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DESIGN OF FINNED TUBE CONDENSERS

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Finned tubes can usually be used to advantage in shell and tube condensing applications where the fin side condensing coefficient is lower than the tube side coefficient. Such a situation normally exists in most organic condensers. This is illustrated by the universal use of finned tubes for condensing refrigerants in the refrigeration industry. How to determine if finned tubes can be used to advantage in petroleum and petrochemical applications is the subject of this second article in a series on finned tube heat transfer.[1]

An understanding of how finned tubes perform in condensing applications is necessary in order to determine whether finned tubes should be specified. A number of papers have been published reporting the performance of finned tubes in condensing applications.[2-8] Nusselt [9] has presented theoretical relationships for condensing of saturated vapors on horizontal and vertical surfaces. These relationships may be applied to the horizontal tube portion (root section) and vertical fin section of a finned tube for predicting the mean coefficient on a horizontal finned tube. Nusselt's relationship for a single horizontal bare tube is:

$$h_r = 0.725 \left[\frac{k_f^3 \rho_f^2 g_c \lambda}{\mu_f D \Delta t_{cf}}\right]^{\frac{1}{4}} \qquad (1)$$

Nusselt's relationship for condensing on a vertical surface is:

$$h_f = 0.943 \left[\frac{k_f^3 \rho_f^2 g_c \lambda}{\mu_f L \Delta t_{cf}}\right]^{\frac{1}{4}} \qquad (2)$$

Equations 1 and 2 were derived by Nusselt by taking into consideration the forces acting on the condensate film, using the assumption that the flow of the condensate is laminar.

Beatty(10) developed a relationship involving an equivalent diameter for finned tubes which incorporates Equation 1 for the horizontal portion (root section) of the finned tube and Equation 2 for the vertical finned portion of the tube. The equivalent diameter for condensation is obtained in the following manner:

Let

$$q_{total} = h_o' A_{eq} \Delta t_{cf} = h_r A_r \Delta t_{cf} + h_f A_f \phi \Delta t_{cf} \qquad (3)$$

Where the subscripts r and f refer to the root and finned portion of the tube respectively and A_{eq} is given by Equation 13 of Reference 1.

Substitution of Equation 1 and Equation 2 into Equation 3 and solving for h_o' assuming that Δt_{cf} in Equation 1 is the same as Δt_{cf} in Equation 2 gives:

$$h_o' = 0.725 \left[\frac{k_f^3 \rho_f^2 g_c \lambda}{\mu_f \Delta t_{cf}}\right]^{\frac{1}{4}} \left(\frac{A_r}{A_{eq} D_r^{\frac{1}{4}}} + \frac{1.30 \phi A_f}{A_{eq} L^{\frac{1}{4}}}\right) \qquad (4)$$

A comparison of Equation 4 with Equation 1 indicates that the equivalent diameter of a finned tube is:

$$\left(\frac{1}{D'_{eq}}\right)^{\frac{1}{4}} = \frac{A_r}{A_{eq}\ D_r^{\frac{1}{4}}} + 1.30\ \frac{\phi\ A_f}{A_{eq}\ L^{\frac{1}{4}}} \qquad (5)$$

where

$$L = \frac{\text{area of one side of one fin}}{\text{diameter of fin}} \qquad (6)$$

Therefore the temperature difference form of Beatty's equation for a single finned tube may be written as:

$$h'_o = 0.725 \left[\frac{k_f^3\ \rho_f^2\ g_c\ \lambda}{\mu_f\ \Delta\ t_{cf}\ D'_{eq}}\right]^{\frac{1}{4}} \qquad (7)$$

In deriving Equation 7 Beatty assumed that the temperature drop across the condensate film on the fin was the same as the temperature drop across the condensate film on the root portion of the tube. A more precise relationship can be derived without making this assumption by allowing for the effect of the fin efficiency on the $\Delta\ t_{cf}$ over the fin portion of the tube. The resulting relationship for a single tube is:

$$h'_o = 0.725 \left[\frac{k_f^3\ \rho_f^2\ g_c\ \lambda}{\mu_f\ \Delta\ t_{cf}}\right]^{\frac{1}{4}} \left(\frac{A_r}{A_{eq}\ D_r^{\frac{1}{4}}} + \frac{1.30\ \phi^{\frac{3}{4}}\ A_f}{A_{eq}\ L^{\frac{1}{4}}}\right) \qquad (8)$$

where

$\Delta\ t_{cf}$ = the temperature drop across the condensate film in the root portion of the tube.

