OPTIMIZING TRANSPORATION CONTRACTS TO SUPPORT JUST-IN-TIME DELIVERIES: THE CASE OF ONE CONTRACTED TRUCK PER SHIPMENT

Candace Arai Yano

Department of
Industrial and Operations Engineering
The University of Michigan
Ann Arbor, MI 48109-2117

February 1989
Technical Report 89-6

OPTIMIZING TRANSPORATION CONTRACTS TO SUPPORT JUST-IN-TIME DELIVERIES: THE CASE OF ONE CONTRACTED TRUCK PER SHIPMENT

ABSTRACT

We consider the problem of setting up transportation contracts for a just-in-time delivery system between a manufacturer and one of its suppliers. The usage of parts fluctuates over time, and if the needed parts do not fit into the contracted vehicle, arrangements are made for an emergency shipment at premium rates. We pose the problem as one of minimizing the total expected cost of contracted and emergency shipments, and where appropriate, inventory holding costs. We derive some properties of the optimal solution and present solution procedures. Computational results and possible research directions are also discussed.

OPTIMIZING TRANSPORATION CONTRACTS TO SUPPORT
JUST-IN-TIME DELIVERIES:
THE CASE OF ONE CONTRACTED TRUCK PER SHIPMENT

1. INTRODUCTION

Manufacturing firms that have established just-in-time policies in conjunction with their suppliers are increasingly using scheduled shipments to ensure the timely arrival of goods. Scheduled shipments serve to nearly eliminate uncertainty about the timing of demands for the supplier, and to dramatically reduce the uncertainty in the transportation lead times by ensuring the availability of a shipper. These scheduled shipments are commonly arranged through a negotiated contract with a trucking or railway company. The stability afforded by such contracts permits the transporter to schedule equipment and personnel in advance, thus reducing the cost of transportation also.

These contracts normally specify a day and time window for the pickup at the supplier and a day (typically the same day as the pickup) and time window for delivery at the purchaser's manufacturing facility for *each* truck trailer or rail car. Typical contracts are established for a one year horizon and are cyclic. For example, the contract might specify that a truck will pick up goods at point A and deliver them to point B every Monday, Wednesday, and Friday morning. The purchaser of the contract is obligated to pay for the contracted truck movements even if some of them are not needed.

On occasion, because of fluctuations in the rates of use of various purchased parts, there is more material to be shipped than will fit into the contracted vehicle. In such instances, additional "emergency" transportation must be arranged, typically immediately and at premium rates. The need for an immediate emergency shipment arises because the supplier does not know which parts are required urgently and which are less critical. Moreover, in a just-in-time setting, inventory levels are low, so most parts are critical a substantial portion of the time. Waiting until the next regular shipment, or even until the next day, would not be an acceptable alternative.

We are implicitly assuming here that the time between shipments is long enough to make an emergency shipment necessary, and that the cost of shortages exceeds the cost of an emergency shipment.. If the time between shipments is short (e.g., a few hours), emergency shipments may not be necessary except in extreme circumstances.

We investigate the problem of determining the optimal time between normal shipments to minimize the total expected cost of normal and emergency shipments. As we will see later, this decision may have an impact on the amount of inventory that must be held. For this reason, we also extend the model to incorporate the cost of inventory. In this paper we consider the case where each shipment consists of one truckload or less, which is common practice in most just-in-time operations. In a sequel, we consider situations in which each shipment may contain more than one truckload.

The remainder of this paper is organized as follows. Assumptions and a brief literature review are presented in Section 2. We then consider a model without inventory holding costs. Some analytic properties of the optimal policy and a sketch of a solution procedure for the case of Brownian motion are given in Section 3. We then generalize this model to include inventory costs and describe a solution technique for it in Section 4. Computational results for the two foregoing models are presented in Section 5. Section 6 concludes with a summary and discussion.

