Directed Study For

DERIVATION OF DYADIC GREEN'S FUNCTION FOR MULTILAYER DIELECTRIC SUBSTRATES

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Abstract

To characterize a field in the multilayer dielectric structure, a generalized full-wave Green's function is derived using two-dimensional spectral-domain technique and it is transformed to space-domain. It is derived using a simple structure where the current source lies between two layers bounded by surface impedance boundaries called the "standard" structure. Reflection and transmission coefficients at the surface impedance boundaries are used to compute the coefficients of the Green's function and this can be achieved by simple iteration technique with the transmission line analogy. The multilayer dyadic Green's function derived in spectral-domain can be converted to space-domain by use of Fourier-Bessel transformation. This space-domain Green's function is consistent with that of Sommerfeld approach for a grounded dielectric geometry excited by a horizontal Hertzian dipole.

This method is versatile and can be used for the either closed or open boundary problem. While the Sommerfeld's approach is difficult to apply in multilayer structure, the method in this report can easily be adapted to multilayer geometry. The Green's function for multilayer dielectric structure as detailed here is used in the numerical modeling of monolithic microwave integrate circuit, dielectric waveguides, and multilayer microstrip antenna structures.

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1 Introduction

The electromagnetic wave propagation in multilayered media, both isotropic and anisotropic, has been studied extensively by the use of full-wave analysis[1]-[5]. In [1], a generalized spectral-domain Green's function for multilayer dielectric is computed with iterative method to find the contribution of all other layers. In [2], a two-dimensional space-domain method of moments treatment of open microstrip discontinuities on multi-dielectric-layer substrates is presented. In [3], a dyadic Green's function in lossy media are investigated. While an operator approach in spectral-domain is presented in [4], in which a TE-TM decomposition and propagation matrices are used.

In this report, a general formulation of the problem of a horizontal dipole in a multilayered environment is presented. The formulation is considerably simplified by seperating Green's function into a transverse-electric(TE) and transverse-magnetic(TM) terms. Moreover, this report contains derivation the Cartesian dyadic components as functions of cylindrical coordinates, which were found to be more handy in many cases related to planar structures which exibit a circular symmetry. This is done by means of a Fourier-Bessel transform, which provides a tractable form of the dyadic Green's function.

The aim of this report, therefore, is to evaluate the dyadic Green's function for the multilayer planar structure, under a planar excitation, in the Fourier domain and, then, transform it to space domain using Fourier-Bessel transform. In fact, the dyadic Green's function by itself can provide useful information about the effects of the substrate, the characteristics of the radiated field, and, finally, the power losses in the layers.

In part 1, the equivalent boundary value problem solved with the use of electric and magnetic vector potentials. In part 2, according to the nonuniqueness of resolution of Hertz vector potentials, dyadic Green's function is derived with magnetic vector potential only and shown that these two approaches produce consistent results.

2 Part 1 : Decomposition of fields using electric and magnetic vector potentials

2.1 Geometry and general formulation of the problem

The electromagnetic study of the structure, Figure 1, can be obtained from Maxwell's equations, with a time variation $e^{j\omega t}$, where only an electric current density $J(\bar{r}')$ is assumed to be present. With such a hypothesis the electric field can be obtained via the following integral equation:

$$\bar{E}(\bar{r}) = \int_v \bar{\bar{G}}(\bar{r}|\bar{r}') \cdot \bar{J}(\bar{r}')\, dv' \tag{1}$$

where $\bar{\bar{G}}(\bar{r}|\bar{r}')$ stands for the dyadic Green's function. In the most general case $\bar{\bar{G}}$ has 9 components, while for planar currents these 9 components are reduced to only 4 components. Moreover, the symmetry of the geometry makes it possibile to find some components of $\bar{\bar{G}}$ in terms of the other components.

The dyadic Green's function is the solution of the fields due to a point source and can be represented, in rectangular coordinades, by

$$\bar{\bar{G}} = \begin{bmatrix} G_{xx}\hat{x}\hat{x} + G_{xy}\hat{x}\hat{y} + G_{xz}\hat{x}\hat{z} \\ +G_{yx}\hat{y}\hat{x} + G_{yy}\hat{y}\hat{y} + G_{yz}\hat{y}\hat{z} \\ +G_{zx}\hat{z}\hat{x} + G_{zy}\hat{z}\hat{y} + G_{zz}\hat{z}\hat{z} \end{bmatrix} \tag{2}$$

where G_{ij} is the i-th component of the field due to a unit j-directed current source $\delta(\bar{r} - \bar{r}')\,\hat{\jmath}$. The well known relation between Hertz vector potentials and the electromagnetic field is given by the following two equations

$$\bar{E} = -jkZ\nabla \times \bar{\Pi}_m + k^2\bar{\Pi}_e + \nabla\nabla \cdot \bar{\Pi}_e \quad (3)$$

$$\bar{H} = jkY\nabla \times \bar{\Pi}_e + k^2\bar{\Pi}_m + \nabla\nabla \cdot \bar{\Pi}_m \quad (4)$$

In addition to these equations, the relations between the Hertz potentials and electric and magnetic vector potentials make the above expressions as it follows:

$$\bar{E} = -\nabla \times \bar{F} - j\omega\mu\bar{A} + \frac{1}{j\omega\epsilon}\nabla\nabla \cdot \bar{A} \quad (5)$$

$$\bar{H} = \nabla \times \bar{A} - j\omega\epsilon\bar{F} + \frac{1}{j\omega\mu}\nabla\nabla \cdot \bar{F} \quad (6)$$

with

$$\bar{\Pi}_e = \frac{1}{j\omega\epsilon}\bar{A} \quad (7)$$

$$\bar{\Pi}_m = \frac{1}{j\omega\epsilon}\bar{F}. \quad (8)$$

These two vector potential functions satisfy the wave equations:

$$\nabla^2\bar{A} + k^2\bar{A} = -\bar{J}_i \quad (9)$$

$$\nabla^2\bar{F} + k^2\bar{F} = -\bar{M}_i \quad (10)$$

where $\bar{J}_i$ and $\bar{M}_i$ are electric and magnetic current sources.

A field in any region can be completely defined by suitable components of $\{A_x, A_y, A_z, F_x, F_y, F_z\}$ and judicious choice of these two components make the field decomposed [1],[6]. In the present chapter, the field is decomposed using (A_z, F_z), that is,

$$\bar{A} = A_z(x, y, z)\hat{z} \quad (11)$$

$$\bar{F} = F_z(x, y, z)\hat{z} \quad (12)$$

For an arbitrary surface current distribution in the xy plane between 11 and 21 layers, with x and y components:

$$\bar{J}(x,y) = J_x(x,y)\,\hat{x} + J_y(x,y)\,\hat{y} \tag{13}$$

the solution to $\bar{E}$ and $\bar{H}$ or $\bar{A}$ and $\bar{F}$ can be written in terms of the known Green's function as follows

$$\Psi(\bar{r}) = \int\!\!\int_s [\, G_{\Psi J_x}(\bar{r}|\bar{r}')J_x(\bar{r}') + G_{\Psi J_y}(\bar{r}|\bar{r}')J_y(\bar{r}')\,]ds \tag{14}$$

