IMPACT OF QUALITY AND PRICING ON THE MARKET SHARES OF TWO COMPETING SUPPLIERS IN A SIMPLE PROCUREMENT MODEL

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Abstract

We investigate the impact of pricing and quality, where the latter is reflected in the yield rate distribution for a product, when a customer has decided to order from two competing suppliers for strategic reasons. This issue is investigated in the context of a two-supplier economic order quantity model, where the customer alternately orders from the two suppliers. For the case of equal yield-adjusted per unit costs, we derive analytical expressions for the relative market shares of the two suppliers and show that it is always optimal to order from both suppliers. For the case of unequal yield-adjusted per unit costs, we obtain an expression that relates the relative order quantities, and derive conditions in which the market share of one or the other supplier will approach zero. Implications of these results for competitive pricing and quality characteristics are discussed.

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INTRODUCTION

In a recent paper, Gerchak and Parlar (1990) describe a continuous-time, constant-demand inventory model in which there are two imperfect suppliers of a single customer. Their model requires that procurement take place at constant intervals, and if it is optimal to use both suppliers, that procurement take place simultaneously from both suppliers.

We use a variation of this scenario as the basis for investigating the impact of supplier quality and pricing on their market shares. Here, we assume that the customer alternately orders from the two suppliers. If the customer's procurement problem were viewed in isolation, it can be shown that it would be preferable to order from one supplier exclusively rather than to alternate orders between two different suppliers. In other words, it would be preferable always to order from the "better" supplier. This is consistent with current trends in the drive for improved quality. For example, there has been an increased emphasis on reducing the number of suppliers and on the development of greater supplier-customer cooperation, often with the intent of improving quality.

On the other hand, major manufacturing companies with semi-captive suppliers (e.g., automobile companies) often have two different suppliers in order to provide "insurance" against unforeseen events such as strikes and major machine failures, and to provide for greater volume flexibility than can be provided by a single supplier. Given that one is constrained to purchase from two suppliers, it is easy to show that one should alternate orders rather than ordering simultaneously if the setup costs are not increased as a consequence of staggering the orders. For any given pair of order quantities, the average inventory is less with alternating orders while the other costs remain unchanged. Indeed, because of these inventory reductions, the alternating order scheme will be less expensive, even with some increase in the total setup cost per cycle, *given that one must purchase from two suppliers*.

We first modify the Gerchak-Parlar model, which assumes equal yield-adjusted unit costs for the two suppliers, to account for alternating orders, and provide some insight into the form of the optimal ordering strategy. We then analyze the alternating order strategy under more general conditions on unit purchase costs than in the Gerchak-Parlar paper. This model allows us to evaluate, in a simple setting, the value (to a customer) of higher average and/or lower variance yields, and the impact of price and the yield rate distribution on the competitive positions of the two suppliers.

We first briefly review the Gerchak-Parlar model. Their assumptions are:

1. a constant demand rate, D, for a single item;
2. instantaneous replenishment ;
3. a joint setup cost, K, for one order from each supplier;
4. inventory holding cost, h, charged on time-weighted average inventory;
5. identical yield-adjusted unit purchase cost for both suppliers;
6. a multiplicative yield model, i.e., the fraction good has a distribution which is independent of the batch size; and
7. there is 100% inspection, with the cost of inspection included in the unit cost, and defective units are disposed following inspection.

Following the notation of Gerchak and Parlar, define:

c_i = purchase cost per unit for supplier i,

U_i = yield rate (fraction good) for supplier i (random variable),

h_i = inventory holding cost per unit per unit time for supplier i,

μ_i = average yield rate (fraction good) for supplier i,

σ_i^2 = variance of the yield rate for supplier i,

$F_i(\cdot)$ = distribution of the yield rate for supplier i,

$f_i(\cdot)$ = density of the yield rate for supplier i,

Q_i = purchase quantity from supplier i during one procurement cycle, and

$Y_{Qi} = Q_iU_i$.

Assumption (5) can be expressed as $c_1/\mu_1 = c_2/\mu_2$. Under this assumption, unit purchase costs are constant and can be excluded from the formulation. The cost per cycle is

$$[K + h\,(Y_{Q1} + Y_{Q2})^2]/2D$$

and the length of the cycle is

$$(Y_{Q1} + Y_{Q2})/D,$$

so the expected cost per unit time is

$$\frac{K + h\,E(Y_{Q1} + Y_{Q2})^2}{2[E(Y_{Q1}) + E(Y_{Q2})]}\ .$$

It is easy to show that

$$E(Y_{Qi}^2) = Q_i^2(\sigma_i^2 + \mu_i^2)$$

and $E(Y_{Qi}) = \mu_iQ_i$. With these substitutions, Gerchak and Parlar show that the first order conditions imply

$$\mu_2\,\sigma_1^2\,Q_1 = \mu_1\sigma_2^2Q_2$$

and that the objective function is convex for Q_1 and Q_2 satisfying this equality. These results indicate that if one source has zero variance, the other will not be used, and if $\sigma_1\sigma_2 > 0$, both sources will be used. They also derive conditions in which it is optimal to use one source (or the other), or both sources simultaneously.

