DEPARTMENT OF CHEMICAL AND METALLURGICAL ENGINEERING
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Progress Report

on

THE PERFORMANCE OF HIGH-FIN TUBES IN COMBINED RADIATIVE AND NATURAL CONVECTION HEAT TRANSFER

Report No. 58

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NOTATION

a, b, c = Constants

A = Outside heat transfer area per foot of length, sq.ft./ft.

A_c = Area of the ultimate receiving plane (sink), sq.ft.

A_{cp} = Area of the continuous plane replacing the row of tubes, sq.ft.

A_f = Area of the flame bundle, sq.ft.

A_o = Total outside area, sq.ft.

A_r = Effective refractory surface area, sq.ft.

A_t = Total area of furnace surfaces in the radiant section = A_{cp} + refractory surface unprotected by tubes, sq.ft.

B, C = Constants

C_C = Correction factor for converting emissivity of carbon dioxide at 1 atm. total pressure to emissivity at pressure P atm.

CFM = Cubic feet per minute

C_p = Heat capacity of gases, BTU/lb.- °F

C_W = Correction factor for converting emissivity of water to values of P_W and P_T other than 0 and 1 atm., respectively

D_o = Outside diameter of bare tubes or diameter over fins of finned tubes, inches

D_r = Root diameter of tube, feet

F_{CF} = Overall exchange factor

$\overline{F}_{cf}$ = Angular emissivity factor

F_D = Heat transfer depth factor

NOTATION (Continued)

F_{ij} = The fraction of all the radiation leaving a surface i in all directions which is intercepted by surface j.

F_{rc} = The fraction of all the radiation emitted from all the refractory surfaces in all directions, which if not absorbed by the gases would hit the net receiving surface

F_s = Angle emissivity

G = Mass flow rate at minimum cross section, lb./hr.-sq.ft.

h_c = Convective heat transfer coefficient, BTU/hr.-sq.ft.-°F

H = The total net heat input into the furnace from all sources, BTU/hr.

k = Thermal conductivity of gases, BTU/hr.-sq.ft.-°F/ft.

ℓ = Fin height, inches

L = Mean length of the radiant beam, ft.

Nu = Nusselt Number ($h_c D_o/k$) for bare tube or ($h_c D_r/k$) for fin tube

P = Partial pressure of carbon dioxide plus the partial pressure of water, atm.

P_c = Emissivity of receiving surface

P_{cp} = Emissivity of ultimate heat-receiving cold plane surface

P_C = Partial pressure of carbon dioxide, atm.

P_f = Flame emissivity

Pr = Prandtl Number, $Cp\,\mu/k$

P_W = Partial pressure of water, atm.

q_c = Convective heat transferred, BTU/hr.

q_f = Net rate of heat transfer from the flame by all mechanisms, BTU/hr.

q_r = Radiative heat transferred, BTU/hr.

q_t = Total heat transferred, BTU/hr.

Re = Reynolds Number $(D_r G / \mu)$ or $(D_o G / \mu)$

s = Fin spacing, inches

t_f = Theoretical flame temperature, °F

t_f' = Pseudo-flame temperature, °F

t_g = Temperature of gas leaving combustion chamber, °F

t_g' = Mean gas temperature in the combustion chamber, °F

t_o = Outside air temperature, °F

t_s = Temperature of the tube wall, °F

T = Temperature, °R

T_g' = Mean temperature of the hot gases in the furnace, °F + 460

T_s = Mean tube skin temperature, °F + 40

U_r = Conductance of the refractory, inside refractory coefficient and outside refractory coefficient, BTU/hr.-sq.ft.- °F

w = Pounds mass of material per hour

W = Width of combustion chamber, feet

Subscripts

C = Subscript indicating carbon dioxide

i = Subscript 1,2, etc., identifying radiating plane

NOTATION (Continued)

ij = Subscript numbers indicating that radiation leaving plane 1 is being received by plane 2, i.e., plane 1 is a radiating plane and plane 2 is a receiving plane

j = Subscript 1,2, etc., identifying receiving plane

W = Subscript indicating water

<u>Greek Letters</u>

α = Absorptivity

α_{CO_2} = Absorptivity of carbon dioxide

α_g = Absorptivity of the stack gas

α_{H_2O} = Absorptivity of water

β = Fraction of total net heat input lost from the external furnace walls

Δ = Incremental value

ϵ = Emissivity

ϵ_{CO_2} = Emissivity of carbon dioxide

ϵ_g = Emissivity of stack gas

ϵ_{H_2O} = Emissivity of water

σ = 0.173×10^{-8} BTU/sq.ft.-hr.-°R^4

ϕ = Overall exchange coefficient

μ = Viscosity, lbs./ft.-hr.

ABSTRACT

The summary of the current status of the laboratory investigation in progress on the performance of high-fin tubes in combined radiative and natural convective heat transfer in direct-fired water heating applications is presented. The significance of all the heat transfer data collected to date is presented in graphical form. The radiant and convective heat transfer design methods of Lobo and Evans and of Hottel are applied to the hot water heater under investigation and compared with experimental data. Comparisons are also made with the work of Zakharikov. A tentative recommended design procedure for designing finned tube hot water heaters using Tube No. 61-0714065-01 is presented. All experimental data covered by this report are for tubes on a 1 15/16 inch square pitch.

OBJECTIVE

The purpose of this investigation is to obtain experimental heat transfer data on high-fin tubes in combined radiative and natural convection in a direct-fired hot water heater. Experimental data is to be analyzed and used to develop a special high-fin tube for this application.

INTRODUCTION

Report No. 57 (1)* dated January 1965 presents a description of the laboratory experimental equipment together with a description of the experimental procedure followed in obtaining test data. The mass spectrometer analytical method used in analyzing the fuel and stack gas samples also appears in this earlier report.

Since January of 1965 this laboratory investigation has been continued. The results of the progress made are presented in this report.

MODIFICATIONS OF EQUIPMENT

Some question arose as to the accuracy of the Rockwell Model No. 4 gas meter, having a capacity of 2,250 cu.ft. per hour at 1/2 inch water pressure or 5,000 cu.ft. per hour at 1 inch water pressure, being used to measure the natural gas flow rate to the furnace. This question was discussed with the Michigan Consolidated Gas Company. Following the discussions, the gas company calibrated a second identical gas meter in their Detroit laboratory and traded the calibrated meter for the one that was being used on the project. Immediately following installation of the new gas meter, the old gas meter was operated in series with the new one in order to obtain a secondary calibration on the old gas meter. The results indicated that the old gas meter was in error from 0.20% to 0.43% over the total operating range. It was concluded that the secondary calibration corrections were negligible and could be ignored. The old gas meter was then returned to the Michigan Consolidated Gas Company.

A total of nine shielded thermocouple assemblies were fabricated in accordance with Figure 12, page 45 of Report No. 57[1]. Four of the shielded thermocouples were placed below the tube bundle and above the gas flame. The other five shielded thermocouples were installed in the stack just above the furnace. This section of the stack was insulated with a fiber glass blanket. This section can be seen in Figure 4, page 14 of Report No. 57[1].

A 100 inch King oil manometer was also installed and connected across the orifice plate used to measure the water flow rate. This manometer was used in conjunction with a standard 30 inch Meriam mercury manometer in order to increase the accuracy of the higher water flow rate measurements.

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*Literature cited is given on page 44.

In order to increase the sensitivity of the steam flow regulation system, the control valve being used was replaced by another valve that would permit operation within the middle range of the flow regulator.

Experimental data collected in the early part of 1965 indicated non-uniform water distribution through the tube bundle. A set of distribution screens were placed in the inlet water header but they tended to clog up. The screens were replaced by a Piezometer ring type manifold distributer. This can be seen in Figure 1* and worked very satisfactorily.

A specially designed and constructed cover plate for the exit water header was installed. This cover plate could accommodate 39 thermocouples eighteen inches long, one for each tube. This assembly permitted individual measurements of the exit water temperature leaving each tube in every row of tubes. Figure 2 shows a view of the thermocouple plate installation.

A second Leeds and Northrup precision portable potentiometer was added to the temperature measurement system in order to speed up thermocouple measurement readings. For some experimental runs, a total of 38 thermocouple readings were taken.

Early experimental data indicated that the horizontal rows of tubes were not being heated uniformly by the hot combustion gases from the burners. It was found that by removing the air inlet baffle plates on the back side of the furnace a more uniform heating could be obtained.

As indicated in Report No. 57[1] difficulty was encountered in measuring the gas side pressure drop across the tube bank by means of a differential manometer. Further attempts were made to measure the stack gas pressure drop but fluctuations in the differential manometer readings precluded satisfactory pressure drop measurements.

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*All figures can be found in the appendix, page 46.

EXPERIMENTAL DATA

To date, 143 experimental runs have been made. These runs have been taken with (a) the tube bundle supplied with the hot water heater; (b) a three row bundle containing 39 finned tubes on a 1 15/16 inch square pitch; (c) a two row bundle containing 26 finned tubes on a 1 15/16 inch square pitch; (d) a one row bundle containing 13 finned tubes spaced 1 15/16 inches apart; (e) a one row bundle of 13 bare tubes on the same spacing as (d). All of the finned tubes were catalog number 61-0714065-01. The bare tubes had a 1.000 inch outside tube diameter.

The water flow rates investigated varied from 40 to 250 gallons per minute. The gas flow rates investigated varied from 15 to 40 cubic feet per minute.

The percent excess air varied from 60 to 200 percent. It was not possible to control the percent excess air. It should be noted that the bulk of the data was obtained with 80 to 100 percent excess air.

Some difficulties were encountered on certain days when the wind was gusty or a strong north wind prevailed, upsetting the draft in the stack.

Report No. 57 [(1)] presented a comparison of the natural gas composition in Table VIII, page 25. At the time of the preparation of that report the mass spectrometer analysis of actual gas samples indicated compositions considerably different from that provided by the Michigan Consolidated Gas Company. An average of the natural gas compositions collected since April 1965 indicate that the gas obtained from the gas main was quite close to the Michigan Consolidated Gas Company's indicated gas composition. The average composition encountered is given in Table I.

A small amount of experimental test data was taken on a one row bundle of bare tubes having the same outside diameter as the root diameter of the finned tubes on the pitch in order to separate out experimentally the net effect of the presence of the fins on radiation heat transfer.

All the experimental data was analyzed using computer programs prepared by project personnel and processed on The University of Michigan IBM 7090 digital computer. The computer printout totals approximately 800 pages. It is impractical to attempt to present all of the experimental test data and computer results in this report. An attempt will be made to present only the significance of the data. All of the experimental test data and the computer analysis of the experimental data are available to Wolverine Tube at any time.

TABLE I

Average Natural Gas Composition and Heating Value

Component	Vol. %
Methane	93.34
Nitrogen	0.54
Ethane	4.36
Carbon Dioxide	0.97
Propane	0.64
N-Butane	0.00
I-Butane	0.15

$(\text{Hydrogen}/\text{Carbon})_{\text{atom ratio}}$ = 3.85

Net Heating Value = 938 BTU/cu.ft.

mols C/mol Fuel = 1.046

HEAT RELEASE CURVES

It is customary in the hot water heater manufacturing industry to think in terms of recovery rate curves or tables. These curves or tables normally present the number of gallons per hour of water that can be heated a prescribed number of °F with a prescribed BTU input to the hot water heating unit. Examination of this type of information indicates that the recovery rate information corresponds to the fixed efficiency of the unit. This means that an extensive recovery rate table provided by a manufacturer can be converted to a single "Heat Release Curve." The manufacturer (Raypak Company, Inc.) of our experimental unit claims 80 percent efficiency for the unit with their tube bundles. Experimental test runs on their unit, Tube Bundle No. 1, indicate that the manufacturer's claim of 80 percent efficiency is correct. Figure 3 presents the "heat release curve" based on the experimental data for Bundle No. 1, substantiating their claim. The slope of the line drawn in Figure 3 corresponds to 80 percent efficiency.

Heat release curves are a convenient way of presenting the amount of energy picked up by the water versus the amount of energy input by burning natural gas. This is purely fortuitous because (a) the temperature driving force for radiant heat transfer is the difference between the fourth power of the absolute temperature of the flame and the fourth power of the absolute temperature of the tube wall[1]; (b) the temperature driving force for convective heat transfer is the difference between the temperature of the hot gases and the temperature of the tube wall; and (c) the combined radiative and convective resistance to heat transfer is so overwhelming controlling that the water side resistance to heat transfer can be neglected. Under (a) above, the fourth power of the absolute temperature of the flame is so large that the fourth power of the absolute temperature of the tube wall is almost negligible in comparison. This means that variation in water flow rate and mean water temperatures have no significant effect on the radiative heat transfer taking place. A similar situation almost exists for (b) above, since ordinary variations in water flow rates and mean water temperature have very small effects on the convective heat transfer taking place. Evidence of the fact that variations in both gas rate and water rate substantiates the above statement is presented in Figure 3 for the Raypak bundle and Figures 4 through 7 for a three row, two row, one row, and one row with "T" bars bundles of finned tubes, respectively. The T-bars used were 9/16 inches high by 1 1/8 inches wide made by spot welding strips of 20 gage steel together.

The slopes of the straight lines in Figures 4 - 7 correspond to the tube bundle efficiencies. For purposes of overall comparison of all tube bundles investigated, the heat release curve method is used in this report. Figure 8 presents the heat releases curves presented in Figures 4 through 7 for comparison purposes.

Figures 9, 10, and 11 present the heat release data for the top tube row (third row), middle tube row (second row), and bottom tube row (first row) of a three tube row bundle, respectively.

Figures 12 and 13 present the heat release data for the top tube row (second row) and the bottom tube row (first row) of a two tube row, respectively.

DISCUSSION OF HEAT RELEASE CURVES

Figures 4 through 7 indicate the following efficiencies for a three row bundle, a two row bundle, a one row bundle, a one row bundle with T-bars, and a one row bare tube bundle.

Three row fin tube bundle	= 83.3% efficiency
Two row fin tube bundle	= 79.2% efficiency
One row fin tube bundle	= 64.0% efficiency
One row fin tube T-bar bundle	= 69.1% efficiency
One row bare tube bundle	= 15.8% efficiency

The one row finned tube bundle has 8.85 times as much outside heat transfer surface as the one row bare tube bundle and transfers four times as much heat.

Figure 14 contains in one figure the curves for (a) the total pickup for a three row bundle (from Figure 4); (b) the pickup by the bottom row of the three rows (from Figure 11); (c) the pickup by the middle row (from Figure 10), and (d) the pickup by the top row (from Figure 9). This figure gives a convenient graphical presentation of the contribution made by each row to the total pickup. The percentage contribution made by each row is:

Bottom row	= 70.3%
Middle row	= 18.0%
Top row	= 11.7%
	100.0%

Obviously, the top row is contributing very little to the total heat transferred.

Figure 15 contains the curves for (a) the total pickup for a two row bundle (from Figure 5); (b) the pickup by the bottom row (from Figure 13); and (c) the pickup by the top row (from Figure 12). The percentage contribution made by each row is:

Bottom row	= 77.4%
Top row	= 22.6%
	100.0%

Figure 8 presents the comparison of the total performance of (a) a three row bundle; (b) a two row bundle; (c) a one row bundle; and (d) a one row bundle with "T" bars. This figure indicates:

(a) The two row bundle gives 94.6% of the performance of a three row bundle.

(b) The one row bundle gives 76.5% of the performance of a three row bundle.

(c) The one row bundle gives 81.0% of the performance of a two row bundle.

(d) The one row bundle with T-bars gives 87.5% of the performance of a two row bundle.

(e) The one row bundle with T-bars gives 107.8% of the performance of the one row bundle without T-bars.

RADIANT AND CONVECTIVE HEAT TRANSFER

A partial literature survey of radiant heat transfer was presented in Report No. 57[(1)]. Because radiant heat transfer is such a significant part of the total heat transfer taking place in this investigation, a summary of the best procedures that can be recommended for predicting radiant heat transfer will be presented in this report.

Method of Lobo & Evans Applied to Bare Tube Data

Lobo & Evans[(2)] have presented a method for calculating the radiant heat transfer taking place in the radiant section of petroleum heaters. The method is based on an analysis of 85 heat transfer tests taken on 19 petroleum heaters differing widely in design. Their method will be outlined by applying it to the radiant section of an experimental hot water heater for a set of conditions corresponding to test run no. 65-A obtained on a single row bundle of bare tubes.

The method of Lobo & Evans uses the following equation (using their notation):

$$q_T = q_r + q_c = 0.173 \left[\left[\frac{T_g'}{100} \right]^4 - \left[\frac{T_s}{100} \right]^4 \right] (\alpha A_{cp}) (\emptyset) + h_c A_o (T_g' - T_s) \qquad (1)$$

where

q_T	=	Total heat transferred, BTU/hr.
q_c	=	Net heat transferred by convection to the tube, BTU/hr.
q_r	=	Net heat transferred by radiation to the tubes, BTU/hr.
T_g'	=	Mean temperature of the hot gases in the furnace, °F + 460
T_s	=	Mean tube skin temperature, °F + 460
αA_{cp}	=	Area of a plane which will absorb the same as the actual cold surface in the furnace, sq.ft.
$\emptyset$	=	An overall exchange factor correcting for flame emissivity, arrangement of the refractory, volume of the combustion chamber, etc.

A major difficulty is encountered when using the above equation. The problem involves determining the mean temperature of the hot gases in the furnace, T_g'. It has been a well known fact for over 25 years that the theoretical flame temperature should not be used for T_g' because it is too high. Prior to 1939, it was common practice to do so. Lobo & Evans' contribution consists partially of a method of determining the correct value of T_g' that should be used. They call this temperature "pseudo-flame temperature." The test data obtained on the 19 petroleum heaters they studied substantiates very well the method they recomment. The method follows.

Reference is made to Figure 16 in the appendix. In this figure the following notation applies:

H	=	The total net heat input into the furnace from all sources, BTU/hr.
q_t	=	The total heat absorbed in the radiant section, BTU/hr.
β	=	Fraction of total net heat input lost from the external furnace walls.
αA_{cp}	=	See Equation (1).

t_g = Temperature of gases leaving radiant section, °F

t'_f = Pseudo-flame temperature, °F

The term $(t'_f - 60)$ is the theoretical temperature the gases would attain (a) if combustion were adiabatic except for the loss of the fraction β of the enthalpy of the fuel and (b) if the products of combustion had a mean specific heat equal to the mean value from t_g down to the base temperature of 60 °F.

In order to use Figure 16, one must have available the total heat absorbed in the radiant section, the total net heat input, and the heat loses other than sensible heat. The experimental test data for Run No. 65-A, appears in Table II. Table II gives $q_{absorbed}$ = 227,150 BTU/hr. and the total net heat input, H = 1,402,500 BTU/hr. The net heat lost must be calculated from the data. First, the Orsat analysis must be converted to a wet basis:

CO_2	= 3.76%
O_2	= 12.85%
N_2	= 76.14%
H_2O	= 7.25%

Mols C/mol stack gas = 0.0397

Sensible heat loss up the stack is determined by use of enthalpy curves for N_2, O_2, CO_2 and H_2O vapor. These are presented in Figures 17, 18, and 19. For the stack temperature of 1020 °F, the stack sensible heat losses are:

CO_2	= 43,250 BTU/hr.
O_2	= 101,300 BTU/hr.
N_2	= 576,000 BTU/hr.
H_2O	= 65,100 BTU/hr.
	785,650 BTU/hr.

The heat loss other than sensible heat is

$$1,175,350 - 785,650 = 389,700 \text{ BTU/hr.}$$

Therefore

$$\beta = \frac{389,700}{1,402,500} = 0.278$$

TABLE II

Experimental Test Data for Run No. 65-A
(Bare Tubes Bundle with One Row of Tubes)

Orsat Stack Gas Analysis:

CO_2 = 4.05%

O_2 = 13.86%

N_2 = 82.09%

100.00%

Lbs./hr. of stack gas = 3,100

$q_{\text{picked up}} = (w\ C_p\ \Delta t)_W = 35,000\ (1)(76.26 - 69.77)$

$= 227,150$ BTU/hour

q_{input} = CFM (60)(net heating value) = 24.92 (60)(938)

= 1,402,500 BTU/hour

Efficiency = 16.18%

$q_{\text{lost}} = q_{\text{input}} - q_{\text{picked up}} = 1,402,500 - 227,150$

= 1,175,350 BTU/hour

Average temperature of stack gas = 1020 °F = t_g

Tube O.D. = 1.000 inch

Tube spacing = 1 15/16 inch

Tube length = 5.21 ft. (62.5 inches)

No. of tubes = 13

A_o = 17.7 sq.ft.

% excess air = 174%

Linear ft. of tube/sq.ft. of floor area = 6.09 ft./sq.ft.-row

A_{cp} is the area of a continuous plane replacing the row of tubes and may be taken as the product of the exposed tube length and center to center distance between tubes and the number of tubes in the exposed radiant row. α is the ratio of reception by the actual surface to reception by a continuous plane. Then the term αA_{cp} is the tube area expressed as equivalent cold plane surface, i.e., the area of a plane which will absorb the same as the actual cold surface in the furnace.

$$A_{cp} = \frac{\text{center to center}}{12} \text{ (exposed length) (no. of tubes), ft.}^2$$

$$= \left[\frac{1\ 15/16}{12}\right] (5.21)(13)$$

$$= 10.94 \text{ sq.ft.}$$

Hottel gives α as a function of the ratio

$$\frac{\text{center to center distance of tubes}}{\text{outside diameter}} = \frac{1.9375}{1.000} = 1.9375$$

The function is presented in Figure 20. Reading from Figure 20, 0.66 is obtained for α.

Therefore, $\alpha A_{cp} = 0.66\ (10.94) = 7.22$ sq.ft.

The pseudo-flame temperature t_f' can now be determined by reference to Figure 14, using the method of similar triangles:

$$\frac{H\ (1 - \beta)}{\alpha A_{cp}} = \frac{1,402,500\ (1 - 0.278)}{7.22} = 140,250$$

$$\frac{q}{\alpha A_{cp}} = \frac{227,150}{7.22} = 31,460$$

Therefore, by similar triangles:

$$\frac{140,250}{31,460} = \frac{(t_f' - 60)}{(t_f' - 60) - (t_g - 60)} = \frac{(t_f' - 60)}{(t_f' - t_g)} = \frac{t_f' - 60}{t_f' - 1020} = 4.46$$

Solving for the pseudo-flame temperature, t_f':

$$t_f' = 1298\ °F$$

The pseudo-flame temperature is always less than the theoretical flame temperature. The theoretical flame temperature curve as a function of "per cent excess air" for the gas fuel composition given in Table I is presented in Figure 21. This figure indicates that for 174% excess air, the theoretical flame temperature is 1698 °F, 400 °F higher than t_f'.

