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DESIGN OF FINNED-TUBE PARTIAL CONDENSERS

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Finned tubes can be used to definite advantage in shell-and-tube partial condenser applications. In vapor-gas cooler-condenser applications the gas film coefficient is usually of such a low magnitude that the added extended surface provided by finned tubes can be used to reduce the size of heat exchanger required. The economic savings are often quite considerable. The purpose of this article is to indicate how such units can be designed using existing finned-tube information which is available in the technical literature.

The design of condensers for the condensation of vapors from noncondensing gases is complicated by unusual conditions of heat transfer not encountered in total condenser design. In condensing part of the vapors in a gas stream, all of the properties of the gas stream vary greatly as the condensable vapor is removed. The heat-transfer coefficient of the gas film, the mass rate of gas flow, and the physical properties of the gas stream can change considerably as condensation proceeds. The condensation depends upon the diffusion of the vapor molecules through the gas mixture to the condensing surface. This involves two types of diffusion: (1) molecular diffusion and (2) eddy diffusion. Molecular diffusion involves the movement of individual molecules of condensable vapor from the main bulk stream onto the condensate film under the influence of a concentration (partial pressure) gradient. Eddy diffusion is the movement of groups of molecules of the bulk stream by turbulent motion to the condensate surface. The main resistance to condensation now occurs between the bulk stream of vapor and the surface of the condensate. Therefore, mass-transfer coefficients must be considered as well as heat-transfer coefficients in the mechanism of condensation rate.

The usual overall heat transfer relationship

$$Q = U_o A \Delta T_m \qquad (1)$$

does not directly apply in this case since no method of calculating mean-temperature differences based upon terminal conditions is applicable. An accurate method of determining such a ΔT_m has long been sought. The usual logarithmic mean of the terminal temperature differences is inapplicable.

The overall heat-transfer coefficient varies widely from point to point in a partial condenser, being high where the condensing vapors are relatively concentrated and low in zones where most of the condensable vapor has been removed. Thus no simple average heat-transfer coefficient is applicable.

The overall heat-transfer relationship for the heat transfer occuring in partial condensers must be written as:

$$dA = \frac{dq}{U\Delta t}$$

where

$\Delta t = (t_c - t_w)$

t_c = condensate-gas interface temperature, °F

t_w = tube wall temperature, °F

U = all resistances but the gas film (U comb. later on)

or

$$A = \int_0^q \frac{dq}{U\Delta t} \qquad (2)$$

In general, it is not possible to integrate this relationship formally using analytical expressions for both U and Δt as functions of q.

The method of Colburn and Hougen[1] is generally accepted as the basis for obtaining rigorous design of cooler-condensers. The method is tedious since it involves successive trial and error substitutions. A number of approximation methods have been published by Cairns[2], Bras[3,4], Mickley[5], and others.

The rate of transfer of sensible heat from the gas stream on the shell side of the exchanger to the outside of the tubes is given by:

$$\frac{dq_s}{dA} = h_o' (t_g - t_c) \qquad (3)$$

The rate of transfer of latent heat from the gas stream on the shell side of the exchanger to the fin side of the tubes due to mass transfer of condensible material to the tube surface is given by:

$$\frac{dq_L}{dA} = K_M(p_v - p_c) \qquad (4)$$

The total rate of heat transfer, given by the sum of (3) and (4) above, must be transferred through the tube and water film:

$$\frac{q}{A} = h_o'(t_g - t_c) + K_M(p_v - p_c) = U_{comb.} \; (t_c - t_w) = U\Delta t \qquad (5)$$

where:

h_o' = heat-transfer coefficient in gas film, $\frac{\text{Btu}}{\text{hr-ft}^2\text{-°F}}$

t_g = temperature of main bulk of the gas vapor mixture, °F

t_c = temperature of condensate at gas-condensate interface, °F

K_M = mass-transfer coefficient, $\frac{\text{lb}}{(\text{hr-ft}^2) \text{ per unit pressure}}$

λ = latent heat of condensation, Btu/lb

p_v = partial pressure of vapor in main bulk of the gas vapor mixture, $\frac{\text{lb}}{\text{sq. in.}}$

p_c = vapor pressure at t_c, lb/sq. in.

t_w = water temperature, °F

$U_{comb.}$ = combined conductances other than the gas film

$$= \frac{1}{\frac{1}{h_{cond.}} + r_o' + r_f + r_m\left(\frac{A_o}{A_m}\right) + \left(\frac{A_o}{A_i}\right) r_i + \frac{A_o}{A_i h_i}} \qquad (6)$$

where:

$h_{cond.}$ = condensing film coefficient.

Other terms defined by Eq. (20) of Part I[9].

The gas film heat transfer and mass transfer coefficients (h_o' and K respectively) are obtained in the following manner. The fin side gas film heat transfer coefficient is obtained from Eq. (1) or Eq. (2) of the third article of this series[6]. The relationship for 19 fin-per-inch tubes in unbored shells is:

$$\left(\frac{h_o' De}{k}\right) = 0.155 \left(\frac{DeG}{\mu}\right)^{0.6} \left(\frac{C_p\mu}{k}\right)^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14} \qquad (7)$$

The fin side gas film mass transfer coefficient is obtained using the heat transfer coefficient obtained from Eq. (7) and the "j" factor relationships for heat and mass transfer. The heat transfer "j" factor is defined as:

$$j_h = \frac{h_o'}{C_p G} \left(\frac{C_p\mu}{k}\right)^{2/3} \left(\frac{\mu_w}{\mu}\right)^{0.14} \qquad (8)$$

The mass transfer "j" factor is defined as:

$$j_m = \frac{K_M M_M P_{gf}}{G\ M_v} \left(\frac{\mu}{\rho D_v}\right)^{2/3} \qquad (9)$$

Equating Eqs. (8) and (9) and simplifying, the mass transfer coefficient is obtained as:

$$K_M = \frac{h_o' M_v}{C_p M_M P_{gf}} \left(\frac{C_p \rho D_v}{k}\right)^{2/3} \left(\frac{\mu_w}{\mu}\right)^{0.14} \qquad (10)$$

where:

j_h = heat-transfer "j" factor (dimensionless)

h_o' = gas-film heat-transfer coefficient, $\frac{\text{Btu}}{\text{hr-ft}^2\text{-°F}}$