Since the multilayer geometry in this case is infinite in x and y, the two-dimensional function $Psi(\bar{r})$ can be defined as

$$\begin{aligned} \tilde{\Psi}(k_x, k_y, z) &= \int\!\!\int_{-\infty}^{\infty} \Psi(x,y,z)e^{j(k_x x + k_y y)}\, dxdy & (15) \\ \Psi(x,y,z) &= \frac{1}{(2\pi)^2}\int\!\!\int_{-\infty}^{\infty} \tilde{\Psi}(k_x, k_y, z)e^{-j(k_x x + k_y y)}\, dk_x dk_y & (16) \end{aligned}$$

where

$$\Psi = (\bar{E}, \bar{H}, \bar{J}, \bar{A}, \bar{F}). \tag{17}$$

Now, the individual components of the electric and magnetic fields can be written in terms of A_z and F_z as shown below:

$$\begin{aligned} E_x &= -\frac{\partial F_z}{\partial y} + \frac{1}{j\omega\epsilon}\frac{\partial^2 A_z}{\partial x \partial y} & (18) \\ E_y &= \frac{\partial F_z}{\partial x} + \frac{1}{j\omega\epsilon}\frac{\partial^2 A_z}{\partial^2 z} & (19) \\ E_z &= \frac{1}{j\omega\epsilon}(\frac{\partial^2 A_z}{\partial z^2} + k^2 A_z) & (20) \\ H_x &= \frac{\partial A_z}{\partial y} + \frac{1}{j\omega\mu}\frac{\partial^2 F_z}{\partial x \partial z} & (21) \end{aligned}$$

$$H_y = -\frac{\partial A_z}{\partial x} + \frac{1}{j\omega\mu}\frac{\partial^2 F_z}{\partial y \partial z} \tag{22}$$

$$H_z = \frac{1}{j\omega\mu}(\frac{\partial^2 F_z}{\partial z^2} + k^2 F_z) \tag{23}$$

2.2 Dyadic Green's function in the spectral-domain

The field components in space-domain can be easily transformed to the spectral-domain using a two-dimensional Fourier transformation.

$$\tilde{E}_x = -jk_y\tilde{F}_z + \frac{k_x}{\omega\epsilon}\frac{\partial \tilde{A}_z}{\partial z} \tag{24}$$

$$\tilde{E}_y = jk_x\tilde{F}_z + \frac{k_y}{\omega\epsilon}\frac{\partial \tilde{A}_z}{\partial z} \tag{25}$$

$$\tilde{E}_z = \frac{1}{j\omega\epsilon}(\frac{\partial^2 \tilde{A}_z}{\partial z^2} + k^2 \tilde{A}_z) \tag{26}$$

$$\tilde{H}_x = \tilde{A}_z + \frac{k_x}{\omega\mu}\frac{\partial \tilde{F}_z}{\partial z} \tag{27}$$

$$\tilde{H}_y = -jk_x\tilde{A}_z + \frac{k_y}{\omega\mu}\frac{\partial \tilde{F}_z}{\partial z} \tag{28}$$

$$\tilde{H}_z = \frac{1}{j\omega\mu}(\frac{\partial^2 \tilde{F}_z}{\partial z^2} + k^2 \tilde{F}_z) \tag{29}$$

in which $\tilde{}$ represents Fourier-transformed component. In source free region, A and F satisfy the homogeneous wave equation

$$\nabla^2 \begin{bmatrix} A_z \\ F_z \end{bmatrix} + k^2 \begin{bmatrix} A_z \\ F_z \end{bmatrix} = 0 \tag{30}$$

In the spectral domain this equation takes the form:

$$(\frac{\partial^2}{\partial z^2} - u^2) \begin{bmatrix} \tilde{A}_z \\ \tilde{F}_z \end{bmatrix} = 0 \tag{31}$$

where

$$u^2 = k_x^2 + k_y^2 - k^2, \;\; k = k_0\sqrt{\epsilon_r} \tag{32}$$

This homogeneous wave equation is not a partial differential equation but an ordinary differential equation and the general solution to the equation can be written as

$$\tilde{A}_z(k_x, k_y, z) = (e^{-uz} + \Gamma_A e^{uz})a(k_x, k_y) \tag{33}$$
$$\tilde{F}_z(k_x, k_y, z) = (e^{-uz} + \Gamma_F e^{uz})f(k_x, k_y) \tag{34}$$

As shown in Figure 2, for the "standard" structure with x directed current source between the interface of the two layer '11' and '21', the general solution to a_{ij} and f_{ij} is given by

$$\tilde{A}_{z_{11}}(k_x, k_y, z) = (e^{u_{11}z} + \Gamma_{A_{11}} e^{-u_{11}z})a_{11}(k_x, k_y) \tag{35}$$
$$\tilde{A}_{z_{21}}(k_x, k_y, z) = (e^{-u_{21}z} + \Gamma_{A_{21}} e^{u_{21}z})a_{21}(k_x, k_y) \tag{36}$$
$$\tilde{F}_{z_{11}}(k_x, k_y, z) = (e^{u_{11}z} + \Gamma_{F_{11}} e^{-u_{11}z})f_{11}(k_x, k_y) \tag{37}$$
$$\tilde{F}_{z_{21}}(k_x, k_y, z) = (e^{-u_{21}z} + \Gamma_{F_{21}} e^{u_{21}z})f_{21}(k_x, k_y) \tag{38}$$

At the interface, tangential electric fields are continuous and tangential magnetic field are discontinuous due to the x directed electric current source as shown by the following equation:

$$\tilde{E}_{x_{11}}(k_x, k_y, z = 0) = \tilde{E}_{x_{21}}(k_x, k_y, z = 0) \tag{39}$$
$$\tilde{E}_{y_{11}}(k_x, k_y, z = 0) = \tilde{E}_{y_{21}}(k_x, k_y, z = 0) \tag{40}$$
$$\tilde{H}_{x_{11}}(k_x, k_y, z = 0) = \tilde{H}_{x_{21}}(k_x, k_y, z = 0) \tag{41}$$
$$\tilde{H}_{y_{21}}(k_x, k_y, z = 0) - \tilde{H}_{y_{11}}(k_x, k_y, z = 0) = 1 \tag{42}$$

With the above 4 boundary conditions the unknown constants Γ_A and Γ_F are determined and the functions a_{11}, a_{21}, f_{11} and f_{21} may be written in the form:

$$f_{11} = \frac{k_y}{k_x^2+k_y^2}\frac{\omega\mu_0(1+\Gamma_{F_{21}})}{[u_{11}(1-\Gamma_{F_{11}})(1+\Gamma_{F_{21}})+u_{21}(1-\Gamma_{F_{21}})(1+\Gamma_{F_{11}})]} \quad (43)$$

$$f_{21} = \frac{k_y}{k_x^2+k_y^2}\frac{\omega\mu_0(1+\Gamma_{F_{11}})}{[u_{11}(1-\Gamma_{F_{11}})(1+\Gamma_{F_{21}})+u_{21}(1-\Gamma_{F_{21}})(1+\Gamma_{F_{11}})]} \quad (44)$$

$$a_{11} = \frac{-k_x}{j(k_x^2+k_y^2)} \frac{\epsilon_{11}u_{21}(1-\Gamma_{A_{21}})}{[\epsilon_{11}u_{21}(1+\Gamma_{A_{11}})(1-\Gamma_{A_{21}})+\epsilon_{21}u_{11}(1+\Gamma_{A_{21}})(1-\Gamma_{A_{11}})]} \quad (45)$$

$$a_{21} = \frac{k_x}{j(k_x^2+k_y^2)} \frac{\epsilon_{21}u_{11}(1-\Gamma_{A_{11}})}{[\epsilon_{11}u_{21}(1+\Gamma_{A_{11}})(1-\Gamma_{A_{21}})+\epsilon_{21}u_{11}(1+\Gamma_{A_{21}})(1-\Gamma_{A_{11}})]} \quad (46)$$

