ &\text{and} \\ \theta_{1\ell} &= \frac{b_o}{a_o^{\ell+1}} \sum_{k=0}^{\infty} F_{1\ell k} x^{n-m+\ell(1-m)+Mk} \end{aligned}\right\} \tag{18}$$

TABLE III

EQUATIONS AND BOUNDARY CONDITIONS WHICH DEFINE A GENERALIZED BLASIUS SERIES SOLUTION FOR VELOCITY IN OSCILLATING FLOW PAST A SHARP-EDGED BODY

For	f	Equation for f	Boundary Condition f(0)	f'(0)	f'(∞)
ψ_0	f_{01}	$f_{01}''' + \frac{m+1}{2} f_{01}f_{01}'' - mf_{01}'^{2} = -m$	0	0	1
	f_{02}	$f_{02}''' + \frac{m+1}{2} f_{01}f_{02}'' - (2m+1)f_{01}'f_{02}' + \frac{m+3}{2} f_{02}f_{01}'' = -(2m+1)$	0	0	1
	f_{031}	$f_{031}''' + \frac{m+1}{2} f_{01}f_{031}'' - (3m+2)f_{01}'f_{031}' + \frac{m+5}{2} f_{01}''f_{031} = -(2m+2)$	0	0	1
	f_{032}	$f_{032}''' + \frac{m+1}{2} f_{01}f_{032}'' - (2m+2)f_{01}'f_{032}' + \frac{m+5}{2} f_{01}''f_{032} = -(m+1) + \frac{m+1}{2} f_{02}'^{2} - \frac{m+3}{2} f_{02}f_{02}''$	0	0	0
ψ_{10}	f_{101}	$f_{101}''' + \frac{m+1}{2} f_{01}f_{101}'' - (n+1)f_{01}'f_{101}' + \frac{2n+1-m}{2} f_{01}''f_{101} = -(m+n)$	0	0	1
	f_{1021}	$f_{1021}''' + \frac{m+1}{2} f_{01}f_{1021}'' - (n+2)f_{01}'f_{1021}' + \frac{2n+3-m}{2} f_{01}''f_{1021} = -(n+m+1)$	0	0	1
	f_{1022}	$f_{1022}''' + \frac{m+1}{2} f_{01}f_{1022}'' - (n+2)f_{01}'f_{1022}' + \frac{2n+3-m}{2} f_{01}''f_{1022} = -\frac{m+3}{2} f_{02}f_{101}'' + (n+2)f_{02}'f_{101}' - \frac{2n+1-m}{2} f_{02}''f_{101} - (n+m+1)$	0	0	0
	f_{1031}	$f_{1031}''' + \frac{m+1}{2} f_{01}f_{1031}'' - (n+3)f_{01}'f_{1031}' + \frac{2n+5-m}{2} f_{01}''f_{1031} = -(m+n+2)$	0	0	1
	f_{1032}	$f_{1032}''' + \frac{m+1}{2} f_{01}f_{1032}'' - (n+3)f_{01}'f_{1032}' + \frac{2n+5-m}{2} f_{01}''f_{1032} = -\frac{m+3}{2} f_{02}f_{1021}'' + (n+3)f_{02}'f_{1021}' - \frac{2n+3-m}{2} f_{02}'f_{1021} - (n+m+2)$	0	0	0
	f_{1033}	$f_{1033}''' + \frac{m+1}{2} f_{01}f_{1033}'' - (n+3)f_{01}'f_{1033}' + \frac{2n+5-m}{2} f_{01}''f_{1033} = -\frac{m+3}{2} f_{02}f_{1022}'' + (n+3)f_{02}'f_{1022}' - \frac{2n+3-m}{2} f_{02}'f_{1022}$	0	0	0
	f_{1034}	$f_{1034}''' + \frac{m+1}{2} f_{01}f_{1034}'' - (n+3)f_{01}'f_{1034}' + \frac{2n+5-m}{2} f_{01}''f_{1034} = -\frac{m+5}{2} f_{03}f_{101}'' + (n+3)f_{03}'f_{101}' - \frac{2n+1-m}{2} f_{03}''f_{101} - (n+m+2)$	0	0	0
ψ_{11}	f_{111}	$f_{111}''' + \frac{m\ 1}{2} f_{01}f_{111}'' - (n-m+2)f_{01}'f_{111}' + \frac{2n-3m+3}{2} f_{01}''f_{111} = f_{101}' - 1$	0	0	0
	f_{1121}	$f_{1121}''' + \frac{m+1}{2} f_{01}f_{1121}'' - (n-m+3)f_{01}'f_{1121}' + \frac{2n-3m+5}{2} f_{01}''f_{1121} = f_{102}' - 1$	0	0	0
	f_{1122}	$f_{1122}''' + \frac{m+1}{2} f_{01}f_{1122}'' - (n-m+3)f_{01}'f_{1122}' + \frac{2n-3m+5}{2} f_{01}''f_{1122} = -\frac{m+3}{2} f_{02}f_{111}'' + (n-m+3)f_{02}'f_{111}' - \frac{2n-3m+3}{2} f_{02}''f_{111}$	0	0	0
	f_{1131}	$f_{1131}''' + \frac{m+1}{2} f_{01}f_{1131}'' - (n-m+4)f_{01}'f_{1131}' + \frac{2n-3m+7}{2} f_{01}''f_{1131} = f_{113}' - 1$	0	0	0
	f_{1132}	$f_{1132}''' + \frac{m+1}{2} f_{01}f_{1132}'' - (n-m+4)f_{01}'f_{1132}' + \frac{2n-3m+7}{2} f_{01}''f_{1132} = -\frac{m+3}{2} f_{02}f_{1121}'' + (n-m+4)f_{02}'f_{1121}' - \frac{2n-3m+5}{2} f_{02}''f_{1121}$	0	0	0
	f_{1133}	$f_{1133}''' + \frac{m+1}{2} f_{01}f_{1133}'' - (n-m+4)f_{01}'f_{1133}' + \frac{2n-3m+7}{2} f_{01}''f_{1133} = -\frac{m+3}{2} f_{02}f_{1122}'' + (n-m+4)f_{02}'f_{1122}' - \frac{2n-3m+5}{2} f_{02}''f_{1122}$	0	0	0
	f_{1134}	$f_{1134}''' + \frac{m+1}{2} f_{01}f_{1134}'' - (n-m+4)f_{01}'f_{1134}' + \frac{2n-3m+7}{2} f_{01}''f_{1134} = -\frac{m+5}{2} f_{03}f_{111}'' + (n-m+4)f_{03}'f_{111}' - \frac{2n-3m+3}{3} f_{03}''f_{111}$	0	0	0
ψ_{12}	f_{121}	$f_{121}''' + \frac{m+1}{2} f_{01}f_{121}'' - (n-2m+3)f_{01}'f_{121}' + \frac{2n-5m+5}{2} f_{01}''f_{121} = f_{111}'$	0	0	0
	f_{1221}	$f_{1221}''' + \frac{m+1}{2} f_{01}f_{1221}'' - (n-2m+4)f_{01}'f_{1221}' + \frac{2n-5m+7}{2} f_{01}''f_{1221} = -f_{112}'$	0	0	0
	f_{1222}	$f_{1222}''' + \frac{m+1}{2} f_{01}f_{1222}'' - (n-2m+4)f_{01}'f_{1222}' + \frac{2n-5m+7}{2} f_{01}''f_{1222} = -\frac{m+3}{2} f_{02}f_{121}'' + (n-m+4)f_{02}'f_{121}' - \frac{2n-5m+5}{2} f_{02}''f_{121}$	0	0	0
	f_{1231}	$f_{1231}''' + \frac{m+1}{2} f_{01}f_{1231}'' - (n-2m+5)f_{01}'f_{1231}' + \frac{2n-5m+9}{2} f_{01}''f_{1231} = f_{123}'$	0	0	0

TABLE III (Continued)

For	f	Equation for f	Boundary Condition		
			f(0)	f'(0)	f'(∞)
	f_{1232}	$f'''_{1232} + \frac{m+1}{2} f_{01}f''_{1232} - (n-2m+5)f'_{01}f'_{1232} + \frac{2n-5m+9}{2} f''_{01}f_{1232} = -\frac{m+3}{2} f_{02}f''_{1221} + (n-2m+5)f'_{02}f'_{1221} - \frac{2n-5m+7}{2} f''_{02}f_{1231}$	0	0	0
	f_{1233}	$f'''_{1233} + \frac{m+1}{2} f_{01}f''_{1233} - (n-2m+5)f'_{01}f'_{1233} + \frac{2n-5m+9}{2} f''_{01}f_{1233} = -\frac{m+5}{2} f_{03}f''_{121} + (n-2m+5)f'_{03}f'_{121} - \frac{2n-5m+5}{2} f''_{03}f_{121}$	0	0	0
	f_{1234}	$f'''_{1234} + \frac{m+1}{2} f_{01}f''_{1234} - (n-2m+5)f'_{01}f'_{1234} + \frac{2n-5m+9}{2} f''_{01}f_{1234} = -\frac{m+3}{2} f_{02}f''_{1222} + (n-2m+5)f'_{02}f'_{1222} - \frac{2n-5m+1}{2} f''_{02}f_{1222}$	0	0	0
ψ_{20s}	f_{20s1}	$f'''_{20s1} + \frac{m+1}{2} f_{01}f''_{20s1} - (2n-m+1)f'_{01}f'_{20s1} + \frac{4n-3m+1}{2} f''_{01}f_{20s1} = -\frac{2n+1-m}{2} f''_{101} + nf'_{101}f'_{101} - n$	0	0	0
	f_{20s21}	$f'''_{20s21} + \frac{m+1}{2} f_{01}f''_{20s21} - (2n-m+2)f'_{01}f'_{20s21} + \frac{4n-3m+2}{2} f''_{01}f_{20s21} = -\frac{m+3}{2} f_{02}f''_{20s1} + (2n-m+2)f'_{02}f'_{20s1} - \frac{4n-3m+1}{2} f''_{02}f_{20s1}$	0	0	0
	f_{20s22}	$f'''_{20s22} + \frac{m+1}{2} f_{01}f''_{20s22} - (2n-m+2)f'_{01}f'_{20s22} + \frac{4n-3m+3}{2} f''_{01}f_{20s22} = -\frac{2n+1-m}{2} f''_{102} + (2n+1)f'_{101}f'_{102} - \frac{2n+3-m}{2} f''_{101}f_{102} - (2n+1)$	0	0	0
	f_{20s31}	$f'''_{20s31} + \frac{m+1}{2} f_{01}f''_{20s31} - (2n-m+3)f'_{01}f'_{20s31} + \frac{4n-3m+5}{2} f''_{01}f_{20s31} = -\frac{m+3}{2} f_{02}f''_{20s21} + (2n-m+3)f'_{02}f'_{20s21} - \frac{4n-3m+3}{2} f''_{02}f_{20s21}$	0	0	0
	f_{20s32}	$f'''_{20s32} + \frac{m+1}{2} f_{01}f''_{20s32} - (2n-m+3)f'_{01}f'_{20s32} + \frac{4n-3m+5}{2} f''_{01}f_{20s32} = -\frac{m+3}{2} f_{02}f''_{20s22} + (2n-m+3)f'_{02}f'_{20s22} - \frac{4n-3m+3}{2} f''_{02}f_{20s22}$	0	0	0
	f_{20s33}	$f'''_{20s33} + \frac{m+1}{2} f_{01}f''_{20s33} - (2n-m+3)f'_{01}f'_{20s33} + \frac{4n-3m+5}{2} f''_{01}f_{20s33} = -\frac{m+5}{2} f_{03}f''_{20s1} + (2n-m+3)f'_{03}f'_{20s1} - \frac{4n-3m+1}{2} f''_{03}f_{20s1}$	0	0	0
	f_{20s34}	$f'''_{20s34} + \frac{m+1}{2} f_{01}f''_{20s34} - (2n-m+3)f'_{01}f'_{20s34} + \frac{4n-3m+5}{2} f''_{01}f_{20s34} = -\frac{2n-m+1}{2} f_{101}f''_{103} + (2n+2)f'_{101}f'_{103} - \frac{2n-m+5}{2} f''_{101}f_{103} - (2n+2)$	0	0	0
	f_{20s35}	$f'''_{20s35} + \frac{m+1}{2} f_{01}f''_{20s35} - (2n-m+3)f'_{01}f'_{20s35} + \frac{4n-3m+5}{2} f''_{01}f_{20s35} = -\frac{2n-m+3}{2} f_{102}f''_{102} + (n+1)f'_{102}f'_{102} - (n+1)$	0	0	0
ψ_{21t}	f_{21t1}	$f'''_{21t1} + \frac{m+1}{2} f_{01}f''_{21t1} - (2n-2m+1)f'_{01}f'_{21t1} + \frac{4n-5m+1}{2} f''_{01}f_{21t1} = 2f'_{20s1} - \frac{2n+1-m}{2} f_{101}f''_{111} + \frac{4n-3m+3}{2} f'_{101}f'_{111} - \frac{2n-3m+3}{2} f''_{101}f_{111}$	0	0	0
	f_{21t21}	$f'''_{21t21} + \frac{m+1}{2} f_{01}f''_{21t21} - (2n-2m+2)f'_{01}f'_{21t21} + \frac{4n-5m+3}{2} f''_{01}f_{21t21} = 2f'_{20s2} - \frac{m+3}{2} f_{02}f''_{21t1} + (2n-2m-2)f'_{02}f'_{21t1} - \frac{4n-5m+1}{2} f'_{02}f'_{21t1}$	0	0	0
	f_{21t22}	$f'''_{21t22} + \frac{m+1}{2} f_{01}f''_{21t22} - (2n-2m+2)f'_{01}f'_{21t22} + \frac{4n-5m+3}{2} f''_{01}f_{21t22} = -\frac{2n+5-m}{2} f_{101}f''_{112} + \frac{4n+5-m}{2} f'_{101}f'_{112} - \frac{2n+5-3m}{2} f''_{101}f_{112} - \frac{2n+3-m}{2} f_{112}f''_{121} + \frac{4n+5-m}{2} f'_{112}f'_{121} - \frac{2n+3-3m}{2} f''_{112}f_{121}$	0	0	0
	f_{21t31}	$f'''_{21t31} + \frac{m+1}{2} f_{01}f''_{21t31} - (2n-2m+3)f'_{01}f'_{21t31} + \frac{4n-5m+5}{2} f''_{01}f_{21t31} = 2f'_{20s3} - \frac{m+3}{2} f_{02}f''_{21t21} + (2n-2m+2)f'_{02}f'_{21t21} - \frac{4n-5m+3}{2} f_{02}f_{21t21}$	0	0	0
	f_{21t32}	$f'''_{21t32} + \frac{m+1}{2} f_{01}f''_{21t32} - (2n-2m+3)f'_{01}f'_{21t32} + \frac{4n-5m+5}{2} f''_{01}f_{21t32} = -\frac{m+3}{2} f_{01}f''_{21t22} + (2n-2m+2)f'_{02}f'_{21t22} - \frac{4n-5m+3}{2} f''_{02}f_{21t22}$	0	0	0
	f_{21t33}	$f'''_{21t22} + \frac{m+1}{2} f_{01}f''_{21t33} - (2n-2m+3)f'_{01}f'_{21t33} + \frac{4n-5m+5}{2} f''_{01}f_{21t33} = -\frac{m+5}{2} f_{03}f''_{21t1} + (2n-2m+3)f'_{03}f'_{21t1} - \frac{4n-5m+1}{2} f''_{03}f_{21t1}$	0	0	0
	f_{21t34}	$f'''_{21t34} + \frac{m+1}{2} f_{01}f''_{21t34} - (2n-2m+3)f'_{01}f'_{21t34} + \frac{4n-5m+5}{2} f''_{01}f_{21t34} = -\frac{2n+1-m}{2} f_{101}f''_{123} + \frac{4n+7-3m}{2} f'_{101}f'_{113} - \frac{2n+7-3m}{2} f''_{101}f_{113} - \frac{2n+5-m}{2} f_{113}f''_{121} + \frac{4n-7-3m}{2} f'_{113}f'_{121} - \frac{2n+3-3m}{2} f''_{113}f_{121}$	0	0	0
	f_{21t35}	$f'''_{21t35} + \frac{m+1}{2} f_{01}f''_{21t35} - (2n-2m+3)f'_{01}f'_{21t35} + \frac{4n-5m+5}{2} f''_{01}f_{21t35} = -\frac{2n+3-m}{2} f_{102}f''_{112} - \frac{4n+7-3m}{2} f'_{102}f_{112} - \frac{2n+5-3m}{2} f''_{102}f_{112}$	0	0	0

TABLE III (Concluded)

For	f	Equation for f	f(0)	f'(0)	f'(∞)
ψ_{22s}	f_{22s1}	$f'''_{22s1} + \frac{m+1}{2} f_{01}f''_{22s1} - (2n-3m+3)f'_{01}f'_{22s1} + \frac{4n-7m+5}{2} f''_{01}f_{22s1} = -\frac{2n+1-m}{2} f_{101}f''_{121} + \frac{4n-5m+5}{2} f'_{101}f'_{121} - \frac{2n-5m+5}{2} f''_{101}f_{121}$	0	0	0
		$+\frac{2n+3-3m}{2} f_{121}f''_{121} - \frac{2n+3-3m}{2} f'_{121}f'_{121}$			
	f_{22s21}	$f'''_{22s21} + \frac{m+1}{2} f_{01}f''_{22s21} - (2n-3m+4)f'_{01}f'_{22s21} + \frac{4n-7m+7}{2} f''_{01}f_{22s21} = -\frac{m+3}{2} f_{02}f''_{22s1} + (2n-3m+4)f'_{02}f'_{22s1} - \frac{4n-7m+5}{2} f''_{02}f_{22s1}$	0	0	0
	f_{22s22}	$f'''_{22s22} + \frac{m+1}{2} f_{01}f''_{22s22} - (2n-3m+4)f'_{01}f'_{22s22} + \frac{4n-7m+7}{2} f''_{01}f_{22s22} = -\frac{2n+1-m}{2} f_{101}f''_{122} + \frac{4n-5m+7}{2} f'_{101}f'_{122} - \frac{2n-5m+7}{2} f''_{101}f_{122}$	0	0	0
		$-\frac{2n+3-m}{2} f_{102}f''_{121} + \frac{4n-5m+5}{2} f'_{102}f'_{121} - \frac{2n-5m+5}{2} f''_{102}f_{121} + \frac{2n+3-3m}{2} f_{111}f''_{112} - (2n+4-3m)f'_{111}f'_{112} + \frac{2n+5-3m}{2} f''_{111}f_{112}$			
	f_{22s31}	$f'''_{22s31} + \frac{m+1}{2} f_{01}f''_{22s31} - (2n-3m+5)f'_{01}f'_{22s31} + \frac{4n-7m+9}{2} f''_{01}f_{22s31} = -\frac{m+3}{2} f_{02}f''_{22s21} - (2n-3m+5)f'_{02}f'_{22s21} - \frac{4n-7m+7}{2} f''_{02}f_{22s21}$	0	0	0
	f_{22s33}	$f'''_{22s33} + \frac{m+1}{2} f_{01}f''_{22s33} - (2n-3m+5)f'_{01}f'_{22s33} + \frac{4n-7m+9}{2} f''_{01}f_{22s33} = -\frac{m+5}{2} f_{03}f''_{22s1} + (2n-3m+5)f'_{03}f'_{22s1} - \frac{4n-7m+5}{2} f''_{03}f_{22s1}$	0	0	0
	f_{22s34}	$f'''_{22s34} + \frac{m+1}{2} f_{01}f''_{22s34} - (2n-3m+5)f'_{01}f'_{22s34} + \frac{4n-7m+9}{2} f''_{01}f_{22s34} = -\frac{2n+1-m}{2} f_{101}f''_{123} + \frac{4n-5m+9}{2} f'_{101}f'_{123} - \frac{2n-5m+9}{2} f''_{101}f_{123}$	0	0	0
		$-\frac{2n+5-m}{2} f_{103}f''_{121} + \frac{4n-5m+9}{2} f'_{103}f'_{121} - \frac{2n-5m+5}{2} f''_{103}f_{121} + \frac{2n+3-3m}{2} f_{111}f''_{113} - (2n-3m+5)f'_{121}f'_{123} + \frac{2n+7-3m}{2} f''_{121}f_{123}$			
	f_{22s35}	$f'''_{22s35} + \frac{m+1}{2} f_{01}f''_{22s35} - (2n-3m+5)f'_{01}f'_{22s35} + \frac{4n-7m+9}{2} f''_{01}f_{22s35} = -\frac{2n+3-m}{2} f_{102}f''_{122} + \frac{4n-5m+9}{2} f'_{102}f'_{122} - \frac{2n-5m+7}{2} f''_{102}f_{122}$	0	0	0
		$+\frac{2n+5-3m}{2} f_{112}f''_{112} - \frac{2n+5-3m}{2} f'_{112}f'_{112}$			
ψ_{22t}	f_{22t1}	$f'''_{22t1} + \frac{m+1}{2} f_{01}f''_{22t1} - (2n-3m+3)f'_{01}f'_{22t1} + \frac{4n-7m+5}{2} f''_{01}f_{22t1} = 2f'_{21t1} - \frac{2n+1-m}{2} f_{101}f''_{121} + \frac{4n-5m+5}{2} f'_{101}f'_{121} - \frac{2n-5m+5}{2} f''_{101}f_{121}$	0	0	0
		$-\frac{2n+3-3m}{2} f'_{111}f'_{121} - \frac{2n-5m+5}{2} f''_{101}f_{121} - \frac{2n+3-3m}{2} f_{111}f''_{111} + \frac{2n+3-3m}{2} f'_{111}f'_{111}$			
	f_{22t21}	$f'''_{22t21} + \frac{m+1}{2} f_{01}f''_{22t21} - (2n-3m+4)f'_{01}f'_{22t21} + \frac{4n-7m+7}{2} f''_{01}f_{22t21} = -\frac{m+3}{2} f_{02}f''_{22t1} + (2n-3m+4)f'_{02}f'_{22t1} - \frac{4n-7m+5}{2} f''_{02}f_{22t1}$	0	0	0
		$-2f'_{21t2}$			
	f_{22t22}	$f'''_{22t22} + \frac{m+1}{2} f_{01}f''_{22t22} - (2n-3m+4)f'_{01}f'_{22t22} + \frac{4n-7m+7}{2} f''_{01}f_{22t22} = -\frac{2n+1-m}{2} f_{101}f''_{22t2} + \frac{4n-5m+7}{2} f'_{101}f'_{22t2} - \frac{2n-5m+7}{2} f''_{101}f_{122}$	0	0	0
		$-\frac{2n+3-m}{2} f_{102}f''_{121} + \frac{4n-5m+7}{2} f'_{102}f'_{121} - \frac{2n-5m+5}{2} f''_{102}f_{121} - \frac{2n+3-3m}{2} f_{111}f''_{112} + (2n+4-3m)f'_{111}f'_{112} - \frac{2n+5-3m}{2} f''_{111}f_{112}$			
	f_{22t31}	$f'''_{22t31} + \frac{m+1}{2} f_{01}f''_{22t31} - (2n-3m+5)f'_{01}f'_{22t31} + \frac{4n-7m+9}{2} f''_{01}f_{22t31} = -\frac{m+3}{2} f_{02}f''_{22t21} + (2n-3m+5)f'_{02}f'_{22t21} - \frac{4n-7m+7}{2} f''_{02}f_{22t21}$	0	0	0
		$+2f'_{21t3}$			
	f_{22t32}	$f'''_{22t32} + \frac{m+1}{2} f_{01}f''_{22t32} - (2n-3m+5)f'_{01}f'_{22t32} + \frac{4n-7m+9}{2} f''_{01}f_{22t32} = -\frac{m+3}{2} f_{02}f''_{22t22} + (2n-3m+5)f'_{01}f'_{22t22} - \frac{4n-7m+7}{2} f''_{02}f_{22t22}$	0	0	0
	f_{22t33}	$f'''_{22t33} + \frac{m+1}{2} f_{01}f''_{22t33} - (2n-3m+5)f'_{01}f'_{22t33} + \frac{4n-7m+9}{2} f''_{01}f_{22t33} = -\frac{m+5}{2} f_{03}f''_{22t1} + (2n-3m+5)f'_{03}f'_{22t1} - \frac{4n-7m+5}{2} f''_{03}f_{22t1}$	0	0	0
		$+2f'_{21t3}$			
	f_{22t34}	$f'''_{22t34} + \frac{m+1}{2} f_{01}f''_{22t34} - (2n-3m+5)f'_{01}f'_{22t34} + \frac{4n-7m+9}{2} f''_{01}f_{22t34} = -\frac{2n+1-m}{2} f_{101}f''_{123} + \frac{4n-5m+9}{2} f'_{101}f'_{123} - \frac{2n-5m+9}{2} f''_{101}f_{123}$	0	0	0
		$-\frac{2n+5-m}{2} f_{102}f''_{121} + \frac{4n-5m+9}{2} f'_{103}f'_{121} - \frac{2n-5m+5}{2} f''_{103}f_{111} - \frac{2n+3-3m}{2} f_{111}f''_{113} + (2n-3m+5)f'_{111}f'_{113} - \frac{2n+7-3m}{2} f''_{111}f_{113}$			
	f_{22t35}	$f'''_{22t35} + \frac{m+1}{2} f_{01}f''_{22t35} - (2n-3m+5)f'_{01}f'_{22t35} + \frac{4n-7m+9}{2} f''_{01}f_{22t35} = -\frac{2n+3-m}{2} f_{102}f''_{122} + \frac{4n-5m+9}{2} f'_{102}f'_{122} - \frac{2n-5m+7}{2} f''_{102}f_{122}$	0	0	0
		$-\frac{2n+5-3m}{2} f_{112}f''_{112} + \frac{2n+5-3m}{2} f'_{112}f'_{112}$			

TABLE IV

EQUATIONS AND BOUNDARY CONDITIONS WHICH DEFINE A GENERALIZED BLASIUS SERIES SOLUTION FOR TEMPERATURE OR CONCENTRATION IN OSCILLATING FLOW PAST A SHARP-EDGED BODY

For	F	Equation for F	Boundary Conditions F(0)	F'(∞)
θ_0	F_{00}	$\frac{1}{Pr} F''_{00} + \frac{m+1}{2} f_{01}F'_{00} = 0$	1	0
	F_{01}	$\frac{1}{Pr} F''_{01} + \frac{m+1}{2} f_{01}F'_{01} - f'_{01}F_{01} = - \frac{m+3}{2} f_{02}F'_{00}$	0	0
	F_{021}	$\frac{1}{Pr} F''_{021} + \frac{m+1}{2} f_{01}F'_{021} - 2f'_{01}F_{021} = - \frac{m+3}{2} f_{02}F'_{01} + f'_{02}F_{01}$	0	0
	F_{022}	$\frac{1}{Pr} F''_{022} + \frac{m+1}{2} f_{01}F'_{022} - 2f'_{01}F_{022} = - \frac{m+5}{2} f_{03}F'_{00}$	0	0
θ_{10}	F_{100}	$\frac{1}{Pr} F''_{100} + \frac{m+1}{2} f_{01}F'_{100} - (n-m)f'_{01}F_{100} = - \frac{2n+1-m}{2} f_{101}F'_{00}$	0	0
	F_{1011}	$\frac{1}{Pr} F''_{1011} + \frac{m+1}{2} f_{01}F'_{1011} - (n-m+1)f'_{01}F_{1011} = - \frac{2n+1-m}{2} f_{101}F'_{01} + f'_{101}F_{01}$	0	0
	F_{1012}	$\frac{1}{Pr} F''_{1012} + \frac{m+1}{2} f_{01}F'_{1012} - (n-m+1)f'_{01}F_{1012} = - \frac{m+3}{2} f_{02}F'_{100} + (n-m)f'_{02}F_{100}$	0	0
	F_{1013}	$\frac{1}{Pr} F''_{1013} + \frac{m+1}{2} f_{01}F'_{1013} - (n-m+1)f'_{01}F_{1013} = - \frac{2n+3-m}{2} f_{102}F'_{00}$	0	0
	F_{1021}	$\frac{1}{Pr} F''_{1021} + \frac{m+1}{2} f_{01}F'_{1021} - (n-m+2)f'_{01}F_{1021} = - \frac{2n+1-m}{2} f_{101}F'_{02} + 2f'_{101}F_{02}$	0	0
	F_{1022}	$\frac{1}{Pr} F''_{1022} + \frac{m+1}{2} f_{01}F'_{1022} - (n-m+2)f'_{01}F_{1022} = - \frac{m+3}{2} f_{02}F'_{1011} + (n-m+1)f'_{02}F_{1011}$	0	0
	F_{1023}	$\frac{1}{Pr} F''_{1023} + \frac{m+1}{2} f_{01}F'_{1023} - (n-m+2)f'_{01}F_{1023} = - \frac{2n+3-m}{2} f_{102}F'_{01} + f'_{102}F_{01}$	0	0
	F_{1024}	$\frac{1}{Pr} F''_{1024} + \frac{m+1}{2} f_{01}F'_{1024} - (n-m+2)f'_{01}F_{1024} = - \frac{m+5}{2} f_{03}F'_{100} + (n-m)f'_{03}F_{100}$	0	0
	F_{1025}	$\frac{1}{Pr} F''_{1025} + \frac{m+1}{2} f_{01}F'_{1025} - (n-m+2)f'_{01}F_{1025} = - \frac{2n+5-m}{2} f_{103}F'_{00}$	0	0
	F_{1026}	$\frac{1}{Pr} F''_{1026} + \frac{m+1}{2} f_{01}F'_{1026} - (n-m+2)f'_{01}F_{1026} = - \frac{m+3}{2} f_{02}F'_{1012} + (n-m+1)f'_{02}F_{1012}$	0	0
	F_{1027}	$\frac{1}{Pr} F''_{1027} + \frac{m+1}{2} f_{01}F'_{1027} - (n-m+2)f'_{01}F_{1027} = - \frac{m+3}{2} f_{02}F'_{1013} + (n-m+1)f'_{02}F_{1013}$	0	0
θ_{11}	F_{110}	$\frac{1}{Pr} F''_{110} + \frac{m+1}{2} f_{01}F'_{110} - (n-2m+1)f'_{01}F_{110} = -F_{100} - \frac{2n+3-3m}{2} f_{111}F'_{00}$	0	0
	F_{1111}	$\frac{1}{Pr} F''_{1111} + \frac{m+1}{2} f_{01}F'_{1111} - (n-2m+2)f'_{01}F_{1111} = - \frac{2n+3-3m}{2} f_{111}F'_{01} + f'_{111}F_{01} - F_{101}$	0	0
	F_{1112}	$\frac{1}{Pr} F''_{1112} + \frac{m+1}{2} f_{01}F'_{1112} - (n-2m+2)f'_{01}F_{1112} = - \frac{m+3}{2} f_{02}F'_{110} + (n-2m+1)f'_{02}F_{110}$	0	0
	F_{1113}	$\frac{1}{Pr} F''_{1113} + \frac{m+1}{2} f_{01}F'_{1113} - (n-2m+2)f'_{01}F_{1113} = - \frac{2n+5-3m}{2} f_{112}F'_{00}$	0	0
	F_{1121}	$\frac{1}{Pr} F''_{1121} + \frac{m+1}{2} f_{01}F'_{1121} - (n-2m+3)f'_{01}F_{1121} = F_{102} - \frac{2n+3-3m}{2} f_{111}F'_{02} + 2f'_{111}f_{02}$	0	0
	F_{1122}	$\frac{1}{Pr} F''_{1122} + \frac{m+1}{2} f_{01}F'_{1122} - (n-2m+3)f'_{01}F_{1122} = - \frac{m+3}{2} f_{02}F'_{1111} + (n-2m+2)f'_{02}F_{1111}$	0	0
	F_{1123}	$\frac{1}{Pr} F''_{1123} + \frac{m+1}{2} f_{01}F'_{1123} - (n-2m+3)f'_{0} F_{1123} = - \frac{2n+5-3m}{2} f_{112}F'_{01} - f'_{112}F_{01}$	0	0

TABLE IV (Continued)

