Fundamental problems confronting the engineer engaged in design of shore protection works are (a) determination of the expected recurrence interval of a specified water level and (b) estimation of the highest water level to be expected in a given interval of time. Tide levels due to astronomical influence are mathematically predictable. Excessively high tides may, however, result from barometric pressure differential, storm winds and combination of the two. Storm tides are not true tides. They are essentially a storm generated wave piling up on a lee shore. In the work presented here it is assumed that the phenomena of storm tides are due solely to wind effects.

Periods of tide records are usually of shorter duration than the design period under study. It is therefore necessary to extrapolate data to longer periods.

The simplest method of accomplishing this is to prepare a plot of cumulative frequency of recorded tides versus the arithmetic or logarithmic values of the tide heights. This is usually most unsatisfactory. A second method involves a plot of the arithmetic values of the tide heights against the probability of exceedance per unit time. The underlying theory is derived from extreme value statistics. This is the method followed by the Rijkswaterstaat in Holland. A third method utilized for investigation of tide heights occurring at certain ports in England also applies extreme value theory. A fourth approach involves plotting a cumulative frequency of occurrence against arithmetic or logarithmic values of the recorded tides on normal probability paper. Gumbel warns against indiscriminate efforts to normalize data. It has been a common practice in engineering to try to fit data to a normal distribution and accept as valid that portion at the end of a plot which deviates from a straight line. This can be a dangerous practice, particularly when data are extrapolated.

In an earlier paper results of an investigation of storm tides along the east coast of the United States were presented. It was suggested that very high storm tides might be described by an extreme value distribution. It was shown that astronom-
Logically generated tides reasonably well follow a log-normal distribution and recorded water levels above the astronomical tides are described by a distribution of the type

\[ p = \exp \left( -e^{-a \ln H/H_{\text{modal}}} \right) \]  

where \( p \) is the probability of occurrence.

\[ \exp = e = \text{base of Naperian logarithms} = 2.71828 \ldots \]

\[ a = \text{constant} \]

\[ \ln = \text{natural logarithm} \]

\[ H = \text{recorded tide height} \]

\[ H_{\text{modal}} = \text{mode (most frequent) value.} \]

Thom has shown that extreme winds in the United States follow a distribution of this type. In this study it is assumed that abnormally high tides are due solely to wind effects. Thom has also shown that the probability of occurrence of a storm in the Biloxi, Miss. area is a function of the probabilities of occurrence of normally occurring storms and of tropical storms.


Monthly highest tides were ranked in intervals of 0.1 of a foot. A histogram was prepared, showing frequency of occurrence and recorded tide height. A typical plot is shown in Figure 1. It can be seen that the left portion appears to follow the general shape of the normal curve. The right part of the curve is skewed.

A cumulative frequency was next prepared and it was assumed that in \( n \) total cases one higher tide could have occurred but had not done so. The cumulative frequency of the \( m \) th case was \( m/n+1 \). It was found that the logarithm of the variable gave much more regular results than when the arithmetic value was plotted. Use of the logarithm does away with the problem of infinite upper and lower limits of the normal curve. This type of data display is shown in Figure 2. A straight line is followed for the majority of the data. However, deviations from the straight line are noted for the higher values of tide heights.

The values in the 'tailed off' portion were treated as a separate distribution and again it was assumed that one possible high value had not yet occurred. Typical results are illustrated in Figure 3, a cumulative frequency plot on extreme value paper.

All monthly highest tides were arrayed in a cumulative frequency distribution on extreme value paper and this was also done for all yearly highest tides. Monthly highs are shown in Figure 4 and the display of yearly highs is presented in Figure 5. Equation 1 was evaluated for the three extreme value distributions for each station.