As a result, the elements of dyadic Green's function for the electric field due to an infinitesimal electric current source in spectral domain are given as follows

$$\tilde{G}^{21}_{E_xJ_x} = -\frac{k_x u_{21}}{\omega\epsilon_{21}}(e^{-u_{21}z}-\Gamma_{A_{21}}e^{u_{21}})a_{21} + jk_y(e^{-u_{21}z}-\Gamma_{F_{21}}e^{u_{21}})f_{21} \quad (47)$$

$$\tilde{G}^{21}_{E_yJ_x} = -\frac{k_y u_{21}}{\omega\epsilon_{21}}(e^{-u_{21}z}-\Gamma_{A_{21}}e^{u_{21}})a_{21} - jk_x(e^{-u_{21}z}-\Gamma_{F_{21}}e^{u_{21}})f_{21} \quad (48)$$

$$\tilde{G}^{21}_{E_zJ_x} = \frac{k_x^2+k_y^2}{j\omega\epsilon_{21}}(e^{-u_{21}z}+\Gamma_{A_{21}}e^{u_{21}})a_{21} \quad (49)$$

for $z > 0$ and

$$\tilde{G}^{11}_{E_xJ_x} = \frac{k_x u_{11}}{\omega\epsilon_{11}}(e^{u_{11}z}-\Gamma_{A_{11}}e^{-u_{11}})a_{11} - jk_y(e^{u_{11}z}+\Gamma_{F_{11}}e^{-u_{11}})f_{11} \quad (50)$$

$$\tilde{G}^{11}_{E_yJ_x} = \frac{k_y u_{11}}{\omega\epsilon_{11}}(e^{u_{11}z}-\Gamma_{A_{11}}e^{-u_{11}})a_{11} + jk_x(e^{u_{11}z}+\Gamma_{F_{11}}e^{-u_{11}})f_{11} \quad (51)$$

$$\tilde{G}^{11}_{E_zJ_x} = \frac{k_x^2+k_y^2}{j\omega\epsilon_{11}}(e^{u_{11}z}+\Gamma_{A_{11}}e^{u_{11}})a_{11} \quad (52)$$

for $z < 0$.

Finally, we note that the elements G_{ij} of the spectral Green's function show very interesting properties, due to the geometrical symmetry of the structure around z-axis, that can be summarized in the followings[5]:

$$\begin{aligned}
\tilde{G}_{E_xJ_y}(-k_y, k_x, z|z') &= -\tilde{G}_{E_yJ_x}(k_x, k_y, z|z') && (53)\\
\tilde{G}_{E_xJ_y}(-k_y, k_x, z|z') &= \tilde{G}_{E_xJ_x}(k_x, k_y, z|z') && (54)\\
\tilde{G}_{E_yJ_y}(-k_y, k_x, z|z') &= \tilde{G}_{E_xJ_x}(k_x, k_y, z|z') && (55)\\
\tilde{G}_{E_xJ_z}(-k_y, k_x, z|z') &= -\tilde{G}_{E_yJ_z}(k_x, k_y, z|z') && (56)\\
\tilde{G}_{E_yJ_z}(-k_y, k_x, z|z') &= \tilde{G}_{E_xJ_z}(k_x, k_y, z|z') && (57)
\end{aligned}$$

Moreover, if there were z directed current source, using the reciprocity theorem and Parseval's theorem we can deduce the following result

$$\begin{aligned}
\tilde{G}_{pq}(k_x, k_y, z_1|z_2) &= \tilde{G}_{qp}(k_x, k_y, z_2|z_1)\\
&\quad \forall\ p \neq q\ (p = x, y; q = z). && (58)
\end{aligned}$$

where z_1 and z_2 are the z coordinates of the chosen sources. We note that the above result express an index permutation property of the spectral dyadic Green's function obtained in the Fourier transform space.

2.3 Space-domain solution of dyadic Green's function

To find space domain Green's function, a two-dimensional inverse Fourier transform is need. The f_{ij} and a_{ij} are functions of k_x and k_y, so a transformation to polar coordinates is made both in coordinate space and k space.

$$x = \rho cos\phi, \quad y = \rho sin\phi \tag{59}$$

$$k_x = \lambda\, cos\zeta, \quad k_y = \lambda\, sin\zeta, \tag{60}$$

with

$$k_x x + k_y y = \lambda\rho cos(\zeta - \phi). \tag{61}$$

Then, the Fourier transform defined in an earlier paragraph becomes

$$\Psi(\rho, \phi, z) = \frac{1}{(2\pi)^2}\int_0^\infty \int_0^{2\pi} \tilde{\Psi}(\lambda, \zeta, z) e^{j\lambda\rho cos(\zeta-\phi)}\, d\zeta\lambda d\lambda. \tag{62}$$

The function $e^{j\lambda\rho cos(\zeta-\phi)-j\omega t}$ represents a plane wave whose propagation constant is λ, traveling in a direction which is normal to the z axis and which makes an angle ζ with the x axis. Each plane wave is multiplied by an amplitude factor $\tilde{\Psi}(\lambda, \zeta, z)$ and then is summed with respect to the propagation constant, or space frequency λ.

If we take a close look at the components of dyadic Green's function, we see that in (λ, ζ)-domain the components can be separated into λ and ζ functions, respectively. That is, for any componens the following is true:

$$\tilde{\Psi}(\lambda, \zeta, z) = A(\lambda, z)\, B(\zeta) \tag{63}$$

and as a result, for a surface current source, the kernal of the Green's function has this form:

$$\begin{bmatrix} -A_1 cos^2\zeta + A_2 sin^2\zeta & -(A_1 + A_2) cos\zeta sin\zeta \\ -(A_1 + A_2) cos\zeta sin\zeta & -A_1 sin^2\zeta + A_2 cos^2\zeta \\ A_3 cos\zeta & A_3 sin\zeta \end{bmatrix} \tag{64}$$