For	F	Equation for F	Boundary Conditions F(0)	F'(∞)
	F_{1124}	$\frac{1}{Pr} F''_{1124} + \frac{m+1}{2} f_{01} F'_{1124} - (n-2m+3) f'_{0} F_{1124} = - \frac{m+5}{2} f_{03} F'_{110} + (n-2m+1) f'_{03} F_{110}$	0	0
	F_{1125}	$\frac{1}{Pr} F''_{1125} + \frac{m+1}{2} f_{01} F'_{1125} - (n-2m+3) f'_{01} F_{1125} = - \frac{2n+7-3m}{2} f_{113} F'_{00}$	0	0
	F_{1126}	$\frac{1}{Pr} F''_{1126} + \frac{m+1}{2} f_{01} F'_{1126} - (n-2m+3) f'_{01} F_{1126} = - \frac{m+3}{2} f_{02} F'_{1112} + (n-2m+2) f'_{02} F_{1112}$	0	0
	F_{1127}	$\frac{1}{Pr} F''_{1127} + \frac{m+1}{2} f_{01} F'_{1127} - (n-2m+3) f'_{01} F_{1127} = - \frac{m+3}{2} f_{02} F'_{1113} + (n-2m+2) f'_{02} F_{1113}$	0	0
θ_{12}	F_{120}	$\frac{1}{Pr} F''_{120} + \frac{m+1}{2} f_{01} F'_{120} - (n-3m+2) f'_{01} F_{120} = F_{110} = \frac{2n-5m+5}{2} f_{121} F'_{00}$	0	0
	F_{1211}	$\frac{1}{Pr} F''_{1211} + \frac{m+1}{2} f_{01} F'_{1211} - (n-3m+3) f'_{01} F_{1211} = F_{111} - \frac{2n-5m+5}{2} f_{121} F'_{01} + f'_{121} F_{01}$	0	0
	F_{1212}	$\frac{1}{Pr} F''_{1212} + \frac{m+1}{2} f_{01} F'_{1212} - (n-3m+3) f'_{01} F_{1212} = - \frac{m+3}{2} f'_{02} F'_{120} + (n-3m+2) f'_{02} F_{120}$	0	0
	F_{1213}	$\frac{1}{Pr} F''_{1213} + \frac{m+1}{2} f_{01} F'_{1213} - (n-3m+3) f'_{01} F_{1213} = - \frac{2n-5m+7}{2} f_{122} F'_{00}$	0	0
	F_{1221}	$\frac{1}{Pr} F''_{1221} + \frac{m+1}{2} f_{01} F'_{1221} - (n-3m+4) f'_{01} F_{1221} = F_{112} - \frac{2n-5m+5}{2} f_{121} F'_{02}$	0	0
	F_{1222}	$\frac{1}{Pr} F''_{1222} + \frac{m+1}{2} f_{01} F'_{1222} - (n-3m+4) f'_{01} F_{1222} = - \frac{m+3}{2} f_{02} F'_{1211} + (n-3m+3) f'_{02} F_{1211}$	0	0
	F_{1223}	$\frac{1}{Pr} F''_{1223} + \frac{m+1}{2} f_{01} F'_{1223} - (n-3m+4) f'_{01} F_{1223} = - \frac{2n-5m+7}{2} f_{122} F'_{01} + f'_{122} F_{01}$	0	0
	F_{1224}	$\frac{1}{Pr} F''_{1224} + \frac{m+1}{2} f_{01} F'_{1224} - (n-3m+4) f'_{01} F_{1224} = - \frac{m+5}{2} f_{03} F'_{120} + (n-3m+2) f'_{03} F_{120}$	0	0
	F_{1225}	$\frac{1}{Pr} F''_{1225} + \frac{m+1}{2} f_{01} F'_{1225} - (n-3m+4) f'_{01} F_{1225} = - \frac{2n-5m+9}{2} f_{123} F'_{00}$	0	0
	F_{1226}	$\frac{1}{Pr} F''_{1226} + \frac{m+1}{2} f_{01} F'_{1226} - (n-3m+4) f'_{01} F_{1226} = - \frac{m+3}{2} f_{02} F'_{1212} + (n-3m+3) f'_{02} F_{1212}$	0	0
	F_{1227}	$\frac{1}{Pr} F''_{1227} + \frac{m+1}{2} f_{01} F'_{1227} - (n-3m+4) f'_{01} F_{1227} = - \frac{m+3}{2} f_{02} F'_{1213} + (n-3m+3) f'_{02} F_{1213}$	0	0
θ_{20s}	F_{20s0}	$\frac{1}{Pr} F''_{20s0} + \frac{m+1}{2} f_{01} F'_{20s0} - (2n-2m) f'_{01} F_{20s0} = - \frac{2n+1-m}{2} f_{101} F'_{100} + (n-m) f'_{101} F_{100} - \frac{4n-3m+1}{2} f_{20s0} F'_{00}$	0	0
	F_{20s11}	$\frac{1}{Pr} F''_{20s11} + \frac{m+1}{2} f_{01} F'_{20s11} - (2n-2m+1) f'_{01} F_{20s11} = - \frac{2n+1-m}{2} f_{101} F'_{101} + (n-m+1) f'_{101} F_{101} - \frac{4n-3m+1}{2} f_{20s1} F'_{01} + f'_{20s1} F_{01}$	0	0
	F_{20s12}	$\frac{1}{Pr} F''_{20s12} + \frac{m+1}{2} f_{01} F'_{20s12} - (2n-2m+1) f'_{01} F_{20s12} = - \frac{m+3}{2} f_{02} F'_{20s0} + (2n-2m) f'_{02} F_{20s0} - \frac{4n-3m+3}{2} f_{20s2} F'_{00}$	0	0
	F_{20s13}	$\frac{1}{Pr} F''_{20s13} + \frac{m+1}{2} f_{01} F'_{20s13} - (2n-2m+1) f'_{01} F_{20s13} = - \frac{2n+3-m}{2} f_{102} F'_{100} + (n-m) f'_{102} F_{100}$	0	0
	F_{20s21}	$\frac{1}{Pr} F''_{20s21} + \frac{m+1}{2} f_{01} F'_{20s21} - (2n-2m+2) f'_{01} F_{20s21} = - \frac{2n+1-m}{2} f_{101} F'_{102} + (n-m+2) f'_{101} F_{102} - \frac{4n-3m+1}{2} f_{20s1} F'_{02} + 2f'_{20s1} F_{02}$	0	0
	F_{20s22}	$\frac{1}{Pr} F''_{20s22} + \frac{m+1}{2} f_{01} F'_{20s22} - (2n-2m+2) f'_{01} F_{20s22} = - \frac{m+3}{2} f_{02} F'_{20s11} + (2n-2m+1) f'_{02} F_{20s11} - \frac{4n-3m+3}{2} f_{20s2} F'_{02} + f'_{20s2} F_{01}$	0	0
	F_{20s23}	$\frac{1}{Pr} F''_{20s23} + \frac{m+1}{2} f_{01} F'_{20s23} - (2n-2m+2) f'_{01} F_{20s23} = - \frac{2n+3-m}{2} f_{102} F'_{101} + (n-m+1) f'_{102} F_{101}$	0	0
	F_{20s24}	$\frac{1}{Pr} F''_{20s24} + \frac{m+1}{2} f_{01} F'_{20s24} - (2n-2m+2) f'_{01} F_{20s24} = - \frac{m+5}{2} f_{03} F'_{20s0} + (2n-2m) f'_{03} F_{20s0}$	0	0
	F_{20s25}	$\frac{1}{Pr} F''_{20s25} + \frac{m+1}{2} f_{01} F'_{20s25} - (2n-2m+2) f'_{01} F_{20s25} = - \frac{2n+5-m}{2} f_{103} F'_{100}$	0	0

TABLE IV (Continued)

For	F	Equation for F	Boundary Conditions F(0)	Boundary Conditions F'(∞)
	F_{20s26}	$\frac{1}{Pr} F''_{20s26} + \frac{m+1}{2} f_{01}F'_{20s26} - (2n-2m+2)f'_{01}F_{20s26} = -\frac{m+3}{2} f_{02}F'_{20s12} + (2n-2m+1)f'_{02}F_{20s12} - \frac{4n-3m+5}{2} f_{20s3}F'_{00}$	0	0
	F_{20s27}	$\frac{1}{Pr} F''_{20s27} + \frac{m+1}{2} f_{01}F'_{20s27} - (2n-2m+2)f'_{01}F_{20s27} = -\frac{m+3}{2} f_{02}F'_{20s13} + (2n-2m+1)f'_{02}F_{20s13}$	0	0.
θ_{21t}	F_{21to}	$\frac{1}{Pr} F''_{21to} + \frac{m+1}{2} f_{01}F'_{21to} - (2n-3m+1)f'_{01}F_{21to} = 2F_{20s0} - \frac{2n+1-m}{2} f_{101}F'_{110} + (n-2m+1)f'_{101}F_{110} - \frac{2n+3-3m}{2} f_{111}F'_{100}$	0	0
		$+ (n-m)f'_{111}F_{100} - \frac{4n-5m+3}{2} f_{22to}F'_{00}$		
	F_{21t11}	$\frac{1}{Pr} F''_{21t11} + \frac{m+1}{2} f_{01}F'_{21t11} - (2n-3m+2)f'_{01}F_{21t11} = 2F_{20s1} - \frac{2n+1-m}{2} f_{101}F'_{111} + (n-2m+2)f'_{101}F_{111} - \frac{2n+3-3m}{2} f_{111}F'_{101}$	0	0
		$+ (n-m+1)f'_{101}F_{101} - \frac{4n-5m+3}{2} f_{21to}F'_{01} + f'_{21to}F_{01}$		
	F_{21t12}	$\frac{1}{Pr} F''_{21t12} + \frac{m+1}{2} f_{01}F'_{21t12} - (2n-3m+2)f'_{01}F_{21t12} = -\frac{3+m}{2} f_{02}F'_{21to} + (2n-3m+1)f'_{02}F_{21to} - \frac{4n-5m+5}{2} f_{21t2}F'_{00}$	0	0
	F_{21t13}	$\frac{1}{Pr} F''_{21t13} + \frac{m+1}{2} f_{01}F'_{21t13} - (2n-3m+2)f'_{01}F_{21t13} = -\frac{2n+3-m}{2} f_{102}F'_{110} + (n-2m+1)f'_{102}F_{110} - \frac{2n+5-3m}{2} f_{112}F'_{100} + (n-m)f'_{112}F_{100}$	0	0
	F_{21t21}	$\frac{1}{Pr} F''_{21t21} + \frac{m+1}{2} f_{01}F'_{21t21} - (2n-3m+3)f'_{01}F_{21t21} = 2F_{20s2} - \frac{2n+1-m}{2} f_{101}F'_{122} + (n-2m+3)f'_{101}F_{112} - \frac{2n+3-3m}{2} f_{111}F'_{102}$	0	0
		$+ (n-m+2)f'_{111}F_{102} + (n-m+2)f'_{110}F_{102} - \frac{4n-5m+3}{2} f_{20t1}F'_{02} + 2f'_{20to}F_{02}$		
	F_{21t22}	$\frac{1}{Pr} F''_{21t22} + \frac{m+1}{2} f_{01}F'_{21t22} - (2n-3m+3)f'_{01}F_{21t22} = -\frac{m+3}{2} f_{02}F'_{21t11} + (2n-3m+2)f'_{02}F_{21t11} - \frac{4n-5m+5}{2} f_{21t2}F'_{01} + f'_{21t2}F_{01}$	0	0
	F_{21t23}	$\frac{1}{Pr} F''_{21t23} + \frac{m+1}{2} f_{01}F'_{21t23} - (2n-3m+3)f'_{01}F_{21t23} = -\frac{2n+3-m}{2} f_{102}F'_{21t1} + (n-2m+2)f'_{102}F_{111} - \frac{2n+5-3m}{2} f_{112}F'_{101} + (n-m+1)f'_{112}F_{101}$	0	0
	F_{21t24}	$\frac{1}{Pr} F''_{21t24} + \frac{m+1}{2} f_{01}F'_{21t24} - (2n-3m+3)f'_{01}F_{21t24} = -\frac{m+5}{2} f_{03}F'_{21to} + (2n-3m+1)f'_{03}F_{21to}$	0	0
	F_{21t25}	$\frac{1}{Pr} F''_{21t25} + \frac{m+1}{2} f_{01}F'_{21t25} - (2n-3m+3)f'_{01}F_{21t25} = -\frac{2n+5n-m}{2} f_{103}F'_{110} + (n-2m+1)f'_{103}F_{110} - \frac{2n+7-3m}{2} f_{112}F'_{100} + (n-m)f'_{112}F_{100}$	0	0
	F_{21t26}	$\frac{1}{Pr} F''_{21t26} + \frac{m+1}{2} f_{01}F'_{21t26} - (2n-3m+3)f'_{01}F_{21t26} = -\frac{m+3}{2} f_{02}F'_{21t12} + (2n-3m+1)f'_{02}F_{21t12}$	0	0
	F_{21t27}	$\frac{1}{Pr} F''_{21t27} + \frac{m+1}{2} f_{01}F'_{21t27} - (2n-3m+3)f'_{01}F_{21t27} = -\frac{m+3}{2} f_{02}F'_{21t13} + (2n-3m+1)f'_{02}F_{21t13}$	0	0
θ_{22s}	F_{22s0}	$\frac{1}{Pr} F''_{22s0} + \frac{m+1}{2} f_{01}F'_{22s0} - (2n-4m+2)f'_{01}F_{22s0} = -\frac{2n+1-m}{2} f_{101}F'_{120} + (n-3m+2)f'_{101}F_{120} + \frac{2n+3-3m}{2} f_{111}F'_{110}$	0	0
		$- (n-2m+1)f'_{111}F_{110} - \frac{2n-5m+5}{2} f_{121}F'_{100} + (n-m)f'_{121}F_{100} - \frac{4n-7m+5}{2} f_{22s1}F'_{00}$		
	F_{22s11}	$\frac{1}{Pr} F''_{22s11} + \frac{m+1}{2} f_{01}F'_{22s11} - (2n-4m+3)f'_{01}F_{22s11} = -\frac{2n+1-m}{2} f_{101}F'_{121} + (n-2m+3)f'_{101}F_{121} + \frac{2n+3-3m}{2} f_{111}F'_{111} - (n-2m+2)f'_{111}f_{111}$	0	0
		$- \frac{2n-5m+5}{2} f_{121}F'_{101} + (n-m+1)f'_{121}F_{101} - \frac{4n-7m+5}{2} f_{22s1}F'_{01} + f'_{22s1}F_{01}$		
	F_{22s12}	$\frac{1}{Pr} F''_{22s12} + \frac{m+1}{2} f_{01}F'_{22s12} - (2n-4m+3)f'_{01}F_{22s12} = -\frac{m+3}{2} f_{02}F'_{22s0} + (2n-4m+2)f'_{02}F_{22s0} - \frac{4n-7m+7}{2} f_{22s2}F'_{00}$	0	0
	F_{22s13}	$\frac{1}{Pr} F''_{22s13} + \frac{m+1}{2} f_{01}F'_{22s13} - (2n-4m+3)f'_{01}F_{22s13} = -\frac{2n+3-m}{2} f_{102}F'_{120} + (n-3m+2)f'_{102}F_{120} + \frac{2n+5-3m}{2} f_{112}F'_{110}$	0	0
		$- (n-2m+1)f'_{112}F_{110} - \frac{2n-5m+7}{2} f_{122}F'_{100} + (n-m)f'_{122}F_{100}$		

TABLE IV (Continued)

For	F	Equation for F	Boundary Conditions F(0)	F'(∞)
	F_{22s21}	$\frac{1}{Pr}F''_{22s21} + \frac{m+1}{2} f_{01}F'_{22s21} - (2n-4m+4)f'_{01}F_{22s21} = -\frac{2n+1-m}{2} f_{101}F'_{122} + (n-3m+4)f'_{101}F_{122} + \frac{2n+3-3m}{2} f_{111}F'_{112}$	0	0
		$- (n-2m+3)f'_{111}F_{112} - \frac{2n-5m+5}{2} f_{121}F'_{102} + (n-m+2)f'_{121}F_{102} - \frac{4n-7m+5}{2} f_{22s1}F'_{02} + 2f'_{22s1}F_{02}$		
	F_{22s22}	$\frac{1}{Pr}F''_{22s22} + \frac{m+1}{2} f_{01}F'_{22s22} - (2n-4m+4)f'_{01}F_{22s22} = -\frac{m+3}{2} f_{02}F'_{22s11} + (2n-4m+3)f'_{03}F_{22s11} - \frac{4n-7m+7}{2} f_{22s2}F'_{01} + f'_{22s2}F_{01}$	0	0
	F_{22s23}	$\frac{1}{Pr}F''_{22s23} + \frac{m+1}{2} f_{01}F'_{22s23} - (2n-4m+4)f'_{01}F_{22s23} = -\frac{2n+3-m}{2} f_{102}F'_{121} + (n-3m+\ )f'_{102}F_{120} + \frac{2n+5-3m}{2} f_{112}F'_{111}$	0	0
		$- (n-2m+2)f'_{112}F_{111} - \frac{2n-5m+1}{2} f_{121}F'_{101} + (n-m+1)f'_{121}F_{101}$		
	F_{22s24}	$\frac{1}{Pr}F''_{22s24} + \frac{m+1}{2} f_{01}F'_{22s24} - (2n-4m+4)f'_{01}F_{22s24} = -\frac{m+5}{2} f_{03}F'_{22s0} + (2n-4m+2)f'_{03}F_{22s0}$	0	0
	F_{22s25}	$\frac{1}{Pr}F''_{22s25} + \frac{m+1}{2} f_{01}F'_{22s25} - (2n-4m+4)f'_{01}F_{22s25} = -\frac{2n+5-m}{2} f_{103}F'_{120} + (n-3m+2)f'_{103}F_{120} + \frac{2n+7-3m}{2} f_{113}F'_{110}$	0	0
		$- (n-2m+1)f'_{113}F_{110} - \frac{2n-5m+9}{2} f_{123}F'_{100} + (n-m)f'_{123}F_{100}$		
	F_{22s26}	$\frac{1}{Pr}F''_{22s26} + \frac{m+1}{2} f_{01}F'_{22s26} - (2n-4m+4)f'_{01}F_{22s26} = -\frac{m+3}{2} f_{02}F'_{22s12} + (2n-4m+3)f'_{02}F_{22s12} - \frac{4n-7m+9}{2} f_{22s3}F'_{00}$	0	0
	F_{22s27}	$\frac{1}{Pr}F''_{22s27} + \frac{m+1}{2} f_{01}F'_{22s27} - (2n-4m+4)f'_{01}F_{22s27} = -\frac{m+3}{2} f_{02}F'_{22s13} + (2n-4m+3)f'_{02}F_{22s13}$	0	0
θ_{22t}	F_{22t0}	$\frac{1}{Pr}F''_{22t0} + \frac{m+1}{2} f_{01}F'_{22t0} - (2n-4m+2)f'_{01}F_{22t0} = 2F_{21t0} - \frac{2n+1-m}{2} f_{101}F'_{120} + (n-3m+2)f'_{101}F_{120} - \frac{2n+3-3m}{2} f_{111}F'_{110}$	0	0
		$+ (n-2m+1)f'_{111}F_{110} - \frac{2n-5m+5}{2} f_{121}F'_{100} + (n-m)f'_{120}F_{100} - \frac{4n-7m+5}{2} f_{12t0}F'_{00}$		
	F_{22t11}	$\frac{1}{Pr}F''_{22t11} + \frac{m+1}{2} f_{01}F'_{22t11} - (2n-4m+3)f'_{01}F_{22t11} = 2F_{21t1} - \frac{2n+1-m}{2} f_{100}F'_{00} + (n-3m+3)f'_{100}F_{121} - \frac{2n+3-3m}{2} f_{111}F'_{111}$	0	0
		$- (n-2m+2)f'_{111}F_{111} - \frac{2n-5m+5}{2} f_{121}F'_{101} + (n-m+1)f'_{121}F_{101} - \frac{4n-7m+5}{2} f_{22t1}F'_{01} + f'_{22t1}F_{01}$		
	F_{22t12}	$\frac{1}{Pr}F''_{22t12} + \frac{m+1}{2} f_{01}F'_{22t12} - (2n-4m+3)f'_{01}F_{22t12} = -\frac{m+3}{2} f_{02}F'_{22t0} + (2n-4m+2)f'_{02}F_{22t0} - \frac{4n-7m+7}{2} f_{22t2}F'_{00}$	0	0
	F_{22t13}	$\frac{1}{Pr}F''_{22t13} + \frac{m+1}{2} f_{01}F'_{22t13} - (2n-4m+3)f'_{01}F_{22t13} = -\frac{2n+3-m}{2} f_{102}F'_{120} + (n-3m+2)f'_{102}F_{120} - \frac{2n+5-3m}{2} f_{112}F'_{110}$	0	0
		$+ (n-2m+1)f'_{112}F_{110} - \frac{2n-5m+7}{2} f_{122}F'_{100} + (n-m)f'_{122}F_{100}$		
	F_{22t21}	$\frac{1}{Pr}F''_{22t21} + \frac{m+1}{2} f_{01}F'_{22t21} - (2n-4m+4)f'_{01}F_{22t21} = 2F_{21t2} - \frac{2n+1-m}{2} f_{101}F'_{122} + (n-3m+4)f'_{101}F_{122} - \frac{2n+3-3m}{2} f_{111}F'_{112}$	0	0
		$+ (n-2m+3)f'_{111}F_{112} - \frac{2n-5m+5}{2} f_{121}F'_{102} + (n-m+2)f'_{120}F_{102} - \frac{4n-7m+5}{2} f_{22t1}F'_{02} + 2f'_{22t1}F_{02}$		
	F_{22t22}	$\frac{1}{Pr}F''_{22t22} + \frac{m+1}{2} f_{01}F'_{22t22} - (2n-4m+4)f'_{01}F_{22t22} = -\frac{m+3}{2} f_{02}F'_{22t11} + (2n-4m+3)f'_{02}F_{22t11} - \frac{4n-7m+7}{2} f_{22t2}F'_{02} + f'_{22t2}F_{02}$	0	0
	F_{22t23}	$\frac{1}{Pr}F''_{22t23} + \frac{m+1}{2} f_{01}F'_{22t23} - (2n-4m+4)f'_{01}F_{22t23} = -\frac{2n+3-m}{2} f_{102}F'_{121} + (n-3m+3)f'_{102}F_{121} - \frac{2n+5-3m}{2} f_{112}F'_{111}$	0	0
		$+ (n-2m+2)f'_{112}F_{111} - \frac{2n-5m+1}{2} f_{122}F'_{101} + (n-m+1)f'_{122}F_{101}$		
	F_{22t24}	$\frac{1}{Pr}F''_{22t24} + \frac{m+1}{2} f_{01}F'_{22t24} - (2n-4m+4)f'_{01}F_{22t24} = -\frac{m+5}{2} f_{03}F'_{22t0} + (2n-4m+2)f'_{03}F_{22t0}$	0	0

TABLE IV (Concluded)

For F	Equation for F	Boundary Conditions F(0)	F'(∞)
F_{22t25}	$\frac{1}{Pr} F''_{22t25} + \frac{m+1}{2} f_{01}F'_{22t25} - (2n-4m+4)f'_{01}F_{22t25} = -\frac{2n+5-m}{2} f_{103}F'_{120} + (n-3m+2)f'_{103}F_{120} - \frac{2n+7-3m}{2} f_{113}F'_{110}$ $+ (n-2m+1)f'_{113}F_{110} - \frac{2n-5m+9}{2} f_{122}F'_{100} + (n-m)f'_{123}F_{100}$	0	0
F_{22t26}	$\frac{1}{Pr} F''_{22t26} + \frac{m+1}{2} f_{01}F'_{22t26} - (2n-4m+4)f'_{01}F_{22t26} = -\frac{m+3}{2} f_{02}F'_{22t12} + (2n-4m+3)f'_{02}F_{22t12} - \frac{4n-7m+9}{2} f_{22t3}F'_{00}$	0	0
F_{22t27}	$\frac{1}{Pr} F''_{22t27} + \frac{m+1}{2} f_{01}F'_{22t27} - (2n-4m+4)f'_{01}F_{22t27} = -\frac{m+3}{2} f_{02}F'_{22t13} + (2n-4m+3)f'_{02}F_{22t13}$	0	0

The functional coefficients $f_{1\ell k}$ and $F_{1\ell k}$ are defined by the equations split up as

$$f_{1\ell 1} = f_{1\ell 11} + \frac{a_1 b_0}{a_0 b_1} f_{1\ell 12}$$

$$f_{1\ell 2} = f_{1\ell 21} + \frac{a_1 b_1}{a_0 b_2} f_{1\ell 22} + \frac{a_1^2 b_0}{a_0^2 b_2} f_{1\ell 23} + \frac{a_2 b_0}{a_0 b_2} f_{1\ell 24}$$

$$\cdots\cdots$$

$$F_{1\ell 1} = F_{1\ell 11} + \frac{a_1}{a_0} F_{1\ell 12} + \frac{b_1}{b_0} F_{1\ell 13}$$

$$\cdots\cdots$$

with the appropriate boundary conditions.

Similarly the solutions to the second-order perturbation are as follows:

$$\left.\begin{aligned} \psi_{2\ell j}(x,\eta) &= \frac{b_0^2}{a_0^{\ell+1}(a_0 Re)^{1/2}} x^{\frac{4n-m(2\ell+3)+(2\ell+1)}{2}} \sum_{k=0}^{\infty} \left(\frac{a_1}{a_0}\right)^k f_{2\ell jk}\, x^{Mk}, \\ \Theta_{2\ell j} &= \frac{b_0^2}{a_0^{\ell+2}} \sum_{k=0}^{\infty} F_{2\ell jk}\, x^{2(n-m)+\ell(1-n)+Mk} \end{aligned}\right\} \quad (19)$$

3. SOLUTIONS UP TO SECOND ORDER

For both cases a and b, the velocity components are

$$\left.\begin{aligned} u &= u_0 + \epsilon[(u_{10}-\omega^2 u_{12})^2 + (\omega u_{11})^2]^{1/2} \cos\left[\omega t + \tan^{-1}\left(\frac{\omega u_{11}}{u_{10}-\omega^2 u_{12}}\right)\right] \\ &+ \frac{\epsilon^2}{2}\left\{(u_{20s}-\omega^2 u_{22s}) + [(u_{20t}-\omega^2 u_{22t})^2 + (\omega u_{21t})^2]^{1/2} \times \right. \\ &\left. \cos\left[2\omega t + \tan^{-1}\left(\frac{\omega u_{21t}}{u_{20t}-\omega^2 u_{22t}}\right)\right]\right\} \end{aligned}\right\} \quad (20)$$

$$v = v_o + \epsilon[(v_{10}-\omega^2 v_{12})^2 + (\omega v_{11})^2]^{1/2} \cos\left[\omega t + \tan^{-1}\left(\frac{\omega v_{11}}{v_{10}-\omega^2 v_{12}}\right)\right]$$

$$+ \frac{\epsilon^2}{2}\left\{(v_{20s}-\omega^2 v_{22s}) + [(v_{20t}-\omega^2 v_{22t})^2 + (\omega v_{21t})^2]^{1/2} \times \right.$$

$$\left. \cos\left[2\omega t + \tan^{-1}\left(\frac{\omega v_{21t}}{v_{20t}-\omega^2 v_{22t}}\right)\right]\right\}$$

The dimensionless shear stress, defined as

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y = 0},$$

is

$$\tau = \left\{\frac{\partial u_o}{\partial y} + \epsilon\left[\left(\frac{\partial u_{10}}{\partial y} - \omega^2 \frac{\partial u_{12}}{\partial y}\right)^2 + \left(\omega \frac{\partial u_{11}}{\partial y}\right)^2\right]^{1/2} \cos\left[\omega t + \tan^{-1}\left(\frac{\omega \frac{\partial u_{11}}{\partial y}}{\frac{\partial u_{10}}{\partial y} - \omega^2 \frac{\partial u_{12}}{\partial y}}\right)\right]\right.$$

$$+ \frac{\epsilon^2}{2}\left(\frac{\partial u_{20s}}{\partial y} - \omega^2 \frac{\partial u_{22s}}{\partial y}\right)$$

$$+ \frac{\epsilon^2}{2}\left[\left(\frac{\partial u_{20t}}{\partial y} - \omega^2 \frac{\partial u_{22t}}{\partial y}\right)^2 + \left(\omega \frac{\partial u_{21t}}{\partial y}\right)^2\right]^{1/2} \times \qquad (21)$$

$$\left. \cos\left[2\omega t + \tan^{-1}\left(\frac{\omega \frac{\partial u_{21t}}{\partial y}}{\frac{\partial u_{20t}}{\partial y} - \omega^2 \frac{\partial u_{22t}}{\partial y}}\right)\right]\right\}$$

The steady-state shear stress is Eq. (21) without the second and fourth terms on the right-hand side of the equation as the periodic terms will contribute nothing when integrated over a whole cycle. The mean position of separation may be obtained by equating the steady-state shear stress to zero.

The temperature is:

$$T = \theta_0 + \epsilon[(\theta_{10}-\omega^2\theta_{12})^2 + (\omega\theta_{11})^2]^{1/2}\cos[\omega t+\tan^{-1}(\omega\theta_{11}/(\theta_{10}-\omega^2\theta_{12}))]$$

$$+ \frac{\epsilon^2}{2}\{(\theta_{20s}-\omega^2\theta_{22s}) + [(\theta_{20t}-\omega^2\theta_{22t})^2 + (\omega\theta_{21t})^2]^{1/2} \times \tag{22}$$

$$\cos[2\omega t + \tan^{-1}(\omega\theta_{21t}/(\theta_{20t}-\omega^2\theta_{22t}))]\}$$

The local Nusselt number, defined as

$$Nu_x = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0}$$

is obtained as:

$$Nu_x = \left\{\frac{\partial\theta_0}{\partial y} + \epsilon\left[\left(\frac{\partial\theta_{10}}{\partial y} - \omega^2\frac{\partial\theta_{12}}{\partial y}\right)^2 + \left(\omega\frac{\partial\theta_{11}}{\partial y}\right)^2\right]^{1/2} \times\right.$$

$$\cos\left[\omega t + \tan^{-1}\left(\omega\frac{\partial\theta_{11}}{\partial y}\Big/\left(\frac{\partial\theta_1}{\partial y} - \omega^2\frac{\partial\theta_{12}}{\partial y}\right)\right)\right] \tag{23}$$

$$+ \frac{\epsilon^2}{2}\left[\left(\frac{\partial\theta_{20s}}{\partial y} - \omega^2\frac{\partial\theta_{22t}}{\partial y}\right)^2 + \left(\omega\frac{\partial\theta_{21t}}{\partial y}\right)^2\right]^{1/2} \times$$

$$\left.\cos\left[2\omega t + \tan^{-1}\left(\omega\frac{\partial\theta_{21t}}{\partial y}\Big/\left(\frac{\partial\theta_{20t}}{\partial y} - \omega^2\frac{\partial\theta_{22t}}{\partial y}\right)\right)\right]\right\}$$

Again the periodic terms contribute nothing to the steady-state value.

4. PERMANENT ALTERATIONS IN THE WALL SHEAR STRESS AND HEAT TRANSFER RATE CAUSED BY FLUCTUATIONS IN THE STREAM VELOCITY

Equations (20) and (23) are interesting and significant results. The former indicates that a flow oscillation induces a steady-secondary or streaming motion; the latter shows that the streaming flow resulting from the oscillation causes a change in the local Nusselt number from that corresponding to steady forced convection. The second-order contributions to the alterations in the wall shear stress and heat transfer rate are obtained from Eqs. (15), (19), (21), and (23) as follows:

For case a

$$\frac{\Delta\tau}{(\mathrm{Re})^{1/2}} = \epsilon^2 a_1 x\left[f''_{20s1}(o) + \frac{4a_3}{a_1} f''_{20s3}(o)\, x^2 + \ldots - \left(\frac{\omega}{a_1}\right)^2 \left(f''_{22s1}(o) + \frac{4a_3}{a_1} f''_{22s3}(o)\, x^2 + \ldots\right)\right] \tag{24}$$

$$\frac{\Delta Nu}{(\mathrm{Re})^{1/2}} = \frac{\epsilon^2}{2}\left[F'_{20s0}(o) + \frac{4a_3}{a_1} F'_{20s2}(o)\, x^2 + \ldots - \left(\frac{\omega}{a_1}\right)^2 \left(F'_{22s0}(o) + \frac{4a_3}{a_1} F'_{22s2}(o)\, x^2 + \ldots\right)\right] \tag{25}$$

For case b

$$\frac{\Delta\tau}{(\mathrm{Re})^{1/2}} = \frac{\epsilon^2}{2}\frac{b_o^2}{(a_o)^{1\ 2}} \sum_{k=0}^{\infty}\sum_{\gamma=0}^{\infty} \left(\frac{a_1}{a_o}\right)^{\gamma} (-1)^{\gamma} \left(\frac{\omega}{a_o}\right)^{2\gamma} f''_{2(2\gamma)sk}(o)\, x^{\frac{4n-m-1}{2}+Mk+2\gamma(1-m)} \tag{26}$$

$$\frac{\Delta Nu}{(\mathrm{Re})^{1/2}} = \frac{\epsilon^2}{2}\frac{b_o^2}{a_o^{3\ 2}} \sum_{k=0}^{\infty}\sum_{\gamma=0}^{\infty} (-1)^{\gamma} \left(\frac{\omega}{a_o}\right)^{2\gamma} F'_{2(2\gamma)sk}(o)\, x^{\frac{4n-3m-1}{2}+Mk+2\gamma(1-m)} \tag{27}$$

Part B. Fluctuating Circulations of Free Stream

This section is devoted to the study of the effects of fluctuating circulation (or free-stream oscillation with a magnitude- and direction-dependent amplitudes) on the transport phenomena. The perturbing force is that due to the flow circulation with a small fluctuating amplitude and low frequency in an otherwise forced convective field. One example is a uniform flow about a heated bluff body with a trailing row of alternating vortices. This induces oscillations in transport phenomena.

The governing differential equations and appropriate boundary conditions are identical with those of Part A for the fluctuation in the free stream. However, the time-average velocity $U_o(x)$ is different from the amplitude of oscillation $U_1(x)$ in the external potential flow. Only the case of flow over a symmetrical blunt body is treated here although the analysis may also be extended to the flow over a sharp-edged body.

When the circulations around a symmetrical blunt body fluctuate, the amplitude follows the formula

$$U_1(x) = \sum_{k=0}^{\infty} b_{2k}\, x^{2k} \,, \tag{28}$$

where the coefficients b_{2k} depend on the nature of the fluctuating circulations. $U_1(x)$ is an even-power series, since only the velocity fluctuations of the potential flow caused by a fluctuating circulation are considered.

The solution to the zeroth-order perturbation, the case of steady-state forced convection, is identical with Eq. (8). The solutions to the first- and second-order perturbations are obtained as follows:

The solution to the first-order perturbation:

$$\begin{aligned} \psi_{1\ell}(x,\eta) &= \frac{1}{(a_1 Re)^{1/2} a_1{}^{\ell}} \left[b_0 f_{1\ell 0} + 3b_2 f_{1\ell 2}\, x^2 + 5b_4 f_{1\ell 4}\, x^4 + \ldots \right] \\ \theta_{1\ell}(x,\eta) &= \frac{2}{a_1{}^{\ell+1}} \left[b_2 F_{1\ell 1}\, x + 2b_4 F_{1\ell 2}\, x^3 + \ldots \right] \end{aligned} \tag{29}$$

where

$$f_{1\ell 2} = f_{1\ell 21} + \frac{b_0 a_3}{a_1 b_2} f_{1\ell 22} \,,$$

$$f_{1\ell 4} = f_{1\ell 41} + \frac{a_3 b_2}{a_1 b_4} f_{1\ell 42} + \frac{a_5 b_0}{a_1 b_4} f_{1\ell 43} + \frac{a_3^2 b_0}{a_1^2 b_4} f_{1\ell 44}$$

and

$$\cdots\cdots$$

$$F_{1\ell 1} = F_{1\ell 1} + \frac{a_3 b_0}{a_1 b_2} F_{1\ell 12}$$

$$F_{1\ell 3} = F_{1\ell 31} + \frac{a_3 b_2}{a_1 b_4} F_{1\ell 32} + \frac{a_5 b_0}{a_1 b_4} F_{1\ell 33} + \frac{a_3^2 b_0}{a_1^2 b_4} F_{1\ell 34}$$

$$\cdots\cdots$$

The solution to the second-order perturbation:

$$\psi_{2\ell j} = \frac{3b_o b_2}{(a_1 Re)^{1/2} a_1^{\ell+1}} \left[f_{2\ell j1}\, x + \frac{2a_3}{a_1} f_{2\ell j3}\, x^3 + \frac{3a_5}{a_1} f_{2\ell j5}\, x^5 + \ldots \right]$$

$$\theta_{2\ell j} = \frac{3b_o b_2}{a_1^{\ell+2}} \left[F_{2\ell jo} + \frac{2a_3}{a_1} F_{2\ell j2}\, x^2 + \frac{3a_5}{a_1} F_{2\ell j4}\, x^4 + \ldots \right]$$

where

$$f_{2\ell j1} = f_{2\ell j11} + \frac{b_o a_3}{a_1 b_2} f_{2\ell j12}$$

$$f_{2\ell j3} = f_{2\ell j33} + \frac{b_o a_3}{a_1 b_2} f_{2\ell j34} + \frac{a_1 b_2}{a_3 b_o} f_{2\ell j32} + \frac{a_1 b_4}{a_3 b_2} f_{2\ell j31} + \frac{a_5 b_o}{a_3 b_2} f_{2\ell j35}$$

. . . .

and

$$F_{2\ell jo} = F_{2\ell jo1} + \frac{b_o a_3}{a_1 b_2} F_{2\ell jo2}$$

$$F_{2\ell j2} = F_{2\ell j21} + \frac{b_o a_3}{a_1 b_2} F_{2\ell j23} + \frac{a_1 b_2}{a_3 b_o} F_{2\ell j22} + \frac{a_1 b_4}{a_3 b_2} F_{2\ell j24} + \frac{a_5 b_o}{a_3 b_2} F_{2\ell j25}$$

. . . .

