ANALYTICAL ESTIMATION OF THROUGHPUT DISTRIBUTION FOR SERIAL MANUFACTURING SYSTEMS WITH MULTI-STATE MACHINES AND ITS APPLICATION

by

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Dedicated to my parents
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LIST OF SYMBOLS

$\lambda_0$ Rate of breakdown in absence of PM

$\mu_0$ Rate of repair

$st$ State of a machine

$T_{up}$ Mean up time

$T_{down}$ Mean down time

$e$ Efficiency of the machine

$\lambda_m$ Rate of PM

$\lambda$ Rate of breakdown

$\mu_m$ Rate of maintenance

$S$ Sensitivity of the rate of breakdown with respect to rate of maintenance

$r$ Ratio of average downtime due to breakdown to average downtime due to PM

$T_m$ Mean time to maintain

$e_{PM}$ Efficiency of the machine in presence of PM

$e_{PM}^d$ Efficiency of the machine in presence of deterministic PM

$e_{PM}^*$ Optimal efficiency of the machine in presence of PM

$\lambda_m^*$ Optimal rate of PM

xi
<table>
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<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$T_{\text{inter-PM}}^d$</td>
<td>Deterministic time between PMs</td>
</tr>
<tr>
<td>$T_m^d$</td>
<td>Deterministic time required to maintain the machine</td>
</tr>
<tr>
<td>$Q$</td>
<td>CTMC rate transition matrix</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Cycle time of the synchronous manufacturing system</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Discretization constant for the CTMC</td>
</tr>
<tr>
<td>$P$</td>
<td>DTMC rate transition matrix of a machine</td>
</tr>
<tr>
<td>$I$</td>
<td>Identity matrix</td>
</tr>
<tr>
<td>$pm_{ij}$</td>
<td>$ij$th element of the machine transition matrix $P$</td>
</tr>
<tr>
<td>$x_j(i)$</td>
<td>Probability of finding the machine in state $j$ at the end of $i$ cycles</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of states of the machine</td>
</tr>
<tr>
<td>$K$</td>
<td>Number of up states for the machine</td>
</tr>
<tr>
<td>$x(i)$</td>
<td>State vector of the machine at the end of $i$ cycles</td>
</tr>
<tr>
<td>$= [x_0(i), x_1(i), ..., x_N(i)]$</td>
<td></td>
</tr>
<tr>
<td>$PU(n)$</td>
<td>Probability of finding a machine in operating state at the end of $n$ cycles</td>
</tr>
<tr>
<td>$PD(n)$</td>
<td>Probability of finding the machine in non operative state at the end of $n$ cycles</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Upstream</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Downstream</td>
</tr>
<tr>
<td>$M^\gamma$</td>
<td>Upstream Machine</td>
</tr>
<tr>
<td>$M^\delta$</td>
<td>Downstream Machine</td>
</tr>
<tr>
<td>$B$</td>
<td>Buffer</td>
</tr>
</tbody>
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$M^\delta$  
Upstream machine

$M^\gamma$  
Downstream machine

$M^i$  
Set consisting all states of $M^i$, where $i \in \{\delta, \gamma\}$

$U^i$  
Set consisting of up states of $M^i$, where $i \in \{\delta, \gamma\}$

$D^i$  
Set consisting of down states of $M^i$, where $i \in \{\delta, \gamma\}$

$x^i(n)$  
State vector for $M^i$ at the beginning of the $n$th cycle, where $i \in \{\delta, \gamma\}$

$y(n)$  
State vector for buffer at the beginning of the $n$th cycle

$s(n)$  
State of the two machine one buffer system at the beginning of the $n$th cycle, i.e. $\{x^\delta(n), x^\gamma(n), y(n)\}$

$S$  
Set containing all possible states of the two machine one buffer system, $s \in S$

$PU^i$, $PD^i$  
Probability of the $i$th machine being in an up and down state respectively, $i \in \{\delta, \gamma\}$

$\mathcal{P}$  
Transition matrix of a buffer over a given cycle

$\alpha$  
Producing states of the system

$\beta$  
Non producing states of the system

$S_\alpha, S_\beta$  
Set containing the producing and non producing states of the system respectively

$D(j, i)$  
Condition that system has produced $j$ parts in $i$ cycles

$\text{th}_{D(j, i)\alpha}$  
State vector consisting of the producing states of the system satisfying the condition $D(a, b)$

$\text{th}_{D(j, i)\beta}$  
State vector consisting of the non producing states of the
system satisfying the condition $D(a, b)$

$td$ Duration of time considered

$\text{Th}(x_i, t_d)$ Probability of the system producing $x_i$ parts in $t_d$ cycles

(Throughput distribution of the system)

$\Phi_1(t_d)$ Average throughput

$\Phi_2(d, t_d)$ Probability of satisfying demand $d$ in $t_d$ cycles

$\Phi_3(d, t_d)$ Risk associated with a system when demand $d$ is to be produced in $t_d$ cycles

$G(x_i, d)$ Penalty function associated with producing $x_i$ parts in presence of demand $d$

$\eta$ Proportionality constant for a linear penalty function

$M_i$ $i$th machine of the serial manufacturing system

$B_i$ $i$th buffer of the serial manufacturing system

$G^i$ $i$th segment of the system

$M^{i\gamma}$ Upstream machine of the $i$th segment

$M^{i\delta}$ Downstream machine of the $i$th segment

$B^i$ Buffer of the $i$th segment

$s^i_j$ $j$th state of the $i$th segment, $s^i_j = (x^i_j, x^{i\delta}_j, y^i_j)$

$n, bl, st, sb$ Normal, Blocked, Starved and Starved & blocked mode of a segment

$P^j$ Machine transition matrix for $j \in \{\gamma, \delta\}$

$P^o$ Buffer transition matrix in mode $o \in \{n, bl, st, sb\}$

$P^{oo}$ System transition matrix in mode $o \in \{n, bl, st, sb\}$
Buffer capacity of the buffer in $i$th segment

Initial number of parts in buffer of $i$th segment

$i,j$th element of the transition matrices $P, P$ and $P$ respectively

Block transition matrix of moving from $i$ type of states to $j$ type of states where $i,j \in \{\text{Producing states}(\alpha)\}$, $\{\text{Non producing states}(\beta)\}$

↑ An up state of a machine

down A down state of a machine

$T^i(t)$ Set containing all elements of the $i$th segment table at the end of $t$ cycles

e$^i_{jk}(t)$ The $jk$th element of the $i$th segment table, at the end of $t$ cycles

Number of parts present in the buffer of segment $i$

Number of parts produced by segment $i$

Probability of blockage

Probability of starvation

Probability of operation mode $o \in \{n, st, bl, sb\}$

Probability

Mode probability $o (o \in \{bl, st, n, sb\})$ of element $(s^i_j, D^i(k-1, b))$ of the $i$th segment table

Condition for feasibility, blockage and starvation respectively
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$T_{\omega_k}^i$</td>
<td>States of the $i$th segment table satisfying the $\omega_k$ condition, where $k \in {f, b, s}$</td>
</tr>
<tr>
<td>$N_s$</td>
<td>Number of states of a system</td>
</tr>
<tr>
<td>$N_{so}$</td>
<td>Number of producing states of a system</td>
</tr>
<tr>
<td>$N_{s\beta}$</td>
<td>Number of non producing states of a system</td>
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CHAPTER I

Introduction

1.1 Motivation

An accurate estimate of a system’s performance is necessary for design, improvement and management of a manufacturing system. A system’s performance can be characterized using many performance measures, several of which include, throughput, work in process (WIP), probability of blockage, probability of starvation and residence time. Of these, throughput, defined as the number of parts produced by the last machine of a manufacturing system over a given period of time, has attracted significant attention over the past fifty years (see reviews by Dallery and Gershwin [1992]; Papadopoulos and Heavey [1996]; Li et al. [2009] and monographs by Buzacott and Shanthikumar [1993]; Gershwin [1994]; Altiok [1997]).

From a manufacturing system modeling perspective, the steady state behavior of a manufacturing system has been well studied, with focus on estimating average (first order) performance of a system (see, for instance, monographs by Buzacott and Shanthikumar [1993]; Papadopoulos et al. [1993]; Gershwin [1994]; Yao [1994]; Altiok [1997]; Liberopoulos et al. [2006]; Li and Meerkov [2009]). Due to uncertainties, such as machine failures, the throughput of a manufacturing system is considered a random variable.
Through factory observations and simulations, Gershwin [1994] reported that the standard deviation associated with the weekly production of a manufacturing system might be over 10% of its mean. Meerkov and Zhang [2008], in their recent study of transient analysis of manufacturing systems, stated that a production line with cycle time of 1 minute, initially empty buffers and operating time of 8 hours (1 shift), could lose up to 10% of the estimated production, within a given shift. Such high variability may result in customer requirements not being met on time, several times. By estimating the throughput distribution of a manufacturing system, higher order estimates of the internal and external performance measures of a manufacturing system can be obtained, using which, higher predictability and dependability of a manufacturing system can be achieved.

Over the past two decades, in a parallel area of research, condition monitoring of machines has developed significantly, largely due to the introduction of low-cost electronics, intelligent sensing devices and data capture equipment, and its successful application to many industries, including processing, services and manufacturing. Condition monitoring aims towards increasing system reliability and reducing maintenance costs, and has resulted in a gradual shift from time based maintenance to condition based maintenance (Fararooy and Allan [1995]).

In contrast to existing manufacturing system machine models, wherein machines are represented as two state models (with operating and failure states); the modern condition based maintenance (CBM) models consider machines as multi state models (Chen and Trivedi [2002]; Chan and Asgarpoor [2006]; Ambani et al. [2009]) with multiple degradation and failure states. Introduction of multi-state machine models in manufacturing system modeling can lead to inclusion of real time condition monitoring within manufacturing system decision making.
Serial manufacturing systems are of economic importance as they are commonly used in high volume production and characterize the inter-relationship of manufacturing stations and buffers, which may be used to model the key features of manufacturing environments with simplifying assumptions (Dincer and Deler [2000]). Further, complex manufacturing systems with assembly, rework, disassembly operations etc., can also be divided into several simpler serial manufacturing systems for analysis (Li [2004]). Hence, the study of serial manufacturing systems is fundamental and essential to analyzing complex manufacturing systems.

Based on the above considerations, the study of throughput distribution of a serial manufacturing system with multi-state machines, can lead to improved predictability and dependability of manufacturing systems, and can aid the development of a real time decision making framework to include condition monitoring of machines. Accomplishing the above tasks will result in improved system reliability and reduced maintenance costs.

1.2 Background and Scope

A manufacturing system consists of material (parts), work stations (machines) and storage areas (buffers). The work stations may consist of machines, work cells or departments within a factory, while the buffers may consist of simple containers, material handling devices or other forms of storage locations. In this dissertation, work areas are referred to as machines, storage areas as buffers and material as parts.

A manufacturing system with machines and buffers arranged in a consecutive order, with buffers present between every two machines, is known as a serial manufacturing system. Figure 1.1 shows the block diagram of a serial manufacturing system, where $M_i$ is the $i$th machine and $B_j$ is the $j$th buffer of the system. Serial
manufacturing systems are also referred to as transfer lines or serial production lines in literature.

Figure 1.1: A Serial Manufacturing System

Depending on the characteristics of the machines and buffers, a manufacturing system may be defined as follows:

1. **Synchronous/ Asynchronous**: If the cycle times of all machines are identical, the manufacturing system is called synchronous. If the cycle times are not identical, the system is called as asynchronous (Li and Meerkov [2009]).

2. **Saturated/ Unsaturated**: If the first machine of the manufacturing system never starves (unlimited supply of raw materials) and the last machine of the system never blocks (infinite demand), then the system is called saturated, otherwise unsaturated (Dallery and Gershwin [1992]). Most practical systems are unsaturated, but in order to focus on the internal dynamics of a system, manufacturing systems are often assumed as saturated.

3. **Reliable/ Unreliable**: Manufacturing systems with 100% reliable machines are referred to as reliable systems. If machines of a manufacturing system can undergo failures, the system is referred to as unreliable.

Further, depending on the nature of machine failures, an unreliable system may be classified to have time dependent failures (TDFs) or operation dependent failures (ODFs). TDFs depend on the time spent by a machine in its up state, while ODFs depend on the number of operations carried out by a machine. Both
TDFs and ODFs are common in manufacturing system modeling, a comparison of which is found in (Buzacott and Hanifin [1978]; Buzacott and Shanthikumar [1993]). TDFs simplify manufacturing system analysis and help in obtaining closed form expressions (Li et al. [2006]), making the system (analytically) more tractable.

4. **Finite/ Infinite buffers**: The size of buffers in a manufacturing system dictates the coupling between machines. For example, in presence of finite buffers, the failure of one machine may lead to downtime of other machines; whereas, in presence of infinite buffers, the machines behave as if in complete isolation.

In this study, a manufacturing system is considered to be synchronous, saturated, unreliable, serial manufacturing systems with finite buffers and time dependent failures. Other features of manufacturing systems that are equally important, but out of the scope of this dissertation are structure and quality. A manufacturing system, depending on the arrangement of machines and buffers (layout), may result in different structures, such as, serial, parallel, closed loop, rework loop, assembly and disassembly (see Li and Meerkov [2009] for detailed description of structural considerations). In presence of material or machine errors, a manufacturing system may produce defective parts, resulting in quality considerations. Several studies have been done to optimally position an inspection station in a manufacturing system (Rebello et al. [1995]; Shin et al. [1995]; Kogan and Raz [2002]; Kalade et al. [2004]; Shiau et al. [2007]; Volsema et al. [2007]), to maximize production of good parts. Another important study related to quality, is determining the relation between quantity and quality of products for a system (Jacobs and Meerkov [1991]; Han et al. [1998]; Chiang [2006]).
Overall, studies related to manufacturing systems can be broadly classified into three areas:

1. **Analysis**: Given a manufacturing system with machine and buffer characteristics, manufacturing system analysis focuses on estimating the performance of a system. Few common performance measures found in literature are throughput, WIP, probability of blockage, probability of starvation and probability of meeting given demand. Depending on the duration of interest, three types of analysis may be pursued:

   (a) **Steady State Analysis**: The steady state analysis of a manufacturing system focuses on the average performance of a system, while the system is in its steady state. Most results and studies in literature focus on this type of analysis (Buzacott and Shanthikumar [1993]; Papadopoulos et al. [1993]; Gershwin [1994]; Yao [1994]; Altiok [1997]; Liberopoulos et al. [2006]; Li and Meerkov [2009]).

   (b) **Transient Analysis**: Study of a manufacturing system during the initial phase of operations, before reaching steady state, is studied in transient analysis. Although equally important, this area has received limited attention in the past and has been recognized as a critical area for future studies (Mitra [1988]; Narahari and Viswanadham [1994]; Mocanu [2005]; Meerkov and Zhang [2008]).

   (c) **Interval Analysis**: Analysis of a manufacturing system over a given finite duration of time, wherein the system may or may not be in steady state, is called as interval analysis. This approach is usually used to obtain the cumulative performance of a system over a given period of time, for
example to obtain the total throughput or the total blockage of a system over a given period of time (Silva et al. [1986]; Dincer and Deler [2000]; Tan [1999]; Csenki [2007]).

2. Continuous Improvement: Redistributing resources within a manufacturing system in order to improve the performance of an existing system is known as continuous improvement. An example of this type of study is bottleneck identification (Lawrence and Buss [1995]; Kuo et al. [1996]).

3. Design: Given a desired performance, obtaining the minimum requirements for machines and buffers is referred to as design of manufacturing systems. Few studies related to this area are found in (Papadopoulos et al. [1993]; Papadopoulos and Heavey [1996]).

This study focuses on the interval analysis of an unreliable, saturated, serial manufacturing system with time dependent failures. More specifically, the main goal of this dissertation is to estimate the throughput distribution and related performance measures of the above described system, over a given period of time.

1.3 Literature Review

1.3.1 Throughput Analysis

Several definitions related to throughput are found in literature. The commonly found definitions are:

1. Throughput is defined as the expected number of parts produced by a production system (Li et al. [2009]).

2. For a system in steady state, the average number of parts produced by the last machine of a production system per unit of time is known as throughput (Li
The number of parts (a random variable) produced by a manufacturing system (transfer line with buffer inventories) per unit time is defined as the throughput rate (Dincer and Deler [2000]).

Another commonly used production related definition is production rate, defined as, the average number of parts produced by the last machine of a manufacturing system in steady state over a given cycle (Li and Meerkov [2009]). As most of the previous studies focused primarily on obtaining average performance of a manufacturing system in steady state, throughput is often considered as the expected or average number of parts produced by a system over a unit time in steady state. For this study, similar to (Dincer and Deler [2000]), throughput is considered as a random variable, and is defined as the total number of parts produced by the last machine of a manufacturing system over a given period of time.

Throughput analysis has been widely studied for many decades now (see Li et al. [2009] for a recent comprehensive review and monographs by Viswanadham and Nara-
hari [1992], Buzacott and Shanthikumar [1993], Papadopoulos et al. [1993], Gershwin [1994], Li and Meerkov [2009] for further details). In this section, we briefly review throughput analysis literature under steady state analysis, transient analysis and interval analysis.

1.3.1.1 Steady State Analysis

Over the years, great emphasis has been laid on the steady state behavior of manufacturing systems (see, for instance, monographs by Buzacott and Shanthikumar [1993]; Papadopoulos et al. [1993]; Gershwin [1994]; Yao [1994]; Altiok [1997]; Liberopoulos et al. [2006]; Li and Meerkov [2009]). Most of these studies, consider machines as a two state system, consisting of an up (operating) and down (repair) state, and having fixed uptime and downtime probability mass distribution (for instance, Bernoulli, geometric, exponential, Rayleigh, Weibull, gamma, log-normal etc.).

The state space of a manufacturing system increases exponentially with increase in the number of machines and with increase in the buffer sizes. As a result, exact analytical results have only been obtained only for two machine one buffer systems, systems with infinite buffers and systems with no buffers (Li et al. [2009]). Gershwin and Berman [1981] developed one of the earliest analytical models for a two machine one buffer system. Since then, based on the machine and buffer characteristics, many two machine one buffer models have been developed. A summary and comparison of eight such models is found in (Li et al. [2006]), and their categorization in Figure 1.3.