C_p = specific heat at constant pressure, $\frac{\text{Bru}}{\text{lb - °F}}$

G = mass velocity, $\frac{\text{lb}}{\text{hr - ft}^2}$ of bulk stream

μ = viscosity, $\frac{\text{lb}}{\text{ft - hr}}$

k = thermal conductivity, $\frac{\text{Btu}}{\text{hr-ft}^2\text{-°F/ft}}$

j_M = mass-transfer "j" factor (dimensionless)

K_M = mass-transfer coefficient, $\frac{\text{lb}}{\text{hr-ft}^2 \text{ per unit of pressure}}$

M_M = molecular weight of gas-vapor mixture (average)

M_v = molecular weight of vapor

P_{gf} = log mean partial pressure of noncondensable gas across the film, lb/sq in.

$$P_{gf} = \left[\frac{(P_g)\text{at } tg - (P_g) \text{ at } tc}{\ln \dfrac{(P_g) \text{ at } tg}{(P_g) \text{ at } tc}}\right]$$

P_g = partial pressure of the noncondensable gas in the main body, lb/sq in.

ρ = vapor density, lb/cu ft

D_v = diffusion coefficient, sq ft/hr

$$D_v = 0.0166 \left[\frac{T^{3/2}}{P(V_A^{1/3} + V_B^{1/3})^2}\right] \sqrt{\frac{1}{M_A} + \frac{1}{M_B}}$$

where:

T = absolute temperature, °Kelvin

for T = °Rankine, change constant to 0.0069

P = pressure, atmosphere

V_A, V_B = molecular volumes[7]

M_A, M_B = molecular weights of gasses and condensing vapors

The combined coefficient, $U_{comb.}$, defined for finned tubes by Eq. 6 is determined in the following manner. The condensing coefficient, $h_{cond.}$, can be determined by the methods given in the second article of this series[8]. The inside and outside fouling factors can be determined in the usual manner. The fin resistance of the tube can be obtained from Table 2 or Fig. 5 of the first article of this series[9]. The inside coefficient and metal resistance are determined in the usual manner. The condensing and water film coefficients vary from the inlet end to discharge end of the exchanger. These resistances are usually small in comparison to the gas film resistance and the variation can ordinarily be neglected.

To obtain an exact design exact design it is necesaary to graphically solve Eq. 2 for the heat transfer area. Partial condensers can have either a saturated vapor-gas feed or superheated vapor-gas feed. A different approach to the problem must be taken for the two cases.

A. DESIGN OF PARTIAL CONDENSERS WITH SATURATED FEED

For the design of partial condensers in which the saturated vapor-gas mixture enters and condensation proceeds along the saturation line without superheating or supercooling, the method of Colburn and Hougen[1] is applicable. Figure 1 shows the cooling-condensing path followed by a saturated feed stream through the heat exchanger. The heat exchanger is broken up into a series of temperature drop zones as indicated in Fig. 1. The total heat transferred in each zone is computed, sensible plus condensing load. The corresponding water temperatures at these zones are obtained from a heat balance. Equation 7 is solved for each zone by a trial and error procedure on t_c. A plot is then made of $\frac{1}{U_{comb.}\Delta t}$ versus q and the area under the curve is obtained to give the required heat transfer area.

B. DESIGN OF PARTIAL CONDENSERS WITH SUPERHEATED FEED

Figure 2 shows the cooling-condensing path followed by a superheated feed stream through the heat exchanger. The problem now consists of determining this path. The most convenient method to use is that of Bras[3] which involves the use of the following relationship.

$$\frac{dp_v}{dt} = \left(\frac{P - p_v}{P_{gf}}\right)\left(\frac{C_p\ \rho\ D_v}{k}\right)^{2/3}\left(\frac{\Delta p}{\Delta t}\right) \qquad (11)$$

in which

$\Delta p = p_v - p_c$, see Fig. 3.

and $\Delta t = t_g - t_c$, see Fig. 3.

Procedure for the design of partial condenser with superheated vapors in an unsaturated condition:

(1) Composition of vapor-gas mixture entering the unit must be known.

(2) Entering temperature and pressure entering the unit must be known.

(3) The partial pressure of the condensible vapor is determined from the composition.

(4) Heat-transfer and mass-transfer coefficients can be calculated by use of Eqs. 7 and 10 (except that P_{gf}, the log mean partial pressure of the noncondensable gas across the film lb/in^2 is unknown). Trial-and-error solution of Eq. 5 is required giving the temperature and vapor pressure at the condensate layer.

(5) $\therefore \Delta t/\Delta p$ can be evaluated and dp/dt calculated from Eq. 11.

(6) Straight lines are drawn with slope of dp/dt and followed for short distances and the process repeated until the outlet gas temperature is reached as indicated in Fig. 3.

(7) About 5 to 10 points are usually sufficient.

(8) The solving of Eq. 5 from point to point gives $U\Delta t$ values for a graphical solution of Eq. 2 as indicated in Fig. 5.

C. APPROXIMATION METHODS

The simplest approximation method is to evaluate the terminal overall coefficients and use an average overall coefficient in conjunction with the logarithmic mean temperature difference. The accuracy of the method rapidly decreases as the difference between the inlet and outlet overall coefficients increases. Also the use of the logarithmic mean temperature difference implies assumptions which are usually not valid for partial condensers.

A more reliable method consists of multiplying a mean gas film heat transfer coefficient by the ratio of the total to sensible heat transfer for the unit as indicated by Carrier and Anderson[10].

$$h = h_o' \left(\frac{q_s + q_L}{q_s} \right) \qquad (12)$$

The outside weighted coefficient, h, is then used in Eq. 20 of the first article[9] in place of h_o'. This overall coefficient can be used with an integrated temperature difference as indicated by Gilmour[11] to obtain the required area.

Cairns has presented a method of formally integrating Eq. 2 if the $U\Delta t$ versus q curve is of parabolic shape[2]. The required heat transfer area is given by:

$$A = \int_{-q_T/2}^{+q_T/2} \frac{dq}{(aq^2 + bq + c)} \qquad (13)$$

$$a = \frac{2\left[(U\Delta t)_2 + (U\Delta t)_1 - 2(U\Delta t)_{mid}\right]}{q_T^{\,2}}$$

$(U\Delta t)_1$ = heat flux at the gas-vapor outlet

$(U\Delta t)_{mid}$ = heat flux at a point in the cooler-condenser where half the total heat has been transferred.

