29 PSEUDO STRAIN-HARDENING DESIGN IN CEMENTITIOUS COMPOSITES

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Abstract

This paper reviews some experimentally determined tensile properties critical to engineering design reported in the literature of continuously aligned and short random fiber reinforced cementitious composites. Only those composites which have (or appeared to have) achieved pseudo strain-hardening are included in this discussion. Some recent development in modelling of the pseudo strain-hardening phenomenon in discontinuous randomly distributed FRC at the University of Michigan is briefly summarized, and comparisons are made between model prediction and experimental measurements. Special issues related uniquely to high fiber volume fraction and random orientation of short (or simply discontinuous) fibers are addressed. Finally, design implications derived from available experimental results and from modelling experience are briefly discussed.

Keywords: Pseudo Strain-Hardening, Fiber Concrete, Composite Design, Tensile Properties, Micromechanics.

High Performance Fiber Reinforced Cement Composites. Edited by H.W. Reinhardt and A.E. Naaman.
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1 Introduction

It is now well known, for continuous aligned fiber reinforced brittle matrix composites (particularly for cementitious and ceramic), that tensile failure strength and strain much higher than that of the matrix alone can be achieved. The pioneering work of Aveston, Cooper and Kelly (1971, hereafter known as ACK) provides much of the theoretical foundation for explaining and predicting this phenomenon. The case for randomly distributed discontinuous fiber reinforcement is not so well documented, and the physical processes connected with tensile failure appear more complicated. The processing ease, relatively low processing cost, and the isotropic mechanical properties of short fiber composites, however, is a significant advantage over continuous reinforcements. A composite employing discontinuous fibers with the attendant processing advantage of discontinuous fiber composites, together with the mechanical performance of continuous reinforcements, would be extremely desirable.

2 Pseudo Strain-Hardening Properties

A tensile stress-strain curve of a fiber reinforced brittle matrix composite which exhibits pseudo strain-hardening is shown schematically in Figure 1. The first crack strength σ_{mu} and first crack strain ε_{mu} are generally higher than the matrix tensile strength and strain due to arresting of microcracks by fibers. Subsequent to first cracking, multiple cracking may ensue with many sub-parallel cracks of small width spreading across the specimen. At the end of multiple cracking, the strain ε_{mc} can be many times larger than the first crack strain. The stress level at this stage σ_{mc} depends on the distribution of initial flaw size. Further increase in loading must be carried by the fibers bridging across the matrix cracks. The ultimate tensile strength σ_{cu} is reached when fiber ruptures by exhausting its tensile strength for the case of continuous aligned fiber composites. The ultimate tensile strain ε_{cu}

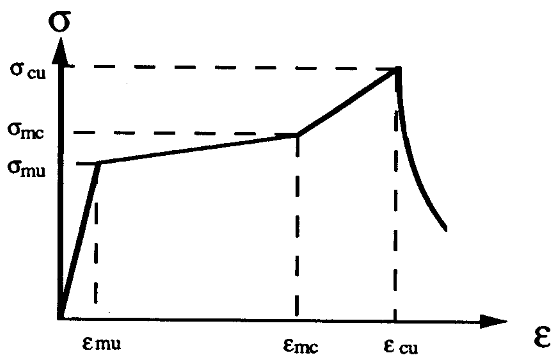

Figure 1. Schematic diagram of tensile stress-strain curve of fiber reinforced brittle matrix showing pseudo strain-hardening behavior.

Table 1. Summary of various fiber reinforced cementitious composites with their tensile properties.

Fiber	d_f (um)	L_f (mm)	V_f (%)	Matrix	σ_{mu} (MPa)	σ_{mc} (MPa)	σ_{cu} (MPa)	ε_{mu} (%)	ε_{mc} (%)	ε_{cu} (%)	Ref
Continuous											
steel	132		2.3	C	9.2	16.3	33.8	0.05	0.23	0.59	Aveston et al,1974
			8.8		23.8	40.0	96.9	0.07	0.21	0.54	Aveston et al,1974
carbon	NA		1.3	C	8.5	12.3	18.5	0.04	0.25	0.57	Aveston et al,1974
			4.4		22.3	31.1	66.2	0.08	0.26	0.74	Aveston et al,1974
			12.3		46.9	68.1	189.2	0.12	0.29	0.88	Aveston et al,1974
PP	340		1.4-8.7	C							Baggott and Gandhi,1981
PP	118		12.0	C	8.0	14.5	>32.0	0.03	~1	>3	Mobasher et al,1990
			13.4		<15.5			<0.057	~1	~2.8	
			8.7		<10.5			<0.046	>1	~4	
carbon	14.5		4.7	C+AD	7.8	10.4	>12.8	0.07	0.63	>0.99	Akihama et al,1986
Discontinuous											
carbon	14.5	10	4	C+AD	4.7	>7.4	>8.1	0.05	>.88	>0.88	Akihama et al,1986
carbon	14.5	3	5	C+AD+SF	7.3		>9.6	0.03	>.06	>0.06	Ohama et al,1985
PP	70	13	5	C+V	0.8	0.9	0.9	0.04	>0.2		Baggott,1983
		26	5		0.6	1.3	1.5	0.04	>0.2		
steel	254	25	1.6-1.9	C			~10				Aveston et al,1974
FRDSP	150	6	0	DSP	6.2			0.01			Tjiptbroto,1991
			3		12.3		11.5	0.02		0.09	
			6		17.7		20.8	0.03		0.18	
			9		23.8		25.6	0.04		0.19	
			12		27.8		36.9	0.05		0.26	
glass strand	12.5	38	4.55	C	5.3	9.9	>17.5	0.03	0.57	>1.37	Oakley and Proctor,1975
glass strand	11	30	6.3	C	8.2	11.4	17.8	0.04	0.38	0.84	Ali et al,1975
			4.4		7.3	9.4	12.4	0.03	0.36	0.72	
		40	4		7.3	9.6	15.3	0.02	0.30	0.90	
		10	4		7.3	9.1	10.0	0.02	0.20	0.51	
glass strand	9.5	11	6.5	gypsum	8.1	10.9	12.5	0.05	0.40	0.58	Laws and Ali,1977
steel	255	25.4	2.5	C	5.5	7.2	7.3	0.03	0.19		Naaman and Shah,1979
SIFCON	500	50	3.6	C+S+FA+SP							Naaman et al,1990
		30	12	C+FA+SP	10.5		21.0				Naaman and Homrich,1989