Next the value of $\emptyset$ is determined.

The partial pressure of CO_2 plus H_2O in atmosphere is

$$P = 0.0376 + 0.0725 = 0.11 \text{ atms.}$$

The mean length of the radiant beam, L, in the combustion chamber is given by Equation 2 for rectangular furnaces.

$$L = 2/3 \sqrt[3]{\text{Furnace Volume}} \qquad (2)$$

The inside dimensions of the combustion chamber are indicated in Figure 22 as 5.21 feet long by 2.21 feet wide by 1.59 feet high. The volume is, = (5.21)(2.21)(1.59) = 18.307 cubic feet.

Therefore

$$L = 2/3 \sqrt[3]{18.307} = 1.76 \text{ feet}$$

The product P L = 0.11 (1.76) = 0.1936 atm.-ft. The flame emissivity is presented in Figure 23 as a function of tube wall temperature, the product P L and gas temperature. This firuge is difficult to read down in the lower right hand corner. As estimate of the flame emissivity, P_f, from this figure is 0.11.

The overall exchange factor ϕ, as defined by Hottel is given by:

$$\phi = \frac{1}{\frac{1}{F_s} + \frac{1}{P_{cp}} - 1} \tag{3}$$

where:

$$F_s = P_f \left[\frac{A_f}{A_t}\right] \left[1 + \left[\frac{A_r}{\alpha A_{cp}}\right] \left[\frac{1}{1 + \left[\frac{P_f}{1 - P_f\left[\frac{A_f}{A_t}\right]}\right] \left[\frac{1}{F_{rc}}\right] \left[\frac{A_f}{A_t}\right]} \right]\right] \tag{4}$$

= angle emissivity factor

where

A_f = Area of the flame bundle, sq.ft.

A_t = Total area of furnace surfaces in the radiant section = A_{cp} + refractory surfaces unprotected by tubes, sq.ft.

A_r = Effective refractory surface, sq.ft. ($A_r = A_t - \alpha A_{cp}$).

P_f = Flame emissivity.

P_{cp} = Emissivity of the ultimate heat-receiving surface, assumed = 0.78.

R_{rc} = Fraction of all the radiation emitted from all the refractory in all directions, which if not absorbed by the gas, would hit cold surface αA_{cp}.

Now

$$A_r = A_t - \alpha A_{cp} \tag{5}$$

A_t is defined as:

$$A_t = A_{cp} + \text{refractory surfaces unprotected by tubes}$$

$$= 10.94 + \text{bare four walls}$$

$$= 10.94 + 2\,(1.589 \times 5.208) + 2\,(1.589 \times 2.208) + 11.499$$

$$= 46.00 \text{ sq.ft.}$$

Substituting into Equation 5:

$$A_r = 46.00 - 9.74$$

$$= 36.26 \text{ sq.ft.}$$

and

$$A_f = (5.208)(2.208) = 11.499 \text{ sq.ft.} = A_3$$

The method of evaluation of the factor F_{rc} is not given by Lobo & Evans, but reference for its exact solution is made to Hottel as presented by him in Perry's Handbook(3). Hottel's method of evaluating F_{rc} follows.

Since the four refractory walls do not all see the same arrangement of tubes, the summation of the product $A_i\ F_{ij}$ for each of the four walls must be determined and divided by the summation of A_i where A_i is the radiation area of the i-th wall and F_{ij} is the view factor from a radiating plane i to a receiving plane j.

Consider the front wall which sees a cold plane replacing the tubes.

$$A_1 = 8.273 \text{ sq.ft.}$$

where A_1 = the radiation-area of the front wall

Reference is made to Figure 24 where plane A_1 is the radiating plane and plane A_2 is the receiving plane, and F_{12} is the angle factor for radiation from plane A_1 to plane A_2. For the front wall and rear wall of the furnace:

$$a = 5.208 \text{ ft.}$$

$$b = 1.589 \text{ ft.}$$

$$c = 2.208 \text{ ft.}$$

Therefore

$$C = \frac{c}{a} = \frac{2.208}{5.208} = 0.424$$

and

$$B = \frac{b}{a} = \frac{1.589}{5.208} = 0.305$$

From Figure 24, $F_{12} = 0.295$

For each side wall

$a = 2.208$

$b = 1.589$

$c = 5.208$

Therefore

$$C = \frac{c}{a} = \frac{5.208}{2.208} = 2.36$$

and

$$B = \frac{b}{a} = \frac{1.589}{2.208} = 0.719$$

from Figure 24, $F_{22} = 0.275$

For the floor:

$a = 1.589$ ft.

$b = 5.208$ ft.

$c = 2.208$ ft.

Therefore

$$C = \frac{c}{a} = \frac{2.208}{1.589} = 1.39$$

and

$$B = \frac{b}{a} = \frac{5.208}{1.589} = 3.28$$

from Figure 25, $F_{32} = 0.40$

For this geometry F_{rc} is defined as:

$$F_{rc} = \frac{2(F_{12})A_1 + 2(F_{22})A_2 + (F_{32})A_3}{2A_1 + 2A_2 + A_3} \qquad (6)$$

Substituting:

$$F_{rc} = \frac{2(0.295)(8.273) + 2(0.275)(3.507) + (0.40)(11.499)}{2(8.273) + 2(3.507) + 11.499}$$

$$F_{rc} = 0.325$$

F_s can now be determined by substituting into Equation 4:

$$F_s = 0.11\left[\frac{11.499}{46.00}\right]\left[1 + \left[\frac{36.26}{9.74}\right]\left[\frac{1}{1 + \left[\frac{0.11}{1 - (0.11)(0.25)}\right]\left[\frac{1}{0.325}\right](0.25)}\right]\right]$$

$$= 0.1229$$

ϕ can now be calculated by substituting into Equation 3.

$$\phi = \frac{1}{\frac{1}{0.1288} + \frac{1}{0.78} - 1} = 0.1187$$

The net heat transferred by radiation to the bare tubes can now be calculated by substitution into Equation 1.

$$q_r = 0.173 \left[\left[\frac{1758}{100}\right]^4 - \left[\frac{560}{100}\right]^4 \right] (7.22)(0.1187)$$

$$q_r = 14{,}030 \text{ BTU/hr. out of a total of } 227{,}150 \text{ BTU/hr.}$$

Lobo & Evans assumes that the convection coefficient, h_c, is 2.0. The convective heat transfer as given in Equation 1 is:

$$q_c = h_c A_o (t'_g - t_s)$$

$$= 2.0\ (17.7)(1298 - 100)$$

$$= 42{,}500 \text{ BTU/hr.}$$

Therefore:

$$q_t = 14{,}030 + 42{,}500$$

$$= 56{,}530 \text{ BTU/hr.}$$

The actual amount of heat transferred was 227,150 BTU/hr.

Method of Hottel Applied to Bare Tube Data

Hottel's method, as described in Perry[3], uses the following equation:

$$q_f = \sigma\ (t_f^4 - t_s^4) A_c\ F_{cf} + h_c A_o (t_f - t_s) + U_r A_r (t_f - t_o) \quad (7)$$

where:

q_f = The net rate of heat transfer from the flame by all mechanisms, BTU/hr.

σ = 0.173×10^{-8} BTU/sq.ft.-hr.-°R^4

t_f = Mean flame temperature, °R

t_s = Temperature of the receiving surface, °R

A_c = Area of the receiving plane (sink), sq.ft. = $\alpha\, A_{cp}$.

F_{cf} = Defined by Equation 8.

h_c = Convective heat transfer coefficient, BTU/hr.-sq.ft.- °F

A_o = Outside area, sq.ft.

U_r = Conductance of the refractory, inside refractory coefficient and outside refractory coefficient, BTU/hr.-sq.ft.- °F

A_r = Refractory area, sq.ft.

t_o = Outside air temperature, °F

This equation states that the net rate of heat transfer from the flame is equal to the radiation to sink plus the convection to sink plus the external loss. In this equation:

$$F_{cf} = \frac{1}{\frac{1}{\overline{F}_{cf}} + \frac{1}{P_c} - 1} \qquad (8)$$

where:

P_c = Emissivity of the receiving surface

and

$$\overline{F}_{cf} = P_f \left[1 + \frac{A_r / A_c}{1 + \left[\frac{P_f}{1 - P_f} \cdot \frac{1}{F_{rc}} \right]} \right] \qquad (9)$$

where the terms are defined following Equation 4.

It should be noted that $\overline{F}_{cf}$ as defined above is applicable to commercial furnaces where A_f may be considered equal to A_t. However, this is not the case with our laboratory furnace. Therefore, Hottel's equation for the exact evaluation of $\overline{F}_{cf}$, Equation 4, must be used (note that F_s will now be used in place of $\overline{F}_{cf}$ in Equation 8).

The flame emissivity, P_f, is evaluated next. The equivalent gray-body emissivity of a flame at temperature t_g interacting with cold surfaces at temperature t_s is defined by:

$$P_f = \frac{\epsilon_g - \alpha_g (t_s / t_g)^4}{1 - (t_s / t_g)^4} \qquad (10)$$

where:

ϵ_g = Emissivity of the gases defined by Equation 11

α_g = Absorptivity of the gases defined by Equation 12.

The pseudo-flame temperature of 1298 °F will be used for t_g.

$$T_g = t_g + 460 = 1298 + 460 = 1758 \text{ °F}$$

The effective beam length, L, for gas radiation was previously calculated to be equal to 1.76 feet. The partial pressure of CO_2 was 0.0376 atm and the partial pressure of the water vapor was 0.0725 atm.

Then

$$P_C L = (0.0376)(1.76) = 0.0662 \text{ atm.-ft.}$$

and

$$P_W L = (0.0725)(1.76) = 0.1276 \text{ atm.-ft.}$$

$$\epsilon_g = (\epsilon_{CO_2}, t_g, P_CL)\ C_C + (\epsilon_{H_2O}, t_g, P_WL)\ C_W - \Delta\epsilon_{t_g} \tag{11}$$

ϵ_{CO_2} is obtained from Figure 26. Entering this figure with a t_g of 1298 °F and a P_CL of 0.0662 atm.-ft., a value of ϵ_{CO_2} = 0.069 is obtained. Similarly, ϵ_{H_2O} is obtained frim Figure 27. At the same t_g and with P_WL equal to 0.1276, a value of ϵ_{H_2O} = 0.073 is obtained.

The correction factor C_C for the effect of total pressure on CO_2 radiation is obtained from Figure 28 and equals 1.0. Similarly, the correction factor for water vapor, C_W, as obtained from Figure 29 is equal to 1.0.

The value of $\Delta\epsilon_{t_g}$ is obtained from Figure 30. First the value of the following ratio must be determermined:

$$\frac{P_W}{P_W + P_C} = \frac{0.0725}{0.0725 + 0.0376} = 0.69$$

Second, the value of $P_CL + P_WL$ must be determined.

$$P_CL + P_WL = 0.0662 + 0.1276 = 0.1938$$

Entering Figure 30 with these two values, $\Delta\epsilon$ = 0.002. Substituting into Equation 11 gives:

$$\epsilon_g = (0.069)(1.0) + (0.073)(1.0) - 0.002$$

$$= 0.14$$

α_g must now be determined by:

$$\alpha_g = \alpha_{CO_2} + \alpha_{H_2O} - \Delta\alpha \tag{12}$$

where

$$\alpha_{CO_2} = \left[\epsilon_{CO_2}, \ t_s, \ P_C L \left[\frac{T_s}{T_g}\right]\right] \left[\frac{T_g}{T_s}\right]^{0.65} \quad (C_C) \tag{13}$$

From Figure 26, the value of ϵ_{CO_2} must be read at:

$$P_C L \left[\frac{T_s}{T_g}\right] = \frac{(0.0662)(560)}{1758} = 0.0211$$

Therefore, the value of ϵ_{CO} is 0.04.

Also

$$\alpha_{H_2O} = \left[\epsilon_{H_2O}, \ t_s, \ P_W L \left[\frac{T_s}{T_g}\right]\right] \left[\frac{T_g}{T_s}\right]^{0.45} \quad (C_W) \tag{14}$$

From Figure 27, the value of ϵ_{H_2O} must be read at

$$P_W L \left[\frac{T_s}{T_g}\right] = \frac{(0.\ 276)(560)}{1758} = 0.0406$$

Therefore, the value of ϵ_{H_2O} is 0.07.

Substituting into Equation 13:

$$\alpha_{CO_2} = 0.04 \left[\frac{1758}{560}\right]^{0.65} (1.0)$$

$$\alpha_{CO_2} = 0.08408$$

Substituting into Equation 14:

$$\alpha_{H_2O} = 0.07 \left[\frac{1758}{560}\right]^{0.45} (1.0)$$

$$\alpha_{H_2O} = 0.117$$

Substituting into Equation 12:

$$\alpha_g = 0.084 + 0.117 - 0.0025$$

$$= 0.198$$

Substituting into Equation 10:

$$P_f = \frac{0.14 - 0.198\ (560 / 1758)^4}{1 - (560 / 1758)^4}$$

$$P_f = 0.138$$

Substituting into Equation 4:

$$F_s = 0.138 \left[\frac{11.499}{46.00}\right] \left[1 + \left[\frac{36.26}{9.74}\right] \left[\frac{1}{1 + \left[\frac{0.138}{1 - 0.138\ (0.25)}\right] \left[\frac{1}{0.325}\right] (0.25)}\right]\right]$$

$$F_s = 0.1502$$

Substituting into Equation 8:

$$F_{cf} = \frac{1}{\frac{1}{0.1502} + \frac{1}{0.87} - 1}$$

$$= 0.147$$

Substituting into the first group of Equation 7:

$$q_r = 0.173 \left[\left[\frac{1758}{100}\right]^4 - \left[\frac{560}{100}\right]^4 \right] (7.22)\ (0.147)$$

$$= 17,350 \text{ BTU/hr.}$$

Substituting into the middle group of Equation 7:

$$q_c = h_c A_o (T_f - T_c)$$

$$= 2\ (17.7)(1758 - 560)$$

$$= 42,500$$

$$q_t = q_r + q_c = 59,850 \text{ BTU/hr.}$$

$$\text{actual } q_t = 227,150 \text{ BTU/hr. exclusive of losses.}$$

Forced Convection

The predicted convective heat transfer coefficient from the literature for gas flowing around bare tubes is given in Figure 31 for heating and cooling air normal to single cylinders[4]. This curve will be used by converting it to a dimensionless equation. The Reynolds number must be first determined for the flow of stack gas past the horizontal row of bare tubes.

$$\text{Minimum free flow area} = \frac{(1\frac{15}{16} - 1)\ (13)\ (62.5)}{144} = 5.29 \text{ sq.ft.}$$

$$G = \text{mass flow rate} = \frac{3100 \text{ lbs./hr.}}{5.29 \text{ sq.ft.}} = 586 \text{ lbs./sq.ft.-hr.}$$

Viscosity, estimated, at 1020 °F for the stack gas mixture

= 0.086 lbs./ft.-hr.

$$\text{Reynolds Number} = \frac{DG}{\mu} = \frac{1.0\ (586)}{12\ (0.086)} = 568$$

The general dimensionless relationship normally used has the following form:

$$\frac{h_c D_o}{k} = C \ (Re)^a \ (Pr)^b \tag{15}$$

where:

Re = Reynolds Number

Pr = Prandtl Number

a, b, c = Constants

Normally, the power on Prandtl Number is taken as 1/3. The value of the power "a" on the Reynolds Number as recommended by McAdams[4] for the Reynolds Number range of 40 to 4,000 is 0.466. Equation 15 becomes:

$$\frac{h_c D_o}{k} = C \ (Re)^{0.466} \ (Pr)^{1/3} \tag{16}$$

McAdams[4] gives the value of B in the following equation when used with air to be equal to 0.615:

$$\frac{h_c D_o}{k} = B \ (Re)^{0.466} \tag{17}$$

A comparison of Equations 16 and 17 indicates that

$$B = C\,(Pr)^{1/3} \qquad (18)$$

The estimated values of C_p and k for the gas mixture are:

C_p = 0.281 BTU/lb.-°F

k = 0.034 BTU/ft.-hr.-°F

μ = 0.086 lbs./ft.-hr.

These values yield a Prandtl Number of 0.71. Substituting into Equation 18 and solving for C:

$$0.615 = C\,(0.71)^{1/3}$$

$$C = 0.69$$

Substituting into Equation 16:

$$\frac{h_c\left(\frac{1}{12}\right)}{0.034} = 0.69\,(568)^{0.466}\,(0.71)^{1/3}$$

from which

$$h_c = 4.82$$

The convective heat transfer now becomes:

$$q_c = h_c A_o (t_g' - t_s)$$

$$= (4.82)(17.7)(1298 - 100)$$

$$= 102{,}100 \text{ BTU/hr.}$$

Using the radiative heat transfer obtained by the method of Hottel, the total heat transferred now becomes

$$q_t = q_r + q_c$$

$$= 17{,}350 + 102{,}100$$

$$= 119{,}450 \text{ BTU/hr.}$$

This value of 119,450 compares with the 59,850 BTU/hr. as calculated previously. The actual heat transferred was 227,150 BTU/hr.

DISCUSSION OF LOBO & EVANS AND HOTTEL ANALYSIS OF BARE TUBE DATA

Table III summarizes the results of analysis of the heat transfer conditions on Run Number 65-A.

TABLE III

Summary of Calculated Results

Method	Radiation (BTU/hr.)	Convection (BTU/hr.)	Total (BTU/hr.)	Percent by Radiation
Lobo & Evans	14,030	42,500	56,530	24.8%
Hottel	17,350	42,500	59,850	29.0%
Modified Hottel	17,350	102,100	119,450	14.5%
Actual			227,150	

Inspection of Table III indicates that the Lobo & Evans method predicts 25% of the actual heat transferred, the Hottel method predicts 26.3% of the actual heat transferred, and the modified Hottel method predicts 52.60% of the actual heat transferred. Only the value of 102,100 BTU/hr. for convection should be used. The convection coefficient of 4.82 is 2.41 times the value of 2.0 assumed by Lobo & Evans and Hottel.

Basically, the main difference between the two methods involves the determination of the flame emissivity, P_f.

A review of the calculation made in the previous section indicates that the pseudo-flame temperature of Lobo & Evans was used as the driving force. Although it was pointed out earlier in this report that it is a well known fact that the theoretical flame temperature has too high a value to be used in the calculations, one might, however, raise the interesting question as to what effect the use of the theoretical flame temperature 1698 °F instead of the pseudo-flame temperature 1298 °F would have on the calculated results. The calculations follow:

(a) Lobo & Evans Method (modified)

$$q_t = 0.173 \left[\left[\frac{2158}{100}\right]^4 - \left[\frac{560}{100}\right]^4 \right] (7.22)(0.1245) + 4.82\,(17.7)\,(1698 - 100)$$

$$= 33{,}730 + 136{,}300$$

$$= 170{,}030 \text{ BTU/hr.}$$

(b) Hottel Method (modified)

$$q_t = 0.173 \left[\left[\frac{2158}{100}\right]^4 - \left[\frac{560}{100}\right]^4 \right] (7.22)\,(0.16) + 4.82\,(17.7)\,(1698 - 100)$$

$$= 43{,}350 + 136{,}300$$

$$= 179{,}650 \text{ BTU/hr.}$$

The results are summarized in Table IV.

TABLE IV

Summary of Calculations Using Theoretical Flame Temperature

Method	Radiation BTU/hr.	Convection BTU/hr.	Total BTU/hr.	Percent Radiation
Lobo & Evans (modified)	33,730	136,300	170,030	19.8%
Hottel (modified)	43,350	136,300	179,650	24.1%
Actual			227,150	

Inspection of Table IV indicates that the modified Lobo & Evans Method predicts 75% of the actual heat transferred, the modified Hottel Method predicts 79.1% of the actual heat transferred. These results indicate that the gas temperature used in the radiation and convection heat transfer calculations is extremely critical.

The actual turbulence level of the furnace gas passing the bare tubes is unknown in spite of the fact that a Reynolds Number of 568 was calculated. This calculated Reynolds Number assumes no turbulence of the furnace gas before it reaches the tube bundle. The hot gasses produced by a fire experience an upward buoyand force. Recent experiments(5) indicate that the hot gas column is fully turbulent. Several investigations have been made concerning turbulent flow in the rising column, Schmidt in 1941(6), Rouse, Yih and Humpheries in 1952(7), Priestly and Ball in 1955(8) and Morton, Taylor and Turner in 1956(9), and Morton(10) in 1957 and 1959(11) have studied this problem in addition to Murgai and Emmons in 1960 and 1961(5, 12).

The stable turbulence level of the furnace gases is not fully established until some distance above the fire. It is believed that this distance is one to two times the fire diameter(5). This distance is analogous to a pipe entrance effect. Unfortunately, the tube bundle base is within this distance to the flame. Actually, at high gas rates the top of the flame plume moves closer to the tube bundle base. The implication is that the turbulence levels and corresponding convection coefficients may be considerably higher than predicted from any available heat transfer correlations. Unfortunately, there appears to be no practical way to experimentally measure the actual turbulence level.

If one assumes that the Hottel or Lobo & Evans method predict the correct amount of radiation taking place, then one might arbitrarily assign the balance of the energy absorbed to convective heat transfer. Using the pseudo-flame temperature and the Hottel method, the computed value of h_c is 9.5 as compared to 4.8 from McAdams curve (twice the coefficient). If, however, the theoretical flame temperature is used, the calculated value of h_c is 7.13 (one and one-half times the coefficient). This needs further investigation.