To estimate the steady state performance of larger manufacturing systems (e.g. serial lines, assembly lines, lines with rework etc.), many approximation techniques have been developed. Most of these techniques can broadly be classified into decomposition based techniques and aggregation based techniques. The underlying ideas of these two approaches are provided below.
1. Decomposition Based Approach

Decomposition has proven to be a robust approximation technique for estimating the steady state performance of manufacturing systems with unreliable machines and finite buffers. The basic principles of this technique were first introduced in (Sevastyanov [1962]). Since, most production lines consist of multiple machines, the decomposition based approach decomposes a longer manufacturing system into smaller analytically tractable two machine one buffer segments. Figure 1.4 shows decomposition of a four machine three buffer system into three two machine one buffer segments. For the $i$th segment of the decomposed system, $M_U(i)$, $b_i$ and $M_D(i)$ represent the upstream machine, buffer and the downstream machine of the segment respectively.

For a given segment, flow of parts into the buffer may stop due to failure of the immediately upstream machine, or failure of a machine further upstream. Similarly, flow of parts out of a segment buffer may be blocked due to failure of the immediately downstream machine, or failure of a machine further down-
stream. The decomposition approach adjusts the parameters of the upstream and downstream machine for each segment, such that, the flow of parts into and out of a buffer for each segment remains same, irrespective of the buffer being in a decomposed segment or the actual line. Further details of this approach are found in (Gershwin [1994]). Several efficient algorithms have also been developed to evaluate the machine parameters of such decomposed system (Gershwin [1987, 1989]).

![Diagram of decomposition approach](image)

Line (1) $\rightarrow \bigcirc \rightarrow \square \rightarrow \bigcirc \rightarrow \square \rightarrow \bigcirc \rightarrow \square \rightarrow \bigcirc \rightarrow$

$m_1 \quad b_1 \quad m_2 \quad b_2 \quad m_3 \quad b_3 \quad m_4$

Line (2) $\rightarrow \bigcirc \rightarrow \square \rightarrow \bigcirc \rightarrow$

$m_{D(1)} \quad b_1 \quad m_{D(1)}$

Line (3) $\rightarrow \bigcirc \rightarrow \square \rightarrow \bigcirc \rightarrow$

$m_{U(2)} \quad b_2 \quad m_{D(2)}$

Figure 1.4: Decomposition Approach (Li et al., 2009)

2. **Aggregation Based Approach**

The principles of aggregation were first introduced by Lim et al. [1990], for asymptotically reliable machines and later generalized by Jacobs and Meerkov [1995]. The fundamental step of an aggregation based approach consists of replacing a two machine one buffer system by an equivalent machine, having the same average throughput in isolation as the two machine one buffer system (Li et al. [2009]). In this approach, a complete manufacturing system is aggre-
gated into a single equivalent machine (aggregating two machines at a time) recursively in the forward and backward direction, till convergence is met.

Consider a manufacturing system with $M$ machines as shown in Figure 1.5. The top half of this figure represents backward aggregation and the bottom half forward aggregation. Backward aggregation is initiated by aggregating the last two machines of the system into an equivalent machine, resulting in a $(M - 1)$ machine system. This equivalent machine is further aggregated with the $(M - 2)$th machine, and so on, till the entire system is represented by a single equivalent machine.

On completing backward aggregation, forward aggregation is carried out. The $i$th step of forward aggregation obtains the $(i+1)$th forward equivalent machine by aggregating the $i$th forward equivalent machine (equivalent of all the machines inclusive and upstream of the $i$th machine) with the $(i+1)$th backward equivalent machine (equivalent of all the machines inclusive and downstream of the $(i+1)$th machine). Similar to backward aggregation, the process is continued till a single equivalent machine is obtained for the entire system. The forward and backward aggregation steps are carried out recursively till the resulting equivalent machines are the same (within a given tolerance).

The aggregation procedure is usually observed to converge quickly (typically within 10 iterations (Li and Meerkov [2009])) and has been proven to be convergent. Further details of the aggregation based approach are found in (Lim et al. [1990]; Jacobs et al. [1999]; Li and Meerkov [2009]).

A brief summary of other approximate techniques is found in the review by Li et al. [2009].
Figure 1.5: Aggregation Approach (Li et al., 2009)
1.3.1.2 Transient Analysis

In a recent review, Li et al. [2009], highlighted transient analysis of manufacturing systems as an important area of future research. As compared to steady state analysis, only few studies have been done to model the transient behavior of manufacturing systems. Few of these studies include (Mitra [1988]; Narahari and Viswanadham [1994]; Mocanu [2005]; Meerkov and Zhang [2008]). Mitra [1988] developed efficient computational procedures and numerically investigated a fluid model for $m$ producing machines, a buffer, and $n$ consuming machines. Narahari and Viswanadham [1994] studied manufacturing situations where transients could have a great impact and discussed two of its applications: 1) computing distribution of time to absorption for Markov based manufacturing system models with deadlocks or failures, and 2) computing cycle time distribution of manufacturing systems in a failure prone environment, over a finite period of time. This analysis focused on the machine-machine interactions, but did not address the machine-buffer interactions found in longer manufacturing systems. Mocanu [2005] studied the transient analysis of manufacturing systems by modeling them as stochastic fluid models. In a recent study, Meerkov and Zhang [2008] studied the transients of buffer occupancy, production rate and work in process for a manufacturing system. They further investigated the impact of machine efficiency, buffer capacity and size of a manufacturing system (number of machines) on the transients. The impact of initial buffer occupancy on the production loss during transients was also studied.

1.3.1.3 Interval Analysis

In a manufacturing system, it is often of interest to compute the cumulative performance of a system over a finite duration of time. For example, the total number
of parts produced by a system or the total blockage occurring within a system, over
a given period of time (e.g., a shift or a week). Such type of analysis is referred to
as interval analysis. During the considered interval, a system may or may not be in
steady state.

Several quantities may be of interest for interval analysis, e.g., throughput distri-
bution, blockage distribution, starvation distribution, WIP distribution etc. Through-
put distribution is critical for manufacturing systems and can lead to important in-
ternal and external performance measures.

Similar to transient analysis, not many studies have been done in the area of in-
terval analysis. Silva et al. [1986] first derived the operational time distribution for
a single repairable system. Dincer and Dele [2000] studied the throughput distribu-
tion of a production system with 100% reliable machines and exponential operating
times. Tan [1997] obtained the variation of a N-station manufacturing system with no
intermediate buffers and time dependent failures, and also obtained the variance of
output of a single and two machine system as a function of time. Csenki [2007] studied
the joint interval reliability for Markov systems with an application in transmission
line reliability. Although some initial work has been done, throughput distribution
of a production system with unreliable machines and finite buffers has not yet been
studied in literature.

1.3.2 Multi State Machine Models / Condition Based Maintenance

All through the development of manufacturing system modeling, the basic ma-
cine model has almost remained unchanged. In most of the literature, a machine is
modeled as a two state system, consisting of up (operating) and down (repair) states.
Depending on the probability mass distribution of the uptime and downtime, a ma-
cine may be modeled in different forms (Li and Meerkov [2009]). The exponential
model for machine reliability is one of the most widely used models in manufacturing systems (see, for instance, Buzacott and Shanthikumar [1993]; Papadopoulos et al. [1993]; Gershwin [1994]; Altiock [1997]; Li and Meerkov [2009]). According to this model, a machine has two states: up and down; and a constant rate of transition from one state to the other. Figure 1.6 shows an exponential machine model with $\lambda_0$ and $\mu_0$ representing the rate of failure and rate of repair respectively.

\[
\begin{array}{c}
\text{Up} \\
\lambda_0 \\
\mu_0 \\
\text{Down}
\end{array}
\]

A major drawback of the two state machine model is that the condition of the machine always needs to be categorized as operating or fail. Often, scenarios may occur (e.g., preventive maintenance (PM)), wherein a machine may not be in an operating or failure mode. Based on this motivation, in a parallel area of research, CBM modeling has received significant attention (Grall et al. [2002]; Wang [2002]; Amari et al. [2006]; Jardine et al. [2006]). CBM refers to a category of maintenance policies, which depend on a quantifiable condition of machine degradation. By identifying the condition of a machine more accurately, effective maintenance and production decisions can be made. Approaches for acquiring and processing data for implementing CBM have been recently reviewed by Jardine et al. [2006].

Many CBM based models have been proposed over the past two decades, several of which are described here. An early multi stage exponential degradation model
relating to condition based maintenance was proposed by Sim and Endrenyi [1988]. Chen and Trivedi [2002] obtained closed form expressions for a single degrading machine. Grall et al. [2002], focused on the mathematical modeling of a condition based inspection/ replacement policy for a continuously degrading system. Jamali et al. [2005] developed a joint optimal periodic and condition maintenance strategy. Chen and Trivedi [2005] further extended the CBM model to semi-Markov based model to include general distributions of sojourn times in different states. Wang et al. [2008] used the Markov model to develop an optimum condition based replacement and spare provisioning policy. Ambani et al. [2009], recently recommended prioritization techniques to develop a multiple machine CBM policy. Further details on maintenance modeling can be found in the following recent reviews (Jardine et al. [2006]; Garg and Deshmukh [2006]).

Although several CBM models have been developed, two shortcomings still exist. First, most CBM models focus only on single machine systems. Although few recommendations have been made in literature for multiple machines, none of the approaches take into consideration the manufacturing system (machine-machine and machine-buffer) interactions. Secondly, most CBM models found in literature, assume maintenance to only improve the degradation state of the machine, while assuming the degradation rate and maintenance rate to be independent.

1.4 Summary of Related Literature and Research Objectives

As discussed in the literature review, most studies related to manufacturing system modeling have focused primarily on obtaining the average performance of manufacturing systems in their steady state. Recent reviews have described transient analysis of manufacturing systems to be a critical area of research, and have expressed the
need for further work. Some preliminary work has been done in the area of interval analysis of manufacturing systems to obtain the cumulative performance of a manufacturing system over a finite period of time. The main accomplishments in this area have been obtaining throughput distribution of manufacturing systems without buffers and obtaining throughput distribution of completely reliable manufacturing systems.

Although a lot of research has been done in the area of manufacturing system modeling, the interval analysis of an unreliable manufacturing system with finite buffers has not been carried out. Further, almost all manufacturing system models in literature, model machines as two state entities consisting of up (operating) and down (failure) states. With advancement in condition monitoring techniques, CBM models have developed rapidly to model machines as multi state entities, to include degradation and failure states. The major limitation of the existing CBM models has been their focus on single and two machine systems only, not taking into consideration the manufacturing system (machine-machine and machine-buffer) interactions.

This study analytically estimates the throughput distribution of an unreliable, synchronous manufacturing system with finite buffers, composed of machines with multiple states and time dependent failures. On achieving the throughput distribution, three performance measures are obtained: average throughput, probability of meeting given demand and the risk associated with given demand.

The above objectives will be realized in three steps:

1. Development of a three state, rate coupled machine model: As discussed earlier, most conventional manufacturing system models consist of machines with only two states (up and down). Further, the breakdown rate of a machine is usually considered independent of the preventive maintenance rate.
In this step, a conventional two state machine model is extended to a three state machine model, by including an additional state of PM. The relation between breakdown rate and preventive maintenance rate is developed, based upon which, the feasibility and robustness conditions for PM are derived. The optimal rate of PM is also obtained and the benefits of extending the two state machine model to a three state model including PM are shown.

2. *Estimating throughput distribution and performance measures for a two (multi state) machine one buffer system*: In this part, an approach is developed to estimate the throughput distribution of a two machine one buffer system, with machines consisting of multiple states and potential couplings. The following performance measures are obtained for such a system: average throughput, probability of meeting given demand and risk associated with fulfilling a given demand. This approach is illustrated using a conventional two state machine system with varied buffer capacity and a degrading system with varied initial conditions. Finally, the performance of the developed analytical estimation approach is compared with the performance of a discrete event simulation approach, and illustrated using two applications.

3. *Extension of the two (multi state) machine one buffer system to a serial manufacturing system*: In this final step of modeling, a segmentation approach is developed to divide a serial manufacturing system into smaller segments. The dynamics of each segment are modeled using four modes: normal, blockage, starvation and blockage & starvation. The probability and dynamics of each mode are obtained to estimate the overall throughput distribution of each segment. The distribution of the last segment of the manufacturing system provides the
overall throughput distribution, using which several performance measures are obtained. A simple illustration of this approach is also provided.

The rest of this dissertation is organized as follows. Chapter 2 extends a conventional two state machine model to a three state machine model with rate coupling. Chapter 3 focuses on the development of an approach to estimate the throughput distribution of a two machine one buffer system, with machines consisting of multiple states. Chapter 4 develops a new segmentation approach to extend the two machine one buffer model to longer serial manufacturing systems. The last chapter, Chapter 5, provides the conclusions and recommendations for future work.
CHAPTER II

Basic Machine Model Development

2.1 Introduction

The exponential model of machine reliability is widely used in manufacturing system modeling (see for instance, Buzacott and Shanthikumar [1993]; Papadopoulos et al. [1993]; Gershwin [1994]; Altıok [1997]; Li et al. [2009]). This model assumes that the machine has two states: up (operating, \( st = 1 \)) and down (under repair, \( st = 0 \)), with constant transition rates between the states. Let the breakdown rate, i.e., the transition rate from \( st = 1 \) to \( st = 0 \), be denoted as \( \lambda_0 > 0 \), and the repair rate, i.e., the transition rate from \( st = 0 \) to \( st = 1 \), as \( \mu_0 > 0 \). Then, the average up and downtime of the machine is given by:

\[
T_{up} = \frac{1}{\lambda_0}, \quad (2.1)
\]

\[
T_{down} = \frac{1}{\mu_0}, \quad (2.2)
\]

and the machine efficiency is given by

---

\[ e = \frac{1}{1 + \frac{\lambda_0}{\mu_0}} = \frac{1}{1 + \frac{t_{\text{down}}}{t_{\text{up}}}}. \] (2.3)

As seen in Equation (2.3), to increase \( e \), either \( \lambda_0 \) should be decreased or \( \mu_0 \) increased. While \( \mu_0 \) may be difficult to change, because of its dependency on the time required to identify faults and the time required to repair, \( \lambda_0 \) can be changed by preventive maintenance (PM). Although PM typically decreases \( \lambda_0 \), it requires additional (scheduled) downtime, which may decrease the machine efficiency to a level below that without PM. Thus, following questions are of interest:

1. When is it beneficial to use PM?

2. How robust are the feasibility conditions for PM?

3. In case PM is feasible, what is the optimal rate of PM?

4. What is the possible improvement achievable by carrying out optimal PM?

A PM activity (planned downtime), often requires less downtime as compared to a repair activity (unplanned downtime). In most manufacturing system models, planned and unplanned downtimes are grouped together as a single down state. To differentiate between planned and unplanned downtime, and to answer the questions listed above, a third state of PM is introduced in the machine model, as shown in Figure 2.1, where \( st = 0, 1 \) and 2 are the failure, operating and PM states of the machine respectively.

Two cases are considered. In the first case, the transitions amongst the three states are considered exponential. For such a machine, the PM feasibility conditions and the
robustness conditions are investigated, and the optimal rate of PM, i.e., the optimal rate of going from \( st = 1 \) to \( st = 2 \), is obtained analytically. Next, a machine is considered with exponential transitions between \( st = 1 \) and \( st = 0 \) and deterministic transitions between \( st = 1 \) and \( st = 2 \). This results in an exponential machine with deterministic PM; such a model may be applicable in practical situations, where a fixed procedure is repeated for maintenance. For such a machine, it is shown by simulations, that the results obtained for the exponential machine with exponential PM, remain valid.

![Figure 2.1: Flow diagram of Exponential Machines with PM](image)

The results are obtained in the framework of a model postulating PM-reliability coupling. According to this coupling, the machine breakdown rate and PM rate are inversely proportional to each other; i.e., related hyperbolically. Thus, the main contribution of this chapter is the theoretical study of introducing a third state of PM to the conventional exponential machine model, along with the PM-reliability coupling.

As far as prior work is concerned, the current literature contains hundreds of publications on PM, alongside numerous important results on PM scheduling and part replacement (see Pierskalla and Voelker [1976]; Gertsbakh [1977, 2005]; Valdez-Flores and Feldman [1989]; Dekker [1996]; Wang [2002]; Nakagawa [2005]; Jardine
and Tsang [2006]). In the context of manufacturing system performance, optimal maintenance policies for single machine systems have been discussed in (Charepnsuk et al. [1997]; Endrenyi et al. [1998]; Chen and Trivedi [2002]; Chan and Asgarpoor [2006]). Among these, Charepnsuk et al. [1997] studied the optimal PM interval that balances expected system cost versus its reliability, while Endrenyi et al. [1998], Chen and Trivedi [2002], and Chan and Asgarpoor [2006] developed methods for evaluating and optimizing PM policies in machines with multiple degradation states.

PM in two-machine production systems was studied in (Schouten and Vanneste [1995]; Meller and Kim [1996]; Kyriakidis and Dimitrakos [2006]). Specifically, Schouten and Vanneste [1995] and Kyriakidis and Dimitrakos [2006] analyzed an optimal PM policy, which minimized the system cost, and Meller and Kim [1996] investigated the optimal inventory level that triggered PM. For production lines with multiple machines, Sun et al. [2008] developed a method to determine the optimal PM strategy for serial lines with no intermediate buffers. Recently, Ambani et al. [2009] developed prioritization based approaches for multiple machine system with multiple degradation states and no intermediate buffers. In spite of these important results, the posed questions listed above, have not been answered in literature.

