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Numerical modeling of cavity flow on bottom of a stepped planing hull

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ABSTRACT

To reduce the friction drag of the hulls of high-speed craft, seaplanes, and hydrofoil boats, steps are made on their undersurfaces. The effect is achieved due to the formation of a gas cavity aft of the step, which reduces the wetted area. The wetted area can also be reduced by striking a compromise between the cavity size and the wetted lengths of the planing surfaces by changing the center-of-mass position of the planing boat and the geometry of the stepped hull. The proper choice of the shape of the stepped bottom and the design parameters may also offer other useful effects, for example, the effect of surface wave energy regeneration on the system of planing surfaces.

This paper presents a solution method for two-dimensional mathematical problem of planing of the stepped air cavity hulls of a high-speed crafts. The method allows predicting and quantifying the above-mentioned effects. The key feature of the proposed approach is that the problem is solved in natural physical formulation. All the required characteristics – the cavity shape and length, the free boundary shape, the wetted lengths of the planing surfaces, and the trim angles are determined from a specified cavitation number, Froude number, and center-of-mass position.

The Froude number is determined from the displacement. When, in addition, the center-of-mass position is specified, additional unknowns appear in the problem – the wetted lengths and the trim angles. In this case, to the singular integral equation of planing must be added the force and the moment balance equations. As a result, a logically closed system of integral equations is obtained. However, the system features parametric nonlinearity in the form of the unknown limits of integration – the cavity length and the wetted lengths. The nonlinear problem is solved by sequential minimization of the residual of the system using nonlinear-programming techniques.

The calculations have shown that the wave amplitude in the wake of the planing boat depends of the cavitation number and the design factors of the step and the planing boat. At negative cavitation numbers, an additional lift develops due to artificial air injection into the cavity under the bottom, which changes the draft and the trim angles. The calculated data suggest that the wake amplitude can be minimized by optimizing the cavitation number and the design and setting angles of the steps.

INTRODUCTION

The designing of high-speed craft, seaplane takeoff/landing systems, hydrofoil boats, and other types of modern vehicles calls for theoretical studies on the planing of variously designed lifting surfaces aimed at improving the hydrodynamic characteristics. The bottom design with a step or a system of steps with air cavities aft of the steps reduces the friction drag and offers some new hydrodynamic qualities. In particular, of both scientific and practical interest is Academician Pavlenko's [1] idea to determine the proper relative position of the surfaces being flown past such that the power consumption is minimized due to wave energy regeneration.

Butuzov [2–4] conducted a theoretical study and calculation of two-dimensional cavities under the bottom of a ship and on a planing surface. In [2], the bottom cavity is modeled by a cavity downstream of a wedge under a solid horizontal wall. A similar model was considered in other works, too, for example, in [5]. In [3, 4], the cavity on the planing surface aft of the step is analyzed using a simplified planing model and Ryabushinsky's cavity scheme to join the solid and the free boundaries. In the simplified planing model in [3, 4], the free liquid boundaries fore and aft of the planing boat are represented as a straight solid wall, and the lengths of the wetted sections, the cavity length, and the trim angle are specified. Note that under such limitations on the model the problem of determination of the planing surface shape optimal in terms of hydrodynamic resistance is considered in [3].

Clearly the actual phenomenon of planing differs greatly from its simplified models. The main reason is that when a water-displacement ship or a planing boat is in motion, its position on the water surface cannot be specified arbitrarily. This position is governed by the speed, the position of the center of mass, and the bottom shape. If there is a cavity under the bottom, its shape and length are not known. The cavity size will mainly depend on the cavitation number and other parameters such as the Froude number and the bottom geometry. The physics of this phenomenon is such that the cavitation number must be specified. For natural vapor cavitation, it is governed by the vapor pressure under given conditions. For artificial ventilation it is governed by the artificially produced pressure. Thus the cavitation number may be both positive and negative. Accordingly, the cavitation-induced physical effects on the planing surface will be different. In particular, at a negative cavitation number an additional upthrust develops thus reducing the draft. Hence the hydrodynamic efficiency of high-speed craft can be controlled with the help of steps, aft of which cavities are formed, and artificial ventilation of bottom sections.

A comprehensive study of the effect of cavities on the hydrodynamic characteristics of planing boats requires computational methods that suit the physics of the phenomenon as fully as possible. Ref. [6] reports a method of analysis of a planing surface, which allows one to determine the wetted length, the pressure distribution, and the trim angle from a specified planing boat displacement, geometry, and center-of-mass position. The method is suitable for the analysis of a system of planing surfaces, and it allows for the presence of cavities between them, which are represented as boundaries with given pressure. A general approach to the study of a system of planing surfaces is presented in [7].

In this work, a method of analysis of a planing hull with a cavity under the bottom aft of the step is described. Examples of calculations that demonstrate the capabilities of the method are provided, and the obtained results and physical effects are discussed.

PHYSICAL PROBLEM FORMULATION

Consider a stepped planing boat moving at constant velocity V_0 over an undisturbed surface of an infinitely deep ideal incompressible liquid (Fig. 1). The boat has two surfaces interacting with the liquid, whose wetted lengths, l_1 and l_2, are not known in advance and have to be found as part of the solution of the problem. The problem is considered in two-dimensional formulation [8]. The spacing between the trailing edges of the surfaces is L. The level of the undisturbed liquid surface coincides with the x-axis. The weight (volume displacement) of the boat is Δ, and its center of mass is situated distance b from the trailing edge of the second planing surface, Δ and b being given quantities.