In equation (64)

$$A_1 = \frac{u_{21}u_{11}}{j\omega}(e^{-u_{21}z} - \Gamma_{A_{21}}e^{u_{21}z})\frac{1-\Gamma_{A_1 1}}{T_A} \tag{65}$$

$$A_2 = j\omega\mu_0(e^{-u_{21}z} - \Gamma_{F_{21}}e^{u_{21}z})\frac{1+\Gamma_{F_1 1}}{T_F} \tag{66}$$

$$A_3 = \frac{-u_{11}}{\omega}(e^{-u_{21}z} + \Gamma_{A_{21}}e^{u_{21}z})\frac{1-\Gamma_{A_1 1}}{T_A} \tag{67}$$

for $z \geq 0$ and

$$A_1 = \frac{u_{21}u_{11}}{j\omega}(e^{u_{11}z} - \Gamma_{A_{11}}e^{-u_{11}z})\frac{1-\Gamma_{A_2 1}}{T_A} \tag{68}$$

$$A_2 = -j\omega\mu_0(e^{u_{11}z} + \Gamma_{F_{11}}e^{-u_{11}z})\frac{1+\Gamma_{F_1 1}}{T_F} \tag{69}$$

$$A_3 = \frac{-u_{21}}{\omega}(e^{u_{11}z} + \Gamma_{A_{11}}e^{-u_{11}z})\frac{1-\Gamma_{A_1 1}}{T_A} \tag{70}$$

for $z < 0$, with

$$T_A = \epsilon_{11}u_{21}(1+\Gamma_{A_{11}})(1-\Gamma_{A_{21}}) + \epsilon_{21}u_{11}(1+\Gamma_{A_{21}})(1-\Gamma_{A_{11}}) \tag{71}$$

$$T_F = u_{11}(1-\Gamma_{F_{11}})(1+\Gamma_{F_{21}}) + u_{21}(1-\Gamma_{F_{21}})(1+\Gamma_{F_{11}}). \tag{72}$$

Now, one may use the following integral

$$\int_0^{2\pi} cos\, m\theta\, e^{-jz\, cos\theta}\, d\theta = 2\pi(-j)^m J_m(z) \tag{73}$$

$$\int_0^{2\pi} sin\, m\theta\, e^{-jz\, cos\theta}\, d\theta = 0 \tag{74}$$

to evaluate the ζ-integration. The final results are in the form:

$$G_{E_xJ_x}(\rho,\phi,z|z'=0) = \frac{1}{4\pi}\int_0^\infty [\,(-A_1+A_2)J_0(\lambda\rho) + (A_1+A_2)\, cos2\phi\; J_2(\lambda\rho)\,]\lambda d\lambda \tag{75}$$

$$G_{E_yJ_x}(\rho,\phi,z|z'=0) = \frac{1}{4\pi}\int_0^\infty (-A_1+A_2)\, sin2\phi\; J_0(\lambda\rho) d\lambda \tag{76}$$

$$G_{E_xJ_x}(\rho,\phi,z|z'=0) = \frac{i}{2\pi}\int_0^\infty A_3\, cos\phi\; J_1(\lambda\rho)\lambda^2 d\lambda \tag{77}$$

$$G_{E_xJ_y}(\rho,\phi,z|z'=0) = G_{E_yJ_x}(\rho,\phi,z|z'=0) \tag{78}$$

$$\begin{aligned} G_{E_yJ_y}(\rho,\phi,z|z'=0) &= \frac{1}{4\pi}\int_0^\infty [\,(-A_1+A_2)J_0(\lambda\rho) \\ &\quad -(A_1+A_2)\, cos2\phi\; J_2(\lambda\rho)\,]\lambda d\lambda \end{aligned} \tag{79}$$

$$G_{E_xJ_y}(\rho,\phi,z|z'=0) = tan\phi\, G_{E_xJ_x}(\rho,\phi,z|z'=0) \tag{80}$$

2.4 Fresnel coefficients at the other boundaries

The Γ's of the field expression are found to be reflection coefficients for a transmission line which is terminated by the load of different characteristic impedance. For the magnetic vector potential($\bar{A}$), the the equivalent transmission line characteristic impedance is equal to β_{ij}/ϵ_{ij} which is identical to the TM wave impedance. For the electric vector potential($\bar{F}$), the equivalent transmission line characteristic admittance is equal to β_{ij}/μ_{ij} which is identical to the TE wave admittance. In addition, the reflection coefficient for $\bar{A}$ /$\bar{F}$ is equivalent to that of a current/voltage wave of a transmission line.

With these analysis and Figure3, the reflection coefficients are determined:

$$\begin{aligned} \Gamma_{A_{ij}} &= \Gamma'_{A_{ij}}\; e^{-2u_{ij}d_{ij}} \\ &= \frac{Z_{A_{ij}} - Z^T_{A_{ij}}}{Z_{A_{ij}} + Z^T_{A_{ij}}}\; e^{-2u_{ij}d_{ij}} \end{aligned} \tag{81}$$

$$\begin{aligned} \Gamma_{F_{ij}} &= \Gamma'_{F_{ij}}\; e^{-2u_{ij}d_{ij}} \\ &= \frac{Y_{F_{ij}} - Y^T_{F_{ij}}}{Y_{F_{ij}} + Y^T_{F_{ij}}}\; e^{-2u_{ij}d_{ij}} \end{aligned} \tag{82}$$

with

$$
\begin{aligned}
Z^T_{ij+1} &= \frac{1-\Gamma_{A_{ij+1}}}{1+\Gamma_{A_{ij+1}}} Z_{A_{ij+1}} && (83)\\
Y^T_{ij+1} &= \frac{1-\Gamma_{F_{ij+1}}}{1+\Gamma_{F_{ij+1}}} Z_{F_{ij+1}} && (84)
\end{aligned}
$$

2.4.1 Grounded-substrate geometry

Since there are no reflection from the upper layer in Figure 4, the reflection coefficients $\Gamma'_{A_{11}}, \Gamma'_{F_{11}}, \Gamma_{A_{11}}, \Gamma_{F_{11}}$ are identical to zero. On a perfect electric conductor(pec) the normal component of electric field is doubled by its image, however, normal component of magnetic field is cancelled by its image. Therefore, the following relations are true:

$$
\Gamma'_{A_{21}} = 1, \;\; \Gamma'_{F_{21}} = -1 \tag{85}
$$

and

$$
\begin{aligned}
\Gamma_{A_{21}} &= e^{-2u_0 d_1} && (86)\\
\Gamma_{F_{21}} &= -e^{-2u_0 d_1} && (87)
\end{aligned}
$$

2.4.2 Substrate-superstrate geometry

For a substrate-superstrate geometry, Figure 5, the Γ's are calculated step by step from the farthest layer as shown below:

$$
\begin{aligned}
\Gamma_{A_{11}} &= e^{-2u_{11} d_1} && (88)\\
\Gamma_{F_{11}} &= -e^{-2u_{11} d_1} && (89)
\end{aligned}
$$

For the upper layer, the reflection coefficients are given by:

$$\Gamma_{A_{21}} = \Gamma'_{A_{21}}\, e^{-2u_{21}d_2} \tag{90}$$

$$\Gamma_{F_{21}} = \Gamma'_{F_{21}}\, e^{-2u_{21}d_2} \tag{91}$$

where

$$\begin{aligned}\Gamma_{A_{21}} &= \frac{Z_{A_{21}} - Z^T_{A_{22}}}{Z_{A_{21}} + Z^T_{A_{22}}} \\ &= \frac{(\beta_{21}/\epsilon_{21}) - (\beta_{22}/\epsilon_{22})}{(\beta_{21}/\epsilon_{21}) + (\beta_{22}/\epsilon_{22})} \qquad (92)\\ \Gamma_{F_{21}} &= \frac{Y_{F_{21}} - Y^T_{F_{21}}}{Y_{F_{21}} + Y^T_{F_{21}}} \\ &= \frac{(\beta_{21}/\mu_{21}) - (\beta_{22}/\mu_{22})}{(\beta_{21}/\mu_{21}) + (\beta_{22}/\mu_{22})} \qquad (93)\\ & \qquad (94)\end{aligned}$$

and

$$u_{ij} = \lambda^2 - k_{ij}^2, \quad k_{ij} = k_0\sqrt{\epsilon_{ij}}. \tag{95}$$

