THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

FUNDAMENTALS OF FINNED TUBE HEAT TRANSFER

Edwin H. Young
Dennis J. Ward

The research from which this paper resulted was carried out through Engineering Research Institute Project 1592, sponsored by the Wolverine Tube Division of Calumet and Hecla, Incorporated, Detroit, Michigan,

August, 1956

IP-172

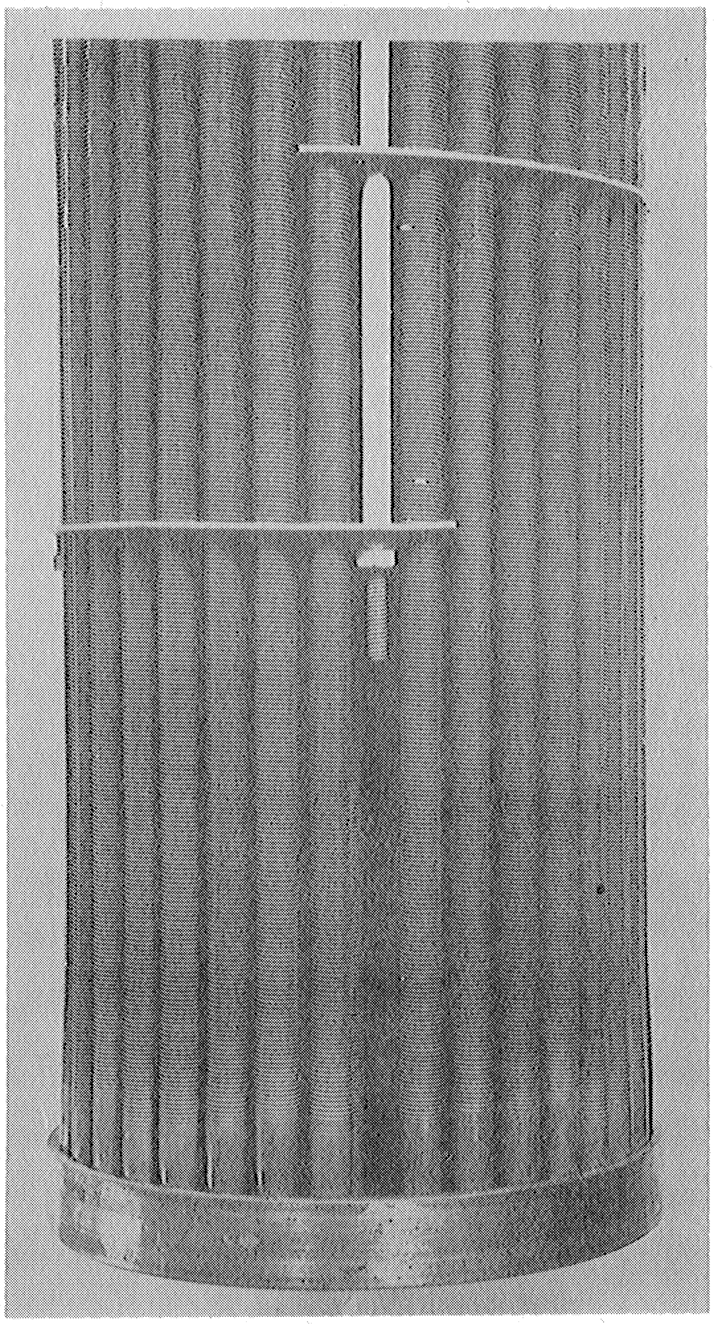

Fig. 1. Typical finned tube bundle.