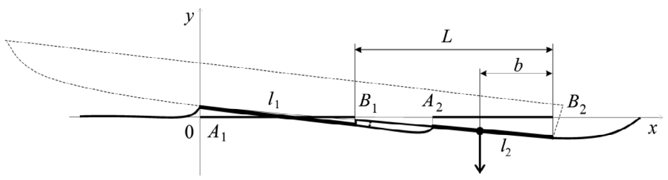


Figure 1: Schematic of flow past a planing boat with a gas cavity under the bottom aft of the step

The motion of the boat is modeled by the motion of a system of two flat plates rigidly joined into an integral structure, the x-projections of the wetted sections of the plates being the segments $[A_1, B_1]$ and $[A_2, B_2]$. It is assumed that the angles of the plates with the travel direction α_1 and α_2 are small, and the premises of the linearized theory of liquid wave motion hold true. The mathematical model of the physical problem is a boundary-value problem for the perturbed velocity potential, and the boundary conditions are transferred to the axis $y = 0$. On the segments $[A_i, B_i]$, $i = 1,2$, the unknown pressure drops – the functions $\gamma_i(x) = (p(x,-0) - p_0)/\rho V_0^2$, $x \in (A_i, B_i)$, $i = 1,2$, are sought, where p_0 is the pressure on the free boundary, $p(x, y)$ is the pressure in the liquid, and ρ is the liquid density; the trim angle α_i and the draft h are not known either. The pressure p_c in the cavity aft of the step is specified by the cavitation number $\sigma = 2(p_0 - p_c)/\rho V_0^2$. No pressure drop occurs on the free surface at $x < A_1$ and $x > B_2$, and the free surface shape is not known. The Froude number $Fr = V_0 / \sqrt{ga}$ is determined by the characteristic length $a = \sqrt[3]{\Delta / \rho g}$ where g is the gravitational acceleration. It is assumed that the free surface boundary and the plate boundaries being flown past constitute a streamline.

MATHEMATICAL FORMULATION

The boundary-value problem for the velocity potential $\varphi(x, y)$ is as follows:

$$\varphi_{xx} + \varphi_{yy} = 0, \quad y < 0 \tag{1}$$

$$\varphi_y(x,-0) = -\frac{d\eta(x)}{dx}, \quad -\infty < x < \infty \tag{2}$$

$$\varphi_x(x,0) - \nu\eta(x) = \gamma(x), \quad -\infty < x < \infty \tag{3}$$

$$\varphi_x, \varphi_y \to 0, \quad y \to -\infty. \tag{4}$$

$$\varphi(-\infty, y) = \varphi_0(x, y), \tag{5}$$

where $\eta(x)$ is the shape of the streamline made up by the free surface boundary and the plate boundaries being flown past, $\nu = 1/Fr^2$, and

$$\gamma(x) = \begin{cases} \gamma_i(x), & x \in [A_i, B_i], \\ 0, & x < A_1, x > B_2, \\ -\sigma/2, & x \in [B_1, A_2], \end{cases}$$

is the function that gives the dimensionless pressure drop along the whole of the liquid surface. The condition (2) is the kinematic condition for smooth flow past the boundary, the condition (3) is the dynamic condition for the pressure on the boundary, which is the Bernoulli equation, the condition (4) is the condition for disturbance attenuation at a great depth, and the condition (5) means that the flow potential is specified at infinity fore of the boat – undisturbed flow or steady-state independent waves.

The kinematic and dynamic conditions written in the form (2) and (3) are not satisfied at points A_1, B_1, A_2, and B_2. However, they are valid in the sense of generalized functions.

The problem (1)–(5) is solved using the Fourier method for the construction of fundamental solutions. Taking the generalized Fourier transform of (1)–(5) gives

$$(|\lambda| - \nu)H(\lambda) = \Gamma(\lambda), \tag{6}$$

where $H(\lambda)=F[\eta(x)](\lambda)$ and $\Gamma(\lambda)=F[\gamma(x)](\lambda)$ are the generalized Fourier transforms of the functions $\eta(x)$ and $\gamma(x)$, respectively.

Eq. (6) is satisfied for the generalized function

$$H(\lambda)=\Gamma(\lambda)\left(reg\frac{1}{|\lambda|-\nu}\right)+$$
$$+\left[A\Gamma(\nu)+A_0\right]\delta(\lambda-\nu)+\left[B\Gamma(-\nu)+B_0\right]\delta(\lambda+\nu),\quad (7)$$

where reg stands for regularization, A, B, A_0, and B_0 are complex constants that specify the free wave amplitudes, and $\delta(\lambda)$ is the delta function. The constants A and B are determined from the emission condition, and their values are as follows: $A=\pi i$, $B=-\pi i$. The constants A_0 and B_0 are the amplitudes of the free independent waves that correspond to the potential $\varphi_0(x,y)$ in (5).

The inverse transformation of (7) gives the free surface shape:

$$\eta(x)=\int_{-\infty}^{\infty}\gamma(s)\left[Q(\nu,x-s)-\sin\nu(x-s)\right]ds+$$
$$+a_0\sin vx+b_0\cos\nu x\quad (8)$$

where

$$Q(\nu,x)=F^{-1}\left[reg\frac{1}{|\lambda|-\nu}\right]=$$
$$=-\frac{1}{\pi}\left[\cos\nu x\,\mathrm{Ci}\,\nu|x|+\sin\nu|x|\left(\frac{\pi}{2}+\mathrm{Si}\,\nu|x|\right)\right],$$

Si and Ci are the integral sine and cosine, and a_0, b_0 are real constants.

