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ANALYSIS OF LAMINAR FILM BOILING IN BOUNDARY-LAYER FLOWS WITH APPRECIABLE RADIATION

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ABSTRACT

This paper presents the theoretical study of the heat transfer and friction characteristics in the natural-convection film boiling on an inclined surface and a sphere, the forced-convection film boiling over a horizontal plate and the stagnation-flow film boiling when radiation is appreciable. The boiling liquid is either at the saturation temperature or subcooled. The two-phase flow and heat transfer problems have been formulated exactly within the framework of boundary-layer theory with the consideration of the shear stress and vapor velocity at the liquid-vapor interface. Through the use of the similarity transformation expressions are obtained to determine the vapor-film thickness, skin friction, and heat transfer rate. It is disclosed that the presence of surface radiation results in an increase in the heat transfer rate and a decrease in the skin friction.

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NOMENCLATURE

a	constant defined as $U_\infty = ax$ for stagnation flow
A	physical parameter, Equation (15-f)
B	physical parameter, Equation (7-g)
B_1	physical parameter, Equation (15-f)
C_p	specific heat
D	physical parameter, Equation (13-c)
E	physical parameter, defined as $h_{fg}\ Pr/C_p(T_W-T_S)$
F	temperature variable, Equations (9-c) and (18-d) for natural-convection film boiling and Equations (25-c) and (28-b) for forced-convection film boiling
f	velocity variable, Equations (9-b) and (18-c) for natural convection film boiling and Equations (25-b) for the forced-convection film boiling
g	gravitational acceleration
h	local heat transfer coefficient, q/T_W-T_S
h_{fg}	latent heat of evaporation
k	thermal conductivity
N_{Nu}	Nusselt number
N_{Pr}	Prandtl number
N_{Re}	Reynolds number
p	pressure
q	local heat flux from wall to vapor
R	radius of sphere
T	temperature: T_W = wall temperature; T_S = saturated temperature; T_∞ = free stream temperature
U_∞	free-stream velocity
u	velocity component of vapor in x-direction

v	velocity component of vapor in y-direction
x	coordinate measuring distance along the plate from leading edge
y	coordinate normal to plate
α	thermal diffusivity
α_r	absorption coefficient of vapor
β	coefficient of thermal expansion
δ	thickness of vapor film, Equations (8) and (18-a) for natural-convection film boiling and Equation (24) for forced-convection film boiling
ϵ	emissivity
η	similarity variable, Equation (9-a) and (18-b) for natural-convection film boiling and Equations (25-a) and (28-a) for forced-convection film boiling
η_δ	dimensionless vapor film thickness
Θ	dimensionless temperature defined as $\frac{T-T_s}{T_W-T_s}$ for vapor film and $\frac{T_L-T_\infty}{T_s-T_\infty}$ for liquid layer.
ν	kinematic viscosity
ρ	density
ρ_r	refractivity
σ	Stefan-Boltzmann constant
ϕ	angle of inclination or x/R
ψ	stream function, Equations (9-b) and (18-c) for natural-convection film boiling and Equations (25-b) and (28-b) for forced-convection film boiling

<u>Subscripts</u>

Unsubscripted quantities--vapor phase

L	liquid phase
r	radiation

s	at saturated state
w	wall surface
∞	free stream

Superscript

$'''$, $''$, $'$ differentiation with respect to η

INTRODUCTION

Film boiling is characterized by a vapor blanketing the entire heated surface. It frequently occurs when the operation of jets or rockets involves the contact of a boiling liquid with high temperature surfaces or in the boiling of mercury especially at high heat fluxes. Film boiling may occur also if cryogenic fluids are used to cool hot surfaces. Since at high temperature differences, the film boiling is the normal type of heat transfer between the heated surface and the liquid, it is therefore of a definite scientific and practical interest.

In stable film boiling regime heat is transferred from a heating surface by conduction through the vapor film and by boiling convection from the surface of the film to the surrounding liquid. Superimposed on this heat-flow path is the contribution of radiation to the total heat transfer. There are a few empirical equations being proposed to estimate the total surface conductance for film boiling when radiation is appreciable. However these equations are poor in accuracy and limited in application. This motivates the study of heat transfer and skin friction characteristics in both natural- and forced-convection film boiling through the application of the boundary-layer theory. Natural-convection film boiling over a vertical plate and forced-convection film boiling over a horizontal plate are investigated in Reference 1. This paper is the extension of Reference 1 to include more two-dimensional and axisymmetrical flows and to demonstrate the generality of the method of analysis for solving laminar film boiling problems.

Previous studies[2-9] of film boiling have been concerned with the situation where all motions are induced by gravity forces and where forced convection is absent. Such a process is usually called the natural-convection film boiling. Bromley[2] and Ellion[3] analyzed laminar film boiling on a vertical plate under the assumption of negligibly small inertia forces and convective effects. Hsu and Westwater[4] studied analytically and experimentally the film boiling in both laminar and turbulent regions. McFadden and Grosh[5] solved the boundary-layer equation for the vapor film and Cess,[6] by means of the integral technique, solved the vapor and liquid boundary-layer equations simultaneously. One feature common to prior analytical work is the assumption of zero interfacial velocity. Koh[7] analyzed the two-phase flow problem with the consideration of the shear stress and vapor velocity at the liquid-vapor interface. The results showed that for water, the effects of the interfacial velocity is small over a wide range of practical interest. Under the assumption of the constancy of vapor properties, the analysis was extended by Sparrow and Cess[8] to include the effects of subcooling and then by Koh and Nilson[9] for the effects of simultaneous action of radiation in saturated film boiling. It is rather unfortunate that the similarity transformation of the conservation equations and the appropriate boundary conditions failed, because the new and old variables coexist in one of the resulting boundary conditions, Equation (26) in Reference 9. In part of the present study, it is attempted to reexamine the problem treated in Reference 9 by introducing a different transformation. Furthermore, consideration is given to the temperature variation of the vapor density.

Cess and Sparrow[10,11] analyzed the film boiling in forced-convection boundary-layer flows for the situation in which the liquid is at the saturation temperature or subcooled. Relative to the case of liquid flow, the skin friction is reduced owing to film boiling. The heat transfer is found to increase as $(\Delta T)^{1/2}$.

In the present work, attention is focused on the natural-convection film boiling on an inclined plate and a sphere, the forced-convection film boiling over a horizontal plate, and the stagnation-flow film boiling. Consideration is given to the convective and radiational exchanges and the associated fluid motions in the vapor film and liquid layer. This is equivalent to solving a two-phase boundary-layer problem. In addition, calculation is carried out for the case of saturated film boiling.

ANALYSIS

Natural-Convection Film Boiling

The physical model and coordinate system selected for natural-convection film boiling is shown in Figure 1-a. It consists of an isothermal inclined plate immersed in a large volume of liquid. It is assumed that the vapor forms a stable film over the surface. The liquid has a bulk temperature T_∞ which is lower than the saturation temperature T_s prevailing at the liquid-vapor interface $y = \delta$. The temperature of the plate surface is prescribed as T_w and $T_w > T_s > T_\infty$.

It is assumed that under a stable film-boiling condition there exists a laminar layer of vapor film adjacent to the plate surface. Since the temperature of the plate and the vapor is relatively high, heat transfer takes place by convection as well as radiation.

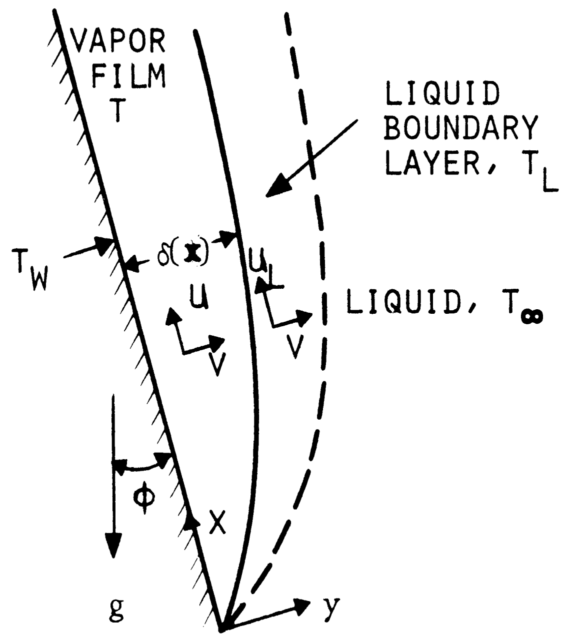
With the assumption, the application of the conservation laws for mass, momentum, and energy to the vapor film produces the following boundary-layer equations for a gravity-induced flow over the surface.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

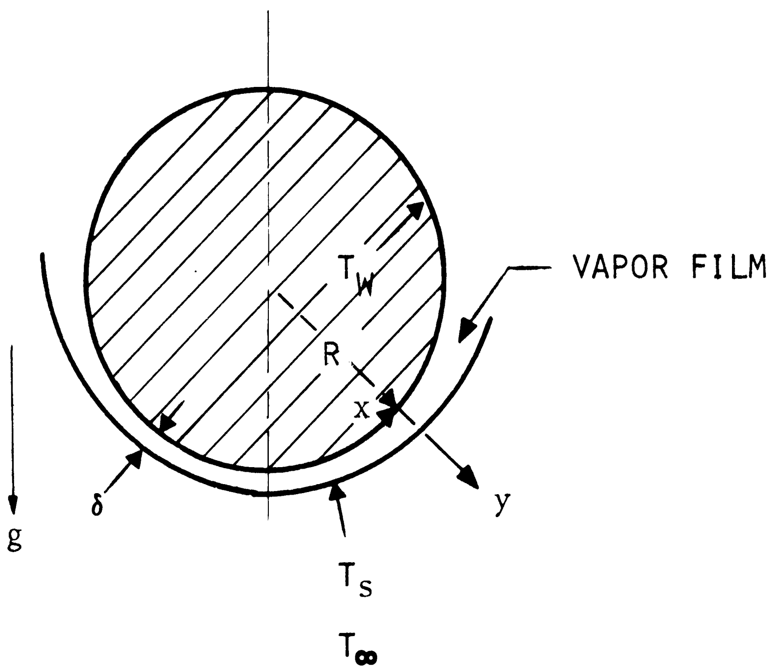
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\rho_L - \rho}{\rho} g \cos\phi \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho C_p}\frac{dq_r}{dy} \tag{3}$$

where dq_r/dy may be determined as follows.



(A) NATURAL-CONVECTION FILM BOILING ON AN INCLINED PLATE



(B) *NATURAL-CONVECTION FILM BOILING OVER A SPHERE*

FIGURE 1. PHYSICAL MODELS AND COORDINATES FOR NATURAL-CONVECTION FILM BOILING.

The vapor is assumed to be in thermodynamic equilibrium and behaves like a gray gas. If the radiation heat transfer between the vapor and the plate surface and liquid-vapor interface is assumed equivalent to that of a slab of gray gas bounded by two parallel black boundaries, then the local radiation flux[9] may be expressed as

$$q_r = 2\int_{\tau}^{\tau_2} \sigma T^4 E_2(t-\tau)dt - 2\int_{0}^{\tau} \sigma T^4 E_2(\tau-t)dt$$
$$+ 2\sigma T_s^4 E_3(\tau_2-\tau) - 2\sigma T_w^4 E_3(\tau)$$

For an optically thin vapor for which τ_2, the product of the absorption coefficient α_r of the vapor and the thickness δ of the vapor film, is much less than unity, the functions E_2 and E_3 may be approximated by $1-O(t)$ and $0.5-t+O(t^2)$ respectively. If the temperature gradient of the vapor in y-direction is much larger than that in x-direction, then the net radiation to unit volume of gas becomes

$$\frac{dq_r}{dy} = 2\,\alpha_r\,\sigma(T_w^4 + T_s^4 - 2\,T^4)$$

Equation (3) may, then, be rewritten as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - 4\alpha_r\sigma\, T_s^4\left[\left(\frac{T}{T_s}\right)^4 - \frac{1}{2} - \frac{1}{2}\left(\frac{T_w}{T_s}\right)^4\right] \qquad (4)$$

The mass and momentum conservation Equations (2) and (3) also apply to the liquid layer, but now a subscript L is employed to identify the physical quantities of the liquid layer. Under assumption that the radiation from the plate surface and vapor to the liquid is completely absorbed at the interface, the energy equation takes the form

$$u_L \frac{\partial T_L}{\partial x} + v_L \frac{\partial T_L}{\partial y} = \alpha_L \frac{\partial^2 T}{\partial y^2} \qquad (5)$$

The appropriate boundary conditions are

$$y = 0: \qquad u = v = 0, \qquad T = T_W \qquad (6\text{-a})$$

$$y = \infty: \qquad u_L = 0, \qquad (6\text{-b})$$

$$T_L = T_\infty \qquad \text{(for subcooled boiling only)}$$

At the liquid-vapor interface, it is required that the continuity of the tangential velocity, the tangential shear, the temperature, the mass-flow crossing interface, and the heat-flow crossing interface be preserved:

$$u_L = u \qquad (7\text{-a})$$

$$\left(\mu \frac{\partial u}{\partial y}\right)_L = \mu \frac{\partial u}{\partial y} \qquad (7\text{-b})$$

$$T_L = T_s \qquad (7\text{-c})$$

$$T = T_s \qquad (7\text{-d})$$

$$\rho_L\left(u \frac{d\delta}{dx} - v\right)_L = \rho\left(u \frac{d\delta}{dx} - v\right) \qquad (7\text{-e})$$

$$\left(k \frac{\partial T}{\partial y}\right)_L - k \frac{\partial T}{\partial y} + B + 2\alpha_r \sigma \int_0^\delta T^4 \, dy = \rho \, h_{fg} \frac{d}{dx} \int_0^\delta u \, dy \qquad (7\text{-f})$$

where

$$B = \frac{\sigma(T_W^4 - T_S^4)}{(\rho/\epsilon)_L + 1 + (\rho/\epsilon)_W} \qquad (7\text{-g})$$

is the net radiation flux between the surface and liquid-vapor interface. Equation (7-f) is an energy balance at the interface which states that the sum of the local heat conduction and net radiation gained at the interface is balanced by the heat of vaporization. The expression

$$\rho \frac{d}{dx} \int_0^\delta u \, dy$$

indicates the rate of vaporization per unit area. The contribution of the vapor radiation is included as the fourth term on the left-hand side of Equation (7-f).