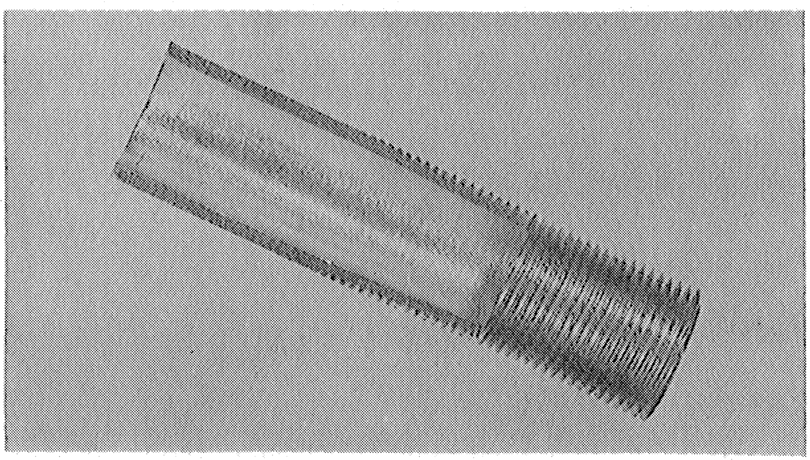

Fig. 2. Cross section of a 19 fin/inch low finned tube.

Fundamentals of Finned Tube Heat Transfer

By EDWIN H. YOUNG And DENNIS J. WARD

Department of Chemical and Metallurgical Engineering
The University of Michigan
Ann Arbor, Michigan

Integral finned tubes can be used to advantage in certain shell and tube heat transfer applications. Finned tube bundles of the type shown in Fig. 1 are becoming widely used in shell and tube heat exchangers. As indicated in Fig. 2 the tube has a plain end for expanding into tube sheets of standard heat exchangers. Heat exchangers for certain applications can be designed so as to take advantage of the extended surface and many existing units can be retubed for the same purpose. The problem of how to determine when finned tubes can be used to advantage is the subject of this series of articles on the use of finned tubes in shell and tube applications. This first article is concerned with the fundamental heat tranfer relationships for determining the required finned tube heat transfer area for shell and tube units. Later articles are concerned with the determination of shell side (fin) coefficients and design of condensers, coolers, and partial condensers. Economic considerations play an important part in determining whether or not finned tube units should be used.

A special Petroleum Processing Report by D. A. Donohue, published in the March, 1956 issue of Petroleum Processing[1] presented an excellent survey of the different types of exchangers, the major characteristics of each, some of the highlights of their design, and some economic considerations. In recent years a number of investigators have published the results of research work on finned tubes.[3,6-16]

The determination of the applicability of finned tubes for a particular service involves the use of ordinary heat transfer considerations plus an understanding of the effect of the fin on the performance of the tube. The basic heat transfer relationship prescribed by the Standards of Tabular Exchanger Manufacturers Association[2] for bare tube surfaces can be modified for extended surface tubes.

Bare Tubes

Equation 1 gives the fundamental relationship for determining the required outside heat transfer tube surface area,

$$A_o = \frac{Q}{U_o \Delta t} \qquad (1)$$

in which:

A_o = required external surface area, sq. ft.,

Q = total heat to be transferred, BTU per hr.,

Δt = the proper mean temperature driving force, and

U_o = overall service coefficient of heat transfer, BTU per (hr.)(°F.)(sq. ft. of outside surface area).

The overall coefficient of heat transfer U_o for bare tubes is further defined as:

$$U_o = \frac{1}{\frac{1}{h_o} + r_o + r_w\left(\frac{A_o}{A_m}\right) + r_i\left(\frac{A_o}{A_i}\right) + \frac{1}{h_i\left(\frac{A_i}{A_o}\right)}} \qquad (2)$$

in which:

h_o = shell side coefficient of fluid medium on outside of tubing,

r_o = fouling resistance on the outside of the tubing,

r_w = resistance of the tube wall (root wall in the case of finned tubes) = X/k,

r_i = fouling resistance on the inside of tubing,

h_i = film coefficient of the fluid medium on inside of tubing,

A_m = mean heat transfer area of metal wall, and

$\left(\frac{A_o}{A_i}\right)$ = the ratio of outside tube surface to inside tube surface.

Finned Tubes

Equation 1 can also be used for determining the required external surface area for finned tube bundles. Equation 2 which defines the overall coefficient of heat transfer for bare tubes may also be used for finned tubes if the outside coefficient, h_o, and outside fouling resistance, r_o, are modified so as to include the efficiency of the finned surface.