The outline of this chapter is as follows: Section 2.2 models exponential machines with exponential PM. Section 2.3 analyzes the performance of an exponential PM machine, deriving the PM feasibility condition and the optimal rate of PM. An exponential machine with deterministic inter-PM time is investigated in Section 2.4. The final section provides the conclusions for this chapter.
2.2 Modeling an Exponential Machine with Exponential PM

Figure 2.1 shows the diagram of an exponential machine with exponential PM, where the states $st = 0, 1$ and $2$ are the down, operating and PM states of the machine respectively.

The transition rate from $st = 1$ to $st = 0$ is given by $\lambda$, which is the rate of breakdown for the machine. The transition rate from $st = 0$ to $st = 1$ is given by $\mu_0$, which is the rate of repair for the machine. The transition rates from $st = 1$ to $st = 2$ and vice versa are given by $\lambda_m$ and $\mu_m$ respectively, indicating the rate of transition from the operating state to the maintenance state and vice versa. The subscript “m” in the rates $\lambda_m$ and $\mu_m$ indicates maintenance.

Of the above defined rates, only the rate of breakdown ($\lambda$) is influenced by the rate of PM. The average time required to maintain the machine (or the average time spent in $st = 2$) and the average time required to repair the machine (or the average time spent in $st = 0$) are both independent of the rate of PM, and are given by

\[
T_{PM} = \frac{1}{\mu_m}, \quad (2.4)
\]
\[
T_{Repair} = \frac{1}{\mu_0}. \quad (2.5)
\]

When no PM takes place, i.e., when $\lambda_m = 0$, it is assumed that $\lambda = \lambda_0$. For values of $\lambda_m > 0$, i.e., when PM is present, $\lambda \leq \lambda_0$, i.e., in presence of PM, the rate of breakdown decreases. As the rate of maintenance increases, the rate of breakdown decreases, tending towards zero for extremely large rate of PM. This relation is modeled using the coupling
\[ \lambda(\lambda_m + \alpha) = \beta, \quad (2.6) \]

where \( \alpha \) and \( \beta \) are positive constants, and illustrated using Figure 2.2. For the graph shown in Figure 2.2, \( \beta \) mainly influences the concavity of the curve and \( \alpha \) influences the initial slope of the curve at \( \lambda_m = 0 \). Since \( \lambda_m = 0 \) implies \( \lambda = \lambda_0 \), Equation (2.6) provides a relation for \( \beta \) as

\[ \beta = \lambda_0 \alpha. \quad (2.7) \]

![Figure 2.2: PM-Reliability Coupling for Machine](image)

To quantify \( \alpha \), the notion of machine sensitivity to PM is introduced as

\[ S = \left| \frac{d\lambda}{d\lambda_m} \right|_{\lambda_m=0}, \quad (2.8) \]

where, \( S \) quantifies the effect of infrequent PM on the machine uptime. Typically, \( S \)
may be determined experimentally or evaluated based on the opinion of equipment specialists. As seen later in this chapter, even “rough” knowledge of $S$ is sufficient to determine whether PM is feasible or not. Using Equation (2.6) and Equation (2.8)

$$S = \left| \frac{\beta}{(\lambda_m + \alpha)^2} \right|_{\lambda_m=0},$$

$\Leftrightarrow S = \frac{\beta}{\alpha^2}.$ \hfill (2.9)

Using Equations (2.6, 2.7 and 2.9)

$$\alpha = \frac{\lambda_0}{S}, \quad (2.10)$$

using which, Equation (2.6) becomes

$$\lambda \left( \lambda_m + \frac{\lambda_0}{S} \right) = \frac{\lambda_0^2}{S}, \quad (2.11)$$

or, equivalently

$$\lambda = \frac{\lambda_0^2}{\lambda_m S + \lambda_0}. \quad (2.12)$$

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The above relationship is referred to as the *PM-reliability coupling*. As far as the relationship between $\mu_0$ and $\mu_m$ is concerned, various assumptions can be made. Typically, $\mu_m \geq \mu_0$ i.e., $T_m \leq T_{\text{down}}$, or in other words, average time required to maintain a machine is less than the average time required to repair a machine. However, since relationships between $T_m$ and $T_{\text{down}}$ are “machine-specific”, the issue is not pursued further, and the efficacy of PM in terms of the downtime ratio is quantified as

$$r = \frac{T_{\text{down}}}{T_m} = \frac{\mu_m}{\mu_0}, r \geq 1.$$  \hspace{1cm} (2.13)

Thus, an exponential machine with exponential PM is defined by $(\lambda_0, \mu_0, S, r, \lambda_m)$ along with Equation (2.12).

**2.3 Performance Analysis and Optimization of an Exponential Machine with Exponential PM**

**2.3.1 Performance Analysis**

The exponential machine with exponential PM, forms a continuous time Markov chain (CTMC), with three states (operating, down and PM). Since the states are communicating and there are “self-loops”, the Markov chain is ergodic. Therefore, there exists a unique steady state probability distribution, which can be obtained using the system’s balance equations Ross [1996]. Let $P_i, i \in \{0, 1, 2\}$ be the steady state probability of the $i$th state of the machine, then balance equations for the exponential machine with exponential PM can be written as
\[ P_0 \mu_0 = P_1 \lambda, \quad (2.14) \]
\[ P_2 \mu_m = P_1 \lambda_m, \quad (2.15) \]
\[ P_1(\lambda + \lambda_m) = P_0 \mu_0 + P_2 \mu_m, \quad (2.16) \]

and the normalization equation can be written as,

\[ P_0 + P_1 + P_2 = 1. \quad (2.17) \]

On solving Equations (2.14- 2.17) we get

\[ P_0 = \frac{\lambda}{\mu_0} \frac{1}{1 + (\frac{\lambda_m}{\mu_m}) + (\frac{\lambda}{\mu_0})}, \quad (2.18) \]
\[ P_1 = \frac{1}{1 + (\frac{\lambda_m}{\mu_m}) + (\frac{\lambda}{\mu_0})}, \quad (2.19) \]
\[ P_2 = \frac{\lambda_m}{\mu_m} \frac{1}{1 + (\frac{\lambda_m}{\mu_m}) + (\frac{\lambda}{\mu_0})}. \quad (2.20) \]

The steady state probability \( P_i \) of the system can be interpreted as: 1) the proportion of time spent by the machine in state \( i \), while the machine is in steady state or 2) the probability of finding the machine in state \( i \) at a given instance of time, while the system is in steady state. Since the machine produces only in the operating state, i.e., when \( s = 1 \), \( P_1 \) is the proportion of time the machine produces parts.
defined as the efficiency of the machine. Let $e_{PM}$ denote the efficiency of the machine in presence of PM, then $e_{PM}$ can be evaluated as

$$e_{PM} = P_1 = \frac{1}{1 + \left(\frac{\lambda_m}{\mu_m}\right) + \left(\frac{\lambda_0}{\mu_0}\right)}, \quad (2.21)$$

$$= 1/ \left(1 + \frac{\lambda_0^2}{\mu_0(\lambda_mS + \lambda_0)} + \frac{\lambda_m}{r\mu_0}\right). \quad (2.22)$$

The behavior of $e_{PM}$ as a function of $\lambda_m$ for different values of $r, S$ and $\mu_0/\lambda_0$ is illustrated in Figure 2.3. As seen from Equation (2.22) and Figure 2.3, $e_{PM}(\lambda_0, \mu_0, S, r, \lambda_m)$ monotonically increases in $\mu_0, S$ and $r$, and decreases in $\lambda_0$. With respect to $\lambda_m$, two patterns are possible: monotonically decreasing and non-monotonic concave. Clearly, in the former case, PM does not lead to machine efficiency improvement, while in the latter there is a range of $\lambda_m$ for which $e_{PM} > e$. This leads to issues of PM feasibility and optimization, which are investigated in the following subsection.

2.3.2 PM Feasibility and Robustness

**Definition 1**: For a given machine, PM is feasible, if the machine efficiency with PM is greater than without PM.

**Theorem 1**: For an exponential machine, exponential PM is feasible if and only if

$$rS > 1 \quad (2.23)$$

**Proof**: As defined, PM is feasible, if $e_{PM} > e$, for $\lambda_m > 0$; i.e.,

30
Figure 2.3: Impact of $\lambda_m$ on $e_{PM}$
\[
\frac{1}{1 + \left(\frac{\lambda_m}{\mu} \right) + \left(\frac{\lambda_0}{\mu} \right)} > \frac{1}{1 + \left(\frac{\lambda_0}{\mu} \right)}
\]

\[\iff \frac{\lambda_m}{r\mu_0} + \frac{\lambda_0^2}{\mu_0(\lambda_mS + \lambda_0)} < \frac{\lambda_0}{\mu_0}\]

\[\iff \lambda_m(\lambda_mS + \lambda_0) + r\lambda_0^2 < rS\lambda_m\lambda_0 + r\lambda_0^2\]

\[\iff \lambda_m < \frac{\lambda_0 rS - 1}{S}.\]

Thus, \(\lambda_m > 0\) exists, if and only if \(rS > 1\).

The feasibility domain for the above condition in the \((S, r)\) plane is given by \(rS = 1\), as shown in Figure 2.4, where for every pair \((S, r)\) above the boundary, exponential PM is feasible and below is not. Since, any point in the domain represents feasibility, even if \(S\) and \(r\) are not precisely known, but if the point \((S, r)\) is sufficiently far from the boundary, PM remains feasible and would improve efficiency of the machine. This implies PM feasibility is robust with respect to \(S\) and \(r\), as long as the point lies well within the feasibility domain.

![Feasibility Domain for Exponential Machines](image)
2.3.3 Optimization

**Theorem 2**: Consider an exponential machine with PM-reliability coupling as defined by Equation (2.6) and $rS > 1$. Its efficiency is maximized if

$$\lambda_m = \lambda^*_m = \lambda_0 \frac{\sqrt{rS} - 1}{S}. \quad (2.25)$$

The resulting machine breakdown rate, $\lambda^*$ and the optimal machine efficiency, $e^*_P M$, are

$$\lambda^* = \frac{\lambda_0}{\sqrt{rS}}, \quad (2.26)$$

$$e^*_P M = \frac{1}{1 + \frac{T_{down}}{T_{up}} \cdot \frac{2\sqrt{rS}-1}{rS}}. \quad (2.27)$$

where $T_{up} = \frac{1}{\mu_0}$ and $T_{down} = \frac{1}{\mu_0}$

**Proof**: On differentiating Equation (2.22) we obtain:

$$\frac{de_{PM}}{d\lambda_m} = \frac{e^2}{\mu_0} \left( \frac{\lambda_0^2}{S(\lambda_m + \lambda_0/S)^2} - \frac{1}{r} \right).$$

Hence, the optimality condition is given by

$$\frac{\lambda_0^2}{S(\lambda_m + \lambda_0/S)^2} = \frac{1}{r},$$

and $e_{PM}$ is maximized by

$$\lambda_m^* = \lambda_0 \frac{\sqrt{rS} - 1}{S}, rS > 1.$$
Substituting $\lambda^*_m$ in Equation (2.21) the optimal efficiency of the machine with optimal PM is obtained as:

$$e^*_PM = 1/\left(1 + \frac{\lambda_0}{\mu_0} \times \frac{2\sqrt{rS} - 1}{rS}\right)$$

$$= 1/\left(1 + \frac{T_{\text{down}}}{T_{\text{up}}} \times \frac{2\sqrt{rS} - 1}{rS}\right)$$

As seen from the PM optimized machine efficiency, $e^*_PM$, and the breakdown rate, $\lambda^*$, both quantities depend on the product of $rS$ rather than on $r$ or $S$ separately, i.e., the sensitivity of $e^*_PM$ to $r$ and $S$ is identical.

2.4 Modeling and Analysis of Exponential Machines with Deterministic PM

Consider an exponential machine with PM, but having deterministic transitions between $st = 1$ and $st = 2$ instead of exponential; i.e., the machine is stopped for maintenance after a fixed period of operation time (either continuous or interrupted by breakdowns) undergoing maintenance for a fixed interval of time. An example of such a policy would be conducting an hour of maintenance on a machine every Monday. The deterministic time between two successive PM activities is referred to as the inter-PM time and denoted by $T^d_{\text{inter-PM}}$, while the deterministic time required to maintain is denoted by $T^d_m$. Thus, an exponential machine with deterministic PM is defined by $(\lambda, \mu_0, T^d_{\text{inter-PM}}, T^d_m)$. It is shown that the efficiency of an exponential machine with deterministic PM is practically the same as that of the corresponding exponential machine with exponential PM.
To accomplish the above, consider an exponential machine with exponential PM defined by \((\lambda_0, \mu_0, S, r, \lambda_m)\) and the PM-reliability coupling given by Equation (2.12). Along with it, consider an exponential machine with deterministic PM defined by \((\lambda, \mu_0, T_{inter-PM}^d, T_m^d)\), where:

\[
\lambda = \frac{\lambda_0^2}{\lambda_m S + \lambda_0}, \tag{2.28}
\]

\[
T_{inter-PM}^d = T_{to-PM}^d = \frac{1}{\lambda_m}, \tag{2.29}
\]

\[
T_m^d = T_m = \frac{1}{\mu_m}, \mu_m = r\mu_0. \tag{2.30}
\]

Equation (2.28) represents the PM-reliability coupling similar to that of exponential machines. Equations (2.29 and 2.30) set the deterministic inter-PM time and deterministic time required to maintain equal to the mean inter-PM time and mean time to maintain of the corresponding exponential machine model with exponential PM. Denote the efficiency of the exponential machine with exponential PM as \(e_{PM}(\lambda_0, \mu_0, S, r, \lambda_m)\) and the efficiency of the exponential machine with deterministic PM as \(e_{dPM}(\lambda, \lambda_0, T_{inter-PM}^d, T_m^d)\).

**Numerical Fact:** Under the above described conditions (Equations (2.28-2.30)), the efficiency of a deterministic PM machine is almost same as efficiency of an exponential PM machine, i.e.,

\[
\frac{|e_{PM}(\lambda_0, \mu_0, S, r, \lambda_m) - e_{dPM}(\lambda, \mu_0, T_{inter-PM}^d, T_m^d)|}{e_{PM}(\lambda_0, \mu_0, S, r, \lambda_m)} = \delta << 1. \tag{2.31}
\]
Justification of Numerical Fact: To justify the numerical fact, a total of 50,000 machines were investigated with parameters $e, \mu_0, S, r, \lambda_m$ selected randomly and equiprobably from the sets:

$$e \in (0.65, 0.95)$$  \hspace{2cm} (2.32)

$$\mu_0 \in (0.02, 0.5),$$

$$r \in (1, 5),$$

$$S \in (0.1, 5),$$

$$\lambda_m \in (0.01, 0.5).$$

These numbers are chosen to replicate practical situations arising in automotive industry. For each machine $\lambda_0$ is evaluated as

$$\lambda_0 = \mu_0 \left( \frac{1}{e} - 1 \right),$$  \hspace{2cm} (2.33)

then $e_{PM}(\lambda_0, \mu_0, S, r, \lambda_m)$ is calculated using the analytical approach using Equation (2.21), while $e_{dPM}^d(\lambda, \mu_0, T_{inter-PM}^d, T_m^d)$ is evaluated using MATLAB code simulating the machine as follows:

**Simulation Procedure**

1. The initial state of each machine is selected up ($st = 1$) with probability $e$ and down ($st = 0$) with probability $1 - e$.

2. For each system under consideration, 20 runs of the simulation code are carried out.
3. In each run, the first 20,000 time slots are used as a warm-up period and the subsequent 200,000 time slots are used to statistically evaluate the performance measures of interest.

4. This results in the estimate of $e_{PM}^d$ with 95% confidence interval 0.001

Equation (2.31) is found to be true for parameters selected randomly from appropriate sets for 50,000 exponential machines and the average, minimum, maximum and standard deviation of $\delta$, are found to be 0.0012, 0.0002, 0.0072, and 0.0009, respectively. Thus, the efficiency of an exponential machine with deterministic PM is practically identical to the efficiency of a corresponding machine with exponential PM. More importantly, the efficiency of the optimal PM exponential machine model is equivalent to the efficiency of the equivalent deterministic model.

2.5 Conclusions

In this chapter, a conventional two state exponential machine model is extended to a three state exponential machine model, by differentiating between a machine’s planned (PM) and unplanned (breakdown state) downtime. A coupling relation between the break down rate and PM rate of the machine is introduced with the help of two physical parameters $r$ (downtime ratio) and $S$ (sensitivity of breakdown rate with respect to preventive maintenance). For such a machine, the feasibility criterion for maintenance is derived as $rS > 1$. It is shown that as long as the machine is well within the feasible region, the feasibility criterion of the machine is robust with respect to $r$ and $S$. The optimal rate of PM and the corresponding optimal efficiency for a machine are also derived. Finally, the efficiency of an exponential machine with exponential PM is found to be almost identical to the efficiency of a corresponding exponential machine with deterministic PM. The next chapter focuses on estimating
the throughput distribution for a system consisting of two machines and one buffer, with machines capable of having multiple states and coupling relations.
CHAPTER III

Two Machine One Buffer System

3.1 Introduction

The previous chapter extended a two state exponential machine model to a three state exponential model, to include PM with coupled rates. The current and the next chapters focus on developing an approach to estimate the throughput distribution and the short term performance measures of a serial manufacturing system, with exponential multi state machines.

Throughput is defined as the number of parts produced by the last machine of a manufacturing system over a given period of time. Many analytical models have been proposed to estimate the throughput of a manufacturing system with unreliable machines and finite buffers. Due to uncertainties, such as machine failures, the throughput of a manufacturing system is considered a random variable. A summary of the throughput analysis models developed over the past 50 years can be found in the following reviews (Koenigsberg [1959]; Buxey et al. [1973]; Dallery and Gershwin [1992]; Govil and Fu [1996]; Papadopoulos and Heavey [1996]; Li et al. [2009]). Although substantial work has been done in the area of throughput analysis, three major limitations still exist:

1. Most existing manufacturing system models estimate only the average (or ex-
pected) throughput of a system, but not the complete throughput (probability) distribution.

