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NUMERICAL ANALYSIS OF THE TEMPERATURE DISTRIBUTION IN THE ROOT OF TWO HIGH-FIN ALL-ALUMINUM TUBES

Report No. 41

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ABSTRACT

The results of this investigation indicate that the effect of the root-wall temperature distribution on the fin efficiency of 9-fin-per-inch aluminum tubes is small for commercial tubes in normal applications. This reduces the fin efficiency by 4 to 5% for a tube with a 0.020-inch root wall and by 11.5% for one with a 0.010-inch root wall.

OBJECTIVE

The purpose of this investigation is to determine the effect of the longitudinal root-wall temperature distribution on the fin efficiency of thin-walled, 9-fin-per-inch, all-aluminum tubes in normal applications.

INTRODUCTION

Finned tubes can be used in many different applications, such as condensing, boiling, cooling, and various combinations of the above. In using the finned tube for transferring heat in a desired application, the heat is transferred under the influence of a "temperature-difference" driving force. This "temperature difference" is distributed across the outside film resistance, the outside fouling resistance, the metal resistance, the inside fouling resistance, and the inside film resistance. The heat is transferred through the metal by conduction under the influence of the temperature drop across the metal.

The performance of a finned tube can be greatly influenced by the distribution of the heat flux within the metal wall. In order to handle high heat transfer rates through a finned tube, it is essential that a large portion of the energy be conducted through the fin itself to the base of the fin and then be dissipated to the film at the inside of the tube. The relative thickness of the fin and root-wall metal is an important factor in the channeling of heat flux to the inner surface of a tube from the surface of the fin.

It is apparent that if for a given fin thickness the root-wall thickness is continually decreased, the heat-flux pattern in the root wall will approach a direction perpendicular to the axis of the tube. Consequently, as the wall thickness decreases, a smaller portion of the heat flux will channel through the fin, resulting in a decrease in the rate of heat transfer. Therefore, a small ratio of root-wall thickness to fin thickness could result in reduced efficiency of the extended surface for a particular set of conditions. Such a situation will be referred to as "crowding of the heat at the base of the fin."

The critical ratio of root-wall to fin thickness is the value below which "crowding" becomes a significant factor in the performance of the tube. This critical value depends on the individual heat transfer coefficients and the thermal conductivity of the metal tube.

This investigation was undertaken in order to determine whether the ratio of root-wall to fin thickness used in a standard one-half-inch fin height, all-aluminum tube having a 0.020-inch root wall is sufficiently above the critical value to prevent occurrence of "crowding of the heat flux at the base of the fin" in ordinary applications of this tube.

In heat transfer theory it can be shown that an adiabatic line or line of constant heat flux is in every point perpendicular to the isotherm passing through that point.[1-3] In other words, once a system of isotherms is known, the heat-flux pattern can be obtained by drawing a system of orthogonal trajectories. Consequently, a fundamental part of this investigation was the determination of the temperature distribution within the root wall and fin. From this distribution a system of isotherms could be obtained. Once the system of isotherms had been determined, the heat-flow pattern could be obtained simply by drawing a system of lines perpendicular to the isothermal lines. This method results in a graphical presentation of the temperature distribution and heat-flux pattern throughout a cross section of the finned tube.

A quantitative measure of the degree to which the root-wall thickness affects the heat transfer to the fin can be made from fin-efficiency determinations.[8] In the paper on fin efficiency by Mr. Karl Gardner,[8] assumptions were made in order to obtain solutions of the equations derived for predicting fin efficiency. Listed among the assumptions was the following:

"The temperature of the base of the fin is uniform."

With the exception of the above assumption, the conditions used in the analyses presented in this report are the same as those used by Mr. Gardner. Therefore, a comparison of the fin efficiencies as obtained by Mr. Gardner with those obtained in these analyses is a test of the above assumption. This comparison gives a direct indication of the degree to which the root-wall temperature distribution affects the efficiency of the fin.

THEORETICAL CONSIDERATIONS

The process of heat transfer from the inside fluid to the inside surface of the tube wall is described by the equation

$$q = h_i (T_{wi} - T_i) A_i \quad . \tag{1}$$

Similarly, the heat transfer from the external surface of the root wall and fin to the outside fluid is described by the equation

$$q = h_o (T_{wo} - T_o) A_o \quad , \tag{2}$$

where

q = rate of heat transferred, Btu/hr,
A_i = inside surface area, ft^2,

A_o = outside surface area, ft^2,
h_i = inside film coefficient, Btu/hr/ft^2/°F,
T_{wi} = temperature of the inside wall, °F,
T_{wo} = temperature of the outside wall, °F,
T_i = mean temperature of the inside fluid,
h_o = outside film coefficient, and
T_o = mean temperature of the outside fluid.

For constant inside and outside coefficients and temperatures with a homogeneous metel wall, the conduction of heat through the metal wall involves a process of two-dimensional heat conduction. If the discussion is restricted to the steady-state condition, it can be said that the temperature at any point in the wall is a function of the coordinates of that point, i.e.,

$$T = T(x,y) \quad . \tag{3}$$

By establishing a heat balance around a differential element of the wall, the following differential equation can be obtained:[1-3]

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad . \tag{4}$$

This is known as Laplace's equation for two-dimensional heat flow. The integration of this equation would give the function $T = T(x,y)$ and, therefore, the temperature distribution in the wall. However, the analytical determination of a particular solution for this equation involves the introduction of both initial and boundary conditions. The complexity and uncertainty of boundary conditions makes it extremely difficult, in practice, to attempt a direct analytical solution of this equation.