2.4.3 Two-layer-substrate geometry

As shown in Figure 6, for an upward looking case, $\Gamma_{A_{21}}$ and Γ_{F21} are equal to zero. For a downward looking case, however,

$$\Gamma_{A_{12}} = e^{-2u_{12}d_2} \tag{96}$$

$$\Gamma_{F_{12}} = -e^{-2u_{12}d_2} \tag{97}$$

and

$$Z^T_{A_{12}} = j\, tanh(u_{12}d_2) Z_{A_12} \tag{98}$$

$$Y^T_{F_{12}} = -j\, coth(u_{12}d_2) Y_{F_12} \tag{99}$$

Finally, the $\Gamma_{A_{11}}$ and $\Gamma_{F_{11}}$ are given as follows:

$$\Gamma_{A_{11}} = e^{-2u_{11}d_1} \frac{Z_{A_{11}} - j\, Z_{A_{12}}\, tanh(u_{12}d_2)}{Z_{A_{11}} + j\, Z_{A_{12}}\, tanh(u_{12}d_2)} \tag{100}$$

$$\Gamma_{F_{11}} = e^{-2u_{11}d_1} \frac{Y_{F_{11}} + j\, Y_{F_{12}}\, coth(u_{12}d_2)}{Y_{F_{11}} - j\, Y_{F_{12}}\, coth(u_{12}d_2)} \tag{101}$$

2.4.4 Substrate-air gap-superstrate geometry

For a downward looking case, Figure 7, the reflection coefficients are given by

$$\Gamma_{A_{11}} = e^{-2u_{11}d_1} \tag{102}$$

$$\Gamma_{F_{11}} = -e^{-2u_{11}d_1}, \tag{103}$$

while, for an upward looking case are identical to the form:

$$\Gamma_{A_{22}} = e^{-2u_{22}d_3} \frac{Z_{A_{22}} - Z_{A_{23}}}{Z_{A_{22}} + Z_{A_{23}}} \tag{104}$$

$$\Gamma_{F_{22}} = e^{-2u_{22}d_3} \frac{Y_{F_{22}} - Y_{F_{23}}}{Y_{F_{22}} + Y_{F_{23}}} \tag{105}$$

with

$$Z^T_{A_{22}} = \frac{1 - \Gamma_{A_{22}}}{1 + \Gamma_{A_{22}}} Z_{A_{22}} \tag{106}$$

$$Y^T_{F_{22}} = \frac{1 - \Gamma_{F_{22}}}{1 + \Gamma_{F_{22}}} Z_{F_{22}} \tag{107}$$

Finally, the Γ's at the air-gap layer are given by the following expressions

$$\Gamma_{A_{21}} = e^{-2u_{21}d_2} \frac{Z_{A_{21}} - Z^T_{A_{22}}}{Z_{A_{21}} + Z^T_{A_{22}}} \tag{108}$$

$$\Gamma_{F_{21}} = e^{-2u_{21}d_2} \frac{Y_{F_{21}} - Y^T_{F_{22}}}{Y_{F_{21}} + Y^T_{F_{22}}} \tag{109}$$

3 Part 2 : Decomposition of fields using magnetic vector potential

3.1 Magnetic vector potential

To derive the generalized Green's function for multilayer structure, a combination of Sommerfeld's resolution[7] of a magnetic vector potential (or Hertz potential Π_e) $\bar{A} = (A_x, 0, A_z)$ for the x-directed dipole source on the imperfect ground plane, and the spectral domain technique are investigated. Moreover, the equivalence between the former approach and this method is also investigated. In contrast to the previous chapter, the potential Green's function is defined as

$$\bar{E}(\bar{r}) = \int_v (k^2 \bar{\bar{I}} + \nabla\nabla) \cdot \bar{\bar{G}}(\bar{r}|\bar{r}') \cdot \bar{J}(\bar{r}')\, d\bar{v}'. \tag{110}$$

The field equations with the vector potential $\bar{A}$ have the following form;

$$\begin{aligned}
E_x &= -j\omega\mu A_x + \frac{1}{j\omega\epsilon}\frac{\partial^2 A_x}{\partial^2 x} + \frac{1}{j\omega\epsilon}\frac{\partial^2 A_z}{\partial x \partial z} && (111)\\
E_y &= \frac{1}{j\omega\epsilon}\left(\frac{\partial^2 A_x}{\partial y \partial x} + \frac{\partial^2 A_z}{\partial y \partial z}\right) && (112)\\
E_z &= -j\omega\mu A_z + \frac{1}{j\omega\epsilon}\frac{\partial^2 A_x}{\partial x \partial z} + \frac{1}{j\omega\epsilon}\frac{\partial^2 A_z}{\partial^2 z} && (113)\\
H_x &= \frac{\partial A_z}{\partial y} && (114)\\
H_y &= \frac{\partial A_x}{\partial z} - \frac{\partial^2 A_z}{\partial x} && (115)\\
H_z &= -\frac{\partial A_x}{\partial y} && (116)
\end{aligned}$$

Using the Fourier transform, the field expressions and wave equation for vector potential can be transformed to the $\bar{k}$-domain. The boundary conditions can also be transformed to the $\bar{k}$-domain.

$$
\begin{aligned}
\tilde{E}_x &= -(j\omega\mu + \frac{k_x^2}{j\omega\epsilon})\tilde{A}_x + \frac{k_x}{\omega\epsilon}\frac{\partial \tilde{A}_z}{\partial z} & (117)\\
\tilde{E}_y &= -\frac{k_x k_y}{j\omega\epsilon}\tilde{A}_x + \frac{k_y}{\omega\epsilon}\frac{\partial \tilde{A}_z}{\partial z} & (118)\\
\tilde{E}_z &= -j\omega\mu\tilde{A}_z + \frac{k_x}{\omega\epsilon}\frac{\partial \tilde{A}_x}{\partial z} + \frac{1}{j\omega\epsilon}\frac{\partial^2 \tilde{A}_z}{\partial z^2} & (119)\\
\tilde{H}_x &= jk_x\tilde{A}_z & (120)\\
\tilde{H}_y &= \frac{\partial \tilde{A}_x}{\partial z} - \tilde{A}_z r & (121)\\
\tilde{H}_z &= -jk_y\tilde{A}_x & (122)
\end{aligned}
$$

In addition to the above, the following equation is true.

$$(\frac{\partial^2}{\partial z^2} - u^2)\tilde{\bar{A}} = 0 \qquad (123)$$

where

$$u^2 = k_x^2 + k_y^2 - k^2, \;\; k = k_0\sqrt{\epsilon_r} \qquad (124)$$

From the geometry of "standard" problem, the solution of the wave equation in the spectral domain in the both side of the interface can be assumed to be

$$
\begin{aligned}
\tilde{A}_{x11}(k_x, k_y, z) &= (e^{u_{11}z} + \Gamma_{x_{11}} e^{-u_{11}z})P_{11}(k_x, k_y) & (125)\\
\tilde{A}_{z11}(k_x, k_y, z) &= (e^{u_{11}z} + \Gamma_{z_{11}} e^{-u_{11}z})Q_{11}(k_x, k_y) & (126)\\
\tilde{A}_{x21}(k_x, k_y, z) &= (e^{-u_{21}z} + \Gamma_{x_{21}} e^{u_{21}z})P_{21}(k_x, k_y) & (127)\\
\tilde{A}_{z21}(k_x, k_y, z) &= (e^{-u_{21}z} + \Gamma_{z_{21}} e^{u_{21}z})Q_{21}(k_x, k_y) & (128)
\end{aligned}
$$

