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Final Summary Report

NADIR POINT OF TRUE VERTICAL

Edward Young

Project 1699-1

DEPARTMENT OF THE AIR FORCE
WRIGHT AIR DEVELOPMENT CENTER
WRIGHT-PATTERSON AIR FORCE BASE, OHIO
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FOREWORD

This report is a resumé of the investigations carried out during the period from August, 1946, to September, 1955, by Project 1699-1 of the Engineering Research Institute of The University of Michigan for the Air Materiel Command, later the Wright Air Development Command, United States War Department, under the terms of Contract No. W33(038)-ac-15318, entitled "Nadir Point of True Vertical."

The project utilized the facilities of the Engineering Research Institute under the directorship of Dr. A. E. White (1946-1953) and Dr. Richard G. Folsom (1953 to date). The investigations were under the direct supervision of Edward Young, Professor of Geodesy and Surveying, The University of Michigan.

Acknowledgments and appreciation are extended to our sponsors, WADC, members of the Engineering Research Institute and The University of Michigan faculty, the United States Coast and Geodetic Survey, other government agencies, and to all project personnel in addition to those mentioned in this report.

The continued cooperation and aid received from Dr. J. E. Clemens, Chief of the Physics Research Branch, Aeronautical Research Laboratory, Directorate of Research, WADC, from the beginning to the end of the work is indeed appreciated. Appreciation is also extended to Mr. Ben B. Johnstone, Assistant Chief, Physics Research Branch, to Dr. C. A. Traenkle, Mr. F. Wazelt, Mr. H. Kroeger, and other scientists of WADC for their aid and cooperation in carrying out the work of this project.

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ABSTRACT

This report outlines procedure and progress over the entire period of the contract, August, 1946, to September, 1955, on the development of photogrammetric methods for a precise determination of the behavior of airborne instruments in flight as well as the reduction of data for the evaluation of these instruments. Since it was desired to obtain a high degree of accuracy, other investigations became necessary for evaluating the work. In the text reference is made to the various technical reports.

OBJECTIVE

The objective of this project was to determine methods of photogrammetric reductions for aerial-defense purposes.

INTRODUCTORY PHASE

Preliminary Work.—Work was started 15 July 1946 on locating space and equipment. A room of 512 sq ft was tentatively assigned in the main hangar at Willow Run Airport. Tables, files, etc., were obtained from variour sources on a loan basis. Temperature controls were adequate for the winter, but a cooling apparatus was needed during the summer. A research associate and two computers were hired and began work on methods and procedures for working up the data to determine nadir points.

A letter, dated 30 August 1946, from the Air Materiel Command stating that the comparators were to be supplied by the contractor necessitated a trip on 4 September 1946 to the Gaertner Scientific Corporation in Chicago. It was found that a comparator suitable for our work could not be delivered in less than five months. Since it was necessary to contact our sponsors, a conference at Wright Field was arranged to discuss the project and obtain control data. The comparator situation was reported, and it was promised that a new 10- x 10-in. comparator ordered by AMC from the Gaertner Scientific Corporation would be loaned to us for this project. This comparator was to be delivered between 1 and 15 October. We were also promised the use of a second comparator, for which the delivery date was not then set, and some computing machines and other equipment from Wright Field.

Control data for the area used in this project were plotted on 40- x 50-in. acetate sheets. These consisted of three polyconic map projections on which ground control was shown by both geographic- and military-grid coordinates. Some additional control points were plotted on a local rectangular projection. Since the computations for the determination of nadir points and other information required were rather long and involved, considerable study was given to methods, computation forms, checks, etc. Miss Margaret Steere, Research Associate, and three part-time student computers were working on this.

In October, 1946, contracts were let for the construction of additional working space adjacent to our room in Hangar No. 1 and for moving a Yorkaire air-conditioning unit from building T-361 to our new rooms. Preliminary work on the project was carried on in Room 205-A, West Engineering Building.

Considerable time was spent on computation forms. The one which we considered the most suitable of those tried by student computers was sent to Wright Field. While the various steps were shown, a number of these could be carried out on the computing machine and the final result recorded. The computation for the angle between the nadir point and the intersection of the cross hairs was based on the assumption that this angle would be small and any displacement of the photographic image on this region of the photograph due to tilt would be negligible.

Precise results could not be obtained when computations were based on military-grid coordinates, at least in the longitude in which we were working. The origin of the "B" grid zone lay in the 81° meridian. The errors, especially in the Y-direction, increased as we departed from the central meridian. In order to check with the local grid, it was necessary to compute plane coordinates from the latitude and longitude positions given. When this was done, the results from the same control points on the two systems agreed.

Until the arrival of the topobase photographs and precise measuring equipment, the work on this project was necessarily preliminary. It was extremely valuable, not only from a training standpoint, but from information gained. While we did not expect precise results under the conditions imposed, we also did not expect discrepancies of the magnitude we obtained when computing nadir points from two different sets of control points. However, we received from Wright Field the dimensions of the focal planes in the aerial cameras used, and by using these dimensions to ratio the measured plate coordinates, thereby correcting for differential changes in the photograph, our results were within the limits one might expect to obtain.

Topobase photographs had much less differential change than the paper base, but it was still necessary to take this into account in order to obtain the desired precision. It was our opinion that this could be accomplished by using the comparator to obtain precise measurements between the collimation marks and then using these measurements in a ratioing process. According to the National Bureau of Standards, tests on topobase (sensitized) showed the hygroexpansion, expressed as approximate percent change in unit dimensions per 10% change in relative humidity, was 0.087 (length) and 0.010 (width). Since it was also stated that the change in size corresponding to a change in relative humidity was approximately constant from 20 to 75% relative humidity, we computed the change for a 9- x 9-in. photograph for changes in relative humidity. A 40% change in relative humidity changed a 9- x 9-in. photograph 0.00087 x 4 x 9 = 0.031 in. (length) and 0.001 x 4 x 9 = 0.036 in. (width), a difference of 0.005 in. (0.125 mm). This differential change was equivalent to giving the photograph two different focal lengths, i.e., one value in the direction of the grain and another across the grain of the film. A similar condition could be caused in processing. Since our problem was not cartographic, corrections could be applied to the plate coordinates rather than to the focal length of the photograph. If this is done accurately, it is equivalent to adjusting the image positions of the ground control points to their corresponding positions on the photographic film at the instant of exposure.

The coefficient of thermal expansion for plain acetate was given by the Bureau of Standards as 8.3×10^{-5}. For a temperature change of 6°F (72° ± 3°), the change over 9 in. of acetate was 0.004482 in. (0.112 mm). While there was no information given on emulsion-coated acetate, it seemed reasonable to assume that the coefficient of expansion would be greater, but less than for topobase, and that sensitized topobase would approximate plain acetate. It was apparent that both temperature and humidity control were important and necessary to hold the dimensions of the photographs constant. We expected that the comparator would be valuable for detecting and measuring the processing, thermal, and humidity changes as well as for measuring plate coordinates.

The necessity for computing plane coordinates from geodetic positions of control points along the flight was explained earlier. An origin in the center of the area (latitude 39°52'30", longitude 84°30'00") was selected and plane coordinates were computed in October, 1946.

In the northwesterly portion of the flight line there were not enough control points to permit computations from two sets of different points, and on some of these photographs the location of the control did not give a good figure. As a result, some tests were made on "strength of figure," using photographs for which the tilt had been determined from two or more sets of control points with maximum discrepancies not exceeding ten minutes of arc.

Our space in Hangar No. 1 at Willow Run Airport was completed in October, 1946, and the air conditioner was being installed. As requested in a letter, photographs were sent at that time and occasionally later on, showing instruments, equipment, and how work was conducted. Installation of the Yorkaire air-conditioning unit was completed in December, 1946, except for the installation of a humidification spray nozzle.

Computations based on scaled-photo coordinates (ratioed for differential change) consistently showed camera No. 414098, C.F.L. 154.3 mm, to be at a higher elevation at exposure than camera No. 414125, C.F.L. 154.0 mm. The difference in elevation between consecutive air stations varied from 60 to 120 ft. The average difference for 23 consecutive photographs was approximately 70 ft. The effect on tilt varied with the size of the tilt angle. When the tilt was one degree, a 70-ft error in elevation of the air station caused an error of approximately 20 seconds of arc. When the tilt was four degrees, the error was about one minute 20 seconds of arc. The above conditions indicated an error in the calibrated focal lengths, which was later found to be true. A 70-ft difference in elevation corresponded to a discrepancy of approximately 0.9 mm between focal lengths.

The project moved to our rooms in Hangar No. 1 at the Willow Run Airport on 4 January 1947. A Thermo-Humidograph was ordered on 10 January and arrived shortly thereafter.

Representatives of the AMC, Maj Williams and Capt Brown, visited the project 15 January 1947. They arrived by plane and brought with them topobase photographs for our preliminary investigations. The comparator had not yet arrived, so computations were based on scaled-photo coordinates of the topobase prints. The prints appeared to have very small, if any, differential changes. However, we did not have the measurements of the fiducial axes of camera No. F41-4139. In order to check the dimensional stability of the topobase photographs under existing conditions, daily measurements were made between certain points on the photographs.

A method of space orientation and resection, as derived by H. P. Underwood in the *Proceedings of the American Society of Civil Engineers*, September, 1946, was tried. A computation form based on this method was prepared and sent to Wright Field.

TOPOBASE PHOTOGRAPHS

In February, 1947, 208 computations were made by using photo coordinates scaled from topobase photographs. Sixty-eight solutions were made for 17 consecutive photographs with four sets of control points. Two sets were based on the plane-coordinate system and two sets on the local-grid system. However, some of the control points were common to the two systems. The distances between these two common points, as computed from the coordinates of the local-grid and plane systems, did not always agree. We found some with discrepancies ranging from 3 to 10 ft. Since some of the distances agreed and some did not, it was our opinion that some of the points were not common to the two systems. The tilt agreement between the two systems for the 17 photographs varied from 3 to 17 minutes of arc with an average of 11 minutes of arc.

Combination of Church and Underwood Methods.—A sample form of this combination had been sent to Wright Field. Shortly thereafter the form was revamped. Investigation brought out the following conclusions.

1. We disregarded analytical approximation for the length of the side LA of the ground pyramid and substituted with a graphical solution which could be made in 15 minutes. There was little doubt that the value of LA could be

determined to within 10 to 30 ft and with care in drafting to within 10 ft or less. This meant that we had small values to converge. If LA was within 10 ft of the correct value, and if a few seconds of arc were negligible in the tilt angle, a second trial was not necessary. When the error in LA exceeded 10 ft, it was necessary to make the second trial to obtain the precision required for this project. However, the error in LA could increase considerably and still be converged with a second trial.

2. The time of computation for the Underwood method did not vary with the numerical value of the tilt when a graphical solution was made for LA. It would vary to some extent if the analytical solution were used for determining the approximate value of LA. In other words, if the error in LA was 10 ft or less, a second trial was not necessary for either high or low tilt.

3. The values of the slope distances between ground control points AB, BC, and CD on the form could be computed and compiled.

4. When tilts were small, this method did not appear to have any advantage over the Church method. The Church method would probably save time. If, however, the error in LA could be kept below that value in which a second trial was required, the Underwood method might save time even on small tilts. For high tilts, the time was not increased when using the Underwood method.

Sources of Error.—

1. Inaccuracy in scaling photo coordinates.

2. Location of control points on photograph because of one of the following reasons.

   a. Some topobase photographs were dark and of low contrast.

   b. Local control spotting was not always clear on the mosaic provided by the AMC.

   c. Some photo points were not adequately described.

   d. There was a lack of sufficient photo points to form good triangles in certain areas along the flight line.

Comparator.—The comparator arrived 26 February 1947. It was unpacked according to directions and seemed in good condition. We checked our operating ability by measurements of our 1/4-meter bar which could be placed on the stage. A copy of the certificate of calibration was sent to the AMC. Our problem was to decide on operating procedure, magnification, and probable errors. We wrote preliminary instructions for the operation of the instrument which called for four readings on the coordinate axes after orientation, two readings on each control point, and two readings on the ends of the lines A and B referred to in Bulletin No. 24, "Calibration Data on T-5 Cameras."

1. Cross hairs:

The intersection of the cross hairs in the telescope did not coincide exactly with the axis of the tube and therefore could not be rotated during a set of measurements. Tests showed that better results were obtained when the cross hairs bisected each road or even when one hair was on one of the roads and the other nearly so, where roads were not quite at right angles. Since many of the control points were at intersections with north and south roads, one cross hair could be set parallel to the north and south roads as an aid in determining the intersection. But once set, the cross hair could not be adjusted for small variations in other roads.

It would have been an advantage if the cross hairs could have been rotated for this type of ground control.

2. Magnification:

Two eyepieces of different focal length and with an extra objective permitted a range in magnification from 4X to 33.6X. Low magnification made it easier for the operator to bisect the image of the control point, but motion of the cross hairs was not readily detected with small changes in vernier readings. We decided that the magnification should not be less than 14X.

3. Probable errors in measurements:

a. Comparator measurements made between the 0- and 230-mm graduations on our meter bar showed the length to be slightly greater at 72°F. A sample of readings was sent to the AMC. The most probable value was 230.0221 mm. The difference of 0.0221 mm was equal to 0.00087 in. The probable error of the mean was ±0.0003 mm and of a single measurement ±0.0009 mm. A length of 230 mm corresponded approximately to the length of the lines A and B referred to above, which we measured for checking dimensional stability and coordinate ratio.

b. Comparator measurements made on the meter bar between the 0- and 110-mm graduations according to our work also showed this length to be slightly greater. A copy of the measurements made to determine this distance was sent to the AMC. The probable value was shown to be 110.0047 mm, or a difference of 0.000185 in. The probable error of the mean was ±0.000996 mm and of a single measurement ±0.003 mm. A distance of 110 mm was equal to 4.3307 in., which approximated the maximum coordinate distance. Precision specification for measuring a distance of 4 in., as set by the AMC, was 0.001 in., which is equal to 0.0254 mm.

c. The above measurements were made on sharply defined lines on the meter bar. In our work the coordinate axes were sharply defined by the fiducial marks, but the photo points were road intersections, considerably larger targets. Some near-average readings on photo point No. 29, which was a road intersection (not at right angles), were sent to the AMC. In making the readings the cross hairs were rotated until one of the hairs was parallel to the north and south roads. The results showed the probable error of the mean to be ±0.0008 mm and of a single observation ±0.00268 mm. This was not a distance and so represented probable errors in pointing only.

d. Similar readings taken on a definite point such as a fiducial mark showed the probable error of a single observation to be ±0.0013 to ±0.0024 mm.

The above measurements were made while using a magnification of 14X. In all fairness to the instrument, a high magnification should have been used on the meter-bar tests. Too definite conclusions could not be drawn from the number of readings taken, but they indicated that photo measurements would be slightly too small. However, since we ratioed all measurements to correct for dimensional change, this also eliminated instrumental error, if the error was uniformly distributed over the measuring range of the instrument. With the constant error of the instrument (b) eliminated, there remained errors (c) and (d). If we consider these to be cumulative and not compensating, the result was ±0.00608 mm. From this one might feel reasonably sure of obtaining the coordinate distance with an accuracy of ±0.010 mm, which is equal to ±0.0004 in.

Ground Control.—The distances between common points in the local-grid control and the plane-coordinate system were computed and sent to the AMC. The discrepancies in distances between points in the two systems varied approximately from 0 to 10 ft. An error in ground distance of tenths of a foot or even a foot did not change the value of tilt appreciably, but the larger discrepancies prevented us from obtaining the desired precision.

The results of our work during March, 1947, showed the plane-coordinate system to be reliable, at least to the extent that we used it, but, as explained earlier, some of the photographs did not have enough points to form good triangles. However, personnel of the AMC advised us that this was being corrected by increased flight altitude, thereby photographing additional terrain containing points which did not appear on earlier photographs.

Computation Forms.—Some changes were made in the computation form for computing by the combined Church and Underwood methods. A copy of this new form and sample tilt computations using this method were sent to the AMC in March, 1947. They were sent in support of earlier statements. A photograph with a five-degree tilt computed by the Church method was used for the combined method. The approximate value of one side of the ground pyramid (LA) was obtained from a graphical solution. The scaled value of LA was 35 ft in error. With this difference, a second trial was necessary to obtain the precision required for this project. The result, which included a second trial but did not take into account the value of K, was 5°02'23". The tilt angle when both the second trial and K were omitted was 5°01'14.9". The tilt angle obtained when the correction K was applied wherever called for after the second trial was the most precise value and was 5°01'59.9". The last tilt angle had the correction K applied only to ΔZ after the second-trial solution. Since the value of Z_L was taken to the nearest foot, the resulting tilt was the same but the computation time was reduced. This final method showed a tilt angle of 5°02'01".

Strength of Figure.—Theoretically, it seemed that the nadir point could be determined from three ground control positions, regardless of their image location on the photograph with respect to the nadir point, with one exception, namely, when the nadir point fell on or near the circle passing through the three control points. In plane-table surveying, this circle is usually referred to as the great circle and the triangle formed by the control points as the great triangle. When the above condition occurred, the point sought was indeterminate. Both graphical and analytical solutions failed. The strength of the ground pyramid depended on the strength of the ground triangle. The strongest ground triangle was one where the point sought lay at the center of gravity of the triangle. The strength of this figure was carried into the ground pyramid, if the nadir point also fell at the center of gravity of the base. In the tilt solution, the ground coordinates of the nadir point did not enter directly into the computations but were used, however, to obtain the values Z coordinate of the air station (Z_L) and the length of one side of the ground pyramid (LA). Moreover, when the nadir point lay approximately at an equal distance from the three control points, the angles formed by the vertical through the nodal point of the camera lens, and the three sides of the ground pyramid, were about equal and of sufficient size so that the cosine varied considerably with small changes in angle. This made the tilt solution less sensitive. Small errors in ground and photo coordinates resulted in relatively small errors in tilt.