C: cement; AD: admixture; SF: silica fume; V: vermiculite; FA: fly ash; SP: superplasizer reflects the (presumably uniform) strain capacity of the composite. Fiber pull-out may follow if rupture of the fiber occurs inside the matrix. For discontinuous fiber systems, the ultimate tensile strength may be associated with fiber pull-out or with fiber rupture (most likely by tensile or bending failure for brittle fibers, in which case it would be an apparent ultimate tensile strength). The 'straining' between end of multiple cracking and reaching ultimate tensile strength is likely to be associated with the opening of a single crack with fiber frictional debonding (Li, 1991). Hence the 'strain' measurement at this stage is expected to be dependent on the gauge length. For structural applications where no visible cracks can be tolerated, such as in leak-proof structures, the first crack strength and strain are the most important tensile properties for design considerations. For those applications where cracks with small opening can be tolerated, multiple cracking may be taken advantage of in load redistribution. This is similar to the strain-hardening behavior of certain metallic material (and hence the title 'pseudo strain-hardening' and sometimes referred to as 'pseudo ductility'). For such applications, the ultimate strength and strain would be the most important tensile properties for design considerations.

There exists several published reports of multiple cracking for both continuous aligned and short random fiber reinforced cementitious composites. Most of the systems studied used one of the following fibers: steel, carbon, polypropylene or glass. Fiber data and reported pseudo strain-hardening properties are summarized in Table 1. Many of the numbers in Table 1 are inferred from published experimental curves and their values are subject to interpretations. Figure 2 plots these data as a function of fiber volume fraction V_f. While these figures show roughly the V_f dependence of these properties, it should be noted that many other factors, such as fiber diameter d_f, aspect ratio L_f/d_f, bond strength τ, elastic modulus E_f, influence the pseudo strain-hardening properties. The intentions of these figures are to give a general impression of the achieved levels of the strain-hardening properties, and to give a feel of the relative performance (for similar volume fractions) between continuous aligned (CA) systems (in solid symbols) and discontinuous random (SR) systems (in open symbols).

For CA systems, all composite strength properties show significant improvements over the matrix strength. The high end has been established by a 12% V_f carbon fiber reinforcement, resulting in a composite with strengths σ_{mu}, σ_{mc}, σ_{cu} of 17, 23 and 57 times respectively that of the matrix material (assumed to have a typical tensile strength of 3 MPa). Strength increase in SR systems are much more modest compared with CA systems. The exceptions are the FRDSP and the SIFCON, with σ_{mu}, σ_{cu} of 5 and 7 times respectively that of the matrix material with 12 % of steel fibers. (FRDSP has a matrix strength of 6 MPa while SIFCON probably has a matrix strength of 3 MPa; σ_{mc} cannot be determined from experimental curves in both cases.)

The story for failure strain development is quite different. Three conclusions may be drawn from figures 2 (d), (e), and (f). 1) Unlike the strength properties, the failure strains between CA and SR systems are similar, particularly for ε_{mc} and ε_{cu}; 2) The first crack strain achieved for SR systems are ε_{mu} ~ 2-5 times, ε_{mc} ~ 10-30 times, ε_{cu} ~ 30-60 times that of the matrix failure strain (typically at 0.03% or less). Thus when multiple cracking is allowed and properly designed for, failure strain of one to two orders of magnitude