Moreover, the boundary conditions in the spectral domain at the interface $z = 0$ are the same as those of the previous chapter. From the electric and magnetic fields boundary conditions, the four potential boundary conditions shown below are obtained.

$$\tilde{A}_z^{21}(k_x, k_y, z = 0) = \tilde{A}_z^{11}(k_x, k_y, z = 0) \tag{129}$$

$$\tilde{A}_x^{21}(k_x, k_y, z = 0) = \tilde{A}_x^{11}(k_x, k_y, z = 0) \tag{130}$$

$$\frac{\partial \tilde{A}_x^{21}}{\partial z} - \frac{\partial \tilde{A}_x^{21}}{\partial z} = 1 \tag{131}$$

$$\left(\frac{1}{\epsilon_{21}}\frac{\partial \tilde{A}_z^{21}}{\partial z} - \frac{1}{\epsilon_{11}}\frac{\partial \tilde{A}_z^{11}}{\partial z}\right) - jk_x\left(\frac{\tilde{A}_x^{21}}{\epsilon_{21}} - \frac{\tilde{A}_x^{11}}{\epsilon_{11}}\right) = 0 \tag{132}$$

As a result, the four unknown function P's and Q's are determined in terms of Γ's in each region.

$$P_{11}(k_x, k_y) = \frac{1 + \Gamma_{x_{21}}}{S_1} \tag{133}$$

$$P_{21}(k_x, k_y) = \frac{1 + \Gamma_{x_{11}}}{S_1} \tag{134}$$

$$\begin{aligned} Q_{11}(k_x, k_y) &= \left(\frac{1}{\epsilon_{11}} - \frac{1}{\epsilon_{21}}\right) \cdot \\ &\quad \frac{jk_x(1 + \Gamma_{x_{11}})(1 + \Gamma_{x_{21}})(1 + \Gamma_{z_{21}})}{S_1(u_{11}, u_{21}, \Gamma_{x_{11}}, \Gamma_{x_{21}})S_2(u_{11}, u_{21}, \Gamma_{x_{11}}, \Gamma_{x_{21}})} \end{aligned} \tag{135}$$

$$Q_{21}(k_x, k_y) = \frac{1 + \Gamma_{z_{11}}}{1 + \Gamma_{z_{21}}} Q_{11}(k_x, k_y) \tag{136}$$

where

$$\begin{aligned} S_1 &= S_1(u_{11}, u_{21}, \Gamma_{x_{11}}, \Gamma_{x_{21}}) \\ &= -[u_{21}(1 - \Gamma_{x_{21}})(1 + \Gamma_{x_{11}}) + u_{11}(1 - \Gamma_{x_{11}})(1 + \Gamma_{x_{21}})] \end{aligned} \tag{137}$$

$$\begin{aligned} S_2 &= S_2(u_{11}, u_{21}, \Gamma_{x_{11}}, \Gamma_{x_{21}}) \\ &= \frac{u_{21}}{\epsilon_{21}}(1 - \Gamma_{z_{21}})(1 + \Gamma_{z_{11}}) + \frac{u_{11}}{\epsilon_{11}}(1 - \Gamma_{z_{11}})(1 + \Gamma_{z_{21}}) \end{aligned} \tag{138}$$

These results are similar in the form with T_A and T_F of the previous chapter, however, the Γ is defined somewhat differently.

3.2 Space-domain solution of dyadic Green's function

The space-domain Green's function of a vector potential $\bar{a}$ with $\hat{x}$ and $\hat{y}$ directed current source is derived using 2-dimensional inverse Fourier transform defined in the previous chapter. Moreover, the symmetry of the structure considered here allows us to deduce some components of Green's function from the other components. The final results are given as following:

$$G^{21}_{A_xJ_x}(\rho,\phi,z) = \frac{1}{2\pi}\int_0^{\infty}(\, e^{-u_{21}z} + \Gamma_{x_{21}}\, e^{u_{21}z})\, P_{21}(\lambda)J_0(\lambda\rho)\,\lambda d\lambda \quad (139)$$

$$G^{21}_{A_zJ_x}(\rho,\phi,z) = \frac{i}{2\pi}cos\phi\int_0^{\infty}(e^{-u_{21}z} + \Gamma_{z_{21}}e^{u_{21}z})Q_{21}(\lambda)J_1(\lambda\rho)\lambda^2 d\lambda \quad (140)$$

for $z > 0$, and

$$G^{11}_{A_xJ_x}(\rho,\phi,z) = \frac{1}{2\pi}\int_0^{\infty}(\, e^{u_{11}z} + \Gamma_{x_{11}}e^{-u_{11}z})\, P_{11}(\lambda)J_0(\lambda\rho)\,\lambda d\lambda \quad (141)$$

$$G^{11}_{A_zJ_x}(\rho,\phi,z) = \frac{i}{2\pi}cos\phi\int_0^{\infty}(e^{u_{11}z} + \Gamma_{z_{11}}e^{-u_{11}z})Q_{11}(\lambda)J_1(\lambda\rho)\lambda^2 d\lambda \quad (142)$$

for $z < 0$.

The P's and Q's are determined in the section 3.1, and Γ_x's and Γ_z's will be determined by the boundary conditions at the other interfaces, that is, tangential electric field must be zero on a pec and the normal component should be enforced. For example, on a perfect electric conductor,

$$\Gamma'_x = -1 \quad (143)$$

$$\Gamma'_z = 1 \quad (144)$$

in which the Γs are reflection coefficients at the interfaces. For a multilayer problem, the Γs are calculated by iterative method as it has been demonstrated at the end of the previous chapter

4 Conclusions

In this report, two types of Green's functions, one is the electric field Green's function and the other is the potential Green's function, are derived for multilayer substrates geometry and shown that these two are equivalent. According to the non-uniqueness of the resolution of the Hertz vector potential, the former approach use TM - TE decomposition with electric($\bar{F}$) and magnetic($\bar{A}$) vector potential. While the later use only magnetic vector potential with two components, in which one component is parallel to the current source and the other is perpendicular to the current source and its surface.

A two dimensional Fourier transform pair and Fourier-Bessel transformation are used to convert the spectral domain Green's function to space domain form. The versatility of the spectral solution for multilayer structure and the possibility of inverse transform to space domain allow us to develop a powerful method of analyzing multi-dielectric layer structure regardless of open or closed geometry. Moreover, the second kind of Green's function can be used to extend ready-made CAD program for a multilayer structure.