Further examination of the calculations made in previous sections indicate that the dimension of the furnace are also extremely critical in the calculation of the mean length of the radiant beam, L. The radial beam length also has a critical effect on the calculated value of the flame or gas emissivity. This emissivity in turn has a great effect on the value of $\emptyset$ or F_{cf}. An examination of Table II and Table III of Lobo & Evans article[2] indicates that the mean radiant beam length for the 19 industrial furnaces they investigated averaged 17.6 ft. as compared with 1.76 ft. in our laboratory furnace. Their corresponding flame emissivities varied from 0.36 to 0.59 with an average value of approximately 0.49 as compared with our values of 0.11 to 0.14. Their corresponding overall exchange factor $\emptyset$ varied from 0.52 to 0.80 as compared to our values of 0.12 to 0.16.

Two factors significantly affect the calculation of the overall exchange factor, $\emptyset$, namely the mean length of the radiant beam, L, and the flame temperature.

Consider a hot water heater with a fire-box ten times the size of our laboratory hot water heater, i.e., 52.1 ft. long by 21.2 ft. wide by 15.9 ft. high. The mean radiant beam length for a box of this size would be:

$$L = 2/3 \sqrt[3]{\text{volume}}$$

$$= 2/3 \sqrt[3]{(52.1)(21.2)(15.9)}$$

$$= 17.6 \text{ ft.}$$

This mean radiant beam length of 17.6 ft. is the same as the average of the 19 furnaces investigated by Lobo & Evans. Further calculations would indicate that the corresponding value of flow emissivity and overall exchange factor $\emptyset$ would also fit the experimental data range of Lobo & Evans. This raises some questions as to whether or not the method of Lobo & Evans, which is a modification of Hottel, is exact for furnaces having 1/1000th the volume of their experimental furnaces.

Method of Lobo & Evans Applied to Finned Tube Data

This method was applied to the set of conditions corresponding to test run no. 44-A-1 obtained on a single row bundle of finned tubes presented in Table V. Table VI summarizes the results of the calculations.

TABLE V

Experimental Test Data for Run 44-A-1
(Fin Tube Bundle With One Row of Tubes)

Orsat stack gas analysis

CO_2 = 5.79%

O_2 = 10.50%

N_2 = 83.71%

Orsat stack gas analysis converted to a wet basis

CO_2 = 5.17%

O_2 = 9.38%

N_2 = 74.80%

H_2O = 10.65%

lbs./hr. of stack gas = 2.900

$q_{picked\ up}$ = 1,306,000 BTU/hr.

q_{input} = 2,030,400 BTU/hr.

efficiency = 63.8%

q_{lost} = 724,400 BTU/hr.

Average temperature of gas = 994 °F

Tube root diameter = 1.000 inch

Tube fin diameter = 1.7914 inch

Tube spacing = 1 15/16 inch

Tube length = 5.21 ft. (62.5 inches)

Number of tubes = 13

A_o = 2.316 sq.ft./linear ft.

= 153.75 sq.ft./row

Percent excess air = 89.4

Linear ft./of tube / sq.ft. of floor area = 6.09

TABLE VI

Summary of the Method of Lobo & Evans Applied to Run 44-A-1

The sensible heat lost with stack gas at 750 °F are:

CO_2	=	38,240 BTU/hr.
O_2	=	48,580 BTU/hr.
N_2	=	375,560 BTU/hr.
H_2O	=	63,630 BTU/hr.
		526,010 BTU/hr.

The heat lost other than sensible = 198,390 BTU/hr.

A_{cp}	=	10.94 sq.ft.
α	=	0.98
αA_{cp}	=	10.72 sq.ft.
L	=	1.76 ft.
t_f'	=	2250 °F
t_f	=	2285 °F
P	=	0.1582 atm.
PL	=	0.2784 atm.-ft.
P_f	=	0.12
F_{rc}	=	0.325
F_s	=	0.1201
ϕ	=	0.116

Using the pseudo-flame temperature:

q_r = 118,200 BTU/hr.

Using the theoretical flame temperature:

q_r = 124,400 BTU/hr.

Method of Hottel Applied to Finned Tube Data

This method will be applied to the set of conditions corresponding to test run. no. 44-A-1 obtained on a single row bundle of finned tubes in Table V. Table VII summarizes the results of the calculations.

TABLE VII

Summary of Hottel Method Applied to Run No. 44-A-1

The sensible heat lost at 750 °F = 526,010 BTU/hr.

Heat lost other than sensible = 198,390 BTU/hr.

A_{cp}	= 10.94 sq.ft.	C_W	= 1.0
α	= 0.98	$P_W/P_C + P_W$	= 0.67
αA_{cp}	= 10.72 sq.ft.	$P_CL + P_WL$	= 0.278 atm.-ft.
L	= 1.76	$\Delta\epsilon$	= 0.031
t_f'	= 2250 °F	ϵ_g	= 0.079
t_f	= 2285 °F	α_{H_2O}	= 0.144
F_{rc}	= 0.325	α_{CO_2}	= 0.12
P_CL	= 0.091 atm.-ft.	$\Delta\alpha$	= 0.03
P_WL	= 0.187 atm.-ft.	α_g	= 0.234
ϵ_{CO_2}	= 0.055	P_f	= 0.079
ϵ_{H_2O}	= 0.055	F_s	= 0.081
C_C	= 1.0	F_{CF}	= 0.08

Using the pseudo-flame temperature t_f' = 2250 °F

q_r = 81,400 BTU/hr.

Using the theoretical flame temperature t_f = 2285 °F

q_r = 85,700 BTU/hr.

Convective Heat Transfer to Finned Tubes

The equation used to predict the convective heat transfer coefficient is[13]:

$$Nu = 0.1378 \, Re^{0.718} \, Pr^{1/3} \, (s/\ell)^{0.296} \tag{19}$$

where:

	Re	=	Reynolds Number $D_r G/\mu$.
	Pr	=	Prandtl Number $C_p \mu/k$.
	s	=	Fin spacing, inches.
	ℓ	=	Fin height, inches.
	Nu	=	Nusselt Number, $h_c D_r/k$.
At 994 °F:	C_p	=	0.285 BTU/lb.- °F.
	μ	=	0.086 lbs./ft.-hr.
	k	=	0.034 BTU/hr.-sq.ft.- °F/ft.

$$Pr = \frac{0.285\,(0.086)}{0.034} = 0.714$$

$$G = \text{Mass flow rate} = 2900/4.355 = 669 \text{ lbs./sq.ft.-hr.}$$

$$Re = \left[\frac{1.0}{12}\right]\left[\frac{669}{6.086}\right] = 648$$

$$s/\ell = 0.1218 \,/\, 0.3957 = 0.3077$$

Substituting into Equation 19:

$$Nu = 0.1378\,(648)^{0.718}\,(0.714)^{1/3}\,(0.3077)^{0.296}$$

$$Nu = 9.032$$

or

$$h_c D_r / k = 9.032$$

$$h_c = \frac{9.032\,(0.034)}{\left[\frac{1.000}{12}\right]}$$

$$= 3.68 \text{ BTU/hr.-sq.ft.- °F}$$

From Figure 34, a value of 0.7 is obtained for F_D, the heat transfer depth factor.

Using the pseudo-flame temperature

$$q_c = A_o h_c F_D (t_f' - t_s)$$

$$= 153.75 (3.58)(0.7)(2250 - 100)$$

$$= 893,200 \text{ BTU/hr.}$$

Similarly:

q_c using theoretical flame temperature = 907,700 BTU/hr.

q_c using the mean gas temperature = 354,600 BTU/hr.

Table VIII presents a summary of the results from the methods of Lobo & Evans and of Hottel for radiation heat transfer and of the convective heat transfer calculations for finned tubes.

TABLE VIII

Summary of Calculations for Finned Tube

Method	Radiation* BTU/hr.	Convection* BTU/hr.	Total BTU/hr.
Lobo & Evans	118,200 (11.7%)	893,200 (88.3%)	1,011,400
Hottel	81,400 (8.4%)	893,200 (91.6%)	974,600
Actual			1,305,700

* Values for Radiation and Convection heat transfer are those resulting from the use of the pseudo-flame temperature t_f'.

Method Based on Actual Measured Hot Gas Temperatures

The actual temperatures of the hot combustion gases approaching the finned tubes and leaving the tube bundle were measured using shielded chromel-alumel thermocouples. For run 44-A-1, these temperatures were 1238 °F and 750 °F, respectively.

If it is assumed that the drop in temperature across the bundle of tubes is essentially due to convective heat transfer, the corresponding amount can be calculated directly to be 432,560 BTU/hr. If the balance of the energy absorbed is attributed to radiation, this would amount to 873,180 BTU/hr. These figures would indicate 33.1% of the energy absorbed is due to convection and 66.9% is due to radiation. These figures can be compared with 90% and 10%, respectively, in Table VIII for the Lobo & Evans method and the Hottel method.

The calculated value of the pseudo-flame temperature, t_g' or t_f', corresponding to 66.9% heat transfer by radiation is 4000 °F. The theoretical flame temperature from Figure 21 is 2250 °F. The 1750 °F difference appears impossible to reconcile. The theoretical flame temperature (Figure 21) assumes complete mixing of all the gaseous fuel and air during combustion. If, however, this does not occur and some of the combustion air by-passes the flame, the actual theoretical flame temperature should be considerably higher. Several attempts were made to measure the temperature of the gas between the flame and flame temperature by using a single unshielded chromel-alumel thermocouple as a probe. The measured temperature of the gases between the flames were from one-half to one-third of the measured flame temperature indicating considerable by-passing of the flame by much of the combustion air. This would tend to indicate that the flame temperature of 2250 °F is too low.

Zakharikov(14) indicates that pre-mixing of the fuel and air may not be complete due to draft effects resulting in air leakage. His studies also indicate that small furnaces attain higher temperatures than large furnaces.

These comparisons indicate the magnitude of the difficulty encountered in attempting to bring the design methods available in the literature into agreement with the experimental data. Stevenson and Grafton encountered similar difficulty in their radiation heat transfer analysis for space vehicles.(15)

CONCLUSIONS

At the present time the design methods of Lobo & Evans and of Hottel do not appear to be sufficiently accurate when combined with available convective correlations and applied to hot water heaters to warrant using them as a basic design method for the design of hot water heaters using bare tubes or finned tubes. A simplified design method based on the laboratory experimental data is recommended for use at the present time. The tentative recommended design method is presented in the next section of this report.

TENTATIVE RECOMMENDED DESIGN PROCEDURE

A simple design method can readily be devised for finned tubes by using the data plotted in Figure 8. This data can be converted to "recovery rate curves" by dividing the heat input by the furnace floor area, 11.499 sq.ft. The total energy picked up is also divided by the furnace floor area. The resulting values are plotted in Figures 32 and 33. These figures present the recovery rate as a function of the heat input both in BTU/hr.-sq.ft. of furnace floor area.

Justification for the usage of Figures 32 and 33 for design purposes is based upon the following assumptions. It is assumed that:

(a) The water side coefficient has negligible effect on the overall heat transfer performance.

(b) Fluctuations in mean water-side temperature have negligible effect on the overall heat transfer performance.

(c) Only tube number 61-0714065-01 is to be used in the water heater.

(d) All the tubes in the tube bundle are on a 1 15/16 inch square pitch.

(e) All normal sized hot water heaters that would be designed would have approximately the same proportions as the laboratory hot water heater on which the experimental data was obtained.

(f) The effect of the dimensions of furnaces of smaller sizes than the laboratory unit would not change the value of the overall exchange factor ϕ significantly (there may be some question as to the validity of this assumption).

(g) Similar strip type burners as used in the laboratory unit would be installed in the designed unit at the same distance below the base of the tube bundle.

(h) Natural gas having a hydrogen/carbon ratio of 3.85 and a net heating value of 938 BTU/cu.ft. is burned as the fuel.

Sample Design Calculations

Several hot water heaters are to be designed to heat 2,000 gallons of water per hour, 30 °F. The heat duty = (2,000)(8.33)(1.0)(30 °)

= 500,000 BTU/hr.

For purposes of design, it will be assumed that the strip burner will provide a heat input of 180,000 BTU per hour, per sq.ft. of combustion chamber furnace floor area.

(a) Design for a 3-row bundle of finned tubes.

Refer to Figure 32.

For a heat input of 180,000 BTU/hr.-sq.ft., a recovery rate of 150,800 BTU/hr.-sq.ft. is obtained.

The required furnace floor area = 500,000/150,000

= 3.32 sq.ft.

Assume the combustion chamber length is twice the width.

$$2\,(W)(W) = 3.32 \text{ sq.ft.}$$
$$W = \sqrt{3.32/2}$$
$$= 1.29 \text{ ft.}$$
$$\text{or} \qquad W = 15.4 \text{ in.}$$

$$\text{The approximate number of tubes per row} = \frac{15.4}{1\frac{15}{16}} = 7.95$$

Therefore, assume 8 tubes per horizontal row. The width of the heat exchanger becomes

$$8\left[1\frac{15}{16}\right] = 15.5 \text{ in.}$$

and the length is

$$(3.32)(144) \;/\; 15.5 = 30.85 \text{ in.}$$

It is suggested that a combustion chamber 16 in. wide by 32 in. long be specified. The tube bundle contains 3 rows of tubes having 8 tubes per row 32 inches long. The total length of tubing required (not including stripped ends) is 64 ft.

(b) Design for a 2-row bundle of finned tubes.

Refer to Figure 32.

For a heat input of 180,000 BTU/hr.-sq.ft., a recovery rate of 142,500 BTU/hr.-sq.ft. is obtained.

$$\text{The required furnace floor area} = 500{,}000 \;/\; 142{,}500$$
$$= 3.51 \text{ sq.ft.}$$

Assume the combustion chamber length is twice the width.

$$2\,W\,(W) = 3.51 \text{ sq.ft.}$$

$$W = \sqrt{3.51/2}$$

$$= 1.325 \text{ ft.}$$

or $$W = 15.9 \text{ in.}$$

The approximate number of tubes per row $= \dfrac{15.9}{1\frac{15}{16}} = 8.21.$

Therefore, assume 8 tubes per horizontal row. The width of the heat exchanger becomes

$$8\left[1\frac{15}{16}\right] = 15.5 \text{ in.}$$

and the length is

$$(3.51)(144) \;/\; 15.5 = 32.6 \text{ in.}$$

It is suggested that the combustion chamber 16 in. wide by 34 in. long be specified. The tube bundle contains 2 rows long 8 tubes per row, 34 inches long. The total length of tubing required (not including stripped ends) is 45.3 ft.

(c) Design for 1 row bundle of finned tubes.

Refer to Figure 32.

For a heat input of 180,000 BTU/hr.-sq.ft., a recovery rate of 115,300 BTU/hr.-sq.ft. is obtained.

The required furnace floor area $= 500,000 \;/\; 115,300$

$= 4.34$ sq.ft.

Assume the combustion chamber length is twice the width.

$$2\,(W)(W) = 4.34 \text{ sq.ft.}$$

$$W = \sqrt{4.34/2}$$

$$= 1.47 \text{ ft.}$$

or $$W = 17.62 \text{ in.}$$

The approximate number of tubes $= \dfrac{17.62}{1\frac{15}{16}} = 9.11.$

Therefore, assume 9 tubes in the horizontal row. The width of the heat exchanger becomes

$$9\left[1\frac{15}{16}\right] = 17.4 \text{ in.}$$

and the length is

$$(4.33)(144) \,/\, 17.4 = 35.9 \text{ in.}$$

It is suggested that a combustion chamber 18 inches wide by 36 inches long be specified. The tube bundle contains 1 row having 9 tubes 36 inches long. The total length of tubing required (not including stripped ends) is 27 ft.

(d) Design for 1 row bundle of finned tubes with "T" bars.

Refer to Figure 33.

For a heat input of 180,000 BTU/hr.-sq.ft., a recovery rate of 124,000 BTU/hr.-sq.ft. is obtained.

The required furnace floor area = 500,000/124,000

= 4.04 sq.ft.

Assume the combustion chamber length is twice the width.

$$2\,(W)(W) = 4.04 \text{ sq.ft.}$$

$$W = \sqrt{4.04/2}$$

$$= 1.43 \text{ ft.}$$

$$\text{or} \qquad W = 17.16 \text{ in.}$$

$$\text{The approximate number of tubes} = \frac{17.16}{1\frac{15}{16}} = 8.9.$$

Therefore, assume 9 tubes in the horizontal row. The width of the heat exchanger becomes

$$9\left[1\frac{15}{16}\right] = 17.4 \text{ in.}$$

and the length is

$$(4.03)(144)\;/\;17.4 = 33.3 \text{ in.}$$

It is suggested that a combustion chamber 18 inches by 34 inches be specified. The tube bundle contains 1 row having 9 tubes 34 inches long. The total length of tubing required (not including stripped end) is 25.5 ft.

Design Summary

The linear foot of tubing required for the four hot water designs above are:

Three row bundle	=	64.0 ft.
Two row bundle	=	45.3 ft.
One row bundle	=	27.0 ft.
One row bundle with "T" bars	=	25.5 ft.

This table indicates that either a one row bundle with "T" bars or a two row bundle are to be preferred over a three row bundle of finned tubes.

It should be noted that the use of the recovery rate curves is not necessary since the percent efficiency can be applied directly to the heat input per sq.ft. of furnace floor area to obtain the recovery rate.

The above design method is considered to be more accurate than any other known design method at the present time.

BOILING

Experimental test runs were made by throttling back the water throughput until boiling occurred. Violent pulsations of the water flow then occurred with an increase in the water-side pressure drop. The burner flame could not be maintained as the safety inter-locks always shut the furnace down. It should be noted that once boiling occurred, boiling could not be quenched by increasing the water flow-rate, presumably because of vapor block. Sagging of the finned tubes occurred as a result of over-heating of the metal. The boiling investigation was discontinued.

FUTURE WORK

The investigation is being continued. A staggered pitch bundle of finned tubes, three rows deep, has been installed (Bundle No. 2b). Data will be taken with three rows, two rows, one row and one row with T-bars.

Bundle No. 2b will be followed by a series of bundles of finned tubes of various sizes having various fin counts. The sequence will be determined from the significance of the data collected.

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APPENDIX

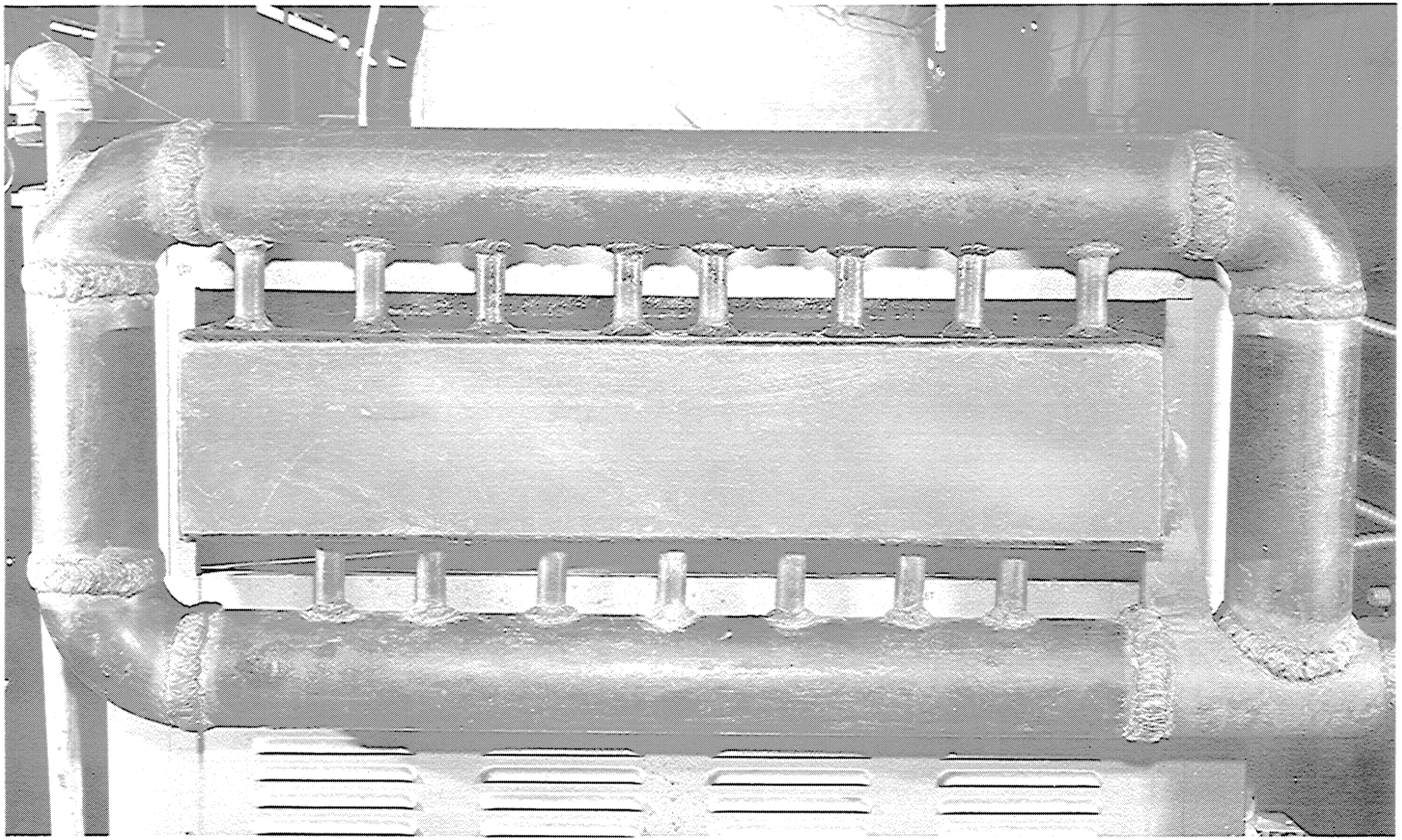

Fig. 1 Water Inlet Side of Experimental Unit Showing Manifold Water Distributing Pipe System to the Inlet Header