The Gerchak-Parlar model has been extended by Parlar and Wang (1990) to consider unequal yield-adjusted costs, maintaining the assumption of simultaneous orders.

MODEL WITH EQUAL YIELD-ADJUSTED VARIABLE UNIT COSTS

In this model, we assume that $c_1/\mu_1 = c_1/\mu_2$, so it is not necessary to include variable unit costs in the formulation. The purchaser alternates between buying Q_1 units from supplier 1 and Q_2 units from supplier 2, where the yield distributions for the suppliers are given by F_i with mean μ_i and variance σ_i^2, $i = 1, 2$. The setup cost per procurement cycle is K, which might be composed of a setup cost for each supplier.

Let Y_{Qi} be the actual yield (good units) if Q_i units are procured and h be the annual cost of holding one unit in inventory. Then, the cost of a cycle is

$$K + h\,(Y_{Q1}^2 + Y_{Q2}^2)/2D$$

and the length of the cycle is

$$(Y_{Q1} + Y_{Q2})/D.$$

Using standard renewal theory arguments (e.g., Ross 1983), the expected cost per unit time is therefore

$$z(Q_1, Q_2) = \frac{2KD + h\,[E(Y_{Q1}^2) + E(Y_{Q2}^2)]}{2[E(Y_{Q1}) + E(Y_{Q2})]}\quad . \tag{1}$$

Observe that for any given Q_1 and Q_2, the cost per unit time is less than or equal to that in the Gerchak-Parlar model. Thus, the cost function for the alternating strategy lies below that of the simultaneous procurement strategy, indicating that the former policy is dominant if the cost parameters are the same. With the alternating strategy, however, the setup cost per cycle may be slightly larger because the two orders are not placed at the same time.

Substituting for the expectations, (1) can be rewritten as

$$\frac{2KD + h\,[Q_1^2(\sigma_1^2 + \mu_1^2) + Q_2^2(\sigma_2^2 + \mu_2^2)]}{2(\mu_1 Q_1 + \mu_2 Q_2)}\quad . \tag{1a}$$

The partial derivatives with respect to Q_1 and Q_2 are:

$$\partial z/\partial Q_1 = \{h\mu_1(\sigma_1^2 + \mu_1^2)Q_1^2 + 2h\mu_2(\sigma_1^2 + \mu_1^2)\,Q_1Q_2$$
$$- h\mu_1(\sigma_2^2 + \mu_2^2)Q_2^2 - 2\mu_1 KD\}/2(\mu_1Q_1 + \mu_2Q_2)^2$$

and $\partial z/\partial Q_2$ is the same, but with subscripts reversed. It is easy to show that the Hessian is positive definite, so the objective function is strictly convex. The denominators of both first order conditions are strictly positive. Thus, the first order necessary conditions involve equating the numerators to zero. Rewriting the expressions, we have

$$(\sigma_1^2 + \mu_1^2)Q_1^2 + \frac{2\mu_2}{\mu_1}(\sigma_1^2 + \mu_1^2)Q_1Q_2 - (\sigma_2^2 + \mu_2^2)Q_2^2 = \frac{2KD}{h} \tag{2}$$

and $$(\sigma_2^2 + \mu_2^2)Q_2^2 + \frac{2\mu_1}{\mu_2}(\sigma_2^2 + \mu_2^2)Q_1Q_2 - (\sigma_1^2 + \mu_1^2)Q_1^2 = \frac{2KD}{h} \quad (3)$$

Equations (2) and (3) represent hyperbolas. We next show that they intersect in a unique point.