The functions $f_{1\ell k}$, and $f_{2\ell jk}$, $F_{1\ell k}$ and $F_{2\ell jk}$ are defined by the differential equations and appropriate boundary conditions presented in Tables V and VI, respectively.

The velocity and temperature profiles and the local wall shear stress and Nusselt number are defined by Eqs. (20), (21), (22) and (23), respectively. The permanent alterations in the local wall shear stress and Nusselt number caused by the streaming motion due to fluctuating circulation are obtained as

$$\begin{aligned} \frac{\Delta\tau}{\sqrt{Re}} = \frac{3\epsilon^2 b_o b_2}{2\sqrt{a_1}}\, x \Bigg\{ & f''_{20s11} + \frac{b_o a_3}{a_1 b_2} f''_{20s12} - \left(\frac{\omega}{a_1}\right)^2 \left(f''_{22s11} + \frac{a_3 b_o}{a_1 b_2} f''_{22s12} \right) + \frac{2a_3}{a_1} \Bigg[f''_{20s33} \\ & + \frac{a_3 b_o}{a_1 b_2} f''_{20s34} + \frac{a_1 b_2}{a_3 b_o} f''_{20s32} + \frac{a_1 b_4}{a_3 b_2} f''_{20s31} + \frac{a_5 b_o}{a_3 b_2} f''_{20s35} - \left(\frac{\omega}{a_1}\right)^2 \Bigg(f''_{22s33} \\ & + \frac{a_3 b_o}{a_1 b_2} f''_{22s34} + \frac{a_1 b_2}{a_3 b_o} f''_{22s32} + \frac{a_1 b_4}{a_3 b_2} f''_{22s31} + \frac{a_5 b_o}{a_3 b_2} f''_{22s35} \Bigg) \Bigg] x^2 + \Bigg\}_{\eta = 0} \end{aligned} \quad (31)$$

and

$$\frac{\Delta Nu}{\sqrt{Re}} = \frac{3\epsilon^2 b_o b_2}{2a_1^2}\left\{F'_{20s01} + \frac{a_3 b_o}{a_1 b_2}F'_{20s02} - \frac{2}{3}\left(\frac{\omega}{a_1}\right)^2\left(F'_{22s01} + \frac{a_3 b_o}{a_1 b_2}F'_{22s02}\right)\right.$$

$$+ \frac{2a_3}{a_1}\left[F'_{20s21} + \frac{a_1 b_2}{a_3 b_o}F'_{20s22} + \frac{a_3 b_o}{a_1 b_2}F'_{20s23} + \frac{a_1 b_4}{a_3 b_2}F'_{20s24} + \frac{a_5 b_o}{a_3 b_2}F'_{20s25}\right. \tag{32}$$

$$\left.\left. - \left(\frac{\omega}{a_1}\right)^2\left(F'_{22s21} + \frac{a_1 b_2}{a_3 b_o}F'_{22s22} + \frac{a_3 b_o}{a_1 b_2}F'_{22s23} + \frac{a_1 b_4}{a_3 b_2}F'_{22s24} + \frac{a_5 b_o}{a_3 b_2}F'_{20s25}\right)\right] x^2 + \right\}_{\eta = 0}$$

Part C. Rotational Oscillation of Cylinder Surface

This part is to investigate momentum, heat and mass transfer from a symmetrical blunt body (a cylinder) which performs a reciprocating harmonic motion about its own axis with small fluctuating amplitude $\epsilon U_1^*(x)$ and low frequency in an otherwise forced convective field.

Imposing on the analysis the same assumptions as given in Part A, the expressions for the governing differential equations and appropriate boundary conditions are obtained as shown in The Fundamental Equations of Part A.

The resulting differential equations for the first- and second-order perturbations are respectively identical with Eqs. (2) and (3) without terms for U_o and U_1. The corresponding boundary conditions are identical except those for u_1 of the first-order perturbation. They should be replaced by $u_1 = U_1(x) \cos \omega t$ at $y = 0$ and $u_1 = 0$ at $y = \infty$.

The external potential flow $U_o(x)$ and the oscillating amplitude $U_1(x)$ of the cylinder surface are expanded into power series as expressed by Eqs. (4) and (28) respectively. Solutions for the components of velocity and temperature are obtained following the same procedure as described in Part A. These solutions in power series are identically expanded as those for the case of fluctuating circulations in Part B: Eqs. (29) and (30) for the first- and second-order perturbations, respectively. The differential equations and boundary conditions which define the functions f and F are given in Tables V and VI. The solutions up to the second-order perturbation are identical with Eqs. (20), (21), (22) and (23) for the velocity components, the dimensionless shear stress, the temperature and the local Nusselt number, respectively. The expressions for the permanent alterations in the wall shear stress and Nusselt number are identical with Eqs. (31) and (32), respectively.

TABLE V

EQUATIONS AND BOUNDARY CONDITIONS FOR THE UNIVERSAL DISTRIBUTION FUNCTIONS OF VELOCITY FOR FLUCTUATING-CIRCULATION AND ROTATIONAL-OSCILLATION CASES

For	f	Equation for f	Boundary Conditions for f: f(0)	f'(0)	f'(∞)	Equation Number Used in the Computer Program
ψ_0	f_{01}	$f'''_{01} = (f'_{01})^2 - f_{01}f''_{01} - 1$	0 or ± V	0	1	1
	f_{03}	$f'''_{03} = 4f'_{01}f'_{03} - 3f''_{01}f_{03} - f_{01}f''_{03} - 1$	0	0	1/4	2
	f_{051}	$f'''_{051} = 6f'_{01}f'_{051} - 5f''_{01}f_{051} - f_{01}f''_{051} - 1$	0	0	1/6	20
	f_{052}	$f'''_{052} = 6f'_{01}f'_{052} + 8(f'_{03})^2 - 5f''_{01}f_{052} - 8f_{03}f''_{03} - f_{01}f''_{052} - \frac{1}{2}$	0	0	0	21
ψ_{10}	f_{100}	$f'''_{100} = f'_{01}f'_{100} - f_{01}f''_{100} - 1$	0 (0)	0 (1)	1 (0)	3
	f_{1021}	$f'''_{102} = 3f'_{01}f'_{1021} - 2f''_{01}f_{1021} - f_{01}f''_{1021} - 1$	0 (0)	0 (1/3)	1/3 (0)	4
	f_{1022}	$f'''_{1022} = 3f'_{01}f'_{1022} - 2f''_{01}f_{1022} - f_{01}f''_{1022} + 4(f'_{03}f'_{100} - f_{03}f''_{100}) - 1$	0	0	0	5
	f_{1041}	$f'''_{1041} = 5f'_{01}f'_{1041} - f_{01}f''_{1041} - 4f''_{01}f_{1041} - 1$	0 (0)	0 (1/5)	1/5 (0)	22
	f_{1042}	$f'''_{1042} = 5f'_{01}f'_{1042} + 12f'_{03}F_{1021} - f_{01}f''_{1044} - 7.2f_{03}f''_{1021} - 4f''_{01}f_{1044} - 4.8f''_{03}f_{1021} - 1$	0	0	0	23
	f_{1043}	$f'''_{1043} = 5f'_{01}f'_{1043} + 6f'_{051}f'_{100} - f_{01}f''_{1043} - 4f''_{01}f_{1043} - 1$	0	0	0	24
	f_{1044}	$f'''_{1044} = 5f'_{01}f'_{1044} + 12f'_{03}f'_{1022} + 6f'_{100}f'_{052} - f_{01}f''_{1044} - 7.2f_{03}f''_{1022} - 4f''_{01}f_{1044} - 4.8f''_{03}f_{1022}$	0	0	0	25
ψ_{11}	f_{110}	$f'''_{110} = f'_{01}f'_{110} - f_{01}f''_{110} + f'_{100} - 1$	0	0	0	6
	f_{1121}	$f'''_{1121} = 3f'_{01}f'_{1121} - 2f''_{01}f_{1121} - f_{01}f''_{1121} + f'_{1021} - \frac{1}{3}$	0	0	0	7
	f_{1122}	$f'''_{1122} = 3f'_{01}f'_{1122} - 2f''_{01}f_{1122} - f_{01}f''_{1122} + f'_{1022} + 4(f'_{03}f'_{110} - f_{03}f''_{110})$	0	0	0	8
ψ_{12}	f_{120}	$f'''_{120} = f'_{110} + f'_{01}f'_{120} - f_{01}f''_{120}$	0	0	0	9
	f_{1221}	$f'''_{1221} = 3f'_{01}f'_{1221} - f_{01}f''_{1221} - 2f''_{01}f_{1221} + f'_{1121}$	0	0	0	10
	f_{1222}	$f'''_{1222} = 3f'_{01}f'_{1222} - 2f''_{01}f_{1222} - f_{01}f''_{1222} + f'_{1122} + 4(f'_{03}f'_{120} - f_{03}f''_{120})$	0	0	0	11
ψ_{20s}	f_{20s11}	$f'''_{20s11} = 2f'_{01}f'_{20s11} + 2f'_{100}f'_{1021} - f_{01}f''_{20s11} + 2f''_{100}f_{1021} - f''_{01}f_{20s11} - \frac{2}{3}$	0	0	0	12
	f_{20s12}	$f'''_{20s12} = 2f'_{01}f'_{20s12} + 2f'_{100}f'_{1021} - f_{01}f''_{20s12} + 2f''_{100}f_{1022} - f''_{01}f_{20s12}$	0	0	0	13
	f_{20s31}	$f'''_{20s31} = 4f'_{01}f'_{20s31} + \frac{10}{3}f'_{100}f'_{1041} - f_{01}f''_{20s31} - \frac{10}{3}f''_{100}f_{1041} - 3f''_{01}f_{20s31} - \frac{2}{3}$	0	0	0	26
	f_{20s32}	$f'''_{20s32} = 4f'_{01}f'_{20s32} + 3(f'_{102})^2 - f_{01}f''_{20s32} - 3f_{1021}f''_{1021} - 3f''_{01}f_{20s32} - \frac{1}{3}$	0	0	0	27
	f_{20s33}	$f'''_{20s33} = 4f'_{01}f'_{20s33} + 8f'_{03}f'_{20s11} + \frac{10}{3}f'_{100}f'_{1042} + 6f'_{102}f'_{1022} - f_{01}f''_{20s33} - 6f_{03}f''_{20s11} - 3f_{1021}f''_{1022}$ $- 3f''_{100}f_{1022} - \frac{10}{3}f''_{100}f_{1042} - 2f''_{03}f_{20s11} - 3f''_{01}f_{20s33}$	0	0	0	28

TABLE V (Concluded)

For	f	Equation for f	Boundary Conditions for f: f(0)	f'(0)	f'(∞)	Equation Number Used in the Computer Program
	f_{20s34}	$f'''_{20s34} = 8f'_{03}f'_{20s12} + 4f'_{01}f'_{20s34} + \frac{10}{3} f'_{100}f'_{1044} + 3(f'_{1022}) - f_{01}f''_{20s34} - 6f_{03}f''_{20s12} - 3f_{1022} - \frac{10}{3} f''_{100}f_{1044} - 2f''_{03}f_{20s12} - 3f''_{01}f_{20s34}$	0	0	0	29
	f_{20s35}	$f'''_{20s35} = 4f'_{01}f'_{20s35} + \frac{10}{3} f'_{100}f'_{1043} - f_{01}f''_{20s35} - \frac{10}{3} f''_{100}f_{1043} - 3f''_{01}f_{20s35}$	0	0	0	30
ψ_{21t}	f_{21t11}	$f'''_{21t11} = 2f'_{20s11} + 2f'_{01}f'_{21t11} + 2f'_{100}f'_{1121} - f_{01}f''_{21t11} - 2f''_{110}f_{1021} - f''_{01}f_{21t11}$	0	0	0	14
	f_{21t12}	$f'''_{21t12} = 2f'_{20s12} + 2f'_{01}f_{21t12} + 2f'_{100}f'_{1122} - f_{01}f''_{21t12} - 2f''_{110}f_{1022} - f''_{01}f_{21t12}$	0	0	0	15
ψ_{22s}	f_{22s11}	$f'''_{22s11} = 2f'_{01}f'_{22s11} - f_{01}f''_{22s11} - f''_{01}f_{22s11} + f'_{100}f'_{1221} + f'_{120}f'_{1021} - f'_{110}f'_{1121} - f''_{120}f_{1021} - f''_{100}f_{1221} + f''_{110}f_{1121}$	0	0	0	16
	f_{22s12}	$f'''_{22s12} = 2f'_{01}f'_{22s12} - f_{01}f''_{22s12} - f''_{01}f_{22s12} + f'_{100}f'_{1222} + f'_{120}f'_{1022} - f'_{110}f'_{1122} - f''_{120}f_{1022} - f''_{100}f_{1222} + f''_{110}f_{1122}$	0	0	0	17
ψ_{22t}	f_{22t11}	$f'''_{22t11} = 2f'_{01}f'_{22t11} - f_{01}f''_{22t11} - f''_{01}f_{22t11} + f'_{100}f'_{1221} + f'_{120}f'_{1021} + f'_{110}f'_{1121} - f''_{120}f_{1021} - f'_{100}f_{1221} - f''_{120}f_{1121} + f'_{21t11}$	0	0	0	18
	f_{22t12}	$f'''_{22t12} = 2f'_{01}f'_{22t12} - f_{01}f''_{22t12} - f''_{01}f_{22t12} + f'_{100}f'_{1222} + f'_{120}f'_{1022} + f'_{110}f'_{1122} - f''_{120}f_{1022} - f''_{100}f_{1222} - f''_{120}f_{1122} + f'_{21t12}$	0	0	0	19

NOTE: The terms underlined in the f equations are for the fluctuating-circulation case only. The boundary conditions in ( ) are those to be replaced for the rotational-oscillation case.

TABLE VI

EQUATIONS AND BOUNDARY CONDITIONS FOR THE UNIVERSAL DISTRIBUTION FUNCTIONS OF TEMPERATURE OR CONCENTRATION FOR FLUCTUATING-CIRCULATION AND ROTATIONAL-OSCILLATION CASES

For	F	Equation for F	Boundary Conditions for F: F(0)	Boundary Conditions for F: F(∞)	Equation Number Used in the Computer Program
θ_0	F_{00}	$\frac{F''_{00}}{Pr} = - f_{01}F'_{00}$	0	1	31
	F_{02}	$\frac{F''_{02}}{Pr} = 2f'_{01}F_{02} - 3f_{03}F'_{00} - f_{01}F'_{02}$	0	0	32
	F_{041}	$\frac{F''_{041}}{Pr} = -f_{01}F'_{041} + 4f'_{01}F_{041} - 5f_{041}F'_{00}$	0	0	47
	F_{042}	$\frac{F''_{042}}{Pr} = - f_{01}F'_{042} + 4f'_{01}F_{042} - 5f_{042}F'_{00} + \frac{8}{3}(2f'_{03}F_{02}-3f_{03}F'_{02})$	0	0	48
θ_{10}	F_{1011}	$\frac{F''_{1011}}{Pr} = f'_{01}F_{1011} - f_{01}F'_{1011} - 3f_{1021}F'_{00}$	0	0	33
	F_{1012}	$\frac{F''_{1012}}{Pr} = f'_{01}F_{1012} + 4f_{100}F_{02} - f_{01}F'_{1012} - 3f_{1041}F'_{00}$	0	0	34
	F_{1031}	$\frac{F''_{1031}}{Pr} = 3f'_{01}F_{1031} - f_{01}F'_{1031} - 5f_{121}F'_{00}$	0	0	49
	F_{1032}	$\frac{F''_{1032}}{Pr} = 2f'_{03}F_{1011} + 3f'_{01}F_{122} + 6f'_{1021}F_{02} - 6f_{03}F'_{1011} - f_{01}F'_{1032} - 6f_{1021}F'_{02} - 5f_{1042}F'_{00}$	0	0	50
	F_{1033}	$\frac{F''_{1033}}{Pr} = 2f'_{03}F_{1012} + 3f'_{01}F_{1033} + 6f'_{100}F_{042} + 6f'_{1022}F_{02} - 6f_{03}F'_{1012} - f_{01}F'_{1033} - 6f_{1022}F'_{02} - 5f_{1044}F'_{00}$	0	0	51
	F_{1034}	$\frac{F''_{1034}}{Pr} = 3f'_{01}F_{1034} + 6f_{100}F_{041} - f_{01}F'_{1034} - 5f_{1043}F'_{00}$	0	0	52
θ_{11}	F_{1111}	$\frac{F''_{1111}}{Pr} = f'_{01}F_{1111} - f_{01}F'_{1111} - 3f_{1121}F'_{00}$	0	0	35
	F_{1112}	$\frac{F''_{1112}}{Pr} = f'_{01}F_{1112} + 4f_{110}F_{02} - f_{01}F'_{1112} - 3f_{1122}F'_{00}$	0	0	36
θ_{12}	F_{1211}	$\frac{F''_{1211}}{Pr} = f'_{01}F_{1211} - f_{01}F'_{1211} - 3f_{1221}F'_{00}$	0	0	37
	F_{1212}	$\frac{F''_{1212}}{Pr} = f'_{01}F_{1212} + 4f_{120}F_{02} - f_{01}F'_{1212} - 3f_{1222}F'_{00}$	0	0	38
θ_{20s}	F_{20s01}	$\frac{F''_{20s01}}{Pr} = \frac{2}{3} f'_{100}F_{1011} - f_{01}F'_{20s01} - f_{20s11}F'_{00}$	0	0	39
	F_{20s02}	$\frac{F''_{20s02}}{Pr} = \frac{2}{3} f'_{100}F_{1012} - f_{01}F'_{20s02} - f_{20s12}F'_{00}$	0	0	40
	F_{20s21}	$\frac{F''_{20s21}}{Pr} = 2f'_{01}F_{20s21} + f'_{1022}F_{1011} + f'_{1021}F_{1012} + 4f'_{20s11}F_{02} - 6f_{03}F'_{20s01} - f_{01}F'_{20s21} - 2f_{1022}F'_{1011} - 2f_{20s11}F'_{02}$ $- 2f_{1021}F'_{1012} + 2f'_{1011}F_{1212} - 3f_{20s33}F'_{00}$	0	0	53
	F_{20s22}	$\frac{F''_{20s22}}{Pr} = 2f'_{01}F_{20s22} + f'_{1021}F_{1011} - f_{01}F'_{20s22} - 2f_{1021}F'_{1011} - 3f_{20s32}F'_{00}$	0	0	54
	F_{20s23}	$\frac{F''_{20s23}}{Pr} = 2f'_{01}F_{20s23} + f'_{1022}F_{1012} + 4F'_{1112}F_{02} - 6f_{03}F'_{20s02} - f_{01}F'_{20s23} - 2f_{1022}F'_{1012} - 2f_{20s12}F'_{02} + 2f'_{100}F_{1033} - 3f_{20s34}F'_{00}$	0	0	55
	F_{20s24}	$\frac{F''_{20s24}}{Pr} = 2f'_{01}F_{20s24} - f_{01}F'_{20s24} + 2f'_{100}F_{1031} - 3f_{20s31}F'_{00}$	0	0	56
	F_{20s25}	$\frac{F''_{20s25}}{Pr} = 2f'_{01}F_{20s25} - f_{01}F'_{20s25} + 2f'_{100}F_{1034} - 3f_{20s35}F'_{00}$	0	0	57

TABLE VI (Concluded)

For	F	Equation for F	Boundary Conditions for F: F(0)	Boundary Conditions for F: F(∞)	Equation Number Used in the Computer Program
θ_{21t}	F_{21to1}	$\frac{F''_{21to1}}{Pr} = \frac{2}{3} f'_{110}F_{1011} + \frac{2}{3} f'_{100}F_{1111} + 2F_{20so1} - f_{01}F'_{21to1} - f_{20s11}F'_{00}$	0	0	41
	F_{21to2}	$\frac{F''_{21to2}}{Pr} = \frac{2}{3} f'_{110}F_{1012} + \frac{2}{3} f'_{100}F_{1112} + 2F_{20so2} - f_{01}F_{21to2} - f_{20s12}F'_{00}$	0	0	42
θ_{22s}	F_{22so1}	$\frac{F''_{22so1}}{Pr} = f'_{120}F_{1011} + f'_{100}F_{1211} - f'_{110}F_{1111} - f_{01}F'_{22so1} - 3f_{22s11}F'_{00}$	0	0	43
	F_{22so2}	$\frac{F''_{22so2}}{Pr} = f'_{120}F_{1011} + f'_{100}F_{1212} - f'_{110}F_{1112} - f_{01}F'_{22so2} - 3f_{22s12}F'_{00}$	0	0	44
θ_{22t}	F_{22to1}	$\frac{F''_{22to1}}{Pr} = f'_{120}F_{1011} + f'_{100}F_{1211} + f'_{110}F_{1111} + 3F_{21to1} - f_{01}F'_{22to1} - 3f_{22s11}F'_{00}$	0	0	45
	F_{22to2}	$\frac{F''_{22to2}}{Pr} = f'_{120}F_{1012} + f'_{100}F_{1212} + f'_{110}F_{1112} + 3F_{21to2} - f_{01}F'_{22to2} - 3f_{22s12}F'_{00}$	0	0	46

III. CONVERGENCE OF THE BLASIUS SERIES

In a brief comparison of the Blasius and Görtler series solutions for steady forced convection, Frössling[38] has pointed out that the stress in considering the merits of various techniques and varients of the method lays primarily on (a) the simplicity of application, (b) the rapidity of convergence of the series, and (c) its generality. This stress will undoubtly apply to the forced convection problems in the unsteady boundary layers. Aside from the Karman-Paulhousen integral technique which yields an approximate solution to the problems, Lighthill,[12] the Blasius technique offers the best simplicity of application. This is essentially due to the fact that the desired result is directly given if the expressions for the outer velocity distribution $U_0(x)$ and its fluctuating amplitude $U_1(x)$ are obtained either from practical measurements or theoretical considerations. The use of a generalized Blasius series as described previously has assured the generality of the Blasius technique. The problem left to be examined is the rapidity of the series convergence. As was pointed out by Frössling,[38] there is no general method available for the determination of the radius of convergence. The quality of the series has to be determined by studying the behavior of the series when the number of terms involved is varied.

The convergence of the series has been examined for the ordinary Blasius series solution for a symmetrical blunt body. No effort has been given to the convergence of the generalized Blasius solution, since it involves parameters m, M and n. The universal functions f and F were evaluated by means of an IBM 7090 digital computer for all three cases using the programs in the Appendix. Their numerical reduction was obtained for Prandtl number from 0, 7 to 10 and V of 0, 0.1 and -0.1. It is observed that the series solutions for all three order perturbations give good convergence after the first two terms, especially for small values of x. A fairly good convergence is obtained near the wall for large values of x. In general the velocity solutions have a better convergence than the temperature solutions for the same number of terms included.

IV. NUMERICAL RESULTS AND DISCUSSION

The influence of external disturbances, such as the flow oscillation and fluctuating circulation of free stream and the rotational oscillation of cylinder surface on the transport phenomena is numerically examined for the simplest and most typical case in two-dimensional flow. It is a flow over a circular cylinder with the external potential flow

$$U_o = 2 \sin(2x).$$

By expanding the sine on the right-hand side of the equation into power series, one obtaines

$$a_1 = 4,$$

$$a_3 = -8/3,$$

$$a_5 = 8/15,$$

$$\cdots\cdots\cdots\cdots$$

The functional coefficients f and F for symmetrical cases, as given in Tables I, II, V, and VI were evaluated by means of an IBM 7090 digital computer. Their numerical reduction was obtained for Prandtl number from 0.7 to 10 and uniform suction of 0, 0.1 and -0.1.

Numerical results are presented in graphical form in Fig. 2 to 8 for the flow-oscillation in free stream, Fig. 9 to 17 for the fluctuating-circulation in free stream and Fig. 18 to 24 for the rotational-oscillation of cylinder surface. The results and discussion on heat transfer to be presented in the following may also be applied to mass transfer by replacing temperature by concentration, Prandtl number by Schmidt number and Nusselt number by Sherwood number.

Figure 2 illustrates the profiles of the first-order velocity at two locations x = 0.05 and 0.2. It shows that the velocity component u_{10} approaches a finite value asymptotically at the outer edge of the boundary layer and negative portion of the profile exists for the velocity component u_{11}. All these profiles are zero at the forward stagnation point and increase in magnitude along the cylindrical surface. Figure 3 is the frequency response of the first order fluid velocity inside the boundary layer. The amplitude of the velocity

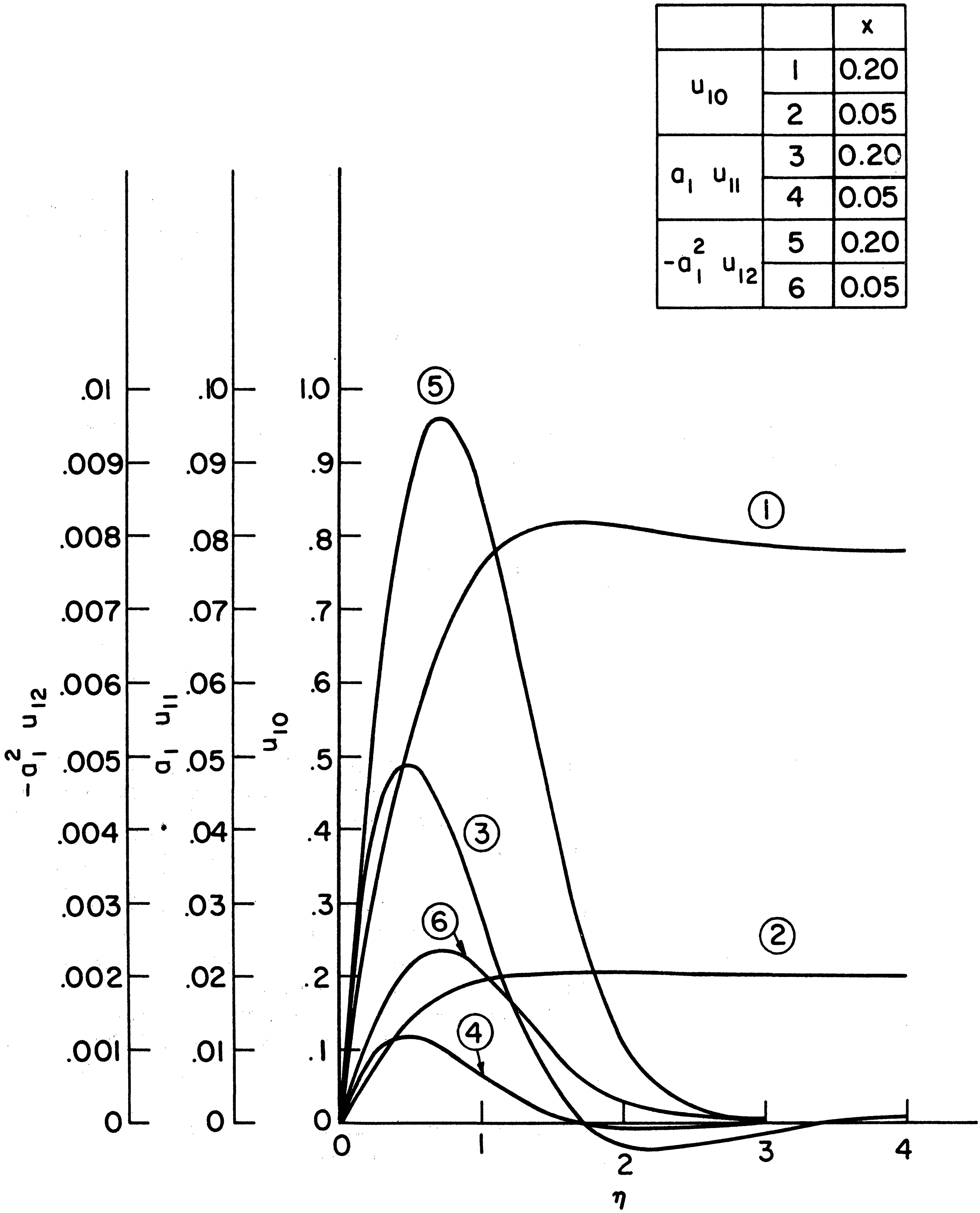
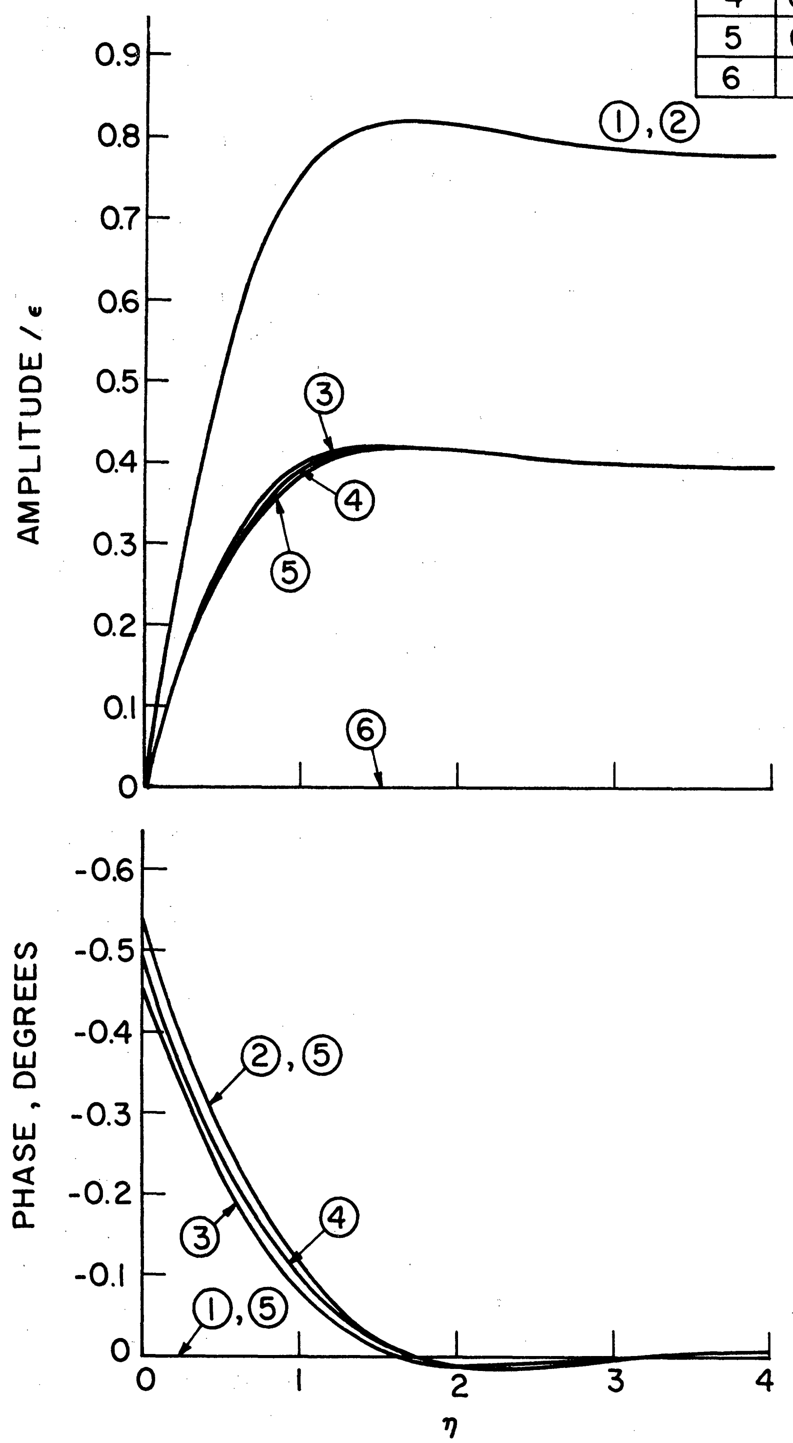


Fig. 2. Profiles of first-order velocities for oscillating flows past a circular cylinder.



	x	ω	V
1	0.2	0	0
2	0.2	0.2	0
3	0.1	0.2	0.1
4	0.1	0.2	0
5	0.1	0.2	-0.1
6	0	ANY	ANY

Fig. 3. Amplitude and phase of fluid velocity for oscillating flows past a circular cylinder

profile is shown to be less affected by the oscillating frequency, but varies very significantly along the surface. However, the phase, which lags the free stream oscillation in the neighborhood of the solid surface and advances in the middle portion of the boundary layer, is a strong function of the oscillating frequency and is less affected by the location x. The removal of decelerated fluid particles from the oscillatory boundary layer by uniform suction results in a slight decrease in the first-order velocity profile in the inner half of the boundary layer. It causes an increase in the phase lag in the inner half and a decrease in the outer half of the boundary layer. The effect of uniform blowing, which supplies additional energy to retarded fluid particles in the oscillatory boundary layer, is opposite to that of uniform suction.

A comparison of the first-order temperature profiles at the forward stagnation point is given here in Fig. 4 for a fluid with the Prandtl number of 0.7. The agreement between the present analysis and the results for the Hiemenz layer obtained by Pohlhausen and Lighthill (1954) is good. The frequency response of the fluid temperature, as illustrated in Fig. 5, shows that both the amplitude in the outer half and the phase lag in the inner half of the oscillatory boundary layer increase along the cylindrical surface. These phenomena are also observed when uniform suction is superimposed. An increase in the fluctuating frequency causes a decrease in the amplitude of the temperature accompanied by an appreciable increase in the phase lag. An increase in the Prandtl number results in the reduction of the thickness of the thermal boundary layer and an increase in the phase lag.

The fluctuations in stream velocity produces a secondary (streaming) flow in the oscillatory boundary layer. The streamline pattern of the steady secondary motion at quasi-steady state is presented in Fig. 6. The stream lines ψ_{2cs} are in the direction of the main flow. The separation of the secondary flow is disclosed at x = 0.8, about 90 degree angle from the forward stagnation point. This separation causes a reversal in the direction for the steady secondary component of the wall shear stress as shown in Fig. 8.

The flow oscillations also induce the distribution of the steady secondary component of temperature in the oscillatory boundary layer. It is seen from Fig. 7 that Θ_{2os} for quasi-steady state its maxima located approximately at the middle of the oscillatory boundary layer. A zero Θ_{2os} line intersects with the solid surface at zero Θ_{2os} at a distance of approximately 0.7 from the forward stagnation point. Downstream from that point is the region of positive Θ_{2os} distribution in the neighborhood of the surface. This results in the alteration of the net change in the local Nusselt number from negative to positive values as illustrated in Fig. 8. In other words, the flow oscillation causes the local heat transfer rate to decrease in the forward region but to increase in the region near the separation point. With an increase in the Prandtl number the zero isothermal line in the fluid shifts toward the forward stagnation point. This provides more solid surface which contributes to an improvement in local heat transfer rate.