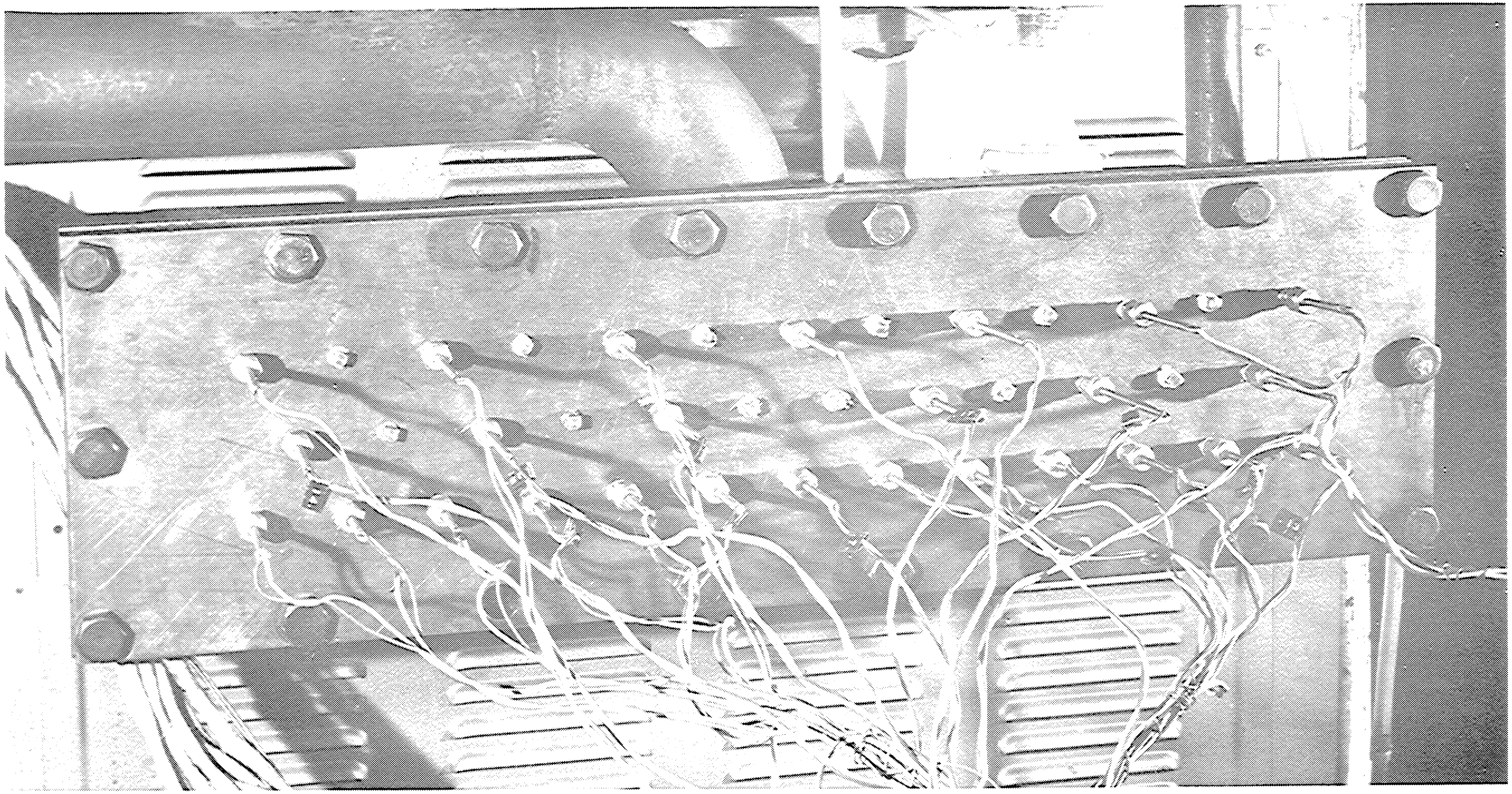

Fig. 2 Water Exit Header of Experimental Unit Showing Cover Plate with Locations for 39 Thermocouples

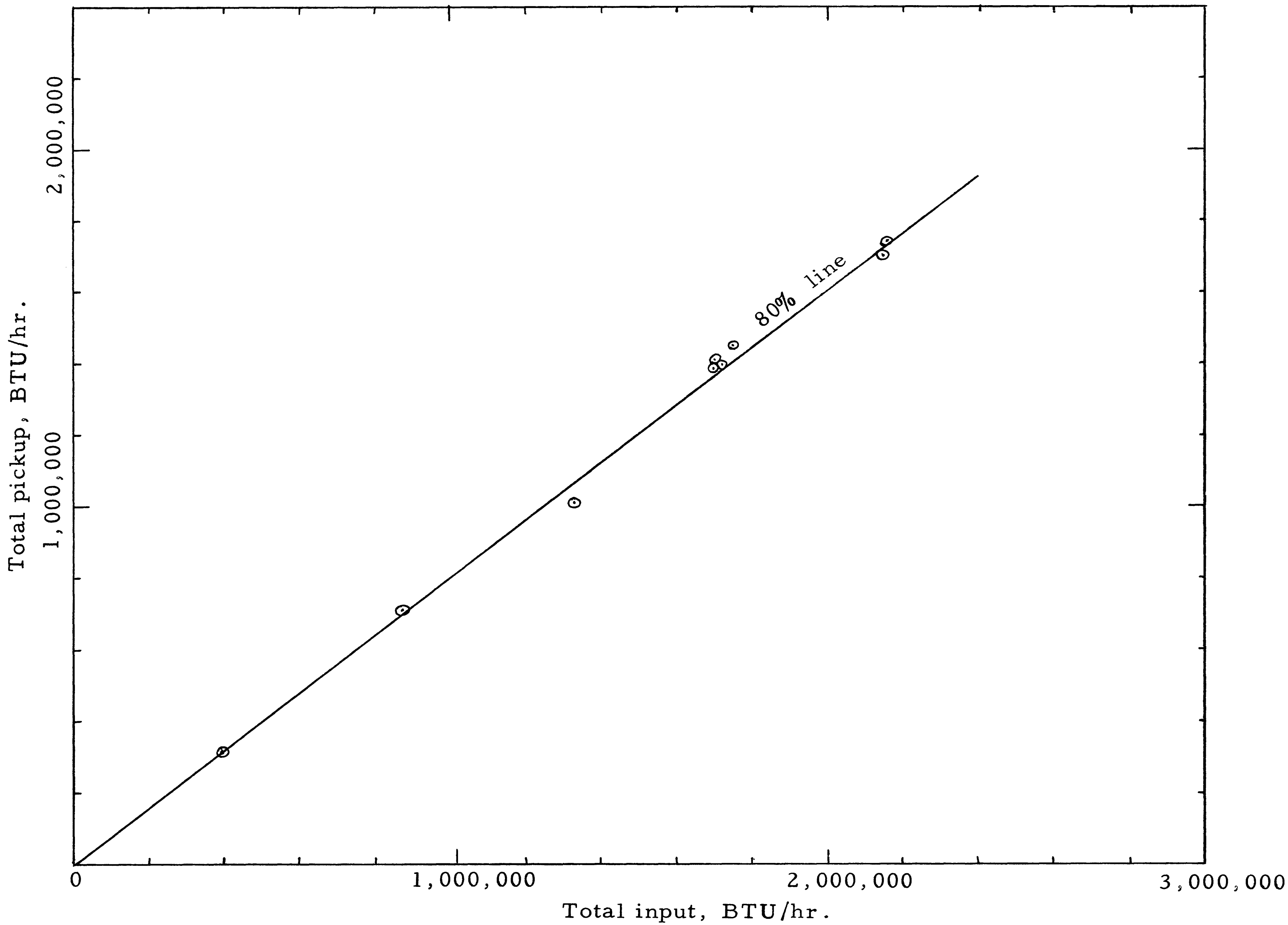

Fig. 3 Heat Release Curve for Raypak Two Row Unit, Bundle No. 1, Indicating 80% Efficiency Line

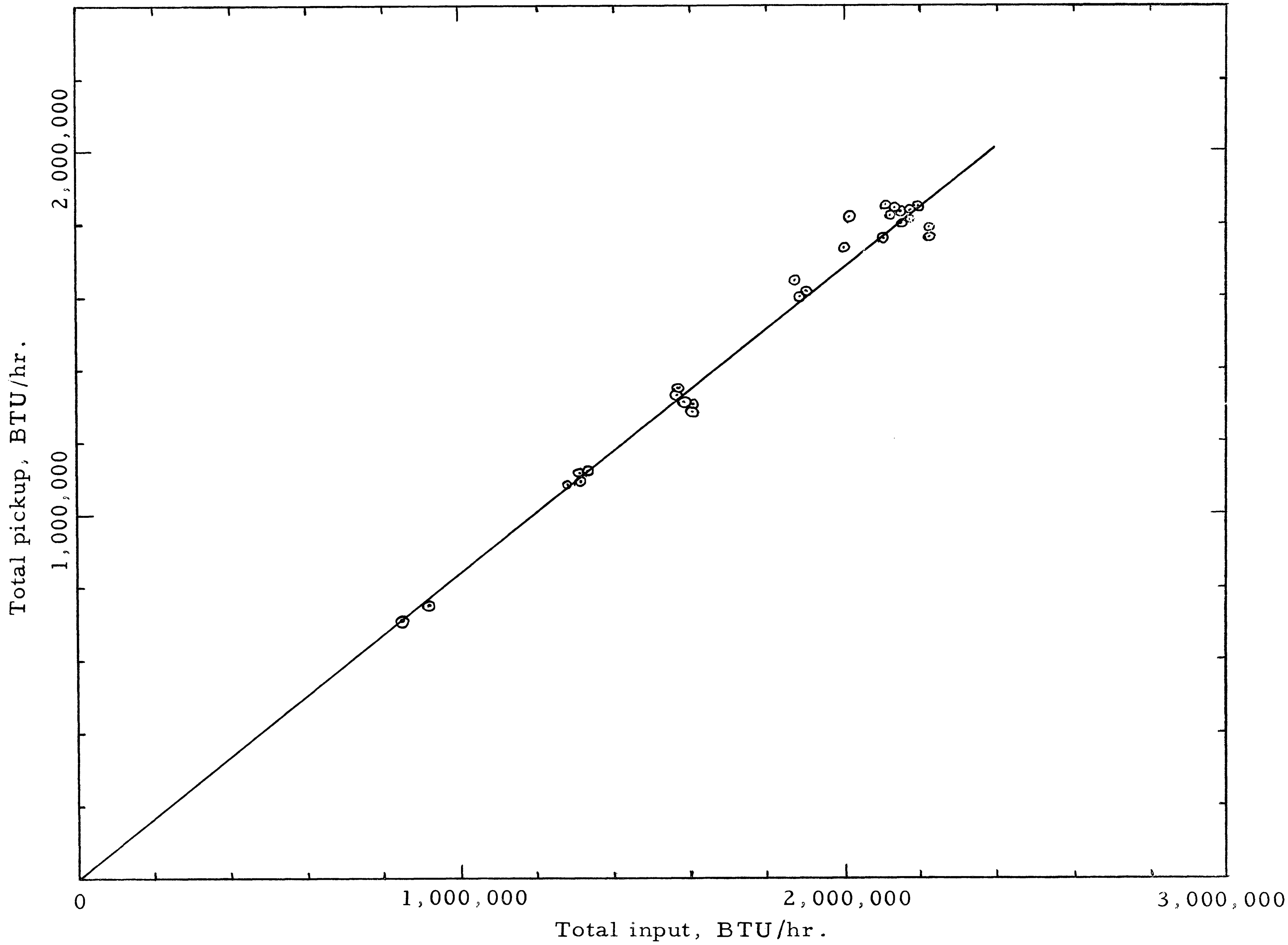

Fig. 4 Heat Release Curve for the Three Row Bundle of Finned Tubes, Bundle No. 2-A, Indicating 83.3% Efficiency

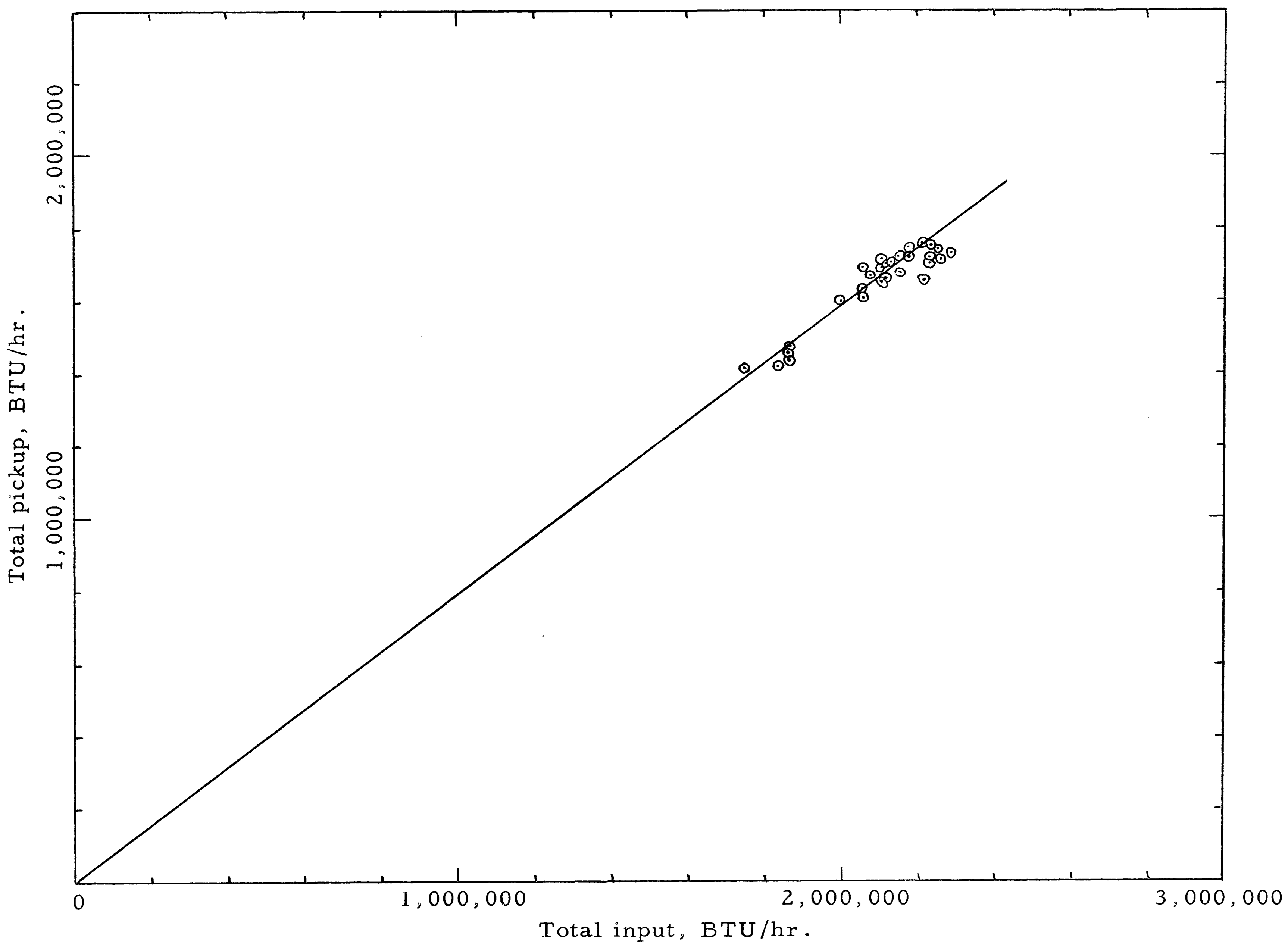

Fig. 5 Heat Release Curve for the Two Row Bundle of Finned Tubes, Bundle No. 2-A, Indicating 79.2% Efficiency

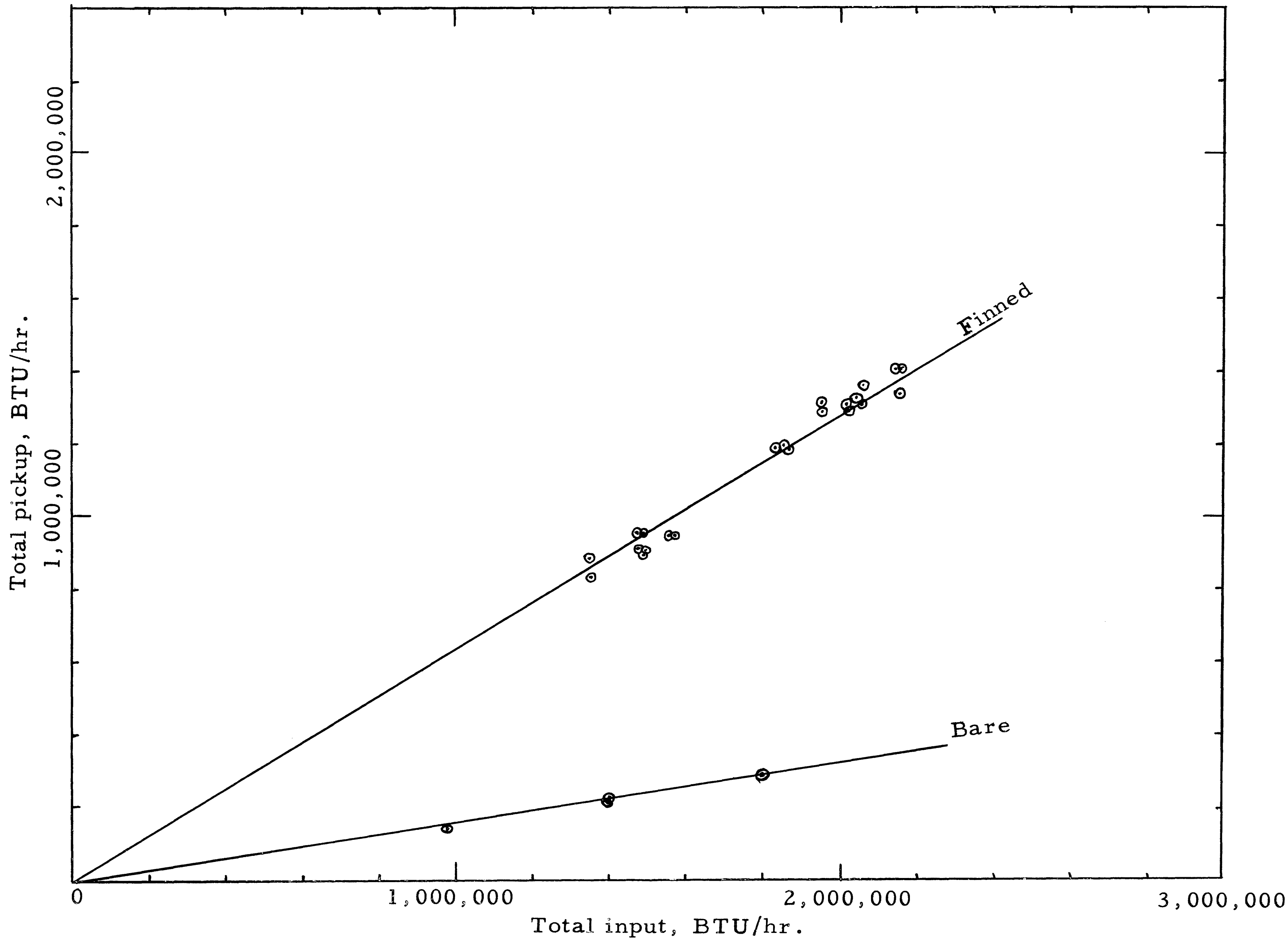

Fig. 6 Heat Release Curves for a One Row Bundle of Finned Tubes, Bundle No. 2-A, Indicating 64.0% Efficiency and a One Row Bundle of Bare Tubes Indicating 15.8% Efficiency

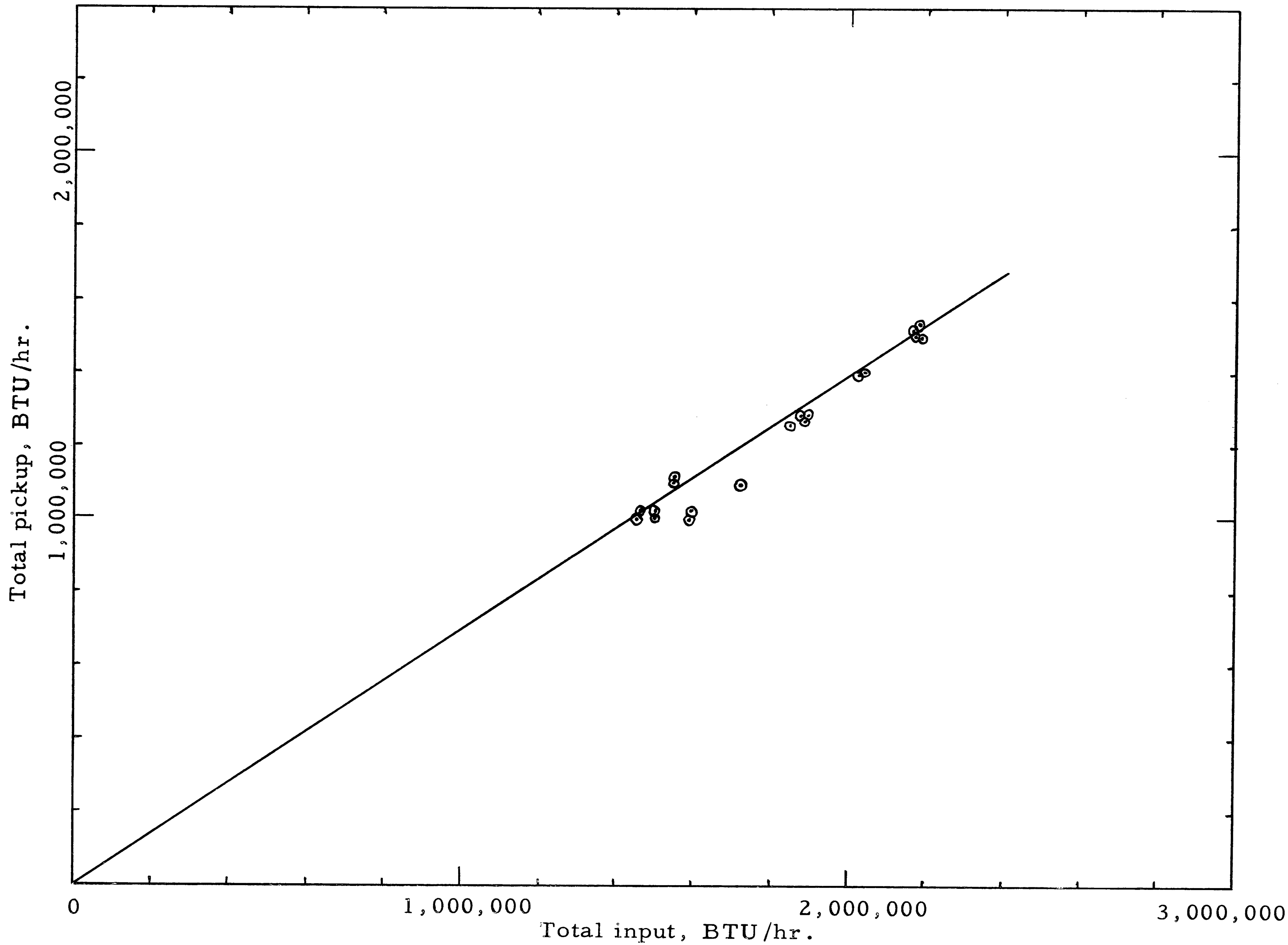

Fig. 7 Heat Release Curve for the One Row Bundle of Finned Tubes with "T" Bars, Bundle No. 2-A, Indicating 69.1% Efficiency