2. Throughput analysis of manufacturing systems is primarily carried out only for systems already in their steady state.

3. Most manufacturing system models consider machines as only two state entities.

Gershwin [1994], through factory observations and simulations, reported that the standard deviation associated with the weekly production of a manufacturing system might be over 10% of its mean. Meerkov and Zhang [2008] in their recent study of transient analysis of manufacturing systems, stated that a manufacturing system with cycle time of 1 minute, initially empty buffers and operating time of 8 hours (1 shift), could lose up to 10% of the estimated throughput within the given shift. Such large variations in throughput makes study of throughput distribution necessary for effective short-term and long-term production and maintenance decision making.

Silva et al. [1986] first derived the operational time distribution of a single repairable system. Dincer and Deler [2000] studied the throughput distribution of a production system with 100% reliable machines and exponential operating times. Tan [1997, 1999] obtained the variation of a N-station manufacturing system with no intermediate buffers and time dependent failures, and further obtained the variance of the output of a single machine and a two machine one buffer system as a function of time. Csenki [2007] studied the joint interval reliability for Markov systems with an application to transmission line reliability. Although some initial work has been done, the throughput distribution of a manufacturing system with unreliable machines and finite buffers has not yet been studied.

Exact analytical results for manufacturing systems exist only for three types of
systems: 1) two machine one buffer systems with unreliable machines and a finite buffer, 2) manufacturing systems with infinite or no buffers and 3) systems with 100% reliable machines. Longer manufacturing systems or assembly line models with unreliable machines and finite buffers, are typically approximated using decomposition (Gershwin [1989]) or aggregation (Li and Meerkov [2009]) based approaches.

Within a manufacturing system, a two machine one buffer segment captures the fundamental principles of machine-buffer interactions. Further, each of the two machine one buffer segments only “loosely” interact with the rest of the manufacturing system. The concept of loose interaction is further introduced in the next chapter. Based on the above motivations, a two machine one buffer segment is modeled in this chapter. Chapter 4 extends the two machine one buffer model to a serial manufacturing system.

Most manufacturing system models consider machines to have only two states: up (operating) and down (failure or repair). In the previous chapter, a three state machine model was introduced, taking into consideration PM and rate coupling. In a parallel research area, condition based maintenance (CBM) decision making models have evolved rapidly to incorporate multiple up and down states (Chen and Trivedi [2002]; Marseguerra et al. [2002]; Chen and Trivedi [2005]; Castanier et al. [2005]; Chan and Asgarpoor [2006]; Ambani et al. [2009]), depicting machine degradation and multiple modes of failure. A major limitation of the existing CBM models is their focus on only single and two machine systems. The machine-buffer interactions for multi state machines has not been studied.

In this chapter, we analytically estimate the throughput distribution of a two machine one buffer system, consisting of machines with multiple states. The throughput distribution is further used to estimate the following performance measures: average
throughput, probability of meeting given demand and the risk associated with a given
demand. The developed approach is illustrated using two examples and the benefits
of using this analytical approach over simulation based approach is also illustrated.

3.2 Modeling

3.2.1 Single Machine Modeling

Consider an exponential machine model, with multiple up and down states, as
shown in Figure 3.1. Such a machine is represented by a CTMC. Let the first $K$
states of the machine be operative (up) and remaining $(N - K)$ states of the machine
be non operative states (down). A state is defined as an operative state of the
machine, if it is capable of processing parts in the given state. On the other hand,
a non operative state of the machine is incapable of producing parts, for example
failure, maintenance, repair states etc. Let $Q$ be the rate transition matrix for the
CTMC, where the $ij$th element of the matrix represents the rate of moving from state
$i$ to state $j$.

![Figure 3.1: Multi State Exponential Machine Model](image)

Let the deterministic cycle time of the machine be $\tau$. Under the assumption of $\tau$
being small compared to the mean time between failure (MTBF) and mean time to
repair (MTTR) of the machine, let $\Lambda = 1/\tau$ be the discretization rate constant for the CTMC. Let $I$ be an identity matrix of dimension $N$ and $P$ be the discretized transition matrix for the described CTMC, describing the dynamics of the machine over a given cycle. Using uniformization (Ross [1996]), the CTMC ($Q$) can be converted to a discrete time Markov chain (DTMC) using the discretization constant $\Lambda$, as shown in Equation (3.1), where $P$ is the discrete time transition matrix of the system. Let $p_{mj}$ represent the $ij$th element of the discretized transition matrix ($P$), where “m” stands for machine.

$$P = I + Q/\Lambda$$

Let $x_j(k)$ be the probability of the machine being in the $j$th state during the $k$th cycle, then $x(k) = [x_0(k), x_1(k), ..., x_N(k)]$ represents the overall $N$-dimensional state vector for the machine at the end of $k$ cycles. Using the DTMC transition matrix ($P$), the evolution of the machine state vector over a given cycle can be written as

$$x(k+1) = x(k)P,$$

and hence, the state of the machine at the end of $k$ cycles, given an initial condition $x(0)$ can be written as

$$x(k) = x(0)P^k.$$
3.2.1.1 Illustration

Consider a condition monitored machine, consisting of three degradation states and one failure state, i.e., $N = 4$ and $K = 3$, as shown in Figure 3.2. Let $st = 1$ depict an “as good as new” state, $st = 2$ a degraded state, $st = 3$ a very degraded state and $st = 4$ the failure state of the machine. Let $\lambda_1$ and $\lambda_2$ represent the rate of degradation from $st = 1$ to $st = 2$ and from $st = 2$ to $st = 3$ respectively. Let $\lambda_3$ and $\lambda_4$ be the rate of failure and rate of repair for the given machine. Let the machine at $t = 0$ be in the as good as new state, then

$$x(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

$$Q = \begin{pmatrix} -\lambda_1 & \lambda_1 & 0 & 0 \\ 0 & -\lambda_2 & \lambda_2 & 0 \\ 0 & 0 & -\lambda_3 & \lambda_3 \\ \lambda_4 & 0 & 0 & -\lambda_4 \end{pmatrix}$$

Figure 3.2: Degrading Machine

Let the cycle time for this machine be $\tau$, hence the discretization constant is given by $\Lambda = 1/\tau$. Hence, the discretized transition matrix can then be obtained by
\[
P = \begin{pmatrix}
1 - \lambda_1/\Lambda & \lambda_1/\Lambda & 0 & 0 \\
0 & 1 - \lambda_2/\Lambda & \lambda_2/\Lambda & 0 \\
0 & 0 & 1 - \lambda_3/\Lambda & \lambda_3/\Lambda \\
\lambda_4/\Lambda & 0 & 0 & 1 - \lambda_4/\Lambda
\end{pmatrix},
\]

The state of the machine at the end of \( k \)th cycle for the given system can hence be obtained using,

\[
x(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} P^k.
\]

### 3.2.2 Two Machine One Buffer Modeling

Figure 3.3 shows a two machine one buffer system, with machines consisting of multiple up and down states. Let \( M^\gamma \) be the upstream machine of the system consisting of \( N^\gamma \) states, of which, first \( K^\gamma \) states are operating (up). Similarly, let \( M^\delta \) represent the downstream machine of the system with \( N^\delta \) states, of which, first \( K^\delta \) states are up. Let \( B \) represent the buffer having a buffer capacity of \( L \).

Let \( x_j^\gamma(k) \) and \( x_j^\delta(k) \) represent the probability of the upstream and downstream machine being in the \( j \)th state during the \( k \)th cycle respectively, and \( y_j(k) \) represent the probability of having \( j - 1 \) parts in the buffer during the \( k \)th cycle. Let \( M^\gamma \) and \( M^\delta \) be the sets containing all possible states of \( M^\gamma \) and \( M^\delta \) respectively. Let \( U^\gamma \) and \( D^\gamma \) be the set of up and down states of the machine \( M^\gamma \), and similarly let \( U^\delta \) and \( D^\delta \) be the set of up and down states of the machine \( M^\delta \). The above definitions can then be stated mathematically as
Let $s(k) = \{x^\gamma(k), x^\delta(k), y(k)\}$ represent a state of the two machine one buffer system at the beginning of the $k$th cycle, where $x^\gamma(k)$ represents the state vector of the upstream machine, $x^\delta(k)$ represents the state vector of the downstream machine and $y(k)$ represents the state vector of the buffer at the beginning of the $k$th cycle. Let $S$ be the set containing all possible states of the system ($s \in S$), then the cardinality of $S$ is $N^\gamma \times N^\delta \times (L + 1)$.

Let the constant deterministic cycle time for each machine be $\tau$. It is assumed
that $\tau$ is small as compared to the mean time between failures and the mean time to repair of the machines. Since cycle time is relatively small, the state of a machine is defined using only the condition of the machine at the beginning of the cycle, i.e., a machine is considered to be in an up or down state during a given cycle, if the machine is in an up or down state at the beginning of the cycle respectively. The states of the two machines are determined independent of each other, according to the machine evolution equations described in Section 3.2.1.

The evolution of the buffer depends on the conditions of the machines during a given cycle, and follow the principles:

1. If $M^\gamma$ and $M^\delta$ are both up or if $M^\gamma$ and $M^\delta$ are both down, the buffer contents do not change

2. If $M^\gamma$ is down and $M^\delta$ is up and if the buffer is not empty, then the buffer contents decrease by one

3. If $M^\gamma$ is down and $M^\delta$ is up and if the buffer is empty, then the buffer continues to remain empty

4. If $M^\delta$ is down and $M^\gamma$ is up and the buffer is not full, then the buffer contents increase by one

5. If $M^\delta$ is down and $M^\gamma$ is up and the buffer is full, then the buffer continues to remain full

Let $PU^\gamma(k)$ and $PD^\gamma(k)$ be the probability of $M^\gamma$ being in an up state or down state respectively, and similarly let $PU^\delta(k)$ and $PD^\delta(k)$ be the probability of $M^\delta$ being in an up or down state respectively, during the $k$th cycle. These quantities are obtained by Equations (3.4-3.6).
\[ PU^\gamma(k) = \sum_{i=1}^{K^\gamma} x_i^\gamma(k), \quad (3.4) \]
\[ PD^\gamma(k) = \sum_{i=K^\gamma+1}^{N_j} x_i^\gamma(k) \quad (3.5) \]
\[ PU^\delta(k) = \sum_{i=1}^{K^\delta} x_i^\delta(k), \]
\[ PD^\delta(k) = \sum_{i=k^\delta+1}^{N_j} x_i^\delta(k) \quad (3.6) \]

To avoid external effects, it is assumed that the first (upstream) machine of the system never starves, i.e., no shortage of raw material, and the last (downstream) machine never blocks, i.e., there is enough storage capacity or very large demand to consume every part produced immediately. Let \( P \) represent the transition matrix for the buffer and \( pb_{ij} \) represent the \( ij \)th element of this transition matrix, where “b” stands for buffer. Based on the principles of buffer evolution and assuming no external effects, the buffer evolution over the \( k \)th cycle can be written as Equation (3.7).

\[ y(k+1) = y(k)P(k), \quad (3.7) \]

where
\[ P(k) = \begin{pmatrix}
\bar{A} & A & 0 & \ldots & 0 & 0 \\
\bar{A}.B & A.B + \bar{A}.\bar{B} & A.\bar{B} & 0 & \ldots & 0 \\
0 & \bar{A}.B & A.B + \bar{A}.\bar{B} & A.\bar{B} & \ddots & 0 \\
0 & 0 & \ddots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \bar{A}.B & A.B + \bar{A}.\bar{B} & A.\bar{B} \\
0 & 0 & \ldots & 0 & \bar{A}.B & A.B + A.\bar{B} + \bar{A}.\bar{B}
\end{pmatrix}, \]

and \( A = PU^\gamma(k), \bar{A} = PD^\gamma(k), B = PU^\delta(k) \) and \( \bar{B} = PD^\delta(k) \).

Let \( \mathbb{P} \) represent the overall system transition matrix for the two machine one buffer system. Let \( ps_{ij}(k) \), where “s” stands for system, represent the \( ij \)th element of the system transition matrix \( \mathbb{P}(k) \), representing the probability of the system going from state \( s_i(k) = \{ x_i(k), x_i^s(k), y_i(k) \} \) to \( s_j(k) = \{ x_j^\gamma(k), x_j^\delta(k), y_j(k) \} \) during the \( k \)th cycle. As the two machines degrade independently and the evolution of buffer depends on the initial conditions of the machines, \( ps_{ij} \) can be written as

\[
ps_{ij}(k) = pm_{x_i^\gamma x_j^\gamma}(k).pm_{x_i^\delta x_j^\delta}(k).pb_{y_i y_j}(k). \quad (3.8)
\]

For a given two machine one buffer system, a part is produced during the \( k \)th cycle, if two conditions are satisfied. First, the buffer is non empty at the beginning of the \( k \)th cycle and second, the downstream machine is in an up state at the beginning of the \( k \)th cycle. If these conditions are satisfied, the system is defined to be in a producing state, else a non producing state. Based on this definition, the overall states of the system can be partitioned into producing states (\( S_\alpha \)) and non producing states (\( S_\beta \)).
Similarly, the system transition matrix $P$ can be partitioned into $P_{\alpha\alpha}$, $P_{\alpha\beta}$, $P_{\beta\alpha}$ and $P_{\beta\beta}$, where, for example $P_{\alpha\beta}$ represents the partition of the transition matrix taking the system from producing states to non producing states.

### 3.2.3 Aggregation Approach

Figure 3.4 shows an aggregation tree for the two machine one buffer system. Let $D(a, b)$ be a node of the aggregation tree, satisfying the condition that the system has produced $a$ parts in $b$ cycles. The components of this node are given as $\text{th}_{D(a, b)\alpha}$ and $\text{th}_{D(a, b)\beta}$, as shown in Figure 3.5, where $\text{th}_{D(a, b)\alpha}$ and $\text{th}_{D(a, b)\beta}$ represent the producing and non producing states of the system respectively, satisfying the condition $D(a, b)$.

At the completion of each cycle, a new row is added to the aggregation tree. For example, a system having undergone $b$ cycles, would have an aggregation tree with $b + 1$ rows. As discussed later in this section, the evaluation of the components of a node in a particular row, only requires the information of the nodes in the previous row.

The aggregation procedure is carried out in three steps:

1. **Initialization**

   The aggregation tree is initiated with the node $D(0, 0)$ using the initial conditions of the system as shown in Equations (3.9-3.10)

   $\text{th}_{D(0, 0)\beta} = s_\beta(0), \quad (3.9)$

   $\text{th}_{D(0, 0)\alpha} = s_\alpha(0), \quad (3.10)$

2. **Iteration**

   50
Figure 3.4: Aggregation Tree
Figure 3.5: Components of an Aggregation Node

As shown in Figure 3.4, an intermediate node $D(a, b)$ can be reached only from two previous nodes. First, if the system had produced $a - 1$ parts in $b - 1$ cycles, i.e., system was in node $D(a - 1, b - 1)$, and produced a part in the $b$th cycle, and second, if the system had produced $a$ parts in $b - 1$ cycles, i.e., was in node $D(a, b - 1)$ and did not produce a part in the $b$th cycle. The components of the intermediate node $D(a, b)$ of the system are given by

$$\text{th}_{D(a,b)\alpha} = \text{th}_{D(a,b)\beta} \alpha$$

1. $D(a, b)$, i.e., has produced exactly $a$ parts in $b$ cycles
2. Producing Conditions, i.e., buffer has non zero parts and downstream machine is up

$$\text{th}_{D(a,b)\beta} = \text{th}_{D(a,b)\alpha} \beta$$

1. $D(a, b)$, i.e., has produced exactly $a$ parts in $b$ cycles
2. Non Producing Conditions, i.e., buffer has zero parts or downstream machine is down

(3.11) $s(b) = \{x^1(b), x^2(b), y(b)\} \in S$ satisfying:

$$\text{th}_{D(a,b)\alpha} = \text{th}_{D(a,b)\alpha} P_{\alpha} + \text{th}_{D(a,b)\beta} P_{\beta}, i \subseteq \{2, 3, ..., t_d\}, j \subseteq \{1, 2, ..., i - 1\}$$

(3.12) $s(b) = \{x^1(b), x^2(b), y(b)\} \in S$ satisfying:

$$\text{th}_{D(a,b)\beta} = t_{D(a,b)\alpha} P_{\alpha} + \text{th}_{D(a,b)\beta} P_{\beta}, i \subseteq \{2, 3, ..., t_d\}, j \subseteq \{1, 2, ..., i - 1\}$$

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The only possible way of reaching the node $D(0, b)$ for the system, is to have produced no parts till the $b-1$th cycle ($D(0, b-1)$) and further produce no part in the $b$th cycle. Hence, the system can arrive at $D(0, b)$ only from one previous node, as shown in Equations (3.13-3.14).

$$\text{th}_{D(0,i)\beta} = \text{th}_{D(0,i-1)\beta} \mathbb{P}_{\beta\beta}, i \subseteq \{1, 2, ..., t_d\} \quad (3.13)$$

$$\text{th}_{D(0,i)\alpha} = \text{th}_{D(0,i-1)\beta} \mathbb{P}_{\beta\alpha}, i \subseteq \{1, 2, ..., t_d\} \quad (3.14)$$

Similarly for the system to produce throughput in every cycle of the system, i.e., to be in node $D(b, b)$, the system should have produced parts in every previous $b-1$ cycles ($D(b-1, b-1)$) and should produce a part in the $b$th cycle, as shown in Equations (3.15-3.16).

$$\text{th}_{D(i,i)\alpha} = \text{th}_{D(i-1,i-1)\alpha} \mathbb{P}_{\alpha\alpha}, i \subseteq \{1, 2, ..., t_d\} \quad (3.15)$$

$$\text{th}_{D(i,i)\beta} = \text{th}_{D(i-1,i-1)\beta} \mathbb{P}_{\alpha\beta}, i \subseteq \{1, 2, ..., t_d\} \quad (3.16)$$

3. Throughput Evaluation

The iteration step is repeated till the desired number of cycles are completed.

On obtaining the desired row of the aggregation tree, the throughput distribution of the system is obtained by summing up components of the node of the aggregation tree. This is carried out as follows
\[ T_h(j, t_d) = \sum_\beta th_{D(j,t_d)\beta} + \sum_\alpha th_{D(j,t_d)\alpha}, j \subseteq \{0, t_d\}, \]

where \( T_h(j, t_d) \) is the probability of producing \( j \) parts in the desired \( t_d \) cycles.