A practical method consists of approximating the solution of Laplace's equation by substituting the differentials by the technique of "finite differences."[2,4,5] This substitution leads to a numerical analysis solution. The method is briefly summarized below.

METHOD OF ANALYSIS

Several approximate methods have been proposed for the solution of the Laplace equation.[2-7] The underlying principle in most of the methods is the substitution of differentials by finite differences.

In order to apply the method of finite differences, a cross section of the tube wall is subdivided into grids as shown in Figs. 1 and 2. Under

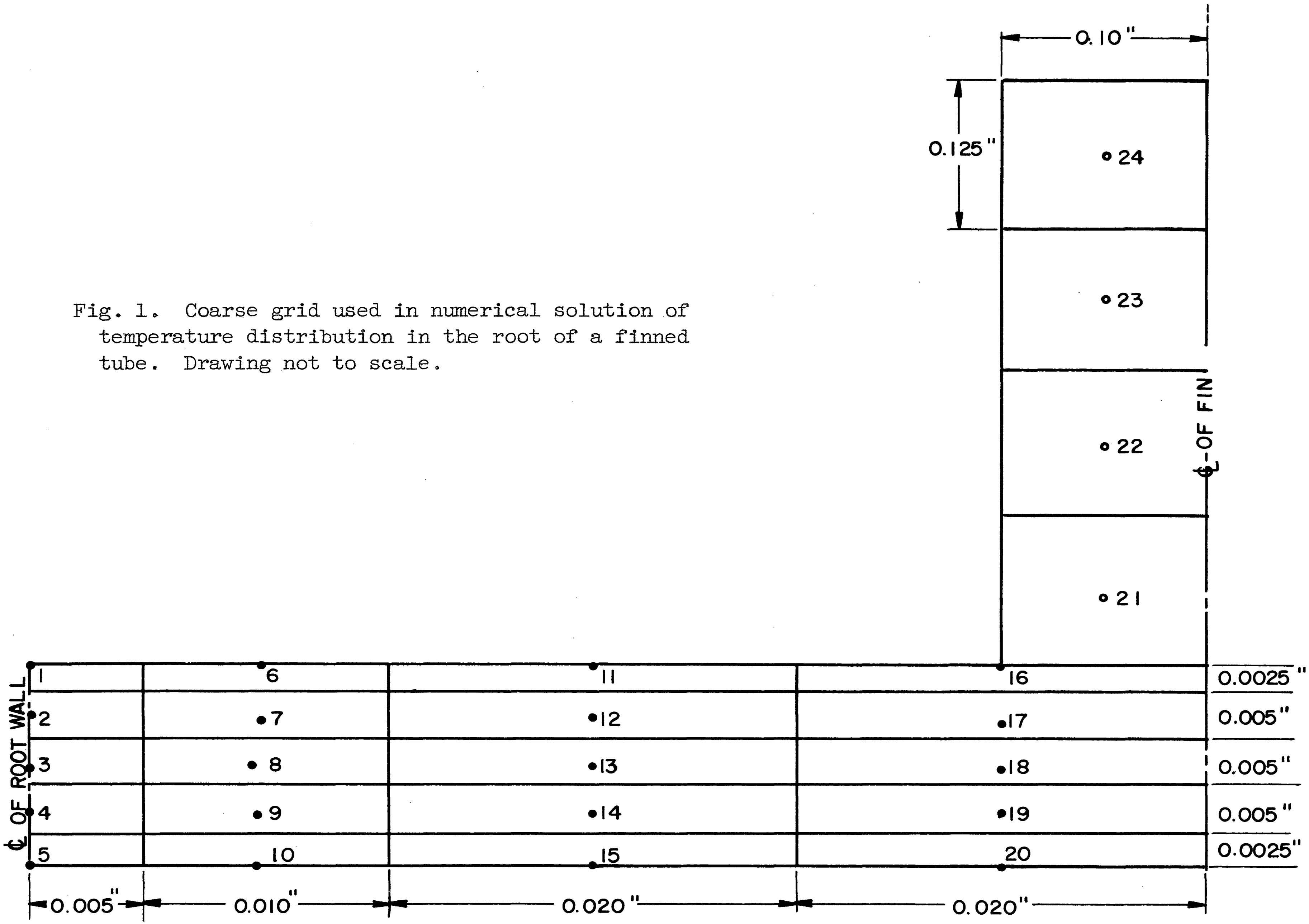
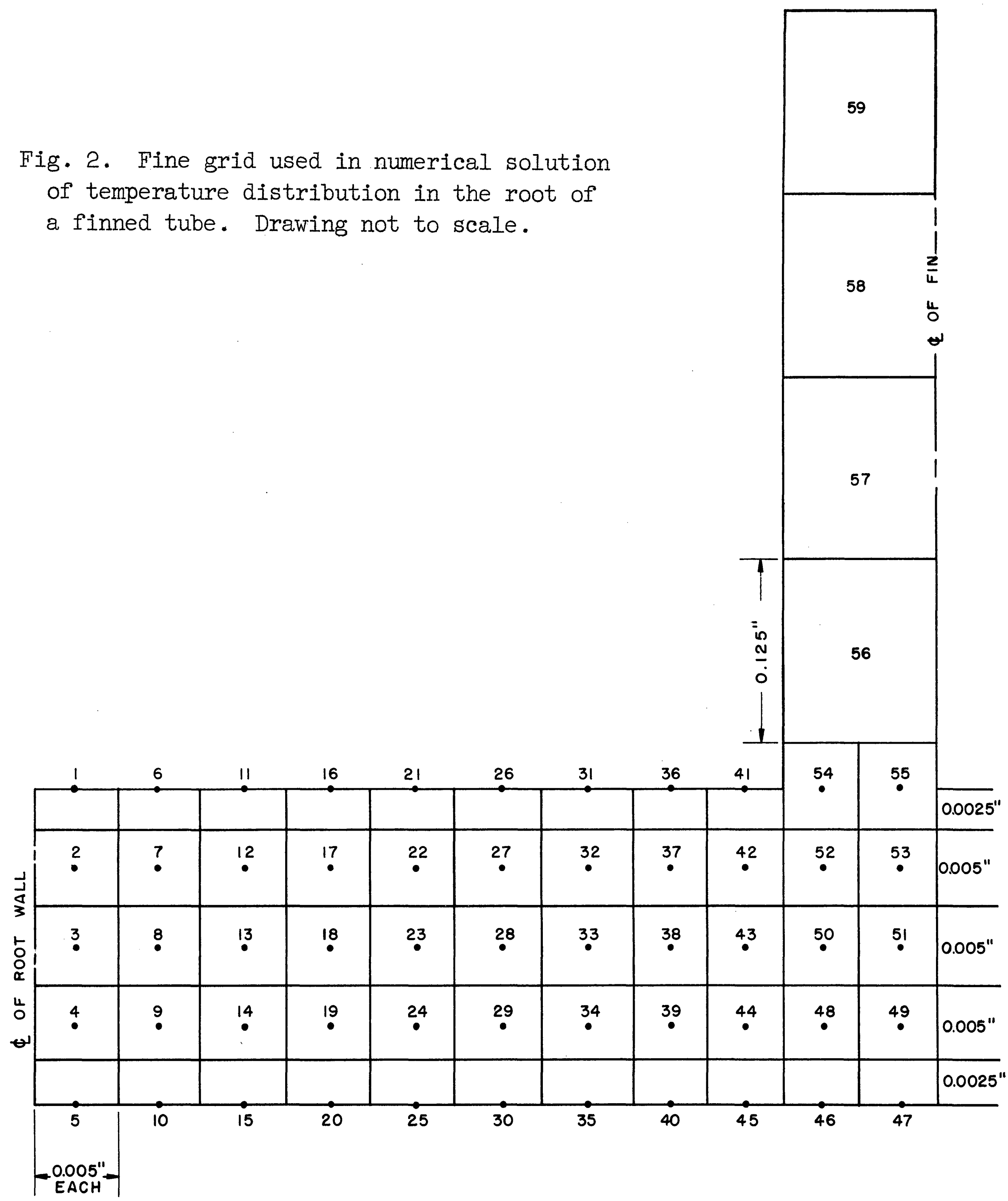