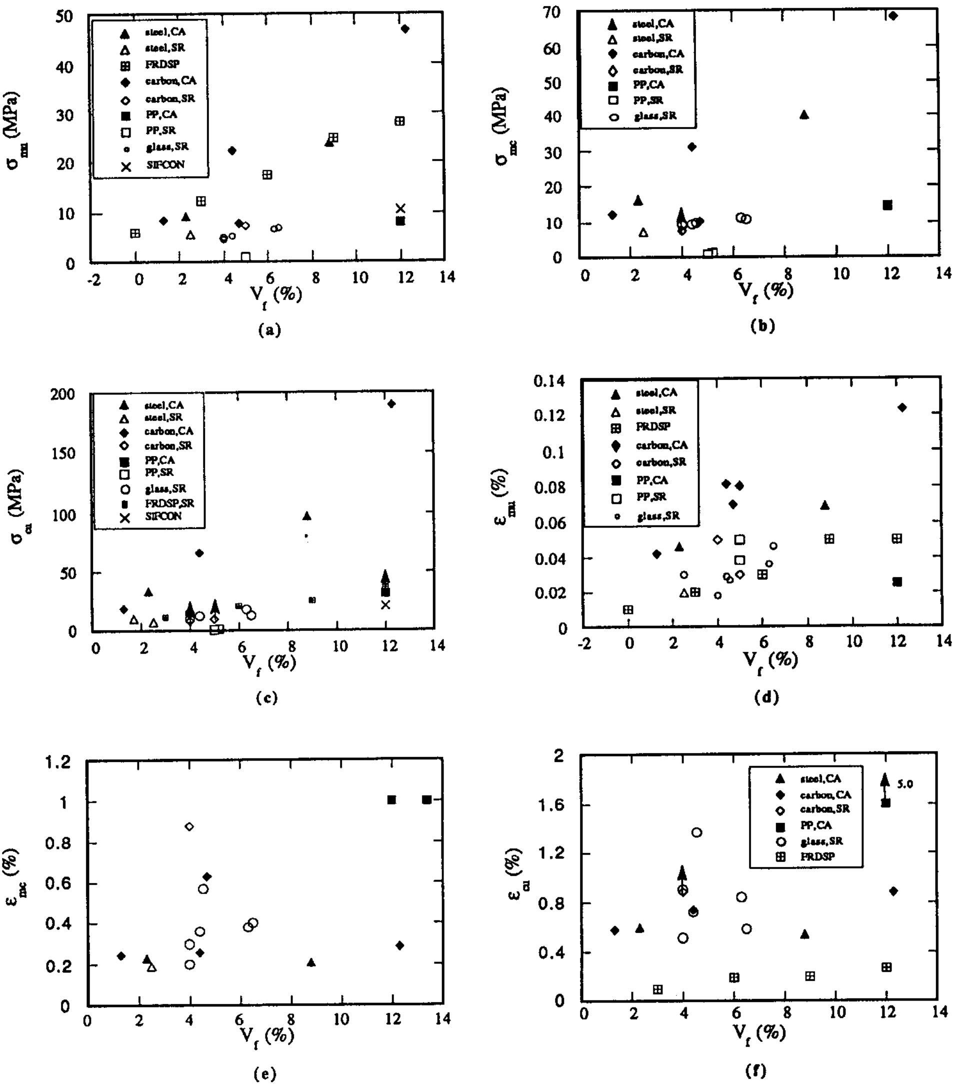

Figure 2. Experimentally determined tensile properties of various fiber reinforced cementitious composites (a) σ_{mu} (b) σ_{mc} (c) σ_{cu} (d) ε_{mu} (e) ε_{mc} (f) ε_{cu}; Solid symbols for continuous aligned fiber systems (CA); Open symbols for short random fiber systems (SR).

improvement over matrix alone is achievable by short random fiber reinforcement. A good reference level would be the yield strain of steel, typically adopted as 0.1%. In SR systems, therefore, strain development can rely on fiber technology. This point is particularly important in view of the fact that high strength binders often have little improvement in strain capacity over low strength binders. It should be noted, however, that some applications may require crack width control rather than high strain capacity alone.

High performance in strength and strain capacity will come from a combination of improved binder technology and fiber technology. The question is, how? More specifically, how does fiber geometric and mechanical properties, matrix properties, and interface properties determine the composite properties? This question is being addressed at the University of Michigan, for the purpose of providing a methodology for high performance engineered cementitious composite design by systematic tailoring of matrix, fiber and interface properties.

3 The U-M Model of Pseudo Strain-Hardening

The U-M Model of Pseudo Strain-Hardening (Li, 1991 ; Li and Leung, 1991, Li and Wu, 1991, Li et al, 1991a,b) is based on the concept of fracture mechanics combined with composite bridging stress crack-opening relationship. This model allows the calculation of all the pseudo strain-hardening properties as a function of fiber, matrix and interface properties. In addition, conditions for occurrence of strain hardening, and fracture energy prediction, are predictable based on the U-M model. Here we compare model predictions of some pseudo strain-hardening properties with experimentally measured values based on the same data set described in the previous paragraphs. For fracture energy prediction and comparison, the reader is referred to Li (1991).

Figure 3 plots the model predicted composite properties against experimentally determined properties. For CA systems (solid symbols), the ACK model is used. For SR systems (open symbols), the U-M model is used. The prediction basis and parametric sets used are described in greater detail in a separate paper (Li and Wu, 1991). The comparisons between theory and measurements in this paper are not meant to verify the theory, but rather to point out deviations from expected behavior. Such deviations often give clues to issues of design of pseudo strain-hardening, to be discussed further in the next section. The error bars for the SR systems indicate uncertain magnitude of a snubbing effect (Li et al, 1990) which is typically not measured nor reported. A typical range of the snubbing coefficient (resulting in an increase of bridging stress by a factor between 1 and 2) has been used. The 45 degree solid lines give the reference exact match between model prediction and measured values. It should be pointed out that many of the parameters required for model prediction are not available, and 'educated guesses' are used in these circumstances. For example, reports of pseudo strain-hardening are often not accompanied by data on fracture toughness of the cementitious matrix. A prediction model for σ_{mc} is presently not available.