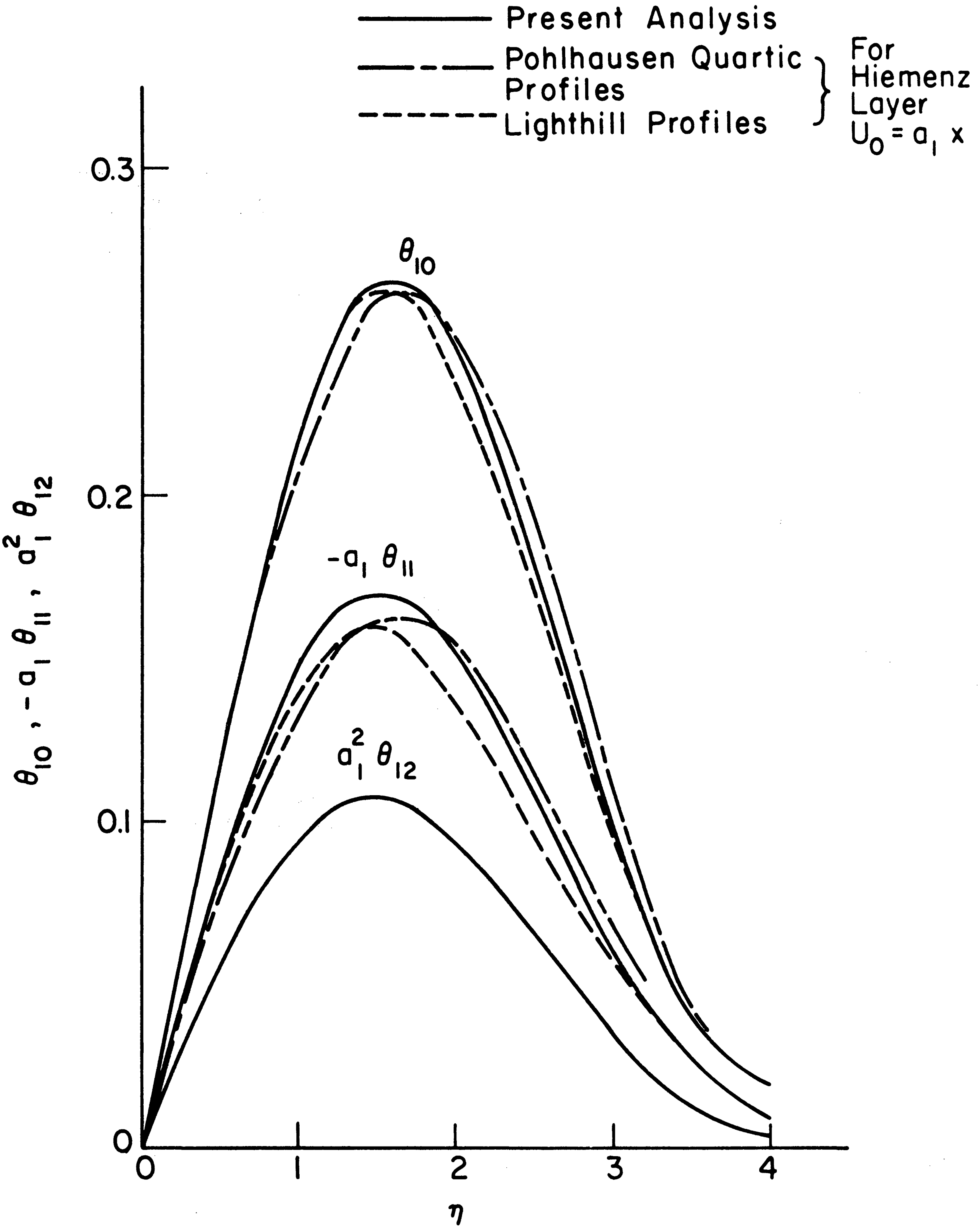


Fig. 4. Profiles of first-order temperatures or concentrations at stagnation point for oscillating flows past a circular cylinder for a fluid with Pr = 0.7 or Sc = 0.7.

	Pr	x	ω	V
1	1	0	0.2	- 0.1
2	1	0	0.2	0
3	1	0	0.2	0.1
4	1	0.2	0.2	0
5	1	0.2	0	0
6	10	0	0.2	0

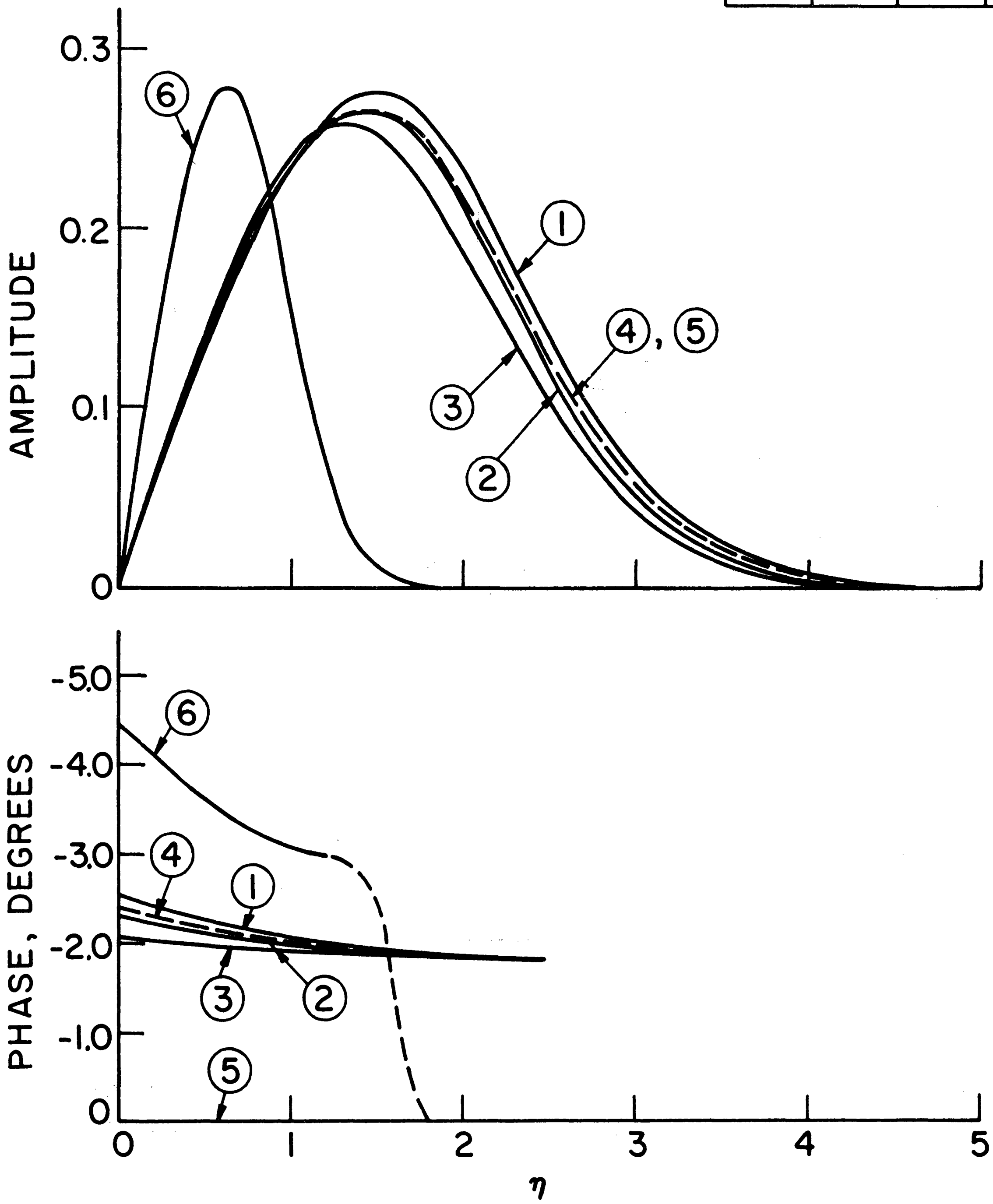


Fig. 5. Amplitude and phase of fluid temperature or concentration for oscillating flows past a circular cylinder.

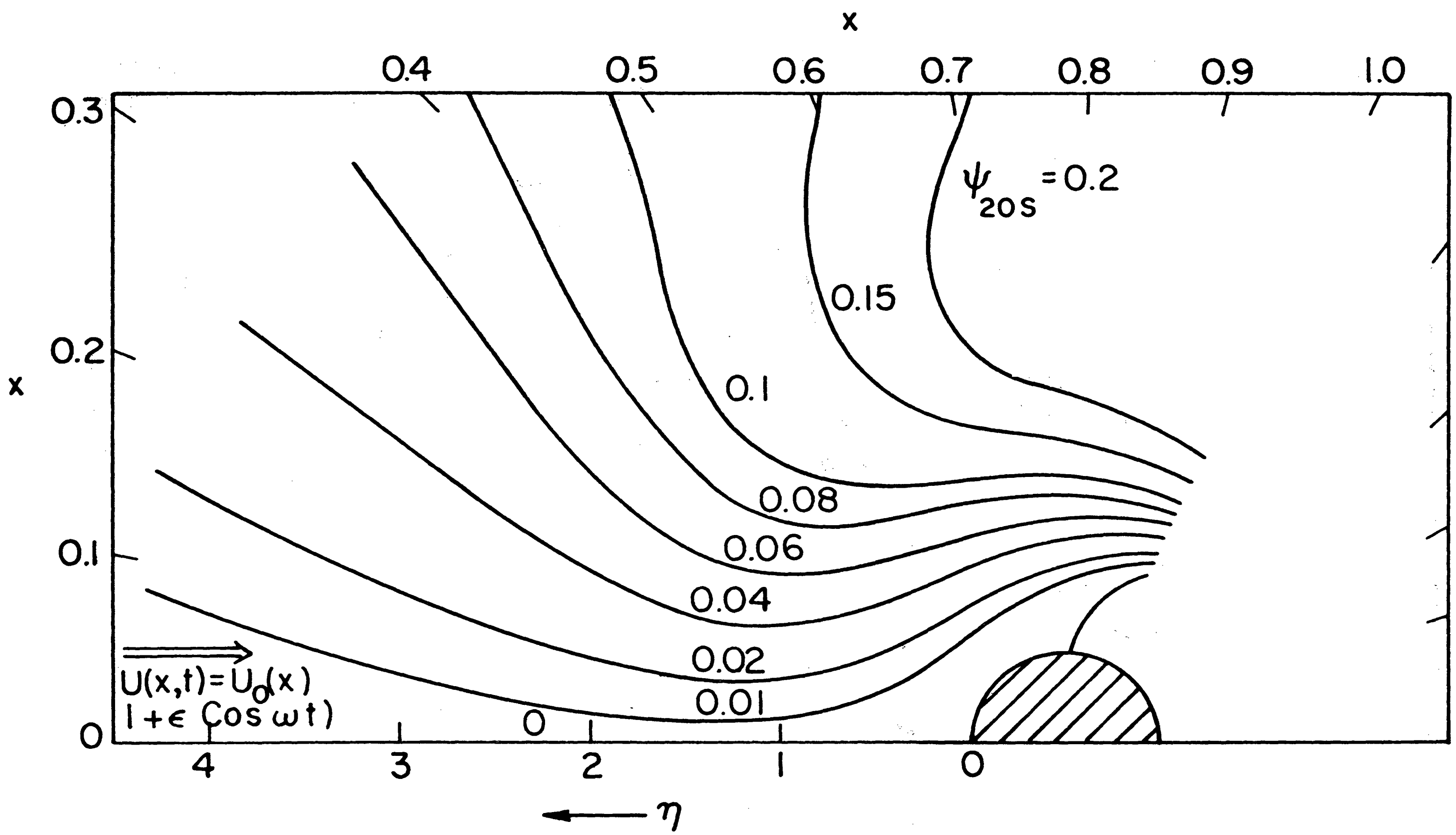


Fig. 6. Streamline pattern of the steady secondary motion produced by oscillating flow past a circular cylinder at quasi-steady state.

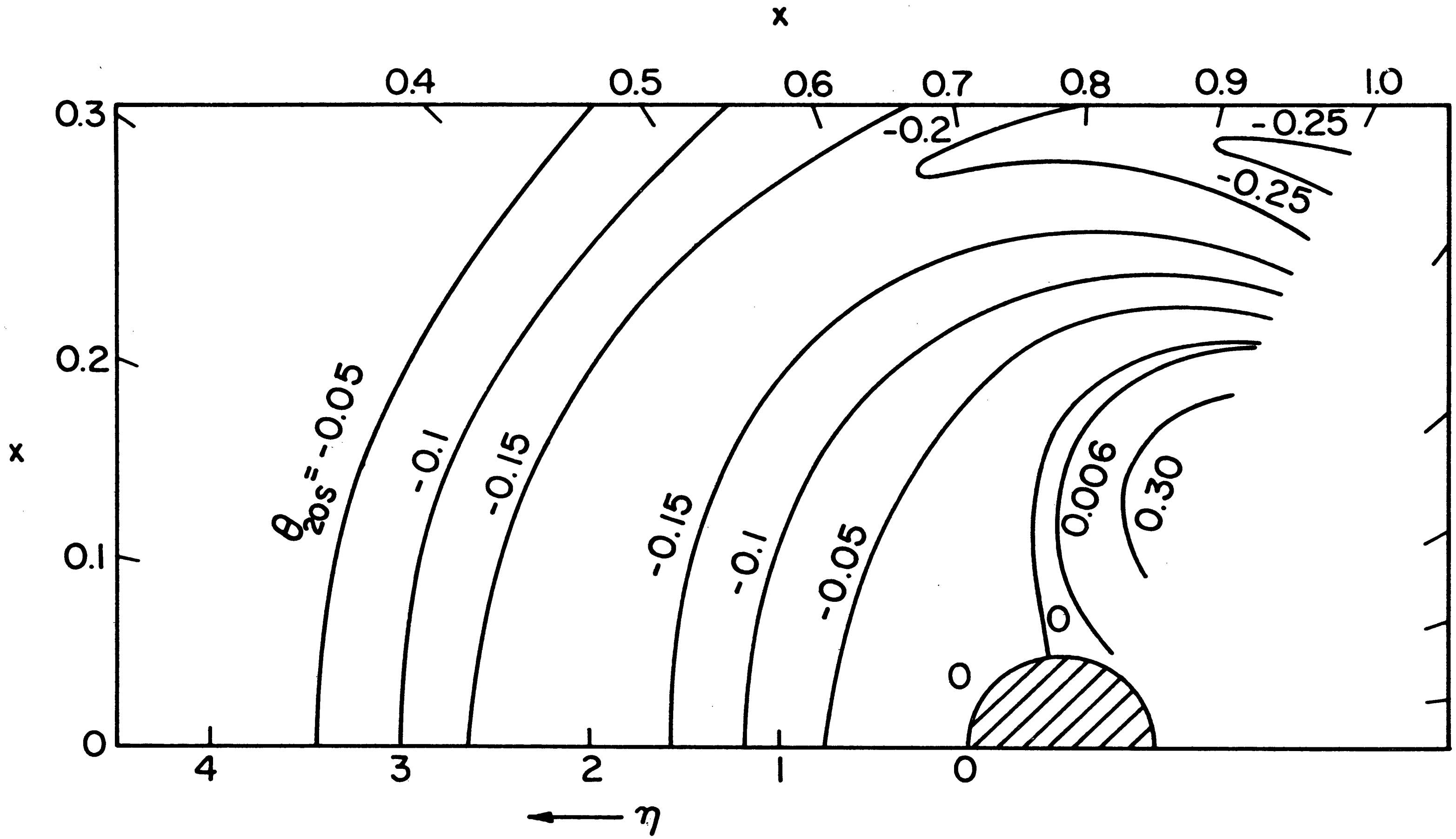


Fig. 7 . Distribution of the steady secondary component of temperature or concentration produced by oscillating flow past a circular cylinder for a fluid with Pr = 1 or Sc = 1 at quasi-steady state.

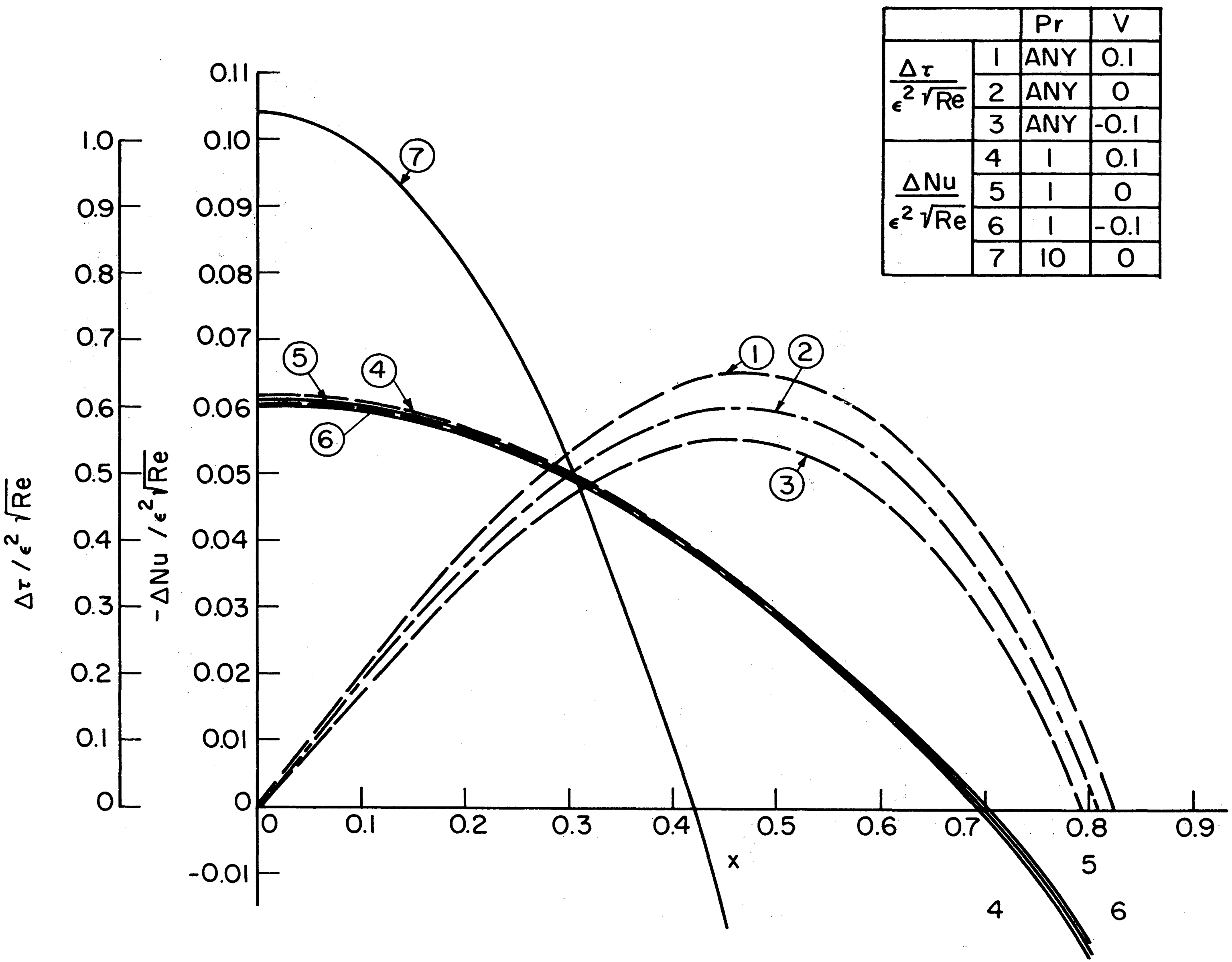


Fig. 8. Effects of flow oscillation on the local wall shear stress and Nusselt number for flow past a circular cylinder at quasi-steady state.

From Fig. 8 one observes that uniform blowing tends to enhance the steady secondary component of wall shear stress. However, it is a well known result that blowing decreases the wall shear stress in a steady flow. Therefore due to the existence of a secondary (streaming) flow in the oscillatory boundary layer, the net effect of blowing on the reduction of wall shear stress becomes less effective in the oscillatory boundary layer than in the steady boundary layer.

It is shown that fluctuations in the stream velocity are capable of causing permanent alterations in both the velocity and temperature profiles in the alminar boundary layer. This provides a secondary flow and permanently alters the wall shear stress and heat transfer rate. These changes are small and are detected from the analysis only when solutions are obtained to at least the second-order approximation beyond this solution for the steady forced convection problem. The superposition of uniform suction tends to increase the wall shear stress and the heat transfer rate in steady forced convection. These effects of uniform suction are suppressed in the neighborhood of the forward stagnation point with the introduction of the fluctuations in the stream velocity. It is disclosed from the numerical studies that the uniform blowing contributes to an increase in the heat transfer rate in a steady forced convection. However, Fig. 8 shows that it causes the steady secondary component of heat transfer rate in an oscillatory boundary layer to decrease near the forward stagnation point followed by an increase along the surface to the point of separation. Therefore the fluctions in stream velocity tend to suppress the contribution of blowing on the increase in heat transfer rate in the neighborhood of the forward stagnation point and to enhance it near the separation point.

A. Fluctuating Circulation

In order to numerically demonstrate the effect of the fluctuating circulation, two different oscillating amplitudes $\epsilon U_1(x)$ are investigated: constant and equal to ϵ and space-dependent and equal to either $\epsilon(1+x^2)$ or $\epsilon(1+x^2+x^4)$. Figure 9 gives the first order velocity profiles. An increase in the magnitude with an increase in x is observed for all three velocity components u_{10}, u_{11} and u_{12}. The effect of the space-dependency of the oscillating amplitude is less evident. This is not the case in the frequency response as illustrated in Fig. 10. The amplitude as well as phase depend on the nature of the oscillating amplitude of the free stream. The effect of the uniform blowing is to increase the amplitude and to decrease the phase lag, although not significantly. The influence of the oscillating frequency is important to an increase in phase lag but less evident to the amplitude. Fig. 11 shows that the first-order temperature profiles Θ_{10}, Θ_{11} and Θ_{12} are all zero at the forward stagnation point and are strongly x-dependency of the amplitude and phase is significant, so is the effect of the Prandtl number. An abrupt change in the phase angle near the outer edge of the boundary layer is observed especially for the low Prandtl number fluid. The streamline pattern of the steady secondary motion produced by the fluctuating circulation

		x	bo	b2
u_{10}	1	0	ANY	ANY
	2	0.2	1	0
	3	0.2	1	1
$a_1\ u_{11}$	4	0	1	ANY
	5	0.2	1	0
	6	0.2	1	1
$-a_1^2\ u_{12}$	7	0	1	ANY
	8	0.2	1	0&1

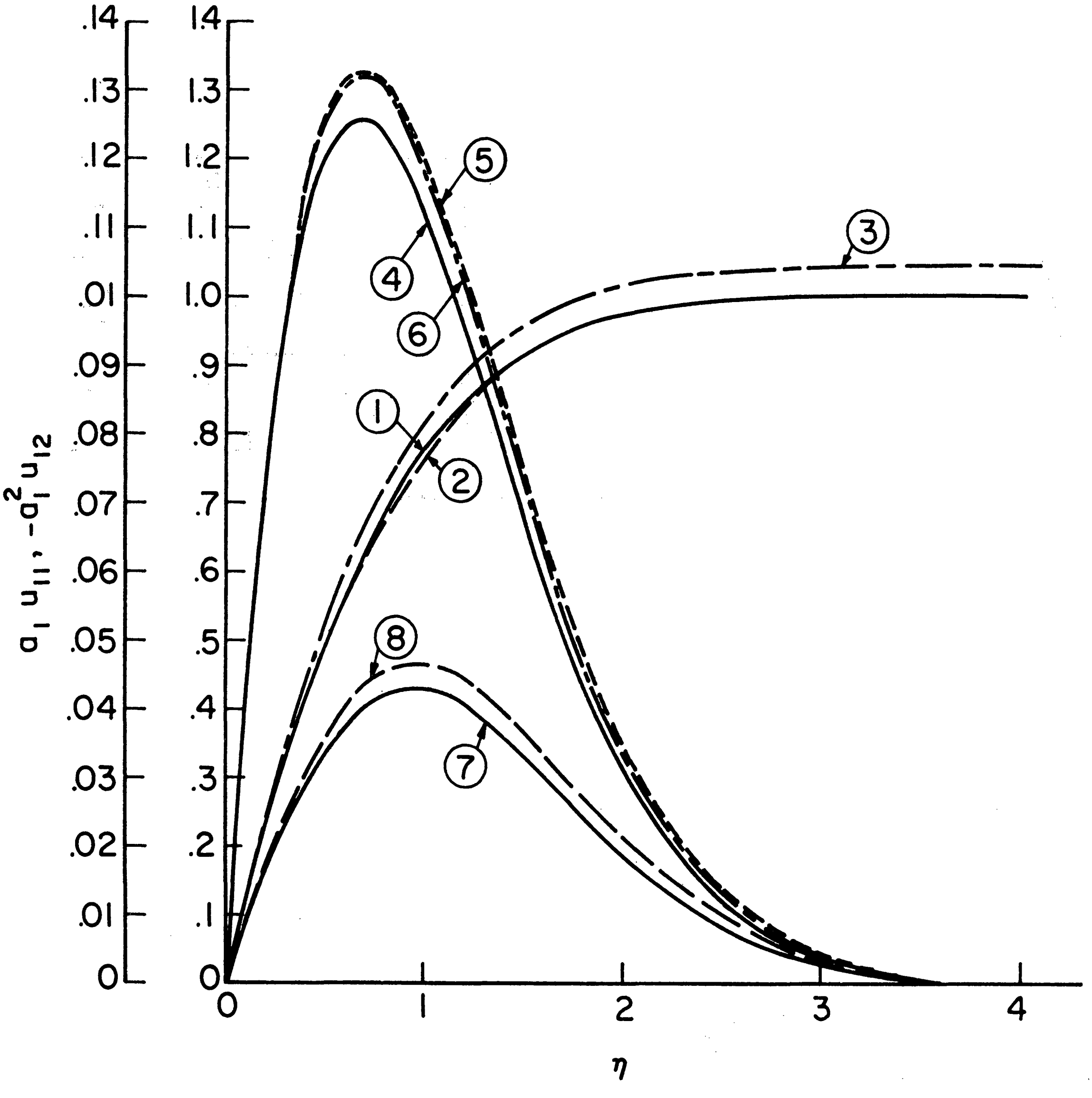
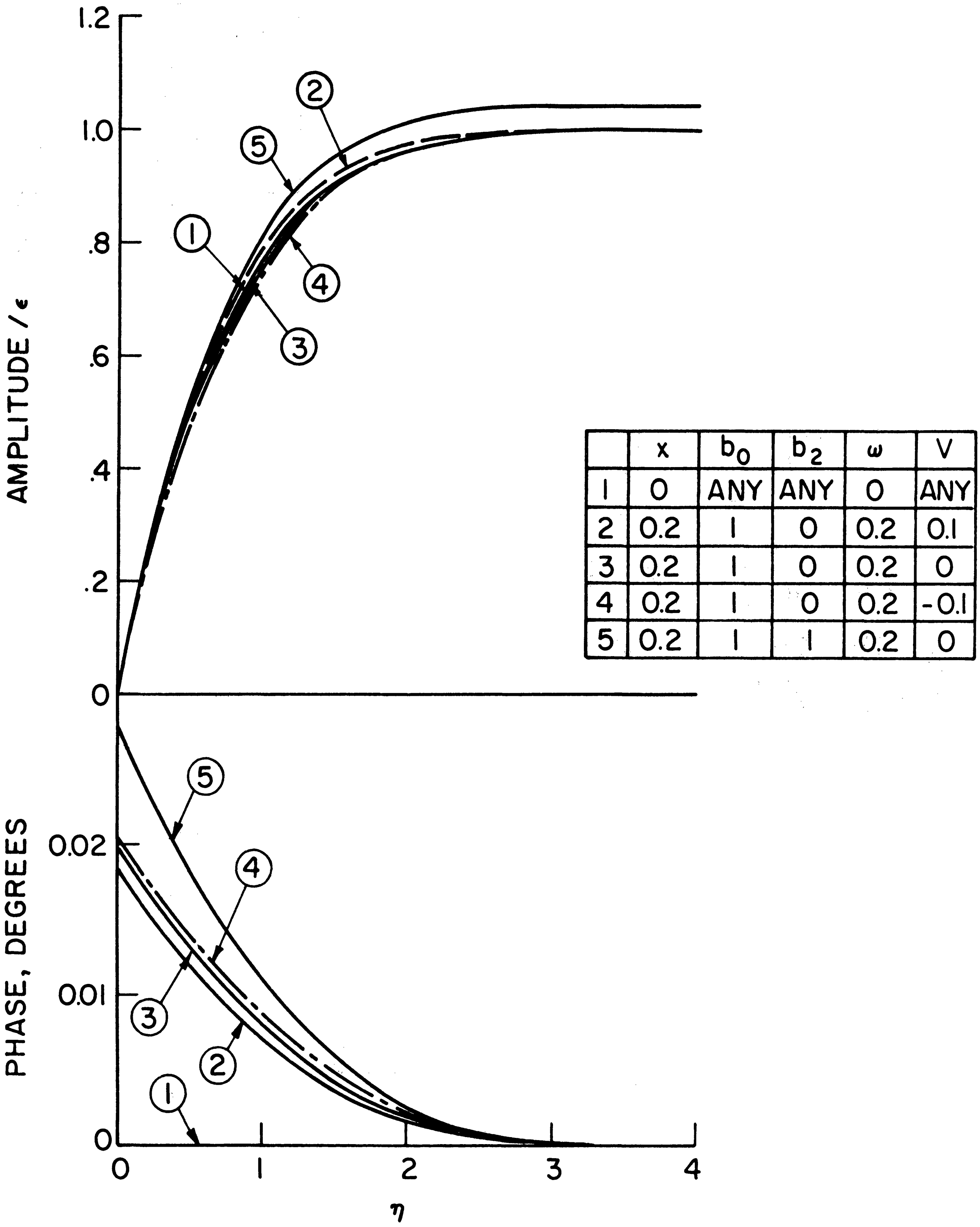


Fig. 9. Profiles of first-order velocities for flow past a circular cylinder with fluctuating circulations.



	x	b_0	b_2	ω	V
1	0	ANY	ANY	0	ANY
2	0.2	1	0	0.2	0.1
3	0.2	1	0	0.2	0
4	0.2	1	0	0.2	-0.1
5	0.2	1	1	0.2	0

Fig. 10. Amplitude and phase of fluid velocity for flow past a circular cylinder with fluctuating circulations.

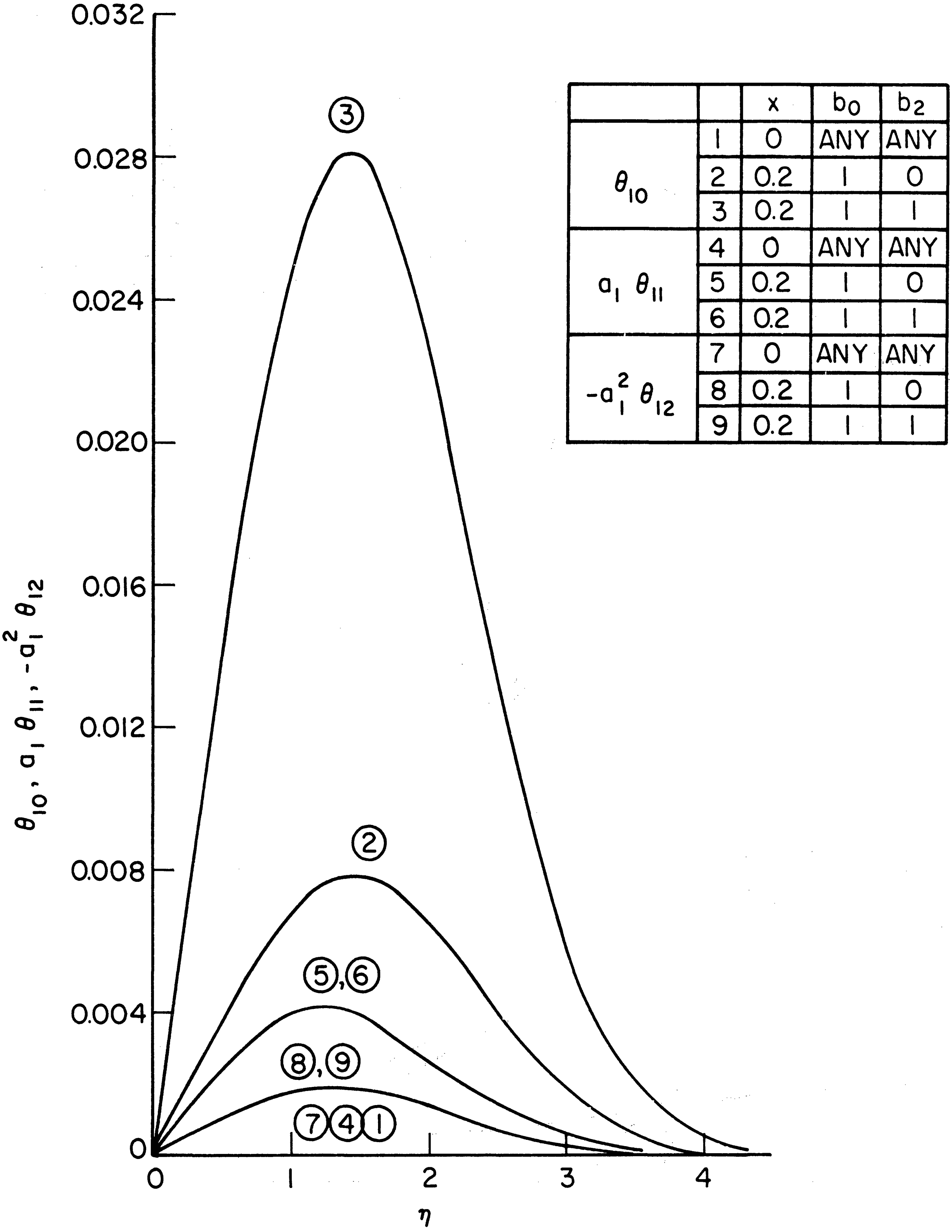
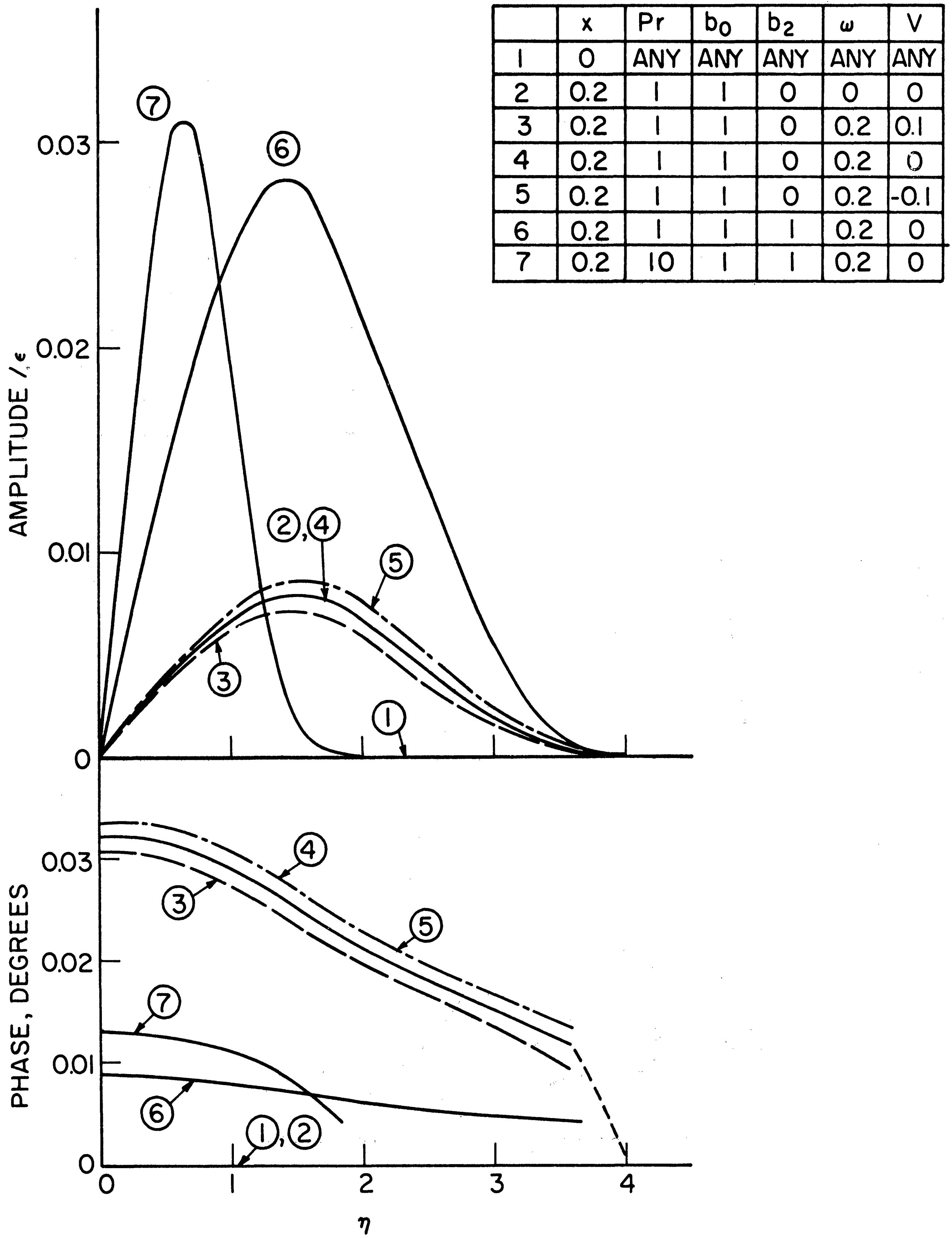


Fig. 11. Profiles of first-order temperatures or concentrations for flow past a circular cylinder with fluctuating circulations for a fluid with Pr = 1 or Sc = 1.



	x	Pr	b_0	b_2	ω	V
1	0	ANY	ANY	ANY	ANY	ANY
2	0.2	1	1	0	0	0
3	0.2	1	1	0	0.2	0.1
4	0.2	1	1	0	0.2	0
5	0.2	1	1	0	0.2	-0.1
6	0.2	1	1	1	0.2	0
7	0.2	10	1	1	0.2	0

Fig. 12. Amplitude and phase of fluid temperatures or concentrations for flow past a circular cylinder with fluctuating circulations.

as shown in Fig. 13 is somewhat different from the one produced by the oscillating flow. A striking distinction is that no separation of the steady secondary flow is observed for the fluctuating circulation case. Instead, the streamlines running in the direction of the main flow get closer to the surface at downstream. For the space-dependent oscillating amplitude of $\epsilon(1 + x^2 + x^4)$ as given in Fig. 14, the streamline pattern is similar to that of a constant amplitude. The corresponding temperature distribution of the steady secondary component Θ_{20s} as respectively presented in Fig. 15 and 16 are also similar in pattern. No positive values exist in the entire boundary layer. The isothermal lines are practically running parallel to the solid surface in case of a constant oscillating amplitude and running closer to the surface along the stream in case of a space-dependent oscillating amplitude. The streamline patterns and temperature distribution as demonstrated in Figs. 13 to 16 are directly related to the effects of the fluctuating circulation on the local values of wall shear stress and Nusselt number as illustrated in Fig. 17. The alternation in the wall shear stress is such as to increase its value while that in the heat transfer rate is to decrease. Both alternations are proporational to the distance from the forward stagnation point except the heat transfer rate under a constant oscillating amplitude. The effect on the heat transfer is more significant for fluids with high Prandtl number.