$(U\Delta t)_2$ = heat flux at the gas-vapor inlet

Equation 13 has three solutions;

when $b^2 = 4ac$

$$A = \left[\frac{-2}{aq + b}\right]_{-q_T}^{+q_T} \tag{14}$$

when $b^2 > 4ac$

$$A = \left[\frac{1}{\sqrt{b^2 - 4ac}} \ln\left(\frac{aq + b - \sqrt{b^2 - 4ac}}{aq + b + \sqrt{b^2 - 4ac}}\right)\right]_{-q_T}^{+q_T} \tag{15}$$

for $b^2 < 4ac$

$$A = \left[\frac{2}{\sqrt{4ac - b^2}} \tan^{-1}\left(\frac{\sqrt{aq + b}}{\sqrt{4ac - b^2}}\right)\right]_{-q_T}^{+q_T} \tag{16}$$

As indicated above, this method requires knowing the values of $(U\Delta t)$ at each end of the cooler-condenser and at a point in the cooler-condenser where half the total heat removed has been transferred. It is then possible to calculate the values of the constants a, b, and c in Eq. 13. The appropriate integrated equation (14, 15, or 16) is then solved depending on whether $b^2 \gtreqless 4ac$. The required area for the conditions specified is thus obtained. The only limitation on this method is that $(U\Delta t)$ when plotted against q must approach a part of a parabola.

Another approximation method also based upon the assumption of a parabolic curve of $(U\Delta t)$ versus q is that involving the use of the graph by Carey and Williamson[12]. The graph gives a factor f which is used to multiply the mid-point heat flux $(U\Delta t)_{mid}$ to obtain the mean heat flux.

An alternate graphical procedure involving polar diagrams has been presented by Bras[4]. The method is limited to the Colburn and Hougen (saturated gas-vapor mixture feed) method[1]. The method gives the exact solution and does not require a parabolic q versus $(U\Delta t)$ curve. The tedious successive approximation computations of the Colburn-Hougen method are greatly reduced by the graphical procedure using the polar diagram.

D. GAS PHASE CONDENSATION

The previous discussions are based on the assumption of surface condensation, i.e., the condensation of the condensable vapor occurs on the surface of the bare or finned tube. The graphical integration method of Colburn and Hougen assumes that the heat transfer and mass transfer mechanisms operate without interaction. As the gas-vapor mixture flows past the cooling surface, mass and sensible heat are transferred at rates proportional to their respective driving forces. In some systems, particularly those involving high molecular weight vapors and low diffusivity, mass transfer is slower than heat transfer. In such a case either supersaturation or condensation in the gas phase then follows.

Schuler and Abell[15] have presented an interesting and useful design method involving a vapor phase condensation correction factor to allow for gas phase condensation or "fog formation". No attempt will be made to review the method here. The reference should be consulted by those encountering the problem in the design of cooler-condensers in which fog formation is expected to occur.

EXAMPLE OF PARTIAL CONDENSER DESIGN INVOLVING SUPERHEATED FEED

A. STATEMENT OF DESIGN PROBLEM

A 50-50 weight percent mixture of ethane and pentane at a total pressure of 160 psia and 225°F is to be cooled at 100°F in shell-and-tube heat exchanger at the rate of 150,000 lb per hour. Treated cooling tower water is available at 90°F and is to be heated to a temperature not to exceed 110°F. Comparable designs of finned-tube units and plain tube units will be provided. The tubes are to be 3/4-inch-OD Admiralty tubes, 14 BWG at the ends.

B. TUBE SPECIFICATIONS

1. Finned-Tube Characteristics

14 BWG Plain End Tube
Trufin 195065-26 Admiralty tube
$N = 19$ fins/in.
$d_o = 0.737$ in.
$d_r = 0.640$ in.
wall thickness = 0.065 in.
mean fin thickness = 0.016 in.
$d_i = 0.510$ in.
$A_o = 0.438$ ft^2/ft
$A_i = 0.1336$ ft^2/ft
$A_o/A_i = 3.28$

$$A_{cs} = \frac{(\pi)(0.510)^2}{(576)} = 0.001418 \text{ ft}^2/\text{tube}$$

$D_e = 0.0559$ ft (for fluid flow)

2. Plain Tube Characteristics

14 BWG Tube
OD = 0.750 in.
ID = 0.584 in.
wall thickness = 0.083 in.

$A_o = 0.1963$ sq. ft/ft
$A_i = 0.153$ sq. ft/ft

$$\frac{A_o}{A_i} = 1.285$$

$$A_{cs} = \frac{\pi(0.584)^2}{576} = 0.00186 \text{ ft}^2\text{/tube}$$

FINNED TUBE UNIT

C. PRELIMINARY HEAT-EXCHANGER ARRANGEMENT

Assume a 43-inch-ID shell containing 1300 tubes on a 1 in.-45° square pitch with a single pass counter-current flow on shell-and-tube sides. The shell side has baffles with 25 per cent window cut on the diameter and spaced at 2.5 ft intervals. The required length of the tubes is to be determined.

D. SAMPLE CALCULATIONS FOR FINNED-TUBE UNIT

1. Flow Areas

Cross-flow areas

$$\text{Number of tubes on centerline} = \left(\frac{\text{shell diameter}}{\text{pitch } \sqrt{2}} - 1\right)$$

$$= \left(\frac{43}{1.414} - 1\right) = 29$$

Number of tubes in flow path

= 2(29) + 1 = 59 (on zig-zag path)

Number of spaces for flow

= 58 plus the ends (due to floating head backing plates)

Cross-flow face area

$$= \left(\frac{1}{12}\right)(2.5)\left[58 + 2\right] = 12.5 \text{ ft}^2$$

Root metal projected area

$$= (D_r)(L)(\text{No. of tubes}) = \left(\frac{0.64}{12}\right)(2.5)(59) = 7.86 \text{ ft}^2$$

Fin metal projected area

$$= (\text{Fin thickness})(D_o - D_r)(\text{fins/ft})(\text{No. of tubes})(L)$$

$$= \left(\frac{0.016}{12}\right)\left(\frac{0.737 - 0.64}{12}\right)(19)(12)(59)(2.5) = 0.362 \text{ ft}^2$$

$$\text{Free cross-flow area} = 12.5 - 7.86 - 0.36 = 4.28 \text{ ft}^2$$

Total cross-sectional area of shell

$$= \frac{\pi(43)^2}{576} = 10.05 \text{ ft}^2$$

Window area[13] $= (0.15355)\dfrac{(43)^2}{144} = 1.97 \text{ ft}^2$

$$\text{Number of tubes in window area} = \frac{1.97}{10.05}(1300) = 255 \text{ tubes}$$