Now, effects may be directed toward finding solutions. It is assumed that the film thickness $\delta(x)$ takes the form

$$\delta(x) \sim x^{1/4}(1 + \sum_{m=1}^{\infty} a_m x^{m/4}) \tag{8}$$

where a_m are the coefficients to be determined. The similarity variable is defined as

$$\eta = cy/x^{1/4} (1 + \sum_{m=1}^{\infty} a_m x^{m/4}) \tag{9-a}$$

where

$$C = \begin{cases} \left[\frac{g \cos \phi}{4\nu^2} \frac{\rho_L - \rho}{\rho}\right]^{1/4} & \text{for subcooled liquid} \\ \left[\frac{g \cos \phi}{4\nu^2} \frac{\rho_L R T_s}{p}\right]^{1/4} & \text{for saturated liquid} \end{cases}$$

when the constancy of vapor properties is assumed. The continuity equation can be satisfied by introducing a stream function such that $u = \partial\psi/\partial y$ and $v = - \partial\psi/\partial x$. We introduce the new variables

$$\psi(\eta) = 4c\nu \sum_{n=o}^{\infty} x^{(n+3)/4} (1 + \sum_{m=1}^{\infty} a_m x^{m/4}) f_n(\eta) \tag{9-b}$$

$$\theta(\eta) = \frac{T-T_s}{T_w-T_s} = F_o(\eta) + x^{1/4} F_1(\eta) + x^{1/2} F_2(\eta) + x^{3/4} F_3(\eta) + \ldots \tag{9-c}$$

From this and the expansion of

$$1/(1 + \sum_{m=1}^{\infty} a_m x^{m/4})$$

in a power series according to the binomial theorem, it follows

$$u = \frac{\partial\psi}{\partial y} = 4c^2\nu\, x^{1/2}\, (f_o' + x^{1/4} f_1' + x^{1/2} f_2' + x^{3/4} f_3' + \ldots) \tag{10-a}$$

$$v = -\frac{\partial\psi}{\partial x} = -4c\nu \sum_{n=o}^{\infty} \sum_{m=o}^{\infty} a_m\, x^{1/4(m+n-1)} [\frac{m+n+3}{4} f_n - \eta f_n' \frac{1}{4}$$

$$(1 + a_1 x^{1/4} + (2a_2 - a_1^2)\, x^{1/2} + (3a_3 - 3a_1a_2 + a_1^2)\, x^{3/4} + \ldots)] \tag{10-b}$$

where the primes represent differentiation with respect to the variable η . Owing to the employment of the binomial theorem in Equation (10-b), the restriction

$$\left| \sum_{m=1}^{\infty} a_m x^{m/4} \right| < 1$$

has been imposed on the solution.

The transformations, Equations (8), (9) and (10), also may be applied to the liquid layer provided that C_L and Θ_L are defined as

$$C_L = \left[\frac{g}{4\nu_L^2} \; \frac{T_s - T_\infty}{T_\infty}\right]^{1/4} \quad \text{for subcooled liquid}$$

$$= \left[\frac{g}{4\nu_L^2} \; \frac{\rho_L - \rho}{\rho}\right]^{1/4} \quad \text{for saturated liquid}$$

$$\Theta_L = \frac{T_L - T_\infty}{T_s - T_\infty} \tag{11}$$

With the introduction of the transformations into the conservation Equations (1) to (5), followed by collecting similar forms in x, one gets the following sets of simultaneous ordinary differential equations for both the vapor film and liquid layer.

For the vapor film:

$$f_o''' + 3f_o f_o'' - 2(f_o')^2 + 1 + \underline{\frac{T_w - T_s}{T_s} F_o} = 0 \tag{12-a}$$

$$f_1''' + 3f_o f_1'' - 5f_o' f_1' + 4f_o'' f_1 + \underline{\frac{T_w - T_2}{T_s}} F_1 = a_1 f_o''' - a_1 f_o f_o'' \tag{12-b}$$

$$f_2''' + 3f_o f_2'' - 6f_o' f_2' + 5f_o'' f_2 + \underline{\frac{T_w - T_2}{T_s}} F_2 = a_1 f_1'' - (a_1 f_o + 4f_1) f_1''$$

$$+ 3(f_1')^2 - a_1 f_o'' f_1 - (3a_1^2 - 2a_2) f_o''' + (a_1^2 - 2a_2) f_o f_o'' \tag{12-c}$$

The underlined terms which result from the buoyancy-force term in the momentum equation contribute to the coupling of the energy and momentum equations, i.e. the coupling of the f and F functions. The terms having the coefficient $(T_W - T_S)/T_S$ are absent if the constancy of fluid properties is assumed in the vapor film. For the liquid layer, the underlined terms in Equations (12-a), (12-b) and (12-c) must be replaced by F_o, F_1 and F_2, respectively.

$$\frac{1}{pr} F_o'' + 3 f_o F_o' = 0 \tag{13-a}$$

$$\frac{1}{N_{P_r}} F_1'' + 3f_o F_1' - f_o' F_1 = \frac{2a_1}{N_{P_r}} F_o'' - F_o'(a_1 f_o - 4f_1) \tag{13-b}$$

$$\frac{1}{N_{P_r}} F_2'' + 3f_o F_2' - 2f_o' F_2 - \underline{D[(1 + \frac{T_w - T_s}{T_s} F_o)^4 - \frac{1}{2} - \frac{1}{2}(\frac{T_w}{T_s})^4]}$$

$$= \frac{2a_1}{N_{P_r}} F_1'' - (a_1 f_o + 4f_1)F_1' + f_1' F_1 - \frac{1}{N_{Pr}}(3a_1^2 - 2a_2)F_o''$$

$$- (5f_2 + a_1 f_1 + 2a_2 f_o - a_1^2 f_o)F' \tag{13-c}$$

where

$$D = 4\alpha_r \sigma \frac{T_s}{(T_w - T_s)\rho C_p \nu C^2}$$

The underlined terms which result from the radiation term in the energy equation are for the vapor film only.

The boundary and matching conditions also may be rephrased in terms of the new variables. It must be noted that since the thickness $\delta(x)$ of the vapor film is small, one may take $\eta_L = 0$ at the liquid-vapor interface for convenience. With the application of the transformations, there results:

Plate surface:

$$f_n(0) = f_n'(0) = 0; \; F_o'(0) = 1 \text{ and } F_n(0) = 0 \tag{14}$$

for all n other than zero

Liquid-vapor interface:

$$f_n'(\eta_\delta) = f_{Ln}'(0) \tag{15-a}$$

$$f_o''(\eta_\delta) = [\frac{\mu_L\rho_L}{\mu\rho}]^{1/2} f_{Lo}''(0)$$

$$f_1''(\eta_\delta) - a_1 f_o''(\eta_\delta) = [\frac{\mu_L\rho_L}{\mu\rho}]^{1/2} [f_{L1}''(0) - a_1 f_{Lo}''(0)]$$

$$f_2''(\eta_\delta) - a_1 f_1''(\eta_\delta) - (a_1^2 - a_2) f_o''(\eta_\delta) = [\frac{\mu_L\rho_L}{\mu\rho}]^{1/2} [f_{L2}''(0) - a_1 f_{L1}''(0) + (a_1^2 - a^2) f_{Lo}''(0)] \qquad (15\text{-b})$$

$F_{Lo}(0) = 1$ and $F_{Ln}(0) = 0$ for all n other than zero (15-c)

$$F_n(\eta_\delta) = 0 \qquad (15\text{-d})$$

$$f_n(\eta_\delta) = [\frac{\mu_L\rho_L}{\mu\rho}] f_{Ln}(0) \qquad (15\text{-e})$$

$$3E\, f_o(\eta_\delta) = -F_o'(\eta_\delta) + AF_{Lo}'(0)$$

$$4E[f_1(\eta_\delta) + a_1 f_o(\eta_\delta)] = -[F_1'(\eta_\delta) - a_1 F_o'(\eta_\delta)] + A[F_{L1}'(0) - a_1 F_{Lo}'(0)] + B_1$$

$$5E[f_2(\eta_\delta) + a_1 f_1(\eta_\delta) + a_2 f_o(\eta_\delta)] = -[F_2'(\eta_\delta) - a_1 F'(\eta_\delta) + (a_1^2 - a_2) F_o'(\eta_\delta)] + A[F_{L2}'(0) - a_1 F_{L1}'(0) + (a_1^2 - a_2) F_{Lo}'(0)] + DP_r \int_0^{\eta_\delta} (1 + \frac{T_w - T_s}{T_s} F_o)^4 d\eta \qquad (15\text{-f})$$

where

$$A = \begin{cases} \dfrac{k_L C_L}{kC} \left(\dfrac{T_s - T_\infty}{T_w - T_s}\right) & \text{for subcooled boiling} \\ 0 & \text{for saturated boiling} \end{cases}$$

$$B_1 = B/k(T_w - T_s)\ C$$

$$E = \rho\, h_{fg}\, \nu/k\, (T_w - T_s) = h_{fg}\, N_{P_r}/C_p\, (T_w - T_s)$$

For subcooled liquid the functions $f_{Ln}(0)$, $f'_{Ln}(0)$ and $f''_{Ln}(0)$ are replaced by

$$\left(\frac{\rho_L}{\rho_L - \rho}\ \frac{T_s - T_\infty}{T_\infty}\right)^{1/4} f_{Ln}(0),$$

$$\left(\frac{\rho_L}{\rho_L - \rho}\ \frac{T_s - T_\infty}{T_\infty}\right)^{1/2} f'_{Ln}(0)$$

and

$$\left(\frac{\rho_L}{\rho_L - \rho}\ \frac{T_s - T_\infty}{T_\infty}\right)^{3/4} f'_{Ln}(0) \quad \text{respectively.}$$

In case the vapor density is assumed constant, $\rho_L(T_s - T_\infty)/(\rho_L - \rho)T_\infty$ in the matching conditions must be replaced by unity.

Free stream:

$f'_{Lo}(\infty) = 0$ and

$f'_{Ln}(\infty) = 0$ for all n other than zero;

$F_{Ln}(\infty) = 0$.

Figure 1(b) shows the physical model for natural-convection film boiling over a sphere with radius R. The governing equations and the boundary and matching conditions are identical with those of the previous case except that the continuity equation now reads

$$\frac{\partial ur}{\partial x} + \frac{\partial ur}{\partial y} = 0 \qquad (17)$$

and ϕ is defined as x/R.

Now, the film thickness and the similarity variable are defined as

$$\delta(x) = 1 + \sum_{m=1}^{\infty} a_{2m}\, \phi^{2m} \qquad (18\text{-a})$$

and

$$\eta = Cy/(1 + \sum_{m=1}^{\infty} a_{2m}\, \phi^{2m}), \qquad (18\text{-b})$$

respectively, where

$$C = \begin{cases} \left[\dfrac{g}{R\nu^2} \dfrac{\rho_L - \rho}{\rho}\right]^{1/4} & \text{for vapor} \\ \left[\dfrac{g}{R\nu_L^2} \dfrac{\rho_L - \rho}{\rho}\right]^{1/4} & \text{for saturated liquid} \\ \left[\dfrac{g}{R\nu_L^2} \dfrac{\rho_L - \rho}{\rho}\right]^{1/4} & \text{for subcooled liquid} \end{cases}$$

The continuity equation may be satisfied by introducing

$$\psi(\eta) = c\nu \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} a_{2m}\, f_{2n+1}\, \phi^{2m + 2n + 1} \qquad (18\text{-c})$$

where $a_o = 1$, such that $u = \frac{1}{r}\frac{\partial\psi}{\partial y}$ and $v = -\frac{1}{r}\frac{\partial\psi}{\partial x}$. The dimensionless temperature is defined as

$$\theta(\eta) = \sum_{n=0}^{\infty} F_{2n}(\eta)\, \phi^{2n}. \qquad (18\text{-d})$$

When the transformations are introduced into the conservation Equations (2), (3) and (4), there results

$$f_1'^2 - 2f_1 f_1'' - H - f_1''' = 0 \qquad (19\text{-a})$$

$$4f_1'f_3' - 2f_1 f_3'' - 4f_1'' f_3 + (\frac{1}{3} - 2a_2) f_1 f_1'' + 2a_2 f_1'' + J - f_3''' = 0 \qquad (19\text{-b})$$

$$3f_3'^2 - 4f_3 f_3'' + \frac{1}{3} f_1 f_3'' + \frac{1}{3} f_1'' f_3 + \frac{1}{45} f_1 f_1'' - K - 2a_2 f_1'' f_3 - 2a_2 f_1 f_3''$$
$$+ 2a_2 f_3'' + 2a_2^2 f_1 f_1'' - 4a_4 f_1 f_1'' + (2a_4 - 3a_2^2) f_1'' - 6f_5 f_1''$$
$$+ 6f_1'f_5' - 2f_1 f_5'' - f_5''' = 0 \qquad (19\text{-c})$$

......

where

$H = 1$, $J = \frac{1}{6}$, $K = \frac{1}{120}$ ... for vapor film

$H = 0$, $J = 0$, $K = 0$ ... for saturated liquid

$H = F_{L0}$, $J = F_{L2} - \frac{1}{6} F_{L0}$,

$K = F_{L4} - \frac{1}{6} F_{L2} + \frac{1}{120} F_{L0}$ for subcooled liquid

$$\frac{1}{N_{Pr}} F_o + 2f_1 F_o' = \begin{cases} D[(\frac{T_w - T_s}{T_s} F_o'' + 1)^4 - \frac{1}{2} - \frac{1}{2} (\frac{T_w}{T_s})^4] \ldots \text{ for vapor} \\ 0 \quad \text{for liquid} \end{cases} \qquad (20\text{-a})$$

$$\frac{1}{N_{Pr}} F_2'' - 2f_1 F_2' + 2f_1 F_2' + 4f_3 F_o' - \frac{1}{3} f_1 F_o' - 2 \frac{1}{N_{Pr}} a_2 F_o'' + 2 a_2 f_1 F_o'$$
$$= \begin{cases} D[-4(\frac{T_w - T_s}{T_s} F_o + 1)^3 \frac{T_w - T_s}{T_s} + 2(\frac{T_w}{T_s})^3 \frac{T_w - T_s}{T_s} b_2] \ldots \text{ for vapor} \\ 0 \ldots \text{ for liquid} \end{cases}$$
$$(20\text{-b})$$

$$\frac{1}{N_{Pr}} F_4'' - 4f_1'F_4 + \underline{2f_1F_4' - 2f_3'F_2 +} 4f_3F_2' - \frac{1}{3} f_1F_2' + 6f_5F_o' - (\frac{1}{3} - 2a_2)f_3F_o'$$

$$- (\frac{1}{45} - 4a_4 + 2a_2^2)f_1F_o' - \frac{1}{N_{Pr}}(2a_4 - 3a_2^2)F_o'' - \frac{2}{N_{Pr}} a_2F_2''$$

$$= \begin{cases} -D[4(\frac{T_w-T_s}{T_s} F_o + 1)^3 \frac{T_w-T_s}{T_s} F_4 + 6(\frac{T_w-t_s}{T_s} F_o + 1)^2(\frac{T_w-T_s}{T_s} F_4)^2] \quad \text{... for vapor} \\ 0 \quad \text{... for liquid} \end{cases} \tag{20-c}$$

....