Adding equations (2) and (3), after some simplification we get

$$[\mu_2^2(\sigma_1^2 + \mu_1^2) + \mu_1^2(\sigma_2^2 + \mu_2^2)]\, Q_1Q_2 = \frac{2KD\mu_1\mu_2}{h} \quad (4)$$

Let

$$\alpha = \mu_2^2(\sigma_1^2 + \mu_1^2) + \mu_1^2(\sigma_2^2 + \mu_2^2)$$

and

$$\beta = 2KD\mu_1\mu_2/h.$$

Then $Q_2 = \beta/\alpha Q_1$. Substituting for Q_2 in (2), we have

$$(\sigma_1^2 + \mu_1^2)Q_1^2 + \frac{2\mu_2(\sigma_1^2 + \mu_1^2)\beta}{\mu_1\alpha} - \frac{(\sigma_2^2 + \mu_2^2)\beta^2}{\alpha^2 Q_1^2} - \frac{\beta}{\mu_1\mu_2} = 0, \text{ or}$$

$$Q_1^2 + \frac{\beta}{\mu_1\mu_2}\left[\frac{2\mu_2^2}{\alpha} - 1\right] - \frac{(\sigma_2^2 + \mu_2^2)\beta^2}{(\sigma_1^2 + \mu_1^2)\alpha^2 Q_1^2} = 0 \quad (5)$$

Let $a = 1$

$$b = \frac{\beta}{\mu_1\mu_2}\left[\frac{2\mu_2^2}{\alpha} - 1\right],$$

$$c = \frac{(\sigma_2^2 + \mu_2^2)\beta^2}{(\sigma_1^2 + \mu_1^2)\alpha^2}, \text{ and}$$

$$y = Q_1^2.$$

Then (5) becomes

$$ay + b + cy^{-1} = 0 \quad (6)$$

Note that we must have $y > 0$ since $y = Q_1^2$ and equation (4) cannot be satisfied by $Q_1 = 0$. Thus, we can multiply (6) by y to get:

$$ay^2 + by + c = 0$$

and $y^* = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Since a is positive, the denominator of the above expression must be positive if y is to be feasible. The sign of b depends on the data. Now $4ac < 0$ since $a > 0$ and $c < 0$, so $(b^2 - 4ac)^{0.5} > |b|$.

Thus, if $b < 0$, $-b - (b^2 - 4ac)^{0.5} < 0$ and the solution is infeasible. Similarly, if $b > 0$, $-b - (b^2 - 4ac)^{0.5} < 0$ and the solution is infeasible. So the unique solution to the first order necessary conditions is:

$$Q_1^{*2} = y^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a}. \tag{7}$$

and Q_2^* is defined by (4). With some algebra, it can be shown that

$$z^*(Q_1^*, Q_2^*) = \frac{h(\sigma_1^2 + \mu_1^2)Q_1^*}{\mu_1}.$$

This solution is quite different from the one obtained by Gerchak and Parlar (1989) for the case in which procurement from the two suppliers occurs simultaneously. More noteworthy is the fact that it is always optimal to order a positive quantity from both suppliers, since $Q_1Q_2 > 0$, as indicated by (4). This contrasts sharply with the result in Gerchak and Parlar that one never orders from an "unreliable" supplier if the other is perfectly reliable ($\sigma_1^2 = 0$). It appears that the procurement policy (simultaneous ordering versus alternating orders) has a dramatic influence on the structure and managerial implications of the optimal policy.

We now turn to the case of different yield-adjusted variable purchase costs.

MODEL WITH UNEQUAL YIELD-ADJUSTED VARIABLE UNIT COSTS

We now consider the case in which $c_1/\mu_1 \neq c_2/\mu_2$ and the holding costs are h_1 and h_2 for parts from the two suppliers, respectively. We continue to assume that the purchaser alternates ordering from the two suppliers. The cost of a cycle is

$$K + (h_1 Y_{Q1}^2 + h_2 Y_{Q2}^2)/2D + c_1Q_1 + c_2Q_2$$

and the length of the cycle is

$(Y_{Q1} + Y_{Q2})/D$

The expected cost per unit time is thus

$$z(Q_1,Q_2) = \frac{2KD + h_1E(Y_{Q1}^2) + h_2E(Y_{Q2}^2) + 2D(c_1Q_1 + c_2Q_2)}{2[E(Y_{Q1}) + E(Y_{Q2})]}.$$

Substituting for $E(Y_{Qi}^2)$ and $E(Y_{Qi})$, we get

$$z(Q_1,Q_2) = \frac{2KD + h_1(\sigma_1^2 + \mu_1^2)Q_1^2 + h_2(\sigma_2^2 + \mu_2^2)Q_2^2 + 2D(c_1Q_1 + c_2Q_2)}{2(\mu_1Q_1 + \mu_2Q_2)}$$