The assumption is made that any excess water level above the astronomical tide height does not occur until the normal tide has risen to its full height. The probability of occurrence of the abnormally high tide is, therefore, the product of the probability that exceed the astronomical tide, \( 1 - P_1 \), and the probability that the wind generated portion will be equalled, \( 1 - P_2 \). The mean recurrence interval of this event will be

\[ R = \frac{1}{12} \left( \frac{1}{1 - P_1} \right) \left( \frac{1}{1 - P_2} \right) \]

\( R \) will be given in years.

As an example, the mean recurrence interval of a water level of 13.3 feet was determined for the Battery and the results presented in Table 1. The choice of the value to be assigned to \( P_1 \) is somewhat of a problem. The point at which the straight line plot of Figure 2 deviates is a matter of judgment. In this case the beginning of the curved portion was taken at a height of 9.4 feet. Therefore, \( P_1 \) is the value for a level of 9.3 feet. This is 0.65.

<table>
<thead>
<tr>
<th>Data treated</th>
<th>Figure(s)</th>
<th>Equation</th>
<th>( R ), years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal x extreme</td>
<td>2 and 3</td>
<td>( R = \frac{1}{12} \left( \frac{1}{1 - P_1} \right) \left( \frac{1}{1 - P_2} \right) )</td>
<td>38.8</td>
</tr>
<tr>
<td>(monthly)</td>
<td></td>
<td>( \frac{1}{12} \left( \frac{1}{1 - P_1} \right) \left( \frac{1}{1 - P_2} \right) )</td>
<td></td>
</tr>
<tr>
<td>Extreme</td>
<td>4</td>
<td>( R = \frac{1}{12} \left( \frac{1}{1 - P_1} \right) \left( \frac{1}{1 - P_2} \right) )</td>
<td>38.0</td>
</tr>
<tr>
<td>(monthly)</td>
<td></td>
<td>( \frac{1}{12} \left( \frac{1}{1 - P_1} \right) \left( \frac{1}{1 - P_2} \right) )</td>
<td></td>
</tr>
</tbody>
</table>
Excellent agreement is noted between the results of the treatment of monthly data. The yearly calculation is the conservative case. Two points must be noted. The yearly array may include two occurrences within the same year but will report only one value. Superimposing the expected recurrence interval of the wind generated surge over the astronomical tide supposes that the highest tide due to astronomical influence occur simultaneously. This may not be the situation. Paape has pointed out that the most conservative case may well happen with a lower astronomical tide and a higher storm surge. This will be the subject of a future investigation. It will be necessary to carefully peruse the records to find the actual predicted astronomical tide on the day of the very high tide.

The choice of the point of deviation of the plot on normal probability paper is most critical and in Figure 6 are presented predicted comparisons of predicted occurrence probabilities and calculated occurrence probabilities from yearly highest tides. The points derived from superposition are calculated assuming that the deviation point was at 9.4 feet. It can be seen that a longer mean recurrence interval would be expected and this would lead the designer to possible underdesign.

The authors wish to acknowledge the aid of Miss Terri Burnett in reduction of data, the kind assistance of Mr. Saul C. Berkman and Dr. A. Paape for his most helpful comments.

References

Paape, A. Private communication.
Figure 2.
Cumulative Distribution of Highest Monthly Tides, 1920-1966
The Battery

Figure 3.
Monthly Highest Tides 9.4' (Plotted as 1.4')
and above. THE BATTERY. 1920 - 1966,

\[ p = \exp\left(-e^{3.99 \ln \left(\frac{H - 8.0}{1.48}\right)}\right) \]
Figure 4.
Monthly Highest Tides. The Battery.
1920 - 1966. Measured Height - 7.0'.

\[ p = \exp(-5.18 \ln (H - 7.0)) \]

Figure 5.
Yearly Highest Tides. The Battery, 1920-1966

\[ p = \exp(-3.42 \ln (H - 8.0)) \]
Figure 6.
Comparison of Recorded Yearly Highest Tides
and Mean Recurrence Intervals by Superposition of Monthly

- Calculated by superimposing the storm surge
  over the highest tide from the normal plot (Month)
- Measured yearly highest tide.