2. PROBLEM ASSUMPTIONS AND FORMULATION

The cost of a transportation contract is highly dependent upon the mode of transportation and the details of the contract. In this paper, we assume that transportation is by truck, principally because most just-in-time deliveries use this mode of transportation. Just-in-time transportation contracts specify the origin, destination, and time windows for pickup and delivery. We assume that because of the time windows, the vehicle must proceed directly from the origin to the destination, and that the contract is priced accordingly. (That is, the pricing does not reflect any interactions with shipments from other origins or to other destinations that might be consolidated onto the same vehicle).

As a consequence, it is reasonable to assume that the transportation cost per vehicle is independent of the utilization of the trailer. This is consistent with the non-consolidation assumption and the fact that a large percentage of the cost of truck transportation is attributable to labor and vehicle operating cost (e.g., gasoline, etc.), both of which are approximately linear functions of the distance traveled and virtually insensitive to trailer utilization. Similarly, we assume that the price per emergency shipment for a given supplier is a constant (i.e., independent of the delivery quantity). We also assume that the order processing or transaction cost per shipment is negligible because of the existence of a shipping contract.

We assume that in the aggregate, the total usage (measured in truckloads) can be represented by a stationary stochastic process. For simplicity, time is measured in years here, but any time unit can be used.

The system uses an order-up-to (base stock) policy and consequently, the order quantity is equal to the usage since the initiation of the previous order We assume that communication between the supplier and manufacturer is instantaneous (e.g., electronic) and that transportation lead times are short in comparison to the time between deliveries. Thus, we do not include safety stocks to accommodate demand variability during the procurement lead time. (These effects can be included but they make the model much more complicated, particularly since safety stock must be specified for each type of part.)

Finally, we assume that the critical constraint is either volume *or* weight , but not both. If the shipment quantity exceeds the capacity of the truck, an emergency shipment is arranged at a premium cost, and is assumed to be done by truck. Because of this, it may be desirable to arrange for more truck capacity than is needed on the average (i.e., to allow for slack capacity in the shipment).

Other research has been done on the issue of emergency shipments, but the contexts differ from the one that we consider here. In these models, an emergency shipment is initiated so that it arrives earlier than the normal shipment, with the intent of reducing potential shortages when the demand is unusually high.

A single period version of this problem was addressed by Barankin (1961). Multiple-period versions of the problem were studied by Daniel (1962), Neuts (1964), Fukuda (1964), Bulinskaya (1964), Veinott (1966), Wright (1968, 1969), and Whitmore and Saunders (1977). Generally, these approaches have involved stochastic, discrete time dynamic programs. Emergency orders have been treated in continuous review contexts by Hadley and Whitin (1962), Allen and D'Esopo (1968), Gross and Soriano (1972), Rosenshine and Obee (1976), and Moinzadeh and Nahmias (1988).

The models developed to date do not consider constraints on the size of the order quantity because of transportation capacity constraints, which is the underlying cause of the emergency shipments in our model. In our model, the emergency shipment arrives concurrently with the the normal shipment to reduce the expected number of shortages during the entire procurement cycle.

3. MODEL FOR PLANNED SHIPMENT SIZE OF ONE TRUCKLOAD OR LESS

As mentioned earlier, we assume that the planned size of the shipment is constrained to be one vehicle or less, as is true in most existing just-in-time operations. We also assume in this section that purchaser either directly or indirectly pays the cost of holding inventory at the supplier's facility as well as at its own manufacturing facility. One example of such a situation occurs when both the manufacturer and supplier are subsidiaries of the same company. Thus, the time between shipments and the size of the shipment affect transportation costs, but do not affect the total amount of inventory at the supplier and purchaser facilities. This is especially true if the supplier, for reasons of scheduling or production economies, produces the parts in batches that are larger than the typical delivery quantities. Of course, there may be an overall reduction of inventory because of the existence of the shipping contract which provides information on demand timing to the supplier. We are simply assuming that, given the existence of a shipping contract, the amount of inventory in the "system" is unaffected by the time between shipments. We also assume that at most one emergency vehicle is required for each normal shipment. This imposes a constraint on the distribution (especially the variance) of the demand process, but this constraint is not restrictive in most just-in-time applications.