The above represented ideal conditions. This did not mean, however, that weaker figures could not give satisfactory results. It seemed logical that if the solution did not fall into the indeterminate class, and if the exact values of all measurements were known, the weakest figure would give correct results. However, the exact value of ground and photo coordinates will never be known. Fortunately, these distances could be determined to a high degree of accuracy; and while one might get by with poor measurements for strong figures, this would not be so for weaker figures, if comparable results were to be obtained.

A judicial selection of triangles would, in general, prevent the weak cases. Triangles in which the principal point fell on or near the great circle had to be avoided. In a German document on space resection (by Dr. C. A. Traenkle, now with WADC) received from the AMC, this condition was referred to as the dangerous cylinder. If the principal point (which approximated the position of the nadir point) was near a control point, the solution might be unstable due to its proximity to the great circle. Also, the angle at the air station between the vertical and the point would be small. The tabular difference for one minute of arc for an angle of two degrees was 0.00001. The tabular difference for one minute of arc increased to 0.00005 for an angle of 10 degrees. An angle of 10 degrees between the vertical through the air station and a line joining the air station to a control point would place the nadir point approximately one inch from the control point on the photograph. In other words, increasing the distance between these points reduced the sensitivity of the tilt solution.

In the West Manchester quadrangle, there were a few places along the flight line where the control was not very well distributed. A proper selection of triangles, with high-precision comparator measurements and ground measurements to tenths of feet, was necessary to obtain the desired accuracy in the weaker triangles.

Gaertner Comparator.—The comparator stage had a slight side shift varying approximately from 0 to 0.020 mm from mid-position to the upper end of travel when moved in the upward direction. This did not occur when the stage was moved in the downward direction. To eliminate this error, all points on which readings were taken were approached in the downward direction of the stage. Some measurements were made using a magnification of 24X. With this power, smaller discrepancies between measurements on the axis and measurements for the overall photo dimensions were more readily obtained. Since these measurements affected all coordinates, we expected a higher overall accuracy. It was stated earlier that the cross hairs could not be rotated due to eccentricity. Upon reconsideration, we decided it was permissible to rotate them one or two degrees wherever it seemed advantageous to determine center lines of roads. A rotation of 10° caused an error of approximately 0.004 mm, while a rotation through an angle of 1° or 2° could scarcely be detected with a magnification of 24X.

As stated earlier, two comparator readings were taken on each control point to safeguard against mistakes in reading as well as to increase the precision of the measurements. The probable error of a single measurement, the probable error of the mean of two measurements, and the PEM of 10 photocoordinate measurements made with the comparator were, respectively, ±0.00417, ±0,00286, and ±00132 mm.

In making the above measurements, the stage was reset for each pair of pointings on the control point. These readings were subtracted from the mean value of the mean of 10 readings taken on each end of the coordinate axis.

Discussions.—Representatives of the AMC, Capt H. L. Brown and Messrs. W. R. Allen and M. E. Warshawsky, visited the project on 9 May 1947. Discussions covered the following topics:

1. Problems encountered in transferring cross hairs from A-4 prints to T-5 prints.

    a. Instrumentation involved.

    b. Expected accuracy of transferring process.

2. Possible means of circumventing cross-hair-transfer difficulties by alternate methods of recording gyro indications.

It was apparent that the transfer process did not approach the precision which both we and the AMC strove to attain and alternate methods of recording gyro indications would have to be tried.

Position Accuracy of Nadir Points.—In investigating the position accuracy of 26 nadir points, both tilt and swing were considered. The following results were based on the computed values obtained from two different sets of control points. The probable error of the mean in position was determined as follows: Let two positions differ by the amount "a," then

Observed Values	Residuals: V	V^2
$M - a/2$	$- a/2$	$a^2/4$
$M + a/2$	$+ a/2$	$a^2/4$
$\frac{2/2 + M}{M}$		$\Sigma V^2 = a^2/2$

$$\text{PEM } E_m = \pm 0.6745 \sqrt{\frac{\Sigma V^2}{n(n-1)}}$$

$$E_m = \pm 0.6745 \sqrt{\frac{a^2/2}{2(2-1)}}$$

$$E_m = \pm 0.33725a$$

Probable error in horizontal position for 26 points varied from 1.2 to 10.9 ft with an average of 5.5 ft. Probable error in vertical position of the air station for 26 photographs varied from 0.0 to 3.0 ft with an average of 1.2 ft.

Space-resection computation showed some variations in the distance and elevation between exposure stations. Assuming that a constant time interval was maintained by the intervalometer, one would expect that decreasing altitude would increase the velocity, thereby increasing the distance between nadir points, and vice versa. If, in general, this had not been so, the computed position of the nadir points might be in question. As a further check on position accuracy, the elevation of the air station was plotted against horizontal distance. With the exception of a couple of stations where the differences were about equal to the probable error, it was seen that curves conformed to the predicted shape. A value of 3.9 seconds between exposures was used in determining the average velocity in plots sent to the AMC. This was obtained from time readings recorded on the photographs and was not precise enough for accurate velocity determination. The actual velocity could have been slightly more or less than that shown by the curve.

Tilt and swing were plotted on a separate strip and also sent to the AMC.

Precision of Comparator Measurement.—The probable error of a single measurement and the probable error of the mean of a number of comparator measurements have been discussed. As a further check on measurements, the following procedure was used in June, 1947. A coordinate distance of a certain point was measured five times. Three days later the same point was remeasured, and the following day the measurements were repeated. This procedure called for orienting and ratioing the photographs for each set of measurements. The maximum difference between the three means was 0.016 mm (0.0005 in.). The largest probable error of a single measurement of the three sets was ±0.003 mm. With respect to ease of point determination, the point selected above was probably below average.

Graphical Solutions.—During July, 1947, improvements were made on graphical solutions for obtaining the length of one side of the ground pyramid and the coordinates of the nadir point. With these values, which were nearly correct, the analytical solution converged more rapidly.

Ultimate Precision of the Photogrammetric Method.—Investigations were begun in July, 1947, to determine the ultimate precision of the photogrammetric method. This was based on the assumption that the camera calibration and lens distortion were known to a high degree of accuracy, as well as the ground control which was spaced to form strong triangles. A least-squares probability analysis was made and based on the probable error of measurements and calibration. Some of these had to be assumed, but the results indicated the calibration precision required. This analysis was supported by mathematical solutions where small errors were introduced into a master solution to obtain the effect on tilt accuracy. Since we were not dealing with a plane surface, there were thousands of combinations in which errors could occur. In preparing the report, "Ultimate Probable Accuracy of the Photogrammetric Method," by Edward Young, Engineering Research Institute, University of Michigan, September 10, 1947, which was sent to the AMC in September, 1947, it was kept in mind that recommendations for improving the accuracy of tilt determination were desired, and, since accuracy on the order of 0.03 milliradians was mentioned, specifications for camera and ground control became more rigid. If the specifications as set up were obtained, the probable tilt accuracy was ±10 seconds of arc. To support the least-squares analysis, problems were set up and solved in which the allowable errors in coordinates, as determined from the least-squares analysis, were introduced in various combinations. The mean value obtained from the solutions differed from the standard problem by 8.5 seconds of arc. This indicated that our analysis for the specifications imposed agreed very well with actual solutions, where the errors of the magnitude determined from the specifications were introduced.

It seemed, then, that the probable ultimate accuracy depended to a large extent on the precision of the camera, with the exception of the focal length, which was not critical so far as the tilt was concerned. Further investigation indicated that where solutions were carried out more precisely, an error of 0.02 mm in the focal length caused an error in tilt of approximately one second of arc.

Dr. F. E. Washer of the National Bureau of Standards was of the opinion that the best distortion measurements obtainable for the outer half of the field of Metrogon lenses with their equipment would have an error of about five microns. If photographs were made on film, even with a grid on glass in front of it to isolate discrepancies, several microns additional error were likely because of processing and difficulties in holding the emulsion accurately in the focal plane. Therefore, it seemed as though a probable error of at least 10 microns would be the best one could possibly hope for with Metrogon lenses and film. Errors of 20 microns or more were easily possible due to irregular and uncertain temperature effects, irregular lack of flatness of focal surface, and many other difficulties when using the existing so-called precision aerial photographic equipment.

Nonuniform distortions in wide-angle lenses of 50 or more microns were not uncommon and were very tedious and expensive to measure. The resolution of the film-lens combination also had a limiting effect on the aerial photographs. Super-XX emulsion and Metrogon lenses had an average resolution of 10 to 15 lines per millimeter from the air, although they resolved about twice as fine in high-contrast targets in collimators in the laboratory.

Since the outer half of the field of a wide-angle lens could be measured with the Bureau of Standards equipment to an accuracy of 0.005 mm, it was suggested that a longer-focal-length lens be considered for the 9- x 9-in. photograph.

The result of processing was dimensional change which was not likely to be equal in the direction of grain and across the grain of the film. However, we thought we could deal with this since our problem was not cartographic. Precise measurements of the camera focal plane and the topobase prints were necessary. Corrections obtained from the ratio of the plate dimensions to the camera dimensions were applied to the plate coordinates. This had the added advantage that it eliminated error in the comparator, if the error was uniformly distributed over the measuring range of the instrument.

In our work, the magnitude of lens distortions was not as important as the accuracy to which these distortions could be measured. The correction for uniform lens distortions was combined with the correction for curvature and refraction to form a single correction curve.

Effect of Introducing a Change of ±0.010 mm in One of the Plate Coordinates.—The effect of introducing a change of ±0.010 mm in one of the three control points was tabulated and sent to the AMC in August, 1947. The change in tilt varied from 7.5 to 35.5 seconds. The change in swing varied from 0°01'42" to 0°08'43". The maximum change in elevation of the air station (Z_L) was 0.7 ft. It was noted that the signs of ΔX_L, ΔY_L, and ΔZ_L for the ground coordinates of the exposure station all reversed when the sign of the numerical change reversed. The same was noted for Δx and Δy of the nadir point, Δtilt and Δswing. The fact that the delta values did not reverse exactly in numerical value was a function of the accuracy of the solution. The magnitude of the residuals showed this to be true.

For a plotted example, the change in the nadir point followed the change in x and opposed the change in y. If vector addition could be applied, tilt could get as high as one minute of arc, if two errors or changes were combined vectorially. It was quite apparent that errors of this magnitude could not be allowed to fall in the plate coordinates where a high precision was desired.

Effect of Introducing a Change of 0.020 mm in the Focal Length of the Camera Lens.—Changes in coordinates of exposure station and of nadir point, and changes in tilt and swing due to an introduced numerical change of 0.020 mm in the focal length of the camera lens, were tabulated and sent to the AMC. They showed the following:

1. 1.4- to 1.6-ft change in elevation of air station.

2. 2- to 4.5-seconds change in tilt.

3. 0°00'26" to 0°01'33" change in swing.

Effect of Curvature and Refraction on Plate-Coordinate Measurements.—Investigations showed that the corrections for both curvature of the earth and the refraction of light were appreciable quantities. However, the combined effect of the two, since they were of opposite sign, caused a smaller error. A report was sent to the AMC, "Report on Curvature and Refraction Corrections for Aerial Photographs," by Edward Young, Engineering Research Institute, University of Michigan, October 10, 1947, showing that errors from these sources were appreciable and that it was necessary to apply a correction to the plate coordinates.

Control-Point Targets.—In August, 1947, some photographs were taken on white circular targets with black centers. The size of the targets was chosen to simulate, at a horizontal distance of 1500 ft, correspondingly larger targets at 12,000 ft. These photographs were made with a 7.5-focal-length lens with a K-2 filter. We were familiar with halation or image spread but had not had a chance to measure this spread. The results were somewhat surprising and were helpful in experimental target selection later.

Mann Comparator.—On 19 September 1947 we received one 8- x 10-in. Mann comparator, No. 42213. Mr. David Mann delivered the comparator and assembled it. Since the comparator stage was not equipped with glass plates to hold the 9- x 9-in. topobase photographs, Mr. Mann said he would send us something suitable for this purpose. Substage illumination over the entire area of the 9- x 9-in. photographs was not necessary for positive prints. It would, however, be necessary for negatives or transparencies. Although control points might fall in the area of substage illumination, the fiducial marks might not.

On 14 October 1947 we received a print holder for this comparator. It consisted of a metal disc in which an area 8-3/4 x 8-3/4 in. was cut out. The disc was fitted with clamps to hold two glass plates with the photograph fitting between these plates.

The maximum magnification obtainable was too low for accurate testing of the instrument. While not stated, the maximum magnification appeared to be about 14X, while the maximum magnification of the Gaertner comparator was 33X.

Reports.—Reports on the following subjects were sent to the AMC in October-November, 1947:

1. Error in Tilt Due to Error in Focal Length of the Lens.

2. Convergence of the Church Method in the Determination of Nadir Points.

3. Convergence of the Underwood Method.

4. Mechanical Method Applied to the Graphical Solution of the Length of the Sides of the Ground Pyramid.

5. New Computation Method and Form. Mathematical justification followed in December, 1947.

6. Correction for Rotation of Cross Hairs in the Gaertner Comparator.

Analysis of High-Altitude Photographs.—Representatives of the AMC, Capt H. L. Brown and Mr. M. E. Warshawsky, visited the project on 3 November 1947. The conference was primarily intended for the discussion of a new project, the analysis of high-altitude photographs. Capt Brown visited us again on 1 December 1947 to discuss this project further. In March, 1949, the report, "Final Report on High-Altitude Visibility on Eight Flights," by Edward Young, Engineering Research Institute, University of Michigan, March, 1949, was completed and sent to the AMC. This report was composed of all the work that had been completed to that date on this subject. All earlier reports were included in this report, with some revisions.

Square-Root Table.—In January, 1948, a square-root table accurate to seven digits and suitable for use with computing machines was computed for use with the new method of tilt determination. This table was similar to the Marchant table,which was accurate to only five digits.

Marsten Morse Method.—In January, 1948, a copy of a short method of tilt determination developed by Marsten Morse was received from the U.S. Coast and Geodetic Survey. A copy of this form, as well as a modified form of the same method, was sent to the AMC. Tests for the accuracy of this method were made by substituting the nadir point (determined by precise methods) for the central point in the Morse method. Relief corrections based on principal-point assumptions were applied to the coordinates of the three outer control points which were measured and used in the precise solution. Under conditions of low relief and low tilt, the Marsten Morse method could be considered precise, if our definitions of precision allowed three or four seconds of arc. The time required for computation was much less than in the other methods which we had been using.

The advantage of the method was its shortness and simplicity. The disadvantage of high relief could be overcome by photographing flat terrain such as the McClure area. The greatest disadvantage, however, was that the method required that one of the control points be close to the principal point. To apply this method generally, points along the flight line would have to be closely spaced; roughly, about every 3000 ft for a flight altitude of 16,000 ft.

Tilt Computations.—Some tilt computations were made from photographs taken at 21,000 ft. This was done to see what improvement would result from the smaller photo-scale road intersections which marked the control-point positions as well as to determine the effect of errors on the local-grid system when used in computations based on higher-altitude photographs.

Corrections to Photo Coordinates.—As a result of discrepancies obtained between different methods, we worked on a different approach to obtaining corrections to photo coordinates. This was based on measurements of 40 control points (one of which was located near the principal point) which were compared with the corresponding ground distances. Displacements due to ground relief were computed. With the values of tilt and swing known, we hoped to obtain an accurate correction curve for photo coordinates.

Four corrections were found which could be applied to the image coordinates of ground control points on the photographs for tilt determination. They were (1) expansion and contraction corrections, (2) distortion correction, (3) relief-displacement correction, and (4) curvature and refraction corrections.

1. The correction for the expansion and contraction of the topobase positive print was applied by taking the ratio of the measured photo distance between the fiducial marks to the true distance (determined from flash plates) and multiplying all coordinates in that direction by this ratio.

2. Data for lens distortion were received from Wright Field. The distortion curves determined by the Aerial Photographic Branch were along each diagonal of the plate. Since the distortion varied considerably between diagonals, an average correction curve for the entire plate would not be accurate; hence, it was necessary to interpolate values of distortion over the range between diagonals. The method of interpolation was to plot distortion vs angular distance around the plate, draw in lines of equal distortion, and take off values at intervals of 15°. The interpolated values were not accurate because the range of interpolation was too great. A distortion-correction sheet was included in material sent to Wright Field in March, 1948, and the distortion values were calculated as explained above.

3. Because of the ease of calculation, the Marsten Morse method was used for computational work. This method required the reduction of all ground elevations to a common datum plane. The values included in the report were obtained by direct calculation for a flying altitude of approximately 20,000 ft. Relief corrections should be applied from the nadir point; however, for small tilts the correction could be applied to the principal point with negligible error.

4. Curvature and refraction curves for an altitude of 12,000 ft had been

computed in September, 1947. By extrapolation of the same method, a new curve was obtained out to 20,000 ft.

A form for applying these corrections for the image coordinates was sent to Wright Field in March, 1948. The ratioing process for expansion and contraction correction, which was missing on this form, was applied on the form used for comparator measurements.

Calibration of T-5 Camera No, 4104.—In April, 1948, preliminary work on the camera calibration of the T-5 camera No. 4104 began. A preliminary report listing the constants of the camera to be determined, as well as the methods to be employed, was sent to Wright Field. For the precise determination of tilt, the following constants of the camera had to be known:

1. Principal distance.

2. Location of the point where the principal axis intersects the focal plane (principal point).

3. Image displacements due to lens aberrations.

4. Angle of the fiducial axes.

5. Dimensions of the focal plane.

6. Flatness of the focal plane.

In May, 1948, work continued on the calibration of the T-5 camera with special emphasis placed upon distortion curves. In June, 1948, the final results of the calibration of camera No. 4104 were sent to Wright Field. The results included the following:

1. Calculation of the principal distance.

2. Location of the point of symmetry.

3. Individual and mean distortion curves.

4. Angles between the fiducial axes.

5. Location of the principal point.

In July, 1948, still further work was done on the calibration, mainly in the measurement of flash plates necessary to determine the values used in ratioing the comparator readings made on the individual plates to correct for expansion and contraction.