Free flow area in baffle window =

$$1.97 - 255\left[\frac{(0.737)^2}{576}\pi\right] = 1.97 - 0.75 = 1.22 \text{ ft}^2$$

$$\text{Geometric mean area} = \sqrt{(1.22)(4.28)} = 2.30 \text{ ft}^2$$

2. Calculation of Heat Load

Assume that the partial pressure of pentane leaving the exchanger will equal the vapor pressure. Then the partial pressure of pentane in the exit gas stream is 15.0 psi (from vapor pressure curve, see Fig. 4). Therefore, the mole fraction of pentane in the exit stream is $\left(\frac{15}{160}\right) = 0.0938$. Assuming that no ethane condenses, moles per hour of ethane in exit stream $= \left(\frac{75{,}000}{30}\right) =$ 2,500. Therefore, moles per hour of pentane vapor in exit gas stream $= \dfrac{2{,}500(0.0938)}{(1.000 - 0.0938)} = 258.$

Lb/hr of pentane vapor leaving = (258)(72) = 18,600 lb/hr.

Lb/hr of pentane liquid leaving = (75,000 - 18,600) = 56,400 lb/hr.

Lb/hr of ethane gas leaving = 75,000 lb/hr.

The path selected for computing the heat removed was to condense the pentane at 225°F, cool the condensate to 100°F, and cool the gases from 225°F to 100°F.

Heat Removed:

Condensing = (56,400)(113) = 6,370,000 Btu/hr

Condensate cooling = (56,400)(0.60)(225-100) = 4,220,000 Btu/hr

Cooling pentane vapor = (18,600)(0.425)(225-100) = 990,000 Btu/hr

Cooling ethane vapor = (75,000)(0.46)(225-100) = 4,310,000 Btu/hr

Total Q = 15,890,000 Btu/hr

3. Determination of Water Requirements

Temp rise of water = (110-90) = 20°F

$$\text{Lb water/hr} = \frac{15,890,000}{20} = 794,500 \text{ lb/hr}$$

$$\text{Ft}^3\text{/hr of water} = \frac{794,500}{62} = 12,800 \text{ ft}^3\text{/hr}$$

4. Water Velocity in Tubes

$$A_{cs} = 0.001418 \text{ ft}^2\text{/tube}$$

Tube side-flow area = (1300)(0.001418) = 1.84 ft^2

Velocity inside tubes at avg water conditions =

$$\frac{12,800}{(3600)(1.84)} = 1.93 \text{ ft/sec}$$

5. Determination of Resistances to Heat Transfer

(a) Fouling, inside = 0.001 (from TEMA)[17]

(b) Fouling, outside = 0.0005 (from TEMA)[17]

(c) Metal Resistance $= \frac{X}{K}\left(\frac{A_o}{A_m}\right)$

$$A_m = \pi\left[\frac{Dr - Di}{\ln\frac{Dr}{Di}}\right] = 3.14\,\frac{(0.0534 - 0.0425)}{\ln\frac{0.0534}{0.0425}} = 0.1535 \text{ ft}^2\text{/ft length.}$$

$$r_m = \frac{(0.065)(0.0438)}{(12)(64)(0.1535)} = 0.00024\,\frac{\text{hr-°F-ft}^2\text{(outside)}}{\text{Btu}}$$

(d) Water side coefficient[14]:

$$h_i = 150\,(1 + 0.011\,t_w)\frac{V_t^{\,0.8}}{d_i^{\,0.2}}$$

where:

V_t = velocity, ft/sec

d_i = inside diameter of tubes, in.

t_w = average water temperature, °F

$$\therefore h_i = \frac{150(2.10)(1.69)}{0.874} = 610\,\frac{\text{hr-°F-ft}^2\text{(inside)}}{\text{Btu}}$$

(e) Condensing Coefficient

The condensing coefficient will vary throughout the exchanger, being lowest at the inlet, where there is a large temperature difference driving force for heat transfer, and highest at the outlet. Often the coefficient may be assumed constant, due to the controlling nature of the gas-film resistances. A calculation is included here to illustrate the procedure.

The determination of the condensing coefficient involves a trial-and-error computation which can be solved only when the gas-condensate interface temperature is known. Since the ΔT is large in this inlet section, assume $h_c = 600$ Btu/hr-°F-ft². This is checked after establishing t_c.

(f) Fin Resistance

For this tube, $r_f = 0.00011$ (see Table 2 or Fig. 5 of Ref. 9).

(g) Determination of (U_{comb}) in Eq. 5.

$$U_{comb.} = \frac{1}{\dfrac{A_o}{A_i h_i} + r_i \dfrac{A_o}{A_i} + r_m + r_f + r_o' + \dfrac{1}{h_{cond.}}}$$

$$U_{comb.} = \frac{1}{\dfrac{3.28}{610} + 0.00328 + 0.00024 + 0.00011 + 0.0005 + \dfrac{1}{600}}$$

$$= \frac{1}{0.01118} = 89.5 \frac{\text{Btu}}{\text{hr-°F-ft}^2\text{(outside area)}}$$

(h) Determination of Gas-Film Coefficient

Reference is made to Eq. 7.

$$\frac{h_o' De}{k} = 0.155 \left(\frac{DeG}{\mu}\right)^{0.6} \left(\frac{C_p \mu}{k}\right)^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14}$$

The physical properties of the gas-vapor mixture are:

$C_p \;=\; 0.478$

$\mu \;=\; 0.0245$

$k \;=\; 0.01445$

Substituting:

$$= 0.155 \left(\frac{0.0559 \times 150{,}000}{2.30 \times 0.0245}\right)^{0.6} \left(\frac{0.478 \times 0.0245}{0.01445}\right)^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14}$$

assuming $\left(\dfrac{\mu}{\mu_w}\right)^{0.14} = 1;$

$$h_o' = \frac{0.01445}{0.0559} \cdot (183.5)$$

$$= 47.6$$

(i) Determination of Mass Transfer Coefficient

From Eq. 10:

$$K_M = \frac{h_o' \, M_v}{C_p \, M_M \, P_{gf}} \left(\frac{C_p \, \rho \, D_v}{k}\right)^{2/3} \left(\frac{\mu_w}{\mu}\right)^{0.14}$$