5 Bibliography

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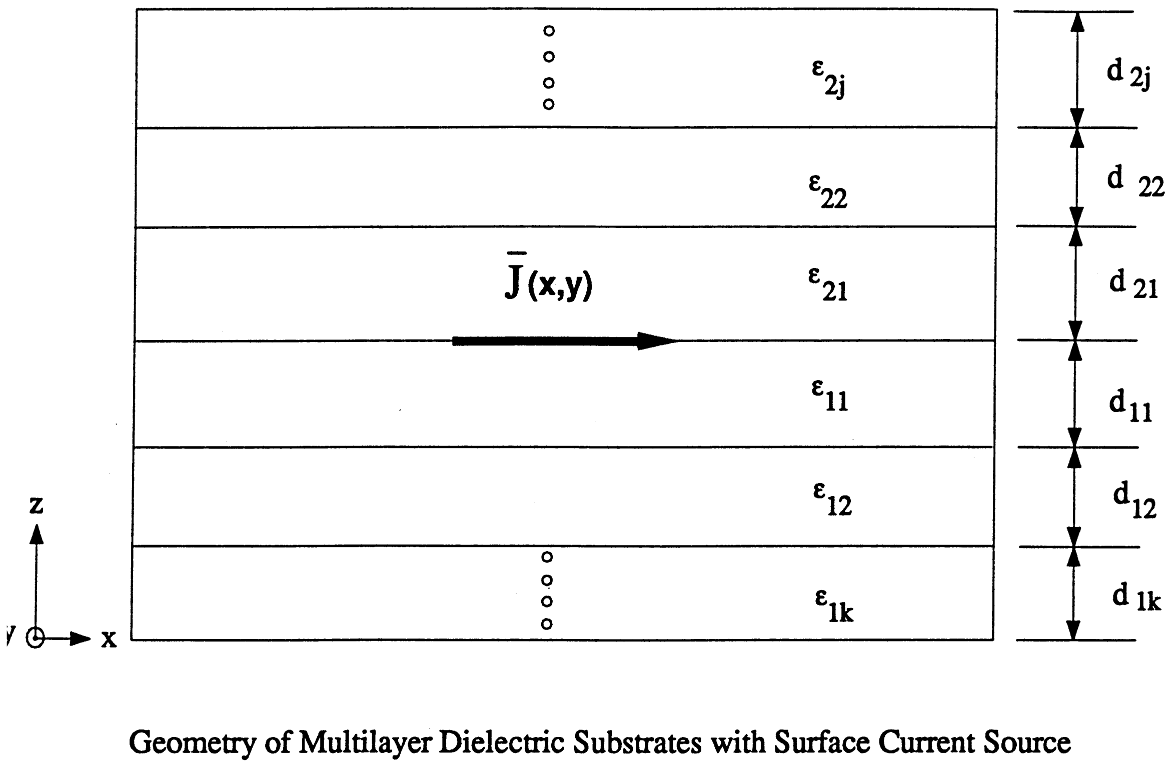


Figure 1: Geometry of multilayer dielectric substrates with surface current source

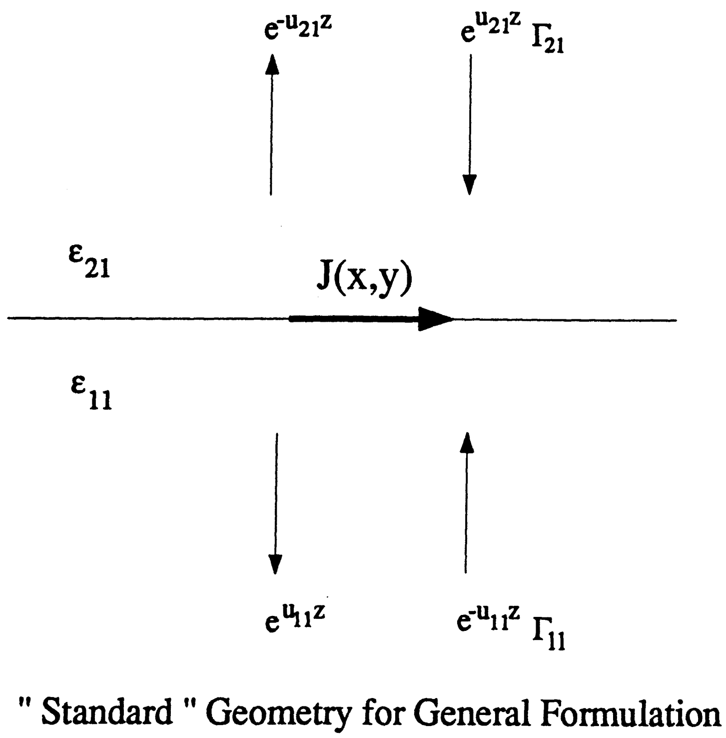
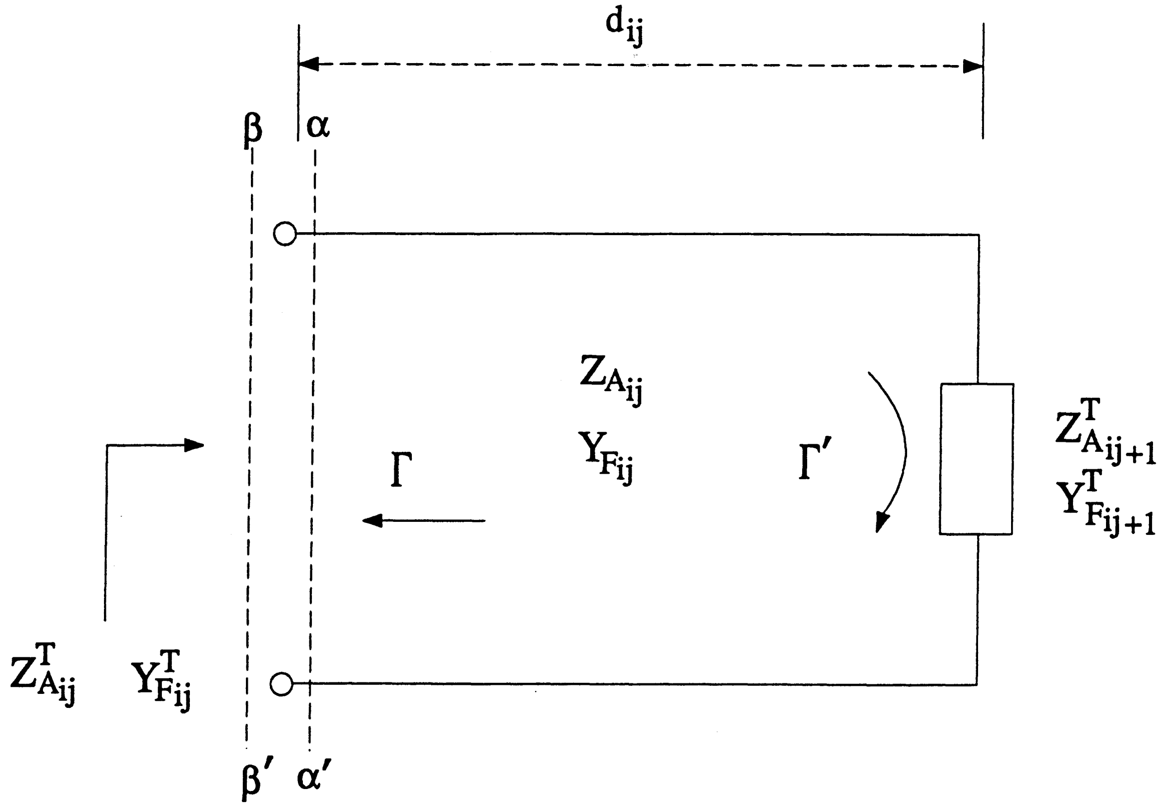