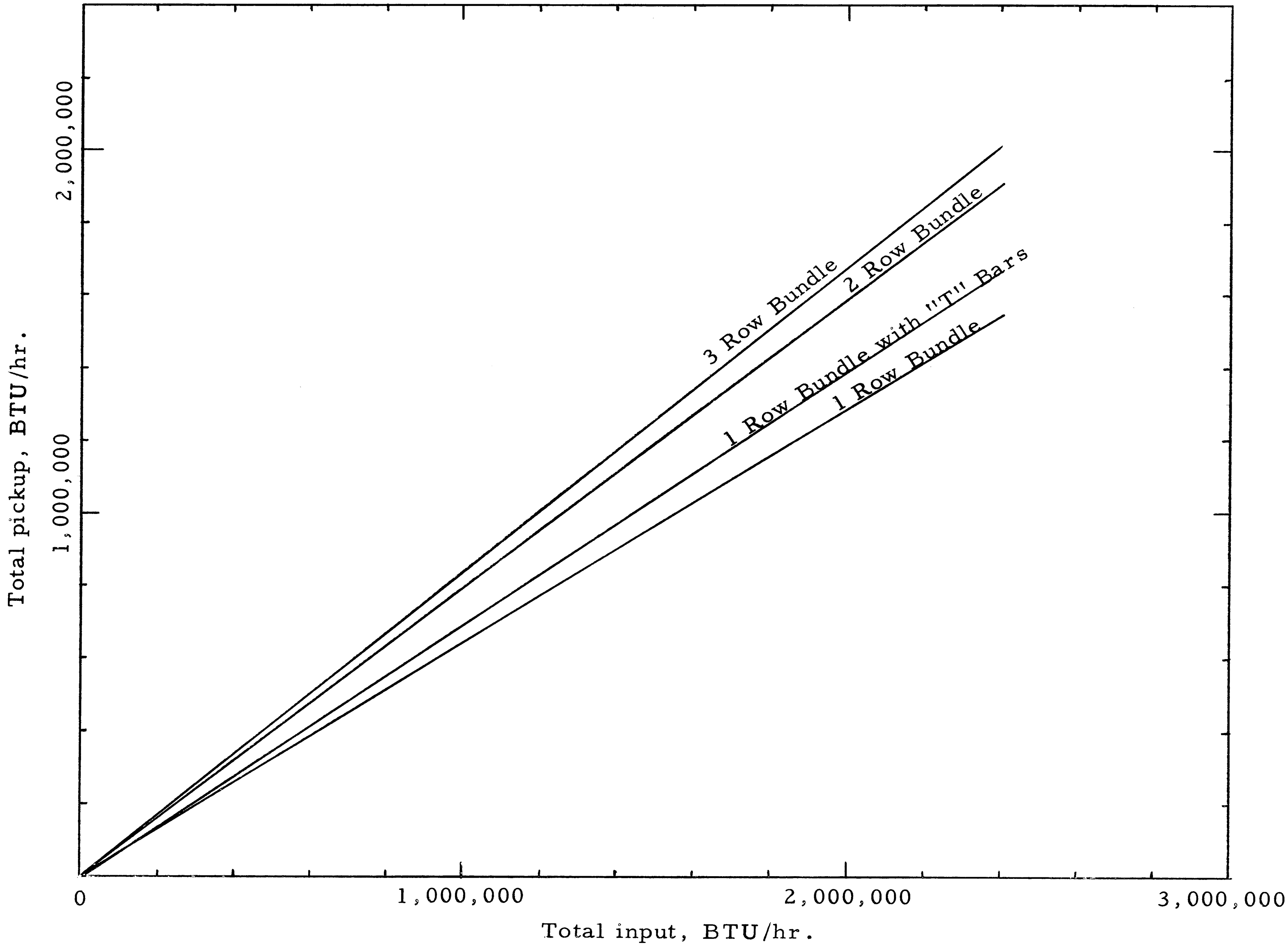

Fig. 8 Comparison of Heat Release Curves Presented in Figures 4 to 7

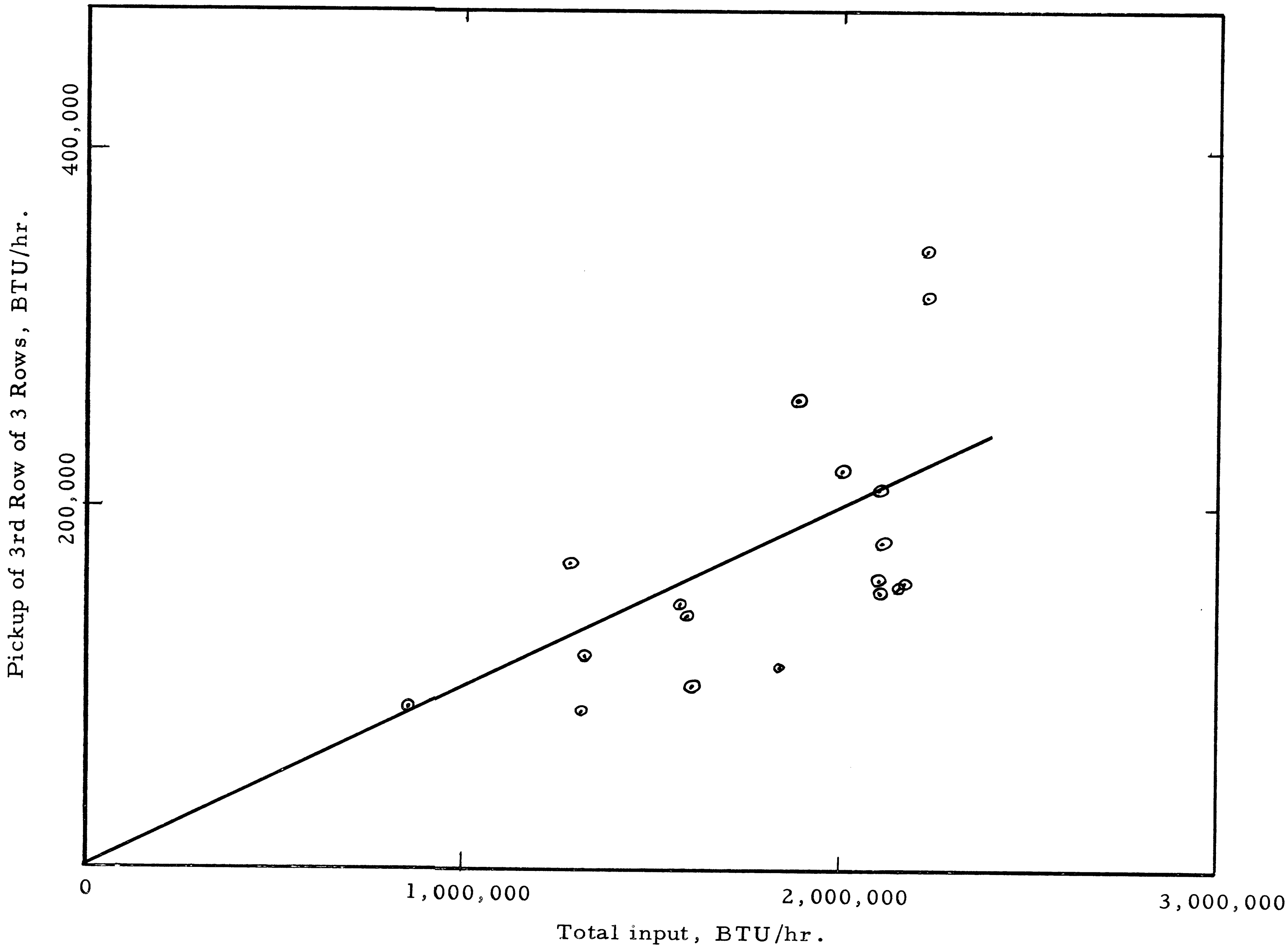

Fig. 9 Heat Release Curve for the Top Tube Row of the Three Tube Row Bundle

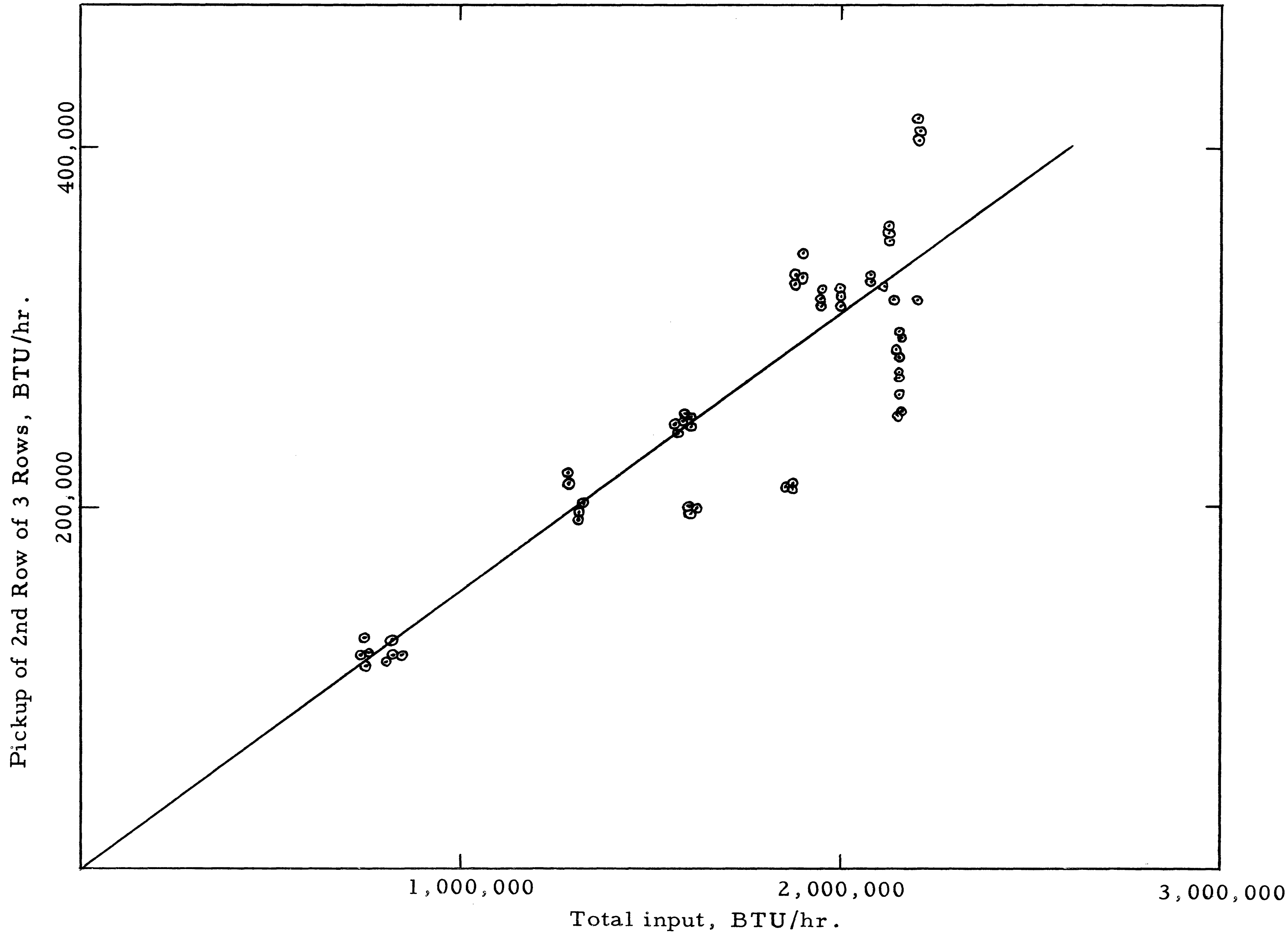

Fig. 10 Heat Release Curve for the Middle Tube Row of the Three Tube Row Bundle

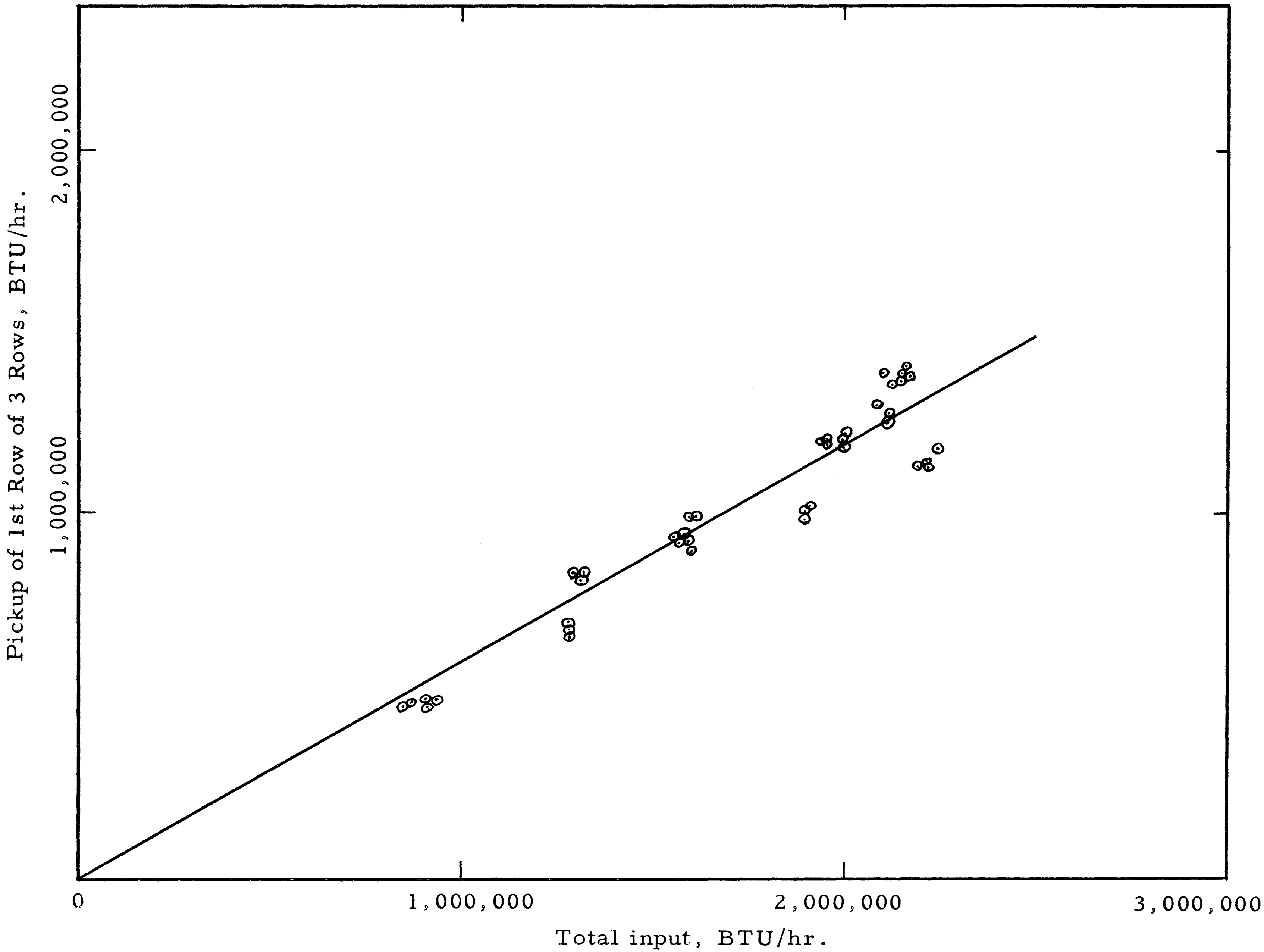

Fig. 11 Heat Release Curve for the Bottom Tube Row of the Three Tube Row Bundle

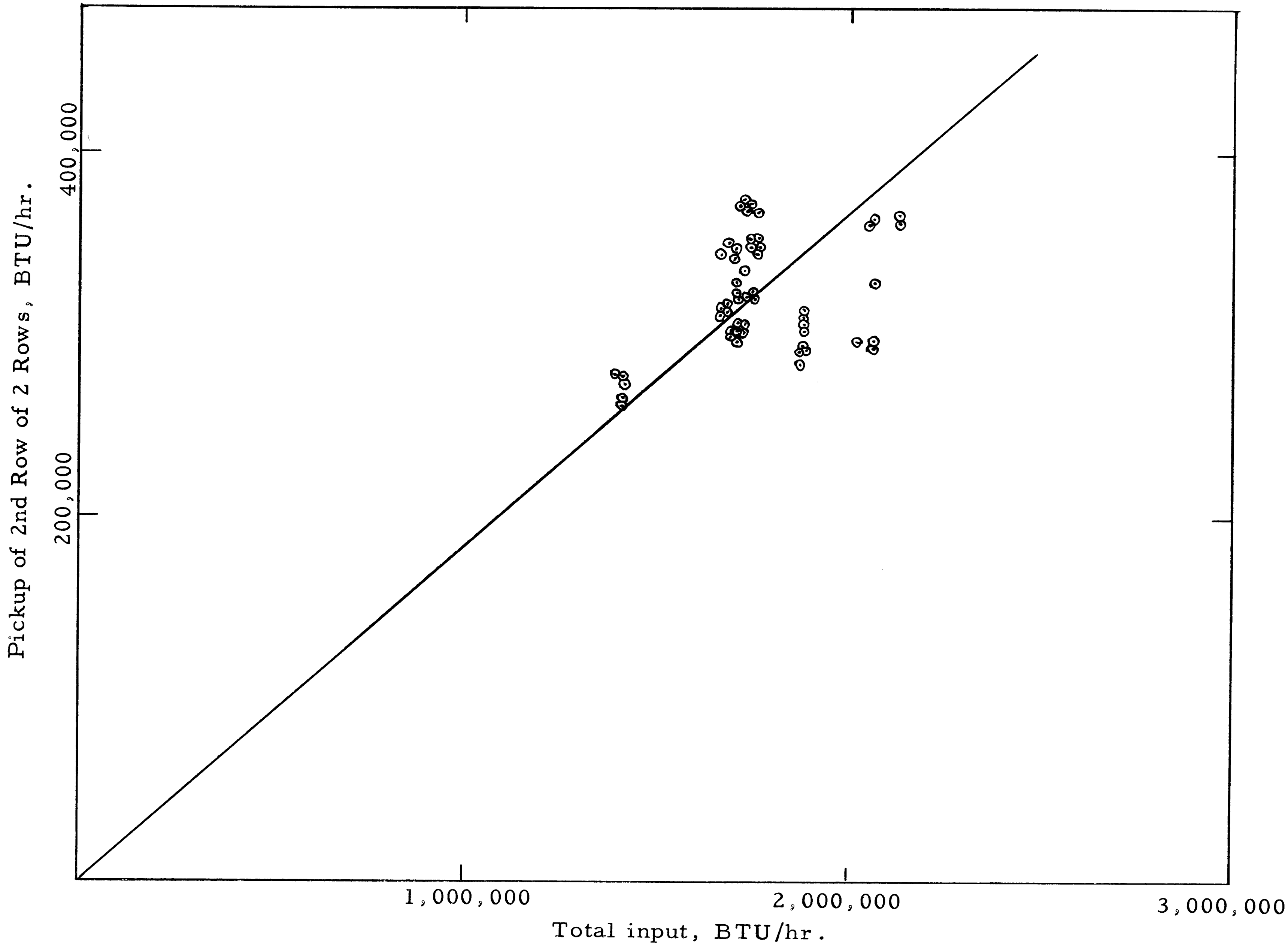

Fig. 12 Heat Release Curve for the Top Tube Row of the Two Tube Row Bundle

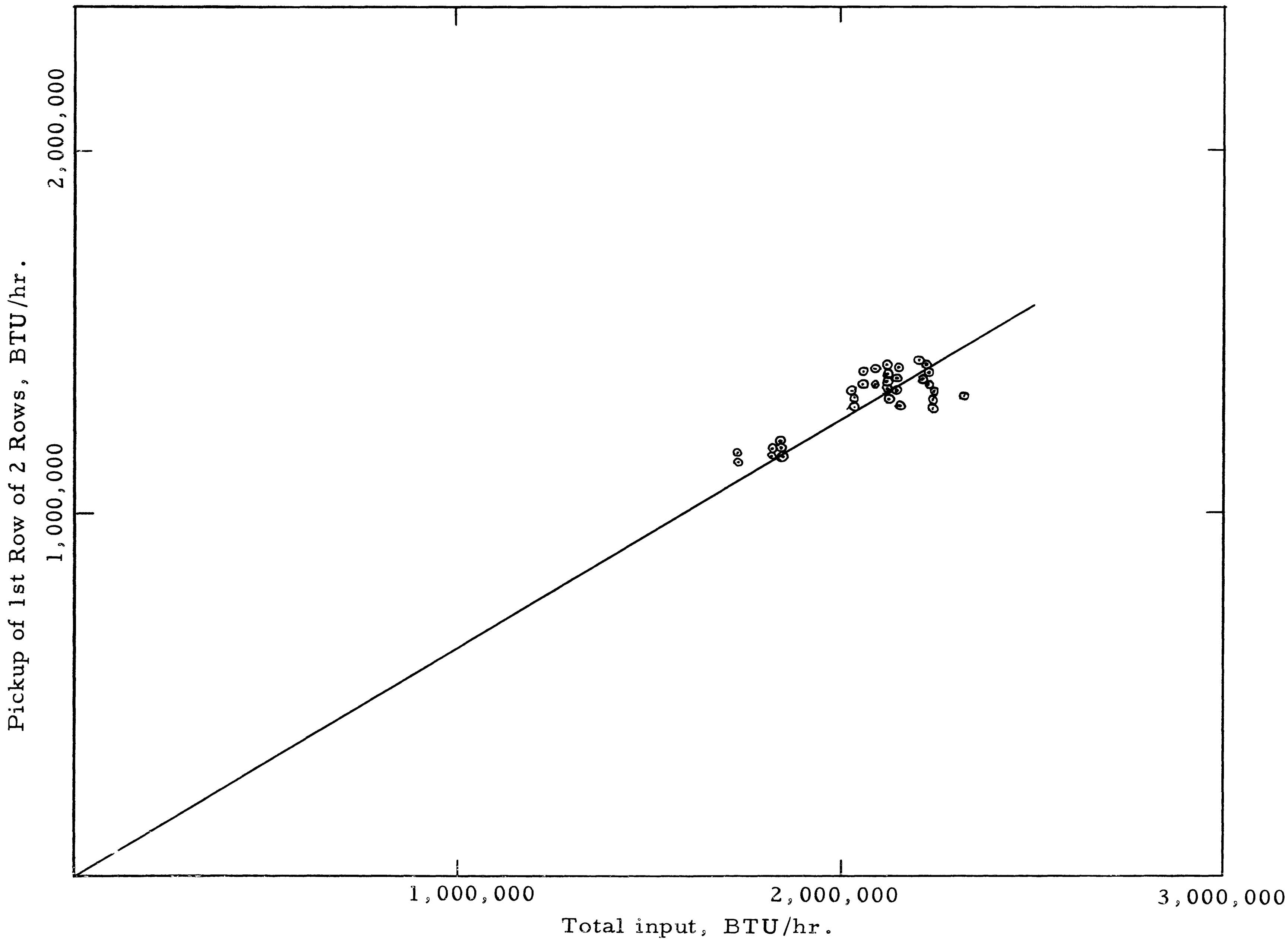

Fig. 13 Heat Release Curve for the Bottom Tube Row of the Two Tube Row Bundle

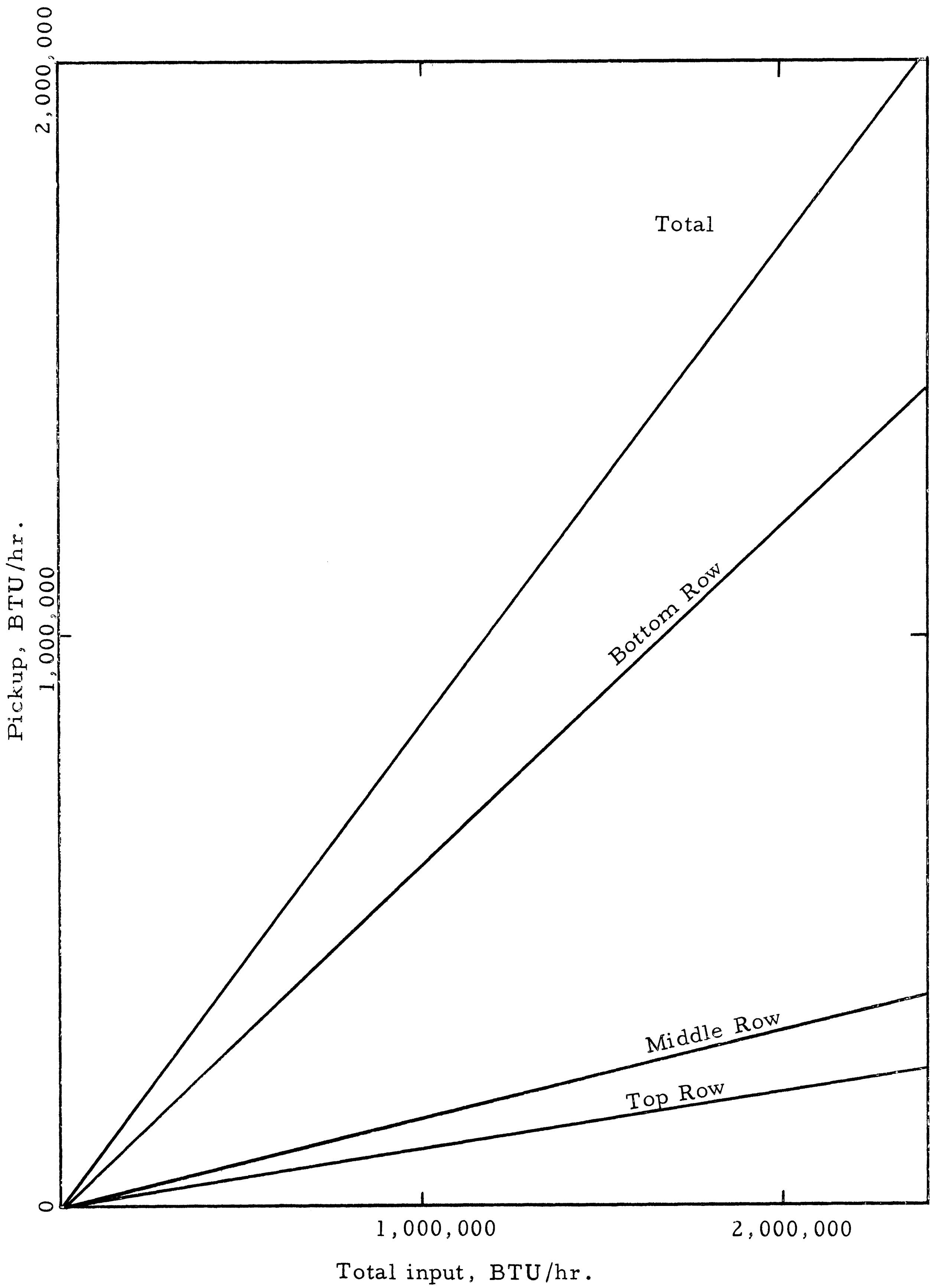

Fig. 14 Comparison of Heat Release Curves for the Three Row Bundle Given in Figures 4, 9, 10 and 11

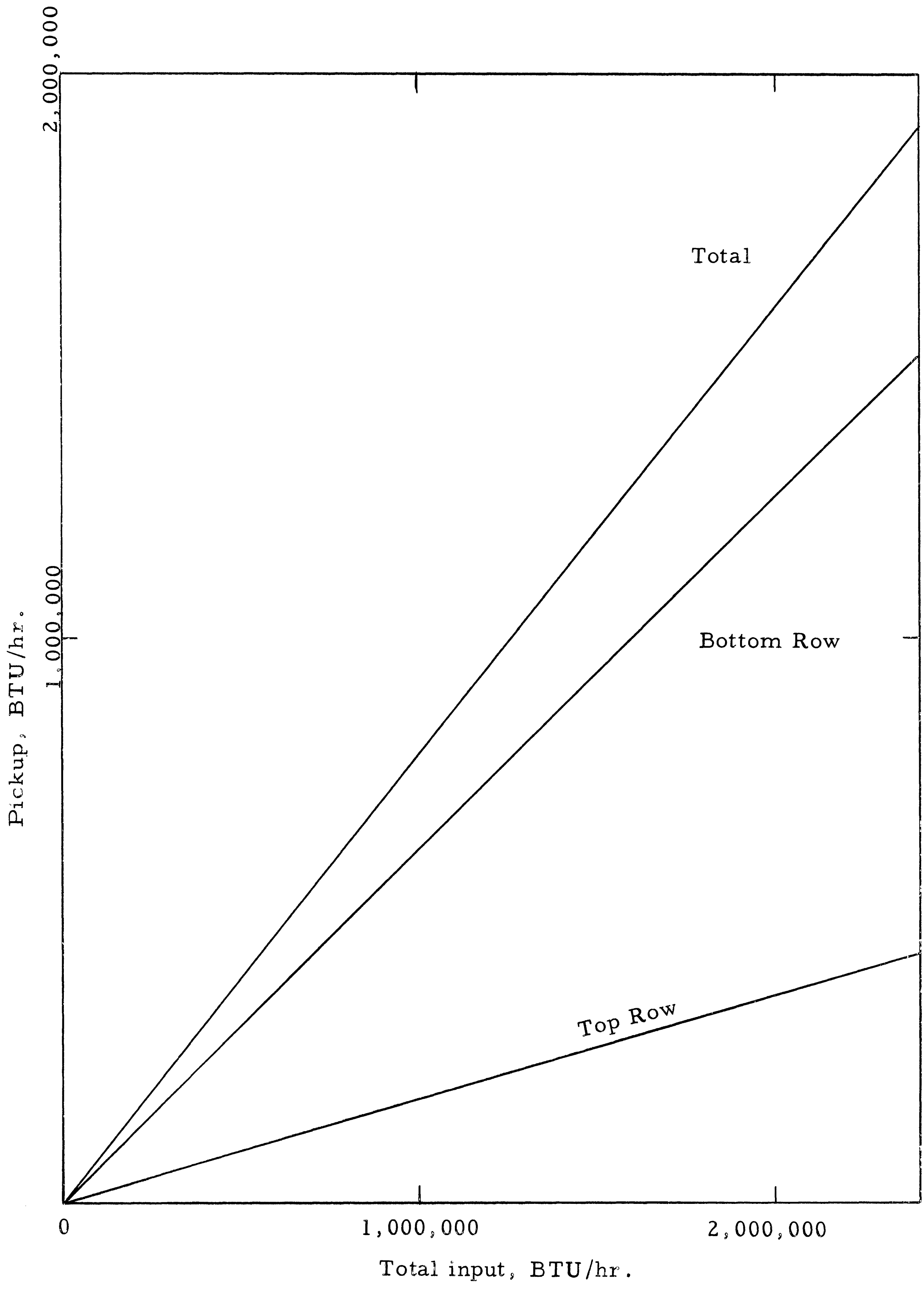

Fig. 15 Comparison of Heat Release for the Two Row Bundle Given in Figures 5, 12 and 13