For most properties for CA systems, the comparisons reconfirm the validity of the ACK theory. Major discrepancies between predicted and experimentally determined

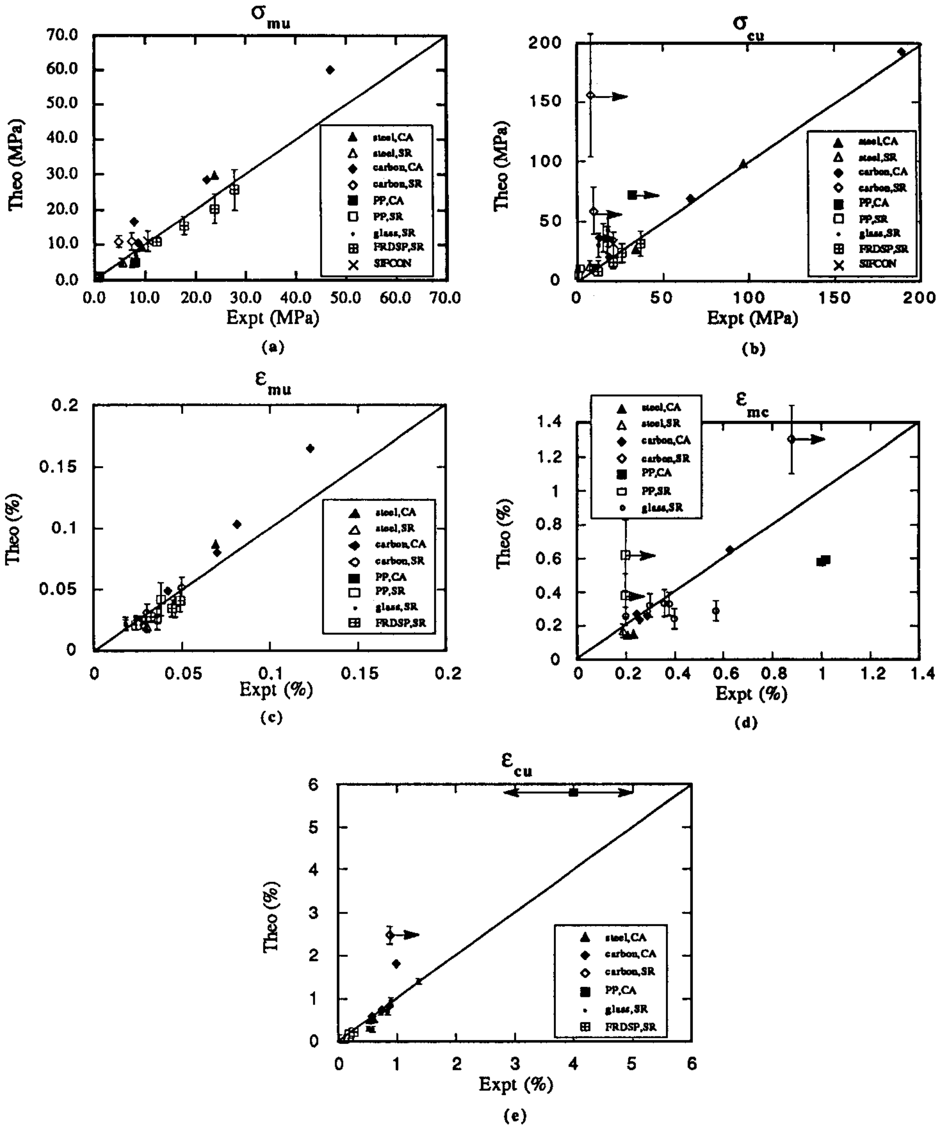

Figure 3. The comparison of tensile properties (a) σ_{mu} (b) σ_{cu} (c) ε_{mu} (d) ε_{mc} (e) ε_{cu} between model predictions and experimental measurements for various cementitious composites.

properties appear for carbon and polypropylene SR systems. In the case of SR polypropylene systems (also for CA systems), composite testing were terminated prior to either fiber pull-out or fiber rupture, and therefore the true property values must be higher than that indicated, and hence the arrows. In the case of carbon fiber systems, it appears that fiber rupture (e.g. reported by Akihama et al, 1986) leads to a limitation on these properties. The arrows on the carbon data points in Figure 3 indicate this effect. In some glass fiber systems, fiber rupture was also reported (e.g. Laws, 1977). Another issue which appears to affect all fiber types in SR systems is the uncertainty in occurrence of multiple cracking. In most published work, no direct observation of multiple cracking is unequivocally provided. In all cases, however, much larger strain, real or apparent, often accompanied by constant or slightly increasing stress is reported. The stress-strain curve therefore give the resemblance of pseudo strain-hardening.