The inside film coefficient, h_i, for finned tubes is determined in the same manner as for ordinary bare tubes. TEMA fouling factors[1] are used for the inside surface fouling. The resistance of the root wall of a finned tube can be computed in the usual manner since the resistance of the fin is taken care of separately. The remaining terms r_o and h_o involve the performance of the fin.

Fouling tests reported by Katz, et al.,[3] indicate that the fouling factors on the outside of finned tubes are about the same as for bare tubes and that solution cleaning of fouled finned tubes can restore heat transfer to its initial condition. Additional finned tube fouling data have been published by Armstrong[4] and by Ames and Newell.[5] The usual normal fouling factors of TEMA can therefore be used with integral helical finned tube surfaces.

The determination of the fin side coefficient has been the subject of a number of papers.[6-16] Gardner has presented fin efficiency curves for several forms of straight fins, annular fins, and spines.[17] Dusinberre has shown that the fin efficiency curves of Gardner can be approximated in simple algebraic form (in the high efficiency region).[18] The fin extends out into the fluid stream and as a result has a "skin" temperature that lies somewhere between the root wall outer surface temperature and the fin side fluid "bulk stream" temperature.

The fin efficiency ϕ is defined as:

$$\phi = \frac{q_{(\text{actual at } \Theta)}}{q_{(\text{if at } \Theta_b)}} = \frac{t_{bs} - t_{mf}}{t_{bs} - t_{mr}} \qquad (3)$$

where

q = actual heat transferred through the fin,

$q_{(at\ \theta_b)}$ = the heat that would have been transferred if the fin surface temperature was at the root wall (or base of fin) temperature,

θ = (temperature of bulk stream - temperature of metal fin),

= $(t_{bs} - t_{mf})$,

θ_b = (temperature of bulk stream - temperature of metal root), and

= $(t_{bs} - t_{mr})$.

Dusinberre's relationship for a circumferential fin having a rectangular cross section is:

$$\phi = \frac{1}{1 + \frac{m^2}{3}\sqrt{\frac{D_o}{D_r}}} \qquad (4)$$

where

$$m = H\sqrt{\frac{2}{\left(\frac{1}{h'_o} + r'_o\right) K_m Y}} \qquad \text{(dimensionless)}$$

where

H = fin height = $(D_o - D_r)/2$,

h'_o = actual fin side coefficient,

r'_o = TEMA fouling resistance on outside of fin tubing,

K_m = thermal conductivity of fin metal,

Y = fin thickness,

D_o = diameter over the fin, and

D_r = root diameter.

The factors that affect the skin temperature of the fin and the fin efficiency are:

1. Fin material (thermal conductivity of fin metal).

2. Fin thickness (if the thickness of a fin is increased, the fin temperature tends to approach the tube root wall temperature rather than the bulk stream temperature).

3. Fin height (if the height of the fin is increased, the fin tends to approach the bulk stream temperature with a loss in temperature driving force).

4. Film coefficient (as the film coefficient rises the fin temperature approaches the bulk stream temperature).

5. Shape of fin.

6. Fouling on the fin side.

The fin efficiency is simply a correction factor which must be included in the design to account for the fact that the temperature drop across the fin film coefficient is different than that across the root surface coefficient. This can be illustrated in the following manner. Let

$$Q_t = Q_f + Q_r \qquad (5)$$

where

Q_t = total heat transferred,

Q_f = heat transferred thru fin, and

Q_r = heat transferred thru prime metal (root metal).

Considering a clean outside surface,

$$Q_f = h'_o A_{fin} (t_{bs} - t_{mf}) \tag{6}$$

and

$$Q_r = h'_o A_{root} (t_{bs} - t_{mr}) \tag{7}$$

where

t_{bs} = temperature of bulk stream,

t_{mf} = temperature of fin metal surface,

t_{mr} = temperature of root metal surface, and

h'_o = actual fin side coefficient.