A comparison of Equation 8 with Equation 4 indicates that the fin efficiency ϕ appears in Equation 8 to the 3/4 power and in Equation 4 to the first power. The $\Delta\, t_{cf}$ of Equation 4 is given by:

$$\Delta\, t_{cf} = \left(\frac{U_o}{h_o'}\right) (\Delta t)_m \qquad (9)$$

whereas the $\Delta\, t_{cf}$ of Equation 8 is given by:

$$\Delta\, t_{cf} = \left(\frac{U_r}{h_r}\right) (\Delta t)_m \qquad (10)$$

where

U_r = the overall coefficient for the root portion of the finned tube,

h_r = given by Equation 1.

In normal organic condensing applications involving low finned tubes the fin efficiency is close to 100% and the use of Equation 8 is not warrented. If low conductivity metal finned tubes are used with relatively high condensing coefficients the use of Equation 8 is recommended since it is more exact and gives slightly higher condensing coefficients. For a fin efficiency of 100% Equation 4 and Equation 8 give identical results. The use of Equation 4 is illustrated in the design of a condenser.

An alternative design procedure is to replace the temperature difference, $\Delta\, t_{cf}$, by a tube loading term, W_1. The necessary relationship is derived in the following manner. Raising both sides of Equation 7 to the fourth power:

$$(h_o')^4 = (0.725)^4 \left(\frac{k_f^{\,3}\ \rho_f^{\,2}\ g_c\ \lambda}{\mu_f\ \Delta\, t_{cf}\ D_{eq}'}\right) \qquad (11)$$

but by Equation 3

$$q = h'_o \; A_{eq} \, \Delta \, t_{cf}$$

and

$$q = W_1 \, \lambda \, L \qquad (12)$$

where

W_1 = pounds per hour per foot of tube length,

L = length of tubing (feet),

λ = latent heat of vaporization, btu per lb.

Substituting Equation 12 into Equation 3 and then substituting the result into Equation 11 and simplifying gives:

$$h'_o = 0.65\left(\frac{k_f^{\,3} \, \rho_f^{\,2} \, g_c \, A_{eq}}{\mu_f \, D'_{eq} \, W_1}\right)^{\frac{1}{3}} \qquad (13)$$

which is the tube loading form of Beatty's equation for finned tubes.

Relationships 7, 8, and 13 are for single tubes. As indicated earlier these relationships assume laminar flow of the condensate film. The theoretical correction factor for the influence of the number of tubes in a vertical row is given by Nusselt as $\left(\frac{1}{N}\right)^{\frac{1}{4}}$ for Equation 7 and 8 and $\left(\frac{1}{N}\right)^{\frac{1}{3}}$ for Equation 13 where N is the average number of tubes in a vertical row. The work of Katz,(5,7) Geist,(5) Short and Brown,(11) and Young and Wohlenberg(12) indicates that the correction factor of Nusselt is too severe and that a turbulence correction factor probably should be included in Nusselt's equations when applied to multitube units. The correction for turbulence C_n as given by Katz, Young, and Balekjian[7] can be included in Equation 7 and Equation 13 as follows:

$$h_o' = 0.725\left(\frac{C_n}{N^{\frac{1}{4}}}\right)\left[\frac{k_f^3\ \rho_f^2\ g_c\ \lambda}{\mu_f\ \Delta\ t_{cf}\ D_{eq}'}\right]^{\frac{1}{4}} \tag{7a}$$

$$h_o' = 0.65\left(\frac{C_n^{\frac{4}{3}}}{N^{\frac{1}{3}}}\right)\left[\frac{k_f^3\ \rho_f^2\ g_c\ A_{eq}}{\mu_f\ D_{eq}'\ W_1}\right]^{\frac{1}{3}} \tag{13a}$$