The boundary and matching conditions are

$$f_{2n+1}(0) = f_{L(2n+1)}(0) = f'_{2n+1}(0) = f'_{L(2n+1)}(0) = 0 \tag{21-a}$$

$$F_o(0) = F_{LO}(0) = 1, \qquad F_2(0) = 0\ , \quad F_4(0) = 0\ , \ ...$$

$$F_{L(2m)}(0) = 0 \qquad \text{for all m other than zero} \tag{21-b}$$

$$F_{2m}(\eta_\delta) = 0 \tag{22-a}$$

$$f'_{L(2n+1)}(0) = (\frac{C}{C_L})^2 \frac{\nu}{\nu_L} f'_{(2n+1)}(\eta_\delta)\ , \tag{22-a}$$

where

$$(\frac{C}{C_L})^2 \frac{\nu}{\nu_L} = \begin{cases} 1 \text{ ... for saturated liquid} \\ [\frac{\rho_L-\rho}{\rho} \frac{T_\infty}{T_s-T_\infty}]^{1/2} \text{ ... for subcooled liquid} \end{cases}$$

$$f''_{L1}(0) = G\, f''(\eta_\delta)$$

$$f''_{L3}(0) = a_2f''_{L1}(0) + G[f''_3(\eta_\delta) - a_2f''_1(\eta_\delta)]$$

$$f''_{L5}(0) = a_2f''_{L3}(0) + (a_4 - a_2^2)f''_{L1}(0) + G[f''_5(\eta_\delta) - a_2f''_3(\eta_\delta)$$

$$- (a_4 - a_2^2)\, f''_1(\eta_\delta) \quad .....,$$

$$\text{where} \quad G = \begin{cases} \left[\dfrac{\mu\rho}{\mu_L\rho_L}\right]^{1/2} & \text{... for saturated liquid} \\[2ex] \left[\dfrac{\mu\rho}{\mu_L\rho_L}\right]^{1/2} \left[\dfrac{\rho_L-\rho}{\rho} \quad \dfrac{T_\infty}{T_s-T_\infty}\right]^{3/4} & \text{... for subcooled liquid} \end{cases}$$

$$AF'_{Lo}(0) - F'(\eta_\delta) + B_1 + \frac{D}{2} N_{Pr} \int_0^{\eta_\delta} (1 + \frac{T_w-T_s}{T_s}\theta_o)^4 \, d\eta = Ef_1(\eta_\delta)$$

$$A[F'_{L2}(0)-a_2F'_{Lo}(0)] - F'_2(\eta_\delta) + a_2F'_o(\eta_\delta)$$

$$+ 2DN_{Pr} \int_0^{\eta_\delta} (1 + \frac{T_w-T_s}{T_s} F_o)^3 \; \frac{T_w-T_s}{T_s} \; F_2 d\eta$$

$$= 3E[a_2f_1(\eta_\delta) + f_3(\eta_\delta)]$$

$$F_{L(2m)}(\infty) = 0 \; , \; f_{L(2m)}(\infty) = 0 \qquad (23)$$

It is important to restate that the analysis may be applied to both the saturated and subcooled film boiling. For the latter case, the functions F_{Ln} and parameter A become identically zero because the liquid temperature is constant and equal to the saturation temperature, i.e., $T_L(\eta_L) = T_\infty = T_{sat}$.

Each set of differential equations for f_n , f_{Ln} , F_n , and F_{Ln} requires ten of eleven boundary and matching conditions. The extra one as expressed by Equation (15-f) may be used for the evaluation of the thickness η_δ and the coefficients a_n .

Now, it is desirable to inspect the physical parameters governing the natural-convection film boiling. There is a total of nine: N_{Pr} , N_{Pr_L}, T_W/T_S , D , $[(\rho\mu)_L/\rho\mu]^{1/2}$, $(\rho_L/\rho_L-\rho)(T_S-T_\infty/T_\infty)$, A , B_1 and E . Of these, the first four arise in connection with the differential equations (13) for the vapor film and liquid vapor, while the last six enter through the interface matching conditions. The parameter which appears only in the natural-convection but not in the forced-convection film boiling is $(\rho_L/\rho_L-\rho)(T_S-T_\infty/T_\infty)$. This results from the consideration given to the temperature dependency of the vapor density. In the absence of the vapor and wall radiation, the governing parameters reduce to five: N_{Pr} , N_{Pr_L} , $[(\rho\mu)_L/\rho\mu]^{1/2}$, A and E .

Forced-Convection Film Boiling

The physical model and coordinate system are shown in Figure 2(A). The situation is, the laminar boundary-layer flow of a liquid with velocity U_∞ over a flat plate. The liquid has a free-stream temperature T_∞ which is lower than the saturation temperature T_S . The plate is maintained at the temperature T_W , sufficiently higher than T_S that film boiling occurs on the plate.

With the same assumption as made in the previous case, the application of the conservation laws for mass, momentum, and energy to both vapor and liquid produces the boundary-layer equations which are identical with Equations (1), (2) with g = 0, (4), and (5). Equation (5) is needed only for the subcooled film boiling since the liquid temperature is essentially constant for the saturated film boiling.

The boundary conditions at the surface of the plate (y = 0) and the matching conditions at the liquid-vapor interface (y = δ) are identical with Equations (6-a) and (7) respectively. However, far from the plate, in the bulk of the liquid, the velocity approaches U_∞ and the temperature approaches its bulk temperature T_∞. Therefore, Equation (6-b) may be used as the boundary conditions provided that u_L is equated to U_∞.

Utilizing prior experience with the free-convection film boiling in the previous section, the new dependent and independent variables for flow over a plate are defined:

$$\delta(x) \sim x^{1/2} \left(1 + \sum_{m=1}^{\infty} a_m x^{m/2}\right) \tag{24}$$

$$\eta = \frac{y}{2} \left[\frac{U_\infty}{\nu x}\right]^{1/2} \Big/ \left(1 + \sum_{m=1}^{\infty} a_m x^{m/2}\right) \tag{25-a}$$

$$\psi(\eta) = 2c\nu \sum_{n=0}^{\infty} x^{(n+1)/2} \left(1 + \sum_{m+1}^{\infty} a_m x^{m/2}\right) f_n(\eta) \tag{25-b}$$

$$\theta(\eta) = \sum_{n=0}^{\infty} x^{n/2} F_n(\eta) \tag{25-c}$$

where C is defined as $(1/2)\ [U_\infty/\nu]^{1/2}$.

From this it follows that

$$u = \frac{\partial\psi}{\partial y} = 2c^2\nu[f_o' + x^{1/2} f_1' + xf_2' + x^{3/2} f_3' + \dots] \tag{26-a}$$

$$v = -\frac{\partial\psi}{\partial x} = -2c\nu \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} x^{1/2(m+n-1)} \left\{\frac{m+n}{2} f_n - \eta f_n' \left[\frac{1}{2}\right.\right.$$

$$\left.\left. + \frac{1}{2} a_1 x^{1/2} + (a_2 - \frac{1}{2} a_1^2)x + \frac{1}{2}(3a_3 - 3a_1a_2 + a_1^3)x^{3/2} + \dots\right)\right\} \tag{26-b}$$

Again, the restriction

$$\left|\sum_{m=1}^{\infty} a_m x^{m/2}\right| < 1$$

has been imposed on the solution owing to the application of the binomial theorem in Equation (26-b). When the transformations defined by Equations (25) and (26) are introduced into the conservation Equations (1), (2), (4) and (5) it yields

$$f_o''' + f_o f_o'' = 0 \qquad (26\text{-a})$$

$$f_1''' + f_o f_1'' - f_o' f_1' + 2f_o'' f_1 = 2a_1 f_o''' - a_1 f_o f_o'' \qquad (26\text{-b})$$

$$f_2''' + f_o f_2'' - 2f_o' f_2' + 3f_o'' f_2 = 2a_1 f_1''' - (2f_1 + a_1 f_o) f_1''$$

$$+ (f_1')^2 - a_1 f_o' f_1 + (2a_2 - 3a_1^2) f_o''' + (\frac{1}{2} a_1^2 - a_2) f_o f_o'' \qquad (26\text{-c})$$

$$\frac{1}{N_{Pr}} F_o'' + f_o F_o' = 0 \qquad (27\text{-a})$$

$$\frac{1}{N_{Pr}} F_1'' + f_o F_1' - f_o' F_1 = \frac{2}{N_{Pr}} a_1 F_o'' - (a_1 f_o - 2f_1) F_o' \qquad (27\text{-b})$$

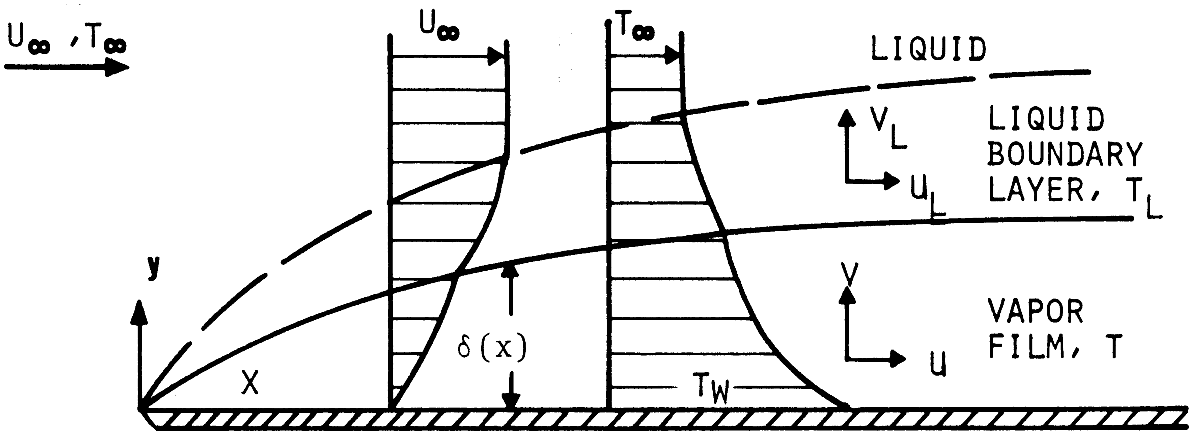
$$\frac{1}{N_{Pr}} F_2 + f F - 2f' F - \frac{D[(1 + \frac{T_w - T_s}{T_s} F_o)^4 - \frac{1}{2} - \frac{1}{2} (\frac{T_w}{T_s})^4]}{\frac{1}{N_{Pr}}}$$

$$= \frac{2}{N_{Pr}} a_1 F_1'' - (a_1 f_o + 2f_1) F_1' + f_1' F_1 + \frac{1}{N_{Pr}} (2a_2 - 3a_1) F_o''$$

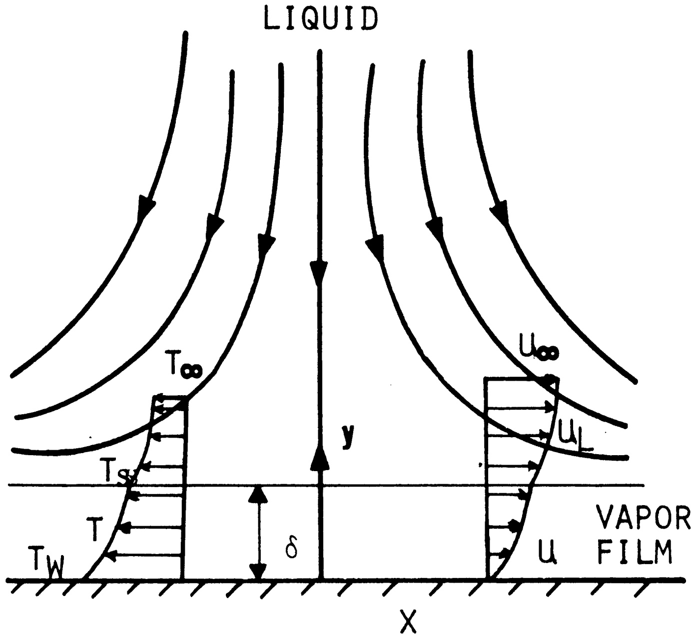
$$- [3f_2 + a_1 f_1 + (2a_2 + a_1^2) f_o] F_o' \qquad (27\text{-c})$$

With the introduction of the transformations, the appropriate boundary and matching conditions become identical with Equations (13), (14) and (15) provided that the coefficients 3E, 4E, 5E,... of the left-side terms of Equation (14-f) are replaced by 1E, 2E, 3E respectively.

The physical model for stagnation film boiling in two-dimensional flow is shown in Figure 2(B). The free stream velocity U_∞ can be expressed as ax, where a is a constant.



(A) FORCED-CONVECTION FILM BOILING ON A FLAT PLATE



(B) STAGNATION FILM BOILING IN TWO-DIMENSIONAL FLOW

FIGURE 2. PHYSICAL MODELS AND COORDINATES FOR FORCED-CONVECTION FILM BOILING.