Differentiating (8) gives the boundary condition (2) required for the determination of the function $\gamma(x)$:

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\gamma(s)\left[\frac{1}{x-s}+\nu R(\nu,x-s)+\nu\pi\cos\nu(x-s)\right]ds=$$
$$=-\eta'(x)-\nu(a_0\cos\nu x-b_0\sin\nu x),\ -\infty<x<\infty,\quad (9)$$

Where

$$R(\nu,x)=\left(\frac{\pi}{2}\mathrm{sgn}(x)+\mathrm{Si}\,\nu(x)\right)\cos\nu x-\mathrm{Ci}\,\nu|x|\sin\nu x.$$

The integrals in (8) and (9) are replaced with the integrals between the finite limits A_1 and B_2.

To Eq. (9) must be added the force and the moment balance conditions [6]:

$$\int_{A_1}^{B_2}\gamma(x)dx=\nu,\quad (10)$$

$$\int_{A_1}^{B_2}\gamma(x)xdx=\nu(B_2-b).\quad (11)$$

The system of Eqs. (9)–(11) forms the basis for the stationary planing theory. In the case of a single planing surface these three equations provide a complete solution to the problem, i.e. make it possible to determine the pressure distribution, the wetted length, and the trim angle.

With a cavity under the bottom, we have two rigidly joined planing surfaces of unknown length. Because of this, Eq (9) is written as a system of two equations for each surface. As a result, the following system of integral equations is obtained from (9)–(11):

$$\frac{1}{\pi}\int_{A_1}^{B_1}\gamma_1(s)K(\nu,x-s)ds-\frac{\sigma}{2\pi}\int_{B_1}^{A_2}K(\nu,x-s)ds+$$
$$+\frac{1}{\pi}\int_{A_2}^{B_2}\gamma_2(s)K(\nu,x-s)ds=$$
$$=-f_1'(x)-\nu(a_0\cos\nu x-b_0\sin\nu x),\ A_1<x<B_1,\quad (12)$$

$$\frac{1}{\pi}\int_{A_1}^{B_1}\gamma_1(s)K(\nu,x-s)ds-\frac{\sigma}{2\pi}\int_{B_1}^{A_2}K(\nu,x-s)ds+$$
$$+\frac{1}{\pi}\int_{A_2}^{B_2}\gamma_2(s)K(\nu,x-s)ds=$$
$$=-f_2'(x)-\nu(a_0\cos\nu x-b_0\sin\nu x),\ A_2<x<B_2,\quad (13)$$

where $K(\nu,x)=\frac{1}{\pi}\left[\frac{1}{x}+\nu R(\nu,x)+\nu\pi\cos\nu x\right]$, $f_1(x)$ and $f_2(x)$ are the functions that describe the shape of the sections being flown past. In our case, $f_i(x)=h_i+k_ix$, $i=1,2$, where h_i are the drafts, $k_i=\tan\alpha_i$, and α_i are the trim angles.

Eqs. (12) and (13) include two unknown functions, $\gamma_1(x)$ and $\gamma_2(x)$, two unknown constants – wetted lengths $l_1=B_1-A_1$ and $l_2=B_2-A_2$, and the unknown trim angle α_1 or α_2 (the second is determined from the rigid geometry of the structure).

The condition (10) has the form

$$\int_{A_1}^{B_1}\gamma_1(s)ds-\frac{\sigma}{2}(L-l_2)+\int_{A_2}^{B_2}\gamma_2(s)ds=\nu.\quad (14)$$

The condition (11):

$$\int_{A_1}^{B_1}\gamma_1(s)sds-\frac{\sigma}{4}\left[(L+l_1-l_2)^2-l_1^2\right]+\int_{A_2}^{B_2}\gamma_2(s)ds=$$
$$=\nu(L+l_1-b).\quad (15)$$

The geometrical condition for cavity closure:

$$\int_{A_1}^{B_1}\gamma_1(s)[(Q(\nu,L+l_1-l_2-s)+\sin\nu(L+l_1-l_2-s))-$$
$$-(Q(\nu,-s)+\sin\nu(-s))]ds-$$
$$-\frac{\sigma}{2}\int_{B_1}^{A_2}[(Q(\nu,L+l_1-l_2-s)+\sin\nu(L+l_1-l_2-s))-$$
$$-(Q(\nu,-s)+\sin\nu(-s))]ds+$$

$$+\int_{A_2}^{B_2} \gamma_2(s)[(Q(\nu, L + l_1 - l_2 - s) + \sin\nu(L + l_1 - l_2 - s)) -$$

$$-(Q(\nu, -s) + \sin\nu(-s))]ds =$$

$$= -(L - l_2)\cdot \tan(\alpha_1 + \Delta\alpha) - l_1 \cdot \tan\alpha_1 + \Delta h \qquad (16)$$

where $\Delta\alpha = \alpha_2 - \alpha_1$ and $\Delta h = f_2(l_1) - f_1(l_1)$ are specified as design factors in conditions of linear approximation.