It is assumed that $\left(\dfrac{\mu_w}{\mu}\right) = 1$

Substituting in the equation for D_v:

$$D_v = \frac{(0.0069)(18{,}000)}{(10.9)(8.64)^2} \sqrt{\frac{1}{30} + \frac{1}{72}} = 0.0332 \text{ ft}^2/\text{hr}$$

The density is computed to be:

$\rho \;=\; 0.961 \text{ lb/ft}^{-3}$

$M_M \;=\; 30(0.706) + 72(0.294) \;=\; 42.4 \text{ lb/lb-mole}$

$M_v \;=\; 72 \text{ lb/lb-mole}$

Substituting

$$K_M = \frac{(47.6)(30)}{(0.478)(42.4)P_{gf}} \left(\frac{0.478 \times 0.961 \times 0.0332}{0.01445}\right)^{2/3}$$

$$= \frac{70.3}{P_{gf}} (1.04) = \frac{73.2}{P_{gf}}$$

(j) Evaluation of UΔt Product

Reference is made to Eq. 5.

$$h_o' \, (t_g - t_c) + \frac{K_M(p_v - p_c)}{P_{gf}} = U_{comb.} \, (t_c - t_w)$$

$$47.6 \, (225-t_c) \frac{(70.3)(132)(47-p_c)}{P_{gf}} = 89.5 \, (t_c - 110)$$

Assume that t_c = 160°F

p_c = vapor pressure = 44 psia (from Fig. 4)

P_{gf} = 114 psia.

Trial, does

$$47.6 \, (225-160) + \frac{(70.3)(132)(47-44)}{114} = 89.5 \, (50)?$$

$$3095 + 244 = 4475 \quad \underline{\text{no}}$$

$$3339 \neq 4475$$

Assume that t_c = 153°F

p_c = 41 psia

P_{gf} = 116

Trial, does

$$47.6 \, (72) + \frac{(70.3)(132)(6)}{116} = (89.5)(43)?$$

$$3430 + 480 = 3850 \quad \underline{\text{no}}$$

$$3910 \neq 3850$$

Apparently, t_c = 154°F approx. Therefore, use a value of

UΔT = 3880

(k) Determination of Point "c" (see Fig. 4)

Eq. 11 is used to determine the slope, $\frac{dp_v}{dt}$ of the line A-C on Fig. 4.

$$\frac{dp_v}{dt} = \frac{(P-p_v)}{P_{gf}} \left(\frac{P_r}{S_c}\right)^{2/3} \frac{\Delta p}{\Delta t}$$

Locating the temperature of 154°F on Fig. 4 the condensate surface temperature, t_c, on the vapor pressure curve, the value of $\frac{\Delta P}{\Delta t}$ is determined as $\frac{6}{72}$.

Then,

$$\frac{dp_v}{dt} = \left(\frac{160-41}{116}\right)\left(\frac{0.88}{0.8155}\right)\left(\frac{6}{72}\right) = 0.092$$

(l) Check of Assumed Condensing Coefficient

$$U\Delta T = 3880$$

$$\Delta T = 225-110 = 115°F$$

$$U_o = 33.8 \text{ Btu/hr-°F-ft}^2 \text{ (outside area)}$$

$$\frac{\Delta T}{\Delta t_c} = \frac{h_c}{U_o}$$

$$\Delta t_c = \frac{33.8}{600}(115) = 6.47°F$$

From Reference 8:

$$h_{cond.} = 0.725 \left(\frac{C_N}{N^{1/4}}\right)\left(\frac{K_f{}^3 \rho_f{}^2 g_c}{u_f}\right)^{1/4} \left(\frac{1}{D_{eq}}\right)^{1/4} \left(\frac{1}{\Delta t_c}\right)^{1/4} (\lambda)^{1/4}$$

For this bundle,

$$N = 0.40\,(X)^{0.54} = 19$$

$\frac{C_N}{N^{1/4}}$ = 0.76 (using Freon 12 line of Fig. 1 of the 2nd article[8])

$$\left(\frac{1}{t_c}\right)^{1/4} = 0.627$$

$$\lambda^{1/4} = 3.35$$

$$\left(\frac{1}{D_{eq}}\right)^{1/4} = 3.5 \quad \text{(from 2nd article[8])}$$

$$\therefore h_{cond} = 4.14 \left(\frac{K_f^3 \, \rho_f^2 \, g_c}{\mu_f}\right)^{1/4}$$

t_f °F	$\left(\frac{K_f^3 \rho_f^2 g_c}{\mu_f}\right)^{1/4}$
90	156
110	159
130	160
150	161
170	161.5

$$t_f = 154 - 3.2 = 150.8°F$$

$$\therefore \left(\frac{K_f^3 \rho_f^2 g_c}{\mu_f}\right)^{1/4} = 161$$

$$\therefore h_{cond} = 4.14\,(161) = 665 \text{ Btu/hr-°F-ft}^2 \text{ (outside area)}$$

(This assumed value was 600.)

F. DETERMINATION OF REQUIRED HEAT TRANSFER AREA

1. Finned-Tube Unit

Calculations were made for other points by the above procedure and are listed in Table I. A plot of $10^4/U\Delta t$ vs $q/10^4$ is given in Fig. 5. The shaded area of the curve represents the solution of Eq. (2).

TABLE I

COMPUTED VALUES FOR FINNED-TUBE UNIT

Gas Temp °F	Condensate Film Temp °F	Water Temp °F	Pentane Vapor lb/hr	$Q/10^4$ Btu/hr	UΔt Btu/hr-ft²	U_o Btu/hr-ft²°F	10^4/UΔt hr-ft²/Btu
225	154	110	75,000		3880	33.8	2.58
200	145	106.6	67,350	266.85	3600	38.6	2.78
175	134	103	58,150	550.4	2930	40.7	3.41
150	121	98.5	45,100	893.0	2150	41.7	4.65
125	104	93.8	29,700	1265	1190	38.4	8.40
110	99	91.1	22,900	1472	690	36.5	14.5
100	94	90	18,500	1590	368	36.8	27.4

TABLE II

COMPUTED VALUES FOR PLAIN TUBE UNITS

Gas Temp °F	Condensate Film Temp °F	Water Temp °F	Pentane Vapor lb/hr	$Q/10^4$ Btu/hr	UΔt Btu/hr-ft²	U_o Btu/hr-ft²	10^4/UΔt hr-ft²/Btu
225	157	100	75,000		6060	52.7	1.65
200	148	107.1	70,900	224.6	5290	57.0	1.89
175	136	103.1	59,200	539.5	4240	59.0	2.36
150	122	98.5	46,000	889.0	3000	58.4	3.33
125	107	93.8	29,700	1265	1750	56.2	5.70
110	100	91.1	22,900	1472	1070	55.4	9.35
100	94	90	18,500	1590	524	52.4	19.10

$$A = \int_0^q \frac{dq}{U\Delta t} = 9840 \text{ ft}^2$$

The area per foot of length = 0.438 ft^2/ft

Length of tubing required = 2240 ft

Using 1300 tubes, the required length is 17.25 ft.