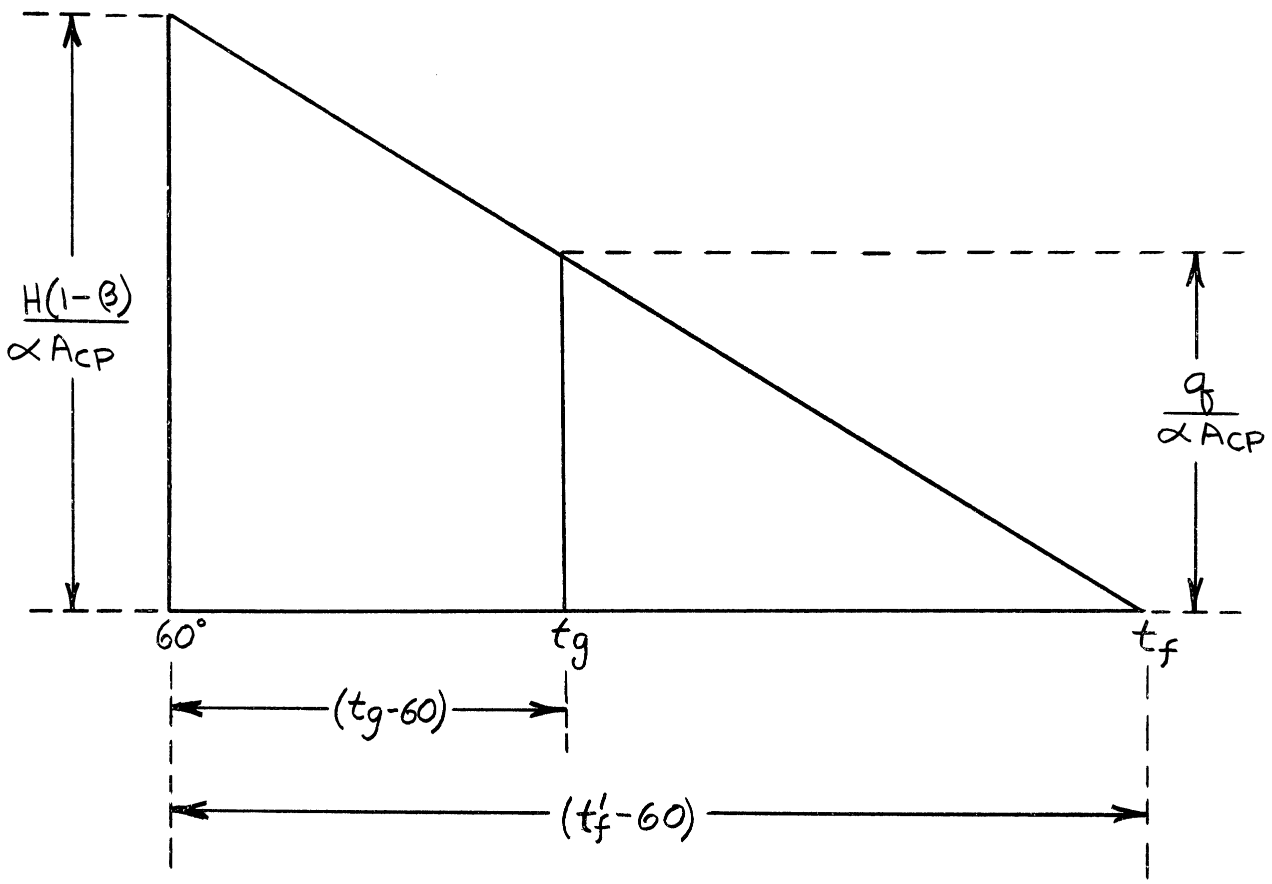

Fig. 16 Graphical Relationship of Lobo & Evans

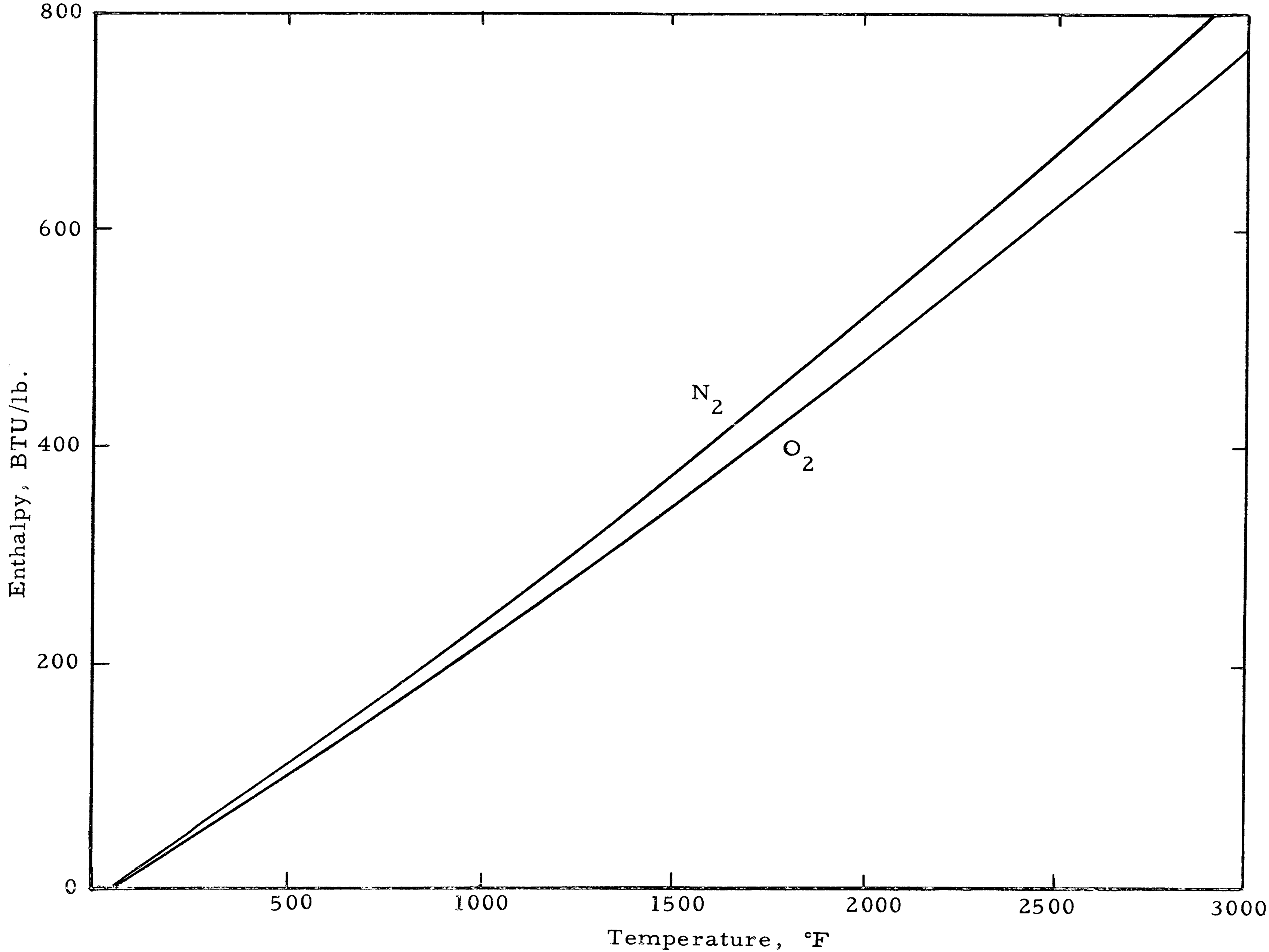

Fig. 17 Enthalpy of Nitrogen and Oxygen Versus Temperature

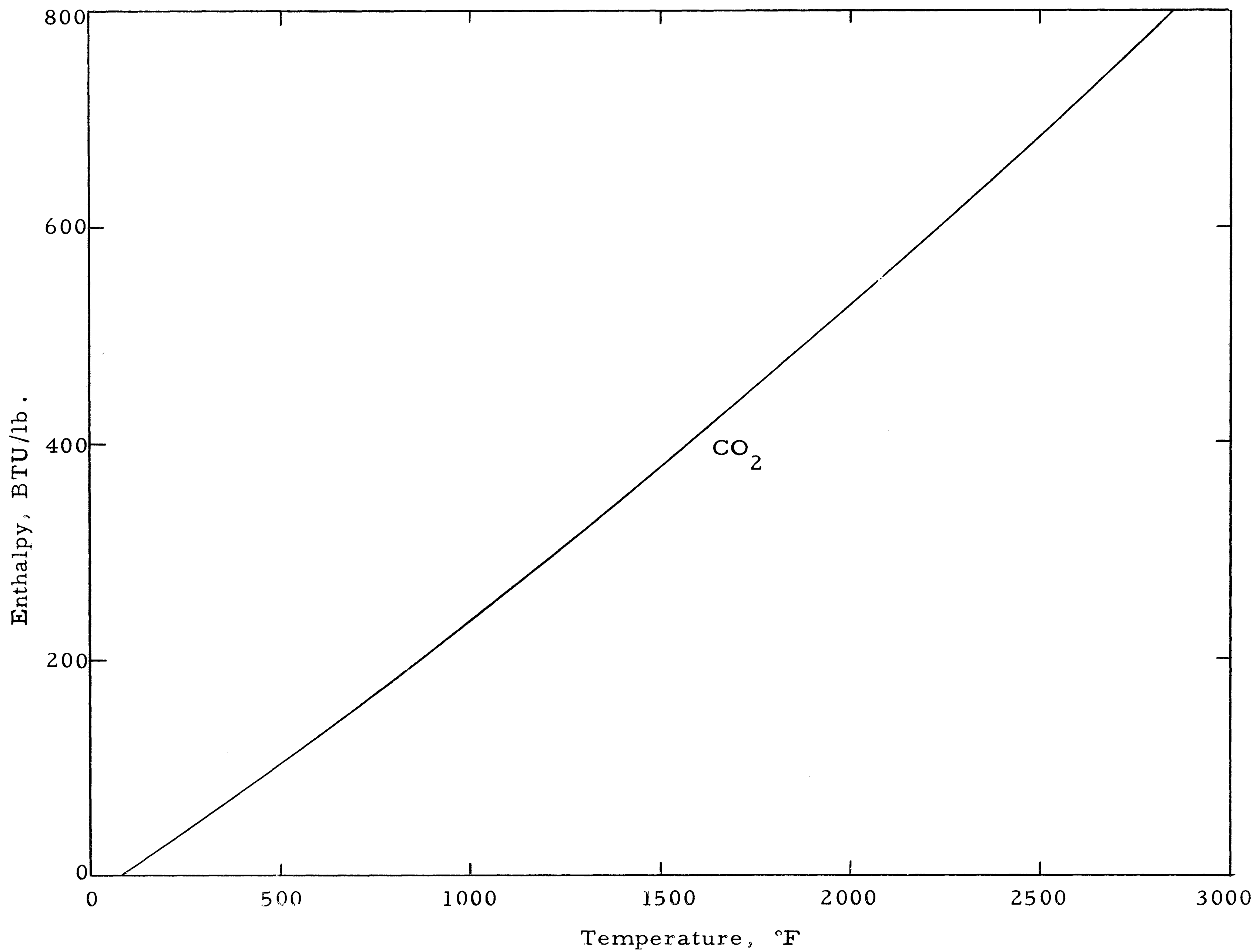

Fig. 18 Enthalpy of Carbon Dioxide Versus Temperature

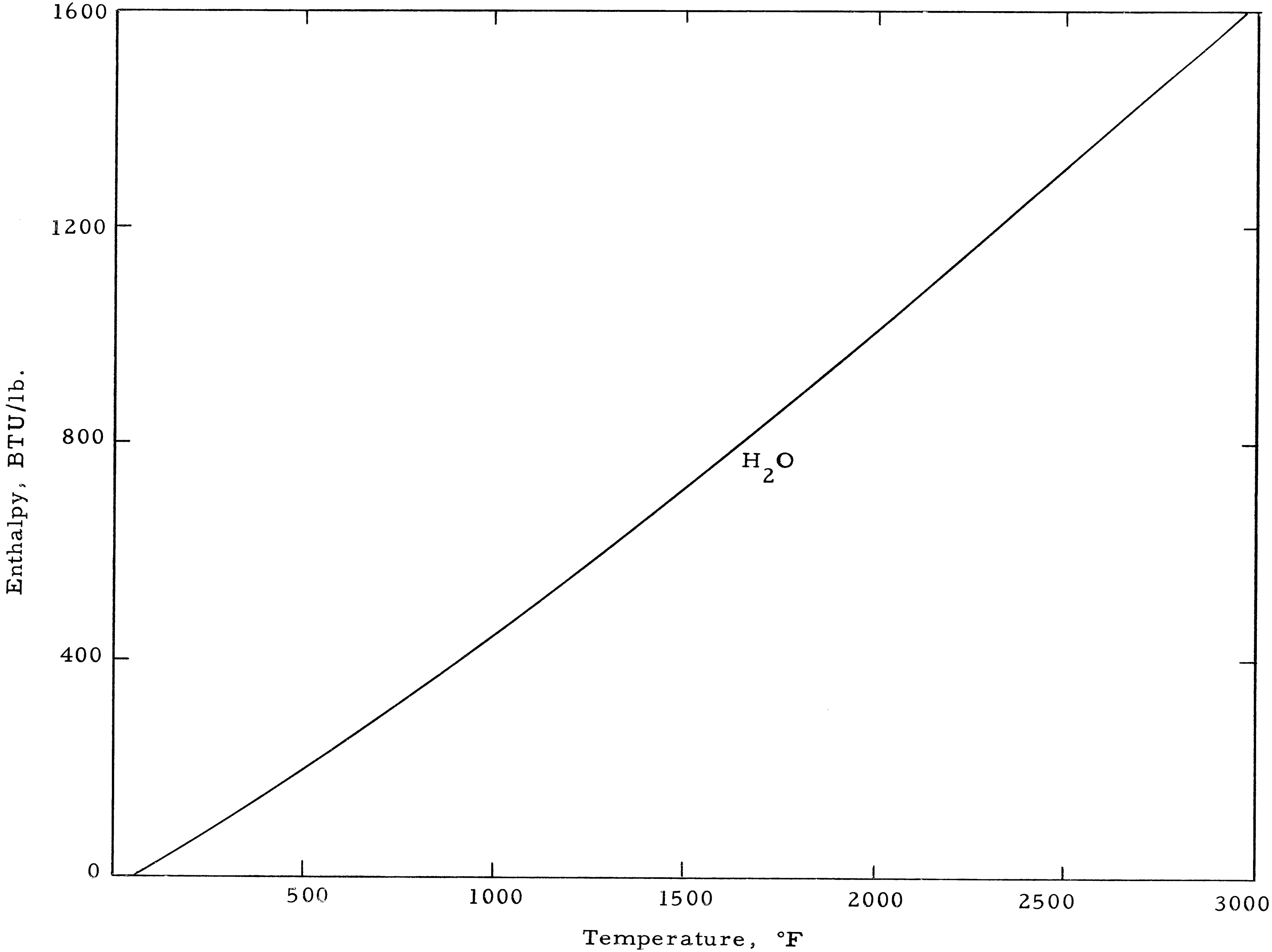

Fig. 19 Enthalpy of Water Vapor Versus Temperature

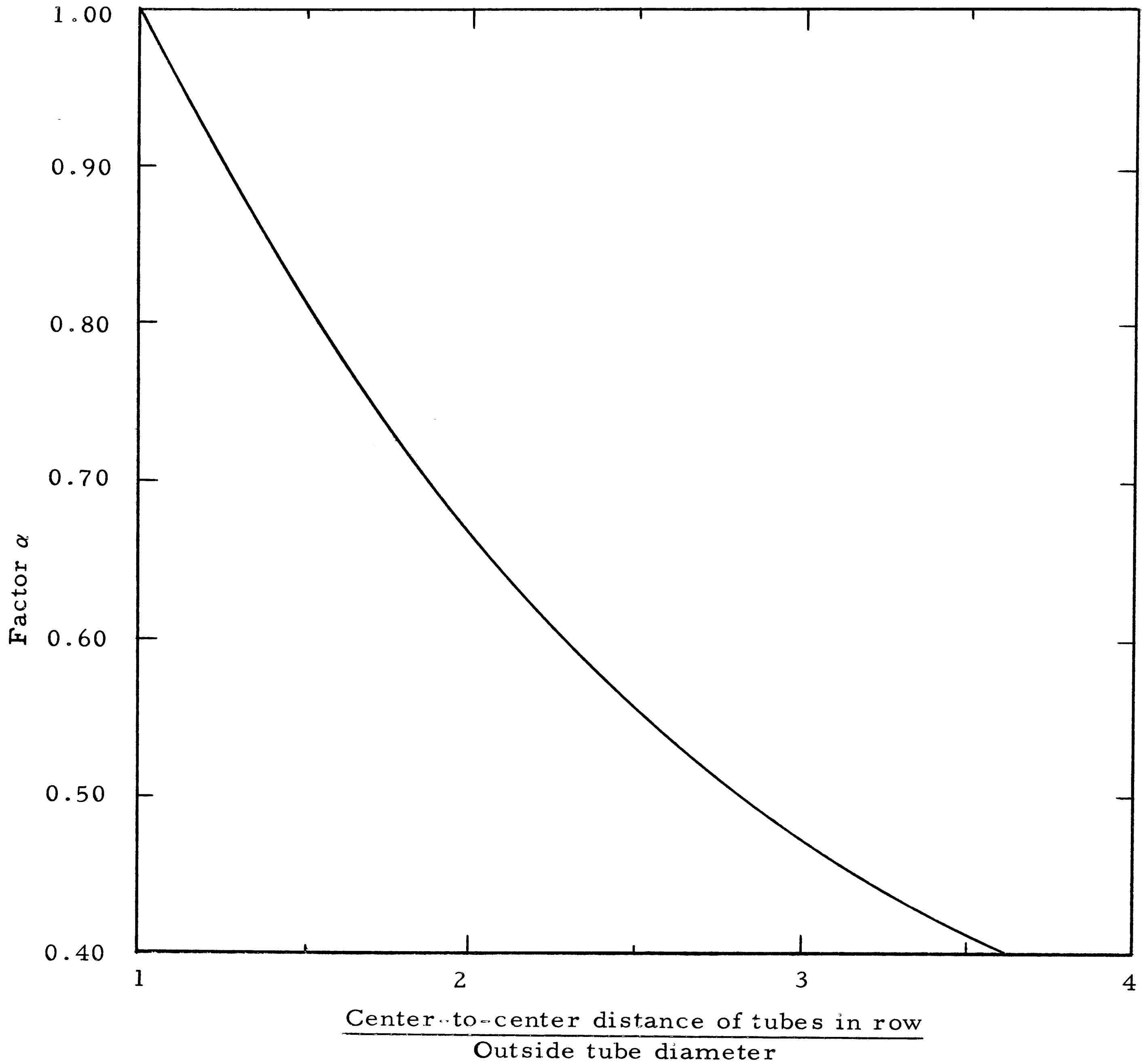

Fig. 20 Absorption Efficiency of the Tube Bank

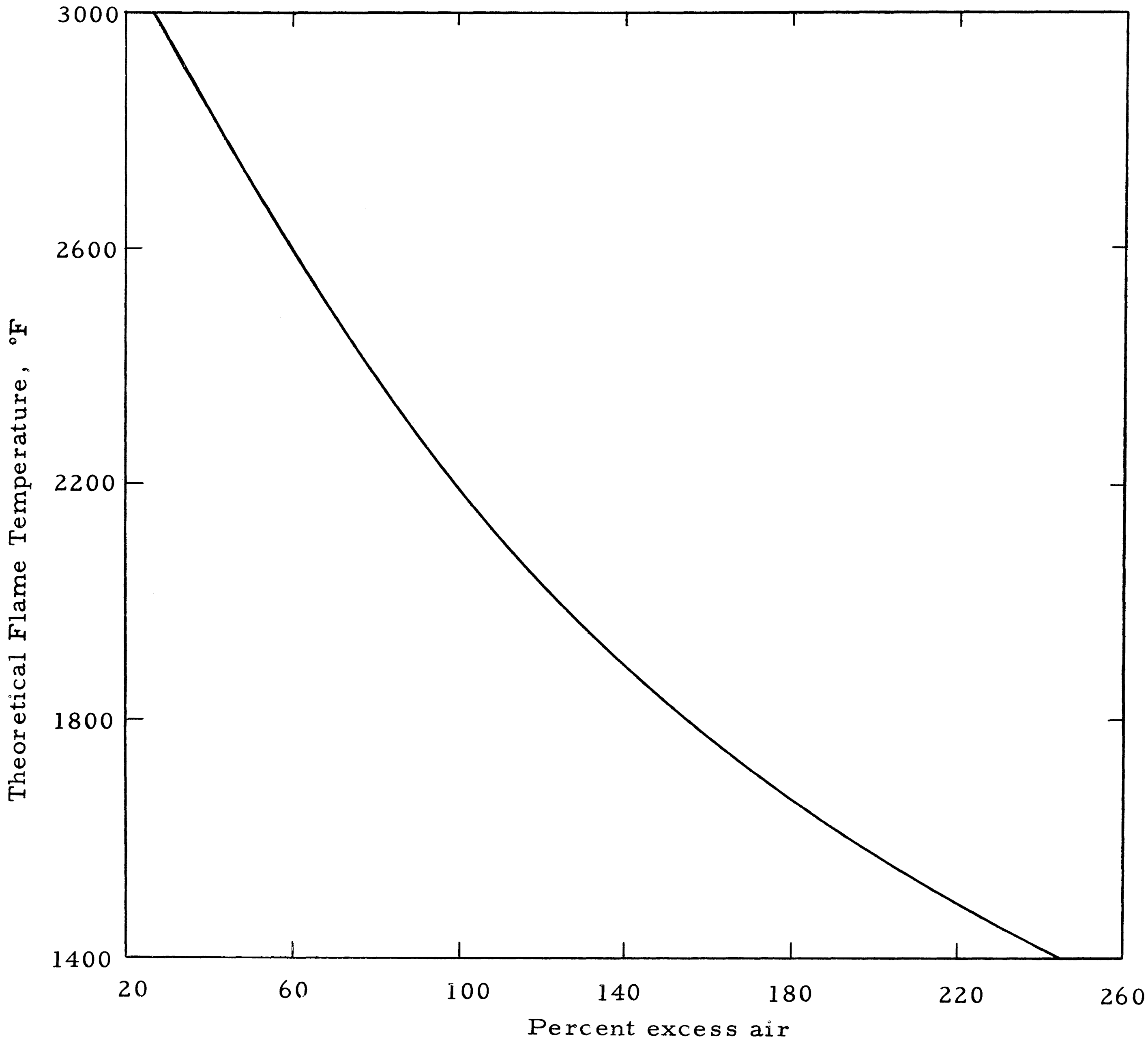

Fig. 21 Theoretical Flame Temperature Versus Percent Excess Air for the Natural Gas Composition Given in Table I