NUMERICAL METHOD

The system of integral equations (12)–(16) may be solved by any of the familiar methods of solution of singular integral equations. Its key feature is that it is parametrically nonlinear in the unknowns l_1 and l_2. The use of any numerical method gives a system of algebraic equations of the form

$$AX = B, \qquad (17)$$

where the vector X is made up by the unknown values of the functions $\gamma_1(x)$, $\gamma_2(x)$, and α_1, which enter into the system linearly, while the elements of the matrix $A = A(l_1, l_2)$ depend on the unknowns l_1 and l_2, which are responsible for nonlinearity. The vector B is made up by elements from the known values of the right-hand sides of Eqs. (12)–(16).

For the solution of systems of this type, it turns out to be efficient to use the familiar method [9] of reduction of the problem (17) to the minimum search problem

$$[A(l_1, l_2)X - B]^{\mathrm{T}}[A(l_1, l_2)X - B] \to \min_{l_1, l_2}. \qquad (18)$$

In this work, the singular integral equations are solved using the discrete singularity method [10], and the problem (18) is solved using the Nelder–Mead flexible polyhedron method (downhill simplex method) [11].

RESULTS

It makes sense to relate the scales of values of the variables that are specified in the problem and define the geometry of the planing boat to the scale of the generated waves. Eq. (7) at $\Gamma(\lambda) = c = const$ is the Fourier transform of the free surface shape generated by the delta-function pressure pulse $\gamma(x) = c\delta(x)$ of strength c traveling at velocity V_0. In this case, $c = \int_{-\infty}^{\infty} \gamma(x)dx = \nu$. Then the inverse transform of (7) will be (without taking into account independent waves, $a_0 = 0$, $b_0 = 0$)

$$\eta(x) = \nu[Q(\nu, x) + \sin\nu x].$$

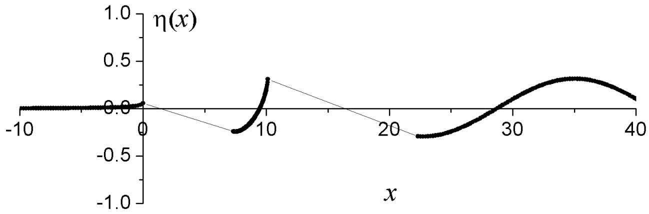
Since $\lim_{x\to\infty} \mathrm{Si}\,\nu|x| = \pi/2$ and $\lim_{x\to\infty} \mathrm{Ci}\,\nu|x| = 0$, $\lim_{x\to\infty} Q(x, \nu) = -\sin\nu|x|$. Hence

$$\lim_{x\to\pm\infty} \eta(x) = \begin{cases} 0, & x < 0, \\ -2v\sin\nu x, & x > 0. \end{cases}$$

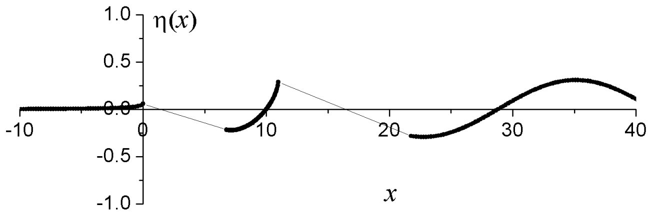
This expression gives an estimate of the scale of the planing-induced waves for the linearized theory. For example, at Froude number $Fr = 2$ the wave amplitude will be 0.5 and the wavelength will be 25.13, and at $Fr = 1.5$ the amplitude and the wavelength will be 0.88 and 14.14, respectively.

With this in mind, presented below are the results of calculations at $L = 15$ (spacing between the trailing edge and the step), $\Delta h = 0.7$ (step height), $\Delta\alpha = 0$ (the planing surfaces fore and aft of the step are parallel), and with the center of mass situated distance $b = 10$ from the trailing edge.

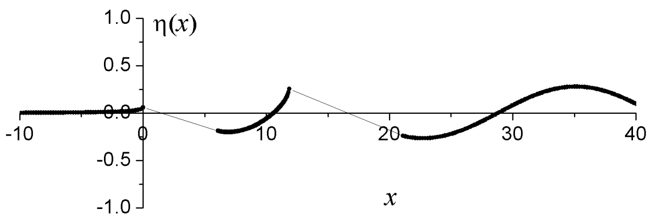
Figs. 2 from a) to h) show the cavity and free boundary shape for the cavitation number ranging from $\sigma = 0.2$ to $\sigma = -0.0224$ at $Fr = 2$. Bold lines in opposite to fig. 1 show the free surface, thing line segments show the wetted boundaries of planing hull. The free surface boundary consists of three areas. The left area begins in minus infinity and ends in a zero point, where a y-axis passes - in the contact point of free surface with beginning of the wetted area of planing hull. The middle area is the free boundary of cavity after step and right-hand area is the wake border.