4 Issues Unique to Short Random Systems

Several issues relate especially to SR systems, but not to CA systems. We discuss these issues in light of pseudo strain-hardening behavior. These issues include the effect of fiber orientation with respect to matrix crack and the fiber end debonding under high fiber volume conditions. Other no less important issues related to specific fiber types and will occur in both CA and SR systems include fiber bundling, non-linear elastic behavior, fibrillation, surface abrasion and deformed fiber geometry. These latter problems will not be discussed in this paper.

4.1 Angle Effect

Limited observations of increase in fiber pull-out resistance as a function of inclined angle has been made for steel fibers (Morton and Grove, 1974) and for polymeric fibers (Li et al, 1990). The increase of pull-out resistance, up to a factor of 2, has been ascribed by Li et al to a snubbing friction, where a localized interaction between fiber and matrix occurs at the exit point. Li (1991) shows that the post-cracking strength (if fiber breakage does not occur) can be used to determine the snubbing factor, assuming that the bond strength is known. Snubbing is beneficial if a higher bridging force across a matrix crack plane is desired. Because of the low bond strength of most polymeric fibers, increase in pull-out resistance may be particularly important in generating pseudo strain-hardening in such composites designed for high first cracking strength. Ongoing studies at the University of Michigan examine the factors, including fiber type, w/c ratio and cement additives such as superplasticizer and microsilica, which influences the magnitude of snubbing. It should be noted that for steel fibers, the pulling out of inclined fibers involve the plastic bending of the fibers. However, Morton and Grove (1976) determined that while this effect existed in their fibers, their contribution amounts to less than 10 % of the pull-out force. Also in steel fibers, in contrast to test results of Morton and Groove, Naaman and Shah (1976) reported no simple correlation between pull-out resistance and inclination angle.

Although failure of the cementitious matrix at the fiber exit point is well known, very little study has been done so far of this phenomenon. Matrix failure can occur by spalling, due to the local high bearing pressure of the fiber (Figure 4), as reported by Li et al (1989).

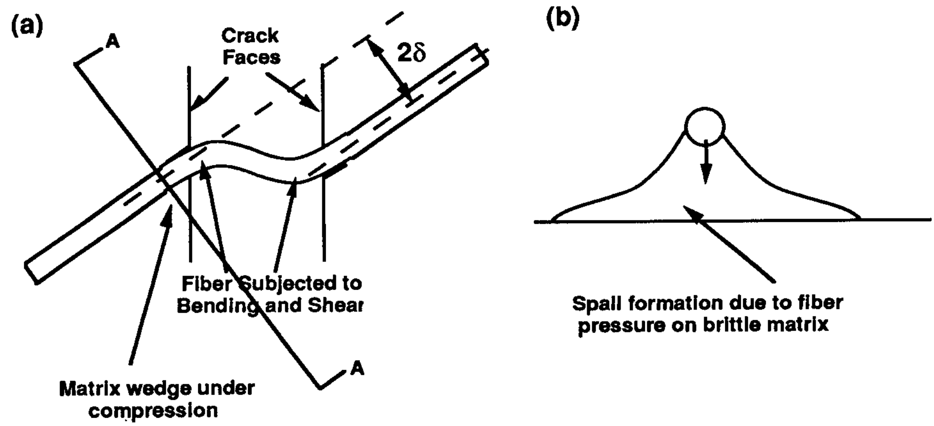

Figure 4. (a) Bending and shearing of a fiber across a crack, (b) Formation of matrix spalling (view of section A-A in (a)).

Larger amount of matrix spalling appears to be associated with higher fiber volume fraction (Wang et al, 1990a,b). Matrix spall failure was studied by Leung and Li (1991) analytically using a beam on foundation model, in combination with a cylinder-tunnel contact model. The foundation stiffness vanishes whenever the matrix strain exceeds its failure strain, thus limiting the amount of bridging stress build-up. Figure 5 shows one such result for a fiber inclined at 30° to the loading direction. As the matrix crack increases, the bridging stress increases to a peak value, then spalling increases and the bridging stress drops. The spall length increases with decreasing matrix failure strain.

Matrix failure can also occur due to a mechanism known as 'plug pull-out', first reported by Naaman and Shah (1976). Although details of this mechanism are as yet unknown, it is plausible that the high fiber-end stress (Leung and Li, 1991b) may initiate matrix cracks (Figure 6a) which join together between adjacent fibers to form the plug. Alternatively, Wang et al (1990b) observed that when the shear bond strength is high, matrix cracks may initiate from fiber or bundle ends to form a conical spall (Figure 6b). These mechanisms are being investigated at the University of Michigan. Matrix failure is expected to allow the fibers to pull-out pre-maturely and hence reduce the bridging stress across the matrix crack. Thus load transfer from the bridging fibers to the cementitious matrix will be limited, and the phenomenon of multiple cracking may be arrested. Indeed, several experimental results involving steel fibers (Naaman and Homrich, 1989) and polymeric fibers (see Figure 7) indicate that after first cracking strength is reached, continued enlargement of a single crack presumably associated with matrix failure occur at more or less constant load. To generate pseudo strain-hardening, it would be critical to design with a matrix strong enough to resist failure due to inclined pull-out of fibers. Unfortunately, a higher quality matrix (e.g. one with high fracture energy) also demands a higher fiber volume fraction to achieve pseudo strain-hardening (Li and Leung, 1991).