Figure 1 presents a plot of $\left(\frac{C_n}{N^{\frac{1}{4}}}\right)$ for condensation of Acetone, N-Butane and Freon 12, on six finned tubes in a vertical row from the data of Katz and Geist.[5] Figure 2 presents $\left(\frac{C_n^{\frac{4}{3}}}{N^{\frac{1}{3}}}\right)$ from the same data for use in the tube loading method. Examination of these figures indicates that the Nusselt correction factor $\left(\frac{1}{N}\right)^{\frac{1}{4}}$ is counterbalanced by a C_n value which for all practicable purposes results in a correction factor virtually independent of the number of tubes in a vertical row. This conclusion based on the data of Katz and Geist substantiates the findings of Short and Brown.[11] A C_n plot based on the data of Young and Wohlenberg[12] for five bare tubes in a vertical row is compared with the C_n curve for Freon 12 condensing on six finned tubes in a vertical row in Figure 3. This figure indicates that the C_n correction for bare tubes is the same as for finned tubes. The data of Young and Wohlenberg[12] supports the conclusion that the correction $\left(\frac{C_n}{N^{\frac{1}{4}}}\right)$ for plain tubes is also virtually independent of the number of tubes in a vertical row. It is recommended that an average value of $\left(\frac{C_n}{N^{\frac{1}{4}}}\right)$ be used in Equation 7a for tube bundles having an average number of tubes in a vertical row less than 10. The recommended values are tabulated in Table 1 for three condensing vapors.

Table 1

Fluid	Average values: $\left(\frac{C_n}{N^{\frac{1}{4}}}\right)$	$\left(\frac{C_n^{\frac{4}{3}}}{N^{\frac{1}{3}}}\right)$
Acetone	0.97	0.96
N-Butane	0.94	.92
Freon-12	0.80	.74

For bundles having more than 10 tubes (average) in a vertical row, it is recommended that the lines in Figures 1 and 2 be extrapolated. The average number of tubes in a vertical row for condensers having more than twenty tubes and single pass arrangements on the shell side can be estimated by the use of Equations 14 and 15:

$$N = 0.815\ (X)^{.52} \quad \text{for square pitch arrangement} \qquad (14)$$

$$N = 0.40\ (X)^{0.54} \quad \text{for triangular pitch arrangement} \qquad (15)$$

where

X = total number of tubes.

Economic Considerations

Industrial bare tube heat exchanger cost curves can be used for estimating finned tube heat exchanger costs by subtracting the cost of the bare tubing and adding the cost of the finned tubing. For example the current cost of a 3/4 inch - 19 fin per inch admiralty tube (.065 inch root wall) costs 20% more per pound or 40% more per foot than a 3/4 inch - 16 gage bare tube. Donohue,(13) Zimmerman and Lavine,(14) and Kern(15) have presented recent cost data on heat exchangers.

Example Design of a Debutanizer Overhead Condenser

Comparable designs of bare tube and finned tube condensers to condense 150,000 pounds per hour of a 20-80 mole per cent mixture of C_3 and N-C_4 at 160 psia are desired. Treated cooling tower water is available at 80°F and is not to exceed an outlet temperature of 120°F. Three-fourths inch O.D. admiralty tubes are to be used in the designs.

Table 2

Tube Specifications

Description	Bare Tube 3/4" - 18 BWG Admiralty	Finned Tube 19 ft/inch - 3/4" O.D. - 18 BWG Admiralty
O.D. inches	0.750	0.737
I.D., inches	0.652	0.541
D_{root} inches	-----	0.641
X = wall thickness, inches	0.049	0.050

Table 2 continued:

Description	Bare Tube	Finned Tube
A_o, sq. ft./ft.	0.1963	0.438
$\frac{A_o}{A_i}$	1.15	3.18
A_{cs}, ft.^{2}/tube	0.00232	0.001605

A. Preliminary Calculations.

1. From the equilibrium values for these compounds[16] the dew point and bubble point of the mixture were computed as:

$$\text{Dew point} = 166°F$$

$$\text{Bubble point} = 154°F$$

2. Assuming a linear temperature drop of the condensate with Q and the maximum temperature rise of the water, the mean temperature difference is:

$$\Delta T_{lm} = \frac{(154-80) - (166-120)}{\ln \frac{74}{46}} = 58.8°F$$

3. The change in enthalpy of the pure components is:[17]

$$\Delta H_{C_3} = 126 \text{ Btu/lb.}$$

$$Q_{C_3} = 126 \text{ Btu/lb} \times 24{,}750 \text{ lb/hr} = 3{,}120{,}000 \text{ Btu/hr.}$$

$$\Delta H_{C_4} = 137 \text{ Btu/lb.}$$

$$Q_{C_4} = 137 \text{ Btu/lb} \times 125{,}250 \text{ lb/hr} = 17{,}180{,}000 \text{ But/hr}$$

for a total heat duty of

$$Q = Q_{C_4} + TQ_{C_4} = 3.12\text{x}10^6 + 17.18\text{x}10^6 = 20.3\text{x}10^6 \text{ Btu/hr}$$

4. The water flow rate is:

$$W = \frac{20{,}300{,}000}{40} = 507{,}500 \text{ lb/hr}$$

or (assuming $\rho = 62 \text{ lb/ft}^3$)

2.27 cu. ft./sec.