Now

$$\partial z(Q_1,Q_2)/\partial Q_1 = [h_1\mu_1(\sigma_1^2 + \mu_1^2)Q_1^2 + 2h_1\mu_2(\sigma_1^2 + \mu_1^2)Q_1Q_2 - 2KD\mu_1 - h_2\mu_1(\sigma_2^2 + \mu_2^2)Q_2^2 + 2D(c_1\mu_2 - c_2\mu_1)Q_2]/2(\mu_1Q_1 + \mu_2Q_2)^2 \qquad (8)$$

and $\partial z(Q_1,Q_2)/\partial Q_2$ is the same but with subscripts reversed. It can be shown that $z(Q_1,Q_2)$ is jointly convex in Q_1 and Q_2 so the optimal solution satisfies the first order necessary conditions. Setting $\partial z(Q_1,Q_2)/\partial Q_i$ equal to zero and multiplying the equation by μ_i, $i=1,2$, then adding the two equations, we can show that

$$Q_1 = (2KD - \beta Q_2)/(\alpha Q_2 + \gamma) \qquad (9)$$

where $\alpha = h_2\mu_1(\sigma_2^2 + \mu_2^2)/\mu_2 + h_1\mu_2(\sigma_1^2 + \mu_1^2)/\mu_1$

$\beta = (c_1\mu_2/\mu_1 - c_2)D$, and

$\gamma = (c_2\mu_1/\mu_2 - c_1)D$.

By substituting for Q_1 in one of the first order conditions, we would obtain a quartic equation in Q_2 which would take some effort to solve. On the other hand, since the objective function is convex, we can perform a single-dimensional search for Q_2, setting the corresponding Q_1 using equation (9).

The question of whether to use both suppliers when one is perfectly reliable still remains. Note that β and γ have opposite signs (unless they are both equal to zero, which is the case discussed earlier). If $\beta < 0$ and $\gamma < 0$, from (9), Q_1 is positive for any nonnegative value of Q_2. This says that if $c_1/\mu_1 < c_2/\mu_2$ we will always purchase a positive quantity from supplier 1, irrespective of the yield rate variances. While it is difficult to determine Q_1 and Q_2 in closed form, it is possible to show that at $Q_2 = 0$ and $Q_1 = 2KD/\gamma$

(from (9) when $Q_2 = 0$), $\partial z(Q_1,Q_2)/\partial Q_2 < 0$ if $2Kh_1(\sigma_1^2 + \mu_1^2) > D\mu_1^2(c_2/\mu_2 - c_1/\mu_1)^2$. If this condition holds, $Q_2^* = 0$ cannot be optimal. If it does hold, $Q_2^* = 0$ is optimal. (Of course, since the customer is constrained to order from both suppliers Q_2 will be positive, but very small, and can easily be driven out of the market by another supplier with slightly better cost-quality characteristics.)

Now consider the case where $\beta > 0$ and $\gamma < 0$. It is easy to show that the numerator and denominator of (9) have the same sign for values of Q_2 in some continuous interval defined by $2KD/\beta$ and $|\gamma|/\alpha$ as the limits. Note that both limits are strictly positive. Thus, if $Q_1^* > 0$, we also have $Q_2^* > 0$. The question that remains is whether $Q_1^* = 0$ can be optimal. By analysis similar to that above, we find that $Q_1^* = 0$ cannot be optimal if $2Kh_2(\sigma_2^2 + \mu_2^2) > D\mu_2^2(c_1/\mu_1 - c_2/\mu_2)^2$, and the converse holds if the condition is violated. (Here again, Q_1 will be positive even if the condition is violated because of the constraint requiring that both suppliers be used.)

These conditions have the attractive feature that one can determine in advance whether one supplier is dominated. They also indicate how low a supplier must price its goods just to stay minimally competitive, even when the customer is constrained to order from both suppliers for other strategic reasons, and what yield characteristics it must have, given current prices, to drive the competitor out of the market except as a secondary (emergency) source.

CONCLUSIONS

It has long been understood that customers are willing to pay more for parts with higher "quality" but it is not always easy to quantify exactly how much more. We have investigated this issue when the "quality" is reflected in the yield rate distribution (fraction good), and where the customer has decided to use two suppliers for strategic reasons beyond the scope of our simple procurement model.

Our analysis indicates that when the two suppliers have equal yield-adjusted unit costs, the optimal policy is for the customer to order from both suppliers. The market shares of the two suppliers can be expressed explicitly in terms of the problem parameters. If the two suppliers have unequal yield-adjusted costs, under certain conditions on costs and yield distributions, the customer will choose to drive one of the supplier's market share to nearly zero, maintaining positive order quantities only because of the external strategic considerations.

These models represent an attempt to understand pricing and quality implications in a simple procurement modelwhen there are multiple potential suppliers. Further research is needed to investigate these issues in other contexts.

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