Fig. 1. Coarse grid used in numerical solution of temperature distribution in the root of a finned tube. Drawing not to scale.

Fig. 2. Fine grid used in numerical solution of temperature distribution in the root of a finned tube. Drawing not to scale.



steady-state heat transfer conditions there will be no accumulation of thermal energy in any one grid. Therefore, the rate of heat transfer to the grid under consideration is equal to the rate of heat transfer from the grid.

The grids are numbered in order and a temperature T_i is assigned to each one, where the subscript "i" is the identification number corresponding to that particular grid. A heat balance can be written around each grid. Since the temperatures of the outside and inside fluids are known, a set of "n" linear equations can be obtained, involving "n" unknown temperatures. Theoretically this system of equations could be solved simultaneously. However, if accurate results are desired, the grids should be very small. Consequently, the number of equations would become very large and the exact solution would become extremely tedious and time consuming.

Professor Southwell[5-7] introduced the idea of relaxation, which gave practicality to the solution of a large number of simultaneous equations. Essentially the method requires a good estimation of the temperatures in the various grids. The residuals in the grids are evaluated from heat-balance equations. A systematic procedure is followed for relaxing the various residuals. The final set of temperatures which reduces all the residuals very close to zero gives a satisfactory solution.

For the solution of this particular problem, the section under consideration was divided into 22 blocks for the first approximation. The resulting system of 22 equations was solved by successive elimination. After the first approximate temperature distribution had been established for the 22-grid system, a new grid of 57 blocks was adopted. Taking as a basis the previous approximate temperatures, this new system of 57 equations was solved by relaxation and the solution was considered complete when all the residuals fell between + 0.05 and -0.05 (Btu/hr). The restriction of the residuals to fall within $\pm$ 0.05 (Btu/hr) results in an average maximum error for the computed temperature of 0.006°F. Once the temperatures at the various points were known, a system of isotherms was drawn in by resorting to linear interpolation between any two temperatures whenever necessary.

For the tube having a root-wall thickness of 0.01 inch, a similar analysis was carried out. The first coarse grid consisted of 14 blocks and the resulting system of equations was solved simultaneously. For the final solution a network of 32 grids was used.

Since no useful purpose would result from reproducing all the above equations in this report, they are not reproduced here. The equations and their solutions are being maintained in project file for future reference.

DESCRIPTION OF SYSTEMS ANALYZED

Longitudinal sections of the tubes were considered, as shown in Figs. 1 and 2. By consideration of symmetry it is permissible to restrict the analysis to a section between the axis of symmetry of the fin and the mid-plane between adjacent fins.