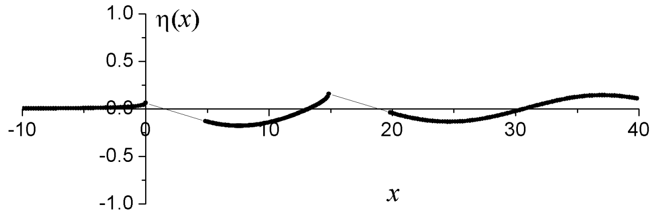
a) $\sigma = 0.2$



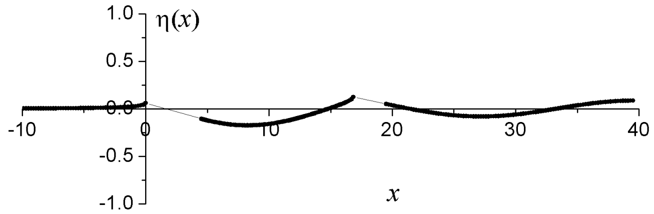
b) $\sigma = 0.1$



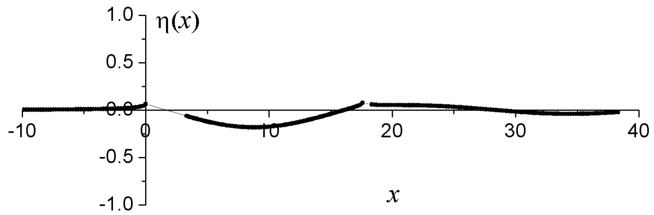
c) $\sigma = 0.05$



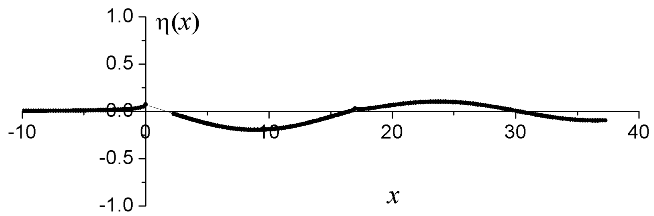
d) $\sigma = 0.0$



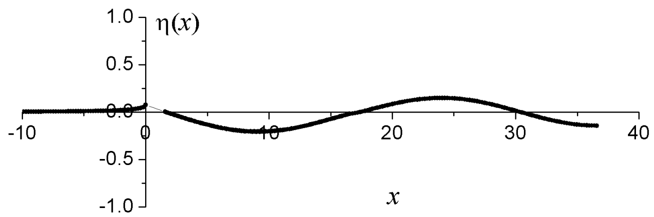
e) $\sigma = -0.01$



f) $\sigma = -0.017$



g) $\sigma = -0.02$

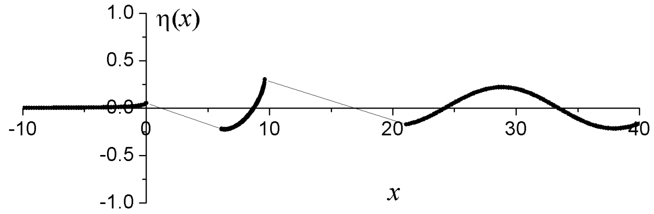


h) $\sigma = -0.0224$

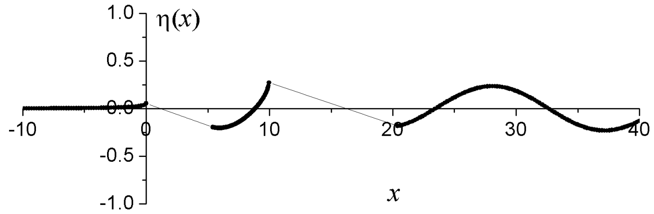
Figure 2: Shape of the free surface, the cavity aft of the step, and the wetted boundaries of the planing boat at $Fr = 2$.

The cavitation number $\sigma = -0.0224$ for parameters mentioned above is close to the value such that second wetted length goes to zero. That means that subsequent increase of pressure in cavity can lead to tearing of stream from back edge and subsequent undesirable unsteady effects.

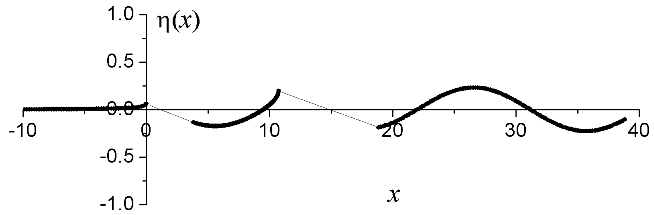
Figs. 3 from a) to f) show the cavity and free boundary shape for the $Fr = 1.7$ and the same center of mass situated distance $b = 10$ from the trailing edge. In this case the critical cavitation number is close to $\sigma = -0.035$.