From a memory perspective, the aggregation procedure has a significant benefit, as only one row of information needs to be saved for a given system. This also makes the computation of longer time intervals feasible, and the overall approach computationally light.

### 3.2.4 Performance Measures

Having obtained the throughput distribution of the given system, several internal and external performance measures for the system can be obtained. For this study, three performance measures are considered.

#### 3.2.4.1 Average Throughput

The average throughput of the system is defined as the average number of parts produced by the system over a given interval of time. Estimating average throughput is critical in the design as well as operations and decision making stages of a manufacturing system. Given the throughput distribution \( (T_h(j, t_d)) \) for a system, Equation (3.17) evaluates the average (expected) throughput of the given system in \( t_d \) cycles. On considering a steady state interval for a system consisting of machines with two states (up and down), the average throughput obtained by the developed approach is similar to the average throughput obtained by the decomposition and aggregation techniques.

\[ \Phi_1(t_d) = \sum_{x_i=0}^{t_d} x_i T_h(x_i, t_d) \]  

(3.17)
3.2.4.2 Probability of Meeting Given Demand

Average throughput provides an internal measure of performance for a manufacturing system. Along with internal performance measures, it is also critical to measure the performance of a system with respect to external factors, such as demand. Equation (3.18) estimates the probability for a given system manufacturing system to meet a customer demand of $d$ within $t_d$ cycles.

$$
\Phi_2(d, t_d) = \sum_{x_i=d}^{t_d} \text{Th}(x_i, t_d) \tag{3.18}
$$

3.2.4.3 Risk Associated with the Systems

Risk is defined as the average loss associated with a given system over a period of time. Often, higher rewards may be achieved at higher risks; hence, it is essential for a decision maker to be able to quantify the risk associated with a given system. Let $\Phi_3(d, t_d)$ be the risk associated with the given manufacturing system, facing a demand of $d$, over a period of $t_d$ cycles. Let $G(x_i, d)$ be defined as the penalty function associated with the system for producing $x_i$ parts, while demand is $d$. The risk associated ($\Phi_3(d, t_d)$) for such a system is evaluated as

$$
\Phi_3(d, t_d) = \sum_{x_i=0}^{d-1} G(x_i, d) \text{Th}(x_i, t_d). \tag{3.19}
$$

For a special case, where the penalty function is proportional (proportionality constant $\eta$) to the number of parts by which production falls short of demand, the risk associated with the given system is

$$
\Phi_3(d, t_d) = \sum_{x_i=0}^{d} \eta(d - x_i) \text{Th}(x_i, t_d). \tag{3.20}
$$
3.3 Case Studies

Two systems are considered to illustrate the approach of estimating the throughput distribution of a given system. The first system consists of machines with only two states (up and down). For this system, the impact of varying buffer capacity on performance measures of the system is studied. The second system consists of degrading machines, with multiple degradation states and a down state. For this system, the impact of initial conditions of machines on the performance measures of the system is studied.

3.3.1 Two State Machine System

3.3.1.1 Estimating Throughput Distribution and Performance Measures

Consider a two machine one buffer system, consisting of machines with only two states: up and down. The CTMC model for such a machine is as shown in Figure 1.6. Three buffer capacities considered for the system are: 1, 10 and 50. Let $\lambda_1$ and $\mu_1$ represent the rate of breakdown and rate of repair for machine 1, and similarly $\lambda_2$ and $\mu_2$ represent the rate of breakdown and rate of repair for machine 2, respectively. Let the duration of time considered be $t_d = 200$ cycles, demand ($d$) be 120, and the penalty for not fulfilling given demand ($\eta$) be $100$ per part. The machine parameters of the system are as follows: $\lambda_1 = 1/70, \mu_1 = 1/30, \lambda_2 = 1/60, \mu_2 = 1/20$. The initial conditions for the system are: \{M_1 = Up, M_2 = Up, Buffer = full\}. Using the developed approach, the throughput distribution for each of the three cases is obtained as shown in Figure 3.6. The performance measures for this system at different buffer capacities are listed in Table 3.1.

As expected, the throughput distribution for the system shifts to the right, with increase in buffer capacity, i.e., the probability of producing higher throughput is
greater for larger buffer capacities. Further, using the above approach the risk associated with the system while attempting to meet a given demand is also quantified. As seen, higher buffer capacities significantly increase the probability of meeting given demand and reduce the risk associated with the system. In a practical system, higher buffer capacity also leads to higher costs, hence a study to find an optimal buffer capacity would be of interest.

Table 3.1: Results for System Consisting of Machines with Two States

<table>
<thead>
<tr>
<th>Performance Systems</th>
<th>Buffer = 1</th>
<th>Buffer = 10</th>
<th>Buffer = 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Th ($\phi_1$)</td>
<td>113.14</td>
<td>122.84</td>
<td>143.26</td>
</tr>
<tr>
<td>Prob ($\phi_2$)</td>
<td>0.45</td>
<td>0.56</td>
<td>0.80</td>
</tr>
<tr>
<td>Risk ($\phi_3$)</td>
<td>$1853$</td>
<td>$1277$</td>
<td>$409$</td>
</tr>
</tbody>
</table>

3.3.1.2 Application: Buffer Design

Installing buffers in a manufacturing system is expensive as it requires space allocations and leads to a higher WIP. Lot of effort in manufacturing system research has gone towards minimizing buffer capacities or eliminating buffers. As the buffer size decreases, the impact of uncertainties, such as machine failures increases and leads to
higher risks (average loss). Hence, finding the right buffer size for a manufacturing system is critical.

Consider the above described system, with a demand of 120 parts over 200 cycles. Using the developed approach for varied buffer capacities, the relation between risk associated with a given system and its buffer capacities can be found. For the considered system, this relation is shown by Figure 3.7. It is observed that increasing buffer capacity leads to decrease in risk, and further, the decrease in risk is much higher at lower buffer capacities as compared to higher buffer capacities.

![Figure 3.7: Relation Between Risk and Buffer Capacity](image)

For a case, where a manufacturer intends to run the operations of a manufacturing within specific risk capacity, the developed relation provides the optimal buffer size to be included within the system. For example, if the manufacturer wished to operate the considered two machine one buffer line within a risk capacity of $1200, the minimum buffer capacity required for the system would be 12. Similar analysis can also be carried out to find the optimal buffer size to ensure a desired average throughput for the system.
3.3.2 Degrading Machine Systems

3.3.2.1 Estimating Throughput Distribution and Performance Measures

The second system considers a two machine one buffer system with degrading machines. The CTMC model for the machines is shown in Figure 3.8, wherein state 1 represents an as good as new state, state 2 represents a marginally degraded state and state 3 represents the most degraded state of the machine. On failing (reaching state 4), the machine is repaired and brought back to as good as new state.

For the considered two machine one buffer system, three initial conditions are considered: \( \{M_1 = 1, M_2 = 1, \text{ Buffer: full} \} \), \( \{M_1 = 1, M_2 = 3, \text{ Buffer: full} \} \) and \( \{M_1 = 3, M_2 = 3, \text{ Buffer: full} \} \). The rest of the system parameters are described as: \( \lambda_{11} = 1/30 \), \( \lambda_{12} = 1/30 \), \( \lambda_{13} = 1/30 \), \( \lambda_{14} = 1/30 \), \( \lambda_{21} = 1/20 \), \( \lambda_{22} = 1/20 \), \( \lambda_{23} = 1/20 \), \( \lambda_{24} = 1/30 \), \( d = 120 \), \( t_d = 200 \), \( \eta = 100 \), buffer capacity = 10, where \( \lambda_{ij} \) is the \( j \)th rate (as shown in figure) of machine \( i \).

The throughput distributions for the three scenarios are shown in Figure 3.9 and the performance measures for the same are listed in Table 3.2. Again, as expected, the throughput distribution for the system shifts towards the right as the condition of the machines improve, i.e., there is a higher probability of producing more parts when the machines are less degraded. Using the developed approach, the impact of condition of the machines on the overall throughput of a system can be studied.

<table>
<thead>
<tr>
<th>Cases</th>
<th>I.C. = {1,1,10}</th>
<th>I.C. = {1,3,10}</th>
<th>I.C. = {3,3,10}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Th (( \Phi_1 ))</td>
<td>127.25</td>
<td>114.35</td>
<td>108.07</td>
</tr>
<tr>
<td>Prob (( \Phi_2 ))</td>
<td>0.614</td>
<td>0.462</td>
<td>0.388</td>
</tr>
<tr>
<td>Risk (( \Phi_3 ))</td>
<td>$967</td>
<td>$1,649</td>
<td>$2,019</td>
</tr>
</tbody>
</table>
Figure 3.8: Degrading Machine

Figure 3.9: Throughput Distribution for Degrading System
3.3.2.2 Application: Risk Based Maintenance Decision Making

Maintenance actions are often modeled in manufacturing system literature as ways to improve the condition (degradation state) of a machine (Chen and Trivedi [2002]; Chan and Asgarpoor [2006]). In this example, a condition based maintenance policy is developed to ensure the operations of the considered system within a desired risk capacity.

As shown in the degrading machine case study, the risk associated with various initial conditions of a system can be evaluated using the developed approach. The total number of possible states for the system are $4 \times 4 \times 11$. Using the analytical approach, the risk associated with the system in each of the initial conditions for producing 120 parts over the next 200 cycles can be obtained. As shown in Figure 3.10 and Figure 3.11. Figure 3.10 shows the impact of different initial conditions of $M_2$ and buffer content on the risk associated with the system, assuming $M_1$ in state 3. From the plot it is observed that the condition of $M_2$ impacts the risk associated with the system more significantly as compared to the initial buffer contents. Figure 3.11 shows the impact of the initial conditions of the two machines on the risk associated with the system, with zero initial buffer contents.

For a given risk capacity of the system, using the above analysis, maintenance decisions can be made to ensure operations continue within desired risk capacity. For example, if the manufacturer wished to operate the manufacturing system within a risk capacity of $2000$, then either $M_1$ or $M_2$ should be maintained in the condition $\{3,3,\text{Empty} \}$, to reduce risk associated with the system.
Figure 3.10: Risk Associated with Degrading System (M1 =3)

Figure 3.11: Risk Associated with Degrading System (Buffer = empty)
3.3.3 Comparison with Simulation

3.3.3.1 Case 1

Another approach to obtain the distribution of a random variable (such as throughput) is by using monte carlo simulations. In such an approach, the system is set to the desired initial condition and allowed to evolve many times. The aggregate behavior of all runs is used to estimate the distribution of the desired random variable. The major drawbacks of this approach are the large set up and simulation times. In this section, the performance of analytically estimating the throughput distribution of a system is compared to the performance of a simulation based estimation approach.

Consider the two state machine system with a buffer capacity of 10, and parameters similar to Section 3.3.1. The discrete event simulation model for the considered system was run 20 times for 1,000, 10,000 and 50,000 iterations. The throughput distributions obtained by both analytical and simulation based approaches for one such run are provided in Figures (3.12-3.14), and the comparison of simulation based approach to analytical based approach with 95 % confidence interval is provided in Table 3.3.

From the results, it is observed that the accuracy and time required for simulations, greatly depend on the number of iterations. In almost all cases, the analytical approach estimates the throughput distribution of the given system more than an order of magnitude faster than simulation. It is expected that the benefits of using an analytical throughput distribution approach will be even greater for estimating performance of larger complex manufacturing systems.

Table 3.3: Comparison with simulations ($T_{\text{analytical}} = 0.28 \pm 0.03$ sec)

<table>
<thead>
<tr>
<th>$T_{\text{sim}}$</th>
<th>2.5 ± 0.6 sec</th>
<th>24.6 ± 1.8 sec</th>
<th>134.1 ± 2.1 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi_1 E_{\text{Err}}$</td>
<td>1.45 ± 0.6 %</td>
<td>0.76 ± 0.25 %</td>
<td>0.36 ± 0.15 %</td>
</tr>
<tr>
<td>$\Phi_2 E_{\text{Err}}$</td>
<td>2.15 ± 0.8 %</td>
<td>2.19 ± 0.76 %</td>
<td>0.7 ± 0.5 %</td>
</tr>
<tr>
<td>$\Phi_3 E_{\text{Err}}$</td>
<td>2.9 ± 0.5 %</td>
<td>1.99 ± 0.4 %</td>
<td>0.6 ± 0.3 %</td>
</tr>
</tbody>
</table>
Figure 3.12: Throughput Distributions (1000 Iterations)

Figure 3.13: Throughput Distributions (10000 Iterations)
3.3.3.2 Case 2

From the previous case, it is seen that simulation and analytical estimates are similar, but the analytical based estimation approach is almost an order of magnitude faster than the simulation based approach. To demonstrate this significant benefit, both the application case studies are analyzed using simulation and analytical based approach, and the time required to estimate throughput distributions was recorded. Table 3.4 shows the comparison of this time over 10 runs with 95 % confidence interval. From the table, it is seen that the analytical based estimation approach can provide significant benefits for decision making analysis.

<table>
<thead>
<tr>
<th>Application</th>
<th>Proposed Analytical Approach</th>
<th>Simulation Based Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buffer Design</td>
<td>$2.1 \pm 0.3$ sec</td>
<td>$250 \pm 15$ sec</td>
</tr>
<tr>
<td>Risk Based Maintenance</td>
<td>$69 \pm 15$ sec</td>
<td>$2345 \pm 205$ sec</td>
</tr>
</tbody>
</table>

Table 3.4: Comparison with Simulation (95% confidence interval)
3.4 Conclusions and Future Work

Three major limitations exist in conventional manufacturing system models. First, most manufacturing system models predict only the average (or expected) throughput of the system. Second, most studies consider only the long-term or steady state behavior of a system and finally, the machines are considered only as two state entities in manufacturing system modeling.

In this chapter, a method was developed to analytically estimate the throughput distribution of a two machine one buffer system, comprising of machines with multiple up and down states. Using this estimate, three performance measures are obtained: average (expected) throughput, probability of satisfying given demand and the risk associated with a given demand. The developed approach is illustrated using two machine one buffer models consisting of simple conventional two state machine models and multiple state degrading machine models. The performance of the analytical based estimation approach is compared with the performance of a simulation based approach. It was observed that the analytical based approach was almost an order of magnitude faster than the simulation based approach. Further benefits of using the proposed estimation approach for decision making analysis was also demonstrated using two application case studies. In the next chapter the developed approach is extended to a serial manufacturing system.
CHAPTER IV

Extension to Serial Manufacturing System

4.1 Introduction

Performance analysis of serial manufacturing systems has been studied for many years now. Most approaches developed in this area depend either on the decomposition approach or the aggregation approach (see, for instance, Gershwin [1989]; De Koster [1987, 1988]; Dallery et al. [1988]; Lim et al. [1990]; Jacobs and Meerkov [1995]; Le Bihan and Dallery [2000]; Li and Meerkov [2009]). The general idea of an aggregation approach is to replace two machine line by a single equivalent machine consisting of the same throughput in isolation. In decomposition approach, the original system is broken into smaller sub systems. Both these approaches have been studied only for the steady state analysis of systems, with an assumption of constant flow through each machine or the same effective throughput for each machine, which does not hold true for a non steady state system. This chapter focuses on the development of a new segmentation approach, which is capable of estimating the throughput distribution of a serial manufacturing system over any given period of time (including non steady state). The developed approach is capable of incorporating multi state machine models.
4.2 Segmentation

Consider a serial manufacturing system as shown in Figure 4.1. Let $M_i$ be the $i$th machine and $B_j$ be the $j$th buffer of the serial manufacturing system. Let the set \{\(M_i, B_i, M_{i+1}\)\} represent a part of the line consisting of $M_i$, $B_i$ and $M_{i+1}$. Using this definition, the manufacturing system is segmented as follows:

$$G^i = \begin{cases} \{M_i\} & \text{if } i = 1 \\ \{M_{i-1}, B_{i-1}, M_i\} & \text{if } i > 1 \end{cases}$$

where $G^i$ is defined as the $i$th segment of the manufacturing system.

The first segment of the system consists of only one machine, while the rest of the segments consist of two machines and a buffer. Figure 4.2 illustrates the segmentation of a four machine three buffer system.

Consider a segment $G^i$ ($i \geq 2$). Let $M^{i\gamma}$ and $M^{i\delta}$ represent the upstream and downstream machine of the segment $G^i$. From the perspective of the actual serial line, $M^{i\gamma}$ is the $i$-th machine ($M_{i-1}$) and $M^{i\delta}$ is the $i$th machine of the system ($M_i$). Let $B^i$ represent the buffer contained in segment $i$, which is the $i$-th buffer of the system ($B_{i-1}$). Further, let $c^i$ be the number of parts (content) present in $B^i$ during a given cycle and $\text{Cap}^i$ be the total capacity of the buffer $B^i$.

Let the throughput distribution of the $i$th segment ($G^i$) at the end of $t_d$ cycles be given by $\text{Th}^i(t_d)$. The overall throughput distribution of a serial manufacturing system can then be obtained by the throughput distribution of the final segment of
the system, which for a N-machine serial line is given by $\text{Th}^N(t_d)$.

Each segment of a manufacturing system interacts with the other segments, as a result, the throughput distribution of a segment depends not only on its own conditions, but also on the condition of the other segments. As will be seen, under the assumption of serial synchronous manufacturing systems, the interactions within a manufacturing system can be simplified significantly.

![Diagram](image)

Figure 4.2: Segmentation of Manufacturing Systems

### 4.3 Manufacturing System Interactions

Chapter 3 estimated the throughput distribution for a two machine one buffer system. Two key assumptions made in this study were: the upstream machine never starves and the downstream machine never blocks. These assumptions isolated the two machine one buffer segment from external effects, such as demand and supply of raw materials. On segmenting a large manufacturing system into two machine one buffer segments, the segments interact amongst each other, as a result, the assumption
of upstream machine not starving and the downstream machine not blocking may not always hold true. This chapter focuses on modeling these interactions to obtain the overall throughput distribution of a system.