The analysis was conducted for two different root-wall thicknesses. Table I summarizes the tube dimensions and heat transfer conditions assumed for the analyses.

TABLE I

DIMENSIONS AND HEAT TRANSFER CONDITIONS ASSUMED FOR ANALYSES

	Tube: A	A	B
	Analysis: I	II	III
Heat Transfer Conditions			
h'_o, Btu/hr/ft^2/°F	10	50	10
k_m, Btu/hr/ft^2/°F	120	120	120
h_i, Btu/hr/ft^2/°F	1000	1000	1000
Inside fluid temperature, °F	240	240	240
Outside fluid temperature, °F	100	100	100
Dimensions			
Fin height, in.	0.50	0.50	0.50
Fin thickness, in.	0.02	0.02	0.02
Fin spacing (center to center), in.	0.110	0.110	0.110
Root-wall thickness	0.02	0.02	0.01

The heat transfer conditions assumed for analyses I and III (Table I) correspond to a typical application of high-finned tubes (such as for condensing steam inside the tubes by air on the outside of the tube in forced convection). The 0.020-inch root wall was selected because a tube having approximately this root-wall thickness gave unusually low heat transfer performance for a monometallic tube. This tube had been tested in a steam-condensing apparatus in which the air coefficient was of the same order of magnitude as assumed in the first analysis (Table I). The second analysis was made in order to determine if crowding of heat flux would occur under more severe heat transfer conditions than were assumed for the first analysis. The third analysis applies to a 0.010-inch root wall. This analysis was made to determine the effect of a thinner root wall on crowding of the heat flux at the base of the fin.

Other assumptions used in the solution of the problem were:

(a) The thermal conductivity and heat capacity of the metal is independent of temperature.

(b) The convective heat transfer coefficients for the inside and outside of the tube remain constant over their entire surfaces.

(c) No heat sources or sinks are present within the metal.

A scale drawing of the fin and root profile considered for the tube with a 0.02-inch root wall is shown by solid lines in Fig. 3.

ANALYTICAL RESULTS

The results of the three analyses are graphically presented in Figs. 4, 5, and 6. These figures show the lines of constant temperature (isotherms) and lines of constant heat flux (adiabats). In all these figures line a-b is the heat-flux line of greatest interest. This line indicates that the heat entering the inner root-wall surface to the right of point "b" channels into the fin. The heat entering the inner root-wall surface to the left of point "b" flows through the root wall only. The position of point "b" therefore indicates the relative portion of the heat that is transferred through the fin and through the outer root wall.

Fin efficiencies were obtained by graphical integration, using the fin and outer root surface temperatures given in Table II and Table III. The fin efficiencies obtained are presented in Table IV. Also included in Table IV are the fin efficiencies obtained from the curves presented by Gardner.[9]

The air-side coefficients actually act on the surfaces with which they are in contact. The fin efficiency is a measure of the reduction in temperature drop across this film encountered along the fin due to the conduction of heat through the fin metal radially toward the root. This means that either a variable temperature difference must in some way be applied, or the coefficient reduced, on the area to obtain an "equivalent area." A widely used method is that of combining the fin efficiency with the outside area to give