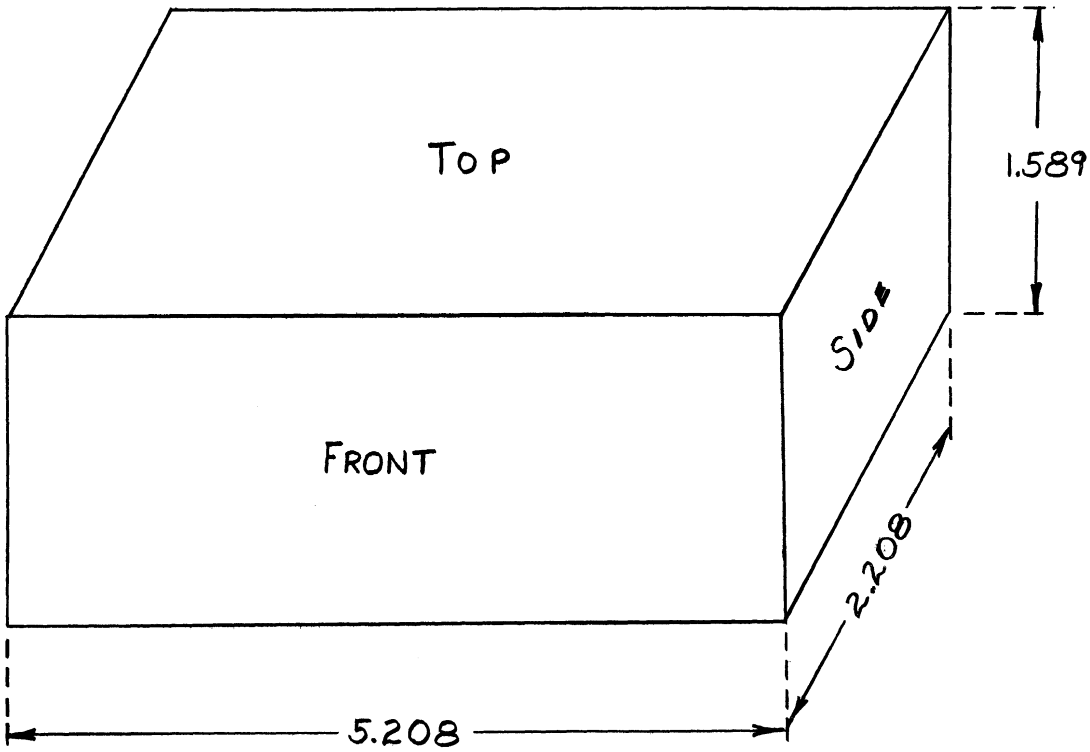

Fig. 22 Schematic Sketch of the Fire Box with Inside Dimensions Given in Feet

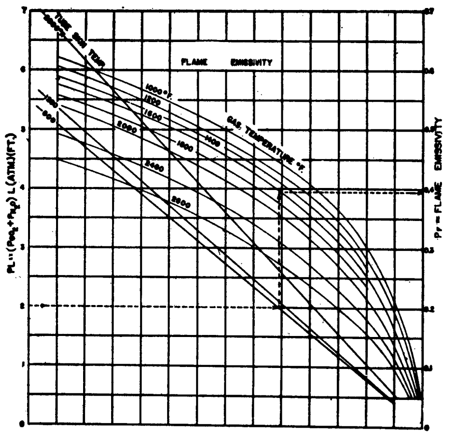

Fig. 23 Flame Emissivity of Lobo & Evans[2]

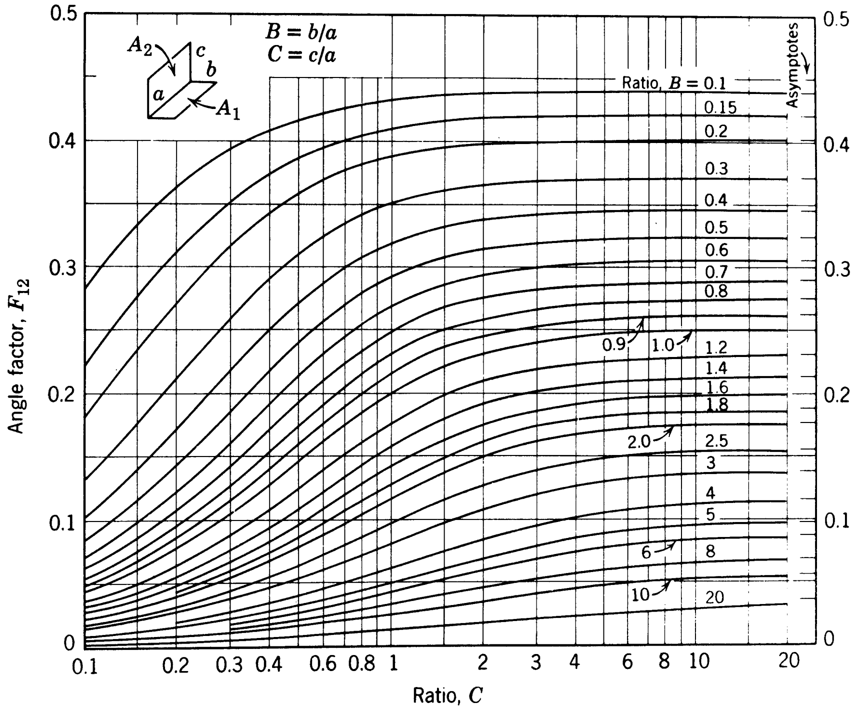

Fig. 24 Angle Factors for a System of Two Rectangular Surfaces Perpendicular to Each Other and Having a Common Edge[16]

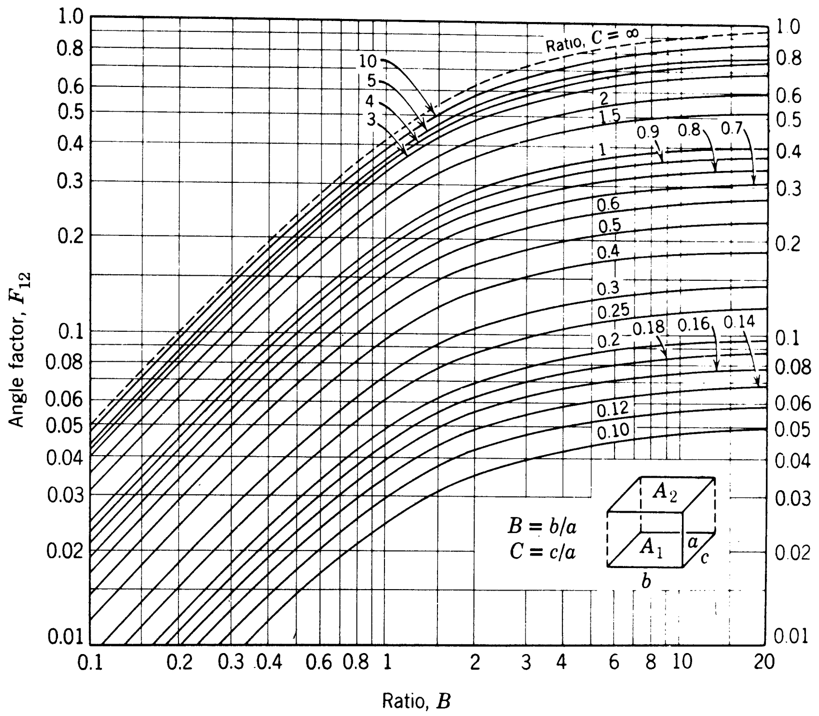

Fig. 25 Angle Factor for a System of Two Equal Parallel Rectangular Surfaces in Opposite Locations[16]

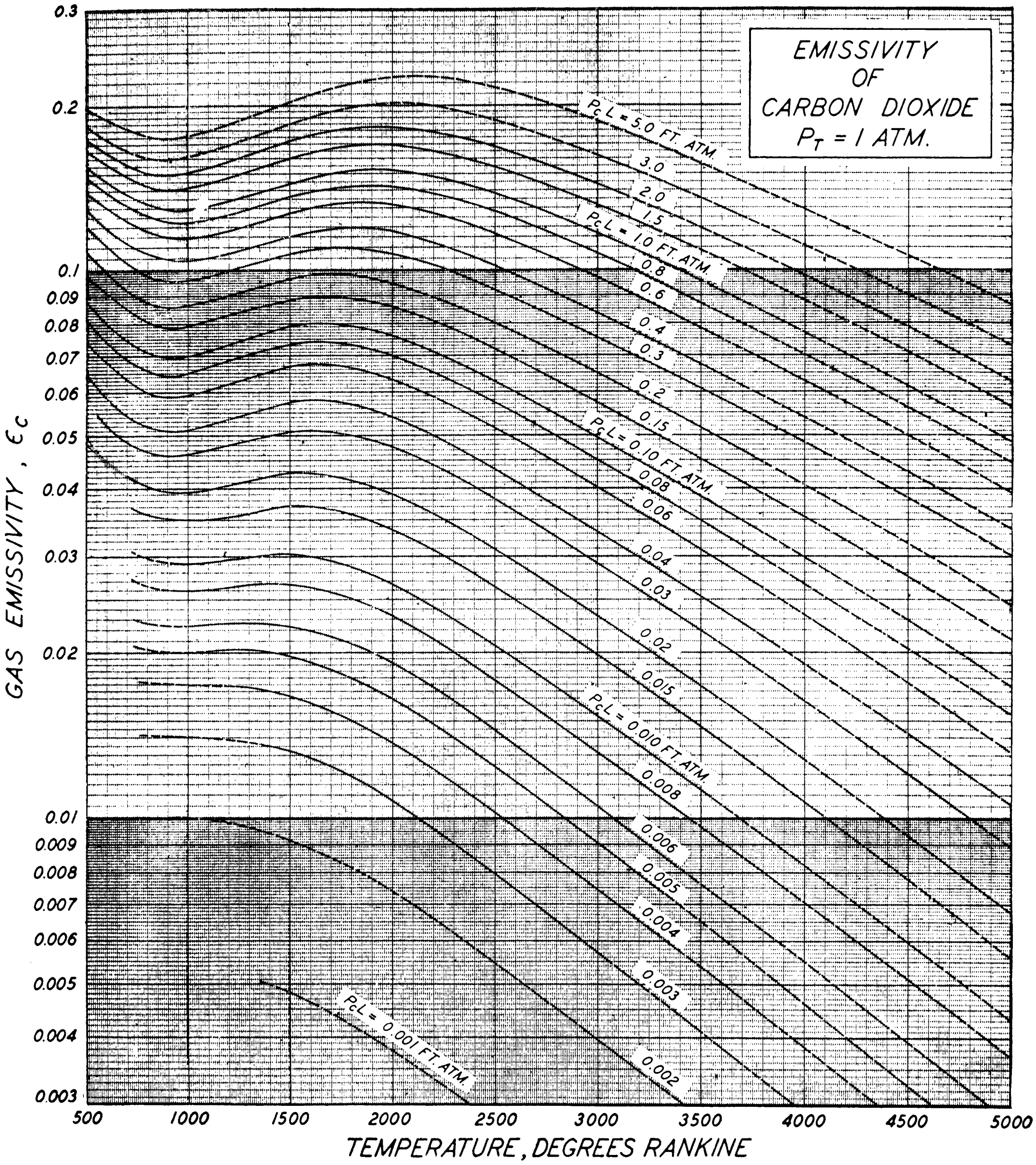

Fig. 26 Emissivity of Carbon Dioxide(17)