B. Design of Finned Tube Condenser.

Assume a 27 inch ID shell containing 656 tubes ten feet long on a 15/16" triangular pitch each with two passes on the tube side.

1. Water flow rate and film resistance

$$A_{flow} = \frac{656}{2} \times 0.001605 = 0.526 \text{ ft.}^2$$

$$V_t = \frac{2.27}{0.526} = 4.31 \text{ ft/sec.}$$

$$V_t^{0.8} = (4.31)^{0.8} = 3.23$$

for water[19]

$$h_i = \frac{150\ (1+0.011t_\omega)\ V_t^{0.8}}{d_i^{0.2}}$$

$$= \frac{150\ (2.1)\ (3.23)}{(0.541)^{0.2}} = 1150 \text{ Btu/hr - °F - sq. ft.}$$

$$\frac{A_o}{A_i h_i} = \frac{3.18}{1150} = 0.00276$$

2. Metal Resistances.

The fin resistance from Table 2 of the first article(1) is:

$$r_f = 0.00011$$

The root metal resistance is:

$$r_m = \frac{X\ A_o}{K\ A_m} = \frac{(0.050)\ (0.438)}{(12)\ (65)\ (0.152)} = 0.00019$$

3. Fouling Resistances.

The outside fouling resistance (from TEMA(18)) is:

$$r_o' = 0.0005$$

The fouling resistance on the inside of the tube is:

$$\frac{A_o}{A_i} r_i = 0.001 \times 3.18 = 0.00318$$

4. Condensing Coefficient.

The condensing coefficient is computed from equation (7a):

$$h_o' = 0.725 \left(\frac{C_n}{N^{\frac{1}{4}}}\right) \left[\frac{k_f^{\,3}\ \rho_f^{\,2}\ g_c}{\mu_f}\right]^{\frac{1}{4}} \left(\frac{1}{D_{eq}'}\right)^{\frac{1}{4}} \left(\frac{1}{\Delta t_c}\right)^{\frac{1}{4}} (\lambda)^{\frac{1}{4}}$$

The Nusselt condensing physical property group, $\left[\frac{k_f^{\,3}\ \rho_f^{\,2}\ g_c}{\mu_f}\right]^{\frac{1}{4}}$

for the condensing mixture is given in Table 3. The $\left(\frac{C_n}{N^{\frac{1}{4}}}\right)$ value obtained from Table 1 or Figure 1 for condensing butane is 0.94. The equivalent condensing diameter, defined by Equation 5 is calculated for various outside resistances. The computed values for the finned tube given in Table 2 are tabulated below (see Table 4).

Table 3

Nusselt Physical Property Group of
Propane - Butane Mixture

Temperature °F	$\left[\frac{k_f^3 \rho_f^2 g_c}{\mu_f}\right]^{\frac{1}{4}}$
100	170
120	171
140	171.5
160	172

Table 4

Equivalent Diameter of Tube

$\left[\dfrac{1}{\dfrac{1}{h'_o} + r'_o}\right]$	$\left(\dfrac{1}{D'_{eq}}\right)^{\frac{1}{4}}$
0	3.53
400	3.52
800	3.49
1200	3.48
Average Value	3.50

Examination of Table 4 indicates that the equivalent condensing diameter of this tube is essentially constant with an average value of 3.50. Substituting in Equation 7a:

$$h'_o = 0.725\ (0.94)\ (3.50)\ (3.4)\ (171.5) \left(\frac{1}{\Delta t_c}\right)^{\frac{1}{4}}$$

$$= 1390 \times \left(\frac{1}{\Delta t_c}\right)^{\frac{1}{4}}$$

The condensing coefficient is first assumed to be:

$$h'_o = 600$$

$$\frac{1}{U_o} - \frac{1}{h'_o} = 0.00011 + 0.0005 + 0.00019$$

$$+ 0.00318 + 0.00276 = 0.00674$$

Substituting for $h'_o = 600$:

$$\frac{1}{U_o} = \frac{1}{600} + 0.00674 = 0.00841$$

$$U_o = 119.0$$

The temperature drop across the condensate film by Equation 9 is:

$$\Delta t_c = \frac{U_o}{h_o'}(\Delta T_m) = \frac{119}{600}(58.8) = 11.65°F$$

Checking the assumed h_o' of 600:

$$h_o' = 1390 \times \left(\frac{1}{11.65}\right)^{\frac{1}{4}} = 756$$

which does not check.