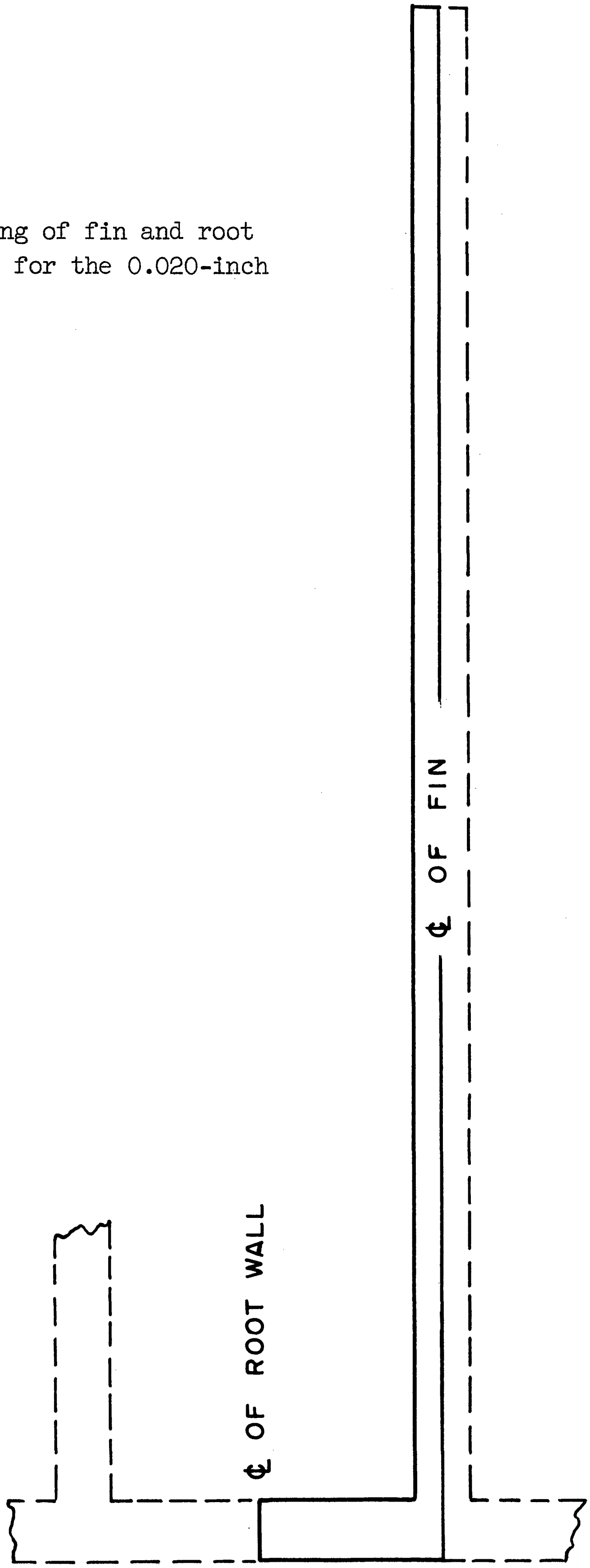


Fig. 3. Scale drawing of fin and root profile considered for the 0.020-inch wall tube.

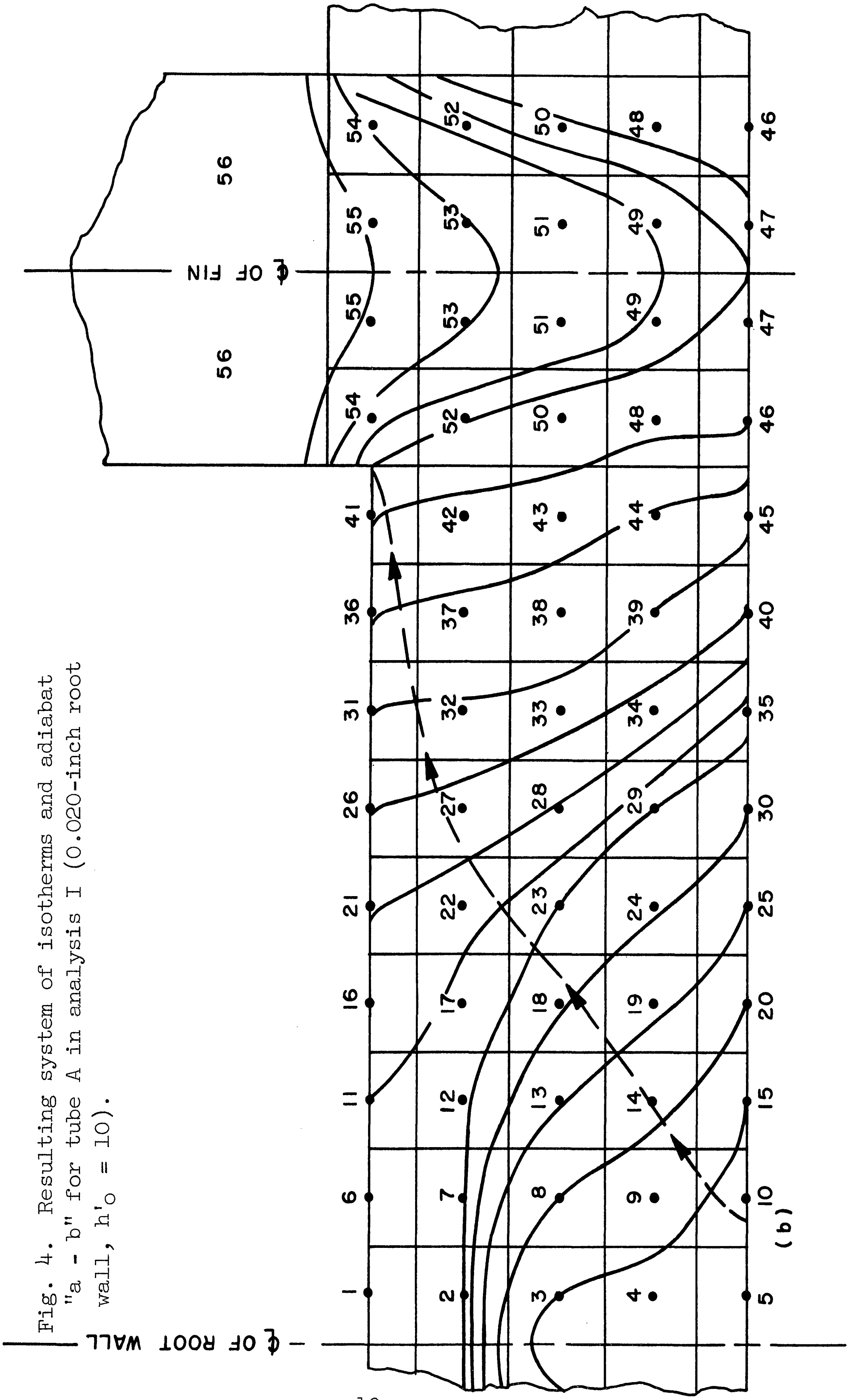


Fig. 4. Resulting system of isotherms and adiabat "a - b" for tube A in analysis I (0.020-inch root wall, $h'_o = 10$).