But

$$(t_{bs} - t_{mf}) = \phi (t_{bs} - t_{mr}) \tag{8}$$

therefore

$$Q_f = h'_o A_{fin} (t_{bs} - t_{mr}) \phi \tag{9}$$

and

$$Q_t = h'_o A_{root} (t_{bs} - t_{mr}) + h'_o A_{fin} (t_{bs} - t_{mr}) \phi \tag{10}$$

An examination of Equation 10 indicates that a choice exists in the application of the fin efficiency ϕ. The fin efficiency can be considered to either (a) reduce the temperature difference $(t_{bs} - t_{mr})$, (b) reduce the fin area, A_{fin}, or (c) reduce the fin side coefficient, h_o'. All three interpretations are used and are of course equally correct. Another alternate design procedure consists of combining the fin efficiency ϕ of the fin with the 100 percent efficiency of the root by taking a weighted average based on the relative fin and root areas and reducing the effective film coefficient, h_o', by this amount after having factored out the h_o' in Equation 10. This method is used by Skiba.[19] Two other alternate methods currently in use are presented below; one is based on the application of fin efficiency to the fin area to give an equivalent area and the second is based on a modification of the fin resistance method of Carrier and Anderson.[20]

Equivalent Area Method

Equation 10 can be factored to give the following relationship:

$$Q_t = h_o' (A_{root} + \phi A_{fin}) (t_{bs} - t_{mr}) \tag{11}$$

or

$$Q_t = h_o' A_{eq} (t_{bs} - t_{mr}) \tag{12}$$

where

$$A_{eq} = A_r + \phi A_{fin} \tag{13}$$

The equivalent area, A_{eq}, and actual fin side coefficient, h_o', are related to the total outside area, A_o, and the design coefficient h_o as follows:

$$h_o A_o = h_o' A_{eq}. \tag{14}$$

The design coefficient h_o can be determined by solving Equation 14 for h_o to give:

$$h_o = h'_o \left(\frac{A_{eq}}{A_o}\right) \tag{15}$$

The fouling resistance r_o in Equation 2 is defined as:

$$r_o = r'_o \left(\frac{A_o}{A_{eq}}\right) \tag{16}$$

where r'_o = TEMA Fouling Resistance. The value of h_o from Equation 15 and the value of r_o from Equation 16 can be substituted directly into Equation 2 and the overall coefficient U_o determined. Substitution of U_o into Equation 1 gives the required external finned tube heat transfer area A_o. An illustration of this procedure is presented following a discussion of the Fin Resistance Method.

Fin Resistance Method

Carrier and Anderson presented the fin resistance method for handling fin efficiency for the case of non-fouled fins in 1944.[20] The resistance of a non-fouled fin is defined as:

$$r_f = \left(\frac{1}{h_o} - \frac{1}{h'_o}\right) \tag{17}$$

where r_f = resistance of the fin.

A relationship including fin efficiency which relates h_o and h'_o is given by Equation 15 in which the definition of A_{eq} is given by Equation 13. If Equation 13 is substituted into Equation 15 and Equation 15 is substituted into Equation 17 for h_o the following relationship of Carrier and Anderson is obtained,

$$r_f = \frac{1}{h'_o}\left[\frac{1-\phi}{\frac{A_r}{A_f}+\phi}\right] \tag{18}$$

It can be shown that if the fin surface is fouled Equation 17 must be modified to include the fouling resistance as follows:

$$r_f = \left[\frac{1}{h'_o} + r'_o\right]\left[\frac{1-\phi}{\frac{A_r}{A_f}+\phi}\right] \tag{19}$$

where r'_o = TEMA outside fouling resistance.

To use the fin resistance method in design applications, Equation 2 must be rewritten in the following form:

$$\frac{1}{U_o} = \frac{1}{h'_o} + r'_o + r_f + r_w\left(\frac{A_o}{A_m}\right) + r_i\left(\frac{A_o}{A_i}\right) + \frac{1}{h_i}\left(\frac{A_o}{A_i}\right) \tag{20}$$

in which

h'_o = actual film coefficient of fluid medium on outside of finned tubing,

r'_o = TEMA fouling resistance on outside of finned tubing,

r_f = resistance of the fin (see Equation 18),

r_w = resistance of root wall,

r_i = fouling resistance on the inside of the tube,

h_i = film coefficient of fluid medium on the inside of tubing, and

$\left(\frac{A_o}{A_i}\right)$ = ratio of outside tube surface to inside tube surface.