Apart from matrix failure, fiber failure can also limit pseudo strain-hardening capacity. Fiber rupture may occur because of fiber length exceeding the critical value, or it may

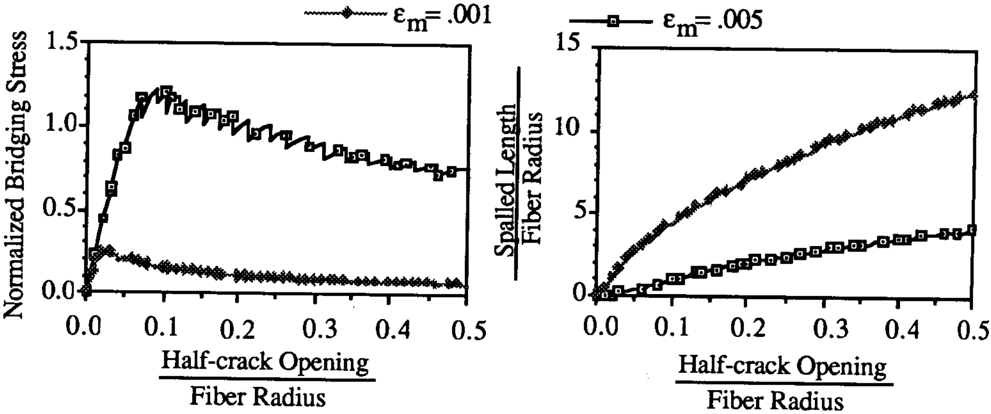

Figure 5. Effect of matrix failure strain on crack bridging by inclined fibers ($\theta = 30^0$, $E_f/E_m=1$).

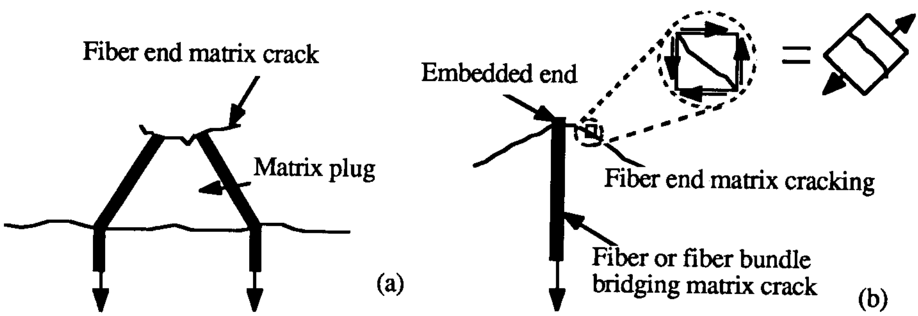

Figure 6. (a) Matrix plug pull-out due to matrix cracks coalescence around adjacent fiber ends; (b) Matrix conical spall due to high stress at fiber ends.

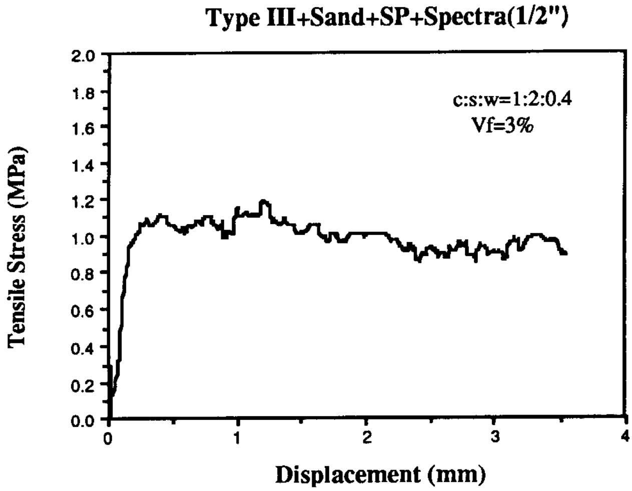

Figure 7. Stress/displacement curve of spectra fiber reinforced cement mortar showing premature fiber pull-out at constant load due to matrix spalling.

happen due to bending failure related to high angle pull out. This is likely to be the case in fibers with relatively low failure strain. There is presently no direct evidence of fiber rupture due to bending, although it may occur in some carbon and glass systems. Leung and Li (1991a) studied fiber rupture due to bending. They found that the combined bridging stress due to interfacial friction and due to bending decreases with inclined angle, as shown in Figure 8. As pointed out in the previous section in relation to carbon fiber cementitious composites experimental data, fiber rupture leads to premature termination of load transfer from fiber to matrix, and therefore limits the pseudo strain-hardening capacity of the composite.

4.2 Fiber End Debonding

It is well-known that the shear stress distribution along the length of an embedded finite length fiber is as shown in Figure 9. In most studies, it is readily assumed that debonding must initiate from the loaded end of the fiber. In reality, however, debonding may initiate from either end and may be joined by another debonding crack from the remaining end. This type of two-way debonding was studied by Leung and Li (1991b). The magnitude of reduction in bridging load due to two-way debonding may be as high as 100 percent and therefore significantly affect the fiber load transfer capacity, again limiting the pseudo strain-hardening properties. Of particular interest here is their finding that high aspect ratio and large fiber volume fraction favors two-way debonding. Figure 10 shows the results of a calculation of the transition from one to two-way debonding, based on parameters typical of steel fiber reinforced cementitious composites. This figure suggests that for SIFCON with fiber volume fraction between 5 to 20 percent, two-way debonding will dominate.