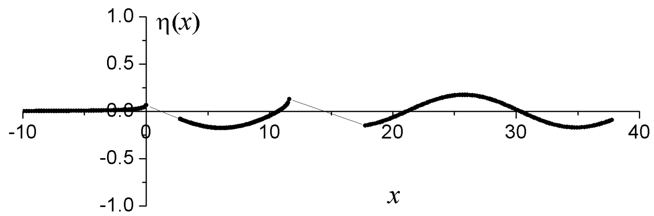
a) $\sigma = 0.1$



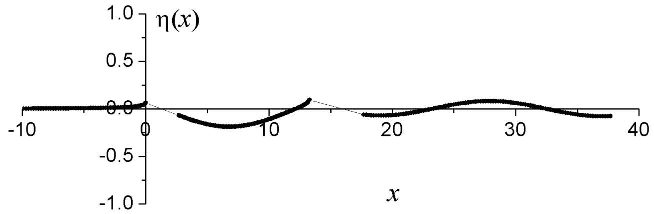
b) $\sigma = 0.05$



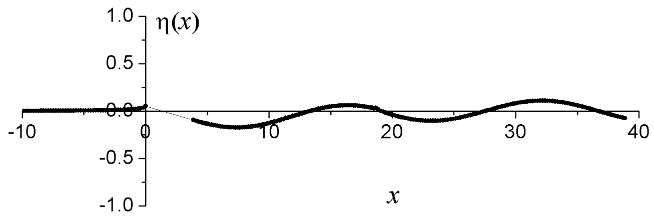
c) $\sigma = 0.0$



d) $\sigma = -0.02$



e) $\sigma = -0.03$

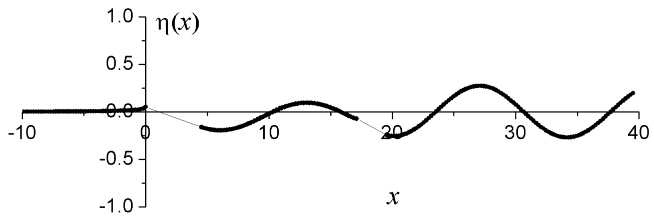


f) $\sigma = -0.035$

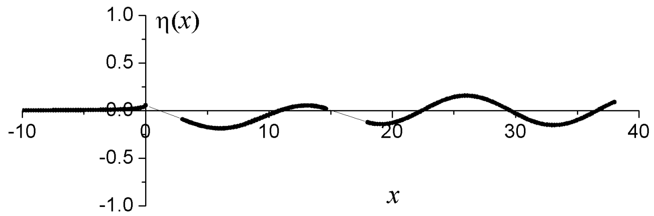
Figure 3: Shape of the free surface, the cavity aft of the step, and the wetted boundaries of the planing boat at $Fr = 1.7$.

Note that design parameter combinations and planing boat and step dimensions do not all allow one to construct a physically feasible flow or make the residual of the system (12)–(16) or the value of the objective function in (18) smaller than a preset small number. However, for all the results presented in the paper the objective function did not exceed 10^{-7}.

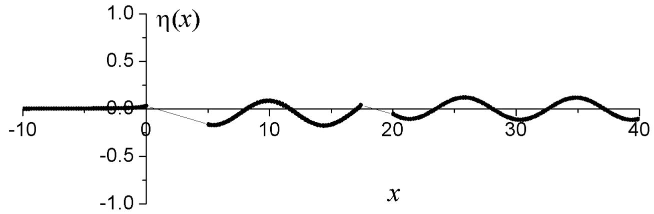
For the rather small Froude numbers for parameters mentioned above, the flows were constructed for only negative cavitation numbers. The samples of such flow for $Fr = 1.5$ and $Fr = 1.2$ are shown on the Figs. 4 from a) to c).



a) $Fr = 1.5$, $\sigma = -0.04$



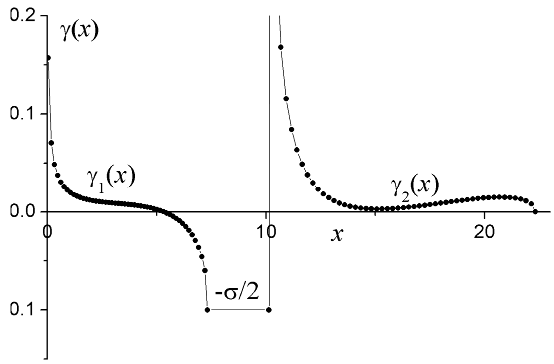
b) $Fr = 1.5$, $\sigma = -0.05$



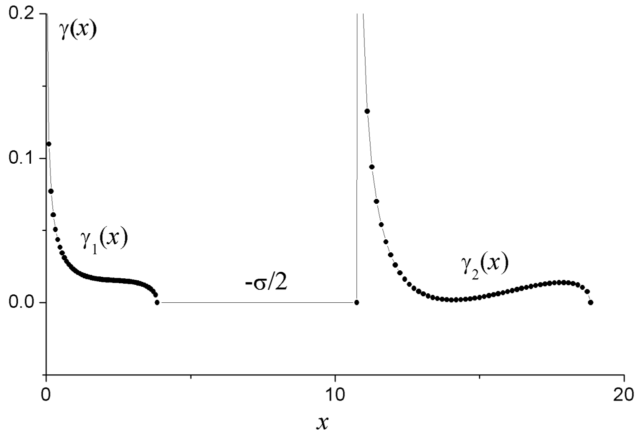
c) $Fr = 1.2$, $\sigma = -0.06$

Figure 4: Shape of the free surface, the cavity aft of the step, and the wetted boundaries of the planing boat at $Fr = 1.5$ and $Fr = 1.2$.

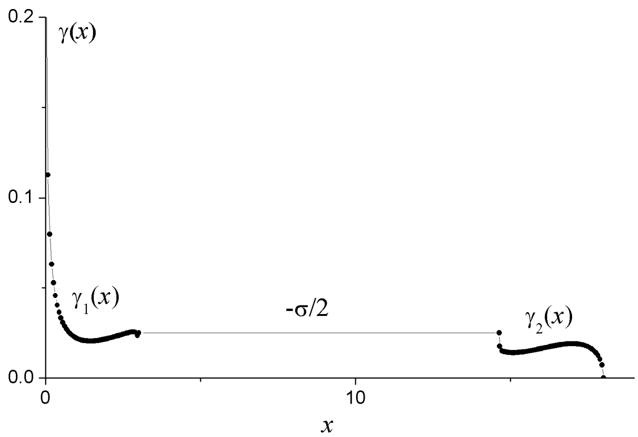
The pressure distributions along the solid boundaries are shown in Figs. 5 from a) to d). They illustrate the pressure distribution for some cases presented in figures 2, 3 and 4. The cases for positive cavitation number, for zero and for two negative values are shown.