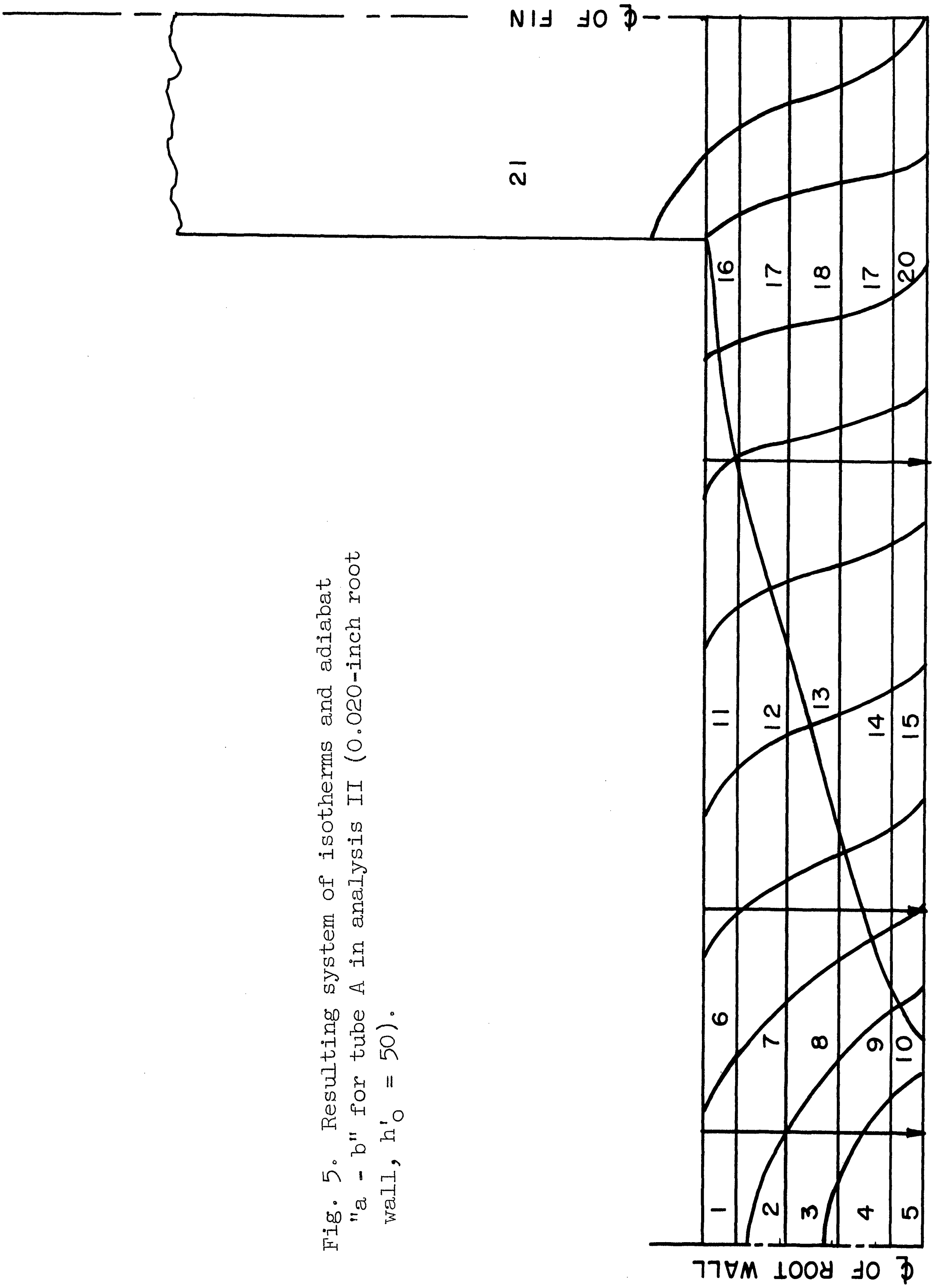


Fig. 5. Resulting system of isotherms and adiabat "a - b" for tube A in analysis II (0.020-inch root wall, h'_o = 50).

Fig. 6. Resulting system of isotherms and adiabat "a - b" for tube B in analysis III (0.010-inch root wall, $h'_o = 10$).