4.3.1 “Loosely” Interacting System

**Definition:** A segment is defined to be loosely interacting with the system, if the state of the segment at the end of a cycle, depends only on the state of the segment and the state of its immediately neighboring segments at the beginning of the cycle, and not on the rest of the system.

The state of a segment is defined by the state vectors of the machines and buffer contained within a segment. Let \( s^i(k) = \{ x_1^i(k), x_2^i(k), y^i(k) \} \) be the state of segment \( i \) during the \( k \)th cycle. Under the assumption of independent degradation of machines and time dependent failures, the transition matrices of the machines can be obtained by the method described in Section (3.2.1).

Within a given cycle, for a segment buffer: 1) either no part enters or leaves the segment, 2) a part enters but no part leaves the segment, 3) no part enters but a part leaves the segment, and 4) a part enters the buffer and another part leaves the buffer.

Without loss of generality, consider segment \( G^3 \) as shown in Figure 4.2. For a part to enter the buffer of \( G^3 \) (i.e., \( B_2 \)), the buffer should be able to accommodate the part (depends on segment condition), the upstream machine of the segment (\( M_2 \)) should be up (depends on segment condition) and the upstream buffer of \( G^2 \) (i.e, \( B_1 \)) should have a part to feed \( M_2 \) segment at the beginning of the cycle (depends on condition of previous segment). Similarly, for a part to leave the buffer of \( G^3 \), the downstream machine of the segment (\( M_5 \)) needs to be up and the downstream buffer of \( G^4 \) (i.e., \( B_3 \)) should have space to accommodate the part. Both the described conditions only depend on the state of \( G^2, G^3 \) and \( G^4 \), i.e., the considered segment and the neighboring
segments. Hence, over a given cycle, the segments of a synchronous manufacturing system are “loosely” connected.

4.3.2 Modes of a Segment

For a segment of the considered manufacturing system, within a given cycle, only two external interactions are possible: blockage and starvation. As discussed in the previous section, these interactions are caused only by the immediate neighbors of the segment. Hence, while modeling the evolution of a segment over one cycle, only the segment and its immediate neighbors are considered.

Due to these interactions, a segment can operate in four possible modes:

1. **Normal mode**: Upstream machine does not starve and the downstream machine does not block. In such a mode, the segment behaves as if in isolation.

2. **Blocked mode**: In case a segment is capable of producing a part (i.e., downstream machine of the segment is up, and the segment buffer consists of non-zero parts), and is unable to deliver the part to the following segment, the segment is blocked. Such a scenario occurs only when the segment following the considered segment has a full buffer, and the downstream machine of the following segment is down. In this mode, the second machine virtually acts as if down, and due to blockage no parts leave the segment.

3. **Starved mode**: In case the upstream machine of the segment is in an up state, but the machine does not receive a part from the previous segment, then the segment is said to be starved. In such a scenario no parts enter the the segment, and the first machine acts as if virtually down.

4. **Starved and Blocked**: In the rare scenario that the system undergoes blockage
and starvation at the same time, the system is assumed to be in starved & blocked mode. As no parts enter or leave the buffer, the total buffer contents do not change.

4.3.3 Mode Dynamics

Similar to the analysis in Chapter 3, two quantities related to the dynamics of a segment are of primary interest: (1) the buffer transition matrix for a given segment; and (2) the throughput distribution at the end of the given cycle.

From Chapter 3, recall $M^\gamma$ and $M^\delta$, $PU^\gamma$ and $PU^\delta$, $PD^\gamma$ and $PD^\delta$ represent the upstream and downstream machines, the up states probabilities of the upstream and downstream machines, and down state probabilities of the upstream and downstream machines respectively. For notational convenience, let $A = PU^\gamma(k), \bar{A} = PD^\gamma(k), B = PU^\delta(k)$ and $\bar{B} = PD^\delta(k)$. Let $c(k)$ and $c(k + 1)$ be the contents of the buffer at the end of $k$ and $k + 1$ cycles respectively, and let $Cap$ represent the capacity of the considered buffer.

Let $P^n, P^bl, P^st, P^sb$ be the buffer transition matrices in the normal, blocked, starved and starved & blocked mode respectively. Similarly, let $P^n, P^bl, P^st, P^sb$ be the system (segment) transition matrix in the normal, blocked, starved and starved & blocked mode.

Let the $ij$th element of the system transition matrix $P^o(k)$ (i.e., $ps^o_{ij}(k)$) give the probability of the system going from state $s_i = \{x_i^g, x_i^d, y_i\}$ to $s_j = \{x_j^g, x_j^d, y_j\}$ in mode $o \in \{n, bl, st, sb\}$, during the $k$th cycle. Similar to Section (3.2.2), under the assumption of independent machine degradation, $ps^o_{ij}(k)$ can be obtained as:
\[ ps^o_{ij}(k) = pm^\gamma_{x_i^jx_j^j}(k).pm^\delta_{x_i^jx_j^j}(k).pb^o_{y_iy_j}(k), o \in \{ n, st, bl, sb \} \tag{4.1} \]

where \( pm_{ij} \) is the \( ij \)th element of the machine transition matrix and \( pb_{ij} \) is the \( ij \)th element of the buffer transition matrix.

### 4.3.3.1 Normal Mode

In a normal mode, a segment does not starve or block. The analysis of such a segment was carried out extensively in Chapter 3. The main characteristic of this mode is that the considered segment does not get influenced by other segments, i.e., it behaves as if in isolation. The buffer dynamics for a segment in normal mode during the \( k \)th cycle are described by

- \( M^\gamma \uparrow M^\delta \uparrow \) or \( M^\gamma \downarrow M^\delta \downarrow \), then \( c(k + 1) = c(k) \)
- \( M^\gamma \downarrow M^\delta \uparrow \) and \( c(k) > 0 \), then \( c(k + 1) = c(k) - 1 \)
- \( M^\gamma \downarrow M^\delta \uparrow \) and \( c(k) = 0 \), then \( c(k + 1) = 0 \)
- \( M^\gamma \uparrow M^\delta \downarrow \) and \( c(k) < \text{Cap} \), then \( c(k + 1) = c(k) + 1 \)
- \( M^\gamma \uparrow M^\delta \downarrow \) and \( c(k) = \text{Cap} \), then \( c(k + 1) = \text{Cap} \)

Using the above described principles, the transition matrix for the buffer in normal mode can be written as
\[
\mathcal{P}^n(k) = \begin{pmatrix}
\bar{A} & A & 0 & \ldots & 0 & 0 \\
\bar{A}.B & A.B + \bar{A}.\bar{B} & A.\bar{B} & 0 & \ldots & 0 \\
0 & \bar{A}.B & A.B + \bar{A}.\bar{B} & A.\bar{B} & \ddots & 0 \\
0 & 0 & \ddots & \ddots & \ddots & \ddots \\
\ddots & \ddots & \bar{A}.B & A.B + \bar{A}.\bar{B} & A.\bar{B} \\
0 & 0 & \ldots & 0 & \bar{A}.B & A.B + A.\bar{B} + \bar{A}.\bar{B}
\end{pmatrix}
\]

The matrix \(\mathcal{P}^n(k)\) (i.e., the buffer transition matrix in normal mode), is found to be a band diagonal matrix, as the transition of the buffer is observed only over one cycle, during which only one part can enter or leave a buffer.

As defined in Section (3.2.3) and illustrated in Figure 3.4, \(D(a, b)\) represents a node of an aggregation tree, satisfying the condition that the system (in this case segment) has produced exactly \(a\) parts in \(b\) cycles. The components of the node \(D(a, b)\) are \(\text{th}_{D(a,b)\alpha}\) and \(\text{th}_{D(a,b)\beta}\), representing the producing states (states containing non zero buffer contents and an up downstream machine) and non producing states (states containing an empty buffer or a down downstream machine) of the segment.

As seen from Figure 4.3, the node \(D(a, b)\) can only be reached from \(D(a-1, b-1)\) or \(D(a, b-1)\), i.e., the condition of producing \(a\) parts in \(b\) cycles, can only be satisfied if the system satisfied \(D(a-1, b-1)\) and produced a part in the \(b\)th cycle, or the system satisfied \(D(a, b-1)\) and did not produce a part in the \(b\)th cycle.

In Figure 4.3, it is observed that a system originally in node \(D(0,0)\) and in a producing state (i.e., \(\text{th}_{D(0,0)\alpha}\)) will produce a part (reach node \(D(1,1)\)) and transition into a producing or non producing system over a given cycle (by \(\mathcal{P}_{\alpha\alpha}^n\) and \(\mathcal{P}_{\alpha\beta}^n\) respectively). Similarly, for a non producing state in \(D(0,0)\) (i.e., \(\text{th}_{D(0,0)\alpha}\)), the
system does not produce a part (reaches node \( D(0, 1) \)) and transition to a producing or non producing state (by \( P^\alpha_{\beta \alpha} \) and \( P^n_{\beta \beta} \) respectively).

![Throughput Distribution in Normal Mode](image)

**Figure 4.3: Throughput Distribution in Normal Mode**

### 4.3.3.2 Blocked Mode

A segment is considered blocked, if the segment is capable of producing a part, but is unable to deliver the part to the next segment. In such a case, the downstream machine of the segment acts as if virtually down. The buffer dynamics for a blocked segment are described as:

- \( M^\gamma \downarrow \) then \( c(k + 1) = c(k) \)
- \( M^\gamma \uparrow \) and \( c(k) < Cap \), then \( c(k + 1) = c(k) + 1 \)
- \( c(k) = Cap \), then \( c(k + 1) = Cap \)

Based on the above principles, the transition matrix for the buffer dynamics is found to be an upper band diagonal matrix.
\[
\mathcal{P}^{bl}(k) = \begin{pmatrix}
\bar{A} & A & 0 & \ldots & 0 & 0 \\
0 & \bar{A} & A & 0 & \ldots & 0 \\
0 & 0 & \bar{A} & A & \ddots & 0 \\
0 & 0 & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \bar{A} & A \\
0 & 0 & \ldots & 0 & 0 & 1
\end{pmatrix},
\]

In the blocked mode, since the segment is unable to deliver the part to the next segment, there is no part produced. Hence, irrespective of the states of the machines and buffer, the segment only moves from node \( D(a, b-1) \) to \( D(a, b) \), as shown in Figure 4.4.

![Figure 4.4: Throughput Distribution in Blocked Mode](image)

### 4.3.3.3 Starved Mode

In a starved mode, the upstream machine of the system is in an operating state, but does not receive a part from the previous segment. This results in the upstream machine of the segment behaving virtually down. The buffer dynamics for a segment in starved mode are described as...
• $M^\uparrow$ and $c(k) > 0$, then $c(k + 1) = c(k) - 1$

• $M^\downarrow$ then $c(k + 1) = c(k)$

• $c(k) = 0$, then $c(k + 1) = 0$

The above principles when written in the form of a transition matrix, form a lower band diagonal matrix.

$$P^{st}(k) = \begin{pmatrix}
1 & 0 & 0 & \ldots & 0 & 0 \\
B & \bar{B} & 0 & 0 & \ldots & 0 \\
0 & B & \bar{B} & 0 & \ddots & 0 \\
0 & 0 & \ddots & \ddots & \ddots & \ddots \\
\vdots & \ddots & B & \bar{B} & 0 \\
0 & 0 & \ldots & 0 & B & \bar{B}
\end{pmatrix}$$

The throughput evolution for a starved system is similar to a normal mode and is shown by Figure 4.5.

Figure 4.5: Throughput Distribution in Starved Mode
4.3.3.4 Starved and Blocked Mode

In rare cases, a segment may get blocked as well as starved. In such a case, both the machines act as if down and hence there is no change in the buffer contents of the system. The buffer dynamics can simply be stated as

- \( c(k+1) = c(k) \),

and the transition matrix is an identity matrix with dimension of \( \text{Cap} + 1 \).

\[
\mathcal{P}^{sb}(k) = \begin{pmatrix}
1 & 0 & 0 & \ldots & 0 & 0 \\
0 & 1 & 0 & 0 & \ldots & 0 \\
0 & 0 & 1 & 0 & \ddots & 0 \\
0 & 0 & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & 0 & 1 \\
0 & 0 & \ldots & 0 & 0 & 1
\end{pmatrix}
\]

Figure 4.6: Throughput Distribution in Starved & Blocked Mode
Since no part is produced by the system, the throughput distribution for a starved and blocked system evolves similar to that of a blocked system, wherein the system in node \( D(a, b - 1) \) can only move to \( D(a, b) \), as shown in Figure 4.6.

4.3.3.5 Illustration: Normal Mode Dynamics

Consider a two machine one buffer system with machines having two states (Up and Down) and buffer having capacity of 1. Let the initial condition of the system be \( \{Up, Up, Full\} \), i.e., both the machines are in up states and buffer is full. This illustration provides the normal mode dynamics of this segment, over the first cycle.

Let the machine transition matrices for for the upstream machine and the downstream machine be given by \( P^\gamma \) and \( P^\delta \):

\[
P^\gamma = \begin{pmatrix}
0.9 & 0.1 \\
0.3 & 0.7
\end{pmatrix}
\]

\[
P^\delta = \begin{pmatrix}
0.9 & 0.1 \\
0.4 & 0.6
\end{pmatrix}
\]

Let \( s(k) = \{x^\delta(k), x^\gamma(k), y(k)\} \) represent a state of the system, such that \( s \in S \). The complete state space for the considered system (\( S \)) consists of \( 2 \times 2 \times 2 \) elements as listed below:

\[
S = \{(1, 1, 1), (1, 1, 0), (1, 0, 1), (1, 0, 0), (0, 1, 1), (0, 1, 0), (0, 0, 1), (0, 0, 0)\}
\]

where, 1 and 0 represent up and down state for machines, and 1 and 0 represent full and empty conditions of the buffer.

As described before, a system is said to be in producing state, if the buffer of the segment has at least one part and the downstream machine is up. Based on this
condition, the state space of the system can be partitioned into producing and non-producing states:

\[ S = S_\alpha | S_\alpha \]

\[ = \{(1, 1, 1), (0, 1, 1) | (1, 1, 0), (1, 0, 1), (1, 0, 0), (0, 1, 0), (0, 0, 1), (0, 0, 0)\} \quad (4.2) \]

Since the initial conditions of both the machines are up, a part would enter the buffer and another part would leave the buffer, keeping the total buffer content same as before. Based on these conditions, the buffer dynamics in normal mode, with both machines up, can be written as:

\[ P_n(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \]

Let \( P^n \) be the transition matrix for the system in normal mode. As the total number of states for the system is 8, \( P \) is of the size 8 × 8. Since the machines degrade independently, the \( ij \)th element of the system is during the first cycle is obtained as:

\[ ps_{ij}^{n}(0) = pm_{x_i x_j}^\gamma(0).pm_{x_i x_j}^\delta(0).pb_{y_i y_j}^n(0) \quad (4.3) \]

Consider the transition from, say, \( s_i = (1, 1, 1) \) to \( s_j = (0, 1, 1) \). For the upstream machine, as seen from the machine transition matrix, the probability of moving from \( st = 1 \) to \( st = 0 \) is 0.1, while for the downstream machine, the probability of moving from \( st = 1 \) to \( st = 1 \) is 0.9. Finally, from the buffer transition matrix (obtained by initial condition of the machines as shown above), the probability of buffer going from full to full is 1. Hence the \( ij \)th element of the system transition matrix can be written as:
\[ ps^n_{ij}(0) = 0.1 \times 0.9 \times 1 \]

Similarly, other elements of the system transition matrix can be found. On grouping and ordering the states of the system as shown in Equation 4.2, and obtaining its transition matrix, the transition matrix can be partition into block matrices as shown below. The dimensions of each block matrix are provided below.

\[
P^n(0) = \begin{pmatrix}
    \begin{pmatrix}
        P^n_{\alpha\alpha}(2 \times 2) & P^n_{\alpha\beta}(2 \times 6) \\
        P^n_{\beta\alpha}(6 \times 2) & P^n_{\beta\beta}(6 \times 6)
    \end{pmatrix},
\end{pmatrix}
\]

### 4.3.4 Mode Probabilities

This section focuses on finding the probability of the occurrence of each mode. Consider the \( i \)th segment of a serial manufacturing system. Except for the first and last segment, each intermediate segment of a serial manufacturing system has two neighboring segments. The \( i-1 \)th segment is referred to as the upstream segment for segment \( i \), while \( i+1 \)th segment as the downstream segment.

Let the number of cycles passed be \( t \) and let \( M_{i^t} \), \( M_{i^d} \) and \( B^i \) (for \( i \geq 2 \)) be the upstream machine, downstream machine and the buffer for the \( i \)th segment. Let the buffer capacity and the initial condition (buffer contents at \( t = 0 \)) for the buffer of the \( i \)th segment be represented by \( Cap^i \) and \( In^i \) respectively. Let \( c^i \) be the number of parts in the \( i \)th buffer during a given cycle.
4.3.4.1 Segment Table

For easier visualization, a table containing all possible scenarios for a segment are listed in the form of a table, called as the segment table. Figure 4.7 represents one such table.

A new table is generated at the end of each time cycle for the system. The \( a + 1 \)th column of the table corresponds to the condition \( D(a, t) \), i.e., the system having completed \( t \) cycles and produced \( a \) parts. Similarly, the first column represents the states of the system wherein the system has produced no parts and the last \( (t + 1) \)th column represents the column wherein the segment has produced a part in each cycle. The rows of the table represent different states of the system. Let the total number of rows for a segment be \( k \) be given by \( N^\gamma \times N^\delta \times (Cap + 1) \) (say \( k \)), where \( N^\gamma \) and \( N^\delta \) represent the total number of states in the upstream and downstream segment of the machine and \( Cap \) is the capacity of the buffer. The states of the segment are arranged in a way, such that the top half of the table contains the producing states of the segment while bottom half contains the non producing parts of the system.