With the introduction of

$$\eta = cy \tag{28-a}$$

$$\psi = \nu cxf(\eta) \tag{28-b}$$

$$\Theta(\eta) = \frac{T-T_s}{T_W-T_s} \tag{28-c}$$

$$u = \frac{\partial X}{\partial Y} = \frac{1}{2} U_\infty f' \text{ and}$$

$$v = - cf \tag{29}$$

the momentum and energy equations, one obtains the following ordinary differential equations for both the vapor film and liquid layer.

$$f''' + ff'' + 4 -(f')^2 = 0 \tag{30}$$

$$\frac{1}{N_{Pr}} \Theta'' + f\Theta' = \frac{D}{2} [(1 + \frac{T_W-T_s}{T_s} \Theta)^4 - \frac{1}{2} - \frac{1}{2} (\frac{T_W}{T_s})^4] \tag{31}$$

where

$$C = [a/2\nu]^{1/2} .$$

The boundary and matching conditions are:

Plate surface: $f(0) = f'(0) = 0 \; ; \; \Theta(0) = 1 ,$ (32)

Liquid-vapor interface:

$$f'(\eta_\delta) = f_L'(0) \; , \; f''(\eta_\delta) = [\frac{\mu_L\rho_L}{\mu\rho}]^{1/2} f_L''(0) \; , \; \Theta_L(0) = 1, \; \Theta(\eta_\delta) = 0$$

$$f(\eta_\delta) = [\mu_L\rho_L/\mu\rho]f_L(0)$$

$$Ef(\eta_\delta) = - \Theta(\eta_\delta) + A \, \Theta_L'(0) + B_1$$

(33)

Free stream:

$$f_L'(\infty) = 2, \; \Theta_L(\infty) = 0 . \tag{34}$$

The stagnation film boiling in three-dimensional flow was analyzed in an analogous manner. In the interest of brevity the results are not presented here.

It is important to examine the physical parameters which govern the transport phenomena, There is a total of eight in the forced-convection film boiling process. They are: N_{Pr} , N_{Pr_L} , T_w/T_s , D, $[\frac{(\rho\mu)_L}{\rho\mu}]^{1/2}$, A , B_1 and E . Of these, N_{Pr} and N_{Pr_L} arise in connection with the differential Equation (27) for the vapor film and liquid layer respectively. The next two arise in connection with the differential Equation (27) for the vapor film induced by the vapor radiation. The last four enter through the matching conditions at the liquid-vapor interface. Among the four, B_1 is related to the surface radiation processes and A is connected to the subcooling of the free stream.

In the absence of subcooling, N_{Pr_L} and A cease to be the governing parameters. When the vapor radiation can be neglected, D must be excluded from consideration as a parameter. When both the vapor and surface radiation can be neglected, T_w/T_s , D , and B_1 must be excluded. This leaves five parameters to govern the transport phenomena.

The dimensionless vapor film thickness η_δ and the coefficients a_m are not considered as independent parameters because there is a unique relation among η_δ , a_m , A , B_1 , D , and E as written by Equation (15-f).

RESULTS

Heat Transfer

The local heat flux contributed by both radiation and convection at the plate surface is

$$q = -k \left(\frac{\partial T}{\partial y}\right)_{y=0} - 2\,\alpha_r \sigma \int_0^\delta T^4\,dy + B$$

When the Reynolds and Nusselt numbers, defined as $N_{Re} = U_\infty x/\nu$ and $N_{Nu} = hx/k$ ($N_{Nu} = hR/k$ for sphere) respectively, are introduced and the heat flux is rephased into the variables of the analysis, there follows:

For natural-convection film boiling one gets

$$\frac{N_{Nu}}{cx^{3/4}} = - F_o'(0) - [F_1'(0) - a_1 F_o'(0) - B_1]x^{1/4} - [F_2'(0)$$
$$-a_1 F_1'(0) + (a_1^2 - a_2)F_o'(0) + \frac{1}{2} D\, N_{Pr} \int_0^{\eta_\delta} (1 + \frac{T_w - T_s}{T_s} F_o)^4\, d\eta]x^{1/2} + \ldots \quad (35)$$

for an inclined surface and

$$\frac{N_{Nu}}{cR} = - F_o'(0) + B_1 - \frac{1}{2} DN_{Pr} \int_0^\eta [B_o^4 + (B_o^4 a_2 + 4B_o^3 B_2)\xi^2 + \ldots]d\eta \quad (36)$$

for a sphere, where

$$B_o = (T_w - T_s)F_o + T_s, \quad B_2 = (T_w - T_s)F_2$$

For forced-convection film boiling on a horizontal surface

$$\frac{N_{Nu}}{N_{Re}^{1/2}} = - \frac{1}{2} F_o'(0) - \frac{1}{2}[F_1'(0) - a_1 F_o'(0) - B_1]x^{1/2} - \frac{1}{2}[F_2'(0)$$
$$-a_1 F_1'(0) + (a_1^2 - a_2)F_o'(0) + D\, N_{Pr} \int_0^{\eta_\delta} (1 + \frac{T_w - T_s}{T_s} F_o)d\eta]x \quad (37)$$

For stagnation film boiling in two-dimensional flow

$$\frac{N_{Nu}}{cx} = - \Theta'(0) + B_1 \tag{38}$$

where $- \Theta'(0)$ and B_1 represent the heat transferred by conduction through the vapor film and by radiation from the surface, respectively. The leading terms of Equations (35) and (37) represent the corresponding heat transfer results in the absence of radiation exchange process. The second terms are the key terms in determining the effects of radiation exchange, since the contribution of the other terms is generally of secondary importance.

An investigation of Equations (35) and (37) reveals that their second terms consist of $F_0'(0)$, $F_1'(0)$, a_1, and B_1 in which $F_1'(0)$ and a_1 are inter-related to the parameter B_1 for the surface radiation by Equations (13-b) and (27-b) and the second expression of Equation (15-f). Since the contribution due to the vapor radiation as represented by the parameter D and its associated quantities a_2 and $F_2'(0)$ first appears in the third terms of Equations (35) and (37), the radiation effects on heat transfer is contributed mainly by the surface radiation, and the vapor radiation plays a rather unimportant role.

Skin Friction

The shear stress exerted by the flowing fluid on the surface may be calculated by Newton's shearing formula $\tau_w = - \mu(\partial u/\partial y)_{y=0}$. A dimensionless representation of the wall shear may be achieved by utilizing a friction coefficient defined as $\tau_w/\frac{1}{2} \rho U_\infty^2$. When this is evaluated in terms of the variables of the analysis, there results

$$\frac{\tau_w \, N_{Re}^{1/2}}{\frac{1}{2}\rho U_\infty^2} = \frac{1}{2}\{ f_o''(0) + [f_1''(0) - a_1 f_o''(0)] \, x^{1/2}$$

$$+ [f_2''(0) - a_1 f_1''(0) + (a_1^2 - a_2) f_o''(0)] \, x + \ldots \} \qquad (39\text{-a})$$

for forced-convection film boiling over a horizontal plate.

$$= \frac{1}{\sqrt{2}} \; f''(0) \qquad (39\text{-b})$$

for stagnation film boiling in two-dimensional flow.

The leading term in Equation (39-a) represents the skin friction in the absence of radiation exchange process. The second term is the most important one in determining the radiation effects.

Based on the similar arguments for heat transfer, the radiation effects on the skin friction are found to be caused mainly by the surface radiation. The vapor radiation exerts a negligible or secondary effect.

NUMERICAL ILLUSTRATIONS

Equations (12) and (13) for natural-convection film boiling over an inclined plate, Equations (26) and (27) for force-convection film boiling over a horizontal plate and Equations (30) and (31) for stagnation film boiling in two-dimensional flow were numerically integrated (Runge-Kutta method) in conjunction with their appropriate boundary conditions by means of an IBM 7090 digital computer. The first step is to prescribe the dimensionless vapor film thickness η_δ . The calculation is carried out for the saturated boiling of water under one atmospheric pressure with the neglect of gas radiation in the vapor film. This is justified as long as the vapor film is thin and the vapor pressure is not high. In other words, all radiation terms in the energy equations are neglected. Only the effects of radiation between the plate surface and the fluid interface, which appear in the boundary conditions, are taken into consideration. The range of the surface temperatures was from 280 to 3225 F (corresponding to η_δ from 0.6 to 1.6) and from 291 to 996 F (corresponding to η_δ from 0.2 to 0.6) respectively, for the natural and forced-convection film boiling. The emissivities of the wall and liquid-vapor interface are taken to be unity. The typical velocity and temperature profiles are shown in Figures 3 - 6.

Figures 3 and 4 show the vapor velocity and temperature profiles in the natural-convection film boiling for the special case of constant vapor property. The terms f_o' and F_o' correspond to the velocity and temperature profiles respectively in the absence of radiation exchange. Since the magnitudes of f_2' and F_2 are rather of secondary importance in comparison with those of f_1' and F_1 , it is observed from Figures 3

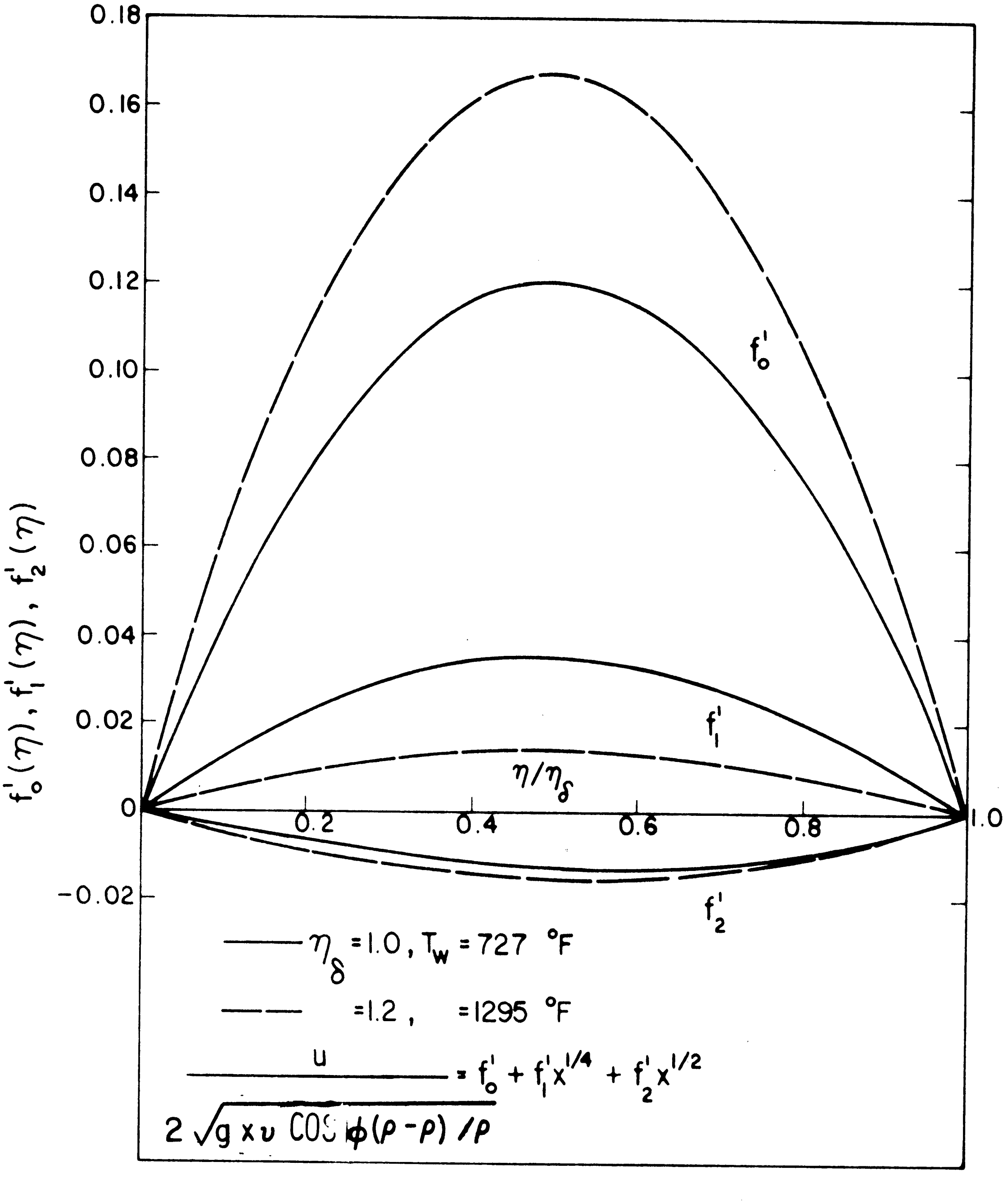


FIGURE 3. FUNCTIONS ASSOCIATED WITH VAPOR VELOCITY IN NATURAL-CONVECTION FILM BOILING ON AN INCLINED PLATE FOR $T_S = T_\infty = 212°F$.