We stated the problem as one of determining the time between normal shipments, T, to minimize the expected overall transportation costs. Since each shipment is constrained to be one vehicle or less, however, it is useful to pose the problem in terms of setting a planned (average) vehicle utilization, u, where $0 \le u \le 1$. Then we set $T = u/\mu$, where μ is the mean aggregate demand rate measured in truckloads. Since μ is a constant, optimizing u is equivalent to optimizing T. The contract is established with T as the time between shipments. The value of using u as the decision variable is that it permits us to readily determine when it is optimal to plan for full truckloads, and when slack capacity is desirable. Other notation follow.

$S =$ normal transportation cost per vehicle ,

$C_e =$ cost of an emergency vehicle ($C_e > S$),

$u =$ planned truck utilization, and

$X(t) =$ demand (in truckloads) in a time interval of duration t (random variable), where $E[X(t)] = \mu t$.

Note that the time between shipments is u/μ.

The problem is to

$$\text{minimize} \quad S\mu/u + C_e\, P[X(u/\mu) > 1]/(u/\mu)$$

$$\text{subject to} \quad 0 \le u \le 1.$$

The first term in the objective function is the cost of contracted shipments. The second term is the expected cost of emergency shipments. It is clear that the first term is monotonically decreasing in u/μ. In the Appendix, we show empirically for a few examples based upon different stochastic processes that the second term is monotonically non-decreasing in u/μ. Thus, a tradeoff exists and $u = 1$ is not always optimal, even in the absence of inventory holding costs.

Throughout the remainder of the paper, we model demand as Brownian motion, which is frequently used in the inventory literature (either explicitly or implicitly) because of its property of infinite divisibility (i.e., $X(t) + X(\Delta t) = X(t + \Delta t)$), and because it permits the modeler to specify two moments. In the following section, we derive some properties of the optimal solution for demand modeled as Brownian motion. Similar results can be obtained for other stochastic processes as well, but we chose Brownian motion for ease of analysis.

3. PROPERTIES OF THE OPTIMAL SOLUTION AND A SOLUTION PROCEDURE

In this section we assume that the demand process is represented by Brownian motion with a mean rate μ and variance σ^2. Thus, the usage in any time interval $[t, t+\Delta t]$ is $N(\mu\Delta t, \sigma^2\Delta t)$. The problem then is to

$$\begin{aligned} &\text{minimize} \quad && S\mu/u + C_e\,(\mu/u)\{1 - \Phi[(1-u)/(u/\mu)^{0.5}\sigma]\} \qquad (1) \\ &\text{subject to} \quad && 0 < u \leq 1, \end{aligned}$$

where ϕ = normal density function, and

Φ = normal distribution function.

Because of the normal distribution function in the objective function, it is not possible to formally establish convexity of the objective function. It seems reasonable, however, that the function is at least unimodal, and we will proceed under this assumption. Setting the first derivative equal to zero we have

$$\partial/\partial u = \quad -S\mu/u^2 - C_e\mu\{1 - \Phi[(1-u)/(u/\mu)^{0.5}\sigma]\}/u^2$$

$$+ 0.5C_e\mu\ \phi[(1-u)/(u/\mu)^{0.5}\sigma](u+1)/u^2(u/\mu)^{0.5}\sigma = 0.$$

Let $z = (1-u)/(u/\mu)^{0.5}\sigma$. We can define z as the number of standard deviations less than full capacity of the vehicle that a given u represents. Note that the relationship between u and z is not linear because the standard deviation of demand in a delivery interval increases as u increases. Solving for u, we have

$$u = \quad [2+\sigma^2z^2/\mu - \sigma z(\sigma^2z^2+4\mu)^{0.5}]/2. \tag{2}$$

(The other root is larger than 1.0, so there is a one-to-one relationship between u and z.) Also, by making a change of variable we have that

$$\partial/\partial z = \quad 0.5\mu\sigma[2\sigma z\mu - (\sigma^2z^2+ 4\mu)^{0.5} - \sigma^2z^2(\sigma^2z^2+ 4\mu)^{-0.5}]\ x$$

$$\{[2+\sigma^2z^2/\mu - \sigma z(\sigma^2z^2+4\mu)^{0.5}]/2\}^{-2}\{-S - C_e[1-\Phi(z)] + 0.5C_e\phi(z)z$$

$$+ C_e\mu^{0.5}\phi(z)\{[2+\sigma^2z^2/\mu - \sigma z(\sigma^2z^2+4\mu)^{0.5}]/2\}^{0.5}\}. \tag{3}$$