Let \( e_{jk}^i(t) \) represent the \( jk \)th element of the \( i \)th segment at the end of \( t \) cycles. This element represents the probability of the segment being in state \( s_j \) while satisfying the condition \( D(a, t) \), i.e., \( e_{jk}^i(t) = pr(s_j, D(k - 1, t)) \), where \( pr \) represents probability. Let the set of all possible states for the \( i \)th segment table at the end of \( t \) cycles be represented by \( T^i(t) \), i.e., \( e_{jk}^i(t) \in T^i(t) \). Since the table contains all possible scenarios for a given segment, the sum of all the elements in a table is equal to 1. This can be stated as

\[
\sum_{T^i(t)} e_{jk}^i(t) = 1. \tag{4.4}
\]
4.3.4.2 Feasibility Conditions

Feasibility condition verifies for a system if two elements of neighboring segment tables can exist at the same time. For example, it could verify if at the end of $t$ cycles, for the $i$th segment in state $s^i = (x^i, x^{i+1}, y^i)$ satisfying condition $D(a^i, t)$, whether an element in segment $i + 1$ in state $s^{i+1} = (x^{i+1}, x^{i+1}, y^{i+1})$, satisfying condition $D(a^{i+1}, t)$ could occur. The feasibility conditions represent the physical constraints of the system, and are represented by two conditions:

1. **Segment Overlap**: Since neighboring segments of the system overlap each other, the state of the common machine should be the same in both the elements, i.e., $x^i = x^{i+1}$.

2. **Conservation of Parts**: For a buffer, the total number of parts must be conserved, i.e., the difference in the final and initial content of the buffer must be...
the same as the difference between the total number of parts entering the buffer and leaving the buffer. For segment $i$ and $i + 1$ the condition can be stated as:

$$c^{i+1} - In^{i+1} = a^i - a^{i+1},$$

(4.5)

where $c^{i+1}$ is the number of parts present in the buffer of segment $i + 1$, while $In^{i+1}$ represents the initial number of parts present in the buffer of segment $i + 1$.

### 4.3.4.3 Blockage and Starvation Condition

**Blockage Condition** As discussed earlier in this chapter, each segment of a manufacturing system only “loosely” interacts with the rest of the system. The blockage conditions of a segment $G^i$ depend only on the condition of the segment itself (internal conditions) and the condition of the downstream segment $G^{i+1}$ (external conditions).

Along with the feasibility conditions, the conditions necessary for $G^i$ to block, while in condition $D(a, b)$ are:

1. *Downstream Buffer Should be Full (external):* The buffer of the segment $G^{i+1}$ ($i.e., B^{i+1}$) should be full, so as to not allow entry of any parts. Let the initial number of parts and buffer capacity of buffer $B^{i+1}$ be $In^{i+1}$ and $Cap^{i+1}$ respectively. The $i+1$th buffer requires additional ($Cap^{i+1} - In^{i+1}$) parts to be full. These additional parts are required to be provided by segment $i$ producing, by producing ($Cap^{i+1} - In^{i+1}$) more parts than segment $i + 1$.  

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\[ a^i = a^{i+1} + (\text{Cap}^i - \text{In}^i) \quad (4.6) \]

In other words, for a condition where \( G^i \) satisfies \( D(a^i, t) \), i.e., the segment \( i \) has produced exactly \( a^i \) parts in \( t \) cycles, segment \( G^{i+1} \) requires to produce

\[ a^{i+1} = a^i - (\text{Cap}^i - \text{In}^i), \]

for blockage.

2. **Downstream Machine of the Downstream Segment Should be Down (external):**
   The downstream machine of the segment \( G^{i+1} \) should be in a down state, i.e., \( M^{i+1\downarrow} \), so that no part is taken out of buffer \( B^{i+1} \) during the \((t+1)\)th cycle, and it continues to remain full.

3. **Segment Producing Conditions (internal):** For blockage of segment \( G^i \), it is essential that \( G^i \) is capable of producing, i.e., buffer \( B^i \) has non zero parts and the downstream machine of segment \( G^i \) is in up state. This condition can be stated as \( x^{i\uparrow} \) is \( \uparrow \) and \( b^i > 0 \).

**Starvation Condition** Similar to the blockage condition, in order for a segment to starve, both internal and external conditions for starvation must be met. The conditions dependant on the segment itself (in this case \( G^i \)) are known as the internal conditions, while conditions dependent on the upstream segment, i.e., \( G^{i-1} \), are known as the external conditions. For starvation of \( G^i \), the following conditions must be satisfied:
1. **Upstream Buffer Must Be Empty (external):** The upstream buffer must be empty, so as to not be able to deliver a part to segment $G^i$, i.e., $c^{i-1} = 0$.

2. **Upstream Machine of the Segment Should Be Up (internal):** To starve, the upstream machine of the segment must be capable of producing a part, i.e., $x^i \uparrow$ should be $\uparrow$.

### 4.3.4.4 Evaluating Mode Probability

Let us consider the $j(k+1)$th element of the $i$th segment table, i.e., $(s^i_j, D^i(k, b))$. All analysis in this section is carried out in terms of this element. For notational convenience $j, k$ and $b$ are dropped for the rest of the section. With simplified notations, let $\omega_f, \omega_b, \omega_s$ represent the feasibility, blockage and starvation conditions for this element. These conditions are obtained based on the discussions above.

Let $T_{w_k}$ be the set of elements of the $i$th segment table satisfying the conditions $\omega_k, k \in \{f, b, s\}$. The sum of all such elements is then obtained by

$$p_k = \sum_{e^i \in T_{w_k}} e^i, k \in \{f, b, s\}. \quad (4.7)$$

#### Blockage

If the internal conditions of blockage are not satisfied, $P_{bl} = 0$ i.e., if $x_j^i$ is down or if $y_j^i = 0$, no part is produced by the segment and hence, the no blockage occurs. If the internal conditions are satisfied, i.e., if a part is produced, segment $i + 1$ may cause blockage for the segment $i$. Let $T^{i+1}_{w_f}$ and $T^{i+1}_{w_b}$ be the set of elements of the $i + 1$th segment table satisfying the feasibility and the external blockage conditions with respect to the considered element. Hence, given an element in segment $i$ satisfying internal blockage condition, using conditional probability, the probability of blockage
caused by the external segment \( G^{i+1} \) can be found by the ratio of sum of probability of states causing blockage to sum of probability of all feasible states with respect to the given element, i.e.,

\[
P_{bl} = \frac{p_b}{p_f}.
\]  

(4.8)

**Starvation**

Similar to blockage, if the internal starvation conditions are not met, \( P_{st} = 0 \). The external starvation conditions are caused due to the upstream segment of the system.

For the considered element with satisfied internal starvation conditions, let \( T_{w_f}^{i-1} \) and \( T_{w_s}^{i-1} \) be the set of elements of the \( i-1 \)th segment table satisfying the feasibility and the external starvation conditions with respect to the considered element. Hence, given an element in segment \( i \) satisfying internal starvation condition, using conditional probability, the probability of starvation caused by the external segment \( (G^{i+1}) \) can be found by the ratio of sum of probability of states causing starvation to sum of probability of all feasible states with respect to the given element, i.e.,

\[
P_{st} = \frac{p_s}{p_f}
\]  

(4.9)

**Mode Probabilities**

As discussed earlier in this chapter, each segment of an exponential synchronous serial manufacturing system interacts only loosely with the rest of the manufacturing system. The blockage of a segment \( G^i \), depends only on the condition of the downstream machine \( (M_i^d) \), condition of the buffer \( (B^i) \) and the downstream segment \( (G^{i+1}) \). Similarly, the starvation of a segment \( G^i \), for a given cycle depends only on the condition of the upstream machine \( (M_i^u) \) and the upstream segment \( (G^{i-1}) \).
Given the condition of the system at the beginning of a cycle, it is observed that the conditions for blockage and starvation are independent of each other.

Let the probability of starvation and blockage for the \(j(k+1)\)th element of the \(i\)th segment table, i.e., \((s^i_j, D^i(k, b))\) segment table be given by \(P_{st}\) and \(P_{bl}\) (simplified notations). Note, the probabilities \(P_{st}\) and \(P_{bl}\) are independent but not mutually exclusive, i.e., there is a possibility that both starvation and blockage may occur at the same time. The mode dynamics described in Section (4.3.3) refer to only starvation, only blockage, normal (no starvation and blockage) and starved & blocked modes. Since \(P_{st}\) and \(P_{bl}\) are independent, probability of starvation & blockage to occur at the same time is obtained by the product of their probabilities, and is denoted by \(P_{isb}^j(D(k, b))\) (with regular notations). To find the probability of only starvation and no blockage, the probability of starvation & blockage is subtracted from the probability of starvation. Similarly, the probability of blockage can also be obtained by subtracting starvation & blockage probability from \(P_{bl}\). Hence, retaining normal notations for the probability of modes, they can be written as shown in Table 4.1.

<table>
<thead>
<tr>
<th>Probability</th>
<th>Blocked</th>
<th>Starved</th>
<th>Mode Name</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_{in}^j(D(k, b)))</td>
<td>No</td>
<td>No</td>
<td>Normal (n)</td>
<td>(1 - P_{bl} - P_{st} + P_{bl}P_{st})</td>
</tr>
<tr>
<td>(P_{bl}^j(D(k, b)))</td>
<td>Yes</td>
<td>No</td>
<td>Blocked (bl)</td>
<td>(P_{bl} - P_{bl}P_{st})</td>
</tr>
<tr>
<td>(P_{st}^j(D(k, b)))</td>
<td>No</td>
<td>Yes</td>
<td>Starved (st)</td>
<td>(P_{st} - P_{bl}P_{st})</td>
</tr>
<tr>
<td>(P_{isb}^j(D(k, b)))</td>
<td>Yes</td>
<td>Yes</td>
<td>Starved &amp; Blocked (sb)</td>
<td>(P_{bl}P_{st})</td>
</tr>
</tbody>
</table>

**Illustration**

Consider a system as shown in Figure 4.8. This illustration provides the probability of blockage for \(G^2\) in state \((1, 1, 1)\), satisfying condition \(D(1, 2)\), at the end of 2 cycles.
Let $Cap^3 = 1$ and $In^3 = 1$, i.e., let the buffer of $G^3$ (i.e., $B_2$) initially (at $t = 0$) have 1 part and let its total capacity be 1. Let the state of $G^3$ be expressed as $s^3 = \{x^3, x^3g, y^3\}$. Let both the machines of $G^3$ have only two states (up or 1, and down or 2) and for the buffer let 0 correspond to empty and 1 correspond to full (i.e., 1 part, which is its capacity). The complete state space of $S^3$ is expressed as $S^3 = \{(1,1,1), (1,1,0), (1,0,1), (1,0,0), (0,1,1), (0,1,0), (0,0,1), (0,0,0)\}$, and the segment table for $G^3$ is as shown in Figure 4.9, where the first two states are producing states.

Figure 4.8: Blockage Probability

Figure 4.9: Illustration of Segment Table for $G^3$
For the element with state (1,1,1) satisfying $D(1,2)$ for $G^2$, $a^2 = 1$ and $t = 2$. For the considered case, internal blockage condition are satisfied, as downstream machine for the segment is up ($x^{2\delta}$ is ↑) and the buffer is not empty ($c^2 > 0$). Hence the system is capable of producing a part, and $G^3$ may cause blocking.

From the obtained feasibility conditions, flow conservation and the segment overlap for $G^3$ can be stated as:

$$ (c^3 - In^3) = a^2 - a^3 $$

$$ \Leftrightarrow c^3 + a^3 = 1 + 1 = 2 $$

and

$$ x^{i+1\gamma} = 1. $$

The conservation of flow is satisfied only by ($c^3 = 0$ and $a^3 = 2$) and ($c^3 = 1$ and $a^3 = 1$) and the segment overlap condition by cases where $x^{i+1\gamma} = 1$. Hence, by referring to the segment table for $G^3$, the sum of probability of all feasible states is obtained as:

$$ p_f = e_{12} + e_{42} + e_{33} + e_{53} \quad (4.10) $$

For blockage, along with the feasibility conditions, the buffer of $G^3$ needs to be full (i.e., $c^3 = 1$) and $M_3$ needs to be down (i.e., $x^3\delta = 0$). Hence of the all possible feasible states, only one state satisfies the condition, hence sum of probability of blockage causing states is

$$ p_b = e_{42} \quad (4.11) $$
Having obtained the sum of probability of the feasible states and the blocked states, the probability of blockage for $G^2$ in state (1,1,1) and condition $D(1,2)$ is obtained as:

$$p_{bl} = e_{42}/(e_{12} + e_{42} + e_{33} + e_{33}).$$

Similarly the blockage and starvation for each of the element of the table can be obtained.

4.4 Aggregation

Having defined the transition matrix and the throughput evolution for each of the mode, this section focuses on the development of an aggregation approach for the segment table. The significant difference between the current approach and that of Chapter 3, is that the aggregation approach developed in Chapter 3 considered only a single (normal) mode of operation, while the current approach considers all the four modes of operations.

To illustrate the procedure, consider an element from the segment table $G^i$, (say $(s^i_j, D^i(a,b))$. As defined earlier, $s^i_j \in S^i$ and recalling that $S^i$ was partitioned as $(S^i_\alpha|S^i_\beta)$ into producing and non producing states.

As only segment $i$ is considered through this section, for notational convenience and easier visualization, the superscript $i$ is dropped from the notations of this section (For example, $s^i_j$ will be referred to as $s_j$ and $P^i_{jn}(D(k,b))$ will be referred to as $P^n_{jn}(D(k,b))$). Let the total number of states of the segment be $N_s$ of which first $N_{sa}$ states are producing and remaining $N_{sb}$ states are non producing. Also, as defined earlier, $th_{D(a,b)\alpha}$ represents a state vector of size $N_{sa}$ representing the producing
states of the segment and \( \mathbf{th}_{D(a,b)\alpha} \) represents a vector of size \( N_{s\alpha} \) representing the non producing states of the system.

For the given element, based on whether \( s_j \) belongs to producing states (\( S_\alpha \)) or non producing states (\( S_\beta \)), over the next cycle, a segment either produces a part and satisfies \( D(a + 1, b + 1) \), or does not produce a part and satisfies the condition \( D(a, b + 1) \). For the \( j \)th element of the state vector, let us define \( \mathbf{th}_{D(a,b)\alpha_j} \) and \( \mathbf{th}_{D(a,b)\beta_j} \) as shown in Figure 4.10, obtained from partitioning a state vector with 1 in the \( j \)th position, and 0 in the remaining positions. This represents the segment being in the \( j \)th state, where \( j \leq N_{s\alpha} \) would belong to a producing state.

![Figure 4.10: Partitioning State Vector](image)

Using the throughput evolution for each mode, Figure 4.11 shows the effect of the four modes on the throughput evolution on an element of the segment. As stated earlier, in the blocked mode (\( bl \)) and the starved & blocked mode (\( sb \)), the system does not produce any part, as a result, irrespective of the state of the system (producing or non producing state) no parts are produced in these modes.

The overall aggregation procedure can be summarized as follows. For each element of the segment table developed at the end of \( b - 1 \) cycles, probability of the four modes (\( st, bl, n, sb \)) is obtained using Section (4.3.4.4). Knowing the probabilities of
Figure 4.11: Evolution of Throughput
occurrence of each mode and the mode dynamics (Section 4.3.3), for every element of the segment table developed at \( b-1 \) cycles, the evolution equations (Equations (4.13- 4.18)) are evaluated, and used to develop a new segment table representing the segment at the end of \( b \) cycles. Finally, at the end of \( t_d \) cycles, column sums of the segment table provide the throughput distribution for the given segment.

Based on the considerations of throughput evolution and the mode probabilities, the state vector consisting of the producing states of the segment over a given duration of \( t_d \) is obtained using Equation 4.13. Similarly, the non producing states of the system are obtained using Equation 4.14.

\[
\text{th}_{D(a,b)\alpha} = \sum_{j=1}^{N_s} \text{th}_{D(a-1,b-1)\alpha j} \mathbb{P}^{\alpha}_{\alpha\alpha} P^n_j (a-1, b-1) + \sum_{j=1}^{N_s} \text{th}_{D(a,b-1)\beta j} \mathbb{P}^{\alpha}_{\beta\alpha} P^n_j (a, b-1) + \\
\sum_{j=1}^{N_s} \text{th}_{D(a-1,b)\alpha j} \mathbb{P}^{\alpha t}_{\alpha\alpha} P^{st}_j (a-1, b-1) + \sum_{j=1}^{N_s} \text{th}_{D(a,b-1)\beta j} \mathbb{P}^{\alpha t}_{\beta\alpha} P^{st}_j (a, b-1) + \\
\sum_{j=1}^{N_s} \text{th}_{D(a,b-1)\alpha j} \mathbb{P}^{bl}_{\alpha\alpha} P^b_j (a, b-1) + \sum_{j=1}^{N_s} \text{th}_{D(a,b-1)\beta j} \mathbb{P}^{bl}_{\beta\alpha} P^b_j (a, b-1) + \\
\sum_{j=1}^{N_s} \text{th}_{D(a,b-1)\alpha j} \mathbb{P}^{sb}_{\alpha\alpha} P^s_j (a, b-1) + \sum_{j=1}^{N_s} \text{th}_{D(a,b-1)\beta j} \mathbb{P}^{sb}_{\beta\alpha} P^s_j (a, b-1),
\]

\( b \in \{1, 2, ..., t_d\}, a \in \{1, 2, ..., b-1\}. \)
\[ \mathbf{th}_{D(a,b)\beta} = \sum_{j=1}^{N_s} \mathbf{th}_{D(a-1,b-1)\alpha_j} P^\alpha_{j}(a-1, b-1) + \sum_{j=1}^{N_s} \mathbf{th}_{D(a,b-1)\beta_j} P^{\beta}_{j}(a, b-1) + \sum_{j=1}^{N_s} \mathbf{th}_{D(a,b-1)\alpha_j} P^{st}_{j}(a-1, b-1) + \sum_{j=1}^{N_s} \mathbf{th}_{D(a,b-1)\beta_j} P^{st}_{j}(a, b-1) + \sum_{j=1}^{N_s} \mathbf{th}_{D(a,b-1)\alpha_j} P^{bl}_{j}(a, b-1) + \sum_{j=1}^{N_s} \mathbf{th}_{D(a,b-1)\beta_j} P^{bl}_{j}(a, b-1) + \sum_{j=1}^{N_s} \mathbf{th}_{D(a,b-1)\alpha_j} P^{sb}_{j}(a, b-1) + \sum_{j=1}^{N_s} \mathbf{th}_{D(a,b-1)\beta_j} P^{sb}_{j}(a, b-1), \]

\[ \forall b \in \{1, 2, \ldots, t_d\}, a \in \{1, 2, \ldots, b-1\}. \] (4.14)

In Equation (4.13) it is seen that for a system to be in a producing state at the end of \(b\) cycles and to have produced \(a\) parts, either the system must have been in \(D(a-1, b-1)\) and in a normal or starved mode producing states; or the system must have satisfied \(D(a, b-1)\) and been in non producing states of any modes, or producing states of blocked or starved & blocked mode (as no parts leave the segment in this mode). The vector \(\mathbf{th}_{D(a,b)\alpha}\) provides the first \(N_{sa}\) rows of the \(D(a, b)\) column of the new segment table at the end of \(b\) cycles. Equation (4.14) provides a similar equation to obtain the non producing states of the system satisfying the condition \(D(a, b)\).