Second Trial

Assume $h_o' = 800$, then

$$\frac{1}{U_o} = \frac{1}{800} + 0.00674 = 125$$

$$\Delta t_c = \frac{125}{800} \times 58.8 = 9.19°F$$

$$\text{or } h_o' = 1390\left(\frac{1}{9.19}\right)^{\frac{1}{4}} = 800$$

which checks exactly the assumed value of 800.

Therefore

$$U_o = 125 \text{ and } \Delta T_m = 58.8°F$$

The required area is

$$A = \frac{Q}{U\Delta T_m} = \frac{20,300,000}{(125)\ (58.8)} = 2670 \text{ sq. ft. (external)}$$

The provided area is

$$A = 656 \times 10 \times 0.438 = 2870 \text{ sq. ft. external}$$

for an excess of

$$\% \text{ X S} = \frac{110}{2760} = 4\%$$

A plain tube condenser, designed using the same procedure, required a 35 inch shell containing 988 tubes ten feet long with six passes on the tube side. A summary and comparison of the plain and finned units are given in Table 5.

Table 5

Summary of Requirements

Item	Plain Tube	Finned Tube
Heat duty, btu/hr	20,300,000	20,300,000
Δt_m, °F	58.8	58.8
$T_{water\ in}$, °F	80	80
$T_{water\ out}$, °F	120	120
Water velocity, ft/sec	5.94	4.31
Outside fouling factor	.0005	.0005
Inside fouling factor	.001	.001
U_o, btu/hr-°F-sq. ft. (outside)	183	125
U_l, btu/hr-°F-ft. of length	35.9	54.7
Shell diameter, inches	35	27
No. of tubes	988	656
Tube length, ft.	10	10
No. tube passes	6	2
Required area, sq. ft.	1890	2760
Area provided, sq. ft.	1941	2870
%XS area, %	2.7	4.0
Exchanger cost	$12,000	$10,400
Savings, dollars	----	1600
Savings, %	----	13.3