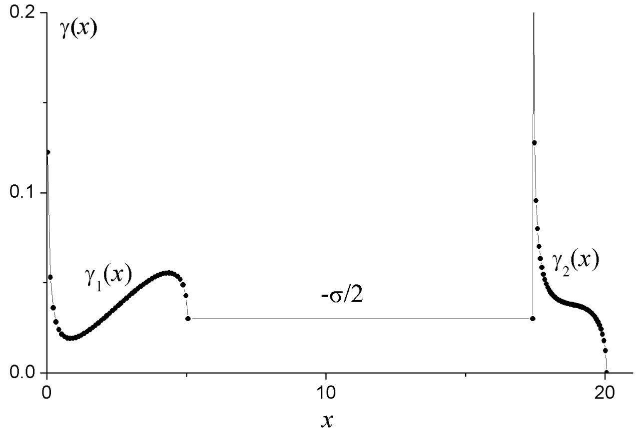
a) $Fr = 2$, $\sigma = 0.2$



b) $Fr = 1.7$, $\sigma = 0.0$



c) $Fr = 1.5$, $\sigma = -0.05$



d) $Fr = 1.2$, $\sigma = -0.06$

Figure 5: Calculated pressure distributions along the solid boundaries

Figure 6 show the cavity length versus cavitation number at the parameters indicated above

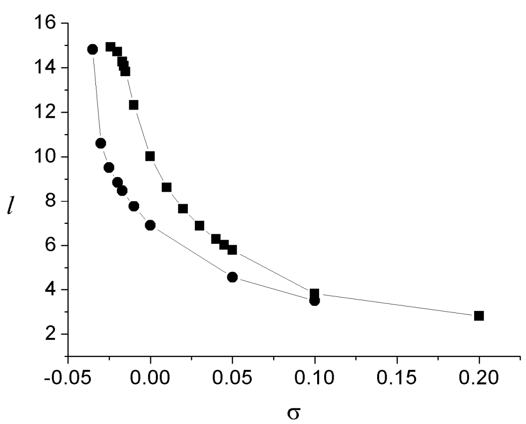


Figure 6: Calculated cavity length versus cavitation number at $Fr = 2.0$ (squares) and $Fr = 1.7$ (circles).

Unfortunately, experimental data for confirmation of calculation model for a stepped planing hulls are unknown to the author. Present in literature information on air cavities under a bottom is relative to the crafts of displacement type, where the task of determination of the wetted surfaces, draft and trim angle is not set. Necessary experimental data can be obtained in specially set experiments with planing models in speed pools.

Adequacy of calculation model of author is received only on the basis of theoretical analysis by comparing with known data in limiting cases. At a zero height of step and approaching of step to the trailing edge calculation data correspond known results for a planing plate with given load and given gravity centre position [6].

DISCUSSION

The calculations show that the shape of the cavity aft of the step is governed by two factors, namely, by the Froude number and the cavitation number. The Froude number governs the cavity curvature, which correlates well with the curvature of the generated waves, and the cavitation number governs the cavity length. At large Froude numbers the cavity curvature is small, and it increases as the Froude number decreases. The waves generated on long cavities at sufficiently small cavitation numbers are of the same length as the waves in the wake of the planing boat.

The pattern of contact of the cavity boundary with the second planing surface depends on the cavity curvature in the vicinity of the contact point. The curvature is governed by the ratio between the cavity length and the wavelength at a given Froude number. Different contact patterns are illustrated in Figure 4.

The charts of pressure distribution on a figure 5 show that local pressure drops along the corps of planing hulls with cavity can be large or small, as for example on a figure 5 c). Calculation information about pressure distribution is necessary for correct determination of strength properties of the corps of planing hulls. Such information is also useful for prediction of character of possible corps deformations.

At negative cavitation numbers the pressure in the cavity is higher than that on the free surface, and thus an additional lift develops under the bottom. It can be seen from the plots of the cavity and free boundary shape that at negative cavitation numbers the planing boat draft decreases. The wave amplitude in the wake decreases too.

If the second planing surface aft of the step finds itself on the trailing wave front, the wake amplitude increases. If the surface finds itself on the leading wave front, the wake amplitude decreases.

The analysis of the calculated data shows that one can select an optimum combination of design parameters and factors such that the wave amplitude in the wake is a minimum. The calculated data suggest that the consumption of energy to form the wake decreases due to fact that the second planing surface aft of the step uses the energy of the wave generated by the first surface.

Multistep planing surfaces with controllable angles of setting may enhance this effect many-fold.

The method of solution of the stationary problem described in this paper may by used for approximate evaluation of the effect of independent waves. Cavity planing over a wave surface may be studied using a quasistationary model. In this case, motion over the wave surface is modeled by a succession of stationary positions of the planing boat relative to the wave. To do so, the phases of the independent waves should be changed in Eqs. (12) and (13).

The above-described approach to the solution of the mathematical problem of the planing of a system of connected surfaces with a cavity, which is based on the method of singular integral equations, is universal. It is suitable for the solution of 3D problems and nonstationary problems in actual nonstationary formulation. The computational algorithms used and tested in this work may form a good basis for the development of engineering design software.