Equations (4.13- 4.14) hold true for \(\forall a \in \{1, 2, \ldots, b-1\}\). For \(a = 0\) and \(a = b\) transitions for the producing and non producing states are given by Equations (4.15-4.16)

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\[ \text{th}_{D(0,b)\alpha} = \sum_{j=1}^{N_s} \text{th}_{D(0,b-1)\beta_j} \mathbb{P}_n^{\alpha} P^n_j (0, b-1) + \sum_{j=1}^{N_s} \text{th}_{D(0,b-1)\beta_j} \mathbb{P}^{st}_n P^n_j (0, b-1) + \sum_{j=1}^{N_s} \text{th}_{D(0,b-1)\beta_j} \mathbb{P}^{bl}_n P^n_j (0, b-1) + \sum_{j=1}^{N_s} \text{th}_{D(0,b-1)\beta_j} \mathbb{P}^{sb}_n P^n_j (0, b-1), \]
\[ \forall b \in \{1, 2, ..., t_d\} \]  

(4.15)

\[ \text{th}_{D(0,b)\beta} = \sum_{j=1}^{N_s} \text{th}_{D(0,b-1)\beta_j} \mathbb{P}_n^{\beta} P^n_j (0, b-1) + \sum_{j=1}^{N_s} \text{th}_{D(0,b-1)\beta_j} \mathbb{P}^{st}_n P^n_j (0, b-1) + \sum_{j=1}^{N_s} \text{th}_{D(0,b-1)\beta_j} \mathbb{P}^{bl}_n P^n_j (0, b-1) + \sum_{j=1}^{N_s} \text{th}_{D(0,b-1)\beta_j} \mathbb{P}^{sb}_n P^n_j (0, b-1), \]
\[ \forall b \in \{1, 2, ..., t_d\}. \]  

(4.16)

\[ \text{th}_{D(b,b)\alpha} = \sum_{j=1}^{N_s} \text{th}_{D(b-1,b-1)\alpha_j} \mathbb{P}_n^{\alpha} P^n_j (b-1, b-1) + \sum_{j=1}^{N_s} \text{th}_{D(b-1,b-1)\alpha_j} \mathbb{P}^{st}_n P^n_j (b-1, b-1) + \sum_{j=1}^{N_s} \text{th}_{D(b-1,b-1)\alpha_j} \mathbb{P}^{bl}_n P^n_j (b-1, b-1) + \sum_{j=1}^{N_s} \text{th}_{D(b-1,b-1)\alpha_j} \mathbb{P}^{sb}_n P^n_j (b-1, b-1), \]
\[ \forall b \in \{1, 2, ..., t_d\}. \]  

(4.17)
\[ \text{th}_{D(b,b)\beta} = \sum_{j=1}^{N_\alpha} \text{th}_{D(b-1,b-1)\alpha_j} P^n_{\alpha_j}(b-1,b-1) + \]
\[ \sum_{j=1}^{N_\alpha} \text{th}_{D(b-1,b-1)\alpha_j} P^{st}_{\alpha_j}(b-1,b-1) \]
\[ \forall b \in \{1, 2, ..., t_d\}. \hspace{1cm} (4.18) \]

Equations (4.15-4.16), represent the condition when the buffer is evolving with no parts produced in the first \(b-1\) cycles and again no part is produced during the \(b\)th cycle. While, Equations (4.17-4.18) represent the conditions where a part is being produced in every cycle of the system.

The throughput distribution at the end of \(t_d\) cycles is obtained by summing the probability of being in up or down state at the end of \(t_d\) cycles and is given by Equation (4.19), wherein \(\text{Th}(a, t_d)\) represents the probability of producing \(a\) parts in \(t_d\) cycles.

\[ \text{Th}(a, t_d) = \sum_{\beta} \text{th}_{D(a,t_d)\beta} + \sum_{\alpha} \text{th}_{D(a,t_d)\alpha}, a \subseteq \{0, t_d\} \hspace{1cm} (4.19) \]

### 4.5 Performance Measures

The throughput distribution can be used to obtain different external and internal performance measures, as described in Chapter 3. More details about the performance measures and their applications are provided in Chapter 3. For a system with throughput distribution \(\text{Th}(a, t_d)\), the following performance measures can be obtained:

1. **Average Throughput:** The average throughput of the serial system over a given
duration of time $t_d$ can be obtained using

$$\Phi_1(t_d) = \sum_{x_i=0}^{t_d} x_i \text{Th}(x_i, t_d)$$  \hfill (4.20)

2. **Probability of Meeting Given Demand**: Given a demand $d$, the probability for the system to satisfy the given demand is given by

$$\Phi_2(d, t_d) = \sum_{x_i=d-1}^{t_d} \text{Th}(x_i, t_d)$$ \hfill (4.21)

3. **Risk Associated with the System**: The risk associated with the system trying to satisfy a demand $d$ is given by

$$\Phi_3(d, t_d) = \sum_{x_i=0}^{d-1} G(x_i, d) \text{Th}(x_i, t_d),$$  \hfill (4.22)

where $G(x_i, d)$ is defined as the penalty function associated with producing $x_i$ parts, in presence of a demand $d$. For a linear penalty function, the risk equation can be stated as

$$\Phi_3(d, t_d) = \sum_{x_i=0}^{d-1} \eta(d - x_i) \text{Th}(x_i, t_d).$$  \hfill (4.23)

### 4.6 Illustration

To illustrate the above developed approach, a 4 machine 3 buffer serial manufacturing system is considered. Each machine of this system is modeled as a 4 state exponential machine consisting of three up and a down state, as shown in Figure 3.8, with rate parameters $\lambda_1 = 1/30$, $\lambda_2 = 1/30$, $\lambda_3 = 1/20$ and $\lambda_4 = 1/20$. The buffers have a capacity of 10 and are initially set to half full and the machines are set to as good as new state. The throughput distribution for this system is estimated over 100 cycles ($t_d = 100$) using the proposed approach and a simulation based technique. The performance measures for the system with a demand $d$ of 60 parts over 100 cycles
is obtained. Let the penalty for not fulfilling demand be linearly proportional with proportionality constant 100.

The 4 machine 3 buffer system is segmented as shown in Figure 4.2. Figure 4.12 shows the plot of the throughput distribution obtained using the analytical based estimation approach and the simulation based approach (using 10,000 iterations). The procedure is repeated 20 times. The 95% confidence interval of the time required by the simulation based approach is 150.2 ± 12.4 seconds, while that of analytical based approach is 1.28 ± 0.21 seconds. The average performance measures obtained using both the techniques are found to be similar as shown in Table 4.2.

<table>
<thead>
<tr>
<th>Performance Measures</th>
<th>Analytical</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Th (ϕ₁)</td>
<td>52.1</td>
<td>53.5</td>
</tr>
<tr>
<td>Prob (ϕ₂)</td>
<td>0.42</td>
<td>0.45</td>
</tr>
<tr>
<td>Risk (ϕ₃)</td>
<td>$910</td>
<td>$950</td>
</tr>
</tbody>
</table>

In Figure 4.12, the vertical axis is probability of obtaining a particular throughput, and the horizontal axis is the throughput of the system. A point \((x, y)\) on the graph, states that the probability of producing \(x\) parts by the system in 100 cycles is \(y\). The total area under the curve is 1. The more the curve shifts towards the right, the higher is the expected throughput of the system, and the better is the system performance.

In both plots, analytical as well as simulation, a sharp spike is seen at \(x = 100\), which may be explained as follows. Initially the machines are in as good as new state and the buffers are full. The time to failure distribution for each machine is exponential, with a mean of 80. Given this form of distribution, there is a significant probability that none of the machines fail during the considered 100 cycles, or inspite of machine failure the buffers compensate so as to result in no loss of throughput, as a result of which there is no loss of throughput and 100 parts are produced. For the
system to produce exactly 99 parts, only one cycle does not produce a part. This scenario is unlikely as machines considered have an average downtime of 20, hence it would be very unlikely to not produce only during one cycle.

![Illustration using 4 machine 3 buffer system](image)

Figure 4.12: Illustration using 4 machine 3 buffer system

### 4.7 Summary and Conclusions

In this section, the method of estimating the throughput distribution for a two machine one buffer system was extended to a serial manufacturing system. The proposed approach and a simulation based approach were applied to a 4 machine 3 buffer system to estimate the throughput distribution and the performance measures of the system. The analytical approach was found to estimate the throughput distribution more than an order faster than the simulation based approach.

In order to carry out the above estimation, the interaction of a two machine one buffer segment with the rest of the manufacturing system was studied. The concept of “loose” interaction of a synchronous exponential segment with the rest of the manufacturing system was introduced and the different modes of operation for the
segment were defined. Mode dynamics and mode probabilities were obtained for each
segment in varying throughput conditions. Having obtained the above, an aggregation
approach was developed to incorporate the four modes of each segment and to obtain
the overall throughput distribution over a given period of time. Using throughput
distribution, an internal performance measure (average throughput) and two external
performance measures were obtained (probability of meeting given demand and the
risk associated with the given system) for the given system. Finally, the approach
was illustrated using a 4 machine 3 buffer system consisting of exponential machines
with four states.
CHAPTER V

Conclusions and Future Work

5.1 Conclusions

Throughput analysis is important for design, operation and management of manufacturing systems, and has been studied for over 50 years. Although a lot of work has been done in the area of throughput analysis, most of the studies have focused on obtaining average performance of manufacturing systems using steady state analysis, while considering machines to have only two states (up and down).

In this study, the modeling of manufacturing systems was reviewed with focus on single machine modeling (including CBM models) and throughput analysis. Throughput analysis was observed to be of three forms: steady state analysis, transient analysis and interval analysis. Most of the studies reported in literature, primarily focused on the steady state analysis of manufacturing systems, although recent reviews have recognized transient and interval analysis as important areas of research.

Manufacturing system modeling has been developed significantly over the years to handle complex manufacturing lines, but the basic model of machine has not changed since its early development. Further, recent advances in condition monitoring techniques have led to the development of multi state machine models, which are capable of representing degradation and multiple failure modes of machines.
To explore a multi state machine, a three state machine model was developed, incorporating a PM state within the conventional two state machine model. A coupling relation was defined to describe the relation between breakdown rate and preventive maintenance rate, using which, feasibility conditions and the optimal PM were derived. Further, it was shown that if the machine parameters were well within the feasible region, then PM is robust with respect to the machine parameters.

In Chapter 3, a two machine one buffer model consisting of machines with multiple up and down states was developed. The model provided a generalized approach, wherein different types of machine (such as the PM model developed in chapter 2) could be incorporated within the two machine one buffer model. An aggregation approach was developed for the two machine one buffer system, using which, the throughput distribution for the system was estimated. Using this distribution, internal and external performance measures such as average throughput, probability of meeting demand and the risk associated with a given demand were estimated.

The proposed approach was demonstrated by two cases: first, studying the impact of buffer capacity on a two machine one buffer system, and second, studying the impact of initial conditions of machines on a two (degrading) machine one buffer system. The application of these approaches to decision making problems was also demonstrated. Finally, the performance of a discrete event simulation based approach was compared with the developed analytical based estimation approach. It was observed that the analytical based estimation approach provided a better estimate of the throughput distribution, more than an order of magnitude faster than the simulation based approach.

In the last part of this study, the two machine one buffer model was extended to a serial manufacturing system. A new segmentation approach was developed to
split the manufacturing system into smaller segments. Modes of operations and the probabilities of their occurrences were derived for each segment of the manufacturing system. By superimposing the modes, the overall transition matrix and throughput evolution for each segment was obtained. Performance measures similar to that of the two machine one buffer systems were obtained for the serial manufacturing system. Again, the time required by simulation was found to be more than an order of magnitude higher than the analytical based estimation approach.

The overall original contributions from this research can be summarized as follows:

1. Development of a new three state exponential machine model, by the introduction of a new PM state and PM-Reliability coupling

2. Estimation of throughput distribution for a two machine one buffer model over a given period of time (wherein system may or may not be steady state), with model consisting of machines with multiple up and down states

3. Development of a new segmentation approach capable of obtaining the throughput distribution of a longer serial manufacturing system, with machines capable of having multiple states, and the system may or may not being in steady state.

5.2 Future Work

The current approach estimates the throughput distribution for synchronous exponential serial manufacturing systems, consisting of machines with multiple (up and down) states. In order to make the approach more broadly applicable and to develop an effective decision support tool, the following future work is recommended.
5.2.1 Structural Modeling

Serial manufacturing systems only provide a fundamental understanding of manufacturing systems. To analyze complex manufacturing systems, the following structural studies are recommended.

5.2.1.1 Aggregation of two machines

The machine model considered in this system consists of multiple up and down states. For a serial manufacturing system, it was assumed that the machines and buffers are placed alternately. There may be scenarios, where two machines may be consecutive without a buffer between them or multiple processes at a station are required to be aggregated. In such a scenario, developing a virtual machine capable of representing the combined effect of the two machines would be valuable towards converting such a system into a serial manufacturing system.

The above idea is illustrated in Figure 5.1. The main tasks of this research would be to identify the number of states and the rate transition matrix for the virtual machine, so as to obtain a similar performance as the two consecutive machines grouped together.

5.2.1.2 Assembly / Disassembly

For complex products (such as an automobile), it is common for manufacturing systems to be composed of multiple manufacturing lines, as shown in Figure 5.2. Such manufacturing systems consist of a merge operation, wherein two components are merged into a single part. In order to analyze assembly lines, the merge operation needs to be modeled. On modeling this operation, an assembly line may be converted into segments of serial manufacturing lines. For example, the assembly line shown in Figure 5.2 could be converted into three serial manufacturing lines (M1 - M2, M3,
M4 - M5) joined by a single merge operation. Similar ideas could also be used to analyze a disassembly process.

5.2.1.3 Quality, Inspection and Rework

Quality is a critical aspect of manufacturing systems. Material or process defects could lead to production of substandard parts, which on inspection may be discarded or reworked. For expensive products or products requiring many hours of processing, it may sometimes make economic sense to rework a product rather than discard. An example of such a rework loop is shown in Figure 5.4. For such a process, it may
be of interest to determine the optimal location of the inspection booths, so as to maximize average production, while maintaining risk within acceptable limits.

By analyzing a manufacturing system for the above structures, the proposed approach may be extended to several types of complex manufacturing systems.

### 5.2.2 Risk Management Framework

As demonstrated in previous chapters, by estimating throughput distribution of a manufacturing system, the risk associated with a given system for a known demand can be obtained. Also, through segmentation, the throughput distribution at the end of each segment of a serial manufacturing system can be achieved.

#### 5.2.2.1 Risk Critical Segment

Using the proposed approach for estimating throughput, it is of interest, to evaluate the risk associated with each segment of a manufacturing system. Further, using
this information, risk critical segments of a manufacturing system can be identified.

Developing a risk management system, capable of reducing and redistributing risk from the risk critical segments, could lead to an increased and more consistent throughput. On obtaining the distribution of risk within a manufacturing system and identifying the risk critical segments, two risk management tools may be developed: risk mitigation and risk transfer.

5.2.2.2 Risk mitigation

Risk mitigation is the process of reducing risk associated with a system. In literature, PM is modeled as a method to improve the condition of a machine. By identifying the critical segments and maintaining the critical machines, the risk associated with a given segment can be reduced. As shown in Figure 5.5, PM pushes up the overall probability distribution of throughput for a given segment by improving the condition of a machine, and reducing its likelihood of failure over the coming intervals. Using this relation maintenance policies based on risk mitigating of risk critical segments may be developed.

Figure 5.5: Risk Mitigation Through Maintenance
5.2.2.3 Risk transfer

In cases, where there is a large mismatch in risks associated with consecutive segments, transfer of buffer contents from one buffer to the other can lead to risk transfer. This transfer, brought about by running machines overtime, leads to an increase in the risk of the upstream machine and reduction of risk in the downstream machine. This form of risk transfer is demonstrated in Figure 5.6

![Figure 5.6: Risk Transfer](image)

Maintenance and overtime are two examples of manipulating risk within a system. An exhaustive study of possible approaches for manipulating risk within a manufacturing system would be beneficial towards development of a condition monitored decision making system. Further, maintenance and overtime, and similarly other methods for risk manipulation, result in additional costs. Taking these costs into consideration, maintenance and production policies can be developed to incorporate risks associated with systems, to increase overall throughput, while ensuring operations remain within desired risk capacity.
5.2.3 Improving Computational Efficiency

The current approach uses a lot of matrix multiplications of sparse and band diagonal matrices. Although the developed approach was an order of magnitude faster than the simulation based approaches, the efficiency of the analytical based approach can further be improved by using algorithms for matrix multiplication customized to sparse and band diagonal matrices. Using such algorithms will further reduce the time required by the analytical approaches.
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