B. Rotational Oscillations

The profiles of the first-order velocities, u_{10}, u_{11}, and u_{12} are shown in Fig. 18. u_{10} decreases from a finite value at the solid surface to zero at the outer edge of the boundary layer. The other two velocities u_{11} and u_{12}, however, have zero values both at the surface and in the free stream and maxima in the inner half of the boundary layer. All three velocities increase in magnitude with an increase in the distance along the surface from the forward stagnation point. An interesting feature is disclosed in Fig. 19 for the frequency response of the fluid velocity. The amplitude has a maximum at the surface diminishing to zero at the outer edge of the boundary layer while the phase is zero both at the surface and the outer edge with a maximum in the outer half of the boundary layer. The amplitude as well as phase lag are magnified by the superposition of the uniform suction at the surface and vice versa.

The profiles of the first-order temperature as given in Fig. 20 resemble those of the fluctuating-circulation case. Θ_{10}, Θ_{11} and Θ_{12} are zero both at the surface and the outer edge with maxima in the inner half of the boundary layer. The magnitudes increase along the surface from zero at the forward stagnation point. The frequency response of the fluid temperature is also similar to those of the two previous cases. With the superposition of the uniform suction, the amplitude as well as phase lag are magnified. The streamline patterns of the steady secondary flow are plotted in Fig. 22. The streamlines in the inner boundary layer adjacent to the solid surface are running opposite to the main flow in direction. However, the streamlines in the outer boundary layer existing in the forward stagnation-point region run in the same

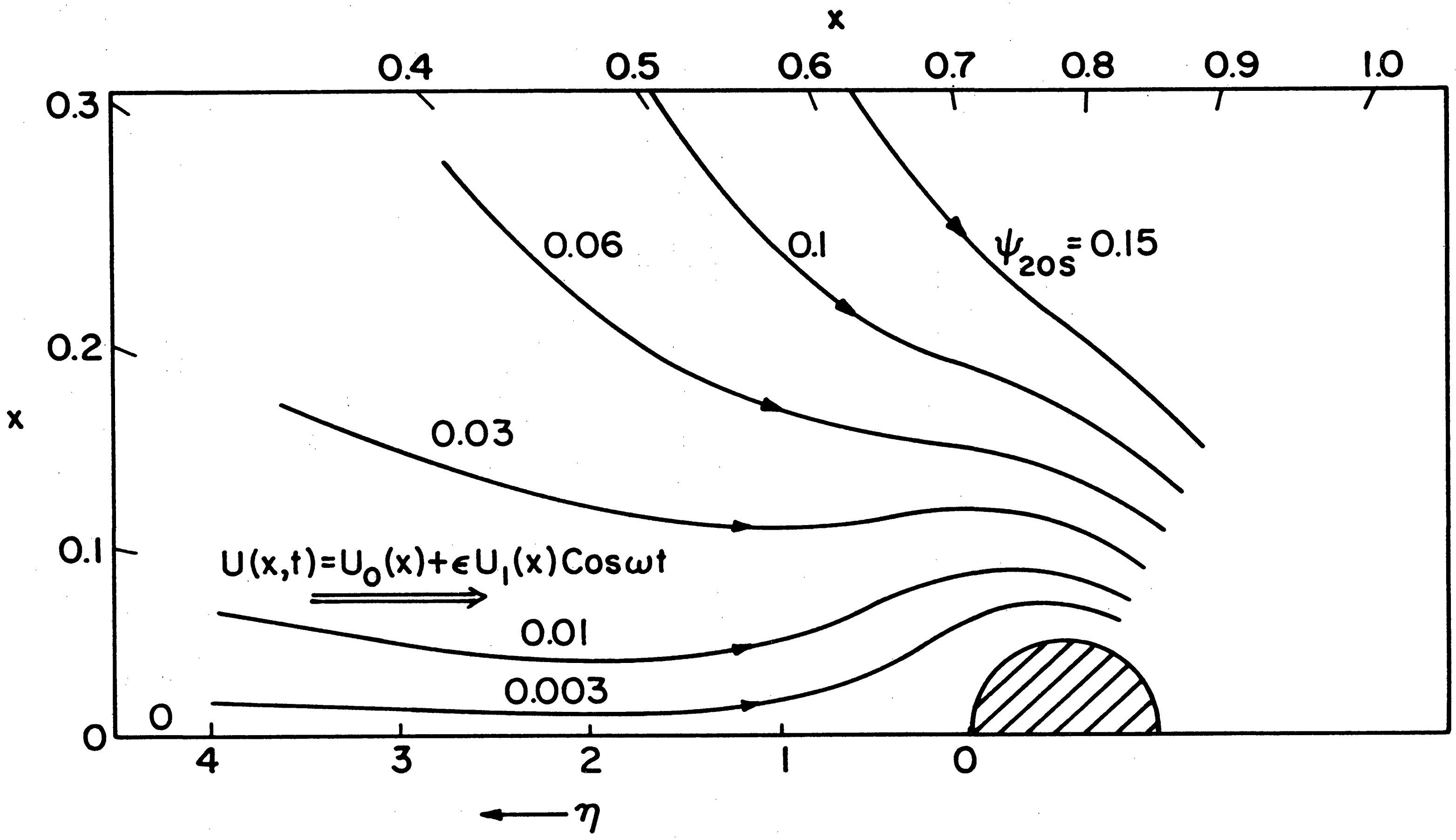


Fig. 13. Streamline pattern of the steady secondary motion produced by flow past a circular cylinder with fluctuating circulation of amplitude $\epsilon U_1 = \epsilon$ at quasi-steady state.

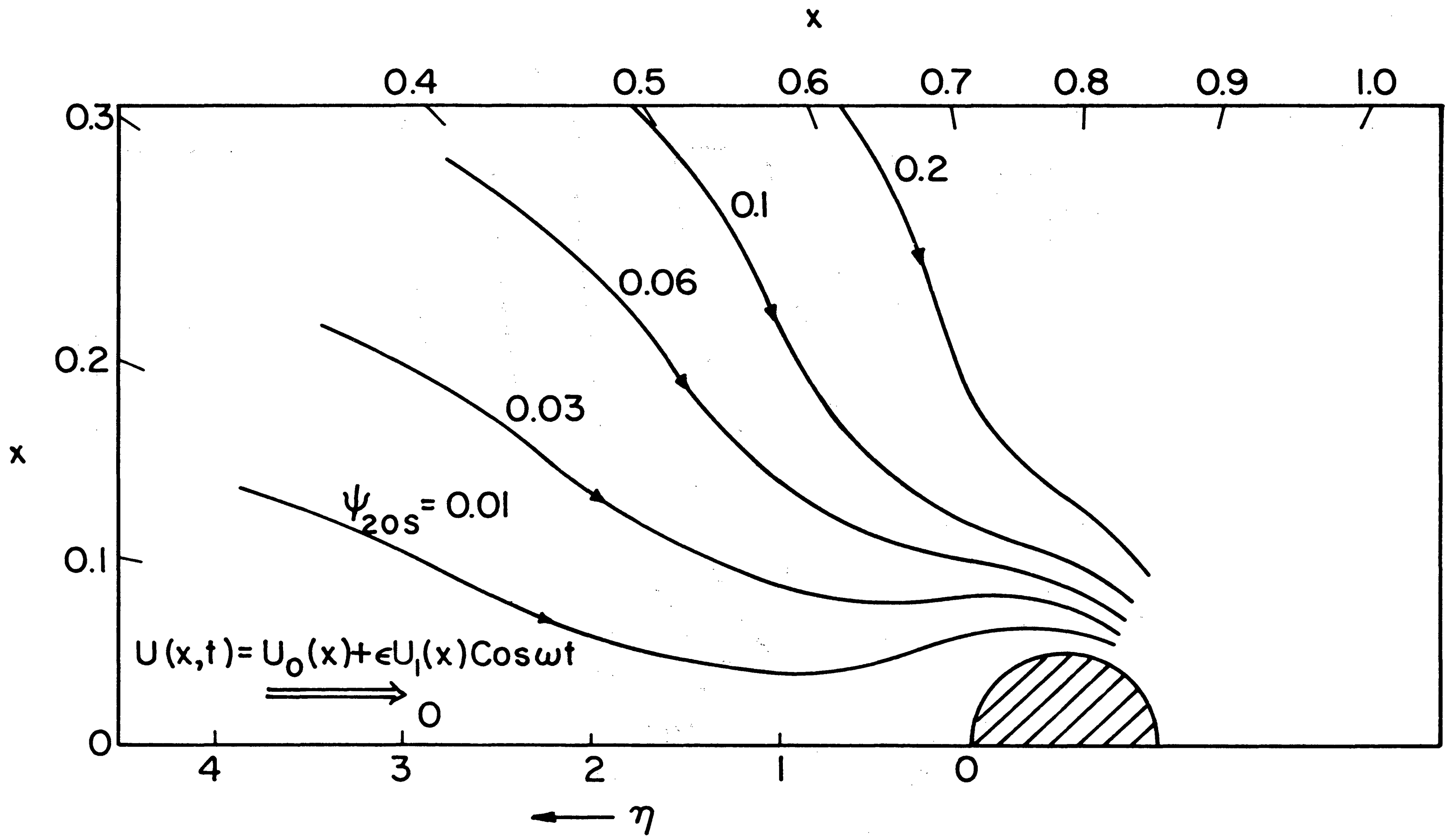


Fig. 14. Streamline pattern of the steady secondary motion produced by flow past a circular cylinder with fluctuating circulation of amplitude $\epsilon U_1(x) = \epsilon(1+x^2+x^4)$ at quasi-steady state.

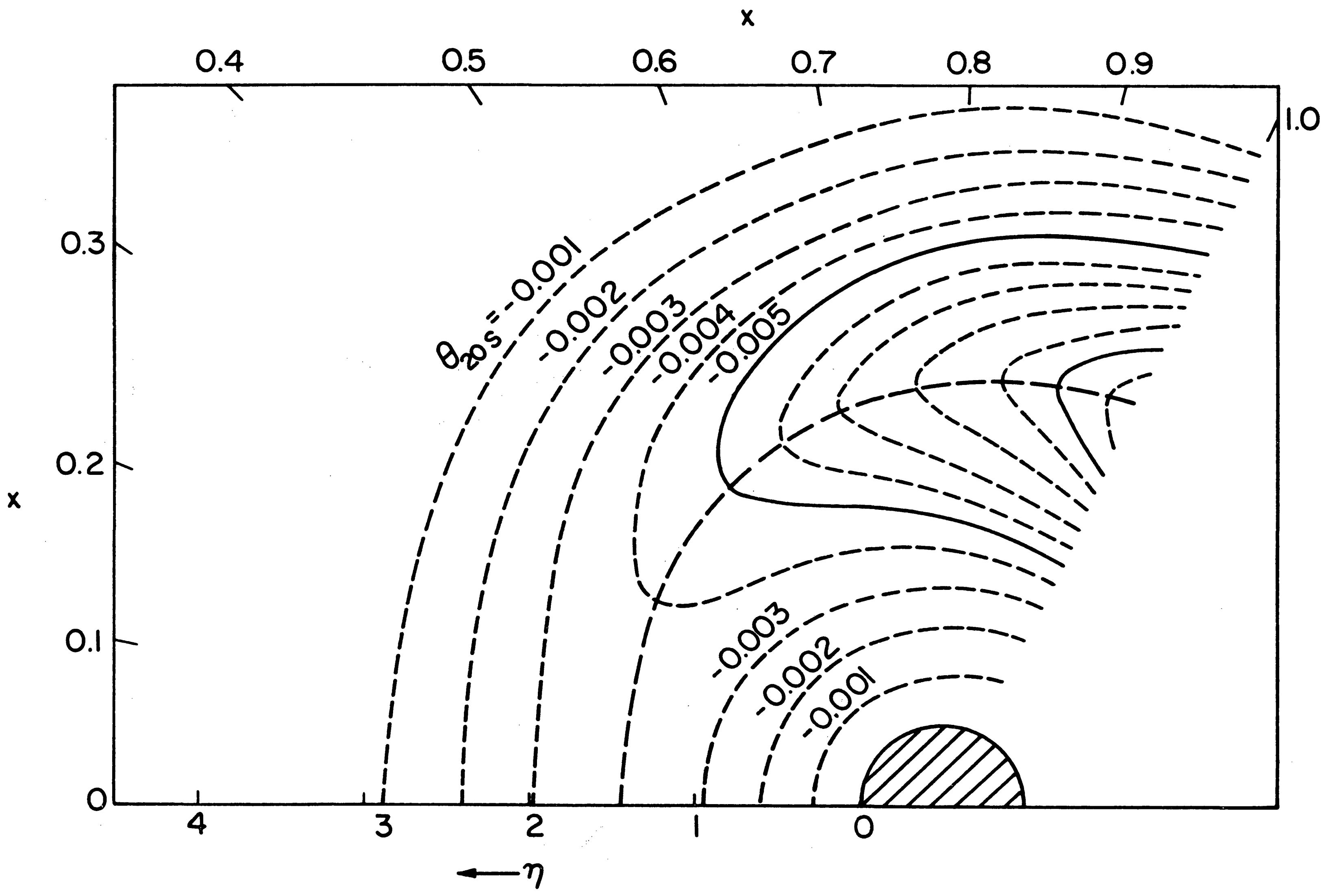


Fig. 15. Distribution of the steady secondary component of temperature or concentration produced by flow past a circular cylinder with fluctuating circulation of amplitude $\epsilon U_1 = \epsilon$ for a fluid having Pr = 1 or Sc = 1 at quasi-steady state.

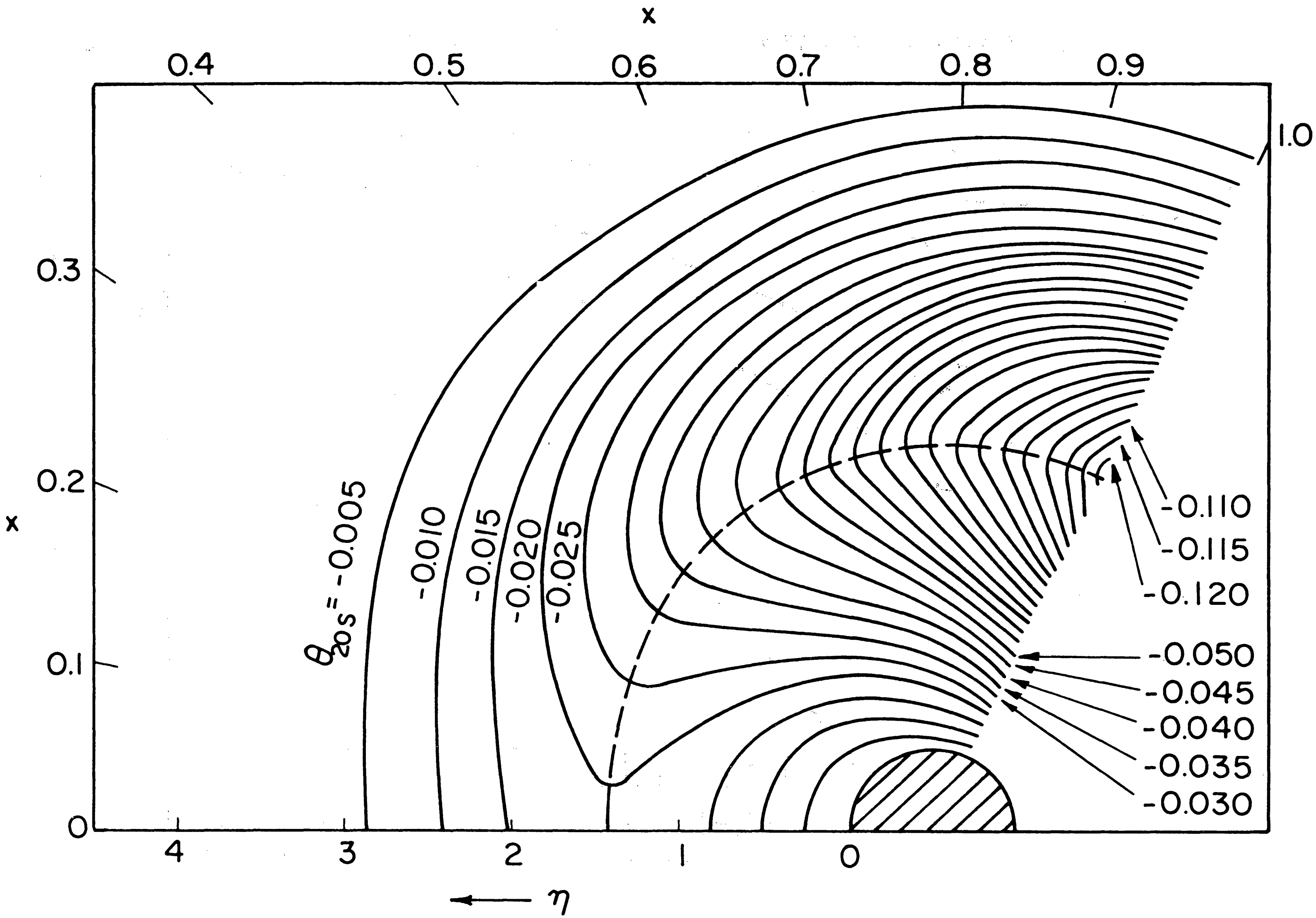
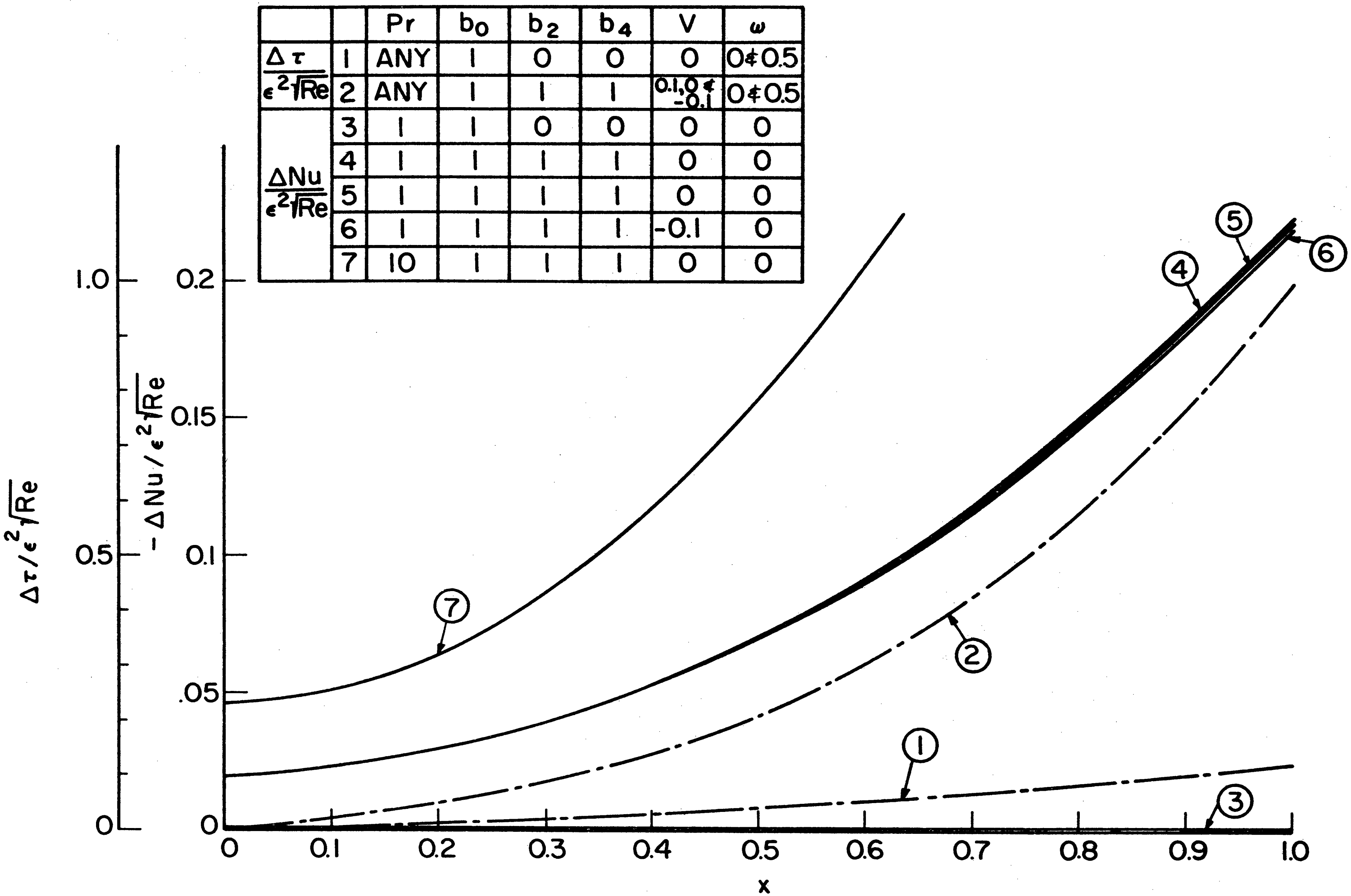


Fig. 16. Distribution of the steady secondary component of temperature or concentration produced by flow past a circular cylinder with fluctuating circulations of amplitude $\epsilon U_1(x) = \epsilon(1+x^2+x^4)$ for a fluid having Pr = 1 or Sc = 1 at quasi-steady state.



		Pr	b_0	b_2	b_4	V	ω
$\frac{\Delta \tau}{\epsilon^2 \sqrt{Re}}$	1	ANY	1	0	0	0	0 ¢ 0.5
	2	ANY	1	1	1	0.1, 0 ¢ -0.1	0 ¢ 0.5
$\frac{\Delta Nu}{\epsilon^2 \sqrt{Re}}$	3	1	1	0	0	0	0
	4	1	1	1	1	0	0
	5	1	1	1	1	0	0
	6	1	1	1	1	-0.1	0
	7	10	1	1	1	0	0

Fig. 17. Effects of fluctuating circulation on the local wall shear stress and Nusselt number for flow past a circular cylinder.

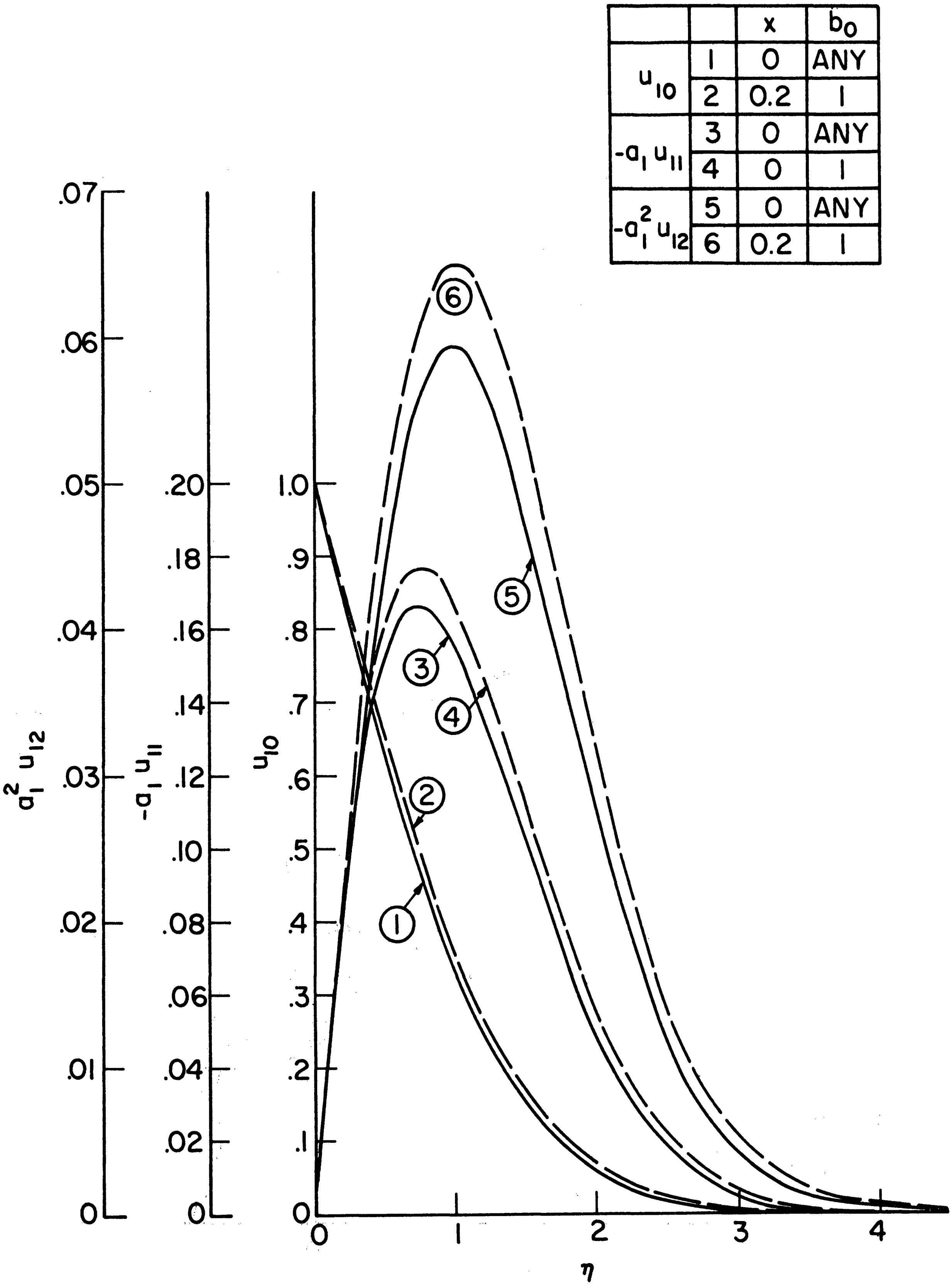


Fig. 18. Profiles of first-order velocities for flow past a circular cylinder in rotational oscillations.

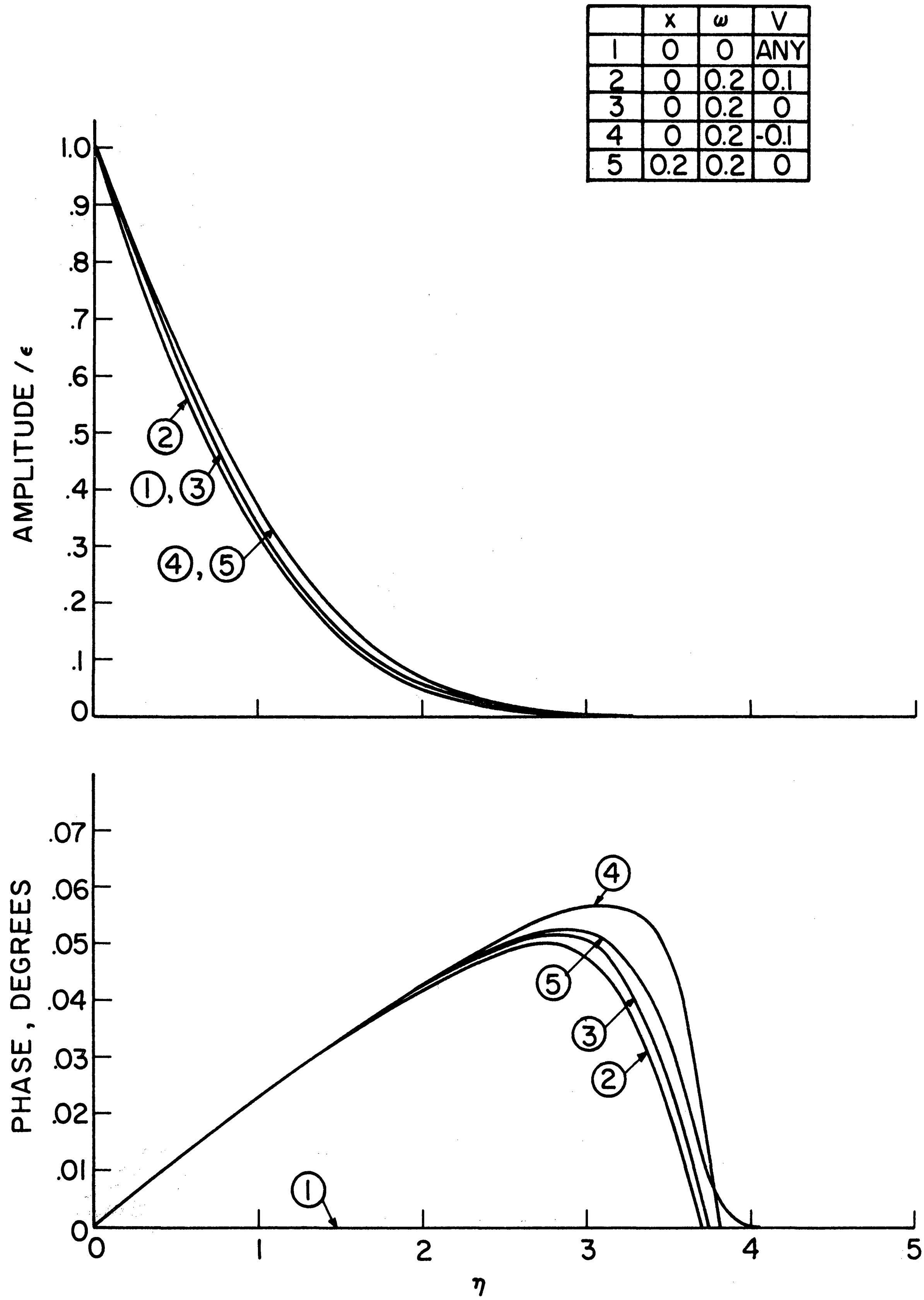


Fig. 19. Amplitude and phase of fluid velocity for flow past a circular cylinder in rotational oscillations.