Using an 18-ft exchanger provides 4.3% excess area.

2. Plain Tube Unit

The required area was found to be 6550 ft for the same shell-and-tube arrangement. This requires 25.7 ft of exchanger; the use of two 14-ft units provides an excess area of 8.95%.

3. Checks of Finned Tube Area by Carey and Williamson Method (see Reference 12)

$$y_1 = (U\Delta t)_{inlet} = 3880$$

$$y_m = (U\Delta t)_{mid} = 2370$$

$$y_2 = (U\Delta t)_{outlet} = 368$$

$$\therefore \frac{y_m}{y_2} = \frac{2370}{368} = 6.45$$

$$\frac{y_m}{y_1} = \frac{2370}{3880} = 0.61$$

Using these values, f from chart is = 0.685.

$\therefore$ Mean driving force = fy_m = (0.685)(2370)

$\therefore (U\Delta t)_{mean}$ = 1625

$$\therefore A = \frac{15{,}900{,}000}{1625} = 9780 \text{ sq ft}$$ as compared with 9840 sq ft

G. COMPARISON OF INITIAL COSTS

1. Finned Tube Unit

The cost of the finned tube exchanger is obtained from a recent article by Kern and Associates[16] as:

Finned tube unit cost = $26,000

2. Plain Tube Unit

The cost of the bare tube exchangers is obtained from the same article[16] as:

Bare tube costs (per shell) = $16,200

or

Total cost = 2 x 16,200 = $32,400

3. Comparison of Costs

Use of the finned tube exchanger yields a saving of:

$32,400 - $26,000 = $6,400

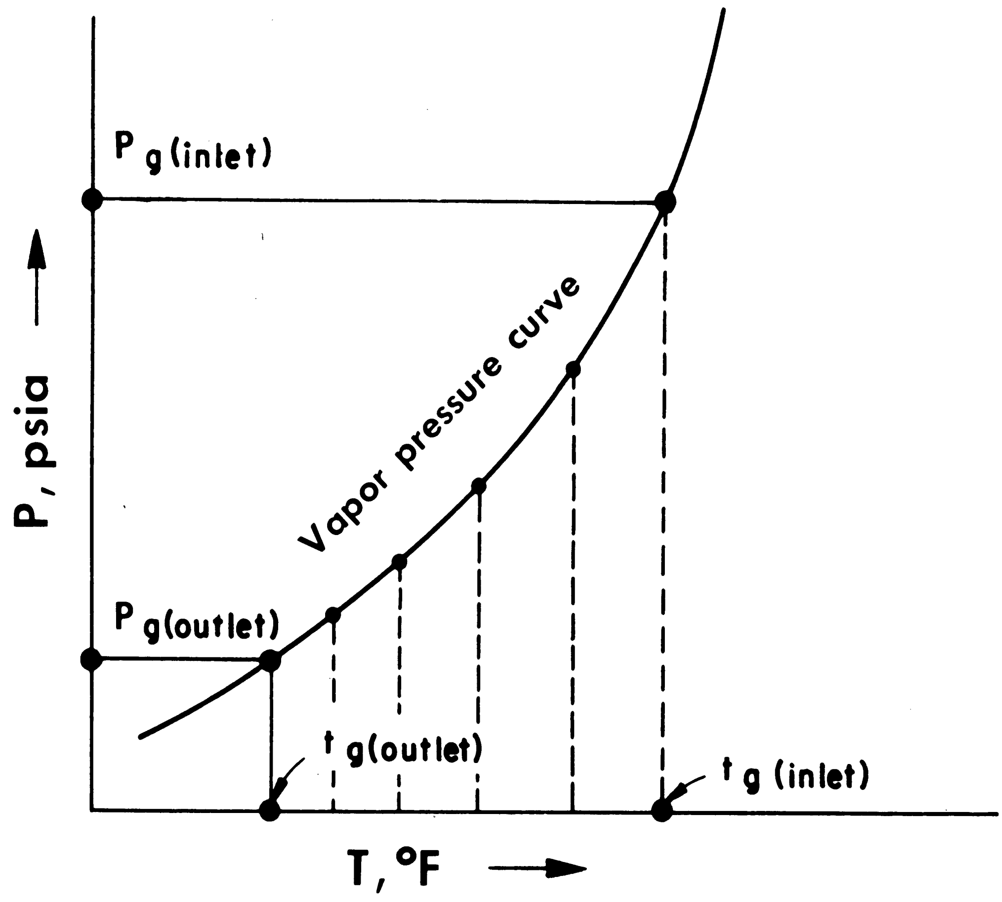

FIGURE 1. VAPOR PRESSURE CURVE SHOWING COOLING-CONDENSING PATH OF SATURATED VAPOR

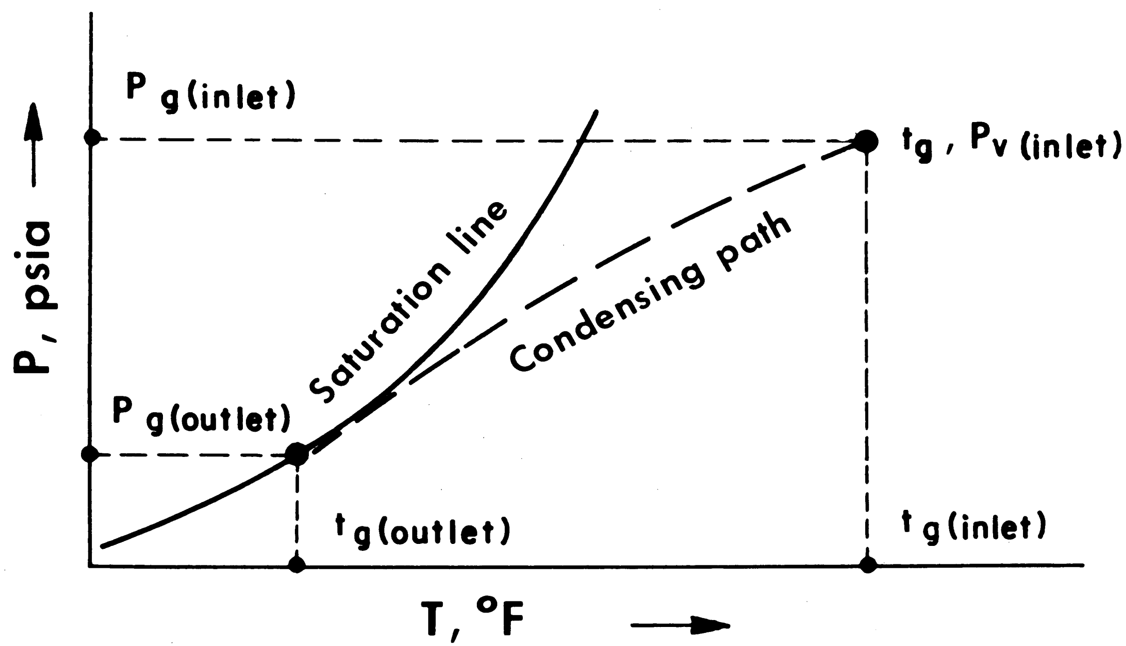

FIGURE 2. VAPOR PRESSURE CURVE SHOWING COOLING-CONDENSING PATH OF SUPERHEATED FEED

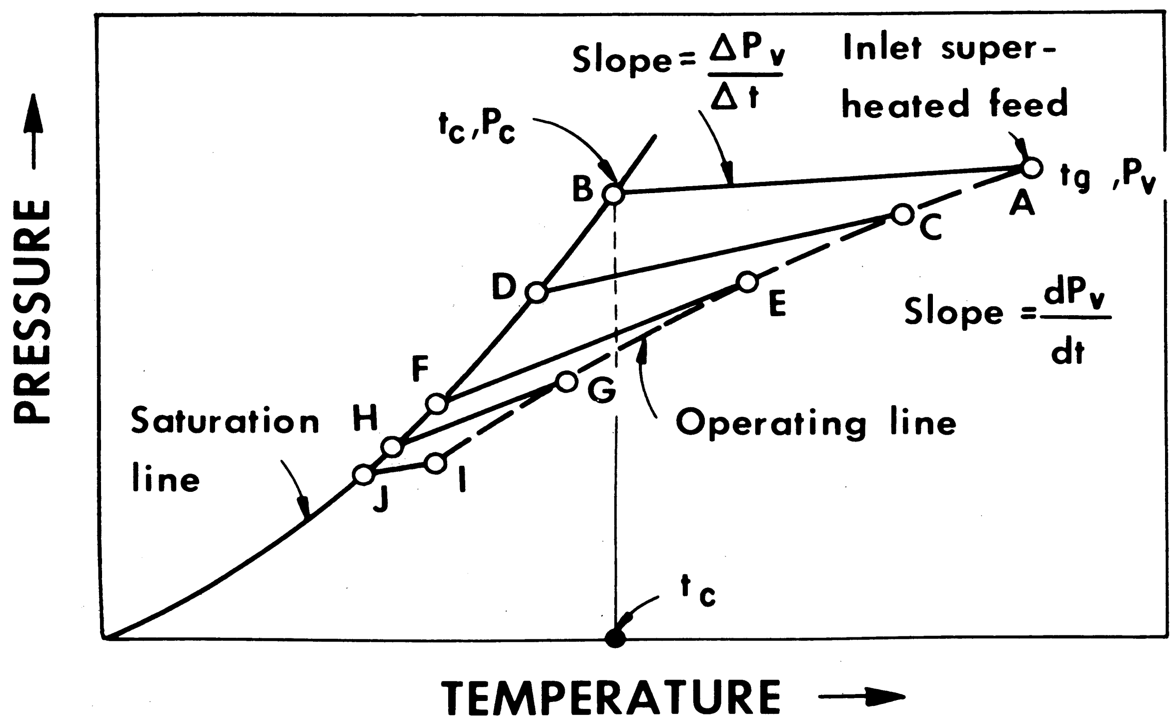