Equation 4 can be substituted into Equation 19 to give:

$$r_f = \left[\frac{1}{h_o'} + r_o'\right]\left[\frac{\frac{m^2}{3}\sqrt{\frac{D_o}{D_r}}}{1 + \frac{A_r}{A_f}\left(1 + \frac{m^2}{3}\sqrt{\frac{D_o}{D_r}}\right)}\right] \tag{21}$$

For given values of h_o' and r_o' the fin resistance r_f can be directly determined for a particular finned tube by use of Equation 21. Table 1 gives the dimensions of a 3/4 inch, 19 fins per inch admiralty tube. The fin resistance r_f corresponding to various values of $\left[\frac{1}{h_o'} + r_o'\right]$ are given in Table 2.

Table 1

3/4 Inch, 19 Fin-per-inch Admiralty Tube Dimensions

A_o sq. ft. per ft. length	0.438
A_o/A_i	3.18
A_{fin} sq. ft. per ft. length, (0.8 A_o)	0.350
A_{root} sq. ft. per ft. length, (0.2 A_o)	0.088
N, number of fins per inch	19
H, fin height, inches	0.048
Y, fin thickness, inches	0.015
x, root wall thickness, inches	0.050
k, thermal conductivity	65
D_o, outside diameter of fins, inches	0.737
D_r, root diameter, inches	0.641
Diameter of plain end, inches	0.750
Wall thickness of plain end, inches	0.068

Table 2

Fin Resistance of 3/4 Inch 19 Fin-Per-Inch Admiralty Tube

$\left[\frac{1}{\frac{1}{h'_o} + r'_o}\right]$	r_f
0	.00011280
10	.00011277
20	.00011274
50	.00011264
100	.00011248
200	.00011217
500	.00011123
1000	.00010971
2000	.00010679

An examination of Table 2 indicates that the fin resistance of this tube is relatively constant over the usual range of fin side coefficients encountered in finned tube applications. A fin resistance curve can be prepared for any particular tube. An illustration of the use of the fin resistance method of design is presented under Example Calculations.

Example Calculations

An illustration of the use of the equivalent area and fin resistance methods for the determination of U_o for a distillate cooler using 3/4 inch - 19 fin-per-inch admiralty tubes are presented as follows:

Table 3

Film Coefficients	
h'_o, distillate film coefficient*	200
h_i, water film coefficient	1000
r'_o, fin side fouling resistance	0.001
r_i, tube side fouling resistance	0.001

Solution:

A. By The Equivalent Area Method

This method involves the use of Equations 2, 13, 15, and 16 and the determination of fin efficiency. Equation 4 indicates that the fin efficiency is a function of:

$$m = H\sqrt{\frac{2}{\left(\frac{1}{h'_o} + r'_o\right) K_m Y}} \quad \text{and} \quad \left(\frac{D_o}{D_r}\right)$$

Substituting the tube dimensions and the outside coefficient and fouling (from Table 1) the expressions are evaluated as:

$$m = H\sqrt{\frac{2}{\left(\frac{1}{h'_o} + r'_o\right) K_m Y}} = \frac{0.048}{12}\sqrt{\frac{2}{\left(\frac{1}{200} + 0.001\right)(65)\left(\frac{0.015}{12}\right)}} = 0.256$$

and

$$\sqrt{\frac{D_o}{D_r}} = \sqrt{\frac{0.737}{0.641}} = 1.072$$

*The determination of fin side film coefficients is the subject of succeeding articles in this series.