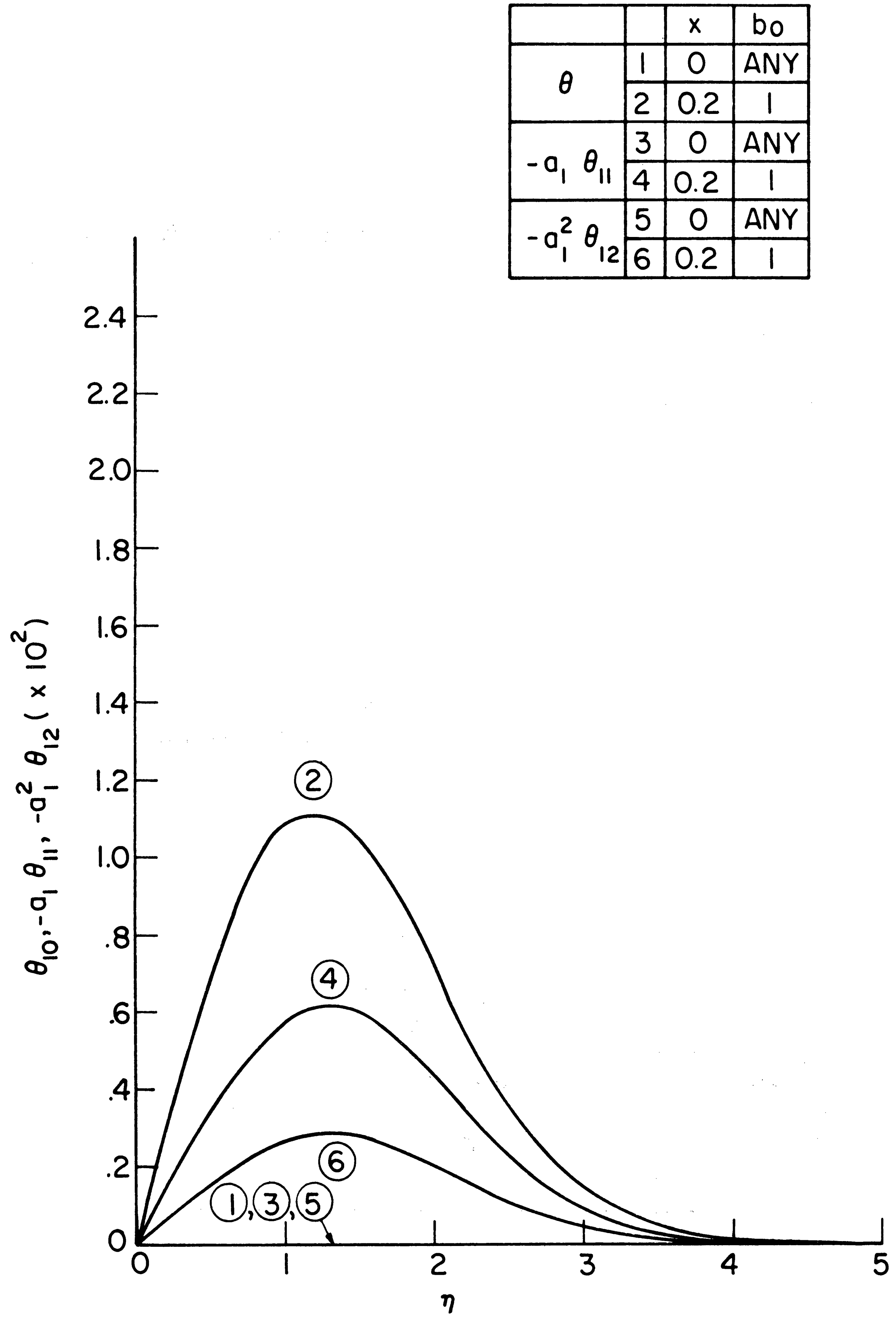


Fig. 20. Profiles of first-order temperatures or concentrations for flow past a circular cylinder in rotational oscillations for a fluid having Pr = 1 or Sc = 1.

	x	Pr	ω	V
1	0	ANY	ANY	ANY
2	0.2	1	0	0
3	0.2	1	0.2	0.1
4	0.2	1	0.2	0
5	0.2	1	0.2	-0.1
6	0.2	10	0.2	0

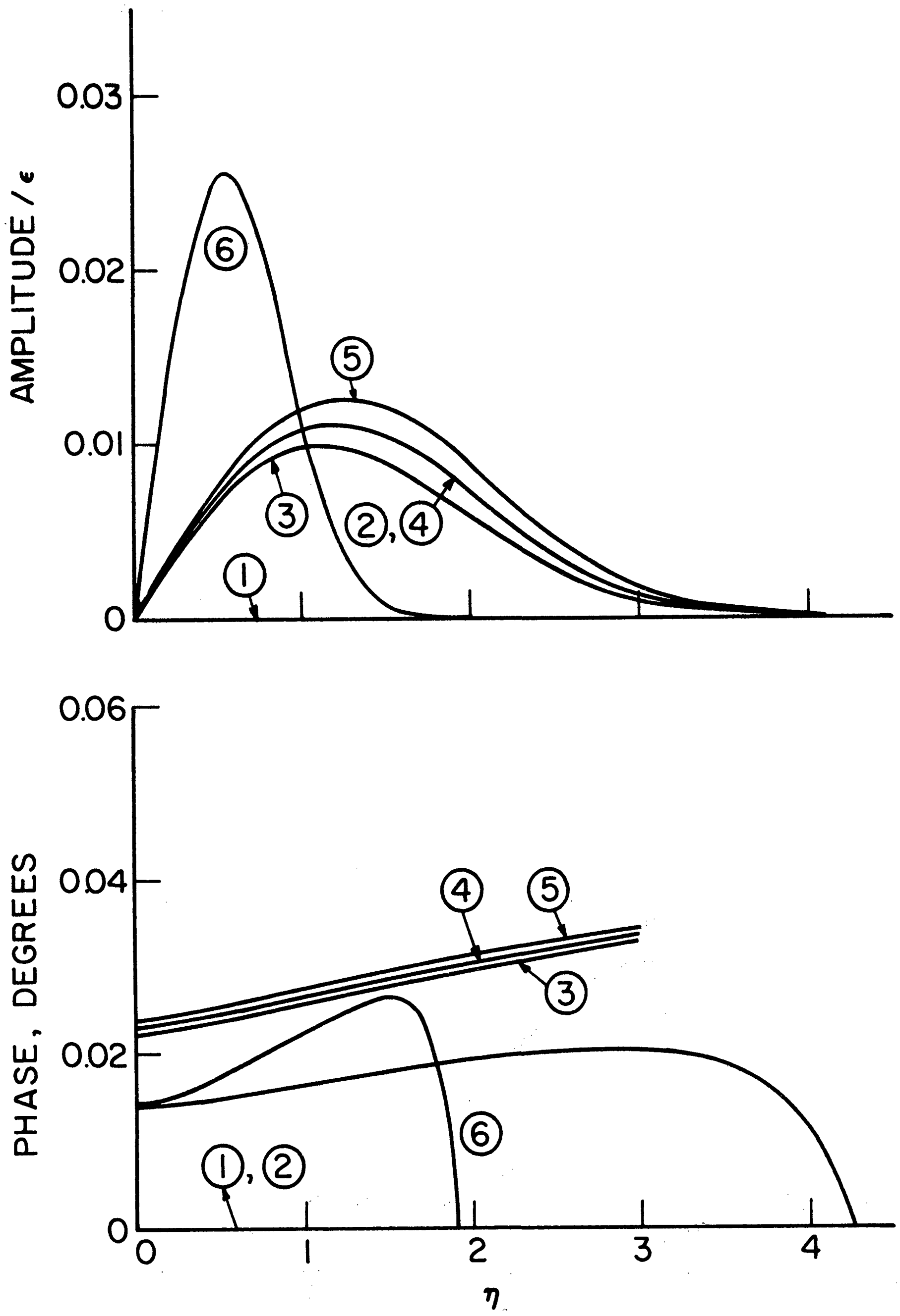


Fig. 21. Amplitude and phase of fluid temperature or concentration for flow past a circular cylinder in rotational oscillations.

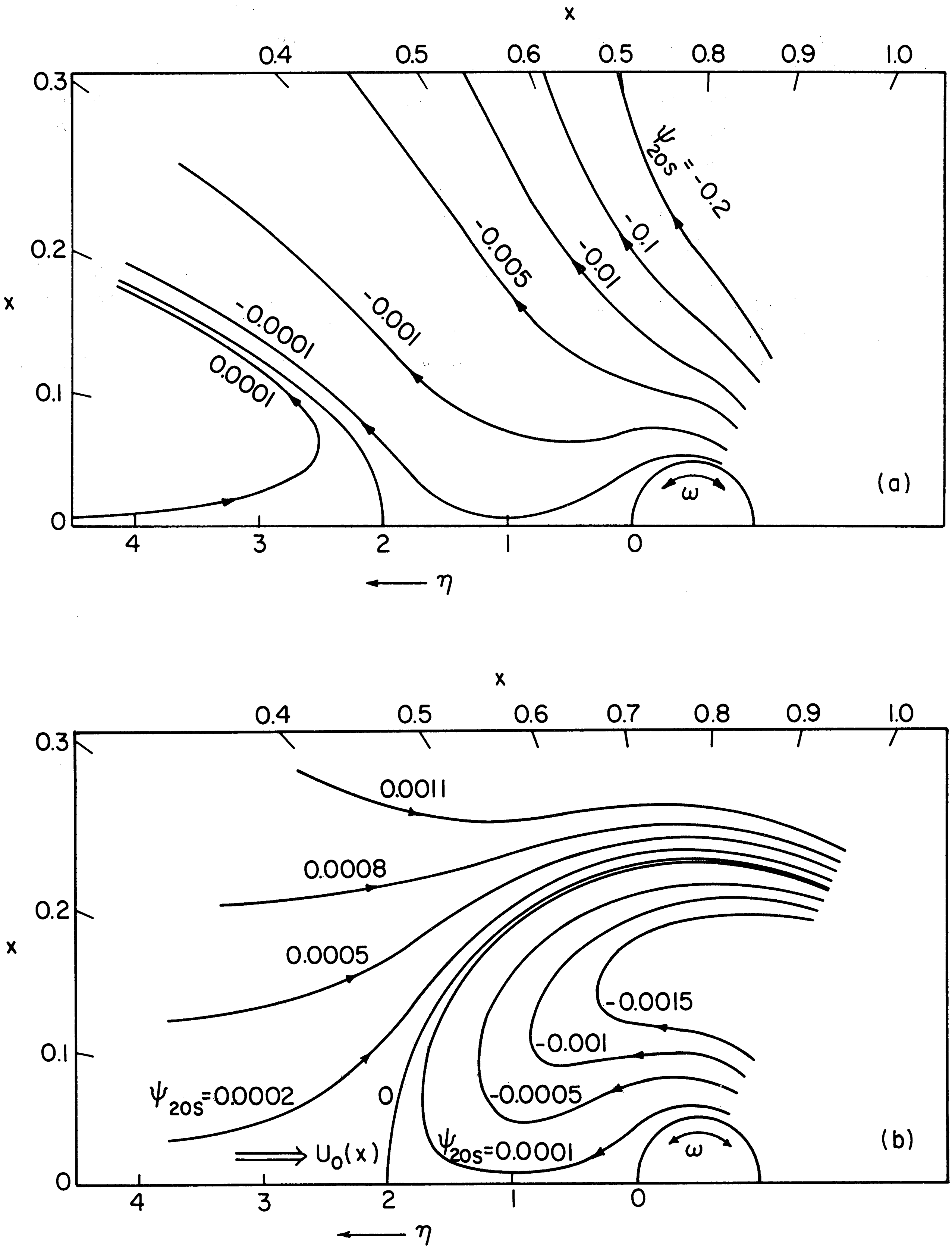


Fig. 22. Streamline patterns of the steady secondary motion produced by flow past a circular cylinder in rotational oscillations with amplitude $\epsilon U_1 = \epsilon$ at quasi-steady state.

direction as the main flow then turn around the direction at somwhere in the middle of the oscillatory boundary layer. Therefore an extra stagnation point is created at x = 0, but in the boundary layer. In spite of the difference in the direction of the steady secondary streamline, the temperature distribution of the steady secondary component as illustrated in Fig. 23 is similar in pattern to the fluctuating-circulation case. The isothermal lines which are negative values in magnitude fall closer to the surface along the surface. From the phenomena observed in Figs. 22 and 23, the alternations in both the local wall shear stress and Nusselt number tend to increase along the surface from the forward stagnation point as illustrated in Fig. 24. It is important to note that the rotational oscillation is the only case in which the net effect on the wall shear stress is negative. Again the superposition of the uniform suction tends to enhance the alternations in both the skin friction and heat transfer.

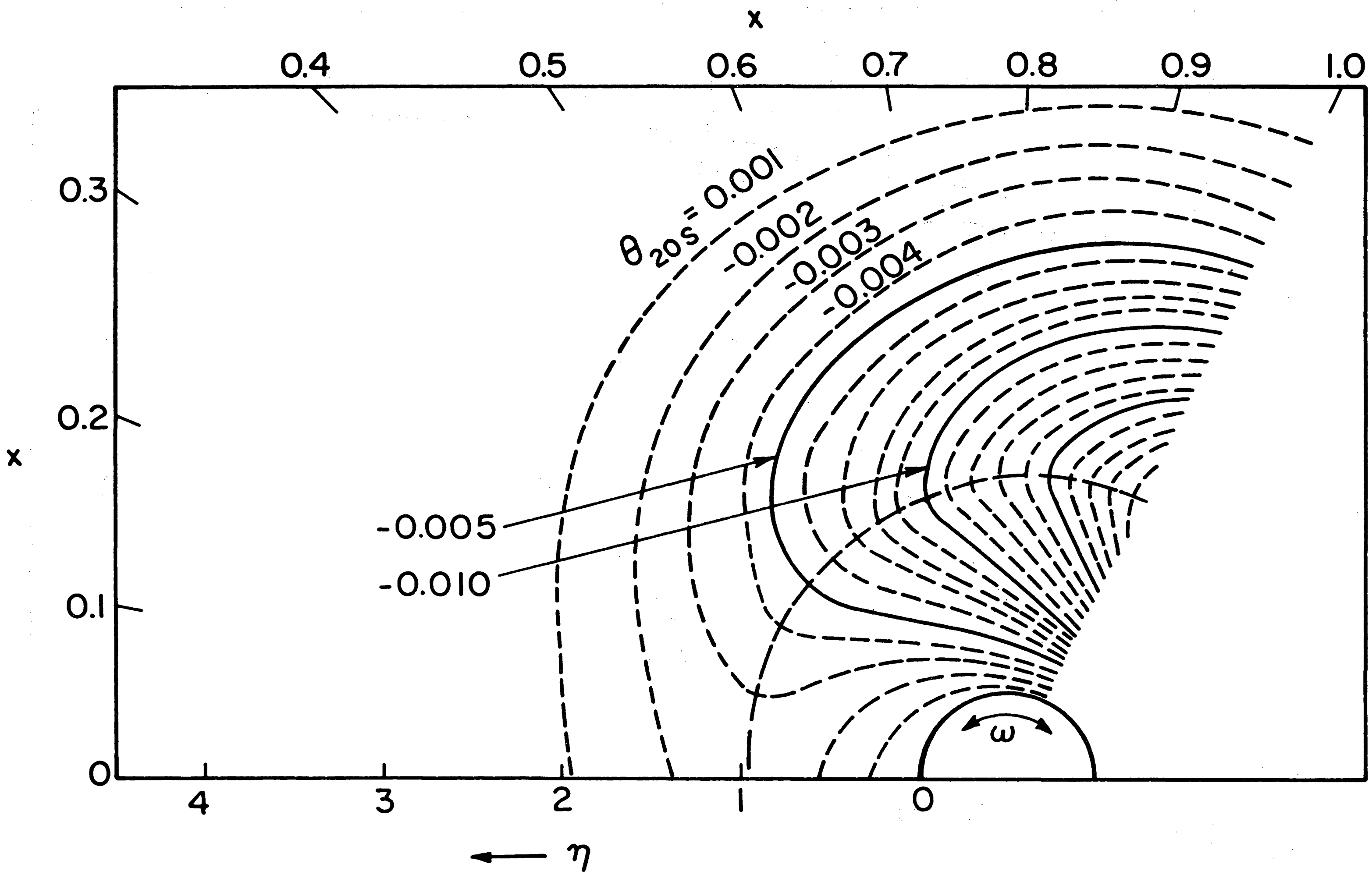


Fig. 23. Distribution of the steady secondary component of temperature or concentration produced by flow past a circular cylinder in rotational oscillations with amplitude $\epsilon U_1 = \epsilon$ for a fluid having Pr = 1 or Sc = 1 at quasi-steady state.

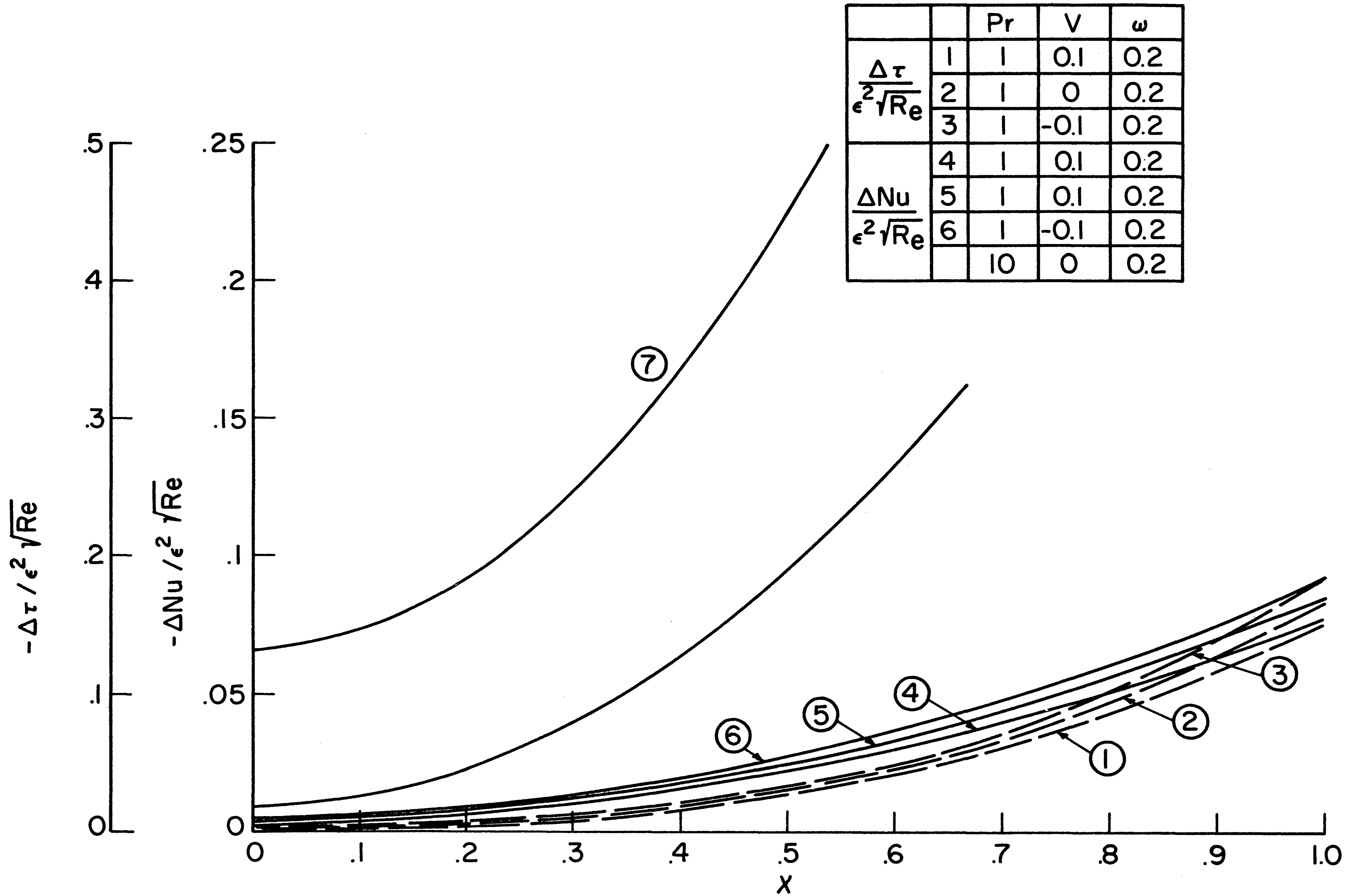


Fig. 24. Effects of rotational oscillations on the local wall shear stress and Nusselt number for flow past a circular cylinder.

V. SUMMARY OF RESULTS

The system of partial differential equations governing fluid motion, heat transfer and mass transfer was solved by a perturbation technique for the cases of fluid flow around a symmetrical blunt body with the flow oscillation and fluctuating circulation in free stream the rotational oscillation of cylinder surface. Flow around sharp-edged body is also analyzed for the flow-oscillation in free stream, although the analysis may be extended to the other two cases with ease. Numerical results for flow past a circular cylinder include the effects of the superposition of uniform suction or blowing.

It is disclosed from the analysis that an increase in the oscillating frequency results in (i) a slight change in the amplitudes of both velocity and temperature, an increase for the flow-oscillation case and a decrease for the rotational-oscillation case; (ii) a significant increase in the phase angle for all cases; and (iii) a slight decrease in the absolute value of the steady alternations in both the wall shear stress and Nusselt number for all cases.

The superposition of the uniform suction contributes to (i) an increase in both the amplitude of the first-order temperature and the phase lag of the first-order velocity and temperature but a decrease in the amplitude of the velocity profile; (ii) a decrease in the absolute value of the steady secondary alternations in the skin friction as well as heat transfer rate.

An increase in the Prandtl number is characterized by a steeper temperature profile in the neighborhood of the solid surface and a decrease in the thickness of thermal boundary layer. This results in an increase in the amplitude and phase lag of the first-order temperature as well as the alternation in the steady, secondary heat transfer rate.

At the forward stagnation point, the net change in the wall shear stress caused by oscillations is zero. Along the surface it increases to a maximum then decreases to zero at the separation point in case of flow oscillation. However for fluctuating-circulation and rotational-oscillation cases, the net change increases monotonically in magnitude along the surface. These changes are positive for the flow oscillation and fluctuation circulation but negative for the rotational oscillation. The alternation in the local Nusselt number is a negative finite value at the stagnation point. It decreases in magnitude along the surface to zero before the separation point and then changes to positive in the first case. For the other two cases a monomatic increase in magnitude is observed.

VI. REFERENCES

1. Lord Rayleigh, "On the Circulation of Air Observed in Kundt's Tubes, and some Allied Acoustic Problems", Phil. Trans., A, 175, pp 1-21 (1883).

2. H. Schlichting, "Berechnung ebener periodischer Grenzschichtstromungen", Phys. Z., 33, pp 327-35 (1932).

3. E. N. Andrade, "On the Circulations Caused by the Vibrations of Air in a Tube", Proc. Roy. Soc., A, 134, pp 445-70 (1931).

4. J. Holtsmark, I. Johnsen, T. Sikkeland and S. Skaulein, "Boundary Layer near a Cylindrical Obstacle in an Oscillating Incompressible Fluid", J. Acoust. Soc. Amer., 26 pp 102 (1954).

5. J. T. Stuart, "On Double Boundary Layers in Oscillatory Viscous Flow", (1960).

6. G. D. West, "Circulations Occurring in Acoustic Phenomena", Proc. Roy. Soc., B., B, 64, p 483 (1951).

7. P. J. Westervelt, "The Theory of Steady Rotational Flow Generated by a Sound Field", J. Acoust. Soc. Amer., 25, pp 60-67, (1953).

8. J. M. Andres and U. Ingard, "Acoustic Streaming at High Reynolds Numbers", J. Acoust. Soc. Amer., 25, pp 928-32 (1953).

9. W. L. Nyborg, "Acoustic Streaming due to Attenuated Plane Waves", J. Acoust. Soc. Amer., 25 pp 68-75 (1953).

10. W. L. Nyborg, "Acoustic Streaming Equations: Laws of Rotational Motion for Fluid Elements", J. Acoustic Soc. Amer., 25, pp 938-44, (1953).

11. L. A. Segel, "Application of Conformal Mapping to Viscous Flow Between Moving Circular Cylinders", Quart. Appl. Math., 18, pp 335-53 (1961).

12. M. J. Lighthill, "The Response of Laminar Skin Friction and Heat Transfer to Fluctuations in the Stream Velocity", Proc. Roy. Soc., Ser. A, vol. 224, pp. 1-23 (1954).

13. C. C. Lin, "Motion in the Boundary-Layer with a Rapidly Oscillating External Flow", Proc. 9th Int. Cong. Appl. Mech., Brussel, vol. 4, pp. 155-167 (1956).

14. W. E. Gibson, "Unsteady, Laminary Boundary-Layer Flow," Ph.D. Thesis in Mathematics, M.I.T. (1957).

15. E. Hori, "Unsteady Boundary Layers", Part I, Bulletin of Japan Soc. Mech. Eng., Vol. 4, No. 16, p. 664 (1961); Part II, Vol. 5, No. 17, p 57 (1962); Part III, Vol. 5, No. 17, p. 64 (1962); Part IV, Trans. Japan Soc. Mech. Eng., Vol. 27, No. 183, p. 1731, (1962).

16. J. T. Stuart, "A Solution of the Navier-Stokes and Energy E Equations Illustrating the Response of Skin Friction and Temperature of an Infinite Plate Thermometer to Fluctuations in the Stream Velocity", Proc. Roy. Soc. Ser. A, 231, pp 116-30 (1955).

17. F. K. Moore, "Unsteady Laminar Boundary-Layer Flow", NACA TN. No. 2471 (1951).

18. S. I. Cheng and D. Elliott, "The Unsteady Laminar Boundary Layer on a Flat Plate", Trans. Amer. Soc. Mech. Eng., 79 pp. 725-33 (1957).

19. C. R. Illingworth, "The Effects of a Sound Wave on the Compressible Boundary-Layer on a Flat Plate", J. Fluid Mech., 3, pp 471-93 (1958).

20. W. Wuest, "Grenzschichten and zylindrischen Korpern mit nichtstationarer Querbewegung", Z. angew. Math. Mech., 32, pp 172-78 (1952).

21. M. B. Blauert, "The Laminar Boundary Layer on Oscillating Plates and Cylinders", J. Fluid Mech., 1, pp 97-110 (1956).

22. N. Rott, "Unsteady Viscous Flow in the Vicinity of a Stagnation Point", Quart. Appl. Math., 13, pp 444-51 (1956).

23. T. W. Jackson, K. R. Purdy and C. C. Oliver, "The Effects of Resonant Acoustic Vibrations on the Nusselt Numbers for a Constant Temperature Horizontal Tube", 1961 Intern. Heat Transfer Conf., II, pp 483-89 (1961).

24. R. M. Fand and J. Kaye, "The Influence of Vertical Vibrations on Heat Transfer by Free Convection from a Horizontal Cylinder", 1961 Intern. Heat Transfer Conf., II, pp 490-98 (1961).

25. F. J. Bayley, P. A. Edwards and P. P. Singh, "The Effects of Flow Pulsations on Heat Transfer by Forced Convection from a Flat Plate," 1961 Intern. Heat Transfer Conf., II, pp 499-509 (1961).

26. J. Kestin, P. F. Maeder and H. H. Sogin, "The Influence of Turbulence on the Transfer of Heat to Cylinders near the Stagnation Point", Zert. angew Math. and Phys., Vol. 12, p 115 (1961).

27. J. Kestin, P. F. Maeder and H. E. Wang, "Influence of Turbulence on the Transfer of Heat from Plates with and without a Pressure Gradient", 1961 Intern. Heat Transfer Conference, ASME Part II, pp. 432 (1961).

28. J. Kestin, P. F. Maeder and H. E. Wang, "On Boundary Layers Associated with Oscillating Streams", Appl. Sci. Res., Sec. A., Vol. 10 (1961).

29. R. J. Schoenhals and J. A. Clark, "Laminar Free Convection Boundary Layer Perturbations Due to Transverse Wall Vibration", J. of Heat Transfer, Trans. ASME, Ser. C, Vol. 84, p. 174 (1962).

30. V. D. Blankenship and J. A. Clark, "Effects of Oscillation on Free Convection from a Vertical Finite Plate", ASME Paper No. 63-HT-31.

31. V. D. Blankenship and J. A. Clark, "Experimental Effects of Transverse Oscillations on Free Convection of a Vertical, Finite Plate", ASME Paper No. 63-WA-123, Presented at the 1963 Winter Annual Meeting of the ASME, to be published in the J. of Heat Transfer ASME, Series C.

32. V. D. Blankenship and J. A. Clark, "Laminar Free Convection from a Vertical Infinite Plate Subjected to Transverse Oscillation," ASME Paper No. (1964).

33. S. Eshghy, "The Effect of Longitudinal Oscillations on Fluid Flow and Heat Transfer from Vertical Surfaces in Free Convection", Ph.D. Thesis, Department of Mechanical Engineering, Univ. of Mich. (1963).

34. T. Y. Na, "Influence of Localized Oscillations on Laminar Flow Over a Flat Plate," Ph.D. Thesis in Mech. Engr., Univ. of Mich., (1964).

35. H. Schlichting, <u>Boundary Layer Theory</u>, 4th Ed., McGraw-Hill (1960).

36. L. Rosenhead, <u>Laminar Boundary Layer</u>, Oxford Press, (1963).

37. N. Frossling, "Evaporation, Heat Transfer, and Velocity Distribution in Two Dimensional and Rotationally Symmetrical Laminar Boundary-Layer Flow ", NACA TM 1432 (1958).

38. N. Frossling, "Problems of Heat Transfer Across Laminar Boundary Layers," <u>Theory and Fundamental Research in Heat Transfer</u>, Edited by John A. Clark, Pergamon Press, New York (1963) pp 181-202.

39. H. Blasius, "Grenzschichten in Flüssigkeiten mit Kliner Reibung," Z. Math. u. Phys., Vol. 56 (1908) and NACA TM 1256.

40. L. Howarth, "On the solution of the laminar boundary layer equations," Proc. Roy. Soc. London, Ser. A, Vol. 164, p. 547 (1938).

41. H. Görtler, "A new series for the evaluation of steady laminar boundary layer flows," J. Math. and Mech., Vol. 6, p. 1 (1957).

42. E. M. Sparrow, "Application of Gortler's series method to the boundary-layer energy equation," J. Aeronaut. Sci., Vol. 25, p. 71 (1958) and "The thermal boundary layer on a non-isothermal surface with nonuniform free stream velocity," J. Fluid Mech., Vol. 4, p. 321 (1958).

43. E. Wrage, "Übertragung der Gorthler'schen Reihe auf die Berechnung von Temperaturgrenzchichten, Teil I, DVL-Bericht Nr 81, (1958).