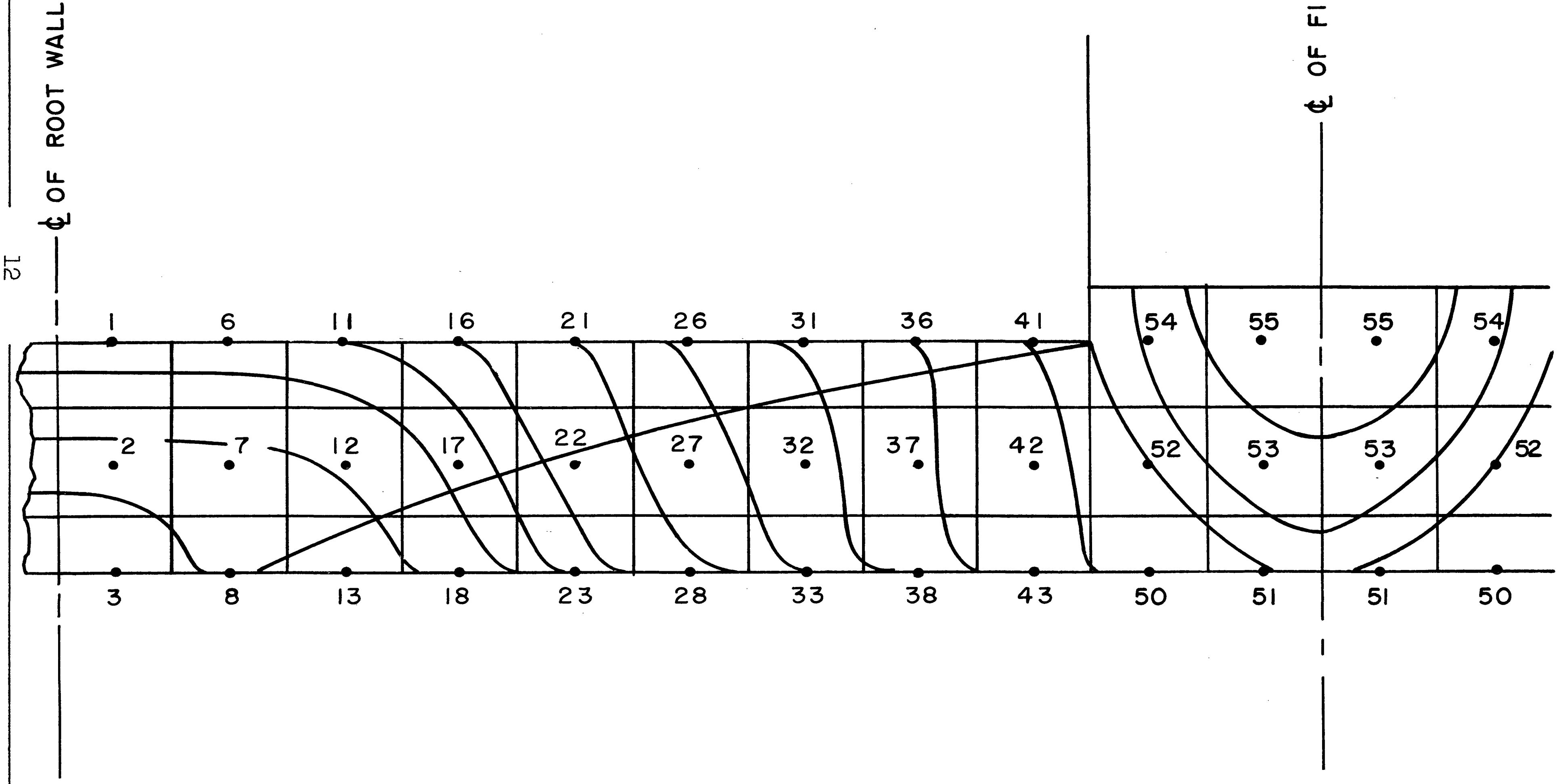


TABLE II

COMPUTED TEMPERATURE DISTRIBUTION IN THE FIN AND ROOT WALLS, USING GRID PRESENTED IN FIG. 1

Position	Tube: A Analysis: I	A II	B III
1	228.181	228.003	227.739
2	228.189	228.031	227.755
3	228.332	228.063	227.788
4	228.341	228.101	---
5	228.369	228.143	---
6	228.212	227.989	227.707
7	228.229	228.017	227.733
8	228.267	228.049	227.767
9	228.303	228.090	---
10	228.340	228.128	---
11	228.140	227.887	227.607
12	228.152	227.914	227.623
13	228.176	227.967	227.657
14	228.210	228.007	---
15	228.247	228.056	---
16	227.734	227.072	227.095
17	227.831	227.375	227.184
18	227.907	227.577	227.243
19	227.967	227.754	---
20	228.015	227.804	---
21	222.352	216.656	224.720
22	219.302	202.153	222.456
23	216.900	192.975	199.152
24	213.624	187.825	187.745

TABLE III

COMPUTED TEMPERATURE DISTRIBUTION IN THE FIN AND ROOT WALLS, USING GRID PRESENTED IN FIG. 2

Position	Tube: A Analysis: I	B III	Position	A I	B III
1	228.181	227.739	31	228.080	227.525
2	228.189	227.755	32	228.095	227.543
3	228.332	227.788	33	228.102	227.566
4	228.341	---	34	228.154	---
5	228.369	---	35	228.181	---
6	228.178	227.722	36	228.011	227.435
7	228.186	227.748	37	228.030	227.445
8	228.303	227.778	38	228.042	227.471
9	228.317	---	39	228.096	---
10	228.353	---	40	228.125	---
11	228.173	227.696	41	227.885	227.275
12	228.183	227.718	42	227.925	227.297
13	228.270	227.758	43	227.963	227.336
14	228.292	---	44	228.016	---
15	228.330	---	45	228.050	---
16	228.163	227.667	46	227.865	---
17	228.175	227.682	47	227.585	---
18	228.235	227.747	48	227.730	---
19	228.262	---	49	227.250	---
20	228.303	---	50	227.650	227.00
21	228.149	227.627	51	227.160	226.53
22	228.160	227.642	52	227.520	226.79
23	228.195	227.685	53	227.920	226.07
24	228.230	---	54	227.265	226.60
25	228.268	---	55	227.405	225.80
26	228.125	227.585	56	223.425	216.548
27	228.137	227.600	57	220.005	202.113
28	228.150	227.634	58	216.95	192.543
29	228.195	---	59	213.620	187.765
30	228.227	---			

TABLE IV

FIN EFFICIENCIES OBTAINED FOR ASSUMED CONDITIONS

	Tube:	A	A	B
	Analysis:	I	II	III
Fin efficiency from graphical integration of temperature distribution	Grid I	92.1%	85.0%	85.2%
	Grid II	92.0%	---	85.0%
Fin efficiency from Gardner curve, Fig. 3A, Reference 9		96 %	89 %	96 %

an equivalent area:

$$A_{eq} = A_r + \epsilon_f A_f \quad ,$$

where

A_{eq} = equivalent area,
A_r = root area,
ϵ_f = fin efficiency, and
A_f = fin area.