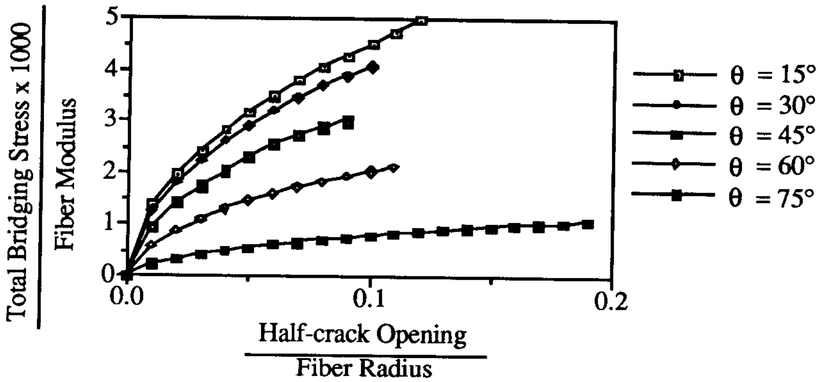

Figure 8. Effects of fiber inclination angle on total crack bridging stress (E_f/E_m =1, σ_m/E_m =0.005, τ/E_f=1/20000).

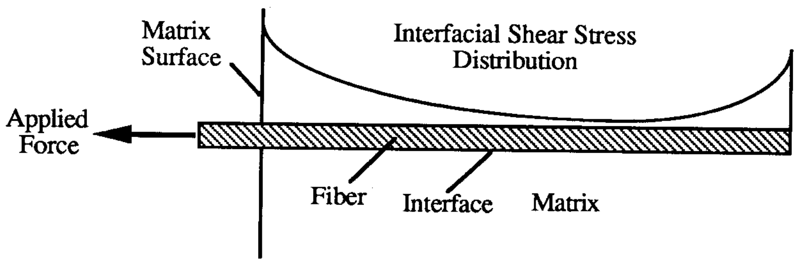

Figure 9. Fiber/matrix interfacial shear stress distribution.

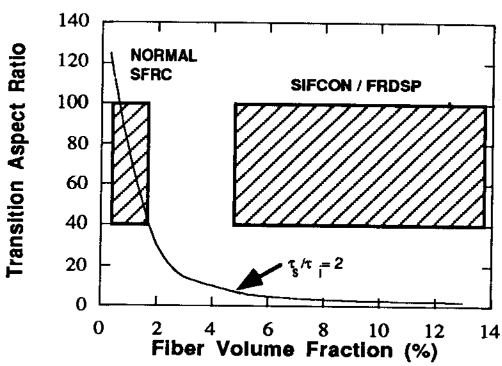

Figure 10. Validity region of one-way or two-way debonding mechanism for steel fiber reinforced cementitious composites.