FIGURE 3. ILLUSTRATION OF PROCEDURE FOLLOWED TO OBTAIN CONDENSING PATH FOR SUPERHEATED FEED

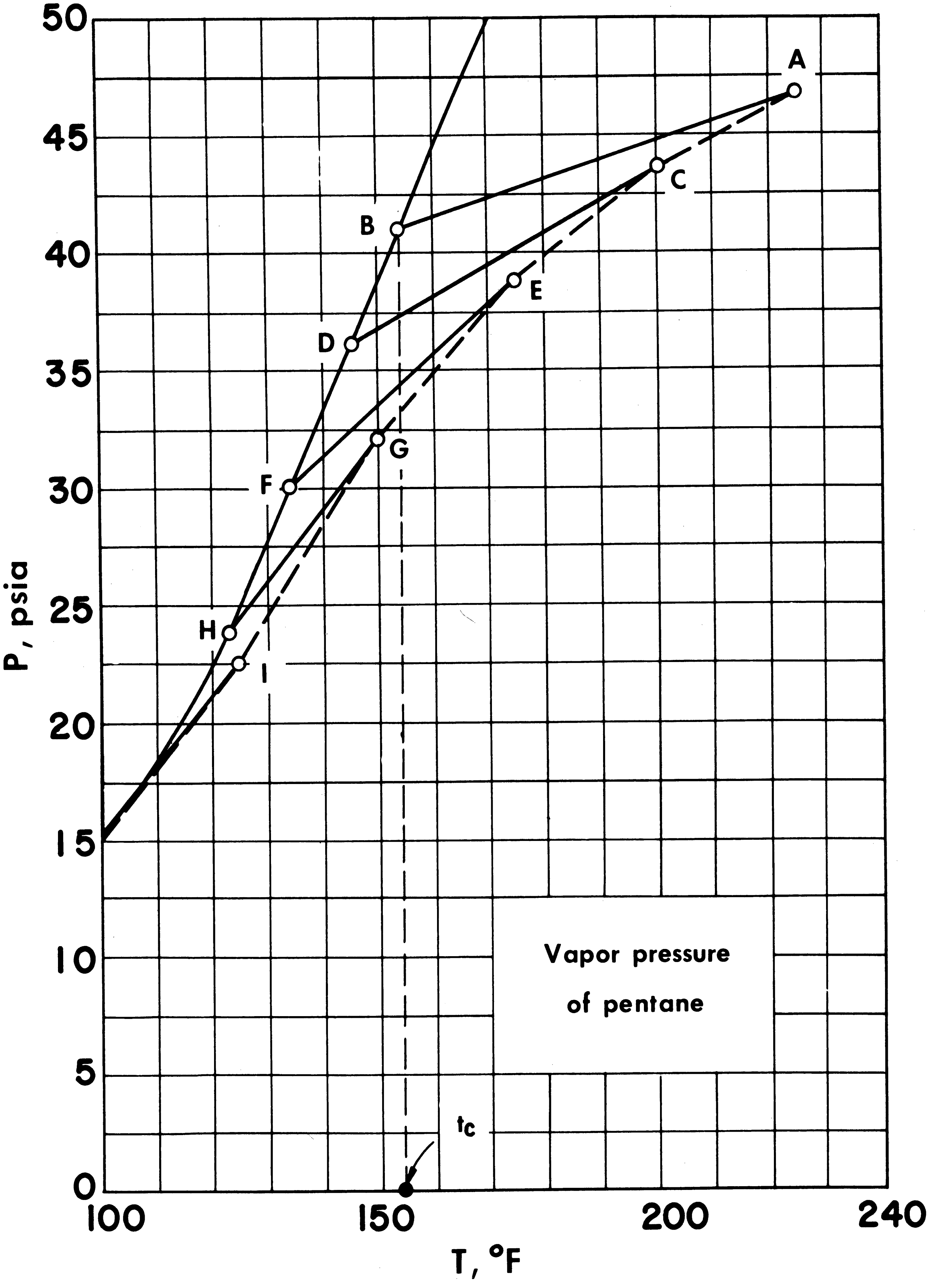

FIGURE 4. CONDENSING PATH IN FINNED TUBE PARTIAL CONDENSERS

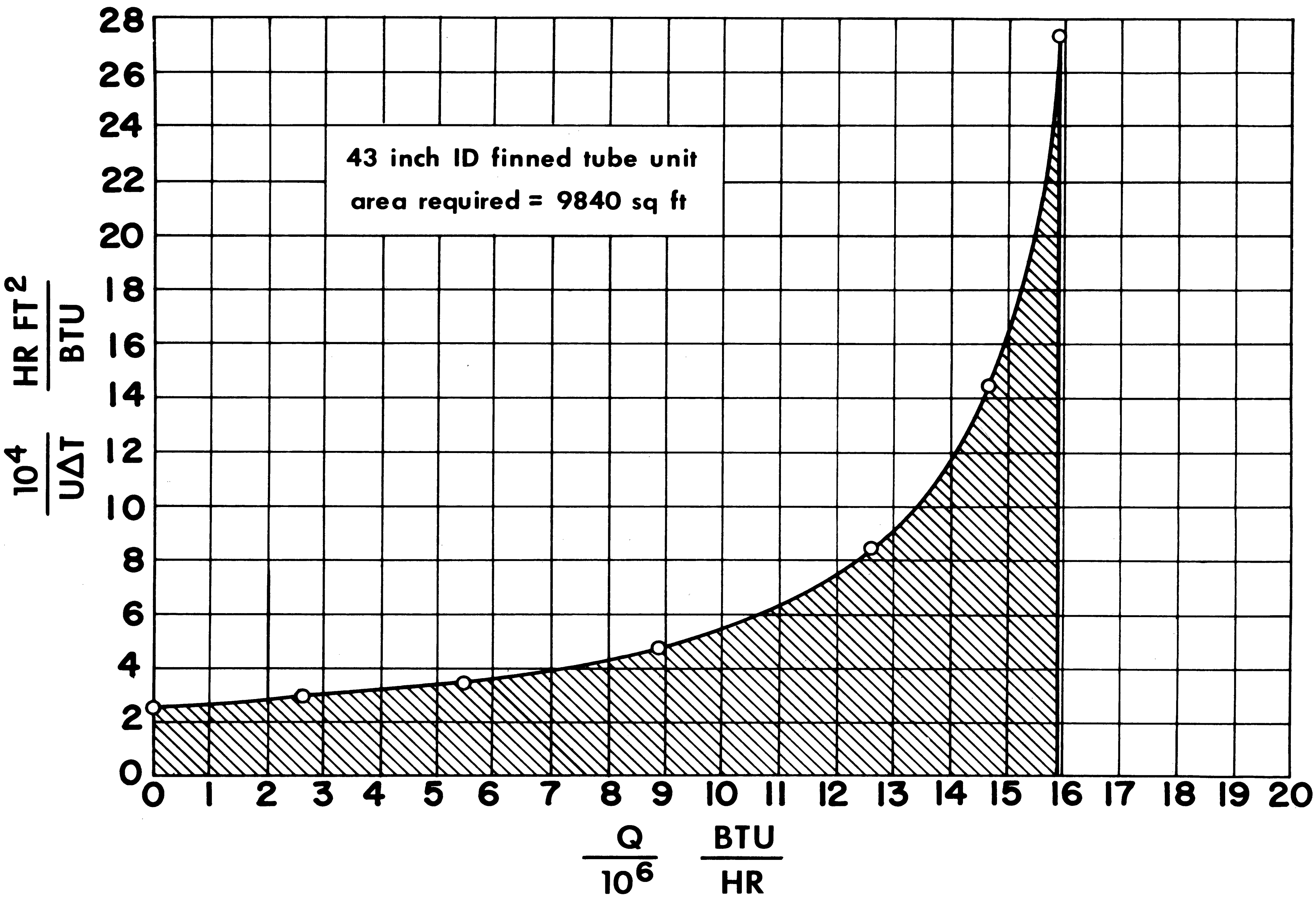

FIGURE 5. GRAPHICAL SOLUTION OF EQ. 2 FOR EXAMPLE DESIGN

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