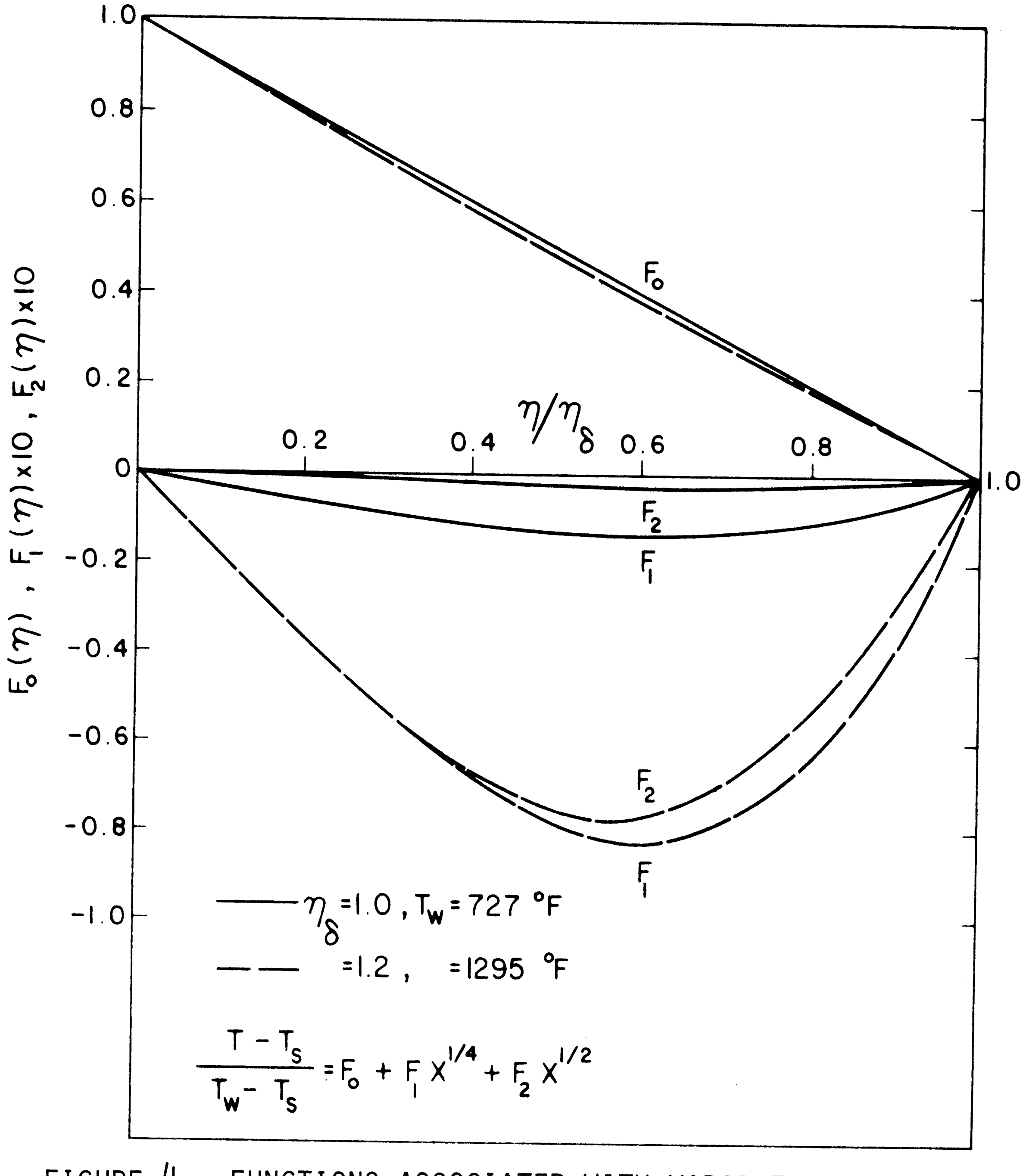


FIGURE 4. FUNCTIONS ASSOCIATED WITH VAPOR TEMPERATURE IN NATURAL-CONVECTION FILM BOILING ON AN INCLINED FLAT PLATE FOR $T_S = T_\infty = 212°F$.

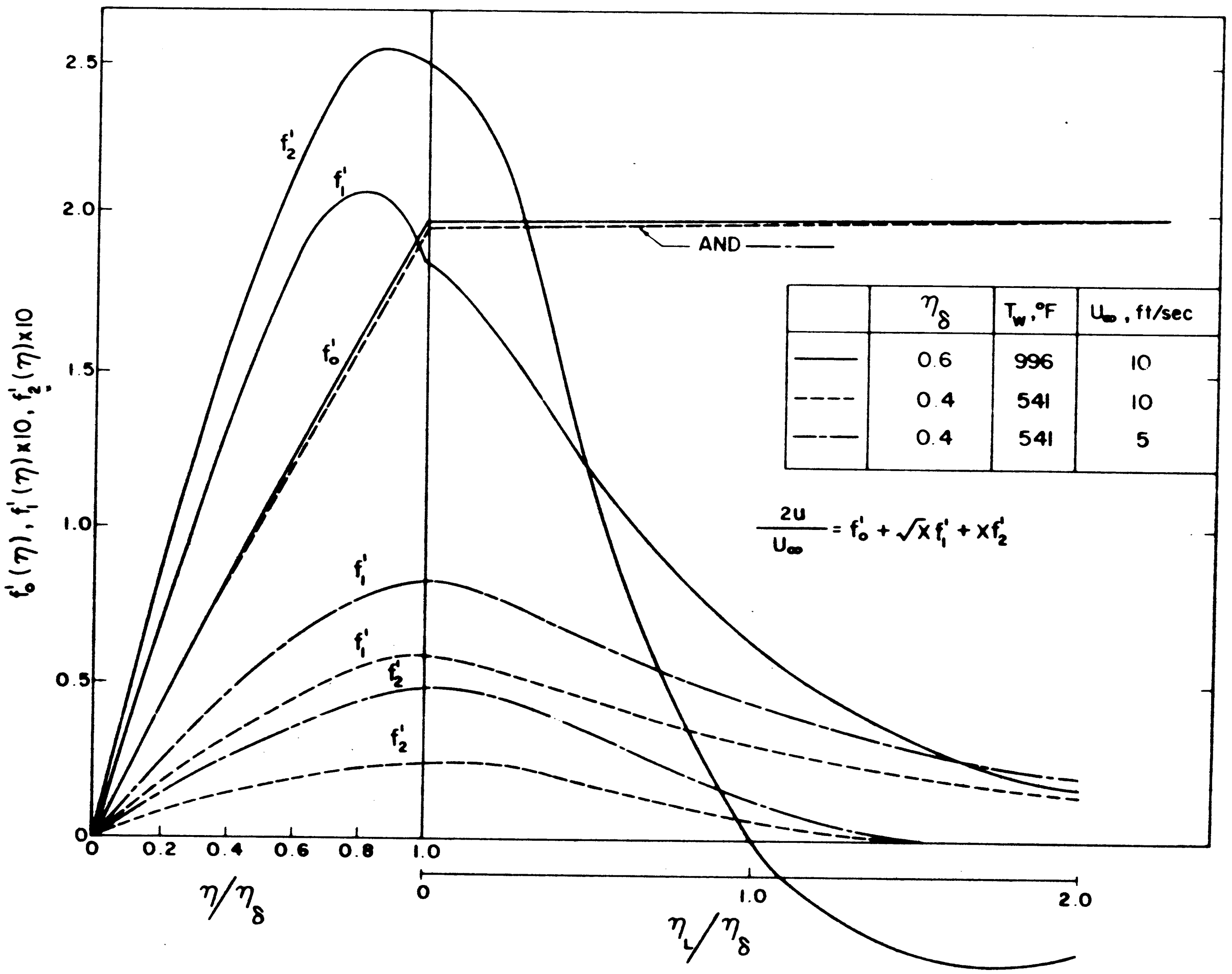


FIGURE 5. FUNCTIONS ASSOCIATED WITH VELOCITY PROFILES IN THE VAPOR AND LIQUID BOUNDARY LAYERS FOR FORCED-CONVECTION FILM BOILING OVER A HORIZONTAL PLATE FOR $T_s = T_\infty = 212°F$.

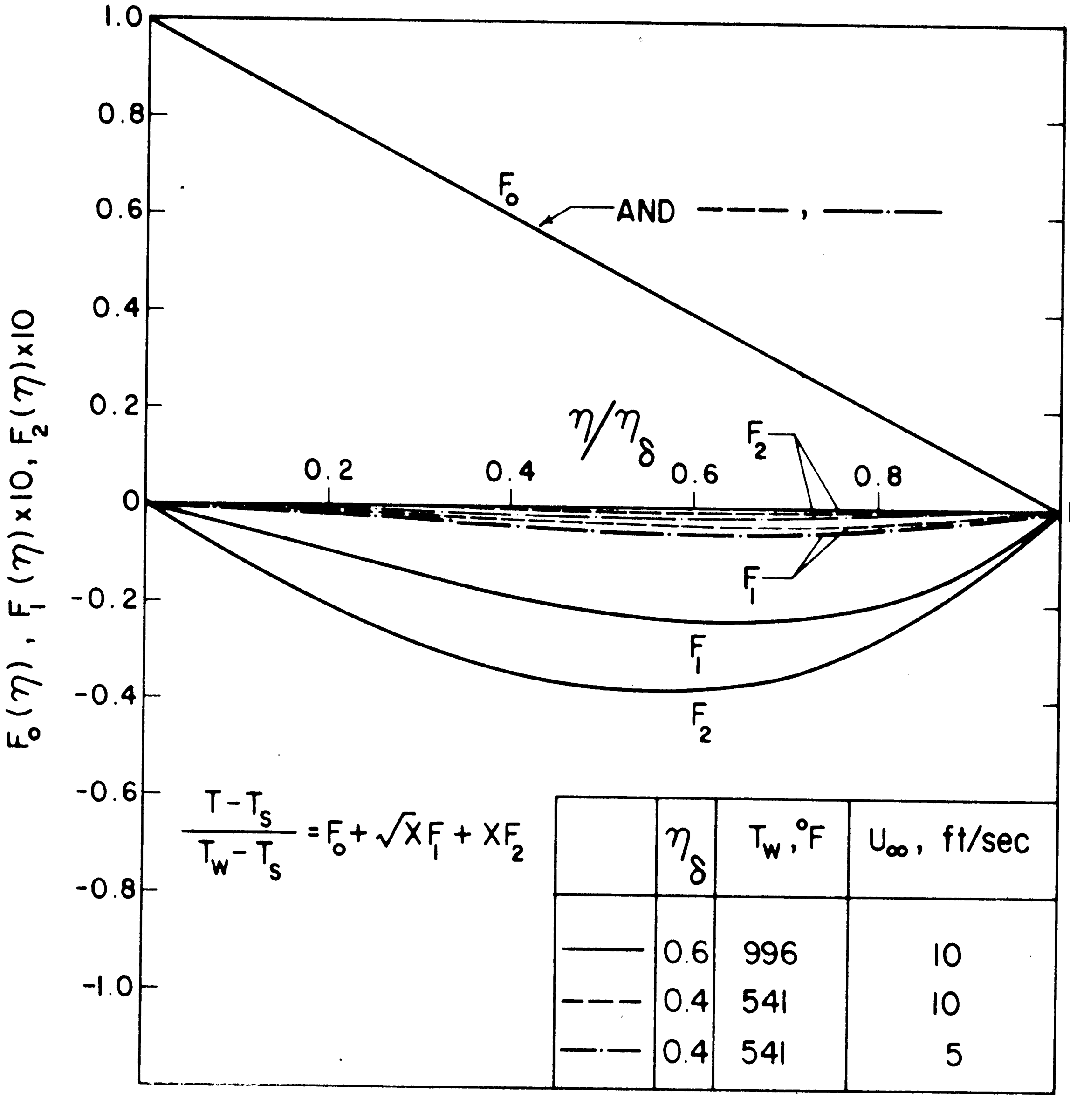


FIGURE 6. FUNCTIONS ASSOCIATED WITH VAPOR TEMPERATURE IN FORCED-CONVECTION FILM BOILING OVER A HORIZONTAL PLATE FOR $T_S = T_\infty = 212°F$.

and 4 that the presence of radiation is to increase the velocity profile and decrease the temperature profile. The effects are greater for higher wall temperature or thicker vapor film.

For the forced-convection film boiling case, Figure 5 illustrates that the velocity profile f'_o in the vapor film is practically linear in the absence of radiation. The presence of radiation is to increase the flow velocities in both vapor film and liquid boundary layer, and hence the skin friction is decreased at the plate surface. An increase in the wall temperature (or the vapor film thickness) or a decrease in the free-stream velocity results in an increase in the radiation effect.

Figure 6 shows that the temperature distribution F_o in the vapor film is practically linear in the absence of radiation process. A simultaneous action of radiation is to decrease the vapor temperature, and hence the heat conduction is increased at the wall surface. As shown in Figure 6, the radiation effects on the temperature profile are larger for thick vapor film (or higher wall temperature) or for lower free-stream velocity.

Figure 7 shows the heat transfer and skin friction characteristics for the stagnation film boiling in two-dimensional flow of water at a velocity of $U_\infty = 10$ x ft/sec under atmospheric pressure. It is seen in the figure that as the surface temperature increases from 212°F, both skin friction $f''(0)$ and conduction through the vapor film $-\theta'(0)$ decrease, while surface radiation B_1 increases. When the surface temperature exceeds 1000°R, B_1 is larger than $-\theta'(0)$ indicating that surface radiation becomes more important than conduction through the vapor film.

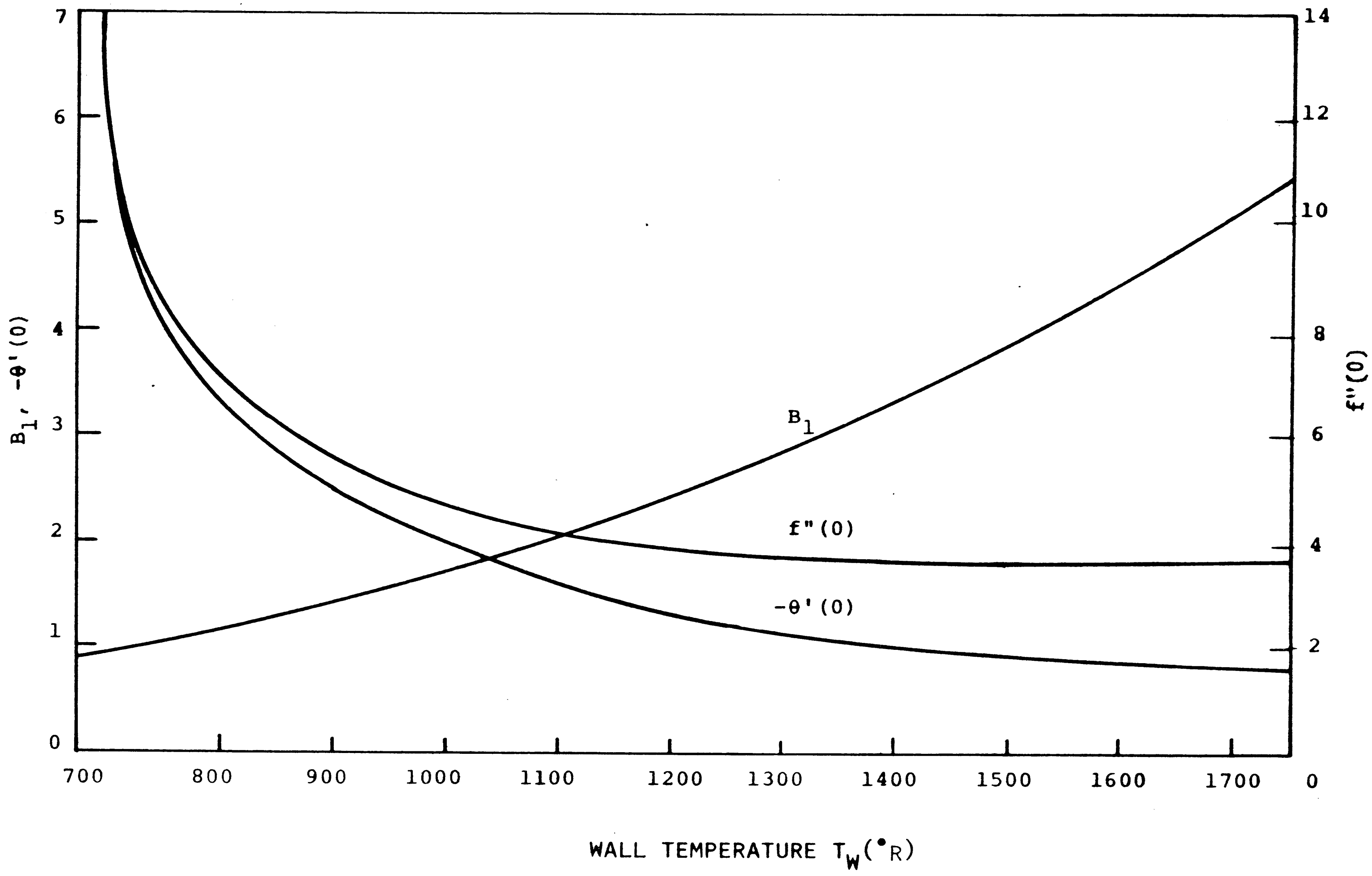


FIGURE 7. HEAT TRANSFER AND SKIN FRICTION CHARACTERISTICS OF STAGNATION FILM BOILING IN TWO-DIMENSIONAL FLOW OF WATER AT ATMOSPHERIC PRESSURE.