Substituting in Equation 4,

$$\phi = \frac{1}{1 + \frac{(0.256)^2}{3}(1.072)} = 0.975$$

The equivalent outside area is calculated using Equation 12:

$$A_{eq} = A_r + \phi_f A_f = 0.088 + (0.975)(0.35)$$

therefore

$$A_{eq} = 0.429 \text{ sq. ft./ft.}$$

Substituting A_{eq}, A_o, and h_o' into Equation 15 gives:

$$h_o = 200\left(\frac{0.429}{0.438}\right) = 196$$

Substituting the outside fouling factor, A_{eq}, and A_o into Equation 16 gives:

$$r_o = .001 \frac{0.438}{0.429} = .00102$$

The tube wall resistance is given by:

$$r_w\left(\frac{A_o}{A_m}\right) = \frac{X A_o}{K A_m} = \frac{(0.050)(0.438)}{(12)(65)(0.153)} = .000184$$

The overall coefficient, U_o, is obtained by using Equation 2,

$$U_o = \frac{1}{\frac{1}{196} + 0.00102 + 0.000184 + 0.001(3.18) + \frac{3.18}{1000}}$$

$$= \frac{1}{0.0127} = 79 \text{ Btu/hr-°F-sq ft (outside surface)}$$

B. By The Fin Resistance Method

The overall coefficient is computed using Equation 20. The fin resistance is given in Table 2. For the value of:

$$\left[\frac{1}{\frac{1}{h'_o} + r'_o}\right] = \frac{1}{\frac{1}{200} + 0.001} = 166.7$$

the fin resistance from Table 2 is:

$$r_f = 0.000112$$

Substituting into Equation 20 gives:

$$\frac{1}{U_o} = \frac{1}{200} + 0.001 + 0.000112 + 0.000184 + 0.001(3.18) + \frac{3.18}{1000}$$

$$= 0.0127$$

therefore

$$U_o = \frac{1}{0.0127} = 79 \text{ Btu/hr-°F-sq ft (outside surface)}$$

Discussion

It is a matter of personal choice as to which method is to be preferred. It is apparent that the corrections for fin efficiency are very slight in the above example. With fin efficiencies of 97 percent or higher the error introduced in assuming 100 percent efficiency will give satisfactory designs for most design purposes. The alternate methods give identical results and indicate how fin efficiency can be used in designing finned tube heat exchangers.

Acknowledgment

Permission by Wolverine Tube Division of Calumet and Hecla, Inc. to publish this paper is appreciated.

References

1. Donohue, D. A., Petroleum Processing, March, 1956.

2. "Standards of the Tubular Exchanger Manufacturers Association", 2nd Ed., TEMA, New York, 1949.

3. Katz, D. L., Knudsen, J. G., Balekjian, G., and Grover, S. S., Petroleum Refiner, Vol. 33, No. 11, 1954.

4. Armstrong, R. M., Trans. ASME, 67 (1945).

5. Ames, G. W., and Newell, R. G., Chemistry in Canada, January, 1954.

6. Kays, W. M., and London, A. L., Trans. ASME, 72 (1950).

7. Katz, D. L., Young, E. H., Williams, R. B., and Balekjian, G., Petroleum Refiner, Vol. 33, No. 11 (1954).

8. Beatty, K. O., Jr., and Katz, D. L., Chem. Eng. Prog., 44 (1948).

9. Myers, J. E., and Katz, D. L., Chem. Eng. Prog. Symposium Series No. 5, Heat Transfer, Vol. 49, 1953.

10. Williams, R. B., and Katz, D. L., Trans. ASME, 74 (1952).

11. Jameson, S. L., Trans. ASME, 67 (1945).

12. London, A. L., Kays, W. M., and Johnson, D. W., Trans. ASME, 74 (1952).

13. Schmidt, T. E., Institute of Mechanical Engineering and ASME, Proc. of the General Discussion on Heat Transfer, Section II, London, 1951.

14. Katz, D. L., and Geist, J. M., Trans. ASME, 70 (1948).

15. Zieman, W. E., and Katz, D. L., Petroleum Refiner, Vol. 26, No. 8, 1947.

16. Katz, D. L., Young, E. H., and Balekjian, G., Petroleum Refiner, 33 (1954)

17. Gardner, K. A., Trans. ASME, 67 (1945).

18. Dusinberre, G. M., Mech. Eng., Vol. 78, No. 6, p. 570, 1956.

19. Skiba, E. J., Chem. Eng., Vol. 61, No. 12, 1954.

20. Carrier, W. H., and Anderson, S. W., Heating, Piping, and Air Conditioning May, 1944.