Figure 2: "standard" geometry for general formulation



Transmission-Line Analogy to Calculate Reflection Coefficients

Figure 3: Transmission line analogy to calculate reflection coefficients

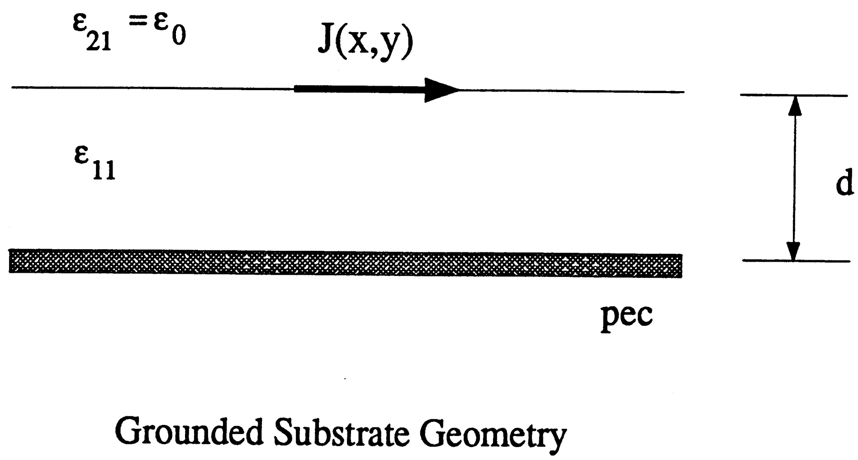


Figure 4: Grounded-substrate geometry

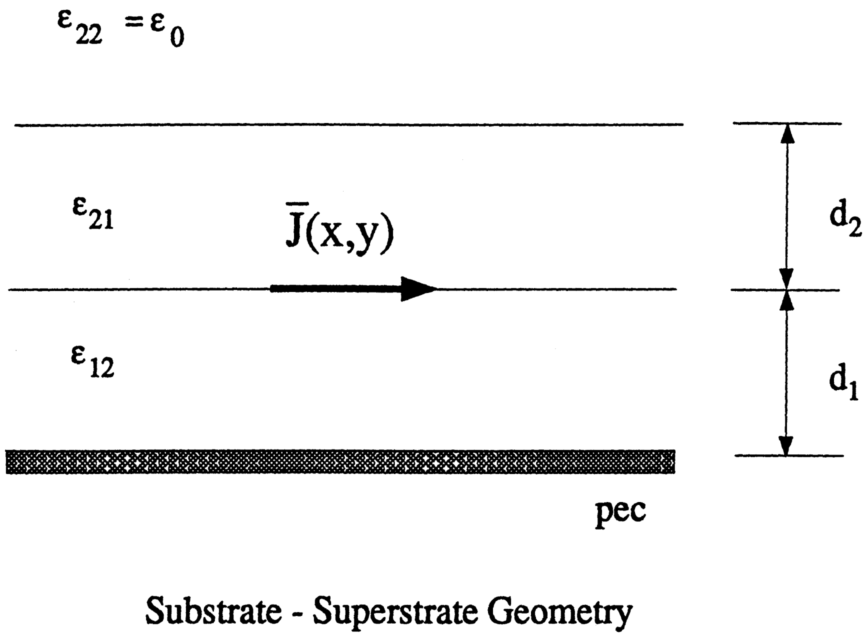


Figure 5: Substrate-superate geometry

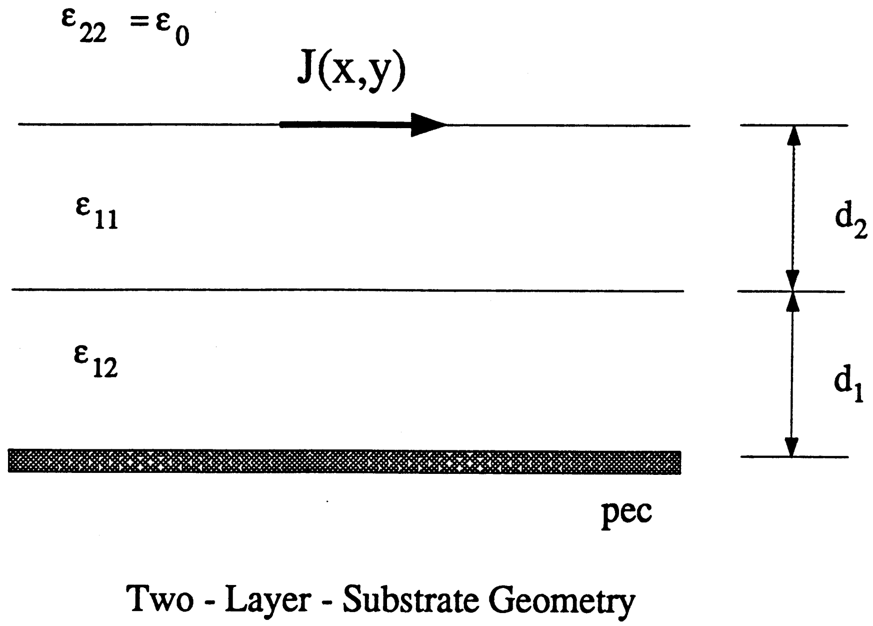


Figure 6: Two-layer-substrate geometry

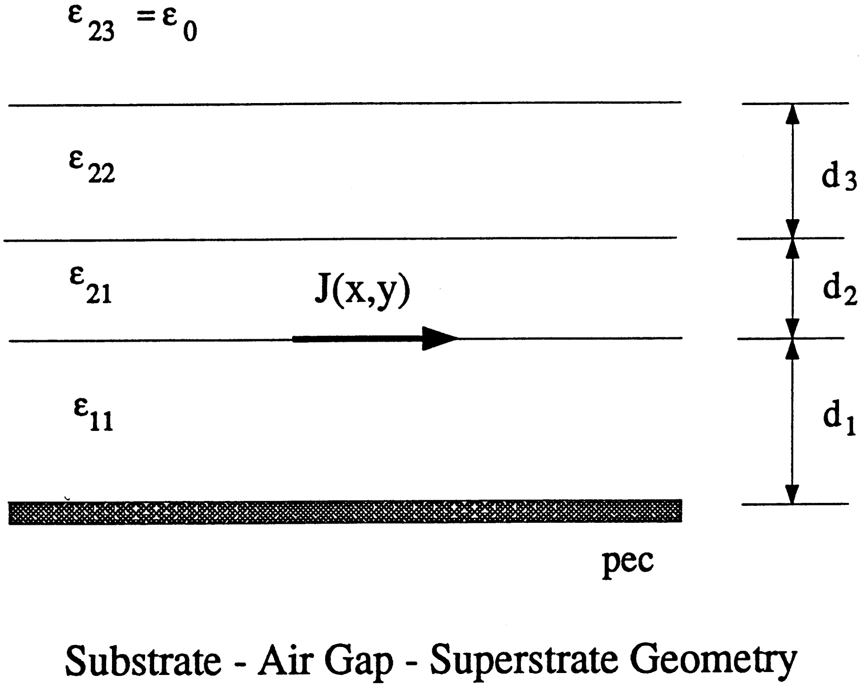


Figure 7: Substrate-air gap-superstrate geometry