At this point, it is useful to point out that, under the assumption that the expected cost function is unimodal, we will have $z^* = 0$ (where z^* denotes the optimal value of z) if the first derivative is positive at $z = 0$. This occurs when

$$\mu < [(S + 0.5C_e)/0.3989C_e]^2.$$

Let $b = C_e/S \geq 1$, where b is the ratio of the cost of an emergency shipment to the cost of a normal shipment. Then the above inequality is equivalent to

$$\mu < [(1 + 0.5b)/0.3989b]^2. \tag{4}$$

The derivative of the right hand side with respect to b is negative, so it achieves its maximum feasible value at $b = 1$, for which the right hand side equals approximately 14. Thus, for small values of μ, it appears that $z^* = 0$, which implies that $u^* = 1.0$. Observe that these expressions are independent of σ. For some values of σ, it is possible that $z^* = 0$ even if μ is larger than the maximum value indicated by (4). Indeed, through empirical investigations (described in further detail in section 5) we have found that one needs to have both a small value of μ (but larger than the right hand side of (4)) *and* a small coefficient of variation (σ/μ) for $z > 0$ (and hence $u < 1$) to be optimal.

These results may, at first, seem counterintuitive. Results from numerical examples indicate that the additional number of contracted vehicles that results from using $u < 1$ has two adverse effects on costs. First, the total number of contracted shipments increases. Second, since the number of contracted shipments increases, so does the number of possible exposures to emergency shipments. These two factors more than offset potential reductions in the cost of emergency shipments, except when a small reduction in the planned utilization permits a substantial reduction in the probability of an emergency shipment (such as when σ is quite small).

4. A MODEL WITH INVENTORY COSTS

The results in the previous section suggest that for medium to high volume suppliers, one should make deliveries as infrequently as possible. One factor that might make such a solution uneconomical is the cost of holding inventory. Recall that in the previous sections we assumed that the purchaser directly or indirectly pays for the cost of holding inventory at both facilities. In this section, we assume that the purchaser is concerned only about inventory at its own facility. This would be applicable to circumstances in which the demand of the purchaser represents only a small portion of the total demand at the supplier, so the amount of inventory at the supplier is not influenced by the amount of inventory held by the purchaser.

We present a model with cycle stock costs only, since in most cases, the amount of cycle stock is much larger than the amount of safety stock. Furthermore, since safety stock would have to be determined for each type of part individually, rather than in the aggregate, the optimization model becomes significantly more complex, making it difficult to infer the qualitative relationships that we seek. Throughout the remainder of this section we define h to be the annual inventory holding cost for one full truckload of goods.

The problem with cycle stock holding costs is to:

$$\text{minimize} \quad S\mu/u + C_e(\mu/u)\{1 - \Phi[(1-u)/(u/\mu)^{0.5}\sigma]\} + 0.5hu \tag{5}$$

$$\text{subject to} \quad 0 \leq u \leq 1.$$

Taking the first derivative of (5) with respect to u and making the same change of variable as in section 2, we have

$$\begin{aligned}\partial/\partial z = \quad & \Big\{\mu\{[2+\sigma^2z^2/\mu - \sigma z(\sigma^2z^2+4\mu)^{0.5}]/2\}^{-2}\{-S - C_e[1-\Phi(z)] + 0.5C_e\phi(z)z \\ & + C_e\mu^{0.5}\phi(z)\{[2+\sigma^2z^2/\mu - \sigma z(\sigma^2z^2+4\mu)^{0.5}]/2\}^{0.5}\} + 0.5h\Big\} \text{ x} \\ & 0.5\sigma[2\sigma z/\mu - (\sigma^2z^2+ 4\mu)^{0.5} - \sigma^3z(\sigma^2z^2+ 4\mu)^{-0.5}] \end{aligned} \tag{6}$$