The relationship between the two areas A_o and A_{eq} is given by

$$h_o A_o = h'_o A_{eq} \quad ,$$

where

h'_o = actual outside film coefficient and
h_o = coefficient compensated for fin efficiency.

This equation can be used for computing h_o from the areas and fin efficiency. This can be done, using either the Gardner fin efficiency or the analytical efficiencies given in Table IV. For comparison purposes the h_o coefficients, using both efficiencies, are tabulated in Table V. A typical calculation is given in Appendix B.

Table V shows that the effect of the root-wall thickness is relatively small for the 0.02-inch tube but becomes a more significant factor for the tube with the 0.01-inch root wall (4% as compared to 11%).

TABLE V

INFLUENCE OF ROOT-WALL TEMPERATURE DISTRIBUTION
ON EFFECTIVE HEAT TRANSFER COEFFICIENT

Analysis	h'_o	ϵ_f, %	$(\epsilon_f)_{Gardner}$ %	h_o	$(h_o)_{Gardner}$	Percent Difference
I	10	92.0	96.0	9.25	9.65	4.32
II	50	85.0	89	43.4	44.9	3.46
III	10	85.0	96	8.68	9.65	11.2

DISCUSSION OF RESULTS

The root-wall thicknesses employed in these investigations were thinner than normally encountered in industrial finned tubes. The heat transfer conditions assumed here were of the same order of magnitude as would be encountered in normal operation using forced air outside high-finned tubes (with condensing steam or water flowing inside the tubes). Since, as shown in Table IV, the effect of increasing the root-wall thickness is to decrease its influence on fin efficiency for normal applications of industrial tubing, the effect of the root-wall temperature distribution would be somewhat less than indicated in this investigation.

The effect of some of the variables on the results of this investigation can be deduced qualitatively. The influence of these variables, keeping all other conditions constant, would be expected to be as follows:

1. Number of Fins per Inch (Fin Spacing).—The effect of reducing the fin spacing (increasing the number of fins per inch) would be to increase the influence of the root-wall temperature distribution, resulting in lower fin efficiencies.

2. Fin Height.—An increase in fin height would increase the influence of the root-wall temperature distribution on fin efficiency, while a decrease in fin height would decrease this influence.

3. Root-Wall Thickness.—The effect of a change of root-wall thickness is shown in Table IV. A comparison of analyses I and III shows that the

thicker the root wall, the closer the fin efficiency to the predicted value given by Gardner's curves.

CONCLUSIONS

The influence of the temperature distribution in the root wall of a high-finned tube on fin efficiency is quite small for a 0.02-inch root wall under the conditions of analysis but becomes more significant for thinner root walls. Since the root-wall thickness of the present commercially manufactured tubing is greater than 0.02 inch, the effect of the root wall on the fin efficiency and the corresponding effective heat transfer coefficient can be considered to be small for air-cooling applications with high-finned tubing.

APPENDIX A

Sample Calculations

Following is a sample of the calculations involved in writing a heat balance around an elementary block.

Consider block 6 of Fig. 2. Heat flows to block 6 from blocks 1, 7, and 11 by conduction, while heat is lost by convection to the outside. Therefore, calling q the net rate of heat flow to the block,

$$q = k\Delta y \frac{(T_1 - T_6)}{\Delta x} + k\Delta x \frac{(T_7 - T_6)}{\Delta y} + k\Delta y \frac{(T_{11} - T_6)}{\Delta x} + h\Delta x\ (100 - T_6) .$$

If steady state is considered,

$$q = 0 .$$

Substituting values from Table I and Fig. 2

$$0 = \frac{120x0.0025}{0.005} (T_1 - T_6) + \frac{120x0.005}{0.005} (T_7 - T_6) + \frac{120x0.0025}{0.005} (T_{11} - T_6) + \frac{10x0.005}{12} (100 - T_6) ,$$

or, after simplification,

$$4.000069\ T_6 - 0.0069 - T_1 - T_{11} - 2T_7 = 0 .$$

APPENDIX B

Calculation of the Effective Heat Transfer Coefficient for Analysis I

The effective outside heat transfer coefficient is defined as

$$h_o = h'_o \frac{A_{eq}}{A_o} ,$$

where

h'_o = outside film coefficient,
A_{eq} = equivalent outside area,
= $A_r + \epsilon_f A_f$
A_o = total outside area,
A_r = outside root area,
A_f = outside fin area, and
ϵ_f = fin efficiency.

The root area for a tube of the following dimensions is 0.28 ft^2/ft.

Tube Dimensions

Outside diameter	2.00 inches
Root diameter	1.04 inches
Fin spacing	0.11 inch
Fin thickness	0.02 inch

The total outside area is 3.59 ft^2/ft; therefore,

$$A_f = A_o - A_r = 3.59 - 0.28 = 3.31 \; ft^2/ft \quad .$$

The fin efficiency obtained in analysis I was 92%; therefore,

$$(h_o) = 10 \left(\frac{0.92 \times 3.31 + 0.28}{3.59} \right) = 9.25 \quad .$$

The fin efficiency from Gardner's curves was 96.0%; therefore,

$$(h_o)_{Gardner} = 10\left(\frac{0.96 \times 3.31 + 0.28}{3.59}\right) = 9.65 \quad .$$

The percent discrepancy is

$$\% = \frac{9.65 - 9.25}{9.25} \times 100 = 4.32\% \quad .$$

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