In connection with the possibility of application of approach to the 3D models, it is necessary to note that the basic restriction is conditions for correctness of application of the linearized theory. A planing body with a step can be axisymmetrical too. However in a model, not taking into account a flow in an air environment only shape of the part of body submerged in a liquid is important.

In 3D case the calculation of wetted surface form before a step and cavity form in a plan, which determines the form of the wetted surface after a step, becomes considerably complicated. In such nonlinear task, instead of two parameters, as in two-dimensional theory, it is necessary to find two curves.

The author does hope that the results and conclusions of this work will stimulate further studies of the general physical regularities of cavitation-assisted planing. This, in its turn, will contribute to the realization of various technical ideas of new types of water-surface high-speed vehicles.

CONCLUSION

The capabilities of the solution method for the problem of planing of a stepped hull with a cavity aft of the step have been demonstrated. The proposed method makes it possible to solve the problem in actual physical formulation – to determine the wetted lengths of the planing surfaces, the trim angle, and the draft from a specified center-of-mass position, cavitation number, and Froude number.

The new results obtained have shown the efficiency of the approach. They have made it possible to elucidate the qualitative features of cavity planing for actual conditions of a specified displacement and a free trim angle. It is shown that the proposed theory can predict possible applications of natural and artificial cavitation to the control of high-speed craft hydrodynamic characteristics. The possibility of providing appropriate technical conditions by means of air injection and step designs that minimize the wake is an example of such a prediction.

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NOMENCLATURE

$\sigma = 2(p_0 - p_c)/\rho V_0^2$ – cavitation number

p_0 – pressure on the free boundary

p_c – pressure in the cavity

ρ – liquid density

V_0 – velocity of planing boat motion

$Fr = V_0/\sqrt{ga}$ – Froude number, $\nu = 1/Fr^2$

g – gravitational acceleration

$a = \sqrt[3]{\Delta/\rho g}$ – characteristic length

Δ – volume displacement

l_1, l_2 – wetted lengths

$\gamma_i(x) = (p(x,-0) - p_0)/\rho V_0^2$ – pressure drops on wetted lengths, $i = 1,2$

$p(x, y)$ – pressure in the liquid

$[A_i, B_i]$ – x-projections of the wetted sections of the plates (segments of wetted lengths), $i = 1,2$

$\eta(x)$ – shape of the streamline (free surface boundary and the plate boundaries being flown past)

$\gamma(x)$ – dimensionless pressure drop along the whole of the liquid surface

$\varphi(x, y)$ – velocity potential

α_i – trim angles, $i = 1,2$, $k_i = \tan\alpha_i$

L – spacing between the trailing edges of the planing surfaces

b – distance from the trailing edge of the second planing surface to center of mass

$f_i(x) = h_i + k_i x$ – shapes of the planing sections, $i = 1,2$

h_i – drafts, $i = 1,2$

a_0, b_0 – real constants, amplitudes of independent waves

$\Gamma(\lambda)$ – generalized Fourier transforms of the function $\gamma(x)$

$H(\lambda)$ – generalized Fourier transforms of the function $\eta(x)$

$\delta(\lambda)$ – delta function

REFERENCES

[1] Pavlenko G.E., 1979, *"Selected transactions."* (in Russian). Kyiv: Naukova dumka, 450 p.

[2] Butuzov, A.A., 1967, "Artificial cavitation flow behind a slender wedge on the lower surface of a horizontal wall." *Fluid Dynamics* 3, 56–58.

[3] Butuzov, A.A., Pakusina, T.V., 1973. "Solution of flow past a planing surface with an artificial cavity." (in Russian) *Trans. Acad. A.N. Krylov TsNII*, No 258, 63–81.

[4] Barabanov, V.A., Butuzov, A.A., Ivanov, A.N., 1973, "Detached cavity flow past hydrofoils in the case of planing and in an infinite stream." (in Russian)*Non-*

Steady Flow of Water at High Speeds/ Proceedings of the IUTAM Symposium Held in Leningrad, June 22-26, 1971. Nauka Publishers, Moscow,. Pp. 113–119.

[5] Matveev K. I. 2003, "On the limiting parameters of artificial cavitation" *Ocean Engineering*. 30. Pp. 1179–1190.

[6] Makasyeyev, M.V., 2003, "Stationary planing of a plate over the surface of a ponderable liquid at a specified load and a free trim angle." (in Russian). *Applied hydromechanics*. 5(77), No 2, 73–75.

[7] Dovgiy, S.A., Makasyeyev, M.V., 2003, "Planing of a system of hydrofoils over the surface of a ponderable liquid." (in Russian) *Dopovidi NAN Ukrainy*. No 9, 39–45.

[8] Sedov L.I., 1967, "*Plane problems of hydrodynamics and aerodynamics*." (in Russian). M.: Nauka, 1981, 448 p.

[9] Roman, V.M., Makasyeyev M.V., 1984, "Calculation of the shape of a cavity downstream of a cavitating finite-span hydrofoil." (in Russian) *Dynamics of a Contunuum with Nonsteady Boundaries*. Cheboksary, Russia. Chuvashia University Publishers. Pp. 103–109.

[10] Belotserkovsky, C.M., Lifanov, I.K., 1985, "*Numerical Methods in Singular Integral Equations*." (in Russian). Nauka Publishers, Moscow. 256 pp.

[11] Himmelblau, D., 1975, "*Applied Nonlinear Programming*." (in Russian). Mir Publishers, Moscow. 450 pp.