GYRO TESTING

Rigid Structure.—On 28 April 1948, suggestions as to a method of reference lines on the screen, which was a component part of the rigid structure designed to carry the gyro test equipment, were sent by letter to the AMC. A report on determining the tilt of the rigid structure was sent in May, 1948. Included were curves showing the relatively small error caused by assuming the rigid structure to rotate about the mirror axis instead of the gimbal axis of the gyro. The general procedure for finding the test-gyro errors was as follows.

1. The true tilt of the structure was determined by the T-5 camera for five-second intervals.

2. The tilt of the reference gyro was determined with respect to the structure for suitable intervals (minimum interval was 1/4 second). The difference between the reference-gyro tilt at the instant the camera operated and the camera tilt was recorded against time.

3. The corrected tilt from the reference gyro was compared with the tilt of the test gyro to get the test-gyro error.

Preliminary work on the tilt determination of the rigid structure proved to be in error. A thorough reconsideration of the tilt analysis of the rigid structure indicated that an error in the basic assumption was made. It was assumed at first that the beam of light reflected from the mirror would move on the screen in a direction parallel to the path of the normal to the mirror; however, this was not the case. An analysis of the rigid structure, a solution for gimbal-axis error, and a tilt-component computation were sent to the AMC in July, 1948.

The tilt components were first computed while neglecting the error due to the gimbal axes. Using these values of the tilt components, the errors were then read from a curve of Δt vs t. Then,

$$t = t_1 + \Delta t.$$

If greater accuracy was desired, the coordinate error, Δx, could be plotted against tilt and recalculations performed until the solution converged:

$$t = \text{arc tan}\left(\frac{x + \Delta x}{M_Z}\right) .$$

Sample computations and curves were included, based on the approximate dimensions of the rigid structure as given on blueprints received from the AMC.

An examination of the 16-mm calibration film received from the AMC on 1 September 1948 indicated that several improvements should have been made in the calibration setup. These improvements were suggested in a telephone call to the Armament Laboratory. Summarized, the suggestions were as follows:

1. Use of a wider-angle lens for calibration filming of normal piercing points in order that the points would fall in a complete square of the screen grid. This would facilitate accurate measurements to allow for lens distortion.

2. Location of a reference point on the screen grid to determine a specific coordinate system.

3. Inclusion of a method of indicating the first T-5 16-mm coincidence. Although the former system of coincidence identification was sufficient for intermediate occurences, a definite indication of the first coincidence was essential.

Personnel of this project connected with the Engineering Research Institute, Professor Edward Young and Mr. Edwin D. Hennessey, visited Wright Field on 18 October 1948. Discussions with Mr. L. M. Warshawsky and other representatives of the AMC covered the following topics:

1. Problems encountered in the calibration of the rigid structure.

2. Methods used in calibration.

3. Accuracy of the calibration.

Computation of gyro tilts with reference to a vertical displaced by a small amount (calibration error) from the true vertical to the collimation spot on the mirror had little effect on the tilt. The allowable range of displacement had not yet been determined. However, a 1/2-inch displacement of the vertical in both the x- and y-directions caused errors in the order of from 15 to 40 seconds for tilts ranging from 0 to 4°. Since this value was small compared with the degree of displacement, it was assumed that the error in tilt caused by the displacement error of the vertical under existing calibration methods was entirely negligible. In February, 1949, a method was devised whereby a front-lighting system could be used to illuminate a grid system and still enable the collimation spots to reproduce on the GSAP film. Front lighting enabled us to obtain finer and more uniform grid lines. A matt-blue background with white grid lines was used for a screen. The screen was illuminated with red or yellow-green light. Since the blue background absorbed the red or yellow-green only, the white grid lines photographed. The blue component of the white light from the collimator was reflected from the blue background and also photographed. With white light excluded from the structure, the grid lines and collimator spots could also be photographed on ordinary panchromatic film. In March, 1949, an enlargement of a typical frame of the GSAP 16-mm film was sent to the AMC showing the need for refinement of the grid system. Also sent were photographs of the grid taken with a 16-mm motion-picture camera of approximately the same focal length as the GSAP camera. These experimental results showed much more uniform lines which were narrow, and, although faint, the grid system was suitable for measurement in the comparator. In April, 1949, the completed report for the proposed grid for rigid structures was sent to the AMC.

While considering the possibility of a specialized device for computing the rigid-structure problem, it was decided that it would be much cheaper and simpler to change the elements in the structure so as to make the gyro-tilt solution a simple problem. The idea was to place the collimators at the normal intercept position with the beam pointing down the mirror vertical when the gyros were in the vertical position. The tilts were then represented by the formulae

$$t_x = \frac{1}{2}\left[\text{arc tan}\,\frac{x'}{M_z} - O_x\right], \quad t_y = \frac{1}{2}\left[\text{arc tan}\,\frac{y'}{M_z}\right],$$

whereas at that time t_x and t_y were represented by much more complex formulae. In order that the spots would not be reflected back to the source when the airplane was horizontal, the entire structure was tipped several degrees in the x-direction of flight. It was realized that other changes would have to be made to increase the field of the GSAP camera since the spots from the reference and test gyros were then widely separated.

Topobase Prints of the McClure Area.—Flight plans for the proposed flying of the USCGS test area near McClure, Ohio, were sent to the AMC on 7 April 1948. We were informed in May, 1948, that the photographic coverage of the McClure test area was accomplished and on 16 June 1948 photographs taken at different altitudes were received from the AMC. A cursory examination of the prints indicated that the targets in the area were quite easily recognized with the aid of descriptions. In May, 1948, ground control data for this area were requested from the USCGS and arrived on 15 June 1948. Coordinates of the control points were based on the Ohio state grid, north zone. The control points were well described and a sketch of each point was given as well as distances to prominent terrain features. In July, 1948, corrections were applied to the coordinates of the McClure test area. The coordinates, which were on a Lambert grid, were corrected for projection and elevation above sea level. Control points in the area were marked with a circle of sprayed lime 12 ft in diameter. Several elevations were taken by the USCGS within the 12-ft circle. The mean elevation was computed for use in our work.

During July, 1948, preliminary work was begun on the topobase prints of the McClure area. A preliminary survey of comparator measurements of targets on these prints indicated a probable error of the mean of ±0.005 mm. This value was obtained by three separate sets of comparator readings made by three operators. Curvature- and refraction-coordinate corrections for photographs at 12,000-ft altitude had been computed earlier and relief and lens-distortion correction tables were made. At an altitude of 12,000 ft, very few ground control points fall near the center of the photographs. For this reason, it was decided not to use the Marsten Morse method of computing tilt in actual production procedure. Another disadvantage of this method was that all solutions for one particular photograph used the same central point, hence the solutions were not independent as was desired. If the central point were in error, all solutions would be in error. A modified Underwood method was used instead, and a sample form was sent to the AMC in July, 1948. Few tilt calculations were then made, however, since the lens-distortion correction for camera No. 4104 was not yet in final form. A large number of problems were computed from the topobase prints of the McClure area in August, 1948. Only those at 12,000-ft altitude were considered. This run had six prints containing control in a useful distribution. The 16,000-ft run had only two useful prints, which was due to the larger time interval between exposures. The 24,000-ft run had only one useful print, as was expected. It was very difficult to make consistent readings on some of these targets with the comparator. This was due, in some cases, to halation of the targets in the light areas of the print, such as roads, fields, etc.; in others, to rather odd deformities of the targets, the origin of which was unknown. The typical accuracy of readings on targets both good and bad was as follows:

Good target - Gail

Probable error of the mean - single observer, ±0.0003 mm

Probable error of the mean - two observers, ±0.0030 mm

Bad target - Eliz

Probable error of the mean - single observer, ±0.0010 mm

Probable error of the mean - two observers, ±0.0060 mm.

In an effort to improve the sharpness of the targets, it was suggested by telephone to the AMC that the negatives be used directly for comparator measurements. This suggestion was followed and the use of photographic negatives proved to be a vast improvement over the topobase prints formerly used. Targets were in much higher contrast than the surrounding terrain and identification was positive, except in a few cases where targets were badly misshapen. The dimensional stability of aerographic film as compared with topobase was about the same. A thorough investigation was begun to determine the effects of small changes in the relative humidity on photographic negatives. The air-conditioning unit on this project allowed a 12% range in the relative humidity, which might have changed the dimensions of the negative while it was being measured, thus making the ratio correction invalid. It is probable, however, that the change was minimized by the fact that the film was held tightly between glass plates where the moisture content of the film would not be influenced by outside conditions.

The corrections to be applied to coordinates for 9400- and 24,000-ft photographs were calculated, as mentioned earlier, and sent to the AMC. In this work, the point of symmetry was being used as the principal point. When using the point of symmetry, the distortion curve calculated from this point was symmetrical for all swing angles and could be added to the curvature and refraction curves since they were all functions of the one variable, radial distance. In the corrections, the distortion curve was added only to the 9400-ft corrections.

In September, 1948, a method was perfected for accurately measuring the angle between the fiducial axes. Results for the angle between the negative x-axis and the positive y-axis from the four calibration plates were:

Exposure No. 1	89°59'55"
Exposure No. 2	89°59'36"
Exposure No. 3	89°59'41"
Exposure No. 4	89°59'27"

The mean value was 89°59'40" and the probable error of the mean was four seconds. The method of determination and a table of corrections to be applied for the axes error was sent to the AMC in September, 1948.

It was stated in a report that no target was placed at Station Grelton in the McClure area. On the way to Wright Field on 17 October 1948, this station and a few others were inspected. Station Grelton was in an orchard and was obscured by one of the trees. Later photographs showed this station. Some of the targets which were placed at the edges of cornfields lost their circular shape due to cultivation. This accounted for some of the odd shapes that we had occasionally encountered in the comparator measurements. All control points that were inspected were marked with white crushed rock.

Because of the large number of points and their distribution over the photographs, a high-altitude photo of the McClure area was thought to be invaluable in the verification of coordinate-correction data. A topobase print of this area at 24,000 ft was on hand; however, the contrast was low and only a few targets were easily identified. It was felt that the negative for this photograph would be satisfactory. Also on hand was a high-altitude negative, but it was somewhat fogged and had been handled considerably and was, consequently, of little use.

Proposed Production Procedure.—Prior to the receipt of regular gyro-test flight negatives and complete calibration data for the rigid structure, a tentative production schedule was organized in October, 1948, for the determination of test-gyro errors. The steps involved were as follows:

1. Cutting and filing of T-5 negatives and 16-mm film. Negatives were filed vertically in separate envelopes.

2. Control spotting. This involved checking all the available control on a photograph and determining which was to be used for computation, taking into account target definition and ground-triangle stability.

3. Ground-triangle computations. Using the modified Underwood tilt-computation method, it was necessary to compute the horizontal distances between control points.

4. Comparator measurements. It seemed advisable to have the photo coordinates of ground control measured by two different operators. This procedure eliminated mistakes which otherwise would not have been apparent until the final tilt computation had been made. Moreover, assuming that the two measurements were made with equal precision, the mean was a better value.

5. Application of corrections to coordinate measurements. The importance of coordinate corrections was well established. This importance was obvious from the observed fact that coordinate corrections generally range in the order of magnitude of 100 microns and that a 45-micron error in the position of the nadir point was equivalent to an error of one minute of arc in the angle. The application of the corrections involved considerable time. It was possible to make a table whereby corrections were applied directly to the x- and y-coordinates instead of the radial distance; however, it was not desirable to compute such a table until corrections were verified and the flying altitude established.

6. Tilt computations. A slight revision of Form M699E was devised to increase computational ease. The method followed in this form was accurate for low tilts; for higher tilts an additional computation was made for precise results. The source of error lay in the fact that relief corrections were assumed to be radial to the principal point instead of the nadir point. The distance between the nadir point and the principal point increased with increasing degree of tilt. The use of flat terrain, such as the McClure area, allowed the adaptation of this comparatively short form for the solution of

the tilt problem.

7. Tabulation of T-5 tilts vs time. A discussion of this in conjunction with reference-gyro tilt appears below in paragraph 12.

8. Measurement of 16-mm-film coordinates. Since the collimator spots were projected on a grid system, it was only necessary to measure them with respect to the nearest set of grid lines, taking into account the location of the set with respect to the origin.

9. Reduction of film coordinates to screen coordinates. The coordinates of the images on the 16-mm film had to be reduced to coordinates as they actually appeared on the screen. A unit conversion and a correction for lens distortion were applied.

10. Test-gyro and reference-gyro tilt computations. With the receipt of the calibration data on the rigid structure, the empirical equations involving the tilt components were set up. The computation of the tilt components t_x and t_y was then a matter of substituting the screen coordinates into the formulae $x - b_x/m_x$ and $y - b_x/m_x$.

11. Plot of reference-gyro tilt and test-gyro tilt vs time.

12. Correction of reference-gyro tilt by T-5 tilt. The procedure was to plot the difference in tilt between the reference gyro and the true tilt (T-5 tilt) at the five-second coincidences and draw a smooth curve through these points. With this curve as an axis, the values of the reference-gyro tilt within the five-second intervals were plotted. Values of Δt_x were then tabulated for each one-half second from this curve. The Δt_x for each one-half second was applied to the corresponding reference-gyro tilt, giving the true tilt over the five-second interval. This corrected reference-gyro tilt was then plotted on the same sheet as the test-gyro tilt. The difference between the two curves was the test-gyro error. The process was then repeated for t_y.

Simplified Tilt-Computation Methods.—Considerable time was spent in October, 1948, in the investigation of shorter methods of computing tilt. Any method that is not a direct analytical solution of the problem must be approximate, and the tendency is that the shorter the method the more approximate the solution. It was felt, however, that an empirical correction which was a function of the approximate tilt could be applied to get the precise tilt. The use of specialized computing devices for calculation of tilt from both T-5 photos and GSAP-film data would represent a considerable conservation of time and funds. Earlier, an estimate was made of 10-1/2 man-hours required for each T-5 photo for the tilt computation. On the basis of existing work, the cost for T-5 and gyro tilt computation was about $10.00 per photo, which corresponded to $600.00 for each individual flight of 60 photos. It was conceivable that the use of a mechanical computer for the tilt and gyro computations would have decreased the expenditure by a large amount as well as shortened the time required to obtain the final result of gyro error. The saving over a number of flights would warrant the initial cost of design and construction. The three types of computers applicable to the problem of tilt were (1) digital, (2) counting, and (3) analog. At that time the problems were being computed on the ordinary digital desk calculator. The cost of construction of a special digital machine which would be adequate to compute the entire tilt problem would be prohibitive. The analog computer, due to its dependence on the measurement of physical quantities, was limited by an accuracy of within 0.1% at best. This was not sufficient for the accuracy required in tilt computations. The counting computer, on the other hand, combined the better properties of the analog and digital computers. It was relatively low in cost and could be designed to nearly any accuracy.

A letter with an accompanying outline of the tilt-analysis problem and requesting cost estimates of a specialized computing mechanism was sent to several of the larger manufacturers of computing machines. A copy of this outline of the problem was sent to the AMC in February, 1949. In view of the fact that specialized digital-type machines generally were not constructed for an individual problem, the estimates that were forthcoming might have seemed unreasonably high. However, this cost might be reduced considerably by eliminating the costly data-handling devices usually included, i.e., punch-card machines, sorters, printers, etc. It was mentioned that the Engineering Research Institute possibly was interested in the design and construction of such a computer at a more reasonable cost. A specialized computer for the problem of photogrammetric reduction was suggested strongly for consideration. Not only would the mechanism be applicable to the general method of the photogrammetric testing of stable platforms, but also to the problem of interior orientation encountered in the broad field of cartography. We were informed that The University of Michigan had added some units to its existing IBM equipment which could be adapted to the tilt-analysis problem.

Contact was made with the IBM Company through two of their representatives, and a letter was sent on 23 March 1949 to IBM requesting further information concerning the setting up of the necessary plug-boards to do the tilt problem. A copy of this letter was sent to the AMC. The local representative of IBM informed us that The University of Michigan would receive the new installation previously mentioned sometime in April, 1949. The 602-A IBM Calculating Punch arrived in April, 1949. Discussions with Dr. Dwyer, who was in charge of the machine, indicated the possibility that the tilt problem could be set up free of charge by the Statistical Research Laboratory of the University. There was some doubt, however, whether actual production could be carried out since the policy of the Statistical Laboratory was not established.

Gyro Flight No. 750.—Data for the flight of 27 October 1948 were received on 16 November and work started immediately with this material. The following comments apply to the data.

1. T-5 negatives. Although the negatives were low in contrast due to haze, the targets were distinguishable. Some of the targets had been repaired at the time of the flight, but many were still in poor condition. All targets were to be remarked before later flights. Sixteen negatives of the flight were usable, No. 256 through No. 271. The first coincidence on the GSAP film was with No. 256.

2. Calibration film. Due to the fact that the grid lines of the screen were nonuniform and faded off to one side, it was decided to use a single reference point near the center of the screen instead of the grid-square system previously devised. Since the tilts on this flight were not large, the collimation spots remained near the center, making the distortion effects of the GSAP lens nearly negligible. Although the positions of the normal intercepts were considerably distorted, they had little effect on the tilt accuracy.

In December, 1948, curves representing the departure from the vertical of the test-gyro axis were sent to the AMC. Tables from which the curves were drawn were also sent; for close study the tables were recommended since the time interval of one-half second was too great for drawing an accurate smooth curve through all the points.

The method for determining the test-gyro error was discussed earlier. In short, the procedure was to correct the reference-gyro tilt every five seconds by the T-5 tilt and, since the T-5 tilt was the true tilt of the structure, the corrected reference-gyro tilt closely represented the true tilt over the total interval. The difference between true tilt and test-gyro tilt was the error in the test gyro.

The maximum probable error in the T-5 tilt was about two minutes. It was expected that data on the vacuum frame of the camera would result in a reduction of this error. A further reduction was expected due to target-repair work carried out after this flight by the USCGS.

Considerable difficulty was experienced in the measurement of the collimation spots on the GSAP film. This was due to the fact that the grid lines were of approximately the same width and intensity as the spots, and, when they coincided, the spot was hard to distinguish. The collimation circles sometimes covered the spots also. It was recommended that these collimation circles be eliminated. The spots themselves were distinguishable by their difference in size.