We will have $z^* = 0$ if (6) is positive at $z = 0$ or

$$0.5h/\mu \ < 1+0.5b - 0.3989b\mu^{0.5},$$

where b is as defined earlier. The left hand side of the inequality is a convex decreasing function of μ and the right hand side is also convex decreasing. Since (i) the left hand side is equal to infinity when $\mu = 0$ and the right hand side takes on a finite value, and (ii) the right hand side eventually reaches zero for sufficiently large μ, the two functions (i.e., left and right hand sides) either do not intersect or intersect in exactly two points. In the former case, the inequality is not satisfied by any value of μ and $z^* > 0$ ($u^* < 1$). In the latter case, the condition is satisfied for some contiguous intermediate range of μ values (between the points of intersection), but not outside of that range. As before, this condition is independent of σ, so it is possible that $z^* = 0$ ($u^* = 1$) in other situations as well. Nonetheless, the condition permits rapid identification of some cases where $z^* = 0$.

5. COMPUTATIONAL RESULTS

We coded a simple procedure to search over u (in steps of .025) to determine approximately optimal utilizations. We considered this to be adequate for practical purposes. For our case with Brownian motion, this is done quite easily with Normal probability tables if one first transforms the trial values of u into the corresponding z values. Empical evidence suggests that the expected cost function is unimodal (although not necessarily convex) in u.

For a range of problem parameters (see Table 1), we used the search procedure to find "optimal" solutions. The parameters were chosen to represent a range of realistic values. The values of μ were chosen to represent the range from roughly one truckload per month up to one truckload per day. The values of the coefficient of variation are representative of those found in manufacturing environments. Values of C_e/S up to 10 were included to permit the possibility of air freight or other types of express service. Finally, the value of h/S (recall that h is the annual holding cost for one truckload of goods) was permitted to vary from 1.0 to 25, where the low end of the spectrum might represent inexpensive and voluminous raw materials (such as raw plastic), and the upper end of the range might represent electronic components.

In the first set of problems we considered the first model (without inventory costs). With few exceptions (e.g, $\mu = 250$ and $\sigma/\mu = 0.05$), the problems with $\sigma/\mu \leq 0.05$ had optimal planned utilization levels between approximately 50% and 80%. On the other hand, for problems with $\sigma/\mu \geq 0.10$, the optimal planned utilization levels were 100%, except in situations with low demand ($\mu \leq 25$), high penalties for emergency shipments ($b \geq 5.0$), and $\sigma/\mu = 0.10$.

Similar patterns were observed when inventory holding costs were added to the model. However, for problems with $\mu = 10$, $b = 10$, and $a = 25$, where the cost of emergency shipments and inventory can easily dominate the cost of normal shipments, optimal planned utilization levels were less than 100% for much larger values of σ/μ (in some cases up to $\sigma/\mu = 0.30$).

These results might suggest that in most circumstances in which trucking contracts would be applicable ($\mu \geq 25$ or 50), the commonly-used policy of 100% planned truck utilization is optimal for $\sigma/\mu \geq 0.10$. On the other hand, it is possible to argue that in a just-in-time environment, the value of σ/μ should be quite small--certainly less than 0.10. (This roughly corresponds to a 95% chance that usage deviations will be less than 20% of the mean.) It is precisely in these instances that planned utilizations less than 100% are optimal.