Tables 1 to 3 furnish important results for radiation effect on heat transfer performance and shear stress at the wall surface and liquid-vapor interface. For the natural-convection film boiling over a vertical plate, Table 1 indicates that the presence of radiation is to increase the heat transfer from the wall to the vapor and from the vapor to the interface. The radiation effects become greater for higher wall temperature or thicker vapor film. Tables 2 and 3 show that for forced-convection film boiling over a horizontal plate, radiation increases the local Nusselt numbers and decreases the shear stresses at the wall surface and liquid-vapor interface. An increase in the wall temperature or vapor film thickness or a decrease in the free-stream velocity may contribute to an increase in the radiation effects.

CONCLUDING REMARKS

To replace the existing empirical equations which have been used for estimating the total surface conductance in film boiling expressions are now obtained for the determination of the heat transfer rate and skin friction in the natural-convection film boiling over an inclined surface and a sphere, the forced-convection film boiling over a horizontal plate, and the stagnation-flow film boiling when radiation is appreciable. The problems have been formulated exactly within the framework of boundary-layer theory with the consideration of the shear stress and vapor velocity at the liquid-vapor interface. The method of analysis may be extended to the natural- and forced-convection film boiling over other surfaces of different geometry. The problems of film boiling on a surface having space-dependent temperature may also be solved by the present method by expanding the surface temperature into an infinite series with respect to the depending space variable.

TABLE 1

CERTAIN PROPERTIES ASSOCIATED WITH HEAT TRANSFER PERFORMANCE IN NATURAL CONVECTION FILM BOILING ON A VERTICAL PLATE FOR $T_s = T_\infty = 212$ F

η_δ	T_w(°F)	B_1	$F'_o(o)$	$F'_1(o)$	$F'_2(o)$	$F'_o(\eta_\delta)$	$F'_1(\eta_\delta)$	$F'_2(\eta_\delta)$	a_1	a_2	b_1	b_2
0.6	280	0.487	-1.674	- .00172	- .0014	-1.65	.0048	.0024	.08	.00205	- .35	- .0068
0.8	424	0.640	-1.267	- .00905	- .00108	-1.209	.0206	.00264	.142	.00896	- .469	- .0143
1.0	727	1.117	-1.031	- .036	- .0102	- .923	.0826	.0223	.315	.0474	- .799	- .053
1.2	1295	2.678	-0.886	- .184	- .0154	- .7095	.375	.271	.948	.466	- 1.92	- .104
1.4	2172	6.751	-0.792	- .774	- 1.87	- .541	1.498	2.89	2.93	5.182	- 5.186	- 2.48
1.5	2707	9.806	-0.757	-1.497	- 6.71	- .469	2.635	7.61	4.73	14.97	- 7.623	- 5.92
1.6	3224	14.63	- .729	-2.796	-21.7	- .406	4.62	20.26	8.15	45.6	-11.49	-34.6

Remarks: $b_1 = F'_1(0) - a_1F'_o(0) - B_1$

$$b_2 = F'_2(0) - a_1F'_1(0) + (a_1^2 - a_2)F'(0)$$

TABLE 2

CERTAIN PROPERTIES ASSOCIATED WITH SHEAR STRESS IN FORCED-CONVECTION FILM BOILING OVER A FLAT PLATE FOR $T_s = T_\infty = 212°F$

$U_\infty(\frac{ft}{sec})$	η_δ	$T_w(°F)$	$f''_o(o)$	$f''_1(o)$	$f''_2(o)$	$f''_o(\eta_\delta)$	$f''_1(\eta_\delta)$	$f''_2(\eta_\delta)$	c_1	c_2
10	0.6	996	3.409	.583	.775	3.018	-.469	-.145	-2.503	3.111
10	0.4	541	5.008	.196	.0093	4.747	-.0096	.0198	-1.13	.317
10	0.2	291	9.792	.0993	.0279	9.66	.0693	.0098	- .67	.071
5	0.4	541	5.008	.278	.172	4.747	-.0134	.044	-1.636	.782
5	0.3	393	6.61	.1856	.069	6.416	.0635	.0346	-1.194	.268
5	0.2	291	9.792	.139	.0249	9.66	.0968	.0204	- .951	.1105

Remarks: $c_1 = f''_1(o) - a_1 f''_o(o)$

$c_2 = f''_2(o) - a_1 f''_1(o) + (a_1^2 - a_2)\, f''_o$

TABLE 3

CERTAIN PROPERTIES ASSOCIATED WITH HEAT TRANSFER PERFORMANCE IN FORCED-CONVECTION FILM BOILING OVER A FLAT PLAT FOR $T_s = T_\infty = 212°F$

$U_\infty(\frac{ft}{sec})$	η_δ	$T_w(°F)$	B_1	$F_o'(o)$	$F_1'(o)$	$F_2'(o)$	$F_o'(\eta_\delta)$	$F_1'(\eta_\delta)$	$F_2'(\eta_\delta)$	a_1	a_2	b_1	b_2
10	0.6	996	4.84	-1.708	-.09	-.1203	-1.537	.288	.301	.899	-.026	-3.301	-1.38
10	0.4	541	2.15	-2.527	-.021	-.00979	-2.414	.0579	.0217	.266	-.00115	-1.505	- .181
10	0.2	291	1.28	-5.013	-.0027	-.00064	-4.958	.0087	.00035	.0788	.001	- .88	- .026
5	0.4	541	3.04	-2.527	-.0266	-.0179	-2.414	.0819	.428	.376	-.00156	-2.141	- .36
5	0.3	393	2.25	-3.35	-.014	-.00575	-3.27	.0355	.0141	.208	.0073	-1.567	- .122
5	0.2	291	1.81	-5.013	-.0039	-.00029	-4.958	.0123	.00398	.111	.00201	-1.254	- .045

Remarks: $b_1 = F_1'(o) - a_1 F_o'(O) - B_1$

$b_2 = F_2'(o) - a_1 F_1'(o) + (a_1^2 - a_2)\, F_o'(o)$

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APPENDIX

COMPUTER PROGRAMS

1. For Stagnation Film Boiling in Two-Dimensional Flow

```
$COMPILE MAD, EXECUTE, PRINT OBJECT, DUMP, PUNCH OBJECT
          PRINT COMMENT $FILM BOILING, STAGNATION FLOW$
          PRINT COMMENT $UNIT IN FT, HOUR, DEGREE R, LBM, BTU$
          DIMENSION Y(3), F(3), Q(3),P(100), FG(100), FGP(100),
         1FGPP(100), FGPPP(100), PL(100), FL(100),FLP(100),FLPP(100),
         2FLPPP(100),T(10), G(10), TR(10), G(10), TWG(10),
         3LP(100), R(100),RP(100),RPP(100), GR(10)
          VECTOR VALUES RESULT = $1H ,5F18.6*$
          VECTOR VALUES OBTAIN = $1H ,4F18.6*$
          INTEGER J, GJMAX, I, N, Z
START     READ AND PRINT DATA
          I=0
S3        J=0
          EXECUTE SETRKD.(3,Y(1),F(1),Q,X,STEP)
          X=0.
          Y(1)=0.
          Y(2)=0.
          Y(3)=FGPP(I)
SAVE      J=J+1
          P(J)=X
          FG(J)=Y(1)
          FGP(J)=Y(2)
          FGPP(J)=Y(3)
          FGPPP(J)=F(3)
          H=(J-1.)/DALJ
          Z=(J-1)/DALJ
          WHENEVER H.E. Z
          PRINT FORMAT RESULT, P(J),FG(J),FGP(J),FGPP(J),FGPPP(J)
          END OF CONDITIONAL
          WHENEVER J .E. GJMAX, TRANSFER TO SS
CALL      S=RKDEQ.(0)
          WHENEVER S .E. 1.0
          F(1)=Y(2)
          F(2)=Y(3)
          F(3)=-FG(J)*FGPP(J)-4.+FGP(J)*FGP(J)
          TRANSFER TO CALL
          END OF CONDITIONAL
          TRANSFER TO SAVE
SS        J=0
          EXECUTE SETRKD.(3,Y(1),F(1),Q,X,LSTEP)
          X=0.
          Y(1)=L*L*FG(GJMAX)
          Y(2)=FGP(GJMAX)
          Y(3)= L*FGPP(GJMAX)
LSAVE     J=J+1
          PL(J)=X
          FL(J)=Y(1)
          FLP(J)=Y(2)
          FLPP(J)=Y(3)
          FLPPP(J)=F(3)
          H=(J-1.)/DALJ
          Z=(J-1)/DALJ
          WHENEVER H .E. Z
          PRINT FORMAT RESULT, PL(J),FL(J),FLP(J),FLPP(J),FLPPP(J)
          END OF CONDITIONAL
          GG=FLP(J)-2.
          WHENEVER .ABS. FLPP(J) .G. EPFLPP, TRANSFER TO SSS
          WHENEVER .ABS. GG .L. EPFLP, TRANSFER TO S8
SSS       WHENEVER J .E. LJMAX, TRANSFER TO S4
LCALL     S=RKDEQ.(0)
```

```
              WHENEVER  S .E. 1.0
              F(1)=Y(2)
              F(2)=Y(3)
              F(3)=-FL(J)*FLPP(J)-4.+FLP(J)*FLP(J)
              TRANSFER TO LCALL
              END OF CONDITIONAL
              TRANSFER TO LSAVE
S4            I=I+1
              WHENEVER I .E. 10, TRANSFER TO START
              T(I)=FGPP(1)
              G(I)=GG
              PRINT RESULTS T(I), G(I)
              WHENEVER I .L. 2, TRANSFER TO S5
              DAL=-G(I)*DAL/(G(I)-G(I-1))
S5            FGPP(1)=FGPP(1)+DAL
              TRANSFER TO S3
S8            WHENEVER J .E. LJMAX, TRANSFER TO S9
              J=J+1
              LP(J)=LP(J-1)+LSTEP
              FL(J)=Y(1)+FLP(J)*LSTEP
              FLP(J)=Y(2)
              FLPP(J)=0.
              FLPPP(J)=0.
              TRANSFER TO S8
S9            PRINT COMMENT $X,FG,FGP,FGPP,FGPPP$
              THROUGH LAST, FOR N=1,1,N.G.GJMAX
LAST          PRINT FORMAT RESULT, P(N),FG(N),FGP(N),FGPP(N),FGPPP(N)
              PRINT COMMENT $XL, FL, FLP, FLPP, FLPPP$
              THROUGH LLAST, FOR N=1,1,N.G.LJMAX
LLAST         PRINT FORMAT RESULT, LP(N),FL(N),FLP(N),FLPP(N),FLPPP(N)
              EXECUTE SETRKD.(2,Y(1),F(1),0,X,STEP)
              I=0
S23           J=0
              X=0.
              Y(1)=1.
              Y(2)=RP(I)
GET           J=J+1
              P(J)=X
              R(J)=Y(1)
              RP(J)=Y(2)
              RPP(J)=F(2)
              H=(J-1.)/DALJ
              Z=(J-1)/DALJ
              WHENEVER H .E. Z
              PRINT FORMAT OBTAIN, P(J),R(J),RP(J),RPP(J)
              END OF CONDITIONAL
              WHENEVER J .E. GJMAX, TRANSFER TO SSSS
CONTI         S=RKDEQ.(0)
              WHENEVER S .E. 1.0
              F(1)=Y(2)
              F(2)=-PR*FG(J)*RP(J)
              TRANSFER TO CONTI
              END OF CONDITIONAL
              TRANSFER TO GET
SSSS          WHENEVER .ABS. R(J) .L. EPR, TRANSFER TO S29
              I=I+1
              GR(I)=R(J)
              TR(I)=RP(1)
              PRINT RESULTS GR(I), TR(I)
              WHENEVER I .L. 2, TRANSFER TO S25
```

```
              RDAL=-GR(I)*RDAL/(GR(I)-GR(I-1))
S25           RP(1)=RP(1)+RDAL
              TRANSFER TO S23
S29           PRINT COMMENT $X, R, RP, RPP$
              THROUGH RLAST, FOR N=1,1,N.G.GJMAX
RLAST         PRINT FORMAT OBTAIN, P(N),R(N),RP(N),RPP(N)
              I=0
TWS3          I=I+1
              E=ROW*NU*HFG/K/(TW-TS)
              BI=SIGMA*(TW+TS)*(TW*TW+TS*TS)/((REFLL/EMITL
             1+1.+REFLW/EMITW)* K)*(2.*NU/A).P.0.5
              TWG(I)=E*FG(GJMAX)+RP(GJMAX)-B1
              PRINT RESULTS TWG(I), TW, B1, E
              WHENEVER I .E. 10, TRANSFER TO START
              WHENEVER .ABS. TWG(I) .L. EPTW, TRANSFER TO START
              WHENEVER I .L. 2, TRANSFER TO TWS5
              TWDAL=-TWG(I)*TWDAL/(TWG(I)-TWG(I-1))
TWS5          TW=TW+TWDAL
              TRANSFER TO TWS3
              END OF PROGRAM
$DATA
DAL=10.,  LSTEP=0.1,  EPR=0.001,  RP(1)=0.5,  HFG=970.3,  TS=672.,
SIGMA=0.172E-08,  REFLL=0.1,  EMITL=0.9,  REFLW=0.,  EMITW=1.,
EPTW=0.1,  RDAL=0.01,  TWDAL=10.,  L=0.00515,
GJMAX=21,  DALJ=5,  LJMAX=21,  EPFLPP=0.01,  EPFLP=0.1,
LJMAX=16,  PR=.94,  L=.00516,  A=36000.,
TW=850.,  STEP=.022, FGPP(1)=5.33, ROW=.0328, NU=1.09,  K=.0171*
TW=1050.,  STEP=.035, FGPP(1)=4.106,  ROW=.0288, NU=1.42, K=.02*
TW=1450., STEP=.065, FGPP(1)=3.592, ROW=.0233, NU=2.2,  K=.0257*
TW=1800.,  STEP=.11,  FGPP(1)=3.542, ROW=.0196, NU=3.08,  K=.0321*
```