Film Stability.—Because a ratioing method was used to allow for expansion and contraction of the negative, changes in the negative size due to processing and relative-humidity variations were assumed not to influence the accuracy of control-point positions. However, the question arose as to whether expansion and contraction were uniform, i.e., whether radial distance vs expansion was a straight-line function. Several negatives were checked under various conditions of humidity, and results indicated that, although the relationship was not linear, the deviation was very small. Facilities were not available for checking changes that occurred during processing. The method would have been to expose an accurate grid system on a film strip and check the grid dimensions after developing, fixing, and drying under standard conditions. In the interests of higher precision, work of this nature should have been undertaken; however, at that time, errors of a greater magnitude due to other unknown causes were apparent and were to be cleared up first.

T-5 Accuracy.—Tilt, as determined by the T-5 camera, did not attain the accuracy expected from the precision of the parameters involved. Two factors for which we had no information were:

1. Filters. Filters with poor optical characteristics seriously impaired the quality of the negatives. It was suggested that they be attached during calibration exposures if they were to be used continuously in flight.

2. Vacuum frame. Data on the flatness of the vacuum frame had never been received from Wright Field. Air Force specifications stated that the overall range in flatness of the vacuum frame be not greater than 0.001 in.; however, information that we had on Camera No. AF 41-4172 showed a range in flatness of 0.0033 in. A range of this magnitude on the vacuum frame of the T-5 used in out work definitely would have affected the tilt accuracy. It was suggested that the condition of the vacuum frame be investigated as soon as possible.

Projection Method for Measuring GSAP Film.—A much faster method was devised in December, 1948, for the measurement of the collimation spots on the GSAP film. The procedure was to project the film to screen size to the underside of a light table by a mirror arrangement. A sheet of paper with grid lines ruled on it was made to correspond with the projected grid lines. The collimation spots were then marked and measured with respect to their zero positions. This eliminated calculations for coordinate reduction and translation involved when measured with the comparator. The difference in accuracy was negligible. In January, 1949, this method underwent further refinement. In order to reproduce the grid image as accurately as possible, the light table was made adjustable over a small range. This eliminated the necessity for moving either the projector or the mirror to obtain the correct grid size. A more accurate grid was obtained by an exact alignment of projector, mirror, and light table. This was accomplished by means of a transit. Two points were aligned with the telescope of the transit. The telescope was then rotated through 180° to point at the mirror. A small mirror was then placed on the viewing screen of the light table. Sighting through the telescope, the telescope image in both mirrors was then made to coincide with the cross hair, completing the alignment. The positions of both the projector and the mirror were then fixed with all further changes being made through the adjustable table.

General Requirements for Higher Accuracy.—With the application of vacuum-frame measurements, the tilt accuracy was the best available, using the current methods of camera calibration and correction for film expansion. The expected range of the probable error of the results was between 1 and 1-1/2 minutes. The probable error was determined from a set of three problems computed from the same photograph, using different control points for each problem.

For the gyro then under test, this gave an accuracy of about 2% in the gyro error, which seemed satisfactory; however, since it was expected that gyro mechanisms of a higher order of accuracy were to be tested, improvements in the photogrammetric tilt determination were necessary to maintain a low percentage of error.

In general, photogrammetric tilt accuracy depends on (1) film characteristics, (2) camera calibration, and (3) exterior conditions, i.e., atmosphere, ground control, etc. Item (3) could be controlled to a sufficient degree of accuracy. Items (1) and (2) were at that time treated as linear symmetrical quantities. Existing information indicated that aerial film experienced not only overall expansion and contraction, but also localized expansions and contractions which were not detectable by ordinary means. Control of these effects could have been established by superimposing an accurate grid on the film at the instant of exposure. A glass plate with a grid etched on it could be placed in front of the film for this purpose. Since the position of a particular grid square containing a control point was known, regardless of film expansion or contraction, the only error due to film in the position of the control point was the incremental change in the grid square. This system was similar in principle to the grid used in the rigid structure for determining the accurate location of the collimation spots on the GSAP film. The Naval Photographic Interpretation Center did considerable work along this line and detailed information was requested from them.

Calibration of the T-5 camera by the field method had several disadvantages. They were: (1) the principle point could not be located satisfactorily; (2) a symmetrical distortion curve was assumed; (3) tangential distortion could not be determined. A precise calibration by either the Bureau of Standards or the Bausch and Lomb Company was required to obtain a complete record of the distortional eccentricities of the lens.

In summary, to improve the photogrammetric tilt accuracy to any considerable extent, it was necessary to (1) correct for film expansion and contraction by the introduction of a grid between the film and lens and (2) recalibrate the camera (with grid and filters in place) by laboratory methods.

Tangential Distortion.—Tangential distortion consists of a displacement of image points from their true positions perpendicular to a radial line from the optical axis. This distortion is caused by a slight decentering of the lens elements, either in manufacture or in use. Compensation for tangential distortion requires a special calibration of the camera. This calibration would consist essentially of making an exposure of a known straight line and then measuring the deviation from a straight line on the exposure. Existing investigations made by the Bausch and Lomb Company on tangential distortion indicated that the magnitude of this error might be large. Work to this time had not considered this error, as no information or data were available; however, it was hoped that subsequent calibrations could be made and errors due to tangential distortion could be corrected.

Correction Tables.—In January, 1949, a table for relief correction for an altitude of 10,000 ft was sent to the AMC. To make this table applicable, it was essential that the pilot hold this altitude within a reasonable degree. In February, 1949, a table for the curvature, refraction, and lens-distortion corrections as a function of two variables, the x- and y-coordinates of the control point on the photograph, was sent to the AMC. Previously, it had been necessary to compute the radial distance, look up the correction, and reduce the correction to x- and y-increments, a long and tedious process.

Film Expansion and Contraction.—In February, 1949, a vector graph representing the film distortion for a particular photograph was sent to the AMC. This work was done by the U.S. Naval Photographic Interpretation Center. The vectors represented the magnitude of the film distortion after it had been ratioed to the camera back. It was seen that the distortion was random in direction and magnitude, although it did follow general patterns. In the margin of the photograph the distortion was over 200 microns. Discrepancies in tilt which had been present in our work could certainly be attributed to this cause.

Details on the installation that PIC used in the F-51 camera were sent to this project and it was expected that the basic setup could be used in a design for a T-5 installation. Detailed drawings of the grid installed in the USCGS F-51 camera were forwarded to the AMC.

The grid was a rectangular-line system etched on an optical-glass plate. PIC reported that the cost of the grid used in their camera was about $200 when installed in 1939. Aside from the greater accuracy obtainable for the testing of gyro mechanisms, the information as to the nature of film distortions which occur under actual flight and processing conditions would be a valuable contribution to photogrammetry.

Gyro Test Flight 800.—The results of Run No. 1, Flight 800 over the McClure test area were sent to the AMC on 23 March 1949. Production problems for obtaining the gyro error had, for the most part, been solved. However, it was thought that the precision of our results could be improved considerably by the installation of a centimeter grid on the focal plane of the T-5 camera and by using front lighting on the grid in the rigid structure to obtain finer grid lines. As in Flight 750, a graph sheet representing the departure from the vertical of the test-gyro axis and tables from which the curves were drawn were included in the report. The following points of interest were noted:

1. The scale used on the graph sheet was slightly smaller than that used in Flight 750.

2. Variations in tilt were considerably less than on Flight 750 and, correspondingly, the test-gyro errors were less and ranged from about plus 12 minutes to minus 12 minutes of arc for both x- and y-tilts.

3. Run No. 1 was shorter than the three following runs due to premature cutoff of the T-5 camera.

Computation of both reference- and test-gyro tilts for Runs 2-4 of Flight 800 was somewhat delayed by the necessity of extending the calibration computations of the rigid structure in order to accommodate the large variations in tilt which arose. This variable behavior during these runs resulted in interesting data for the test-gyro error. Runs 2-4 had 16 to 18 exposures over the test area, whereas Run 1 had only 10 exposures.

Experimental results indicated that the difference between the T-5 tilt components and the reference-gyro tilt components was not a constant, which might have been assumed upon casual inspection. A detailed survey of the geometry of the aircraft in flight indicated, however, that this variable difference was completely consistent with the physical picture. For the preliminary discussion, several assumptions were made:

1. The T-5 tilt was precise.

2. The reference tilt was precise.

3. The reference gyro was stable, i.e., did not precess and was not affected by the acceleration of the aircraft.

The variations in the difference were due to the fact that the direction and magnitude of the angle maintained by the reference gyro were fixed rigidly to the frame of the earth as a frame of reference, whereas the angular computations were made with respect to the aircraft as a frame of reference (actually, the normal to the focal plane of the T-5 camera and the rigid-structure screen). An angular attitude was assumed for the reference gyro. This angular attitude was given by the angle θ and its NE direction with respect to the earth. It was also referred to the aircraft coordinate system by the components θ_x and θ_y.

A tilt was assumed for the aircraft where the angles R_x and R_y were those computed as the components of the reference-gyro tilt with respect to the normal of the structure screen. The relations $t_x - R_x = \theta_x$ and $t_y - R_y = \theta_y$ were determined by the components of θ. Therefore, since it was doubtful that either a precise heading or the same amount of crab was maintained throughout a run, the difference between the T-5 tilt components and the gyro tilt components was not a constant. Since the vector sums of the components θ_x and θ_y were constants, as one increased the other decreased. Thus, mathematically $\sqrt{\tan \theta_x^2 + \tan \theta_y^2} = \tan \theta$. This relation was tabulated for Run No. 1 of Flight 800 and included in the report. Later it was also tabulated for succeeding runs in Flight 800. The differences in the values of θ were accounted for in the inaccuracies inherent in the existing methods of computation and possible eccentricities of the reference gyro, since all errors were accumulated in this value.

Run No. 2 of Flight 800 was completed in June, 1949, and the report sent to the AMC. The probable accuracy of the T-5 tilt computations for this run was shown in a table in the report. The table was based on the theory of least-squares relation; $\text{PEM} = 0.6745 \sqrt{v^2/n(n-1)}$. The average probable accuracy was summarized as follows:

$$\text{PEM } t_x = \pm 0.014° = 50 \text{ seconds of arc}$$

$$\text{PEM } t_y = \pm 0.012° = 43 \text{ seconds of arc.}$$

Since the error of the gyro computations was relatively small, these values represented nearly the total accuracy of the final results.

Run No. 3 of Flight 800 was completed in July, 1949, and the report was sent to the AMC on 11 August 1949. The probable accuracy of the T-5 tilt computations for this run was shown in a table in the report. The average probable accuracy was summarized as follows:

$$\text{PEM } t_x = 0.015° = 52 \text{ seconds of arc}$$

$$\text{PEM } t_y = 0.013° = 48 \text{ seconds of arc.}$$

If reference was made to Table I in this report, it was noted that several of the individual probable errors were considerably larger than the mean. This was caused by the necessity of choosing very unstable triangles for computation, which in turn was forced by the scarcity of control on these negatives.

Run No. 4 in Flight 800 was completed in September, 1949, and the report was sent to the AMC. The probable accuracy of the T-5 tilt computations for this run was shown in a table in the report and was summarized as follows:

$$\text{PEM } t_x = 0.012° = 43 \text{ seconds of arc}$$

$$\text{PEM } t_y = 0.014° = 50 \text{ seconds of arc.}$$

Referring to Table II in this report, it was noted that the average deviation of the reference-gyro axis with the vertical was considerably larger than on previous runs. This was explained by examining the following table:

TABLE OF MEAN DEVIATIONS

Run	V_{θ_x}	V_{θ_y}	V_θ
I	0.056	0.038	0.026
II	0.144	0.055	0.050
III	0.091	0.043	0.036
IV	0.213	0.067	0.192

In general, the roll component, θ_x, was larger than the pitch component, introducing larger deviations in this direction. In Run No. 4 the magnitude of θ_x was much greater with respect to θ_y than in the previous runs. Since in this case the greater deviation occurred with the largest component of θ, the mean deviation of θ was large.

A final report summarizing all runs of Flight 800 was completed in September, 1950, and sent to the AMC. The report, "Operational Testing of Airborne Gyroscopes by Photogrammetric Methods," by Edward Young, Engineering Research Institute, University of Michigan, September, 1950, summarized the investigations on gyroscope-verticality testing carried out by this project from August, 1946, to September, 1950.

Target Definition.—There was considerable variation in the definition of the targets in the McClure area. Many of the targets were badly deformed and, therefore, had indeterminate centers. In April, 1949, a set of seven photomicrographs of several targets in the area was sent to the AMC. Some of the targets appeared to have been plowed up or washed away, while a few were nearly obliterated. Under these conditions, comparator operators were unable to determine with any degree of certainty the position of the control point. Due to halation or image spread, the 12-ft targets in the test area appeared on the film to be approximately 16 ft in diameter. A 10% error in centering introduced an error of approximately 45 seconds of arc. It was decided that future precision would be increased if the targets were repaired.

Tilt-Determination Methods.—In addition to the precise method of tilt determination being used by this project, in April, 1949, the AMC stated its desire for a method which could be solved in much less time and still maintain a reasonable degree of accuracy. With this in mind, we investigated a number of methods which were considerably faster than our existing method. The methods investigated were as follows:

1. Direct graphical

2. AMS

3. Scale check

4. Vanishing point

5. Projection

6. Rapid calculation

The discussion of these methods included a survey of accuracy and speed. Only three of these methods were sufficiently accurate to meet the requirements set forth by the AMC. They were the vanishing-point, projection, and rapid-calculation methods. The following points were noted concerning each of the six methods.

1. This method was relatively long due to the time required to adjust the ground distances to the pyramid sides. A test photo of known tilt was run, taking care for accuracy. The resultant error in t_x was 13 minutes and in t_y was 3 minutes. It was estimated that an average of ± 15 minutes in accuracy could be maintained.

2. The AMS method used graphical aids; the AMS ratiograph for enlarging the scale of the photograph, and a three-armed protractor for angular measurement. With these aids it was conceivable that this method would be fairly rapid. However, several solutions performed with only a straight edge, compasses, and scale proved quite lengthy and exacting. It was probable that the use of the AMS aids by an experienced operator could increase the speed of the solution considerably. The accuracy of this method was limited by the following factors:

a. Limitations of the drawing board and graphical methods.

b. No corrections for lens distortions or film eccentricities. The accuracy of this method was probably no better than ±10 minutes.

3. The advantages of this semigraphical method were its simplicity and speed. The accuracy was restricted by the following factors:

a. Limitations of drawing board and graphical methods.

b. No corrections for lens distortions or film eccentricities.

c. Location of scale points.

Due to these limitations, the accuracy to be expected with this method was ±10 minutes. This method was faster than the two previous methods, however.

4. This method appeared to be a very fast and simple procedure for tilt determination. Its accuracy was believed better than graphical methods because of the following factors:

a. Lines instead of points were used for control.

b. Distortion over the length of the line was compensating.

c. Altitude was not a factor.

5. This project had no projection equipment, hence, no actual tests were made. However, we believed that projection methods could fill the gap in tilt accuracy through the ranges of from five to ten minutes. Projection methods have the advantage of being rapid and simple.

6. There were several approximate methods for tilt calculation when the photographic coordinates of the control point were known. Two of these methods were thoroughly investigated earlier and found to have small discrepancies for low tilts. These were the Marsten Morse method and the simplified Church method. Due to the fact that comparator measurements require considerable time, these methods could not be considered rapid. However, since it was not necessary to measure control points to comparator accuracy, a less accurate device could have been used.

<u>Traenkle's Projection Method</u>.—Members of this project visited Wright Field on 26-27 July 1949 for a conference with representatives of the AMC. Dr. Traenkle's projection method of tilt analysis was discussed. It was generally conceded that analytical methods are more precise than graphical or projection methods. Yet, the results of these methods, when used in tilt de-

termination, varied from indeterminate results to some degree of accuracy. Higher accuracy (among other things) results from a judicial selection of ground control points. This increased the strength of the figure which was solved. Analytically made solutions did not insure high precision in the case of adverse data.

The following points were noted concerning Dr. Traenkle's projection method.

1. Based on the assumption that the distortions in the Multiplex projection lens were similar to the distortions in the camera lens, error in the camera lens would be canceled by the Multiplex lens in projection.

2. This approach to the tilt problem employed a strong figure.

3. Errors in the position of control points caused by differential changes in the film due to processing, temperature, and humidity, if uniform, could be corrected in both the analytical and projection methods by a ratioing process. In the projection method, the projection of the fiducial axis was measured. Nonuniform changes in the film caused errors in any method. A study and correction for these errors could best be made with the use of a precise grid in the focal plane of the aerial camera.

4. Curvature and refraction had little effect in the projection method, so far as tilt was concerned. This was due to the fact that the control points selected were equally spaced with respect to the principal point of the photograph. Since all image points were displaced toward the center an equal distance, this was equivalent to using a slightly shorter focal lens, which in turn called for a small change in the elevation of the air station. However, the small image-position change caused by curvature and refraction did not cause more than a few seconds-of-arc error in tilt.

Control Layout for Projection Method.—Based on the assumption that the central point was within one-half inch of the principal point, it was estimated that about 30 additional control points were required to establish a flight line in the McClure area for Dr. Traenkle's projection method. It was suggested that Dr. Traenkle determine to what extent the control for his method could depart from an orthogonal arrangement. In September, 1949, a sketch of the McClure test area, showing 38 additional control points required for the projection method, was sent to the USCGS for a cost estimate. The additional control was also to fill in undesirable gaps in the existing control system which had caused a loss in precision in the determination of T-5 tilts.

Camera Calibration and Testing.—In August, 1949, two methods were under consideration for testing and calibrating the aerial camera. A short summary of these methods is as follows.