This result, unfortunately, does not exactly reconcile with most current practices. Some manufacturers plan for 90% to 95% utilization, but few plan for an average utilization of 80% or less. There are several reasons for this. First, most manufacturing facilities carry some safety stock which can provide a buffer against "short shipments." Thus, the manufacturer and supplier may discuss what can be omitted when the shipment quantity exceeds the truck capacity. Second, if the manufacturer-supplier discussion is impractical, or if the shipment quantity plus safety stock is still inadequate, the manufacturer may choose to modify the production schedule to produce what *can* be made rather than what *should* be made, or to make shorter production runs than originally planned. The economic impact of these schedule modifications is difficult to assess, and therefore difficult to include in an optimization model. Finally, in many instances, the organization responsible for setting up transportation contracts is not responsible for arranging (or paying for) emergency shipments. Thus, there is no incentive for it to optimize the overall cost of transportation.

The models discussed here can be used to quantify the impact of local optimization and inappropriate incentives. The economic impact of modifying the schedule in response to short shipments is quite difficult to quantify, and additional research is needed along these lines. The presence of safety stock can be incorporated into our models, but this is greatly complicated by the impact of the truck capacity on the ability to replenish safety stock. Finally, when demand is random, it may be desirable to have shipments consisting of more than one truckload, even when the transportation cost consists only of a fixed charge per truck. We are currently investigating these extensions.

Acknowledgement

The author gratefully acknowledges support from a U.S. automobile manufacturer under a research contract with the University of Michigan.

Table 1

Parameter	Values
μ	10, 25, 50, 100, 250
σ/μ	0.025, 0.05, 0.10, 0.20, 0.30
C_e/S	1.25, 2.5, 5.0, 10.0
h/S	1.0, 4.0, 9.0, 16.0, 25.0

REFERENCES

Allen, S.G. and D'Esopo, A. (1968), "An Ordering Policy for Stock When Delivery Can be Expedited," *Operations Research* 16 (4), 880-883.

Barankin, E.W. (1961), "A Delivery-Lag Inventory Model with an Emergency Provision," *Naval Research Logistics Quarterly* 8 (3), 285-311.

Bulinskaya, E. (1964), "Some Results Concerning Optimal Inventory Policies," Theory of Probability and Its Applications 9 (3), 502-507.

Daniel, K.H. (1962), " A Delivery-Lag Inventory Model with Emergency," Chapter 2 in Multistage Inventory Models and Techniques, Scarf, Gilford, Shelly (Eds.), Stanford University Press, Stanford, California.

Fukuda, Y. (1964), "Optimal Policies for the Inventory Problem with Negotiable Leadtime," *Management Science* 10 (4), 690-708.

Gross, D. and Soriano, A. (1972), "On the Economic Application of Airlift to Product Distribution and Its Impact on Inventory Levels," *Naval Research Logistics Quarterly* 19 (3), 501-507.

Hadley, G. and Whitin, T.M. (1962), "An Inventory-Transportation Model with N Locations," Chapter 5 in Multistage Inventory Models and Techniques, Scarf, Gilford, Shelly (Eds.), Stanford University Press, Stanford, California.

Moinzadeh, K. and Nahmias, S. (1988), "Continuous Review Model for an Inventory System with Two Supply Modes," *Management Science* 34 (6), 761-773.

Neuts, M.F. (1964), "An Inventory Model with Optional Time Lag," *SIAM J. Applied Math.* 12 (1), 179-185.

Rosenshine, M. and Obee, D. (1976), "Analysis of a Standing Order Inventory System with Emergency Orders," *Operations Research* 24 (6), 1143-1155.

Veinott, A.F., Jr. (1966), "The Status of Mathematical Inventory Theory," *Management Scence* 12 (11), 745-777.

Whittmore, A.S. and Saunders, S. (1977), "Optimal Inventory Under Stochastic Demand with Two Supply Options," *SIAM J. Applied Math.* 32 (2), 293-305.

Wright, G.P. (1968), "Optimal Policies for a Multi-Product Inventory System with Negotiable Lead Times," *Naval Research Logistics Quarterly* 15 (3), 375-401.

Wright, G.P. (1969), "Optimal Ordering Policies for Inventory Systems with Emergency Ordering," *Operational Research Quarterly* 20 (1), 111-123.