APPENDIX

A. THE COMPUTER PROGRAM FOR FLOW OSCILLATION IN FREE STREAM

```
$COMPILE MAD, EXECUTE, PRINT OBJECT, DUMP, PUNCH OBJECT
           PRINT COMMENT $ THE COMPUTER PROGRAM FOR OSCILLATING FLOW $.
           DIMENSION Y(3), F(3), Q(3)
           INTEGER J, I, N, M, K, Z, T, ZET, PRI
           VECTOR VALUES DEL=$1H ,F6.4,4F15.8*$
           VECTOR VALUES ROH=$1H ,F6.4,3F15.8*$
           VECTOR VALUES ALP=$1H ,F6.4,3F15.8*$
           VECTOR VALUES BET=$1H ,F6.4,12F10.6*$
           VECTOR VALUES EPS=$1H ,1F50.4*$
           DIMENSION P(45), Y1(45), Y2(45), F3(45), R(55),Y3(45),
          1US(45)TUT(45),UU(45),Q1(45),Q1S(45),Q2(45),Q2S(45),AMQS(45),
          2AMQ(45)TANQ(45),PHAQ(45),TS(45),TT(45),TU(45),S1(45),S1S(45),
          3S2(45)TS2S(45),AMSS(45),AMS(45),ANS(45),PHAS(45),YP3(10),
          4XY(10),WA(10),YP2(10),NTAU(10),W(10),NNUS(10),SSR(45),
          5SST(45)
           DIMENSION A(1260,AD),B(1260,BD),C(1260,CD),D(630,DD)
           VECTOR VALUES AD=2,1,45
           VECTOR VALUES BD=2,1,45
           VECTOR VALUES CD=2,1,45
           VECTOR VALUES DD=2,1,45
BEGIN      READ AND PRINT DATA
           EXECUTE SETRKD. (3, Y(1), F(1), Q, X, STEP)
           T=1
 SW6       ZET=0
 SW10      ZET=ZET+1
           WHENEVER ((-1).P.ZET).L.0, YX=YDP+ZET-1
           WHENEVER ((-1).P.ZET).G.0, YX=YDP-ZET+1
           R(0)=1.5
           R(1)=1
           N=1
 SW2       Q=CF*(R(N)/R(N-1)-1)
           WHENEVER Q.G.87.20, Q=87.20
           WHENEVER Q.L.(-87.20), Q=-87.20
           DX=-DY*((2.35040/(EXP.(Q)-EXP.(-Q)))+1)
           DY=DX
           N=N+1
           YX=YX+DY
           WHENEVER T.L.TMAX
           Y(3)=YX
           OTHERWISE
           Y(2)=YX
           END OF CONDITIONAL
           J=0
           X=0
           WHENEVER T.E.1,Y(1)=CONST
           WHENEVER T.G.1,Y(1)=0
           WHENEVER T.L.TMAX, Y(2)=0
 START     J=J+1
           P(J)=X
           Y1(J)=YU1)
           Y2(J)=Y(2)
           WHENEVER T.L.TMAX
           Y3(J)=Y(3)
           F3(J)=FU3)
           OTHERWISE
           F3(J)=F(2)
           END OF CONDITIONAL
 SW1       F(1)=Y(2)
           WHENEVER T.L.TMAX, F(2)=Y(3)
```

```
 WHENEVER T.E.1, F(3)=(Y(2).P.2)-Y(1)*Y(3)-1
 WHENEVER T.E.2, F(3)=4*B(1,J)*Y(2)-3*C(1,J)*Y(1)-A(1,J)*Y(3)-
11
 WHENEVER T.E.3, F(3)=B(1,J)+0.5*X*C(1,J)-1-A(1,J)*Y(3)+2*B(1,
1J)*Y(2)-Y(1)*C(1,J)
 WHENEVER T.E.4, F(3)=0.5*X*C(2,J)+B(2,J)-3*A(2,J)*C(3,J)+4*B(
12,J)*B(3,J)-C(2,J)*A(3,J)-0.25-A(1,J)*Y(3)+4.0*B(1,J)*Y(2)-3.
20*C(1,JD*Y(1)
 WHENEVER T.E.5, F(3)=2*B(1,J)*Y(2)-A(1,J)*Y(3)-C(1,J)*Y(1)+B(
13,J)
 WHENEVER T.E.6,F(3)=B(4,J)-3.*A(2,J)*C(5,J)+4.*B(2,J)*B(5,J)
1-A(5,J)MC(2,J)-A(1,J)*Y(3)+4.*B(1,J)*Y(2)-3.*C(1,J)*Y(1)
 WHENEVER T.E.7, F(3)=2*B(1,J)*Y(2)+(B(1,J)+0.5*X*C(1,J)).P.2-
1A(1,J)*0(3)-0.25*(A(1,J)+X*B(1,J))*(3.0*C(1,J)+X*D(1,J))-Y(1)
2*C(1,J)-1.0
 WHENEVER T.E.8, F(3)=B(2,J)*B(7,J)+3*B(1,J)*Y(2)+4*(B(1,J)+0.
15*X*C(1,J))*(B(2,J)+0.5*X*C(2,J))+B(1,J)*Y(2)-3*A(2,J)*C(7,J)
2-A(1,J)MY(3)-0.75*(3*C(1,J)+X*D(1,J))*(A(2,J)+X*B(2,J))-0.25
3*(A(1,J)+X*B(1,J))*(3.0*C(2,J)+X*D(2,J))-3.0*C(1,J)*Y(1)-A(7,
4J)*C(2,J)-1.+3.*B(2,J)*B(7,J)
 WHENEVER T.E.9, F(3)=2*B(7,J)+2*B(1,J)*Y(2)+B(3,J)*(B(1,J)+0.
15*X*C(1TJ))-A(1,J)*Y(3)-0.5*(A(1,J)+X*B(1,J))*C(3,J)-C(1,J)*Y
2(1)
 WHENEVER T.E.10, F(3)=2*B(8,J)+B(2,J)*B(9,J)+3*B(1,J)*Y(2)+B(
13,J)*(B(2,J)+0.5*X*C(2,J))+3*B(4,J)*(B(1,J)+0.5*X*C(1,J))+B(1
2,J)*Y(2D+3*B(2,J)*B(9,J)-A(1,J)*Y(3)-3*A(2,J)*C(9,J)-0.5*C(4,
3J)*(A(1,J)+X*B(1,J))-1.5*C(3,J)*(A(2,J)+X*B(2,J))-C(2,J)*A(9,
4J)-3.0*C(1,J)*Y(1)
 WHENEVER T.E.11, F(3)=2.0*B(1,J)*Y(2)-A(1,J)*Y(3)-C(1,J)*Y(1)
1+2.0*B(5,J)*(B(1,J)+0.5*X*C(1,J))-((B(3,J)).P.2)-0.5*C(5,J)*(
2A(1,J)+X*B(1,J))-0.5*A(5,J)*(3.0*C(1,J)+X*D(1,J))+A(3,J)*C(3,
3J)
 WHENEVER T.E.12, F(3)=4*B(2,J)*B(11,J)+4*B(1,J)*Y(2)-3*A(2,J)
1*C(11,JD-A(1,J)*Y(3)-3*C(1,J)*Y(1)-C(2,J)*A(11,J)+4*B(6,J)*(B
2(1,J)+0.5*X*C(1,J))+4*B(5,J)*(B(2,J)+0.5*X*C(2,J))-4*B(3,J)*B
3(4,J)-1C5*C(5,J)*(A(2,J)+X*B(2,J))-0.5*C(6,J)*(A(1,J)+X*B(1,J
4))-1.5MA(6,J)*(3.0*C(1,J)+X*D(1,J))-0.5*A(1,J)*(3.0*C(2,J)+X*
5D(2,J))+3.0*C(3,J)*A(4,J)+A(3,J)*C(4,J)
 WHENEVER T.E.13, F(3)=2.0*B(1,J)*Y(2)-A(1,J)*Y(3)-C(1,J)*Y(1)
1+2.0*B(5,J)*(B(1,J)+0.5*X*C(1,J))+((B(3,J)).P.2)-0.5*C(5,J)*(
2A(1,J)+X*B(1,J))-A(3,J)*C(3,J)+2.0*B(9,J)-0.5*A(5,J)*(3.0*C(1
3,J)+X*DU1,J))
 WHENEVER T.E.14, F(3)=-A(1,J)*Y(3)+B(2,J)*B(13,J)+4*B(1,J)*Y(
12)+3*B(2,J)*B(13,J)-3*A(2,J)*C(13,J)-3*C(1,J)*Y(1)+4*B(5,J)*(
2B(2,J)+0.5*X*C(2,J))+4*B(6,J)*(B(1,J)+0.5*X*C(1,J))+4*B(3,J)*
3B(4,J)-1.5*C(5,J)*(A(2,J)+     X*B(2,J))-0.5*C(6,J)*(A(1,J)+
4  X*B(1,J))-1.5*A(6,J)*(3*C(1,J)+X*D(1,J))-0.5*A(1,J)*(3*C(2,
5J)+X*D(2,J))-3*C(9,J)*A(4,J)-A(3,J)*C(4,J)+2*B(10,J)
 WHENEVER T.E.15, F(2)=-PR*A(1,J)*Y(2)
 WHENEVER T.E.16, F(2)=PR*(2.0*B(1,J)*Y(1)-3.0*A(2,J)*B(15,J)-
1A(1,J)*Y(2))
 WHENEVER T.E.17, F(2)=-PR*(A(1,J)*Y(2)+A(3,J)*B(15,J)-0.5*X*B
1(15,J))
 WHENEVER T.E.18, F(2)=PR*(2.0*B(1,J)*Y(1)+2.0*A(16,J)*B(3,J)-
13.0*A(2,J)*B(17,J)-B(16,J)*A(1,J)+0.5*X*B(16,J)-3.0*A(4,J)*B(
215,J)*A(3,J))
 WHENEVER T.E.19, F(2)=-PR*(A(5,J)*B(15,J)+A(1,J)*Y(2)-A(17,J)
1)
 WHENEVER T.E.20, F(2)=-PR*(3.0*A(2,J)*B(19,J)+3.0*A(6,J)*B(15
```

```
        1,J)+B(16,J)*A(5,J)-2.0*B(1,J)*Y(1)-2.0*B(5,J)*A(16,J)+A(1,J)*
        2Y(2)-A(18,J))
         WHENEVER T.E.21, F(2)=-PR*(A(1,J)*Y(2)+0.25*(B(15,J)+X*C(15,J
        1))*(A(1TJ)+X*B(1,J))+A(7,J)*B(15,J))
         WHENEVER T.E.22, F(2)=PR*(2.0*B(1,J)*Y(1)+B(16,J)*(B(1,J)+0.5
        1*X*C(1,J))+2.0*A(16,J)*B(7,J)-3.0*A(2,J)*B(21,J)-A(1,J)*Y(2)-
        20.75*(B(15,J)+X*C(15,J))*(A(2,J)+X*B(2,J))-0.25*(B(16,J)+X*C(
        316,J))*UA(1,J)+X*B(1,J))-3.0*A(8,J)*B(15,J)-A(7,J)*B(16,J))
         WHENEVER T.E.23, F(2)=-PR*(0.5*A(3,J)*(B(15,J)+X*C(15,J))+0.5
        1*B(17,JD*(A(1,J)+X*B(1,J))-X*B(15,J)+A(9,J)*B(15,J)+A(1,J)*Y(
        22))
         WHENEVER T.E.24, F(2)=-PR*(-2*B(1,J)*Y(1)-2*B(9,J)*A(16,J)+3*
        1A(2,J)MB(23,J)+A(1,J)*Y(2)+3*A(10,J)*B(15,J)+A(9,J)*B(16,J)-X
        2*B(1,J)MB(16,J)-2*A(18,J)*(B(1,J)+0.50*X*C(1,J))+1.5*A(4,J)*(
        3B(15,J)+X*C(15,J))+0.5*A(3,J)*(B(16,J)+X*C(16,J))+1.5*B(17,J)
        4*(A(2,JD+0.5*X*B(2,J))+0.5*B(16,J)*(A(1,J)+0.5*X*B(1,J))-X*B(
        516,J))
         WHENEVER T.E.25, F(2)=-0.5*PR*(2.0*A(1,J)*Y(2)+2.0*A(11,J)*B(
        115,J)+A(5,J)*(B(15,J)+X*C(15,J))+(A(1,J)+X*B(1,J))*B(19,J)-2.
        20*A(3,JD*B(17,J))
         WHENEVER T.E.26, F(2)=PR*(2.0*B(11,J)*A(16,J)-3.0*A(2,J)*B(25
        1,J)-A(1TJ)*Y(2)-3.0*A(12,J)*B(15,J)-A(11,J)*B(16,J)+2.0*B(1,J
        2)*Y(1)+2.0*(B(1,J)+0.5*X*C(1,J))*B(20,J) -2.0*B(3,J)*B(18,J)-
        31.5*A(6TJ)*(B(15,J)+X*C(15,J))-0.5*A(5,J)*(B(16,J)+X*C(16,J))
        4-2.0*(A(2,J)+X*B(2,J))*B(15,J)-0.5*(A(1,J)+X*B(1,J))*B(20,J)+
        53.0*A(4TJ)*B(17,J)+A(1,J)*B(18,J))
         WHENEVER T.E.27, F(2)=-0.5*PR*(A(5,J)*(B(15,J)+X*C(15,J))+(A(
        11,J)+X*B(1,J))*B(19,J)+2.0*A(3,J)*B(17,J)-A(23,J)+2.0*A(1,J)*
        2Y(2)+2.0*A(11,J)*B(15,J))
         WHENEVER T.E.28, F(2)=-PR*(-2.0*B(11,J)*A(16,J)+3.0*A(2,J)*B(
        127,J)+A(1,J)*Y(2)+3.0*A(12,J)*B(15,J)+A(11,J)*B(16,J)-2.0*B(1
        2,J)*Y(1D-X*B(5,J)*B(16,J)-2.0*(B(1,J)+0.5*X*C(1,J))*B(20,J)-2
        3.0*B(3,J)*B(18,J)+1.5*A(2,J)*(B(15,J)+X*C(15,J))+0.5*A(5,J)*(
        4B(16,J)+X*C(16,J))+3.0*A(4,J)*B(17,J)+A(3,J)*B(18,J)-0.5*A(24
        5,J)+0.5*(A(1,J)+X*B(1,J))*B(20,J) +2.0*(A(2,J)+X*B(2,J))*B(19
        6,J))
CALC     S=RKDEQ.(0)
         WHENEVER S.E.1.0, TRANSFER TO SW1
         WHENEVER J.E.GRSIZ, TRANSFER TO SW3
         TRANSFER TO START
SW3      WHENEVER T.E.1, TRANSFER TO SW8
         WHENEVER T.E.2, TRANSFER TO SW11
         WHENEVER 2.L.T .AND. T.L.TMAX, TRANSFER TO SW7
         WHENEVER T.E.TMAX, TRANSFER TO SW12
         WHENEVER T.G.TMAX, TRANSFER TO SW13
SW7      WHENEVER .ABS.Y(2).G.0.001 .AND. N.L.NMAX
         R(N)=Y(2)
         TRANSFER TO SW2
         OTHERWISE
         TRANSFER TO SW9
         END OF CONDITIONAL
SW8      WHENEVER .ABS.(Y(2)-1).G.0.001 .AND. N.L.NMAX
         R(N)=Y(2)-1
         TRANSFER TO SW2
         OTHERWISE
         TRANSFER TO SW9
         END OF CONDITIONAL
SW11     WHENEVER .ABS.(Y(2)-0.2500).G.0.001 .AND. N.L.NMAX
         R(N)=Y(2)-0.2500
```

```
          TRANSFER TO SW2
          OTHERWISE
          TRANSFER TO SW9
          END OF CONDITIONAL
SW12      WHENEVER .ABS.(Y(1)-1).G.0.001 .AND. N.L.NMAX
          R(N)=Y(1)-1
          TRANSFER TO SW2
          OTHERWISE
          TRANSFER TO SW15
          END OF CONDITIONAL
SW13      WHENEVER .ABS.Y(1).G.0.001 .AND. N.L.NMAX
          R(N)=Y(1)
          TRANSFER TO SW2
          OTHERWISE
          TRANSFER TO SW15
          END OF CONDITIONAL
SW15      WHENEVER .ABS. Y(2).G.0.001 .AND. ZET.L.ZMAX, TRANSFER TO SW1
         10
SW9       WHENEVER .ABS. Y(3).G.0.001 .AND. ZET.L.ZMAX, TRANSFER TO SW1
         10
          PRINT RESULTS T
          PRINT RESULTS ZET
          WHENEVER ZET.E.ZMAX, PRINT COMMENT $ EQ. CAN NOT BE MADE TO S
         1ATISIFY BOUNDARY CONDITIONS $
          WHENEVER PR.G.5.,TRANSFER TO SW40
          TRANSFER TO SW41
SW40      WHENEVER N.E.NMAX,GRSIZ=GRSIZ-5
          WHENEVER N.E.NMAX .AND.GRSIZ.L.25,TRANSFER TO SW14
SW41      WHENEVER N.E.NMAX,TRANSFER TO SW14
          PRINT COMMENT $ R(M) $
          THROUGH SW5, FOR M=2, 1, M.G.N
SW5       PRINT FORMAT EPS, R(M)
          THROUGH SW4, FOR I=1, 1, I.G.GRSIZ
          A(T,I)=Y1(I)
          B(T,I)=Y2(I)
          WHENEVER T.L.TMAX
          C(T,I)=Y3(I)
          D(T,I)=F3(I)
          OTHERWISE
          C(T,I)=F3(I)
          END OF CONDITIONAL
          WHENEVER T.L.TMAX
          PRINT FORMAT DEL, P(I), Y1(I), Y2(I), Y3(I), F3(I)
          OTHERWISE
          PRINT FORMAT ROH, P(I), Y1(I), Y2(I), F3(I)
SW4       END OF CONDITIONAL
          WHENEVER T.L.TMAX
          PRINT FORMAT DEL, X, Y(1), Y(2), Y(3), F(3)
          OTHERWISE
          PRINT FORMAT ROH, X, Y(1), Y(2), F(2)
          END OF CONDITIONAL
          Z=GRSIZ
          A(T,Z)=Y(1)
          B(T,Z)=Y(2)
          WHENEVER T.L.TMAX
          C(T,Z)=Y(3)
          D(T,Z)=F(3)
          OTHERWISE
          C(T,Z)=F(2)
```

```
            END OF CONDITIONAL
            I=I+1
            WHENEVER T.E.TMAX, EXECUTE SETRKD. (2, Y(1), F(1), Q, X, STEP
           1)
            WHENEVER T.LE.TEND,TRANSFER TO SW6
            THROUGH SW21,FOR J=1,1,J.G.JMAX
            THROUGH SW21,FOR K=1,1,K.G.KMAX
            THROUGH SW20,FOR I=1,1,I.G.GRSIZ
            YP3(J)=4.*A3*(XY(J).P.3)
            S=STEP*(I-1)
            P(I)=S
            US(I)=A1*XY(J)*(B(1,I)+0.5*S*C(1,I))+YP3(J)*(B(2,I)+0.5*S*C(2
           1,I))
            UT(I)=A1*XY(J)*B(3,I)+YP3(J)*B(4,I)
            UU(I)=A1*XY(J)*B(5,I)+YP3(J)*B(6,I)
            WA(K)=WUK)*W(K)/(A1*A1)
            Q1(I)=WA(K)*UU(I)-US(I)
            Q1S(I)=Q1(I)*Q1(I)
            Q2(I)=W(K)*UT(I)/A1
            Q2S(I)=-2(I)*Q2(I)
            AMQS(I)=Q1S(I)+Q2S(I)
            AMQ(I)=SQRT.(AMQS(I))
            ANQ(I)=Q2(I)/Q1(I)
            PHAQ(I)=ATAN.(ANQ(I))*57.2958
            A1S=0.5*SQRT.(A1)
            YP2(J)=4.*A3*XY(J)*XY(J)/A1
            SSR(I)=2.*A1S*XY(J)*(A(7,I)+YP2(J)*A(8,I))
            TS(I)=0C5*S*(B(15,I)+YP2(J)*B(16,I))
            TT(I)=A(17,I)+YP2(J)*A(18,I)
            TU(I)=AU19,I)+YP2(J)*A(20,I)
            S1(I)=WA(K)*TU(I)-TS(I)
            S1S(I)=S1(I)*S1(I)
            S2(I)=W(K)*TT(I)/A1
            S2S(I)=S2(I)*S2(I)
            AMSS(I)=S1S(I)+S2S(I)
            AMS(I)=SQRT.(AMSS(I))
            ANS(I)=S2(I)/S1(I)
            PHAS(I)=ATAN.(ANS(I))*57.2958
            SST(I)=A(21,I)+YP2(J)*A(22,I)
SW20        PRINT FORMAT BET,P(I),US(I),UT(I),UU(I),AMQ(I),PHAQ(I),SSR(I)
           1,TS(I),TT(I),TU(I),AMS(I),PHAS(I),SST(I)
            NTAU(K)3A1S*(A1*XY(J)*C(7,1)+YP3(J)*C(8,1)-WA(K)*(A1*XY(J)*
           1C(11,1)+YP3(J)*C(12,1)))
            NNUS(K)3A1S*(B(21,1)+YP2(J)*B(22,1)-WA(K)*(B(25,1)+YP2(J)*B(
           126,1)))
SW21        PRINT FORMAT ALP,XY(J),W(K),NTAU(K),NNUS(K)
            TRANSFER TO BEGIN
SW14        END OF PROGRAM
$DATA
ZMAX=6,TMAX=15,TEND=28,CF=1,NMAX=50,STEP=0.1,GRSIZ=45,PR=1.,
DY=0.001,YDP=1.000,CONST=-0.1,A1=4.,A3=-2.6666,XY(1)=0,XY(2)=0.05,
XY(3)=0.1,XY(4)=0.2,XY(5)=0.3,XY(6)=0.4,XY(7)=0.5,XY(8)=0.8,JMAX=8,
W(1)=0,KMAX=1 *
```

B. THE COMPUTER PROGRAM FOR FLUCTUATING CIRCULATION IN FREE STREAM

```
$COMPILE MAD, EXECUTE, PRINT OBJECT, DUMP, PUNCH OBJECT
           PRINT COMMENT $ THE COMPUTER PROGRAM FOR FLUCTUATING CIRCULAT
           ION $
           DIMENSION Y(3), F(3), Q(3)
           INTEGER J, I, N, M, K, Z, T, ZET, PRI
           VECTOR VALUES DEL=$1H ,F6.4,4F15.8*$
           VECTOR VALUES ROH=$1H , F6.4, 3F15.8*$
           VECTOR VALUES ALP=$1H ,F6.4, 3F15.8*$
           VECTOR VALUES BET=$1H ,F6.4,12F10.6*$
           VECTOR VALUES EPS=$1H , 1F20.4*$
           DIMENSION P(45), Y1(45), Y2(45), F3(45), R(55), Y3(45),
          1US(45)TUT(45),UU(45),Q1(45),Q1S(45),Q2(45),Q2S(45),AMQS(45),
          2AMQ(45)TANQ(45),PHAQ(45),TS(45),TT(45),TU(45),S1(45),S1S(45),
          3S2(45),S2S(45),AMSS(45),AMS(45),ANS(45),PHAS(45),YP3(10),
          4XY(10),WA(10),YP2(10),NTAU(10),W(10),NNUS(10),SSR(45),A2(10),
          5YP1(10),YP4(10),A14(10),A4(10),A12(10),XYS(10),SST(45)
           DIMENSION A(2565,AD),B(2565,BD),C(2565,CD),D(2565,DD)
           VECTOR VALUES AD=2, 1, 45
           VECTOR VALUES BD=2, 1, 45
           VECTOR VALUES CD=2, 1, 45
           VECTOR VALUES DD=2, 1, 45
BEGIN      READ AND PRINT DATA
           EXECUTE SETRKD. (3, Y(1), F(1), Q, X, STEP )
           T=1
SW6        ZET=0
SW10       ZET=ZET+1
           WHENEVER ((-1).P.ZET).L.0, YX=YDP+ZET-1
           WHENEVER ((-1).P.ZET).G.0, YX=YDP-ZET+1
           R(0)=1.5
           R(1)=1
           N=1
SW2        Q=CF*(R(N)/R(N-1)-1)
           WHENEVER Q.G.87.20, Q=87.20
           WHENEVER Q.L.(-87.20), Q=-87.20
           DX=-DY*U(2.35040/(EXP.(Q)-EXP.(-Q))+1)
           DY=DX
           N=N+1
           YX=YX+DY
           WHENEVER T.L.TMAX
           Y(3)=YX
           OTHERWISE
           Y(2)=YX
           END OF CONDITIONAL
           J=0
           X=0
           WHENEVER T.E.1, Y(1)=CONST
           WHENEVER T.G.1, Y(1)=0
           WHENEVER T.L.TMAX, Y(2)=0
START      J=J+1
           P(J)=X
           Y1(J)=YU1)
           Y2(J)=Y(2)
           WHENEVER T.L.TMAX
           Y3(J)=Y(3)
           F3(J)=FU3)
           OTHERWISE
           F3(J)=F(2)
           END OF CONDITIONAL
SW1        F(1)=Y(2)
```

```
 WHENEVER T.L.TMAX, F(2)=Y(3)
 WHENEVER T.E.1, F(3)=(Y(2).P.2)-Y(1)*Y(3)-1
 WHENEVER T.E.2, F(3)=4*B(1,J)*Y(2)-3*C(1,J)*Y(1)-A(1,J)*Y(3)-
11
 WHENEVER T.E.3, F(3)=B(1,J)*Y(2)-A(1,J)*Y(3)-1
 WHENEVER T.E.4, F(3)=3*B(1,J)*Y(2)-2*C(1,J)*Y(1)-A(1,J)*Y(3)-
11
 WHENEVER T.E.5, F(3)=3*B(1,J)*Y(2)-2*C(1,J)*Y(1)-A(1,J)*Y(3)-
11+4*(B(2,J)*B(3,J)-A(2,J)*C(3,J))
 WHENEVER T.E.6, F(3)=B(1,J)*Y(2)-A(1,J)*Y(3)+B(3,J)-1
 WHENEVER T.E.7, F(3)=3*B(1,J)*Y(2)-2*C(6,J)*Y(1)-A(1,J)*Y(3)+
1B(4,J)-0.333
 WHENEVER T.E.8, F(3)=3*B(1,J)*Y(2)-2*C(1,J)*Y(1)-A(1,J)*Y(3)+
1B(5,J)+4.0*(B(2,J)*B(6,J)-A(2,J)*C(6,J))
 WHENEVER T.E.9, F(3)=B(1,J)*Y(2)-A(1,J)*Y(3)+B(6,J)
 WHENEVER T.E.10, F(3)=3*B(1,J)*Y(2)-2*C(1,J)*Y(1)-A(1,J)*Y(3)
1+B(7,J)
 WHENEVER T.E.11, F(3)=3*B(1,J)*Y(2)-2*C(1,J)*Y(1)-A(1,J)*Y(3)
1+B(8,J)+4*(B(2,J)*B(9,J)-A(2,J)*C(9,J))
 WHENEVER T.E.12, F(3)=2.0*B(1,J)*Y(2)+2.0*B(3,J)*B(4,J)-A(1,J
1)*Y(3)+2.0*C(3,J)*A(4,J)-C(1,J)*Y(1)-0.666
 WHENEVER T.E.13, F(3)=2.0*B(1,J)*Y(2)+2.0*B(3,J)*B(5,J)-A(1,J
1)*Y(3)+2.0*C(3,J)*A(5,J)-C(1,J)*Y(1)
 WHENEVER T.E.14, F(3)=2.0*B(1,J)*Y(2)+2.0*B(12,J)+2.0*B(3,J)*
1B(7,J)-A(1,J)*Y(3)-2.0*C(6,J)*A(4,J)-C(1,J)*Y(1)
 WHENEVER T.E.15, F(3)=2.0*B(13,J)+2.0*B(1,J)*Y(2)+2.0*B(3,J)*
1B(8,J)-A(1,J)*Y(3)-2.0*C(6,J)*A(5,J)-C(1,J)*Y(1)
 WHENEVER T.E.16, F(3)=2.0*B(1,J)*Y(2)-A(1,J)*Y(3)-C(1,J)*Y(1)
1+B(3,J)MB(10,J)+B(9,J)*B(4,J)-B(6,J)*B(7,J)-C(9,J)*A(4,J)-C(3
2,J)*A(10,J)+C(6,J)*A(7,J)
 WHENEVER T.E.17, F(3)=2.0*B(1,J)*Y(2)-A(1,J)*Y(3)-C(1,J)*Y(1)
1+B(3,JD*B(11,J)+B(9,J)*B(5,J)-B(6,J)*B(8,J)-C(9,J)*A(5,J)-C(3
2,J)*A(11,J)+C(6,J)*A(8,J)
 WHENEVER T.E.18, F(3)=2.0*B(1,J)*Y(2)-A(1,J)*Y(3)-C(1,J)*Y(1)
1+B(3,J)MB(10,J)+B(9,J)*B(4,J)+B(6,J)*B(7,J)-C(9,J)*A(4,J)-C(3
2,J)*A(10,J)-C(6,J)*A(7,J)+B(14,J)
 WHENEVER T.E.19, F(3)=2.0*B(1,J)*Y(2)-A(1,J)*Y(3)-C(1,J)*Y(1)
1+B(3,J)*B(11,J)+B(9,J)*B(5,J)+B(6,J)*B(8,J)-C(9,J)*A(5,J)-C(3
2,J)*A(11,J)-C(6,J)*A(8,J)+B(15,J)
 WHENEVER T.E.20,F(3)=6.*B(1,J)*Y(2)-5.*C(1,J)*Y(1)-A(1,J)*Y(3
1)-1.
 WHENEVER T.E.21,F(3)=6.*B(1,J)*Y(2)+8.*B(2,J)*B(2,J)-5.*C(1,J
1)*Y(1)- .*A(2,J)*C(2,J)-A(1,J)*Y(3)-0.5
 WHENEVER T.E.22,F(3)=5.*B(1,J)*Y(2)-A(1,J)*Y(3)-4.*C(1,J)*Y(1
1)-1.
 WHENEVER T.E.23,F(3)=5.*B(1,J)*Y(2)+12.*B(2,J)*B(4,J)-A(1,J)
1*Y(3)-7C2*A(2,J)*C(4,J)-4.*C(1,J)*Y(1)-4.8*C(2,J)*A(4,J)-1.
 WHENEVER T.E.24,F(3)=5.*B(1,J)*Y(2)+6.*B(3,J)*B(20,J)-A(1,J)*
1Y(3)-4.MC(1,J)*Y(1)-1.
 WHENEVER T.E.25,F(3)=5.*B(1,J)*Y(2)+12.*B(2,J)*B(5,J)+6.*B(3,
1J)*B(21TJ)-A(1,J)*Y(3)-7.2*A(2,J)*C(5,J)-4.*C(1,J)*Y(1)-4.8
2*C(2,J)*A(5,J)
 WHENEVER T.E.26,F(3)=4.*B(1,J)*Y(2)+3.3333*B(3,J)*B(22,J)-A(1
1,J)*Y(3)-3.3333*C(3,J)*A(22,J)-3.*C(1,J)*Y(1)-0.6667
 WHENEVER T.E.27,F(3)=4.*B(1,J)*Y(2)+3.*B(4,J)*B(4,J)-A(1,J)*
1Y(3)-3.*A(4,J)*C(4,J)-3.*C(1,J)*Y(1)-0.3333
 WHENEVER T.E.28,F(3)=8.*B(2,J)*B(12,J)+4.*B(1,J)*Y(2)+3.3333*
1B(3,J)MB(23,J)+6.*B(4,J)*B(5,J)-A(1,J)*Y(3)-6.*A(2,J)*C(12,J)
2-3.*A(4TJ)*C(5,J)-3.*C(3,J)*A(5,J)-3.3333*C(3,J)*A(23,J)-2.*C
```

```
3(2,J)*AU12,J)-3.*C(1,J)*Y(1)
 WHENEVER T.E.29.F(3)=8.*B(2,J)*B(13,J)+4.*B(1,J)*Y(2)+3.3333*
1B(3,J)*B(25,J)+3.*B(5,J)*B(5,J)-A(1,J)*Y(3)-6.*A(2,J)*C(13,J)
2-3.*C(5,J)*A(5,J)-3.3333*C(3,J)*A(25,J)-2.*C(2,J)*A(13,J)-3.*
3C(1,J)*Y(1)
 WHENEVER T.E.30.F(3)=4.*B(1,J)*Y(2)+3.3333*B(3,J)*B(24,J)-A(1
1,J)*Y(3D-3.3333*C(3,J)*A(24,J)-3.0*C(1,J)*Y(1)
 WHENEVER T.E.31. F(2)=-PR*A(1,J)*Y(2)
 WHENEVER T.E.32, F(2)=PR*(2.0*B(1,J)*Y(1)-3.0*A(2,J)*B(31,J)-
1A(1,J)*Y(2))
 WHENEVER T.E.33, F(2)=PR*(B(1,J)*Y(1)-A(1,J)*Y(2)-3.0*A(4,J)*
1B(31,J))
 WHENEVER T.E.34, F(2)=PR*(B(1,J)*Y(1)+4.0*B(3,J)*A(32,J)-A(1,
1J)*Y(2)-3.0*A(5,J)*B(31,J))
 WHENEVER T.E.35, F(2)=PR*(B(1,J)*Y(1)-A(1,J)*Y(2)-3.0*A(7,J)*
1B(31,J))
 WHENEVER T.E.36, F(2)=PR*(B(1,J)*Y(1)+4.0*B(6,J)*A(32,J)-A(1,
1J)*Y(2)-3.0*A(8,J)*B(31,J))
 WHENEVER T.E.37, F(2)=PR*(B(1,J)*Y(1)-A(1,J)*Y(2)-3.0*A(10,J)
1*B(31,J))
 WHENEVER T.E.38, F(2)=PR*(B(1,J)*Y(1)+4.0*B(9,J)*A(32,J)-A(1,
1J)*Y(2)-3.0*A(11,J)*B(31,J))
 WHENEVER T.E.39, F(2)=PR*(0.666*B(3,J)*A(33,J)-A(1,J)*Y(2)-A(
112,J)*B(31,J))
 WHENEVER T.E.40, F(2)=PR*(0.666*B(3,J)*A(34,J)-A(1,J)*Y(2)-A(
113,J)*B(31,J))
 WHENEVER T.E.41, F(2)=PR*(0.666*B(6,J)*A(33,J)+0.666*B(3,J)*A
1(35,J)+2.0*A(39,J)-A(1,J)*Y(2)-A(14,J)*B(31,J))
 WHENEVER T.E.42, F(2)=PR*(0.666*B(6,J)*A(34,J)+0.666*B(3,J)*A
1(36,J)+2.0*A(40,J)-A(1,J)*Y(2)-A(15,J)*B(31,J))
 WHENEVER T.E.43, F(2)=PR*(B(9,J)*A(33,J)+B(3,J)*A(37,J)-B(6,J
1)*A(35,J)-A(1,J)*Y(2)-3.0*A(16,J)*B(31,J))
 WHENEVER T.E.44, F(2)=PR*(B(9,J)*A(34,J)+B(3,J)*A(38,J)-B(6,J
1)*A(36,J)-A(1,J)*Y(2)-3.0*A(17,J)*B(31,J))
 WHENEVER T.E.45,F(2)=PR*(B(9,J)*A(33,J)+B(3,J)*A(37,J)+B(6,J)
1*A(35,J)-A(1,J)*Y(2)-3.*A(16,J)*B(31,J)+3.*A(41,J))
 WHENEVER T.E.46,F(2)=PR*(B(9,J)*A(34,J)+B(3,J)*A(38,J)+B(6,J)
1*A(36,J)-A(1,J)*Y(2)-3.*A(17,J)*B(31,J)+3.*A(42,J))
 WHENEVER T.E.47,F(2)=PR*(-A(1,J)*Y(2)+4.*B(1,J)*Y(1)-5.*A(20,
1J)*B(31,J))
 WHENEVER T.E.48,F(2)=PR*(-A(1,J)*Y(2)+4.*B(1,J)*Y(1)-5.*A(21,
1J)*B(31,J)+2.6667*(2.*B(2,J)*A(32,J)-3.*A(2,J)*B(32,J)))
 WHENEVER T.E.49,F(2)=PR*(3.*B(1,J)*Y(1)-A(1,J)*Y(2)-5.*B(31,J
1)*A(22,J))
 WHENEVER T.E.50,F(2)=PR*(2.*B(2,J)*A(33,J)+3.*B(1,J)*Y(1)+6.*
1A(32,JD*B(4,J)-6.*A(2,J)*B(33.J)-A(1,J)*Y(2)-6.*A(4,J)*B(32,J
2)-5.*A(23,J)*B(31,J))
 WHENEVER T.E.51,F(2)=PR*(2.*B(2,J)*A(34,J)+3.0*B(1,J)*Y(1)+6.
1*B(3,J)MA(48,J)+6.*B(5,J)*A(32,J)-6.*A(2,J)*B(34,J)-A(1,J)*Y(
22)-6.*B(32,J)*A(5,J)-5.*A(25,J)*B(31,J))
 WHENEVER T.E.52,F(2)=PR*(3.*B(1,J)*Y(1)+6.*B(3,J)*A(47,J)-A(1
1,J)*Y(2)-5.*B(31,J)*A(24,J))
 WHENEVER T.E.53,F(2)=PR*(2.*B(1,J)*Y(1)+B(5,J)*A(33,J)+B(4,J)
1*A(34,J)+4.*B(12,J)*A(32,J)-6.*A(2,J)*B(39,J)-A(1,J)*Y(2)-2.*
2A(5,J)*B(33,J)-2.*A(12,J)*B(32,J)-2.*A(4,J)*B(34,J)+2.*B(3,J)
3*A(50,J)-3.*A(28,J)*B(31,J))
 WHENEVER T.E.54,F(2)=PR*(2.*B(1,J)*Y(1)+B(4,J)*A(33,J)-A(1,J)
1*Y(2)-2.*A(4,J)*B(33,J)-3.*A(27,J)*B(31,J))
 WHENEVER T.E.55,F(2)=PR*(2.*B(1,J)*Y(1)+B(5,J)*A(34,J)+4.*B(1
```

```
          13,J)*A(32,J)-6.*A(2,J)*B(40,J)-A(1,J)*Y(2)-2.*A(5,J)*B(34,J)-
          22.*A(13,J)*B(32,J)+2.*B(3,J)*A(51,J)-3.*A(29,J)*B(31,J))
           WHENEVER T.E.56,F(2)=PR*(2.*B(1,J)*Y(1)-A(1,J)*Y(2)+2.*B(3,J)
          1*A(49,J)-3.*A(26,J)*B(31,J))
           WHENEVER T.E.57,F(2)=PR*(2.*B(1,J)*Y(1)-A(1,J)*Y(2)+2.*B(3,J)
          1*A(52,J)-3.*A(30,J)*B(31,J))
CALC       S=RKDEQ.(0)
           WHENEVER S.E.1.0, TRANSFER TO SW1
           WHENEVER J.E.GRSIZ, TRANSFER TO SW3
           TRANSFER TO START
SW3        WHENEVER T.E.1, TRANSFER TO SW8
           WHENEVER T.E.2, TRANSFER TO SW7
           WHENEVER T.E.3, TRANSFER TO SW8
           WHENEVER T.E.4, TRANSFER TO SW16
           WHENEVER T.G.4 .AND.T.L.20,TRANSFER TO SW11
           WHENEVER T.E.20,TRANSFER TO SW17
           WHENEVER T.E.21,TRANSFER TO SW11
           WHENEVER T.E.22,TRANSFER TO SW18
           WHENEVER T.G.22 .AND.T.L.TMAX,TRANSFER TO SW11
           WHENEVER T.E.TMAX,TRANSFER TO SW12
           WHENEVER T.G.TMAX,TRANSFER TO SW13
SW7        WHENEVER .ABS.(Y(2)-.25).G.0.001 .AND. N.L.NMAX
           R(N)=(YU2)-.25)
           TRANSFER TO SW2
           OTHERWISE
           TRANSFER TO SW9
           END OF CONDITIONAL
SW8         WHENEVER .ABS. (Y(2)-1).G.0.001 .AND. N.L.NMAX
           R(N)=Y(2)-1
           TRANSFER TO SW2
           OTHERWISE
           TRANSFER TO SW9
           END OF CONDITIONAL
SW11       WHENEVER .ABS. Y(2).G.0.001 .AND. N.L.NMAX
           R(N)=Y(2)
           TRANSFER TO SW2
           OTHERWISE
           TRANSFER TO SW9
           END OF CONDITIONAL
SW16       WHENEVER .ABS.(Y(2)-0.333).G.0.001 .AND. N.L.NMAX
           R(N)=Y(2)-0.333
           TRANSFER TO SW2
           OTHERWISE
           TRANSFER TO SW9
           END OF CONDITIONAL
SW12       WHENEVER .ABS. (Y(1)-1 ).G.0.001 .AND.N.L.NMAX
           R(N)=Y(1)-1
           TRANSFER TO SW2
           OTHERWISE
           TRANSFER TO SW15
           END OF CONDITIONAL
SW13       WHENEVER .ABS.Y(1).G.0.001    .AND.N.L.NMAX
           R(N)=Y(1)
           TRANSFER TO SW2
           OTHERWISE
           TRANSFER TO SW15
           END OF CONDITIONAL
SW17       WHENEVER .ABS.(Y(2)-0.16667).G.0.001 .AND.N.L.NMAX
           R(N)=Y(2)-0.16667
```

```
            TRANSFER TO SW2
            OTHERWISE
            TRANSFER TO SW9
            END OF CONDITIONAL
SW18        WHENEVER .ABS.(Y(2)-0.2).G.0.001 .AND.N.L.NMAX
            R(N)=Y(2)-0.2
            TRANSFER TO SW2
            OTHERWISE
            TRANSFER TO SW9
            END OF CONDITIONAL
SW15        WHENEVER .ABS. Y(2).G.0.001    .AND. ZET.L.ZMAX, TRANSFER TO
           1SW10
SW9         WHENEVER .ABS. Y(3).G.0.001  .AND. ZET.L.ZMAX, TRANSFER TO SW
           110
            PRINT RESULTS T
            PRINT RESULTS ZET
            WHENEVER ZET.E.ZMAX, PRINT COMMENT $EQ. CAN NOT BE MADE TO SA
           1TISFY BOUNDARY CONDITONS $
            WHENEVER N.E.NMAX,TRANSFER TO SW14
            PRINT COMMENT $ R(M) $
            THROUGH SW5, FOR M=2, 1, M.G.N
SW5         PRINT FORMAT EPS, R(M)
            THROUGH SW4, FOR I=1, 1, I.G.GRSIZ
            A(T,I)=Y1(I)
            B(T,I)=Y2(I)
            WHENEVER T.L.TMAX
            C(T,I)=Y3(I)
            D(T,I)=F3(I)
            OTHERWISE
            C(T,I)=F3(I)
            END OF CONDITIONAL
            WHENEVER T.L.TMAX
            PRINT FORMAT DEL, P(I), Y1(I), Y2(I), Y3(I), F3(I)
            OTHERWISE
            PRINT FORMAT ROH, P(I), Y1(I), Y2(I), F3(I)
SW4         END OF CONDITIONAL
            WHENEVER T.L.TMAX
            PRINT FORMAT DEL, X,Y(1), Y(2), Y(3), F(3)
            OTHERWISE
            PRINT FORMAT ROH, X, Y(1), Y(2), F(2)
            END OF CONDITIONAL
            Z= RSIZ
            A(T,Z)=Y(1)
            B(T,Z)=Y(2)
            WHENEVER T.L.TMAX
            C(T,Z)=Y(3)
            D(T,Z)=F(3)
            OTHERWISE
            C(T,Z)=F(2)
            END OF CONDITIONAL
            T=T+1
            WHENEVER T.E.TMAX, EXECUTE SETRKD.(2,Y(1),F(1),Q,X,STEP)
            WHENEVER T.LE.TEND,TRANSFER TO SW6
            THROUGH SW21,FOR Z=1,1,Z.G.ZMA
            THROUGH SW21,FOR M=1,1,M.G.MMAX
            THROUGH SW21,FOR J=1,1,J.G.JMAX
            THROUGH SW21,FOR K=1,1,K.G.KMAX
            THROUGH SW20,FOR I=1,1,I.G.GRSIZ
            P(I)=STEP*(I-1)
```

```
          XYS(J)=XY(J)*XY(J)
          YP1(J)=3.*A2(M)*XYS(J)
          YP2(J)=3.*A0*A3*XYS(J)/A1
          US(I)=A0*B(3,I)+YP1(J)*B(4,I)+YP2(J)*B(5,I)
          UT(I)=A0*B(6,I)+YP1(J)*B(7,I)+YP2(J)*B(8,I)
          UU(I)=A0*B(9,I)+YP1(J)*B(10,I)+YP2(J)*B(11,I)
          WA(K)=W(K)*W(K)/(A1*A1)
          Q1(I)=WA(K)*UU(I)-US(I)
          Q1S(I)=Q1(I)*Q1(I)
          Q2(I)=W(K)*UT(I)/A1
          Q2S(I)=-2(I)*Q2(I)
          AMQS(I)=Q1S(I)+Q2S(I)
          AMQ(I)=SQRT.(AMQS(I))
          ANQ(I)=Q2(I)/Q1(I)
          PHAQ(I)3ATAN.(ANQ(I))
          A1S=1.5*A0/(SQRT.(A1))
          A2S=A0*A3/A1
          A05=A0*A5/A3
          A14(Z)=A1*A4(Z)/A3
          A12(M)=A1*A2(M)*A2(M)/(A0*A3)
          SSR(I)=2.*A1S*XY(J)*(A2(M)*A(12,I)+A2S*A(13,I)+2.*A3*(A2(M)*A
         1(28,I)+A14(Z)*A(26,I)+A12(M)*A(27,I)+A2S*A(29,I)+A05*A(30,I))
         2*XYS(J)/A1)
          YP3(J)=2.*A2(M)*XY(J)/A1
          YP4(J)=2.*A0*A3*XY(J)/(A1*A1)
          TS(I)=YP3(J)*A(33,I)+YP4(J)*A(34,I)
          TT(I)=YP3(J)*A(35,I)+YP4(J)*A(36,I)
          TU(I)=YP3(J)*A(37,I)+YP4(J)*A(38,I)
          S1(I)=WA(K)*TU(I)-TS(I)
          S1S(I)=S1(I)*S1(I)
          S2(I)=WUK)*TT(I)/A1
          S2S(I)=S2(I)*S2(I)
          AMSS(I)=S1S(I)+S2S(I)
          AMS(I)=SQRT.(AMSS(I))
          ANS(I)=S2(I)/S1(I)
          PHAS(I)=ATAN.(ANS(I))
          SST(I)=3.*A0*(A2(M)*A(39,I)+A2S*A(40,I)+2.*A3*(A2(M)*A(53,I)
         1+A12(M)*A(54,I)+A2S*A(55,I)+A14(Z)*A(56,I)+A05*A(57,I))*XYS(J
         2)/A1)/(A1*A1)
SW20      PRINT FORMAT BET,P(I),US(I),UT(I),UU(I),AMQ(I),PHAQ(I),SSR(I)
         1,TS(I),TT(I),TU(I),AMS(I),PHAS(I),SST(I)
          NTAU(K)=A1S*XY(J)*(A2(M)*C(12,1)+A2S*C(13,1)-WA(K)*(A2(M)*C(
         116,1)+A2S*C(17,1))+2.*A3*(A2(M)*C(28,1)+A14(Z)*C(26,1)+A12(M)
         2*C(27,1)+A2S*C(29,1)+A05*C(30,1))*XYS(J)/A1)
          NNUS(K)3A1S*(A2(M)*B(39,1)+A2S*B(40,1)-WA(K)*(A2(M)*B(43,1)+
         1A2S*B(44,1))+2.*A3*(A2(M)*B(53,1)+A12(M)*B(54,1)+A2S*B(55,1)+
         2A14(Z)*B(56,1)+A05*B(57,1))*XYS(J))/A1
SW21      PRINT FORMAT ALP,XY(J),W(K),NTAU(K),NNUS(K)
          TRANSFER TO BEGIN
 SW14     END OF PROGRAM
$DATA
ZMAX=6,TMAX=31,TEND=57,CF=1,NMAX=50,STEP=0.1,GRSIZ=45,PR=1.,
DY=0.001,YDP=1.000TCONST=0    ,A1=4.,A3=-2.6666,XY(1)=0,XY(2)=0.05,
XY(3)=0.1,XY(4)=0C2,XY(5)=0.3,XY(6)=0.4,XY(7)=0.5,XY(8)=0.8,XY(9)=1.,
W(1)=0,KMAX=2,A0=1C,MMAX=1,JMAX=9,A5=0.53333,A4(1)=0  ,ZMA=1,A2(1)=0,
W(2)=0.2 *
```