1. Laboratory control area. The plan was to set the camera on a platform in the roof trusses of one of the Willow Run hangars and to lay out the control area on the floor. The object distance was to be approximately 36 ft. At this distance the camera would not be in focus and some adjustment was to be made to the camera to obtain sharp definition. However, there was some possibility that satisfactory image sharpness could be obtained by stopping down the lens aperture. The camera could be calibrated from photographs of the control area made with the focal-plane level. For checking analytical and projection tilt-determination methods, known tilts could be introduced into the camera and the results compared. A more detailed discussion of this method was sent to the AMC in October, 1949, in the report, "Proposed Laboratory Type Methods of Determining the Absolute Values of Error in Vertical Obtained by Photogrammetric Methods," by Edward Young, Engineering Research Institute, University of Michigan, September 9, 1949.

2. Star trails. By photographing the stars from a stationary platform, accurate control could be obtained. The method was to leave the shutter open for a period of time and make short exposure breaks of four seconds at one-minute intervals. The resultant pattern on the photograph appeared as a series of curved dashed lines. Knowing the position of the stars, the position of the camera station, and the chronometer time, it was possible to locate accurately the angular positions of the trail breaks with respect to the zenith. From the zenith angles the camera tilt could be computed and, if the focal plane was set horizontally, the camera could be calibrated by selecting breaks that fell along radial lines. A much more detailed discussion of this application of star-trail photography was given in the report, "Aerial Camera Calibration and Testing Using Star Trails," by Edward Young, Engineering ReSearch Institute, University of Michigan, September 30, 1949, which was sent to the AMC in October, 1949. No actual tilt analysis or calibration was done at that time. However, considerable time was spent in developing practical methods for photographing star trails, constructing shutter mechanisms, and working out the mathematical relationships.

In either of these methods it would have been necessary to determine the tilt of the focal plane to the highest possible accuracy in some direct manner. The value thus obtained would be used as a standard for comparison with computed values.

Time Reduction for Comparator Measurements.—Using a data-recording camera, the comparator vernier was photographed. Recording by the operator was eliminated, as well as errors in reading the vernier. It was expected that the photographic recording of two pointings on each target taken by one operator would suffice. Assuming the discrepancy between two vernier readings to be normal, the mean value was used in the coordinate reduction. Large discrepancies were an indication of the operator's inability to bisect the target, and they were checked. A method of identifying the vernier reading recorded on 35-mm film for a given control point for the T-5 negative was worked out in November, 1949, using a coding system with lights. Since the comparator operator was relieved of reading and recording the vernier, the photographic process reduced the operating time and increased the production capacity of the comparator.

Test-Gyro Tilt as an Aid in Precise Tilt Computation.—Using the tilt as indicated by the test gyro, a value for the first approximation of the length of one side of the ground pyramid LA_g was obtained with sufficient accuracy to permit the ground pyramid to be solved with one solution. The approximation was

$$LA_g = \frac{L_aH}{x_a \tan t_x' + y_a \tan t_y' + f} .$$

H was the flying height as determined by the first photograph of a run. Examination of previous runs showed that the flying height was fairly consistent over a short period. The values t_x' and t_y' were the tilts in the x- and y-directions as indicated by the test gyro at the instant of the T-5 exposure.

Targets in the McClure Test Area.—In an earlier report, photomicrographs were used to illustrate the overall poor condition of targets. The following paragraphs explain from the mathematical standpoint why it was highly desirable to improve the control by placing black centers on each control point. At this date, many of the limestone targets had lost their circular shape.

At an altitude of 10,000 ft, six seconds of arc is equivalent to 1/4 ft on the ground. Six seconds of arc was the accuracy in tilt which the AMC desired. To obtain this accuracy, it was logical that the accuracy of all the components affecting tilt computation should be less than an equivalent six seconds, i.e., less than four microns on the plate, less than 1/4 ft on the ground, etc. The estimated accuracy of various factors affecting tilt accuracy at that time were as follows:

1. Ground-control accuracy - four inches (five microns)

2. Atmospheric effects - five microns, estimated

3. Comparator accuracy - three microns

4. Calibration accuracy - 30 to 40 microns

5. Film accuracy - 10 microns

6. Target definition - six microns

Compared to usual techniques for improving accuracy, the technique for improving targets was extremely easy and straightforward. Black target centers were made from tar paper or plywood and staked to the control point. Existing targets were circles of crushed limestone 12 ft in diameter.

To determine what size of target would be required to overcome halation effects, several experimental targets made from tar paper were placed on control in the McClure area 29 October 1949. It was hoped that these targets would be photographed on the T-5 manual-erection flights in order that optimum target size could be ascertained before the F-51 flight. While at the area, it was noted that many of the control targets were virtually hidden by the overgrowth of weeds.

On 26 November 1949 a trip was made to the McClure area to inspect all the targets. Those obscured by weeds were cleared. Mr. Ernest Reimond of McClure, employed by the USCGS in the construction of the targets, helped in this work. Sketches were made and photographs on 35-mm film were taken of most of the control points. In November, 1949, four photographs were sent to the AMC to illustrate target conditions.

Norden-Bombsight Test Flights.—Runs 1, 2, and 8 in Flight 1208 were measured in January, 1950. Run 2 in Flight 1183 was also measured for a comparison analysis with Dr. Traenkle's work. In this run, large discrepancies appeared which apparently were caused by vacuum failure in the T-5 camera. Work continued on these flights in the succeeding months. A report entitled "Manual Stabilization Error in the Norden Bombsight" was sent to the AMC on 21 August 1950. This report included only Flight 1208. Flight 1183 consisted of two runs and was of doubtful value due to apparent vacuum failure. However, a report, "Report on Tilt Values for Photographs 480-486 in Flight 1183," by Edward Young, Engineering Research Institute, University of Michigan, February 6, 1940, was sent to the AMC on 16 October 1950.

Gyro-Error Evaluation for Flights 1183 and 1208.—Evaluation of the GSAP 16-mm negatives for Flights 1183 and 1208 continued during January, 1950. Experimentation to improve the lighting of the rigid-structure grid awaited the arrival of a concentrated zirconium arc lamp. The grid lines on the GSAP film produced by the former lighting were too wide and variable for accurate reading. A two-watt zirconium arc lamp was received on 13 February 1950, but it was defective and was returned to the manufacturer and a new lamp procured. This nearly point source of light (0.003 in.) proved to be suitable for projecting the collimator spots on the grid screen of the rigid structure. The lamp proved successful in experiments simulating actual conditions, and details of the experimental setup were sent to the AMC on 20 March 1950.

Projection Method.—A positive method for testing the accuracy of Dr. Traenkle's projection method was developed in February, 1950. The basic idea was to set up a simulated photograph of known tilt with precalculated image positions from fictitious ground control. Photographs for use on a rigid test of the method were made in April, 1950. On 7 June 1950, personnel from this project visited Wright Field and delivered the report, "A Positive Method for Testing the Accuracy of the Projection Method of Tilt Analysis," along with the glass plates of the photographs marked with crosses representing ground-control images.

Camera Calibration and Tilt Determination by the Star Method.—The rigid structure for mounting the camera was designed in February, 1950. Construction was completed in May, 1950, on the platform consisting of a steel tower 15 ft in height, set on a reinforced-concrete slab, the bottom of which was 3 ft below ground surface. The tower was placed in a low building on the east side of Willow Run Airport after tests demonstrated that landing aircraft would not set up vibrations great enough to affect the precision of the work. The camera was mounted upside down on the top of the structure with provisions for making the focal plane horizontal as well as inducing small tilt angles. It was estimated that the maximum error of the entire system would not exceed 10 seconds of arc. Photographs and plans of the setup were forwarded in May, 1950. Tests were initiated in July, 1950, with a mock T-5 camera having the same outside dimensions and an F-7, 6.83-in., Zeiss Protar lens. The instrumentation was shown in a photograph forwarded in July, 1950, and was as follows: a BC348 short-wave receiver to obtain the time signals from Station WWV, a Brush oscillograph used to record time signals as well as the time and duration of the shutter operation, a one-rpm synchronous motor fitted with two cams, and microswitches to stop and start the oscillograph. In photographing star trails, the shutter is left closed for about five seconds per minute of time, thereby causing small breaks to appear in the star trail. Since the true position of each break can be determined, the breaks provide the necessary control for calibration or tilt determination. The T-5 camera arrived by plane on 8 August 1950. A test exposure on star trails failed to record the fiducial marks, but four one-volt miniature bulbs set at each fiducial mark and connected in parallel in a four-volt circuit and flashed by a timer were successfully used. The bulbs were painted black, with the exception of the tip, and no fogging or stray light reached other parts of the film. Several exposures were made in September, 1950, with super-pan press plates, but they lacked the desired speed. Tri-X panchromatic plates were ordered, and 13 plates were exposed during October, 1950. The T-5 camera was returned to Wright Field on 24 October 1950. Preliminary investigation on these star-trail plates indicated that the method of exposing the fiducial marks made accurate measurements nearly impossible.

In order to have a comparison for the precise stellar calibration, a calibration from the plates made on the field-calibration range at Wright

Field was carried out. These were the plates from the calibration reported in the camera-calibration report of 8 September 1950 by A. A. Garra. These plates were received from Mr. Eldon D. Sewell on 15 February 1951. A request was made for a four-exposure calibration on the Wright Field range to afford a more precise comparison. Investigation of tangential distortion from the two plates received from Mr. Garra revealed a side shift of 20 microns in the slide of the Gaertner comparator for motion in a downward direction. Initial tests made when the comparator was first received did not show this shift, indicating that it may have been caused by wear. A representative of the Gaertner Scientific Corp. was sent to the project and the contact pads were scraped until tests showed that the error was eliminated. The report on camera calibration was completed in July, 1951, and discussed the theoretical aspects of camera calibration as they affect precise tilt determinations. It also included a summary of the methods used by this project in the various phases of calibration, together with examples based on the two calibration plates loaned from Wright Field.

The four calibration plates of T-5 camera No. 41-4104 that were requested arrived on 22 August 1951. The method of R. Roelofs was employed, as discussed in the report, "Report on Camera Calibration," by J. C. Rowley, Engineering Research Institute, University of Michigan, September, 1951. A separate plate was used for each diagonal and along each fiducial axis in the Wright Field calibration plates. To avoid the use of more than one plate, an experimental calibration was attempted on one of the star-trail plates made in October, 1950. As mentioned previously, it was impossible to make accurate measurements on this plate due to poor fiducial marks. These errors, however, amounted only to a translation and rotation of the plate axes (fiducial axes). This did not affect the symmetry of the distortion upon which the calibration was based, but, unfortunately, these axes deviated from orthogonality by enough to affect tangential distortion.

F-51 Camera Grid.—A study of aerial negatives, taken with the F-51 by PIC, in conjunction with grid measurements made by PIC, was inconclusive. However, the following facts were apparent:

1. The grid lines were not of uniform width and sharpness, which would seem to indicate that the film was not in complete contact with the glass grid.

2. Film-distortion curves were distinctly nonlinear, as is assumed by a ratioing process. Nonlinear displacement could also have been caused by failure of contact between the film and glass grid.

McClure Area.—Personnel from this project visited the McClure area on 4 May 1950. Problems encountered in laying out the new control points were discussed with Mr. John M. Neal of the USCGS who was in charge of the work. All new control stations were examined and found highly satisfactory. The new targets were six ft in diameter and well covered with crushed limestone. The addition of the new targets provided suitable control for the projection method as well as for the analytic method. Reduction of the coordinates of the additional ground control stations in the McClure area was completed in September, 1950, and copies of the coordinates were sent to the AMC. Project personnel inspected and reconditioned targets in the McClure area on 7 November 1950. As requested, negatives for the incomplete run over the McClure area with the T-5 camera were sent to us. We were unable to use them since one fiducial mark was not recorded on the film. From a later request, we received the F-51 negatives of the same run. Since very few of the F-51 negatives were taken over the test area, and since the investigation could be carried out in conjunction with flights for other purposes, it was postponed. Targets in the McClure area were inspected and reconditioned by this project on 5 April 1951.

In order to check the theoretical calculations as to the required light intensity on ground control points for 10,000-ft night photography over the McClure area, several ground photographs were made on lights two miles from the camera. Since unfavorable weather conditions were encountered in this work, more tests were necessary to reach a reliable conclusion. The results of these additional tests and the first tests were sent to Wright Field in April, 1951. On 1 August 1951, personnel from this project visited the McClure area to evaluate the condition of the 19 experimental targets. It was observed that the asphalt targets would not be as satisfactory as concrete, with respect to both performance and cost of installation. Some of the factors pertinent to these conclusions were included in a report, "Experimental Targets in the McClure Photogrammetric Test Area," by Edward Young, Engineering Research Institute, University of Michigan, July 15, 1951. In view of the flights proposed for September, Mr. Reimond was instructed to clean up all the targets in the three central north-south rows of the area. In September, 1951, Mr. Reimond reported that he had installed 24 concrete targets in the McClure area. This meant that all our targets and a few of the USCGS targets on the three central rows were hard surfaced. Most of the remaining targets (limestone surfaced) on these rows were in fair-to-good condition. On 5 October 1951 personnel from this project visited the McClure area to inspect the newly installed concrete targets. A summary of target conditions was sent to the AMC, and a copy was sent to the USCGS on 15 October 1951.

Two glass plates of the McClure area at altitudes of approximately 10,000 and 20,000 ft were received. They were used to check the consistency of the tilt evaluation for a run with good ground control and for theoretical investigations. Inspection of the two glass plates of the McClure area indicated that the 10,000-ft plate could not be used because one fiducial mark was not exposed. However, on this plate the three remaining fiducial marks were very clear, so it was decided to measure this plate by determining the axis of the missing fiducial mark, using the two perpendicular to it, with the horizontal ways of the comparator. On the 20,000-ft plate, this fiducial mark was faint but discernible and investigations were made on this plate. On the 20,000-ft plate, 20 problems were evaluated using one set of plate measurements and 30 problems using a second set of plate measurements. On the 10,000-ft plate, 20 problems were evaluated. Work was completed on these two plates in February, 1952. The probable error of the mean of the computed tilts was about 7 seconds of arc on the 20,000-ft plate and 13 seconds of arc on the 10,000-ft plate. The distribution was approximately Gaussian, suggesting that the major sources in error were random in nature. These values fell very close to the hoped-for probable error of six seconds. However, in this investigation a large number of solutions were made from each glass-based photograph, which tended to reduce the probable error below that obtained from only three or four solutions made from film-based photographs. On the other hand, a large number of solutions could be obtained only by using all available ground triangles, although some of these would not form a strong figure. In addition, it became necessary to use the same control point in more than one triangle. As a result of this procedure, all problems were weighted and the weighted values used in the reduction of data. A report, "Distribution of Tilt Under Flying Conditions," by Eldon Schmidt, Engineering Research Institute, University of Michigan, March, 1952, discussed these results and was sent to the AMC.

Altitude Curve.—In May, 1950, a curve of the altitude vs time for a 55-second run over the McClure test area was sent to the AMC. This curve illustrated the accuracy in space position which could be attained by precision photogrammetric analysis. A study of the curve showed that the deviation from the smooth curve for a particular point did not exceed one foot. This was equivalent to an accuracy of one part in 10,000. There were, however, constant errors in the system which had the effect of displacing the entire curve up or down. The primary error of this type was probably the calibration error in the principal distance. It was estimated that the maximum error in the principal distance would not exceed 20 microns, which was equivalent to about one foot at the altitude of this flight.

Holloman Air Base Flight.—At a discussion in the Office of Air Research at Wright Field on 7 June 1950, a flight over the Holloman Air Base ground control was proposed for the purpose of determining the space position of the aircraft in flight. It was decided that after recommendations had been sent to the AMC several runs at a flying height of 5500 ft over the selected area would be satisfactory for the test. The preliminary plans for the proposed flight were as follows:

1. Several daylight runs over the control area using film in the T-5 camera with exposures synchronized with the ground tracking system. Space position of the T-5 camera in the aircraft was to be determined by aerial photogrammetry and compared with the values obtained from the ground tracking method.

2. Several night runs over the control area were to be made, using one photographic glass plate per run in the T-5 camera. This method would require lights on the ground control points as well as synchronization with the ground tracking system. This test would give an indication of the gain in precision resulting from the elimination of film distortion, platen irregularity, and poor target definition.

A representative of this project, Mr. J. C. Rowley, met with representatives from Wright Field, Messrs. Lubing, Kroeger, Waegon, and Furnish, at Holloman Air Force Base on 3 January 1951. The purpose of this meeting was to carry out the day and night flights over the Holloman launching range in synchronization with the ground tracking system as outlined previously. Under the coordination of Dr. Steinhoff of the Office of Plans at Holloman, members of the party conferred with the interested groups at the base. Discussions were with Land Air Inc., which was to track the aircraft and provide the synchronization equipment, Mr. Bagley, resident engineer, who was to provide the ground control data and aid in the placement of ground targets, and with the Projects Office personnel, who were to release the use of the launching range for the flights. As a result of these discussions, several facts were brought out:

1. The photo-theodolite ground tracking system was not suitable for night tracking of the aircraft.

2. An untracked night flight using glass photographic plates could not be made for the following reasons:

   a. Target lights available for this work were inadequate.

   b. Examination of the shutter mechanism designed for exposing image trails and breaks of the ground control lights on the photographic glass plates revealed that it would be difficult to mount on the T-5 camera; therefore, it was decided that a night flight was not feasible at that time and would be discussed at a later date.

It was decided that the day flights were to be carried out as soon as the synchronization equipment was installed. Ground control points were selected and those requiring targets (about 22 points) were marked with cloth panels. In March, 1951, a request was made for the negatives of the Holloman Air Force Base flights of January, 1951. It was felt that even though the cloth-panel targets were not resolved, sufficiently accurate tilt analyses could be made on several of these negatives by utilizing some of the more prominent targets. Preliminary work on these negatives would facilitate analyses of future flights. Only those photographs which gave consistent results were to be compared with a Holloman Air Force Base data reduction, if it seemed advisable to do so.