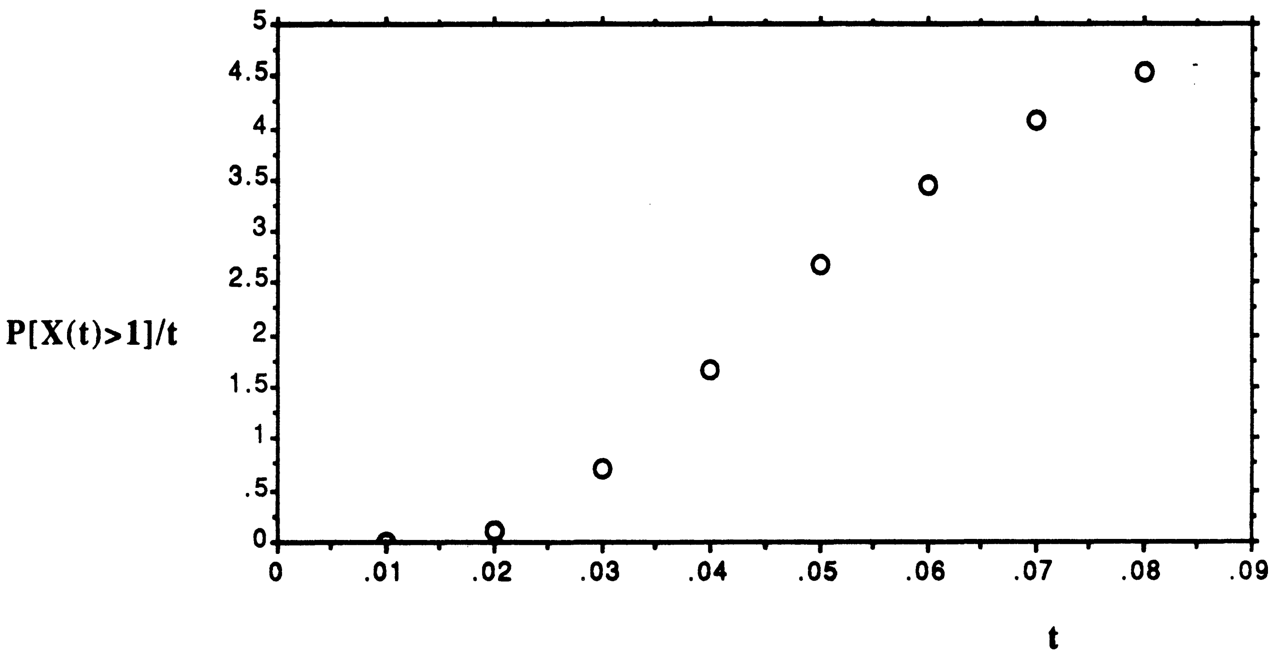
APPENDIX

Examples to Illustrate P[X(t) > 1]/t

Recall that we assume that the variance of the demand process is sufficiently small to ignore the possibility of a second emergency vehicle. Thus, for the following examples, we plot P[X(t) > 1]/t only for values of T for which the assumption is satisfied. (For large values of t, the ratio clearly approches zero since the numerator is bounded above by 1.0.)

<u>Example 1</u>

Brownian motion with $\mu = 10$, $\sigma^2 = 4$.



It should be noted that for Brownian motion, P[X(t) > 1]/t always has this general shape. The horizontal position of the curve is affected principally by μ, while the gradient over the domain in which the function increases is affected by the coefficient of variation (σ/μ).

Example 2

Poisson Process.

For this case $P[X(t) > 1]/t$ can be expressed analytically as

$$(1 - e^{-\lambda t} - \lambda e^{-\lambda t})/t.$$

The derivative with respect to t is

$$[(\lambda t + 1)(1 + \lambda)\, e^{-\lambda t} - 1]/t^2.$$

It can be shown analytically that this expression is positive for all $t \leq 1/\lambda$ (which encompasses the relevant range here, i.e., a truckload or less) when $\lambda > e-1$. Thus, for reasonable values of λ, $P[X(t) > 1]/t$ is strictly increasing over its entire relevant range.