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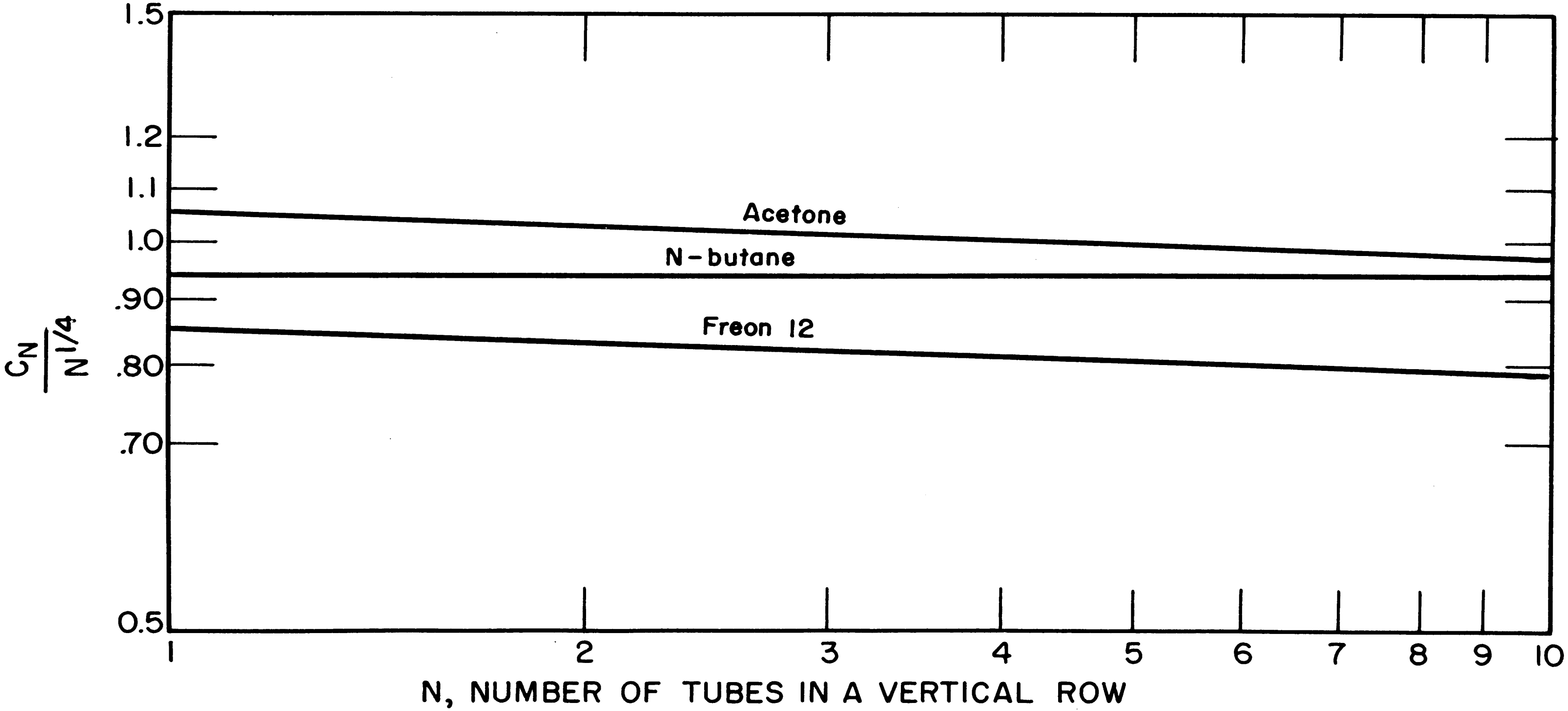
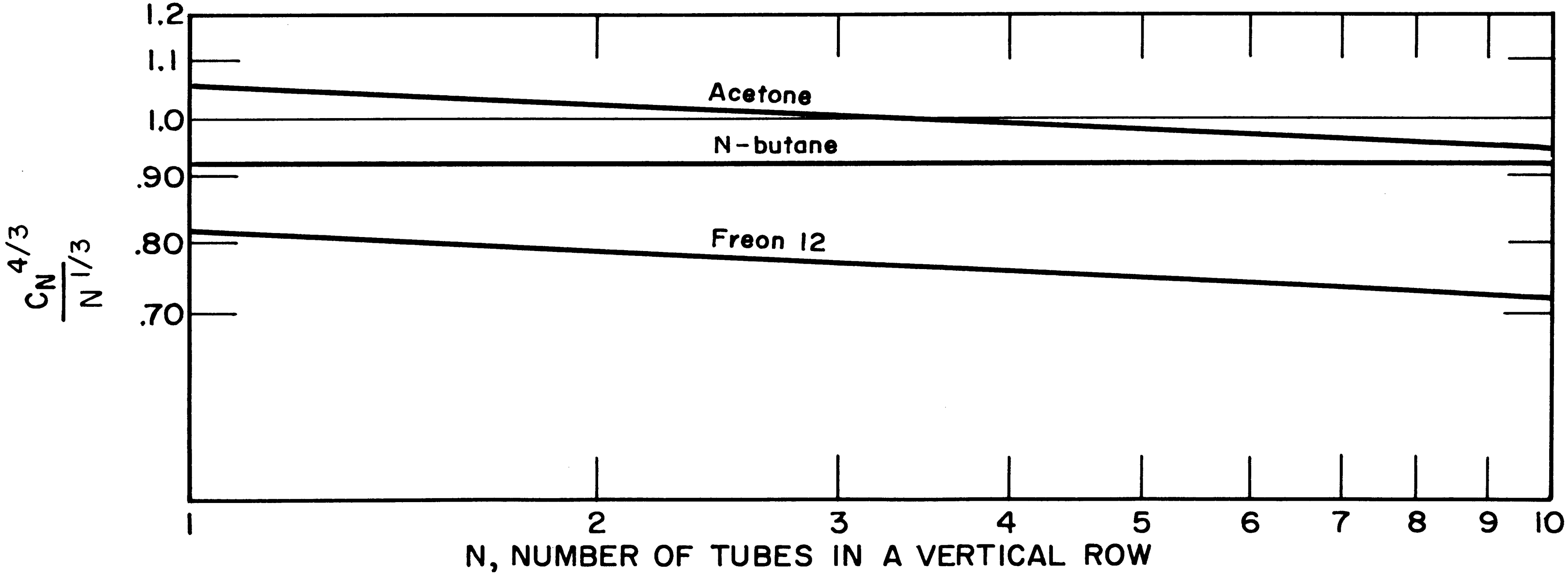
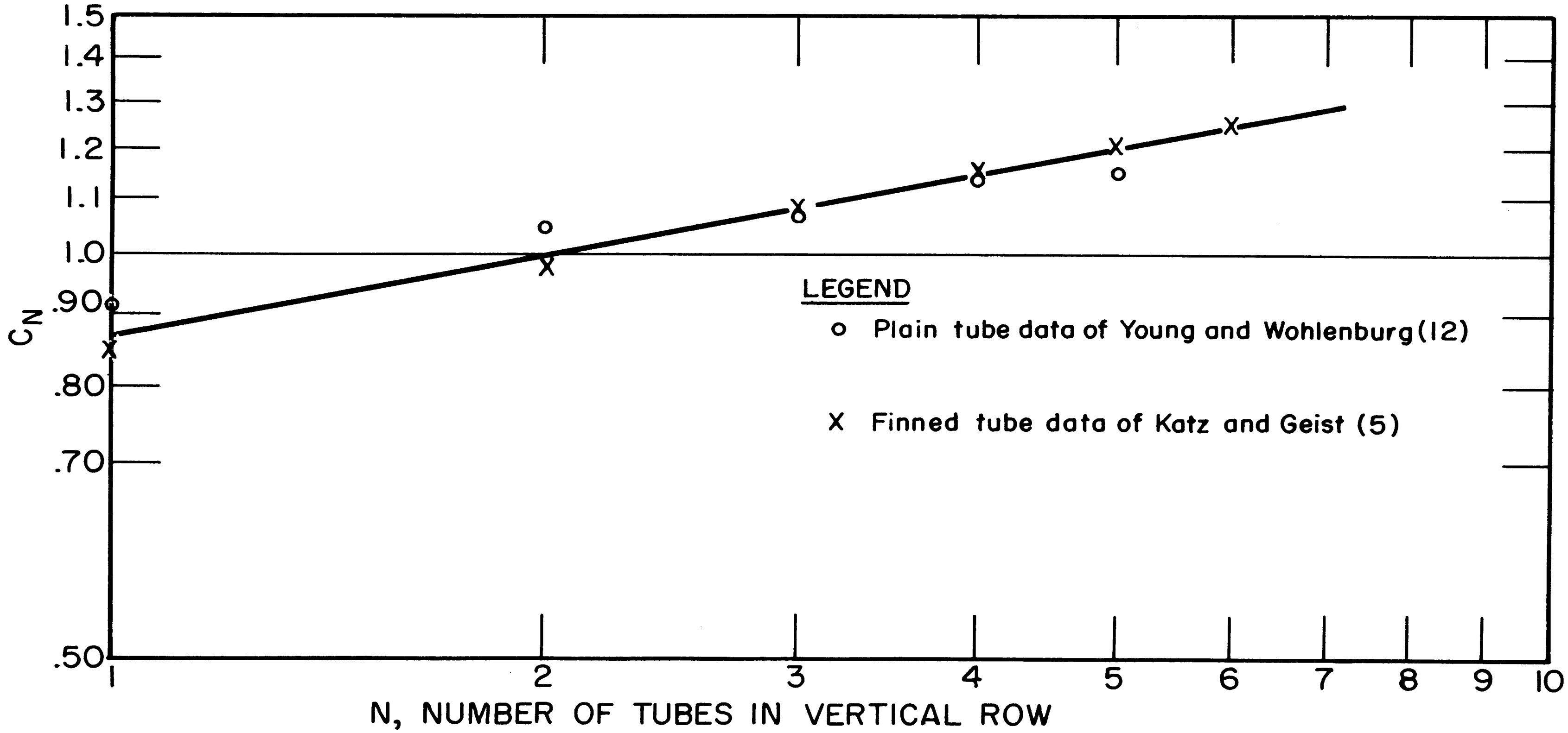


1. Correction for number of finned tubes in a vertical row for use with temperature difference method.



2. Correction for number of finned tubes in a vertical row for tube loading method.



3. Comparison of C_n values for Freon 12 condensing for plain and finned tubes.