Phoenix Area.—A reconnaissance flight over the Phoenix area had been considered, but it was decided not to make this flight because of the road-intersection type of control points in this area. Further study of the area was made with the aid of photographs and films taken over the area and which were sent to us from Wright Field. Flights at 10,000 ft over this area were considered in September, 1949, but the idea was abandoned on account of the low position accuracy of the control as well as the type of control which, in general, consisted of road intersections. However, since there were about 1000 control points in the 30-mile-square area, it seemed advisable to investigate the area more thoroughly with the prospect of making some flights at 20,000 ft above the terrain. Moreover, the size of the area permitted considerable choice in flight lines from which the optimum line with respect to control and relief could be selected. Photographs of control points with control data and film taken over the Phoenix area were received 4 January 1951. All control points were plotted on four 15-minute quadrangle sheets at 1/4000 scale. From these sheets a flight line was selected. The choice was based primarily on minimum relief variation which was found in the two easterly quadrangles. At 20,000 ft above the terrain, the flight strip was 30,000 ft in width. The geographic coordinates of control points in this 30-mile strip (216 points) were reduced to a plane-coordinate system with origin at the center of the Phoenix control area, latitude 33°15' N, longitude 112° W, and plotted on a rectangular grid. The proposed flight line was drawn on USCGS quadrangle sheets as well as on the above-mentioned sheets. The following data were sent to the AMC in February, 1951.

1. Copies of the four 15-minute quadrangles which comprise the area and showed the positions of all ground control stations.

2. Copies of the two easterly quadrangles, showing the control stations along the proposed flight line.

3. A tabulation of plane coordinates and elevations of control stations along the proposed flight line.

4. Two U.S. Geological Survey maps showing the location of the proposed flight line.

Atmospheric Refraction.—A report concerning a complete consideration of atmospheric refraction in photogrammetry was begun in February, 1951. A comprehensive investigation was carried out, including a survey of the literature and a complete tabulation of refraction corrections for various altitudes, as well as a discussion of the limits of error in the correction and the variation with atmospheric conditions. The report on atmospheric refraction was completed in May, 1951, and forwarded to Wright Field. Completion of the report was delayed by the difficulties in obtaining the reference material. The report presented data for correction of image displacement due to atmospheric refraction. The data were in the form of curves and gave the image correction as a function of radial distance for various altitudes. An error consideration included in the report indicated that the correction data were of sufficient accuracy to afford the high absolute precision of orientation analysis by this project.

Fictitious Photographs.—On 5 February 1951, a brief discussion of the mathematical analysis and supporting data, which were the basis of the fictitious photographs developed by this project as a means of testing the accuracy of Dr. Traenkle's multiplex method of tilt analysis, was transmitted to Wright Field. Reference also was made to a previous report, "A Positive Method for Testing the Accuracy of the Projection Method of Tilt Analysis," by this project. A complete resurvey of the computations for the fictitious photographs began in March, 1951. A preliminary consideration of this resurvey indicated that there was no overall calculation inconsistency that would account for the contradictions in the y-values as noted in Dr. Traenkle's multiplex result. The resurvey was completed in April, 1952, and the report, "A Resurvey of Fictitious Photograph Calculations," by John C. Rowley, Engineering Research Institute, University of Michigan, April 10, 1951, was sent to Wright Field. From this report it was concluded that no calculation inconsistency existed, but rather that the derivation of Dr. Traenkle's geometrical relation between the tilt angle and nadir errors had caused the apparent contradictions in signs for the y-values. A search for a possible source of the larger deviations in the y-values by means of measurements on the contact-negative copies did not reveal any systematic errors or mistakes. In particular, a study of Photograph III, which retained the sign contradiction between tilt angle and nadir error, showed only a random deviation between measured and calculated plate coordinates. However, the contact plate was found to be 30 microns oversize in the y-direction. The tilt analysis was made for both measured and calculated plate coordinates and demonstrated the effect of random errors on the values of the orientation elements.

Photographic plates ordered from the Eastman Kodak Company arrived on 17 September 1951. The changes requested by Dr. Traenkle in his letter of 31 May 1951 were made and the plates were prepared. As requested, photo points and fiducial marks on the plates were indicated by crosses composed of lines about 0.1 mm wide. Points I-3, II-8, and III-23 were moved in and point V-31 was added. Also, a central cross (principal point) was added to each photograph. Each plate was measured carefully on the Gaertner comparator. The average difference between measured and computed coordinates was about five microns. The greatest difference was about 14 microns, and this was an isolated case. Although all the plates were processed carefully and were dried in a horizontal position, shifts in the emulsion were evident. A summary of the measurements, as well as the revised ground and plate (computed) coordinates, was sent with the plates to the AMC. A copy of the orientation data was also included. (On 14 March 1953, three reports, RS-4, RS-5, and RS-6, were received from Dr. Traenkle. Study began on them immediately, and a report on our findings as to their application to the type of evaluation undertaken by this project was sent to the WADC.)

Coordinates for the Arizona Area.—In June, 1951, the Universal Transverse Mercator Coordinates for the Southern Arizona control area were examined. It was concluded that, due to the remoteness of the origin, the grid error of the coordinates was too large for our application and would have required large corrections which could not have been made without further information as to the scale of the zone central meridian. The plane coordinates of a proposed flight line in the two easterly quadrangles of the area were reduced from latitude and longitude positions by this project and were satisfactory. These coordinates were reduced to sea level and required correction to the elevation of the control area.

Effect of Systematic Error on Tilt Determination.—The fictitious photographs afforded a base of absolute data for the tilt-orientation elements. Such data were mathematically correct and thus were ideal for a study of the effects of plate-coordinate errors on the determination of orientation elements.

The systematic errors with which the photogrammetric system is usually burdened are such that image points are displaced radially, this displacement being a function of the radial distance only. It was demonstrated in May, 1951, in a report sent to the AMC that systematic errors that displaced image points radially and were functions of the radial distance had little effect on resulting tilt values, but did disturb the altitude determination to a considerable degree.

Tilt-Rectification Transformation.—A transformation was derived in July, 1951, to obtain directly the coordinates of points on the equivalent vertical photograph from the measured coordinates of a tilted photograph for which the orientation elements had been obtained. The usual tilt transformations familiar to photogrammetrists for the numerical rectification of tilted photographs were rather complex, requiring several translations and rotations of coordinates. Thus, they were lengthy and very liable to mistakes in computation. It was felt, therefore, that a definite need existed for a direct transformation which could be given by a single relation between the coordinates of the tilted plane, the coordinates on the equivalent vertical photograph, and the elements of exterior orientation. Such a transformation was derived and presented in the report, "A Rectification Transformation for Tilted Photographs," by John C. Rowley and Eldon Schmidt, Engineering Research Institute, University of Michigan, July, 1951.

Random-Error Analysis.—A report on the effect of random error in plate coordinates on the tilt problem was begun in September, 1951. Work proceeded from both the numerical and analytical points of view. This was important in obtaining any desired tilt accuracy, as it would put quantitative limits on allowable random errors such as residual film distortion, emulsion creep, and measurement errors. It was hoped that a solution of the form $dt_x = f(c_1 \ldots c_n, dx_a, dy_a \ldots dy_c)$ could be arrived at, where f is a known function, $c_1 \ldots c_n$ are known constants of the problem, dx_a and $dy_a \ldots dy_c$ are errors in plate coordinates, and dt_x is the error in tilt.

Among the results obtained by March, 1952, it was of interest to note that a typical triangle solution involving three control points gave the following formula for differential change:

$$dt = -0.35dx_a - 0.21dy_a + 0.35dx_b - 0.23dy_b - 0.03dx_c + 0.42dy_c,$$

where dt is the change in tilt in degrees and $dx_a \ldots dy_c$ are the changes in the respective plate coordinates in mm. Thus, if the coordinate changes were all equal (dx_i) and the signs selected so that all terms were additive,

$$dt = \pm 1.69\ dx_i.$$

Now if $dx_i = 0.010$ mm, $dt = \pm 0.0169° = \pm 60.84$ seconds of arc. This placed an extreme limit on the error in tilt due to a known uncertainty in plate coordinates, in this case one minute of arc for a 10-micron uncertainty. Of course, if the error in plate coordinates was random, there were only 2^n possible sign distributions for n coordinates. As we were not interested in the sign of dt, it was reduced to 2^{n-1}. In our case, therefore, n = 6 and there was only one chance in 32 of obtaining the worst case. So with a random selection of the dx_i, dt almost certainly would be less than one minute of arc; e.g.,

	dx	dy
a	-0.010	+0.010
b	-0.010	-0.010
c	+0.010	-0.010

$dt = 0.0043° = 15.48$ seconds of arc.

The foregoing was, of course, a specific example. Work proceeded on other triangles and also on the new Area-Distortion method where four control points were involved. It was found in April, 1952, that the error in tilt was inversely proportional to the square of the linear distances involved in the plate or ground triangles. This relation was found to be very close but not precise. Thus, using the largest possible triangle for a T-5 camera (plate width 230 mm, f = 154.5 mm), we got

$$dt = 0.31\ dx_a + 0.05\ dy_a - 0.18\ dx_b + 0.24\ dy_b - 0.15\ dx_c - 0.29\ dy_c.$$

This was the absolute upper limit attainable for accuracy with the T-5 camera, using a three-point pyramid solution for the tilt. The mean-square error from this relation was

$$dt_m = 0.54\ dx_i,$$

where dt is in degrees and the dx_i are in mm. This amounted to 0.94×10^{-4} radians per 0.010-mm error in coordinates.

Emulsion Creep, Two-cm Grid.—While the fictitious photographs were being made, an overall shrinkage in the emulsion after processing was noted. It was decided to investigate this creepage, using the fictitious photographs for data, and to supplement these data by making a two-cm grid in a similar manner which could be contact printed readily with emulsions on glass as well as on an acetate base. However, it was found that a very slight difference in measurements for the Mann and Gaertner comparators amounting to about 10 microns in 230 mm existed. For most uses this error was negligible, but for our work in emulsion creep the error was of the same order of magnitude as the creepage. This difference also necessitated the disregarding of the data on emulsion creep from the fictitious photographs, as the points were placed by using the Mann comparator and measured by using the Gaertner comparator.

Two two-cm grids were made in December, 1951, one positive and one negative. The method employed was first to make a standard two-cm grid by ruling lines at two-cm intervals on a glass-based photographic plate in a manner similar to that used in making the fictitious photographs for Dr. Traenkle's investigation. This was done by projecting a fine beam of light back through the optical system of a Mann comparator. The measuring screw was used to obtain the desired distance (two cm), and the perpendicular ways were used as a guide. This plate was developed and carefully calibrated, using the x- and y-coordinates of the grid intersections on the Gaertner comparator. This instrument measured to a least count of 0.001 mm, and these and all subsequent measurements were made by three independent observers. The axes of the coordinate system were selected by taking the mean of all points on the three central rows (39 points in each case). These axes were perpendicular to within 0.01°, which is the finest division on the circular scale of the Gaertner comparator.

Now, using this standard grid, two other grids were prepared as follows. Using Kodak Aerographic Positive Plates, Contrast, 9.5 x 9.5 x 0.220 in. (which were extremely fine-grained, blue-sensitized, anti-halation plates for making diapositives of low-to-medium-contrast aerial negatives), two contact prints were made. The first was contact printed from the standard grid and was negative, the second was contact printed from the negative grid and was positive again. Both plates were developed in D-11 for five minutes at 68°F, fixed in F-5, washed for 20 minutes, and carefully dried in a horizontal position to eliminate gravitational creep. Then each grid was carefully measured in the same manner as the original grid. The differences in x- and y-coordinates were now considered to be the emulsion creep. This was so because the plates were contact printed with no optical distortion. The differences were shown in two vector graphs which were sent to the AMC. Graph I is the creep from the negative to the positive contact grid, while Graph II is the creep from the original grid to the negative grid.

From the two graphs, there were apparently two separate effects. One was a systematic expansion away from the center of the plate, while the other was a random creep, which was usually consistent in a small area of the plate. The expansion seemed to be greatest at the edges of the plate, while the random effect showed no similar distribution in magnitude. The expansion was apparently due to physical characteristics of the emulsion, and the local effects were probably due to adhesions to the glass base underneath. The graphs, however, represented only two specific cases and, therefore, no definite conclusions were made.

Tilt-Rectification Transformation.—In July, 1951, a direct tilt transformation of points from a tilted photograph to the equivalent vertical photograph was developed by this project. In November, 1951, work began on a report illustrating possible uses of this transformation in both theoretical and practical applications involving tilt distortions. It may be of some interest to photogrammetrists to see how it may be applied in deriving the familiar properties of the tilted photograph and also its application to practical problems; in particular, how straight-line figures are deformed by distortions introduced through tilt of the photographic plane, how areas are altered, and what happens to curved figures. This report was intended to supplement and complete the earlier report, "A Rectification Transformation for Tilted Photographs." The new report, "Applications of the Rectification Transformation," by Eldon Schmidt, Engineering Research Institute, University of Michigan, December, 1951, was completed and sent to the AMC.

New Tilt-Calculation Procedure, Area-Distortion Method.—While working on the tilt transformation, a new method for calculating the coordinates of the plate nadir point was found. Previously, the method of tilt determination used by this project was the Church pyramid method with slight modification. This was an analytic, precise procedure. However, it had several drawbacks with reference to the purposes of this project. First, the procedure was iterative and, consequently, cumbersome and time consuming in computation. This was especially evident when many solutions were sought, as in the case of flights over control areas for instrument calibration. In addition, this method was oversensitive to errors in plate coordinates because only three control points were used in one solution. Such coordinate errors resulted from errors in measurement, emulsion creep, or film distortion, lens distortion, etc.

A new method of tilt determination was developed by personnel of this project which, to a large extent, reduced the above difficulties. The calculations were easier, as they were noniterative and contained no irrational operations such as square roots. A single solution used a minimum of four control points, and, if desired, an unlimited number of control points may be used in one solution. The more points used, the less the error in tilt due to errors in plate coordinates.

This method was based on area distortion and using the Jacobian of the transformation, which is the ratio of equivalent differential areas on the tilted and untilted photographs and is well suited to the type of ground control used by the Church method. Also, this method, like the latter, is theoretically precise and subject only to errors in the given data.

In order to obtain an evaluation of this method under practical conditions, six independent tilt problems were worked on the 20,000-ft McClure-area plate. The mean solution compared with the mean solution of 17 independent problems worked by the modified Church method is as follows:

Area-Distortion method	$t_x = 5.4972°$	$t_y = -2.8816°$	H = 19,965.99 ft
Church method	$t_x = 5.4908°$	$t_y = -2.8824°$	H = 19,967.45
Difference	$dt_x = 0.0064°$	$dt_y = 0.0008°$	
	= 23 seconds	= 3 seconds	

This was very close agreement, especially since the modified Church method employed 27 more control points than the Area-Distortion method. To compare the spread of answers, the greatest deviation from the mean was 0.0454° for the Area-Distortion method and 0.2381° for the modified Church method.

Derivation of the formulae used in this method and a sample solution are given in the report, "Tilt by Area Distortion," by Eldon Schmidt, Engineering Research Institute, University of Michigan, February, 1952.

Reconsideration of Relief Correction.—It had been standard procedure on this project, when the relief was low as in the McClure area, to apply the relief correction $dr = -r/Hh$ radially to the principal point (where dr is the radial correction, r the radial distance, H the altitude, and h the relief over the ground-datum plane). This was, of course, the correct value for an untilted photograph. The assumption had been that this value would change very little for low tilts. As one of the purposes of this project was to attain the highest possible accuracy in solving for the plate nadir point, we reconsidered this problem and found that, for our purposes, the error in the assumption was too large to be neglected.

From computations from a fictitious photograph, it was found that errors as great as 19 microns at a five-degree tilt arose for points near the isometric parallel and halfway out on the plate. It was also noted that along the principal line the approximate formula was very good because the radial distance remained almost proportional to the relief correction.

We were able to avoid this difficulty with the analytical pyramid method by incorporating the relief into the problem (with considerable increase in computational difficulty). This was not possible with the Area-Distortion method. Here we made a provisional solution for the camera station and then corrected the ground coordinates for relief as viewed from the camera station. It was found, with reference to the accuracy of the correction, that a single solution for the camera station was sufficient for this purpose.

Missile Trajectories.—The requested report on missile trajectories was sent to the WADC in April, 1952. This report was prepared by Professor G. E. Hay of the Mathematics Department of The University of Michigan.

Camera-Constant Error.—As previously mentioned, the error due to the measuring of the plate width for the purpose of ratioing was $dt_x = -t_x dR_x$. For the T-5 camera, with a 0.020-mm error in measuring, $dR_x = 0.000087$, and when t_x was less than five degrees, dt_x was less than 1.5 seconds of arc. Formerly, as Dr. Traenkle pointed out, $dt = 0.00024$ radians or about 50 seconds of arc for the same error. Thus, the Area-Distortion method practically eliminated this source of error. Our precise results were $x_n = x'_n/R_x$, $dx_n = -x'_n \, dR_x/R_x^2$, and $dx_n/f = (-x'_n/f)(dR_x/R_x^2)$. For small x_n, $\tan^{-1}(x_n/f) = t_x \cong x_n/f$ radians; thus, $dt_x = -t_x \, dR_x/R_x^2$, and, as $R_x \cong 1$, $dt_x \cong -t_x \, dR_x$ very closely.

New Test Runs.—On 9 July 1952, Mr. H. Kroeger and Mr. H. L. Weigand from the WADC visited this project to discuss future test runs. They brought with them the films and target identification of the Arizona area and the test film from the T-11 camera. Also discussed was the possibility of enlarging the McClure test area. The area around McClure was well suited for this purpose, as the relief was very low. A rough estimate of the cost was given to the client by telephone on 11 July 1952.

Tests on Sperry Compensated Vertical Gyro.—A preliminary examination was made of the films over the Arizona area. Of the first 57 photographs, only nine were found to be over the strip of reduced ground control and several were not over the area at all. Before those not over the strip of reduced ground control could be evaluated, the additional ground control had to be reduced.

To test the Area-Distortion method on this area, two tests were evaluated experimentally, giving the following results:

Test 48, Photo 243	Pitch 2°45'56.16"	PEM 8'41.28"
	Roll 0°16'49.44"	PEM 2'09.60"

Test 53, Photo 27	Pitch 2°38'52.44"	PEM 4'06.60"
	Roll 0°13'04.80"	PEM 4'22.44"

The error in the relief correction was about four minutes. It was larger than in the McClure area, as the relief was larger. As the desired accuracy was 30 minutes, the present method was adequate.