2. For Forced-Convection Film Boiling Over a Horizontal Plate

```
$COMPILE MAD, EXECUTE, PRINT OBJECT, DUMP, PUNCH OBJECT
           PRINT COMMENT $FILM BOILING, HORIZONTAL$
           DIMENSION Y(3),F(3),Q(3),      P(100),
          1       Y1(300,WA),Y2(300,WB),Y3(300,WC),F3(300,WD),
          2R(300,WE),RR(300,WF),RRR(300,WG),T(30 ,WH),G(30 ,WI),
          3LY1(300,WJ),LY2(300,WK),LY3(300,WL), LF3(300,WM),
          4LR(300,WN), LRR(300,WP),LRRR(300,WQ), LP(100),
          5A(30,WR),H(30,WS)
           VECTOR VALUES  WA  =2,1,100
           VECTOR VALUES  WB  =2,1,100
           VECTOR VALUES  WC  =2,1,100
           VECTOR VALUES  WD  =2,1,100
           VECTOR VALUES  WE  =2,1,100
           VECTOR VALUES  WF  =2,1,100
           VECTOR VALUES  WG  =2,1,100
           VECTOR VALUES  WH  =2,1,10
           VECTOR VALUES  WI  =2,1,10
           VECTOR VALUES  WJ  =2,1,100
           VECTOR VALUES  WK  =2,1,100
           VECTOR VALUES  WL  =2,1,100
           VECTOR VALUES  WM  =2,1,100
           VECTOR VALUES  WN  =2,1,100
           VECTOR VALUES  WP  =2,1,100
           VECTOR VALUES  WQ  =2,1,100
           VECTOR VALUES  WR  =2,1,10
           VECTOR VALUES  WS  =2,1,10
           VECTOR VALUES RESULT = $1H ,5F18.6*$
           INTEGER I,J,N,U,M
START      READ AND PRINT DATA
           M=0
S1         I=0
           M=M+1
           WHENEVER M.E.2
           U=0
           A1=B1/(2*E*Y1(1,JMAX) -RR(1,JMAX))
           END OF CONDITIONAL
           WHENEVER M.E.3
           U=0
           A2=(A1*RR(2,JMAX)-A1*A1*RR(1,JMAX)-3*E*A1*Y1(2,JMAX))/
          1(3*E*Y1(1,JMAX)-RR(1,JMAX))
           END OF CONDITIONAL
S3         J=0
           EXECUTE SETRKD.(3,Y(1),F(1),Q,X,STEP)
           X=0.
           Y(1)=0.
           Y(2)=0.
           Y(3)=Y3(M,1)
SAVE       J=J+1
           P(J)=X
           Y1(M,J)=Y(1)
           Y2(M,J)=Y(2)
           Y3(M,J)=Y(3)
           F3(M,J)=F(3)
           WHENEVER J.E.JMAX, TRANSFER TO SSS
CALL       S=RKDEQ.(0)
           WHENEVER S.E.1.0
           F(1)=Y(2)
           F(2)=Y(3)
           WHENEVER M.E.1
           F(3)= -Y(1)*Y(3)
           END OF CONDITIONAL
           WHENEVER M.E.2
           F(3)=-  Y1(1,J)*Y(3)+  Y2(1,J)*Y(2)-2*Y3(1,J)*Y(1)+2
          1*A1*F3(1,J)-A1*Y1(1,J)*Y3(1,J)
           END OF CONDITIONAL
           WHENEVER M.E.3
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```
          F(3)=-Y1(1,J)*Y(3)+2*Y2(1,J)*Y(2)-3*Y3(1,J)*Y(1)+A1*2*F3(2,J)
         1-(A1*Y1(1,J)+2*Y1(2,J))*Y3(2,J)+  Y2(2,J)*Y2(2,J)-A1*Y1(2,J)
         2*Y3(1,J)-(3*A1*A1-2*A2)*F3(1,J)+(0.5*A1*A1-A2)*
         3Y1(1,J)*Y3(1,J)
          END OF CONDITIONAL
          TRANSFER TO CALL
          END OF CONDITIONAL
          TRANSFER TO SAVE
SSS       J=0
          EXECUTE SETRKD.(3,Y(1),F(1),0,X,LSTEP)
          X=0.
          Y(1)=0.
          Y(2)=Y2(M,JMAX)
          WHENEVER M.E.1
          Y(3)=L*Y3(1,JMAX)
          END OF CONDITIONAL
          WHENEVER M.E.2
          Y(3)=A1*LY3(1,1)+L*  (Y3(2,JMAX)-A1*Y3(1,JMAX))
          END OF CONDITIONAL
          WHENEVER M.E.3
          Y(3)=A1*LY3(2,1)-(A1*A1-A2)*LY3(1,1)
         1+L*(Y3(3,JMAX)-A1*Y3(2,JMAX)-(A1*A1-A2)*Y3(1,JMAX))
          END OF CONDITIONAL
LSAVE     J=J+1
          LP(J)=X
          LY1(M,J)=Y(1)
          LY2(M,J)=Y(2)
          LY3(M,J)=Y(3)
          LF3(M,J)=F(3)
          WHENEVER M.E.1
          GG=Y(2)-2.
          OTHERWISE
          GG=Y(2)
          END OF CONDITIONAL
          WHENEVER .ABS.Y(3).G.EPSIH, TRANSFER TO SS
          WHENEVER .ABS.GG  .L.EPSII, TRANSFER TO S8
SS        WHENEVER J.E.LJMAX, TRANSFER TO S4
LCALL     S=RKDEQ.(0)
          WHENEVER S.E.1.0
          F(1)=Y(2)
          F(2)=Y(3)
          WHENEVER M.E.1
          F(3)=-Y(1)*Y(3)
          END OF CONDITIONAL
          WHENEVER M.E.2
          F(3)=-LY1(1,J)*Y(3)+LY2(1,J)*Y(2)-2*LY3(1,J)*Y(1)+2*
         1A1*LF3(1,J)- A1*LY1(1,J)*LY3(1,J)
          END OF CONDITIONAL
          WHENEVER M.E.3
          F(3)=-LY1(1,J)*Y(3)+2*LY2(1,J)*Y(2)-3*LY3(1,J)*Y(1)+2*A1*
         1LF3(2,J)-(A1*LY1(1,J)+2*LY1(2,J))*LY3(2,J)+LY2(2,J)*
          END OF CONDITIONAL
          TRANSFER TO LCALL
          END OF CONDITIONAL
          TRANSFER TO LSAVE
S4        I=I+1
          T(M,I)=Y3(M,1)
          G(M,I)=GG
          PRINT RESULTS G(M,I), T(M,I)
          WHENEVER I.L.2, TRANSFER TO S5
          WHENEVER G(M,I).G.0, TRANSFER TO S6
          WHENEVER G(M,I-1).G.0., TRANSFER TO S7
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```
             TRANSFER TO S15
             DAL=DALTA
             Y3(M,1)=Y3(M,1)+DAL
             TRANSFER TO S3
S6           WHENEVER G(M,I-1).L.0., TRANSFER TO S7
             TRANSFER TO S15
S7           DAL=-G(M,I)*DAL/(G(M,I)-G(M,I-1))
             Y3(M,1)=Y3(M,1)+DAL
             TRANSFER TO S3
S15          DAL=-G(M,I)*DAL/(G(M,I)-G(M,I-1))
             Y3(M,1)=Y3(M,1)+DAL
             TRANSFER TO S3
S8           WHENEVER J.E.LJMAX, TRANSFER TO S9
             J= J+1
             LP(J)=LP(J-1)+LSTEP
             LY1(M,J)=Y(1)
             LY2(M,J)=Y(2)
             LY3(M,J)=0.
             LF3(M,J)=0.
             TRANSFER TO S8
S9           PRINT RESULTS M
             THROUGH LAST, FOR N=1.1.N.G.JMAX
LAST         PRINT FORMAT RESULT,P(N),Y1(M,N),Y2(M,N),Y3(M,N),F3(M,N)
             THROUGH LLAST, FOR N=1,1,N.G.LJMAX
LLAST        PRINT FORMAT RESULT,LP(N),LY1(M,N),LY2(M,N),LY3(M,N),LF3(M,N)
             EXECUTE SETRKD.(2,Y(1),F(1),Q,X,STEP)
             I=0
S23          J=0
             X=0
             WHENEVER M.E.1
             Y(1)=1.0
             OTHERWISE
             Y(1)=0.
             END OF CONDITIONAL
             Y(2)=RR(M,1)
GET          J=J+1
             P(J)=X
             R(M,J)=Y(1)
             RR(M,J)=Y(2)
             RRR(M,J)=F(2)
             WHENEVER J.E.JMAX, TRANSFER TO SSSS
CONTI        S=RKDEQ.(0)
             WHENEVER S.E.1.0
             F(1)=Y(2)
             WHENEVER M.E.1
             F(2) =-PR  *Y1(1,J)*Y(2)
             END OF CONDITIONAL
             WHENEVER M.E.2
             F(2)=PR*(-Y1(1,J)*Y(2)+Y2(1,J)*Y(1)+(2/PR)*A1*RRR(1,J)-(
             1A1*Y1(1,J)-2*Y1(2,J))*RR(1))
             END OF CONDITIONAL
             WHENEVER M.E.3
             F(2)=PR*(-Y1(1,J)*Y(2)+2*Y2(1,J)*Y(1)+(2/PR)*A1*RRR(2,J)
             1-(A1*Y1(1,J)-2*Y1(2,J))*RR(2,J)-Y2(2,J)*R(2,J)+(1/PR)*
             2(2*A2-3*A1*A1)*RRR(1,J)-(Y2(3,J)+6*Y1(3,J)+A1*
             3Y1(2,J)+(2*A2-   A1*A1)*Y1(1,J))*RR(1,J))
             END OF CONDITIONAL
             TRANSFER TO CONTI
             END OF CONDITIONAL
             TRANSFER TO GET
SSSS         WHENEVER .ABS. Y(1).L.EPSID, TRANSFER TO S29
             TRANSFER TO S24
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```
S24         I=I+1
            G(M,I)=Y(I)
            T(M,I)=RR(M,1)
            PRINT RESULTS G(M,I), T(M,I)
            WHENEVER I.L.2, TRANSFER TO S25
            WHENEVER G(M,I).G.0., TRANSFER TO S26
            WHENEVER G(M,I-1).G.0., TRANSFER TO  S27
            TRANSFER TO S35
S25         DAL =DALTA
            RR(M,1)=RR(M,1)+DAL
            TRANSFER TO S23
S26         WHENEVER G(M,I-1).L.0., TRANSFER TO S27
            TRANSFER TO S35
S27         DAL=-G(M,I)*DAL/(G(M,I)-G(M,I-1))
            RR(M,1)=RR(M,1)+DAL
            TRANSFER TO S23
S35         DAL=-G(M,I)*DAL/(G(M,I)-G(M,I-1))
            RR(M,1)=RR(M,1)+DAL
            TRANSFER TO S23
S29         WHENEVER M.E.1
            E=-RR(1,JMAX)/  Y1(1,JMAX)
            TW=ROWG*HFG*NU/(KA*E)+TS
            B1=SIGMA*(TW+TS)*(TW*TW+TS*TS)/((REFL/ EMITL
           1+1+RELW/EMITW)*KA*UINF.P.0.5)*2*NU.P.0.5
            THROUGH AAA, FOR N=1,1,N.G.J
           2LY2(2,J)-A1*LY1(2,J)*LY3(1,J)-(3*A1*A1-2*A2)*LF3(1,J)+ (A1
           3  *0.5*A1-A2)*LY1(1,J)*LY3(1,J)
AAA         PRINT RESULTS P(N),R(M,N),RR(M,N),RRR(M,N)
            PRINT RESULTS E,TW,B1
            TRANSFER TO S1
            END OF CONDITIONAL
            WHENEVER M.E.2
            AA1=(B1-RR(2,JMAX)-2*E*Y1(2,JMAX ))/(2*E*Y1(1,JMAX)
           1-RR(1,JMAX))
            WHENEVER .ABS.(A1-AA1).L.EPSIA,TRANSFER TO S50
            U=U+1
            H(M,U)=AA1-A1
            A(M,U)=A1
            PRINT RESULTS H(M,U), A(M,U),A1
            WHENEVER U.L.2, TRANSFER TO S65
            WHENEVER H(M,U).G.0., TRANSFER TO S66
            WHENEVER H(M,U-1).G.0., TRANSFER TO  S67
            TRANSFER TO S75
S65         DA  =DALTAA
            A1=A1+DA
            TRANSFER TO S3
S66         WHENEVER H(M,U-1).L.0.,TRANSFER TO S67
            TRANSFER TO S75
S67         DA =-H(M,U)*DA /(H(M,U)-H(M,U-1))
            A1=A1+DA
            TRANSFER TO S3
S75         DA =-H(M,U)*DA /(H(M,U)-H(M,U-1))
            A1=A1+DA
            TRANSFER TO S3
            END OF CONDITIONAL
            WHENEVER M.E.3
            AA2=(-RR(3,JMAX)+A1*RR(2,JMAX)-A1*A1*RR(1,JMAX)
           1-3*E*Y1(3,JMAX)-3*E*A1*Y1(2,JMAX))/(3*E*Y1(1,JMAX)
           2-RR(1,JMAX))
```

```
          WHENEVER .ABS.(A2-AA2).L.EPSIB,TRANSFER TO S51
          U=U+1
          H(M,U)=AA2-A2
          A(M,U)=A2
          PRINT RESULTS H(M,U), A(M,U),A2
          WHENEVER U.L.2, TRANSFER TO S85
          WHENEVER H(M,U).G.0., TRANSFER TO S86
          WHENEVER H(M,U-1).G.0., TRANSFER TO  S87
          TRANSFER TO S95
S85       DA  =DALTAA
          A2=A2+DA
          TRANSFER TO S3
S86       WHENEVER H(M,U-1).L.0., TRANSFER TO S87
          TRANSFER TO S95
S87       DA =-H(M,U)*DA /(H(M,U)-H(M,U-1))
          A2=A2+DA
          TRANSFER TO S3
S95       DA =-H(M,U)*DA /(H(M,U)-H(M,U-1))
          A2=A2+DA
          TRANSFER TO S3
          END OF CONDITIONAL
S50       PRINT RESULTS A1
          THROUGH ASS, FOR N=1,1,N.G.J
ASS       PRINT RESULTS P(N),R(2,N),RR(2,N),RRR(3,N)
          TRANSFER TO S1
S51       PRINT RESULTS A2
          THROUGH ASSS, FOR N=1,1,N.G.J
ASSS      PRINT RESULTS P(N),R(3,N),RR(3,N),RRR(3,N)
          TRANSFER TO START
          END OF PROGRAM
$DATA
Y3(1,1)=0.5,Y3(2,1)=0.5,Y3(3,1)=0.5,RR(1,1)=0.5,RR(2,1)=0.5,RR(3,1)=0.5,
DALTA=0.2,PR=0.9,EPSIA=0.001,EPSIB=0.01,              EPSID=0.001,
REFL=0.1,EMITL=0.9,RELW=0.,EMITW=1.,
EPSIH=0.01,EPSII=0.001,LJMAX=40,L=0.00556,DALTAA=-100.,LSTEP=0.1,
ROWG=0.02,HFG=970.3,NU=3.60,KA=0.035,TS=672,SIGMA=0.172E-8,
JMAX=101,
UINF=36000,STEP=0.006*
UINF=36000,STEP=0.004*
UINF=18000,STEP=0.004*
```