C. THE COMPUTER PROGRAM FOR ROTATIONAL OSCILLATION OF CYLINDER SURFACE

```
$COMPILE MAD, EXECUTE, PRINT OBJECT, DUMP, PUNCH OBJECT
            PRINT COMMENT $ THE COMPUTER PROGRAM FOR ROTATIONAL OSCILLATI
           1ON $
            DIMENSION Y(3), F(3), Q(3)
            INTEGER J, I, N, M, K, Z, T, ZET, PRI
            VECTOR VALUES DEL=$1H ,F6.4,4F15.8*$
            VECTOR VALUES ROH=$1H , F6.4, 3F15.8*$
            VECTOR VALUES ALP=$1H ,F6.4, 3F15.8*$
            VECTOR VALUES BET=$1H ,F6.4,12F10.6*$
            VECTOR VALUES EPS=$1H , 1F20.4*$
            DIMENSION P(45), Y1(45), Y2(45), F3(45), R(55), Y3(45),
           1US(45),UT(45),UU(45),Q1(45),Q1S(45),Q2(45),Q2S(45),AMQS(45),
           2AMQ(45D,ANQ(45),PHAQ(45),TS(45),TT(45),TU(45),S1(45),S1S(45),
           3S2(45),S2S(45),AMSS(45),AMS(45),ANS(45),PHAS(45),YP3(10),
           4XY(10)TWA(10),YP2(10),NTAU(10),W(10),NNUS(10),SSR(45),A2(10),
           5YP1(10)TYP4(10),A14(10),A4(10),A12(10),XYS(10),SST(45)
            DIMENSION A(2565,AD),B(2565,BD),C(2565,CD),D(2565,DD)
            VECTOR VALUES AD=2, 1, 45
            VECTOR VALUES BD=2, 1, 45
            VECTOR VALUES CD=2, 1, 45
            VECTOR VALUES DD=2, 1, 45
BEGIN       READ AND PRINT DATA
            EXECUTE SETRKD. (3, Y(1), F(1), Q, X, STEP )
            T=1
SW6         ZET=0
SW10        ZET=ZET+1
            WHENEVER ((-1).P.ZET).L.0, YX=YDP+ZET-1
            WHENEVER ((-1).P.ZET).G.0, YX=YDP-ZET+1
            R(0)=1.5
            R(1)=1
            N=1
SW2         Q=CF*(RUN)/R(N-1)-1)
            WHENEVER Q.G.87.20, Q=87.20
            WHENEVER Q.L.(-87.20), Q=-87.20
            DX=-DY*((2.35040/(EXP.(Q)-EXP.(-Q)))+1)
            DY=DX
            N=N+1
            YX=YX+DY
            WHENEVER T.L.TMAX
            Y(3)=YX
            OTHERWISE
            Y(2)=YX
            END OF CONDITIONAL
            J=0
            X=0
            WHENEVER T.E.1, Y(1)=CONST
            WHENEVER T.G.1, Y(1)=0
            WHENEVER T.E.1,Y(2)=0
            WHENEVER T.E.2,Y(2)=0
            WHENEVER T.E.3,Y(2)=1.0
            WHENEVER T.E.4,Y(2)=0.3333
            WHENEVER T.G.4 .AND.T.L.22,Y(2)=0
            WHENEVER T.E.22,Y(2)=0.2
            WHENEVER T.G.22 .AND.T.L.TMAX,Y(2)=0
START       J=J+1
            P(J)=X
            Y1(J)=Y(1)
            Y2(J)=Y(2)
            WHENEVER T.L.TMAX
```

```
         Y3(J)=Y(3)
         F3(J)=F(3)
         OTHERWISE
         F3(J)=F(2)
         END OF CONDITIONAL
SW1      F(1)=Y(2)
         WHENEVER T.L.TMAX, F(2)=Y(3)
         WHENEVER T.E.1, F(3)=(Y(2).P.2)-Y(1)*Y(3)-1
         WHENEVER T.E.2, F(3)=4*B(1,J)*Y(2)-3*C(1,J)*Y(1)-A(1,J)*Y(3)-
        11
         WHENEVER T.E.3, F(3)=B(1,J)*Y(2)-A(1,J)*Y(3)
         WHENEVER T.E.4, F(3)=3*B(1,J)*Y(2)-2*C(1,J)*Y(1)-A(1,J)*Y(3)
         WHENEVER T.E.5, F(3)=3*B(1,J)*Y(2)-2*C(1,J)*Y(1)-A(1,J)*Y(3)+
        14.*(B(2TJ)*B(3,J)-A(2,J)*C(3,J))
         WHENEVER T.E.6, F(3)=B(1,J)*Y(2)-A(1,J)*Y(3)+B(3,J)
         WHENEVER T.E.7, F(3)=3*B(1,J)*Y(2)-2*C(6,J)*Y(1)-A(1,J)*Y(3)+
        1B(4,J)
         WHENEVER T.E.8, F(3)=3*B(1,J)*Y(2)-2*C(1,J)*Y(1)-A(1,J)*Y(3)+
        1B(5,J)+4.0*(B(2,J)*B(6,J)-A(2,J)*C(6,J))
         WHENEVER T.E.9, F(3)=B(1,J)*Y(2)-A(1,J)*Y(3)+B(6,J)
         WHENEVER T.E.10, F(3)=3*B(1,J)*Y(2)-2*C(1,J)*Y(1)-A(1,J)*Y(3)
        1+B(7,J)
         WHENEVER T.E.11, F(3)=3*B(1,J)*Y(2)-2*C(1,J)*Y(1)-A(1,J)*Y(3)
        1+B(8,J)+4*(B(2,J)*B(9,J)-A(2,J)*C(9,J))
         WHENEVER T.E.12, F(3)=2.0*B(1,J)*Y(2)+2.0*B(3,J)*B(4,J)-A(1,J
        1)*Y(3)+2.0*C(3,J)*A(4,J)-C(1,J)*Y(1)
         WHENEVER T.E.13, F(3)=2.0*B(1,J)*Y(2)+2.0*B(3,J)*B(5,J)-A(1,J
        1)*Y(3)+2.0*C(3,J)*A(5,J)-C(1,J)*Y(1)
         WHENEVER T.E.14, F(3)=2.0*B(1,J)*Y(2)+2.0*B(12,J)+2.0*B(3,J)*
        1B(7,J)-A(1,J)*Y(3)-2.0*C(6,J)*A(4,J)-C(1,J)*Y(1)
         WHENEVER T.E.15, F(3)=2.0*B(13,J)+2.0*B(1,J)*Y(2)+2.0*B(3,J)*
        1B(8,J)-A(1,J)*Y(3)-2.0*C(6,J)*A(5,J)-C(1,J)*Y(1)
         WHENEVER T.E.16, F(3)=2.0*B(1,J)*Y(2)-A(1,J)*Y(3)-C(1,J)*Y(1)
        1+B(3,J)MB(10,J)+B(9,J)*B(4,J)-B(6,J)*B(7,J)-C(9,J)*A(4,J)-C(3
        2,J)*A(10,J)+C(6,J)*A(7,J)
         WHENEVER T.E.17, F(3)=2.0*B(1,J)*Y(2)-A(1,J)*Y(3)-C(1,J)*Y(1)
        1+B(3,J)*B(11,J)+B(9,J)*B(5,J)-B(6,J)*B(8,J)-C(9,J)*A(5,J)-C(3
        2,J)*A(11,J)+C(6,J)*A(8,J)
         WHENEVER T.E.18, F(3)=2.0*B(1,J)*Y(2)-A(1,J)*Y(3)-C(1,J)*Y(1)
        1+B(3,J)MB(10,J)+B(9,J)*B(4,J)+B(6,J)*B(7,J)-C(9,J)*A(4,J)-C(3
        2,J)*A(10,J)-C(6,J)*A(7,J)+B(14,J)
         WHENEVER T.E.19, F(3)=2.0*B(1,J)*Y(2)-A(1,J)*Y(3)-C(1,J)*Y(1)
        1+B(3,JD*B(11,J)+B(9,J)*B(5,J)+B(6,J)*B(8,J)-C(9,J)*A(5,J)-C(3
        2,J)*A(11,J)-C(6,J)*A(8,J)+B(15,J)
         WHENEVER T.E.20,F(3)=6.*B(1,J)*Y(2)-5.*C(1,J)*Y(1)-A(1,J)*Y(3
        1)-1.
         WHENEVER T.E.21,F(3)=6.*B(1,J)*Y(2)+8.*B(2,J)*B(2,J)-5.*C(1,J
        1)*Y(1)- .*A(2,J)*C(2,J)-A(1,J)*Y(3)-0.5
         WHENEVER T.E.22,F(3)=5.*B(1,J)*Y(2)-A(1,J)*Y(3)-4.*C(1,J)*Y(1
        1)
         WHENEVER T.E.23,F(3)=5.*B(1,J)*Y(2)+12.*B(2,J)*B(4,J)-A(1,J)
        1*Y(3)-7C2*A(2,J)*C(4,J)-4.*C(1,J)*Y(1)-4.8*C(2,J)*A(4,J)
         WHENEVER T.E.24,F(3)=5.*B(1,J)*Y(2)+6.*B(3,J)*B(20,J)-A(1,J)*
        1Y(3)-4.MC(1,J)*Y(1)
         WHENEVER T.E.25,F(3)=5.*B(1,J)*Y(2)+12.*B(2,J)*B(5,J)+6.*B(3,
        1J)*B(21TJ)-A(1,J)*Y(3)-7.2*A(2,J)*C(5,J)-4.*C(1,J)*Y(1)-4.8
        2*C(2,J)*A(5,J)
         WHENEVER T.E.26,F(3)=4.*B(1,J)*Y(2)+3.3333*B(3,J)*B(22,J)-A(1
        1,J)*Y(3)-3.3333*C(3,J)*A(22,J)-3.*C(1,J)*Y(1)
```

```
 WHENEVER T.E.27,F(3)=4.*B(1,J)*Y(2)+3.*B(4,J)*B(4,J)-A(1,J)*
1Y(3)-3.MA(4,J)*C(4,J)-3.*C(1,J)*Y(1)
 WHENEVER T.E.28,F(3)=8.*B(2,J)*B(12,J)+4.*B(1,J)*Y(2)+3.3333*
1B(3,J)*B(23,J)+6.*B(4,J)*B(5,J)-A(1,J)*Y(3)-6.*A(2,J)*C(12,J)
2-3.*A(4,J)*C(5,J)-3.*C(3,J)*A(5,J)-3.3333*C(3,J)*A(23,J)-2.*C
3(2,J)*AU12,J)-3.*C(1,J)*Y(1)
 WHENEVER T.E.29,F(3)=8.*B(2,J)*B(13,J)+4.*B(1,J)*Y(2)+3.3333*
1B(3,J)*B(25,J)+3.*B(5,J)*B(5,J)-A(1,J)*Y(3)-6.*A(2,J)*C(13,J)
2-3.*C(5,J)*A(5,J)-3.3333*C(3,J)*A(25,J)-2.*C(2,J)*A(13,J)-3.*
3C(1,J)*Y(1)
 WHENEVER T.E.30,F(3)=4.*B(1,J)*Y(2)+3.3333*B(3,J)*B(24,J)-A(1
1,J)*Y(3D-3.3333*C(3,J)*A(24,J)-3.0*C(1,J)*Y(1)
 WHENEVER T.E.31, F(2)=-PR*A(1,J)*Y(2)
 WHENEVER T.E.32, F(2)=PR*(2.0*B(1,J)*Y(1)-3.0*A(2,J)*B(31,J)-
1A(1,J)*Y(2))
 WHENEVER T.E.33, F(2)=PR*(B(1,J)*Y(1)-A(1,J)*Y(2)-3.0*A(4,J)*
1B(31,J))
 WHENEVER T.E.34, F(2)=PR*(B(1,J)*Y(1)+4.0*B(3,J)*A(32,J)-A(1,
1J)*Y(2)-3.0*A(5,J)*B(31,J))
 WHENEVER T.E.35, F(2)=PR*(B(1,J)*Y(1)-A(1,J)*Y(2)-3.0*A(7,J)*
1B(31,J))
 WHENEVER T.E.36, F(2)=PR*(B(1,J)*Y(1)+4.0*B(6,J)*A(32,J)-A(1,
1J)*Y(2)-3.0*A(8,J)*B(31,J))
 WHENEVER T.E.37, F(2)=PR*(B(1,J)*Y(1)-A(1,J)*Y(2)-3.0*A(10,J)
1*B(31,J))
 WHENEVER T.E.38, F(2)=PR*(B(1,J)*Y(1)+4.0*B(9,J)*A(32,J)-A(1,
1J)*Y(2)-3.0*A(11,J)*B(31,J))
 WHENEVER T.E.39, F(2)=PR*(0.666*B(3,J)*A(33,J)-A(1,J)*Y(2)-A(
112,J)*B(31,J))
 WHENEVER T.E.40, F(2)=PR*(0.666*B(3,J)*A(34,J)-A(1,J)*Y(2)-A(
113,J)*B(31,J))
 WHENEVER T.E.41, F(2)=PR*(0.666*B(6,J)*A(33,J)+0.666*B(3,J)*A
1(35,J)+2.0*A(39,J)-A(1,J)*Y(2)-A(14,J)*B(31,J))
 WHENEVER T.E.42, F(2)=PR*(0.666*B(6,J)*A(34,J)+0.666*B(3,J)*A
1(36,J)+2.0*A(40,J)-A(1,J)*Y(2)-A(15,J)*B(31,J))
 WHENEVER T.E.43, F(2)=PR*(B(9,J)*A(33,J)+B(3,J)*A(37,J)-B(6,J
1)*A(35,J)-A(1,J)*Y(2)-3.0*A(16,J)*B(31,J))
 WHENEVER T.E.44, F(2)=PR*(B(9,J)*A(34,J)+B(3,J)*A(38,J)-B(6,J
1)*A(36,J)-A(1,J)*Y(2)-3.0*A(17,J)*B(31,J))
 WHENEVER T.E.45,F(2)=PR*(B(9,J)*A(33,J)+B(3,J)*A(37,J)+B(6,J)
1*A(35,J)-A(1,J)*Y(2)-3.*A(16,J)*B(31,J)+3.*A(41,J))
 WHENEVER T.E.46,F(2)=PR*(B(9,J)*A(34,J)+B(3,J)*A(38,J)+B(6,J)
1*A(36,J)-A(1,J)*Y(2)-3.*A(17,J)*B(31,J)+3.*A(42,J))
 WHENEVER T.E.47,F(2)=PR*(-A(1,J)*Y(2)+4.*B(1,J)*Y(1)-5.*A(20,
1J)*B(31,J))
 WHENEVER T.E.48,F(2)=PR*(-A(1,J)*Y(2)+4.*B(1,J)*Y(1)-5.*A(21,
1J)*B(31,J)+2.6667*(2.*B(2,J)*A(32,J)-3.*A(2,J)*B(32,J)))
 WHENEVER T.E.49,F(2)=PR*(3.*B(1,J)*Y(1)-A(1,J)*Y(2)-5.*B(31,J
1)*A(22,J))
 WHENEVER T.E.50,F(2)=PR*(2.*B(2,J)*A(33,J)+3.*B(1,J)*Y(1)+6.*
1A(32,JD*B(4,J)-6.*A(2,J)*B(33,J)-A(1,J)*Y(2)-6.*A(4,J)*B(32,J
2)-5.*A(23,J)*B(31,J))
 WHENEVER T.E.51,F(2)=PR*(2.*B(2,J)*A(34,J)+3.0*B(1,J)*Y(1)+6.
1*B(3,J)MA(48,J)+6.*B(5,J)*A(32,J)-6.*A(2,J)*B(34,J)-A(1,J)*Y(
22)-6.*B(32,J)*A(5,J)-5.*A(25,J)*B(31,J))
 WHENEVER T.E.52,F(2)=PR*(3.*B(1,J)*Y(1)+6.*B(3,J)*A(47,J)-A(1
1,J)*Y(2)-5.*B(31,J)*A(24,J))
 WHENEVER T.E.53,F(2)=PR*(2.*B(1,J)*Y(1)+B(5,J)*A(33,J)+B(4,J)
1*A(34,J)+4.*B(12,J)*A(32,J)-6.*A(2,J)*B(39,J)-A(1,J)*Y(2)-2.*
```

```
          2A(5,J)MB(33,J)-2.*A(12,J)*B(32,J)-2.*A(4,J)*B(34,J)+2.*B(3,J)
          3*A(50,JD-3.*A(28,J)*B(31,J))
           WHENEVER T.E.54,F(2)=PR*(2.*B(1,J)*Y(1)+B(4,J)*A(33,J)-A(1,J)
          1*Y(2)-2C*A(4,J)*B(33,J)-3.*A(27,J)*B(31,J))
           WHENEVER T.E.55,F(2)=PR*(2.*B(1,J)*Y(1)+B(5,J)*A(34,J)+4.*B(1
          13,J)*A(32,J)-6.*A(2,J)*B(40,J)-A(1,J)*Y(2)-2.*A(5,J)*B(34,J)-
          22.*A(13,J)*B(32,J)+2.*B(3,J)*A(51,J)-3.*A(29,J)*B(31,J))
           WHENEVER T.E.56,F(2)=PR*(2.*B(1,J)*Y(1)-A(1,J)*Y(2)+2.*B(3,J)
          1*A(49,J)-3.*A(26,J)*B(31,J))
           WHENEVER T.E.57,F(2)=PR*(2.*B(1,J)*Y(1)-A(1,J)*Y(2)+2.*B(3,J)
          1*A(52,J)-3.*A(30,J)*B(31,J))
CALC       S=RKDEQC(0)
           WHENEVER S.E.1.0, TRANSFER TO SW1
           WHENEVER J.E.GRSIZ, TRANSFER TO SW3
           TRANSFER TO START
SW3        WHENEVER T.E.1, TRANSFER TO SW8
           WHENEVER T.E.2, TRANSFER TO SW7
           WHENEVER T.G.2 .AND.T.L.20,TRANSFER TO SW11
           WHENEVER T.E.20,TRANSFER TO SW18
           WHENEVER T.G.20 .AND.T.L.TMAX,TRANSFER TO SW11
           WHENEVER T.E.TMAX,TRANSFER TO SW12
           WHENEVER T.G.TMAX,TRANSFER TO SW13
SW7        WHENEVER .ABS.(Y(2)-.25).G.0.001 .AND. N.L.NMAX
           R(N)=(Y(2)-.25)
           TRANSFER TO SW2
           OTHERWISE
           TRANSFER TO SW9
           END OF CONDITIONAL
SW8         WHENEVER .ABS. (Y(2)-1).G.0.001 .AND. N.L.NMAX
           R(N)=Y(2)-1
           TRANSFER TO SW2
           OTHERWISE
           TRANSFER TO SW9.
           END OF CONDITIONAL
SW11       WHENEVER .ABS. Y(2).G.0.001 .AND. N.L.NMAX
           R(N)=Y(2)
           TRANSFER TO SW2
           OTHERWISE
           TRANSFER TO SW9
           END OF CONDITIONAL
SW12       WHENEVER .ABS. (Y(1)-1 ).G.0.001 .AND.N.L.NMAX
           R(N)=Y(1)-1
           TRANSFER TO SW2
           OTHERWISE
           TRANSFER TO SW15
           END OF CONDITIONAL
SW13       WHENEVER .ABS.Y(1).G.0.001    .AND.N.L.NMAX
           R(N)=Y(1)
           TRANSFER TO SW2
           OTHERWISE
           TRANSFER TO SW15
           END OF CONDITIONAL
SW18       WHENEVER .ABS.(Y(2)-0.16667) .G.0.001 .AND.N.L.NMAX
           R(N)=Y(2)-0.16667
           TRANSFER TO SW2
           OTHERWISE
           TRANSFER TO SW9
           END OF CONDITIONAL
SW15       WHENEVER .ABS. Y(2).G.0.001    .AND. ZET.L.ZMAX, TRANSFER TO
```

```
           1SW10
SW9         WHENEVER .ABS. Y(3).G.0.001  .AND. ZET.L.ZMAX, TRANSFER TO SW
           110
            PRINT RESULTS T
            PRINT RESULTS ZET
            WHENEVER ZET.E.ZMAX, PRINT COMMENT $EQ. CAN NOT BE MADE TO SA
           1TISFY BOUNDARY CONDITONS $
            WHENEVER N.E.NMAX,TRANSFER TO SW14
            PRINT COMMENT $ R(M) $
            THROUGH SW5, FOR M=2, 1, M.G.N
SW5         PRINT FORMAT EPS, R(M)
            THROUGH SW4, FOR I=1, 1, I.G.GRSIZ
            A(T,I)=Y1(I)
            B(T,I)=Y2(I)
            WHENEVER T.L.TMAX
            C(T,I)=Y3(I)
            D(T,I)=F3(I)
            OTHERWISE
            C(T,I)=F3(I)
            END OF CONDITIONAL
            WHENEVER T.L.TMAX
            PRINT FORMAT DEL, P(I), Y1(I), Y2(I), Y3(I), F3(I)
            OTHERWISE
            PRINT FORMAT ROH, P(I), Y1(I), Y2(I), F3(I)
SW4         END OF CONDITIONAL
            WHENEVER T.L.TMAX
            PRINT FORMAT DEL, X,Y(1), Y(2), Y(3), F(3)
            OTHERWISE
            PRINT FORMAT ROH, X, Y(1), Y(2), F(2)
            END OF CONDITIONAL
            Z= RSIZ
            A(T,Z)=Y(1)
            B(T,Z)=Y(2)
            WHENEVER T.L.TMAX
            C(T,Z)=Y(3)
            D(T,Z)=F(3)
            OTHERWISE
            C(T,Z)=F(2)
            END OF CONDITIONAL
            T=T+1
            WHENEVER T.E.TMAX, EXECUTE SETRKD.(2,Y(1),F(1),Q,X,STEP)
            WHENEVER T.LE.TEND,TRANSFER TO SW6
            THROUGH SW21,FOR M=1,1,M.G.MMAX
            THROUGH SW21,FOR Z=1,1,Z.G.ZMA
            THROUGH SW21,FOR J=1,1,J.G.JMAX
            THROUGH SW21,FOR K=1,1,K.G.KMAX
            THROUGH SW20,FOR I=1,1,I.G.GRSIZ
            P(I)=STEP*(I-1)
            XYS(J)=XY(J)*XY(J)
            YP1(J)=3.*A2(M)*XYS(J)
            YP2(J)=3.*A0*A3*XYS(J)/A1
            US(I)=A0*B(3,I)+YP1(J)*B(4,I)+YP2(J)*B(5,I)
            UT(I)=A0*B(6,I)+YP1(J)*B(7,I)+YP2(J)*B(8,I)
            UU(I)=A0*B(9,I)+YP1(J)*B(10,I)+YP2(J)*B(11,I)
            WA(K)=W(K)*W(K)/(A1*A1)
            Q1(I)=WA(K)*UU(I)-US(I)
            Q1S(I)=Q1(I)*Q1(I)
            Q2(I)=W(K)*UT(I)/A1
            Q2S(I)=Q2(I)*Q2(I)
```

```
          AMQS(I)=Q1S(I)+Q2S(I)
          AMQ(I)=SQRT.(AMQS(I))
          ANQ(I)=Q2(I)/Q1(I)
          PHAQ(I)3ATAN.(ANQ(I))
          A1S=1.5*A0/(SQRT.(A1))
          A2S=A0*A3/A1
          A05=A0*A5/A3
          A12(M)=A1*A2(M)*A2(M)/(A0*A3)
          A14(Z)=A1*A4(Z)/A3
          SSR(I)=2.*A1S*XY(J)*(A2(M)*A(12,I)+A2S*A(13,I)+2.*A3*(A2(M)*A
         1(28,I)+A14(Z)*A(26,I)+A12(M)*A(27,I)+A2S*A(29,I)+A05*A(30,I))
         2*XYS(J)/A1)/A1
          YP3(J)=2.*A2(M)*XY(J)/A1
          YP4(J)=2.*A0*A3*XY(J)/(A1*A1)
          TS(I)=YP3(J)*A(33,I)+YP4(J)*A(34,I)
          TT(I)=YP3(J)*A(35,I)+YP4(J)*A(36,I)
          TU(I)=YP3(J)*A(37,I)+YP4(J)*A(38,I)
          S1(I)=WA(K)*TU(I)-TS(I)
          S1S(I)=S1(I)*S1(I)
          S2(I)=WUK)*TT(I)/A1
          S2S(I)=S2(I)*S2(I)
          AMSS(I)=S1S(I)+S2S(I)
          AMS(I)=SQRT.(AMSS(I))
          ANS(I)=S2(I)/S1(I)
          PHAS(I)=ATAN.(ANS(I))
          SST(I)=3.*A0*(A2(M)*A(39,I)+A2S*A(40,I)+2.*A3*(A2(M)*A(53,I)
         1+A12(MD*A(54,I)+A2S*A(55,I)+A14(Z)*A(56,I)+A05*A(57,I))*XYS(J
         2)/A1)/(A1*A1)
SW20      PRINT FORMAT BET,P(I),US(I),UT(I),UU(I),AMQ(I),PHAQ(I),SSR(I)
         1,TS(I),TT(I),TU(I),AMS(I),PHAS(I),SST(I)
          NTAU(KD=A1S*XY(J)*(A2(M)*C(12,1)+A2S*C(13,1)-WA(K)*(A2(M)*C(
         116,1)+A2S*C(17,1))+2.*A3*(A2(M)*C(28,1)+A14(Z)*C(26,1)+A12(M)
         2*C(27,1)+A2S*C(29,1)+A05*C(30,1))*XYS(J)/A1)
          NNUS(K)3A1S*(A2(M)*B(39,1)+A2S*B(40,1)-WA(K)*(A2(M)*B(43,1)+
         1A2S*B(44,1))+2.*A3*(A2(M)*B(53,1)+A12(M)*B(54,1)+A2S*B(55,1)+
         2A14(Z)*B(56,1)+A05*B(57,1))*XYS(J))/A1
SW21      PRINT FORMAT ALP,XY(J),W(K),NTAU(K),NNUS(K)
          TRANSFER TO BEGIN
 SW14     END OF PROGRAM
$ DATA
ZMAX=6,TMAX=31,TEND=57,CF=1,NMAX=50,STEP=0.1,GRSIZ=45,PR=1.,
DY=0.001,YDP=1.000TCONST=0    ,A1=4.,A3=-2.6666,XY(1)=0,XY(2)=0.05,
XY(3)=0.1,XY(4)=0C2,XY(5)=0.3,XY(6)=0.4,XY(7)=0.5,XY(8)=0.8,XY(9)=1.,
W(1)=0,KMAX31,A0=1C,MMAX=1,JMAX=9,A5=0.53333,A4(1)=1.,ZMA=1,A2(1)=1. *
ZMAX=6,TMAX=31,TEND=57,CF=1,NMAX=50,STEP=0.1,GRSIZ=45,PR=1.0,
DY=0.001,YDP=1.000TCONST=0.1 ,A1=4.,A3=-2.6666,XY(1)=0,XY(2)=0.05,
XY(3)=0.1,XY(4)=0C2,XY(5)=0.3,XY(6)=0.4,XY(7)=0.5,XY(8)=0.8,XY(9)=1.,
W(1)=0,KMAX=1,A0=1C,MMAX=1,JMAX=9,A5=0.53333,A4(1)=0 ,ZMA=1,A2(1)=0 *
```