T-11 Films.—The films from the T-11 camera were examined for the new type of fiducial mark. These appeared to be a distinct improvement over those used in the T-5 camera. Before any films could be evaluated properly, using this camera, it was necessary to have the complete calibration data.

Error in Photo Axes.—Due to the rather poor quality of the fiducial-mark edges in the T-5 films, it was of interest to investigate the error in tilt due to inaccurate measurement of these edges. In effect, this resulted in a small shift in photo axes on the film. To check this, we traced a differential shift through the area-type problem with the following results:

$$dt_x \times 10^6 = -1.98\, dx + 11.36\, dy \text{ and } dt_y \times 10^6 = 3.01\, dx + 7.22\, dy,$$

where dt_x and dt_y were tilt errors in radians and dx and dy were axis shifts in mm. For $dx = dy = 0.050$ mm, $dt_x = 0.096$ seconds of arc and $dt_y = 0.106$ seconds of arc. As this was certainly negligible, this error did not contribute to the given results.

Sperry Compensated Vertical-Gyro Tests.—Evaluation of the Arizona area for the Sperry Compensated Vertical-Gyro began in September, 1952. Many of the tests could not be evaluated due to one of the following reasons: test not flown over area of ground control, test flown on very edge of area and included insufficient ground control, film was fogged, and ground control was obliterated or not surveyed in the area. However, Universal Transverse Mercator coordinates for the Arizona area, compiled by the Army Map Service, proved to be sufficient for the purpose. Thus, the reduction of latitude and longitude positions to a plane-coordinate system with origin within the area in order to avoid UTM-grid corrections was not necessary on this project. Also, since the difference in elevation between ground control points in the Arizona area was considerably more than in the McClure area, an investigation was made of the error in using the Area-Distortion method in areas of higher relief.

During October, 1952, to January, 1953, work was suspended on the Arizona-area tests in order to complete the K-system tests. In the first 20 tests, 11 could not be evaluated for one of the afore-mentioned reasons. Work resumed on these tests in January, 1953, after completion of the K-system tests. During February, 1953, the remaining 37 of the total 57 tests were completed. Of these, seven tests could not be evaluated, due to insufficient or very poor control in five cases and photographs not over the control area in the remaining two cases. Due to the nature of control in the Arizona area, the accuracy of these results was rather poor but was within the requested accuracy of 30 minutes of arc.

K AND K1 SYSTEMS

Test Runs with the K1 System.—On 7 May 1952, Mr. Schmidt of this project visited the WADC to discuss the evaluation of 35 runs over the McClure area for the purpose of testing the K1 system. The best calculation procedure, with respect to both accuracy and time of calculation, was apparently the Area-Distortion method with a relief correction very similar to that of Dr. Traenkle. Two photographs from Flights 1 and 2 were evaluated by this method with the following results:

Flight 1, Photo 18

$t_x = 1.4225°$ PEM = 0.0044° = 15.84 seconds
$t_y = -0.3192°$ Pem = 0.0012° = 4.32 seconds

Flight 2, Photo 38

$t_x = 1.4225°$ PEM = 0.0016° = 5.76 seconds
$t_y = -0.1960°$ PEM = 0.0112° = 40.32 seconds

In the above, three independent problems were worked which involved 12 control points. The error in using the relief correction in all cases was less than 10 seconds for these two photos. The mean error due to the relief-correction method was 3.24 seconds. The remarkably small probable errors were due to the fact that with the Area-Distortion method the ratioing error was completely systematic. It was shown that if the photograph was ratioed by R_x and R_y along the x- and y-axes, respectively, the tilts were altered by the factors $1/R_x$ and $1/R_y$ in tan t_x and tan t_y. Thus, an error in ratioing would not contribute in any way to the spread of results.

The above, together with a very large spread of results by the triangle method, suggested that a rather large error was being made in ratioing. Also suggesting this was the unexplained fact that all photos in the three rolls received here expanded from the calibrated size, while all previous photos received by this project had shown shrinkage. The four most likely explanations were: (1) very much different conditions of exposure or in developing procedures, (2) the use of a different type of film, (3) an alteration in the fiducial-mark distances, and (4) vacuum failure. Of these, only the third explained how the Area-Distortion method could remain consistent while the triangle method did not. With this in mind, it was requested that a flash plate be made with the camera employed in the K1-system tests to reaffirm the calibrated fiducial-mark distances.

On 12 August 1952, Lt Schaeffer from the WADC visited this project to discuss the progress of the K1-system tests. He brought with him the requested flash plate for the T-5 camera used on this project. This plate yielded camera constants which differed slightly from those obtained from previous flash plates. The axis of the large fiducial mark was unaltered, while the other axis was 40 microns larger than before. This introduced an error of about three seconds of arc when the tilt component was five degrees and less when the tilt component was less than five degrees. Therefore, no correction of the previous data was necessary.

The evaluation of the 35 test runs over the McClure area was completed in September, 1952. Runs 3, 5, and 7 were evaluated as accurately as possible. The results on these three runs were not as good as the others because of the film distortions apparently caused by vacuum failure. It was also possible that the results contained a systematic error which could not be detected with the information we had available.

K-System Tests.—The K-system test with the T-11 and B-47-002 began in October, 1952. Results were better than the K1-system tests due to improved control (concrete targets) and the lack of vacuum failure in the T-11 camera. Runs 1-8 were completed in October, 1952, Runs 9-19 were completed in November, 1952, and the final runs were completed in December, 1952, and the first two weeks of January, 1953. Inasmuch as the K1-system tests with the T-5 camera were impaired by camera failure (apparently vacuum failure), the K-system tests with the T-11 camera gave us our first opportunity to investigate our methods under normal operating conditions.

In these tests with 187 photographs evaluated, it was found that the total time of one photograph evaluation amounted to about one hour. The probable error of the mean for three independent solutions was 13.51 seconds of arc in tilt component. This represented the average of 187 photographs and corresponded to a random positional error in control of about 0.015 mm in the plane of the photograph. The weakest and strongest solutions were 36 seconds and one second, respectively. Comparing this with the mean showed at once that errors as large as 30 seconds were rare. In fact, only 11% of the photographs exceeded 20 seconds of arc and only one photograph exceeded 30 seconds of arc. The majority of these larger errors were from photographs where the control distribution was very poor (in some cases, less than 50% of the photograph contained any control at all). The above results could be improved slightly either by increasing the control area or, if possible, by flying more directly over the three present central rows of the McClure test area.

Taking the above into consideration, the following estimates were made. Inasmuch as the random error is of the order of 0.015 mm, the measuring precision could be safely lowered to about the order of 0.010 mm. This could be accomplished by one observer with well-defined control points. Two solutions were sufficient for most work, since there was still a check on the result. All remaining reductions could be done by programmed punch-card machines or by automatic digital computers such as MIDAC. Allowing about 10 and 15 minutes for measuring (±0.010 mm), the time for evaluating one photograph was 25 minutes for two solutions and 35 minutes for three solutions. This did not include time for the setting up and operation of the comparator (probably less than 10 minutes). It did include an independent check on the final results, which was the agreement between two or three independent solutions. In a production-type evaluation, the effective measuring time might be reduced by about 50%, as two comparators were used. This greatly speeded up evaluation but still retained the accuracy of the K-system tests quite closely.

IBM Calculation for Tilt Determination.—On 27 October 1952, representatives of IBM visited this project to investigate the possibility of using punch-card calculators to solve the tilt problems used in our and similar investigations. However, The University of Michigan had at that time added an electronic calculator (CPC-2) to their tabulation service, and its use was investigated and found practical for the test evaluations of this project. The use of this service reduced the evaluation work to measuring and reducing the data and, hence, materially speeded up the rate of operation. Expansion of this service included reduction of data and provided a reasonable estimate of from five to six photographs processed per hour.

Mann Comparator.—In November, 1952, the Mann comparator was equipped to handle film negatives by the addition of a stage on which the photograph was clamped between two glass plates. The addition of this instrument contributed greatly to the speeding up of the evaluation process. Later, the possibility of motorizing this comparator was considered by this project. This would have facilitated greatly the operation of the comparator, which had no means for rapid motion.

Space Resection.—To complete the solution by the Area-Distortion method, it was necessary to know the coordinates of the space station and the azimuth of the principal plane. A four-point space-resection solution was worked out and served, in conjunction with the tilt solution by the Area-Distortion method, to give a complete solution for the exterior orientation. The Area-Distortion method gave the tilt and swing, while the space-resection solution gave space coordinates and the azimuth of the principal plane. It should be noted that inasmuch as the space resection was dependent on the tilt solution, it was necessarily a low-relief method, as was the Area-Distortion method. For the development of this method of space resection, which is primarily one of position circles, see the report, "Some Methods in Photogrammetric Reduction of Exterior Orientation," by Eldon Schmidt, Engineering Research Institute, University of Michigan, October, 1954.

Visitors from Eglin Field.—During the week of January 19, 1953, Mr. Roger Daly and Mr. James Replogle from the U.S. Air Force Base at Eglin Field visited the project. During the week, they were brought up-to-date on the methods used for the determination of the nadir point.

T-5 Calibration.—The calibration of the T-5 camera was completed in May, 1953. The Wright Field Calibration Range was used in these tests. Four plates were exposed, one for each diagonal and one for each fiducial axis. The method of data reduction used in this calibration was that of R. Roelofs as outlined in "Report on Camera Calibration," by J. C. Rowley, Engineering Research Institute, University of Michigan, September, 1951. The final results were in very close agreement with previous calibrations of this camera. (Note: Due to the serrated edge of fiducial mark No. 2, the plate size was measured in a different place than that used at Wright Field.)

CAMERA CALIBRATION AND TILT DETERMINATION USING STAR TRAILS

On 16 February 1953, the T-11 camera was received by this project and work was started on the attachments necessary to proceed with our investigations in the general field of star photography. For this purpose, we constructed a plate-holder attachment, an external shutter mechanism, and a camera mount. Fiducial-mark illumination built into the camera was well suited for our work and did not require further adaptation. The tower for mounting the camera and the remaining equipment for this work were available from the setup employed with the T-5 camera in earlier experiments. The necessary equipment was installed during March, 1953, but poor weather conditions delayed exposures until April, 1953.

Preliminary short trial runs gave good agreement with the calibration of the lens by Fairchild. During April, 1953, several plates were exposed with good results. The star trails were fairly well distributed over the full field of the lens, which was a major advantage in this type of field calibration. The use of the T-11 camera with self-illuminating fiducial marks avoided the greatest experimental difficulty encountered in previous work with the T-5 camera, which was fiducial-mark illumination in cameras that ordinarily use through-the-lens type of illumination.

A method of camera calibration is outlined in "Report on Camera

Calibration and Absolute Check on Photogrammetric Tilt Determination Using Star Trails," by Eldon Schmidt and Edward Young, Engineering Research Institute, University of Michigan, August, 1955. Since the error considerations were too complex for theoretical investigation, we can best find the errors by repeating the calibration several times and comparing results. With this in mind, the calibration was repeated four times, giving the following results:

Manufacturer's Calibration

Camera	T-11 Serial No. 51-193
Cone	Serial No. SF220
Lens	Serial No. SF220
Calibrated f (72°)	153.210 mm
A-B (72°)	237.07 mm
C-D (72°)	235.39 mm
Ratioing Markers	153.210 ± 0.050 mm
Angle A	90°1'
Point Symmetry	(0.007, 0.010) mm

Our results for the same data were:

Plate	f_c (72°) mm	(dx, dy) mm	A (90°±)	A-B	C-D	M_x	M_y
1	153.181	-0.011, 0.024	7.4"	237.071	235.390	153.219	153.192
2	153.180	-0.016, 0.009	- 0.4"	237.092	235.406	153.229	153.206
3	153.175	-0.014, 0.029	-20.7"	237.080	235.424	153.232	153.205
4	153.173	-0.004, 0.012	-29.9"	237.094	235.375	153.218	153.206
Avg	153.177	-0.011, 0.019	-10.9"	237.084	235.399	153.225	153.202
PE	0.002	0.004, 0.006	12.0"	0.007	0.014	0.005	0.005

The most significant thing to notice is the probable error, for it gives a representative value for accuracy. Unfortunately, the actual values for distances cannot be properly compared since the temperature coefficients were only typical values for the material used. Temperature coefficients were requested from the manufacturer, but they did not arrive. The fact that the calibrations were not made at a uniform temperature was the major difficulty in this type of field calibration. However, dx and dy, and also the angle A, were unaffected by temperature, and the angle A was in good agreement with the original specifications. As the calibration certificate did not give errors for dx and dy, we could only assume that they were at least as large as ours and, hence, that the two error ranges overlapped, giving agreement.

It was still possible to compare f_c, A-B, and C-D, even without temperature coefficients. Our A-B and C-D averaged larger than the certificate's by about 0.011 mm. It was then reasonable to argue that our f_c was also larger since the effects of temperature were uniform. Actually, our f_c was smaller by 0.033 mm. Since both values were selected to give the same distortions, this indicated that the f_c's were not in agreement. For the sake of comparison, the calibration by the manufacturer listed the probable error in the calibrated focal length as less than 0.050 mm. As the exact values were not given, a direct comparison could not be made. However, it could be estimated safely that our values were considerably under those obtained by the manufacturer. It should be emphasized here that the errors listed by the manufacturer were within the requirements for a cartographic camera such as the T-11. However, when such a camera was used for purposes where the highest possible accuracy was required, it followed that the highest possible accuracy was also desirable in the camera calibration. Such was the case in the various K-system tests where the evaluations were carried out by this project.

As explained in Section III of "Report on Camera Calibration and Absolute Test on Tilt Determination by Using Star Trails," the probable error of the distortion curve was seven or eight microns. It is important to notice that seven microns is the average position error due to measuring and emulsion errors. This was an indication that this method of calibration was at least equal in accuracy to the limits imposed by the emulsion and measuring errors. No calibration procedure could hope to exceed these limits.

For tangential distortion, we found that almost all the points were within the error limits of seven microns. We were unable to find a significant pattern and had to conclude that this lens had negligible tangential distortion. This was very unfortunate for our purposes since the method was devised with the purpose in mind of finding the tangential-distortion pattern along with the radial-distortion pattern from one exposure. It would be very interesting to try this method on a decentered-lens system where the tangential distortion is large.

We concluded from the experiment that one could expect the following accuracies from this type of calibration:

Component	Approximate Accuracy (PE)
Distortion	0.007 mm
f_c	0.002 mm
Point of symmetry in coordinates	0.005 mm
Angle between axes	12 seconds
Ratioing distances	0.007 mm

This method of calibration could be adapted to practical calibration procedure when the indicated accuracy is desired. As in most field calibrations, the lack of equipment is made up for by the necessity of more elaborate data-reduction processes. However, if such a method were to be employed in the large-scale calibration of cameras, these reductions could be programmed for automatic computation.

Another very important aspect of star-trail photography is that it offers an ideal opportunity to check photogrammetrically computed tilt against physically measured tilt. The computed tilt was found with the usual photogrammetric methods, using reduced zenith coordinates of trail breaks as "ground control." The measured tilt was found by measuring the angle between the back surface of the photographic plate and level surface with an accurate theodolite. The following remarks set forth the general principles involved in these measurements. The theodolite is used much as an autocollimator by setting it on its own image in a reflecting surface. The collimating error introduced by not using true autocollimation is canceled out since the difference between two similar-type settings on two reflecting surfaces is used. The level surface is obtained by using a liquid surface and access to the photographic plate is obtained by cutting a small aperture in the bottom of the plate holder. The theodolite is positioned so that it is approximately equidistant from both surfaces. The measuring axis of rotation of the telescope is perpendicular to the principal plane of the photograph. Thus, the total tilt is measured directly.

In November, 1953, a star-trail plate was exposed and measured for tilt with a theodolite. This plate was excessively fogged, due to the presence of the moon and the fact that too much light was admitted through the aperture in the plate holder. However, enough trails appeared for working three Church-type solutions with a tilt of 3°06'43.3" with a probable error of 15.4 seconds. The measured tilt was 3°06'57.5" with a PE of 1.7 seconds. Inasmuch as the measured value fell within the PE of the computed value, the agreement was very good. The PE of 15.4 seconds was remarkably small compared with Church solutions on film and showed the advantage of using a combination of clear control points and glass plates. Several more plates were exposed later with added refinements to cut down the fogging effects and to make possible the use of the more stable Area-Distortion method. In December, 1953, a more satisfactory star plate was exposed, but it still lacked sufficient trails in the corners of the plate for working strong four-point Area-Distortion-method solutions. On this tilt plate, four independent solutions were worked with a PEM of eight seconds of arc. As the distribution of stars on the plate was only fair, these four solutions were not strong. Therefore, the above accuracy probably could be improved slightly. However, as this accuracy was already in the range of emulsion and calibration errors, no great improvement was expected. Another tilt plate was exposed on 8 April 1954. The astronomical position of the camera station was observed and compared with the geodetic position to determine the deflection of the plumb line. This deflection correction was applied to subsequent plates. The results of the 8 April 1954 plate were as follows. The mean tilt for five independent observations was 3°06'59.66" with a PEM of 5.17 seconds. From theodolite measurements, the mean tilt for four independent observations was 3°06'57.27" with a PEM of 1.18 seconds. The difference of 2.39 seconds between the two tilts was well within the probable errors involved. Two more plates were exposed during June, 1954. The PEM for these two plates was about 2.5 seconds of arc for five solutions, which was very good agreement. However, the tilt values differed from the theodolite readings by about one minute. The theodolite reading was too large in one case and too small in the other. Since in the plate of 8 April 1954 the agreement was very close, the possibility of some systematic experimental error arose. In order to find whether this one-minute discrepancy was accidental or not, another plate was exposed on 3 August 1954. The results for this plate were as follows: theodolite tilt was 3°03'50.45" with a PEM of 3.41 seconds; photogrammetrically calculated tilt was 3°04'02.94" with a PEM of 2.88 seconds. This was a difference of only 12 seconds, which seemed to indicate that the previous plates exposed in June, 1954, were subject to some experimental error.