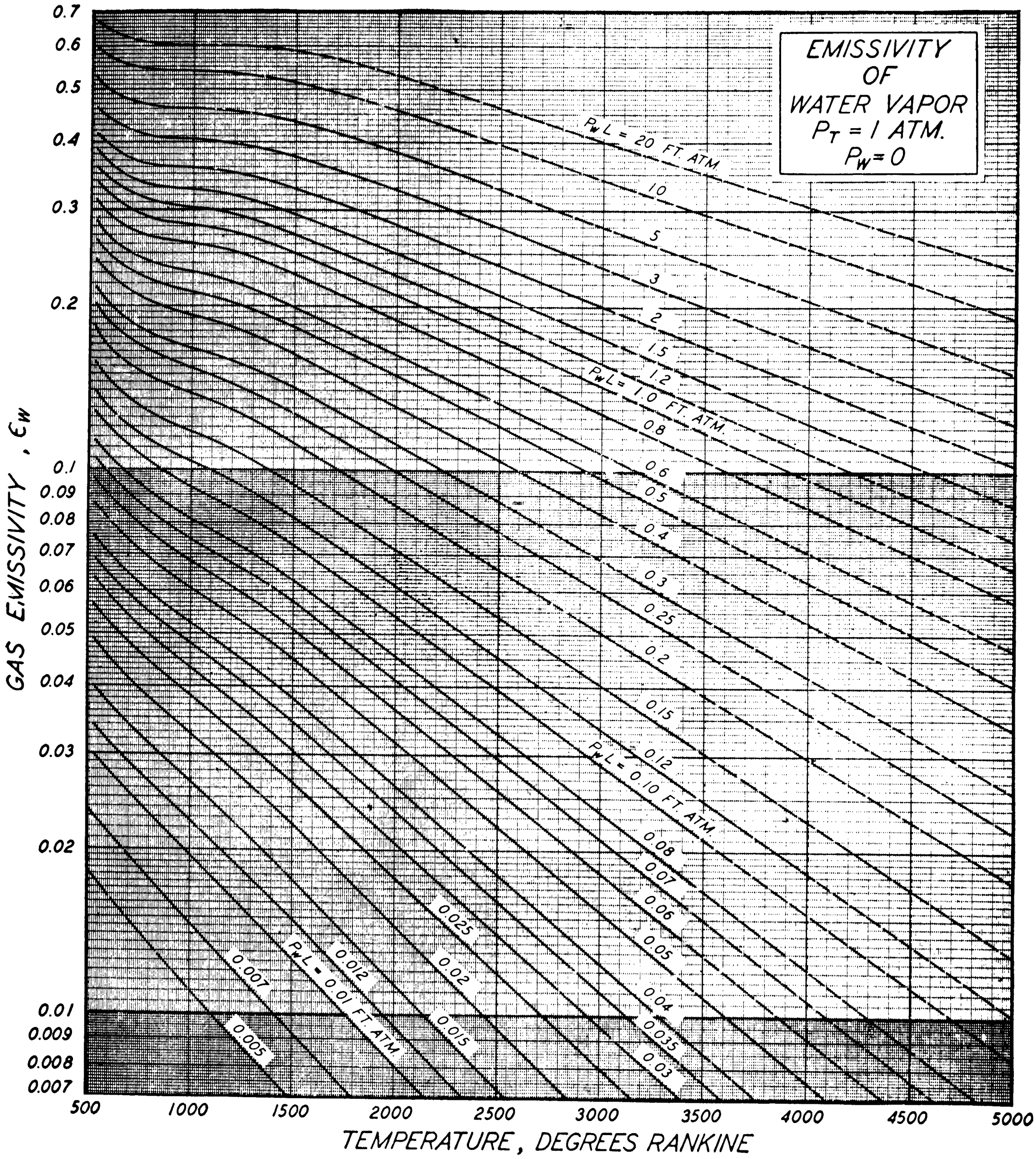

Fig. 27 Emissivity of Water Vapor(17)

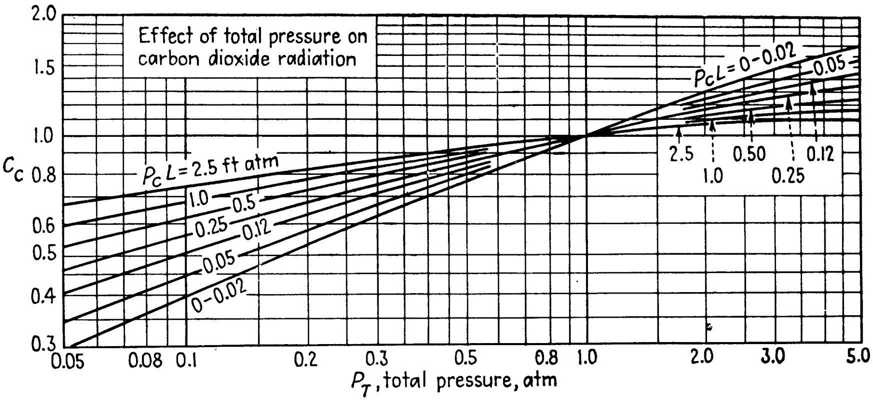

Fig. 28 Correction Factor C_C for Converting Emissivity of CO_2 at 1 atm. Total Pressure to Emissivity at P_T atm.[17]

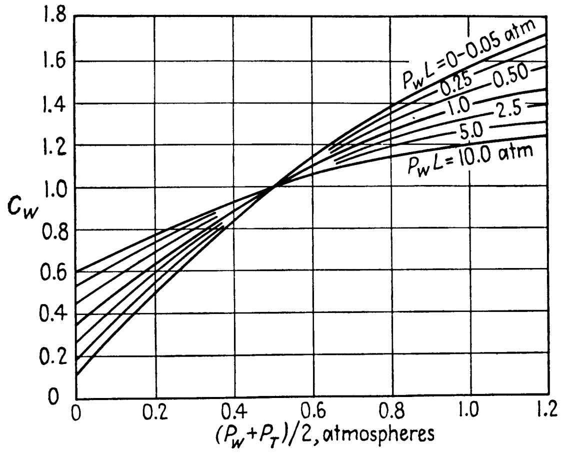

Fig. 29 Correction Factor C_W for Converting Emissivity of Water Vapor to Values of P_W and P_T Other Than 0 and 1 atm., Respectively[17]

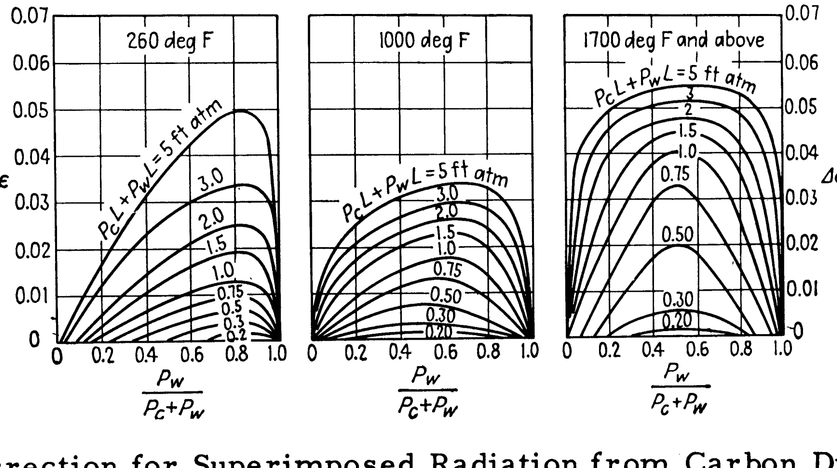

Fig. 30 Correction for Superimposed Radiation from Carbon Dioxide and Water Vapor[17]

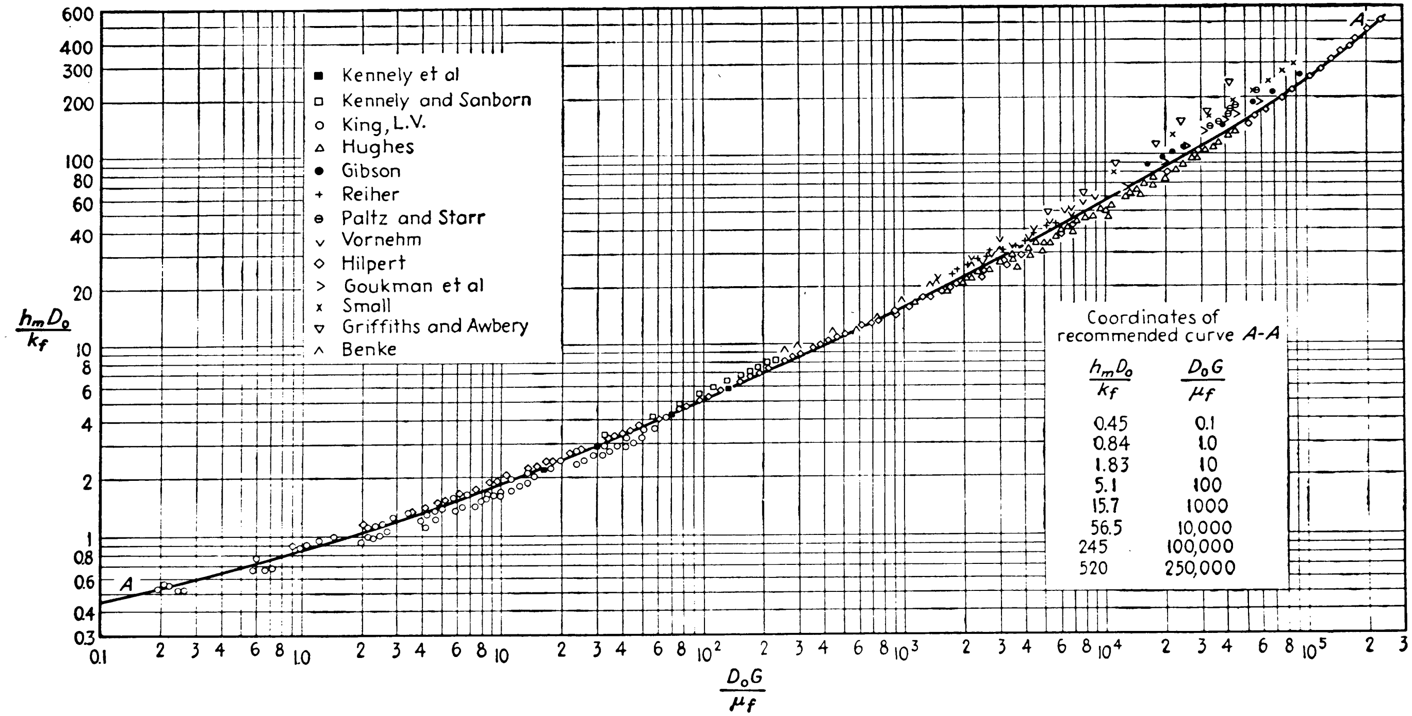

Fig. 31 Data for Heating and Cooling Air Flowing Normal to Single Cylinder, Corrected for Radiation to Surroundings(4)

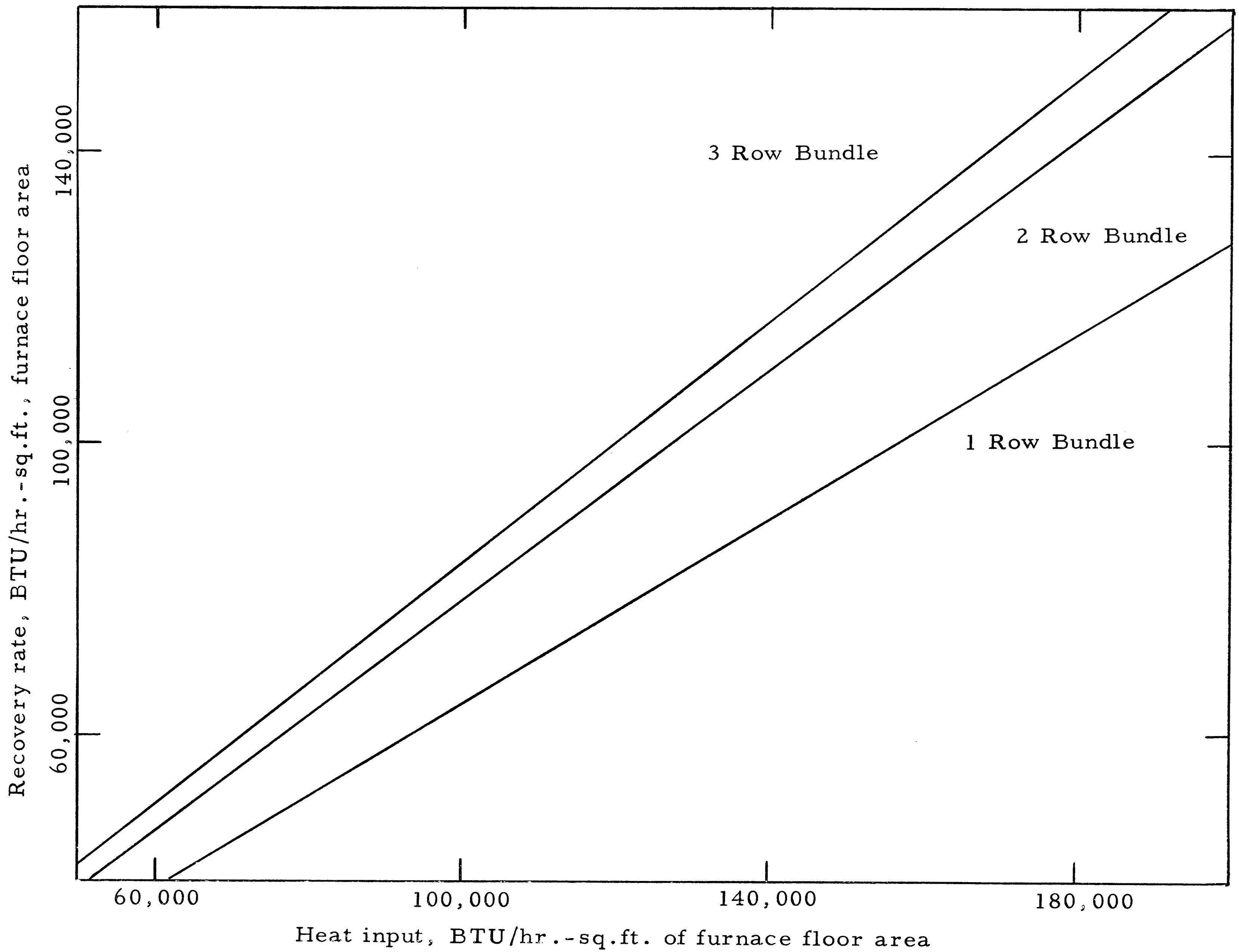

Fig. 32 Recovery Rate Curves for the Three Row, Two Row and One Row Bundles

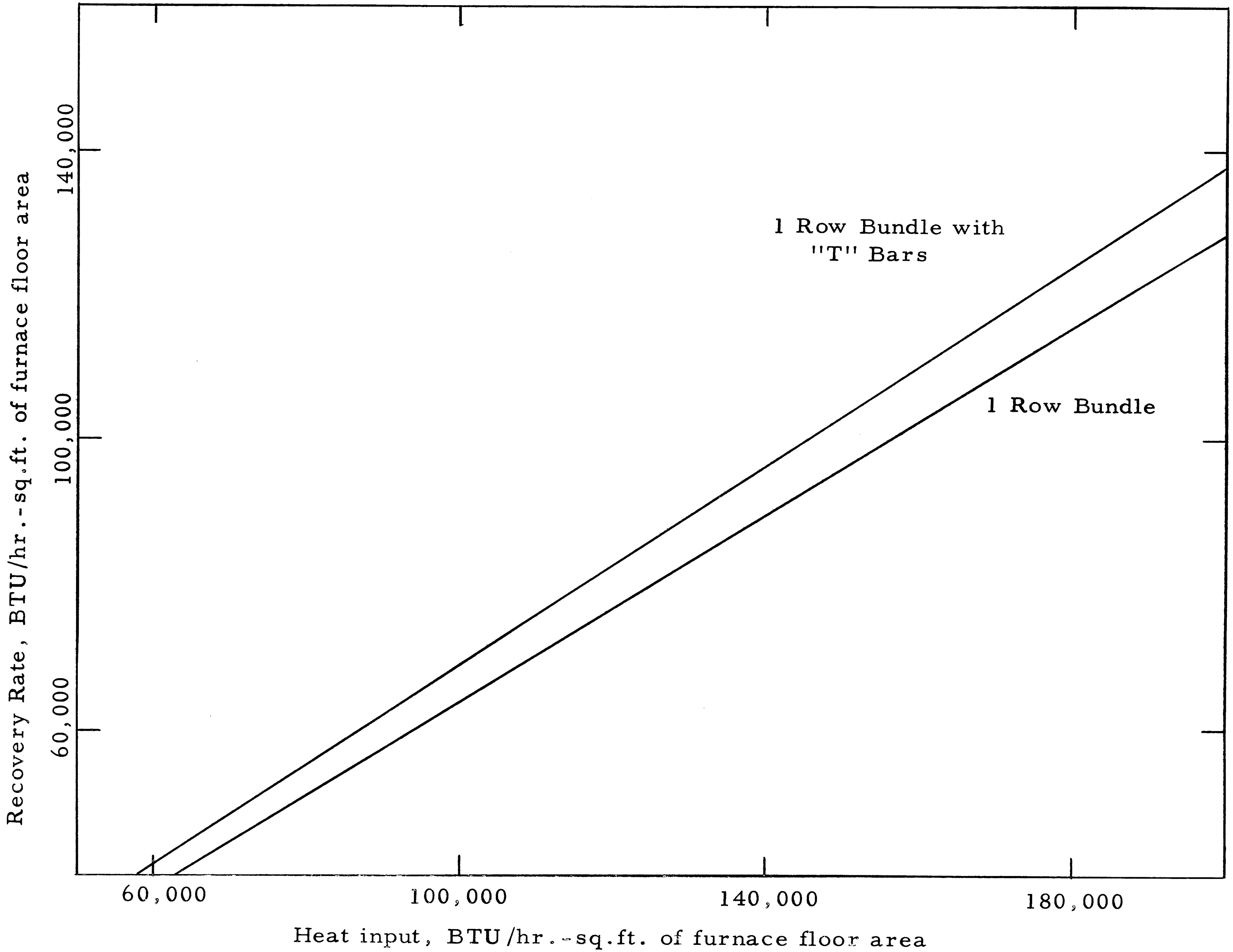

Fig. 33 Recovery Rate Curves for the One Row and One Row with "T" Bars Bundles

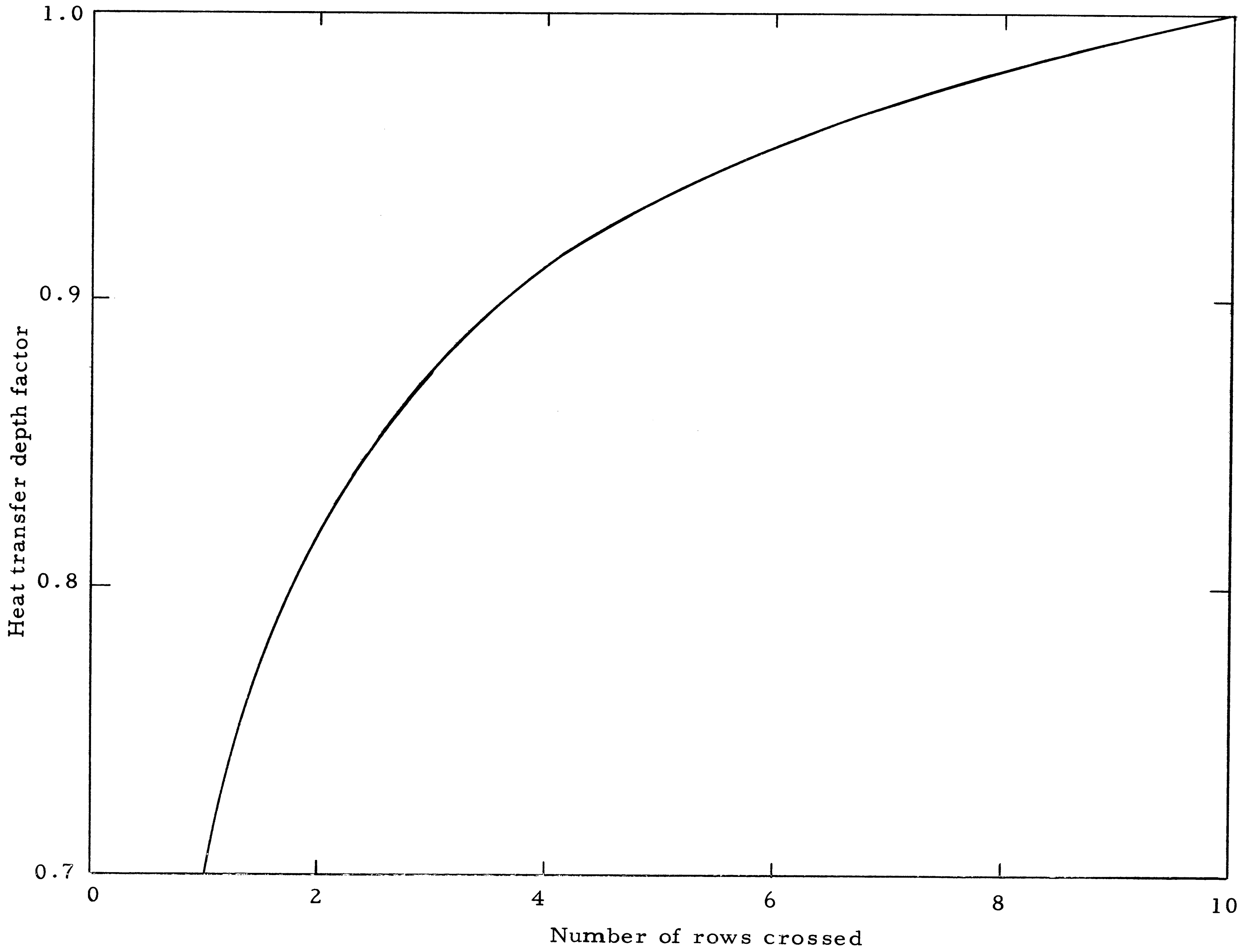

Fig. 34 Heat Transfer Depth Factor Versus Number of Rows Crossed