3. **For Natural-Convection Film Boiling Over a Vertical (or Inclined) Plate**

```
$COMPILE MAD, EXECUTE, PRINT OBJECT, DUMP, PUNCH OBJECT
            PRINT COMMENT $FILM BOILING, VERTICAL $
            DIMENSION Y(3),F(3),Q(3),        P(200),
           1        Y1(600,WA),Y2(600,WB),Y3(600,WC),F3(600,WD),
           2R(600,WE),RR(600,WF),RRR(600,WG),T(60 ,WH),G(60 ,WI),
           3A(60,WJ),H(60,WK)
            VECTOR VALUES  WA   =2,1,200
            VECTOR VALUES  WB   =2,1,200
            VECTOR VALUES  WC   =2,1,200
            VECTOR VALUES  WD   =2,1,200
            VECTOR VALUES  WE   =2,1,200
            VECTOR VALUES  WF   =2,1,200
            VECTOR VALUES  WG   =2,1,200
            VECTOR VALUES  WH   =2,1,20
            VECTOR VALUES  WI   =2,1,20
            VECTOR VALUES  WJ   =2,1,20
            VECTOR VALUES  WK   =2,1,20
            VECTOR VALUES RESULT = $1H ,5F18.6*$
            INTEGER I,J,N,U,M
START       READ AND PRINT DATA
            M=0
S1          I=0
            M=M+1
            WHENEVER M.E.2
            U=0
            A1=B1/(4*E*Y1(1,JMAX) -RR(1,JMAX))
            END OF CONDITIONAL
            WHENEVER M.E.3
            U=0
            A2=(A1*RR(2,JMAX)-A1*A1*RR(1,JMAX)-5*E*A1*Y1(2,JMAX))/
           1(5*E*Y1(1,JMAX)-RR(1,JMAX))
            END OF CONDITIONAL
S3          J=0
            EXECUTE SETRKD.(3,Y(1),F(1),Q,X,STEP)
            X=0.
            Y(1)=0.
            Y(2)=0.
            Y(3)=Y3(M,1)
SAVE        J=J+1
            P(J)=X
            Y1(M,J)=Y(1)
            Y2(M,J)=Y(2)
            Y3(M,J)=Y(3)
            F3(M,J)=F(3)
            WHENEVER J.E.JMAX, TRANSFER TO SSS
CALL        S=RKDEQ.(0)
            WHENEVER S.E.1.0
            F(1)=Y(2)
            F(2)=Y(3)
            WHENEVER M.E.1
            F(3)=-3*Y(1)*Y(3)+2*Y(2)*Y(2)-1
            END OF CONDITIONAL
            WHENEVER M.E.2
            F(3)=-3*Y1(1,J)*Y(3)+5*Y2(1,J)*Y(2)-4*Y3(1,J)*Y(1)
           1+A1*F3(1,J)-A1*Y1(1,J)*Y3(1,J)
            END OF CONDITIONAL
            WHENEVER M.E.3
            F(3)=-3*Y1(1,J)*Y(3)+6*Y2(1,J)*Y(2)-5*Y3(1,J)*Y(1)+A1*F3(2,J)
           1-(A1*Y1(1,J)+4*Y1(2,J))*Y3(2,J)+3*Y2(2,J)*Y2(2,J)-A1*Y1(2,J)
           2*Y3(1,J)-(3*A1*A1-2*A2)*F3(1,J)+(A1*A1-2*A2)*Y1(1,J)*Y3(1,J)
            END OF CONDITIONAL
            TRANSFER TO CALL
            END OF CONDITIONAL
            TRANSFER TO SAVE
SSS         WHENEVER .ABS.Y(2).L.EPSIC,TRAN
            TRANSFER TO S4
S4          I=I+1
            T(M,I)=Y3(M,1)
            G(M,I)=Y(2)
            PRINT RESULTS G(M,I), T(M,I)
            WHENEVER I.L.2, TRANSFER TO S5
            WHENEVER G(M,I).G.0, TRANSFER TO S6
            WHENEVER G(M,I-1).G.0., TRANSFER TO S7
            TRANSFER TO S15
```

```
S5          DAL=DALTA
            Y3(M,1)=Y3(M,1)+DAL
            TRANSFER TO S3
S6          WHENEVER G(M,I-1).L.0., TRANSFER TO S7
            TRANSFER TO S15
S7          DAL=-G(M,I)*DAL/(G(M,I)-G(M,I-1))
            Y3(M,1)=Y3(M,1)+DAL
            TRANSFER TO S3
S15         DAL=-G(M,I)*DAL/(G(M,I)-G(M,I-1))
            Y3(M,1)=Y3(M,1)+DAL
            TRANSFER TO S3
S9          PRINT RESULTS M
            THROUGH LAST, FOR N=1,1,N.G.J
LAST        PRINT FORMAT RESULT,P(N),Y1(M,N),Y2(M,N),Y3(M,N),F3(M,N)
            EXECUTE SETRKD.(2,Y(1),F(1),Q,X,STEP)
            I=0
S23         J=0
            X=0
            WHENEVER M.E.1
            Y(1)=1.0
            OTHERWISE
            Y(1)=0.
            END OF CONDITIONAL
            Y(2)=RR(M,1)
GET         J=J+1
            P(J)=X
            R(M,J)=Y(1)
            RR(M,J)=Y(2)
            RRR(M,J)=F(2)
            WHENEVER J.E.JMAX, TRANSFER TO SSSS
CONTI       S=RKDEQ.(0)
            WHENEVER S.E.1.0
            F(1)=Y(2)
            WHENEVER M.E.1
            F(2) =-PR*3*Y1(1,J)*Y(2)
            END OF CONDITIONAL
            WHENEVER M.E.2
            F(2) =PR*(-3*Y1(1,J)*Y(2)+Y2(1,J)*Y(1)+2*A1*RRR(1,J)/PR
           1-A1*Y1(1,J)*RR(1,J)-4*Y1(2,J)*RR(1,J))
            END OF CONDITIONAL
            WHENEVER M.E.3
            F(2) =PR*(-3*Y1(1,J)*Y(2)+2*Y2(1,J)*Y(1)+2*A1*RRR(2,J)/PR
           1-(A1*Y1(1,J)+4*Y1(2,J))*RR(2,J)+Y2(2,J)*R(2,J)-(3*A1*A1-2*A2)
           2*RRR(1,J)/PR-(5*Y1(3,J)+A1*Y1(2,J)+2*A2*Y1(1,J)-A1*A1
           3*Y1(1,J))*RR(1,J))
            END OF CONDITIONAL
            TRANSFER TO CONTI
            END OF CONDITIONAL
            TRANSFER TO GET
SSSS        WHENEVER .ABS.Y(1).L.EPSID , TRANSFER TO S29
            TRANSFER TO S24
S24         I=I+1
            G(M,I)=Y(1)
            T(M,I)=RR(M,1)
            PRINT RESULTS G(M,I), T(M,I)
            WHENEVER I.L.2, TRANSFER TO S25
            WHENEVER G(M,I).G.0., TRANSFER TO S26
            WHENEVER G(M,I-1).G.0., TRANSFER TO  S27
            TRANSFER TO S35
S25         DAL =DALTA
            RR(M,1)=RR(M,1)+DAL
            TRANSFER TO S23
S26         WHENEVER G(M,I-1).L.0., TRANSFER TO S27
            TRANSFER TO S35
S27         DAL=-G(M,I)*DAL/(G(M,I)-G(M,I-1))
            RR(M,1)=RR(M,1)+DAL
            TRANSFER TO S23
S35         DAL=-G(M,I)*DAL/(G(M,I)-G(M,I-1))
            RR(M,1)=RR(M,1)+DAL
            TRANSFER TO S23
S29         WHENEVER M.E.1
            E=-RR(1,JMAX)/(3*Y1(1,JMAX))
            TW=ROWG*HFG*NU/(KA*E)+TS
            B1=SIGMA*(TW+TS)*(TW*TW+TS*TS)/((REFL/ EMITL
           1+1+RELW/EMITW)*KA*C)
            THROUGH AAA, FOR N=1,1,N.G.J
```

```
AAA       PRINT RESULTS P(N),R(M,N),RR(M,N),RRR(M,N)
          PRINT RESULTS E,TW,B1
          TRANSFER TO S1
          END OF CONDITIONAL
          WHENEVER M.E.2
          AA1=(B1-RR(2,JMAX)-4*E*Y1(2,JMAX ))/(4*E*Y1(1,JMAX)
         1-RR(1,JMAX))
          WHENEVER .ABS.(A1-AA1).L.EPSIA,TRANSFER TO S50
          U=U+1
          H(M,U)=AA1-A1
          A(M,U)=A1
          PRINT RESULTS H(M,U), A(M,U),A1
          WHENEVER U.L.2, TRANSFER TO S65
          WHENEVER H(M,U).G.0., TRANSFER TO S66
          WHENEVER H(M,U-1).G.0., TRANSFER TO  S67
          TRANSFER TO S75
S65       DA  =DALTAA
          A1=A1+DA
          TRANSFER TO S3
S66       WHENEVER H(M,U-1).L.0.,TRANSFER TO S67
          TRANSFER TO S75
S67       DA =-H(M,U)*DA /(H(M,U)-H(M,U-1))
          A1=A1+DA
          TRANSFER TO S3
S75       DA =-H(M,U)*DA /(H(M,U)-H(M,U-1))
          A1=A1+DA
          TRANSFER TO S3
          END OF CONDITIONAL
          WHENEVER M.E.3
          AA2=(-RR(3,JMAX)+A1*RR(2,JMAX)-A1*A1*RR(1,JMAX)
         1-5*E*Y1(3,JMAX)-5*E*A1*Y1(2,JMAX))/(5*E*Y1(1,JMAX)
         2-RR(1,JMAX))
          WHENEVER .ABS.(A2-AA2).L.EPSIB,TRANSFER TO S51
          U=U+1
          H(M,U)=AA2-A2
          A(M,U)=A2
          PRINT RESULTS H(M,U), A(M,U),A2
          WHENEVER U.L.2, TRANSFER TO S85
          WHENEVER H(M,U).G.0., TRANSFER TO S86
          WHENEVER H(M,U-1).G.0., TRANSFER TO  S87
          TRANSFER TO S95
S85       DA  =DALTAA
          A2=A2+DA
          TRANSFER TO S3
S86       WHENEVER H(M,U-1).L.0., TRANSFER TO S87
          TRANSFER TO S95
S87       DA =-H(M,U)*DA /(H(M,U)-H(M,U-1))
          A2=A2+DA
          TRANSFER TO S3
S95       DA =-H(M,U)*DA /(H(M,U)-H(M,U-1))
          A2=A2+DA
          TRANSFER TO S3
          END OF CONDITIONAL
S50       PRINT RESULTS A1
          THROUGH ASS, FOR N=1,1,N.G.J
ASS       PRINT RESULTS'P(N),R(2,N),RR(2,N),RRR(3,N)
          TRANSFER TO S1
S51       PRINT RESULTS A2
          THROUGH ASSS, FOR N=1,1,N.G.J
ASSS      PRINT RESULTS P(N),R(3,N),RR(3,N),RRR(3,N)
          TRANSFER TO START
          END OF PROGRAM
$DATA
Y3(1,1)=0.5,Y3(2,1)=0.5,Y3(3,1)=0.5,RR(1,1)=0.5,RR(2,1)=0.5,RR(3,1)=0.5,
DALTA=0.2,       EPSIA=0.001,EPSIB=0.001,EPSIC=0.001,EPSID=0.001,
          HFG=970.3,          TS=672,SIGMA=0.172E-8,DALTAA=-100.,
REFL=0.1,EMITL=0.9,RELW=0.,EMITW=1.,
STEP=0.01,PR=0.91,ROWG=0.0169,NU=4.170,KA=0.0388,C=144,JMAX=141*
STEP=0.010,PR=0.89,ROWG=0.0145 ,NU=5.94,KA=0.0508,C=125,JMAX=151*
STEP=0.01,PR=0.87,ROWG=0.0120,NU=7.710,KA=0.0610,C=106,JMAX=161*
                                                     659 LINES PRINTED
```