At the same time, work was proceeding on the determination of the deflection of the plumb line at the camera station. In order to find the astronomical position of the camera station, it was necessary to equip a Wild T-2 theodolite with five-second bubbles for the required observations. In addition, the possibility of finding the camera position, while using present equipment, was investigated. The astronomical longitude was determined by timing the meridian passage of a group of stars and the latitude was determined by measuring the zenith distance. Latitude was also determined by measuring the small difference in zenith distance between pairs of selected stars which were opposite sides of the zenith. Unfortunately, work was curtailed somewhat during September, 1954, due to unfavorable weather conditions. Because of the large number of observations required to attain accuracy, and favorable weather conditions being needed for each set of observations, the investigation necessarily spread over a considerable time. At this time, preliminary results were in fair agreement with the plumb-line deflection at The University of Michigan Observatory, which is some ten miles from the camera station. The geodetic position of the dome of the meridian circle of the observatory was determined from a nearby geodetic station and compared with the astronomical position of the observatory. This indicated that our results were in the expected range. Geometric considerations indicated that the change introduced by the plumb-line deflection would reduce the slight discrepancy between photogrammetric and theodolite tilt. As this correction was the last of the systematic errors in the procedure that could be measured and removed, the above-mentioned calculations completed the experimental phase of this investigation. Final results established the error introduced by plumb-line deflection at about six seconds with a PEM of approximately one second.

It was further found, however, that there was still one significant systematic error that appeared in the results of the tilt plates. This resulted from the fact that the plates used were not exactly plane parallel. This wedge angle was measured with the theodolite in much the same manner as the main measurements were made. The reading error was correspondingly the same (about two seconds). As this error was random, it added to the other errors in reading. This addition must be by the law of error propagation. Thus, the errors that contributed to the difference between photogrammetric and measured tilt can be summarized as follows:

Photogrammetric tilt
  Random errors - five seconds
  Systematic errors - negligible

Measured tilt
  Random errors - three seconds
  Systematic errors - one second

This gave a total error of six seconds of arc. The random errors represented the upper limits of the PEM, and in practice one would expect the errors to be less than given. The total error represented the predicted difference between the two tilt determinations. The four plates used in the experiment to test the validity of this method gave the final results listed below. The correction for wedge appears in the value for photogrammetric tilt.

Plate	Photogrammetric Tilt	PEM	Measured Tilt	PEM	Difference
8 April 1954	3°07'16.3"	5.2	3°06'57.3"	1.2	9.0
22 June 1954	3°04'13.8"	3.3	3°04'18.4"	1.5	4.6
23 June 1954	3°02'14.3"	2.3	3°02'18.0"	0.3	3.7
3 August 1954	3°03'49.6"	3.1	3°03'50.5"	3.4	0.9
	Avg	3.5		1.6	4.6

For a more complete discussion of the use of stellar photography, see "Report on Camera Calibration and Absolute Test on Tilt Determination Using Star Trails," which illustrates how the extreme accuracies of stellar positions can be utilized in photogrammetric applications. This involves methods of reproducing star images in an advantageous manner and the reduction of the stellar positions to corresponding planar positions. There are given in addition to this, two specific photogrammetric applications using these star images. One is a method of checking tilt value in a physical sense and the other is a method of camera calibration.

INVESTIGATION OF TILT WITH HIGH RELIEF

With known tilt and altitude, the precise relief correction in terms of the nadir point (x_n, y_n), altitude (H), relief over datum plane of the control point (h), the plate coordinates of the control point (x_a, y_a), and the focal length (f) is

$$\Delta x_a = - \frac{x_a - x_n}{1 + \frac{(H - h_A) f'^2}{h_A N_a}} \quad \text{and} \quad \Delta y_a = - \frac{y_a - y_n}{1 + \frac{(H - h_A) f'^2}{h_A N_a}}$$

where $f'^2 = x_n^2 + y_n^2 + f^2$ and $N_a = x_a x_n + y_a y_n + f^2$.

This suggested a method of solving for tilt with high relief without having to solve for the space geometry, i.e., either the space station or the legs of the space pyramid. Tilt by the Area-Distortion method solves for x_n, y_n and H, which are precisely the elements needed to compute the relief corrections. By successive application of the relief correction to the initial data of the tilt problem, the precisely correct values of x_n, y_n and H will be obtained.

Tests made on the iterative process during February, 1953, indicated that with relief differences as high as 1/5 H and high tilt (about 15°) the iteration closed to a three-second error in tilt in five iterations, with low tilt (about 5°) in four iterations, and with lower relief (1/50 H) in three iterations. Tests were made on the possibility of using IBM calculators for the above procedures. Also, tests were made on other iterative procedures used in high-relief methods (Church and Underwood). It was found that machine computers, such as the CPC-2 and the newer MIDAC, were highly practical and were strongly recommended for this type of procedure. The loss of the services of the CPC-2 led this project to investigate the uses of MIDAC, which is an all-electronic computer completed at this time by The University of Michigan Willow Run Research Center. Preliminary discussions led us to believe that this computer was well-suited to the tilt-problem calculations. In all aspects MIDAC offered a great improvement over the CPC-2. The use of such electronic computers is highly recommended for data-reduction procedures as employed in the evaluation process.

GYRO TESTS IN B-29-671

During March, 1953, Mr. Ernest Reimond of McClure, Ohio, started the work of cleaning up the control targets in the McClure test area. Targets were reconditioned prior to the flight with the T-5 camera in a B-29. The flight had been discussed in a telephone conversation with Mr. H. Kroeger on 30 March 1953.

Flight 1 of this series was received on 6 July 1953, while Flights 2, 3, and 4 were received later. Flight 1 was evaluated during March, 1953, by modified Church-type solutions. Flight 2 and later flights were evaluated with the program punch-card computer (CPC-2). Flight 2 was completed in August, 1953, and Flight 3 was completed in the latter part of August, 1953, and early part of September, 1953. In Flight 3 there were two runs, Nos. 1 and 10, that could not be evaluated because they were too far off the control area. It was also noted that the photographs marked "not evaluated," although in the control area, did not contain sufficient control for useful evaluation due to the distribution of control in the McClure area, which was designed for higher altitudes than were employed in these tests. Flight 4 was completed in October, 1953. The evaluation was held up due to discontinuation of the CPC-2 calculator by The University of Michigan Tabulating Services on 14 October 1953. Arrangements were made to complete the work on this flight by using another CPC-2 in the Detroit area. This completed the test in this series.

The completion of this series of tests permitted a comparison with former work of a similar nature. In all, 517 photographs were evaluated. The mean PEM was found to be 18.82 seconds in pitch and 21.28 seconds in roll. This was a mean error in tilt component of 20.05 seconds, as compared with 13.51 seconds in the K-system tests. The reason for the difference lay in the altitude of the two series of flights. The gyro tests were flown too high for the type of control distribution at McClure. As a result, about half the solutions were quite weak and none of them could be considered strong. This demonstrated the need for flying at altitudes prescribed by the design of the photogrammetric range, when the highest accuracy is desired.

DRIFTMETER TESTS

On 22 July 1953, Professor Young and other personnel from this project went to McClure, Ohio, to discuss the evaluation of the driftmeter tests with Mr. F. Wazelt and Mr. Rudolf Opitz from Wright Field. Of the 26 photographs to be evaluated, only seven had sufficient control for immediate evaluation. The remaining needed the addition of from one to six control points. Much of this control could be located by photogrammetric methods with sufficient accuracy for this work. However, this was not possible for the six photographs in the Malinta area. For this, an additional ground survey was required. On 28-29 July 1953, Professor Young and other personnel of this project made the necessary survey to tie in the needed control with the McClure test area. The evaluation of the photographs using Grelton area was completed in August 1953. The photographs in the Grelton area required some additional new control. This was put in by photogrammetric means with sufficient accuracy for this purpose. The work on these tests was completed during September, 1953, and the results sent to the WADC. On photographs 1, 2, 3, 5, and 6, 27 ground objects were located in the horizontal and vertical positions by ground-surveying methods and used in the evaluation. Photograph 4 was not evaluated because it was isolated from the others and would have required considerable time to locate control for a single photograph.

A sidelight to these tests was a demonstration of the usefulness of control extension by photogrammetric means. Since other photographs in these tests, not mentioned above, lacked sufficient control, an analytic procedure was used together with a knowledge of the ground elevations obtained from a contour map of the McClure area (this area has extremely low relief). By testing with known controls, the accuracy of this method was found to be less than one foot in horizontal position. Due to the unexpectedly good results obtained here, a further investigation of this method was undertaken.

B-17 AFAC TEST

On 19 November 1953, Mr. H. Kroeger and Mr. F. Wazelt from Wright Field visited this project and brought a series of flights over the new Eglin Field photogrammetric range as well as coordinates of the control points. Preliminary investigation showed some discrepancies in the reduced ground coordinates of the control points as compared with the USCGS coordinates. Also, most of the photographs in the runs showed extensive areas of poor focus, which was probably due to vacuum failure. This being the case, very accurate results from these runs were not possible. Even if there had been no vacuum failure, the poor focus rendered the targets invisible and, consequently, useless for accurate work. This condition also made it difficult to evaluate the usefulness of the target pattern. The highest of the runs received indicated that, up to this altitude (about 26,000 feet), the targets were usable and showed a visible pattern. However, for greater altitudes, this need not be true. From the point of view of lens-film resolution, 30-ft targets with 10-ft sections are no better than any one of the 10-ft sections. The actual usefulness of the pattern was mainly in identification. To take full advantage of the large targets, they should be a solid color that would give high contrast with the adjacent terrain. It is also of some advantage in measuring to have uniform symmetry in shape. Circular targets best satisfy this condition, although the square targets installed were satisfactory in this respect. The first two runs were evaluated by hand computation in December, 1953. In Flight 29 only two solutions could be worked from each photograph while, in Flight 50, three solutions were worked from each photograph. The results were as follows:

Flight 26 (two solutions each)
Mean PEM in minutes of arc
Pitch 2.23
Roll 2.24

Flight 50, R-1-W (three solutions each)
Mean PEM in minutes of arc
Pitch 3.73
Roll 3.10

In addition, one Release photograph was evaluated. The PEM for three solutions in the ground nadir point was 46.54 ft in one component and 23.23 ft in the other. In altitude the PEM was 15.96 ft and in the azimuth of the heading it was 5.04 seconds of arc.

All these runs showed varying amounts of distortion, which was probably due to vacuum failure. Because of this, the above accuracy level was about that of the remainder of this series of tests.

The remaining runs were computed by MIDAC during February, 1954, although considerably more difficulty than expected was encountered in the programming of MIDAC. MIDAC makes the complete computation, including all six orientation elements, in about two seconds. When only the tilt components are desired, the computation time is less than one second. However, the computation time is only one part of the total machine time, most of which is used in printing out the result. In practice, MIDAC runs the six-element computation in about 25 seconds and the tilt-component solution alone in about 15 seconds. As the computation time with CPC-2 was already a very small part of the total evaluation time, no great improvement was achieved in this respect. However, the problem as programmed with MIDAC is much more complex, giving a precise solution for any relief and printing out only the result desired. An equivalent solution using CPC-2 would take about 10 minutes. The use of computations by MIDAC not only increases the general application of photogrammetric methods, but also greatly reduces calculation time over other automatic computers and, hence, reduces the overall expense.

During the period of 15-23 March 1954, Mr. J. E. McGaughy, Mr. Fred Davis, and Mr. Kenneth Woolsey, from Eglin Air Force Base, Florida, visited this project. At that time, the entire reduction process was covered with special emphasis on computation procedures. These gentlemen took a copy of the B-17 AFAC Tests with them as well as the negatives used in that series of tests. Formal copies were sent to them when the work on this series was completed. Copies of the results of these tests were also sent to the WADC.

The question of a possible systematic error between the photogrammetric results obtained at this project and the independent results obtained by personnel at Eglin Air Force Base was raised by the latter in April, 1954. This amounted to about 95 ft in the ground nadir point. The discrepancy was studied and resolved and the information given to Eglin Air Force Base.

The B-17 AFAC tests gave the opportunity to test the determination of heading under actual conditions. Heading is not one of the standard photogrammetric terms and it was found necessary, in connection with the B-17 AFAC tests, to define this quantity as well as to find a method for its computation. We defined heading as the azimuth measured from the north, in the direction of flight of the orthogonal projection of that fiducial axis, parallel to the axis of the aircraft, measured about the ground nadir point. This, of course, assumed that one fiducial axis had been so positioned that it paralleled the heading axis of the aircraft. An orthogonal projection was used as azimuth and is measured in a horizontal plane only.

It was found that in a four-point solution the four values did not exactly agree. This was true because four points form an overdetermination of the plane of the photograph. Actually, a four-point solution arrived at a mean position of the plane for any four given points. The final value for α_H is the mean of the four calculated values. As the computation is not very sensitive to the exact value of tilt, there should be very small errors between individual problems on one photograph. Such was the case in the Release photograph evaluated where the PEM was only 5.04 seconds of arc. Therefore, although in the B-17-AFAC test series of photographs the tilt accuracy was very poor, the accuracy in heading was quite high. In all, fourteen photographs were evaluated for heading. From these, the mean PEM for three solutions was 37.8 seconds of arc, corresponding to a mean tilt error of about six minutes 32.4 seconds of arc. This bore out very well the theoretical contention that was sent to the WADC in December, 1953.

DISCUSSIONS WITH BOEING AIRCRAFT COMPANY

From 17-20 November 1953, Mr. Warder, Mr. Houser, and Mr. Gregory, from Boeing Aircaaft in Wichita, Kansas, visited this project for the purpose of familiarizing themselves with photogrammetric-reduction processes of the type employed here for the various gyro tests. On 12 March 1954, Mr. Warder visited this project to discuss certain details pertaining to the evaluation process to be employed by that company.

On 18-20 May 1954, Mr. Schmidt, of this project, visited the Boeing Aircraft Company in Wichita, Kansas, for the purpose of consultation on the reduction procedures used by that company. The procedures were obtained from this project as mentioned above.

On 11 June 1954, a request for special information on the reduction procedure was received from Mr. George Warder of the Boeing Aircraft Company. The information requested was transmitted.

CONCLUDING REPORTS

Report on Photogrammetric Reduction.—A consolidated report on the reduction procedures used by this project was considered in June, 1954. Work immediately started on this report which, it is hoped, will save time and effort for those groups considering the possible use of this method. The technical report, "Some Methods in Photogrammetric Reduction for Exterior Orientation," by Eldon Schmidt, Engineering Research Institute, University of Michigan, October, 1954, was completed in October, 1954, and sent to the WADC. This report summarized briefly the methods and procedures used by this project in obtaining a complete photogrammetric reduction of an aerial photograph. A complete reduction must contain all six coordinates necessary to orient a rigid body (the camera) in space. The methods of obtaining each of these six coordinates are given along with an example. A discussion of photogrammetric errors and their effects on the above six elements is given with actual values as found in practice. The working procedures used at this project and the application of automatic computers to these procedures are also given.

Atmospheric Refraction.—In connection with the investigation requested on oblique photogrammetry, it was necessary to expand the refraction corrections up to angles including the horizon. Work began in August, 1954, on the evaluation of the necessary integral. A report of the results was to be made on the completion of this work. The calculations involved in this investigation proved quite cumbersome and time consuming; because of this, the possibility of using automatic computation was investigated. The computing work on this investigation was given to MIDAC in October, 1954, in order to speed up the work and to get the highest possible accuracy. The calculations by MIDAC were completed in December, 1954, and the results were prepared graphically. Also, an attempt was made to find a simple analytical expression giving these results as a function of altitude and elevation angle. Such an expression, if suitably accurate, would be more appropriate for automatic computation work. The work was temporarily interrupted by the move to new quarters but was resumed in March, 1955. A report, "Atmosphere Refraction for Oblique Photography," by Eldon Schmidt, was sent to the WADC in August, 1955, in the form of a supplement to the report, "Atmospheric Refraction," by John Rowley, 1951. J. C. Rowley's report was restricted to low-tilt photography where the light rays were inclined at no more than 45° from the vertical. For high tilt, this restriction cannot be made. The supplement extends the ideas to angles up to and including the horizon.

Control Extension by Analytic Methods.—During the work on the driftmeter tests in 1954, some work was done on control extension by photogrammetric means. This was a limited procedure involving the location of points within a well-controlled area. It would be of much interest to develop a method to extend both horizontal and vertical control into areas of no control by means of a strip of overlapping photographs starting over known control points. An investigation to develop procedures that would be suited to all tilts and restrict flying conditions as little as possible was started by this project in September, 1954. Preliminary work showed that there was at least one direct method of obtaining control extension by analytic methods that was well suited for automatic computation. With the use of fictitious photographs, it was demonstrated that the exact horizontal and vertical control could be obtained in strip flying from known control, subject to the condition that the aircraft must undergo no large changes in altitude during the interval between two adjacent exposures. The limits that these changes are subject to were not known at that time, but it was quite certain that a normal flight pattern is well within these limits. The nature of these limits as well as the effects of normal photogrammetric errors on the accuracy was the subject of further investigations. The method was tested by using photographs of the McClure range. In this way it was hoped to find the accuracy of the procedure under actual conditions. The investigation was also continued from a theoretical standpoint with fictitious photographs. This procedure, when proved practical, would constitute a considerable expansion of the possible use of analytic photogrammetry. A report on the method of control extension as developed by this project was sent to the WADC in June, 1955. This report gave only theoretical background since the complete investigation requires extensive calculations.

Toss-Bombing.—A report entitled, "Theoretical Toss-Bombing Study," by G. E. Hay, Department of Mathematics, The University of Michigan, was completed in September, 1955, and is being distributed.

Report on Camera Calibration and Absolute Check on Photogrammetric Tilt Determination.—A report on this subject was completed by Eldon Schmidt and Edward Young in August, 1955, and is summarized in the present report on page 56 and following.