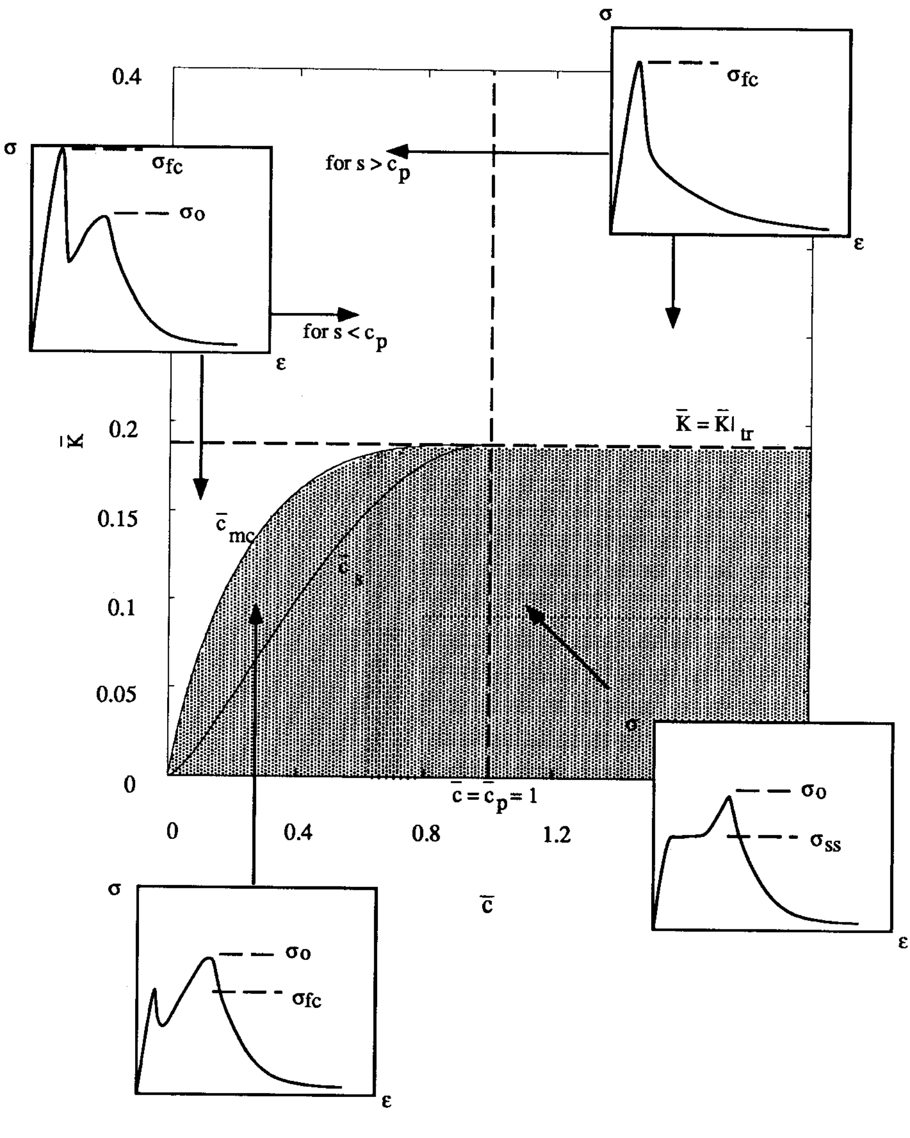

Figure 11. The $\overline{K}$-$\overline{c}$ failure mechanism map.

5 Design Implications

A driving force behind the investigation of the relation between material structure and composite properties is to apply the new understanding to composite design. This is the concept behind the Engineered Cementitious Composites Program at the University of Michigan. The Program attempts to address the following issues: For a given fiber and matrix system, what fiber volume and fiber aspect ratio is necessary to create pseudo strain-hardening? What pseudo strain-hardening properties can be achieved and with what fiber, matrix and interface combinations? For a given required set of properties, what material components should one use to engineer a composite that meets the performance target (i.e. the inverse problem)? We focus on the first question here.

We found (Li and Leung, 1991) that the type of failure in a fiber reinforced cementitious composite may be expressed conveniently in the form of a mechanism map (Figure 11). The desirable design regime for pseudo strain-hardening is shaded. This map indicates that an important criterion for multiple cracking is that the parameter $\overline{K}$ is less than a transition value $\overline{K}\big|_{tr}$ which may be expressed in terms of fiber, matrix and interface properties. Figure 12 shows an interpretation of this criterion in terms of a critical fiber volume fraction. Each line represents a combination of minimum bond strength and fiber aspect ratio that must be met to create multiple cracking, for the indicated critical volume fraction, calculated using parameters suitable for a polyethylene fiber in a normal strength cement matrix system. As expected, a large aspect ratio reduces the need of good bond and requires less fibers to achieve pseudo strain-hardening. On the other hand, we must recognize that high aspect ratio fibers creates more difficulty in a mixing process. The solid line indicates the combination of fiber bond strength and aspect ratio such that fiber rupture will occur. For any given choice of fiber volume fraction, the design point (combination of fiber bond strength and aspect ratio, with all other parameters fixed) must lie above the broken line of the specified volume fraction and below the solid line so as to achieve pseudo strain-hardening but without any fiber rupture.

As an illustration of the composite design process, we took an as-received Spectra fiber which has a bond strength of approximately 1 MPa. For a fiber length of 12.7 mm, the aspect ratio was 334. This design point was located in Figure 12 to be just above the V_f = 3 % broken line. Thus if we use 3 % volume fraction we should be able to achieve pseudo strain-hardening, according to this chart. A preliminary series of uniaxial test was carried out on just such a composite at the University of Michigan. All three specimens of this composite show clear multiple cracking, as expected. An image of multiple cracking from part of a tested specimen is shown in Figure 13. During multiple cracking, it was observed that the load was more or less maintained and may even increase slightly. Further investigations are being conducted.

6 Summary and Conclusion

We discuss in this paper the possibility of achieving pseudo strain-hardening behavior in short random fiber reinforced cementitious composites, and indicate the magnitude of

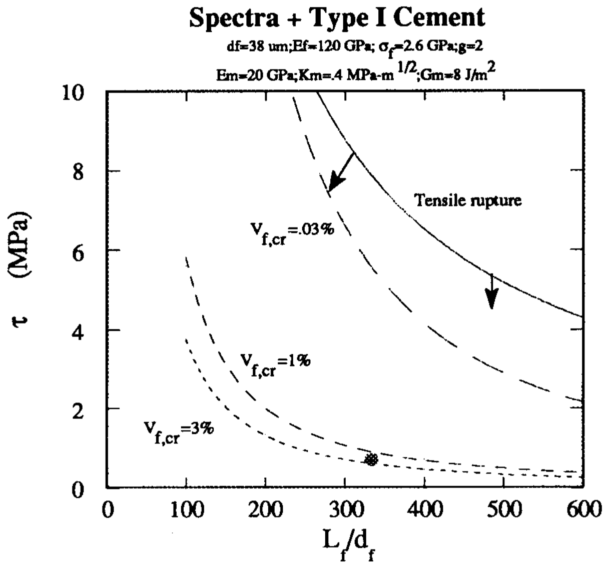

Figure 12. The τ - Lf / df relationship of achieving multiple cracking for various critical fiber volume fraction.

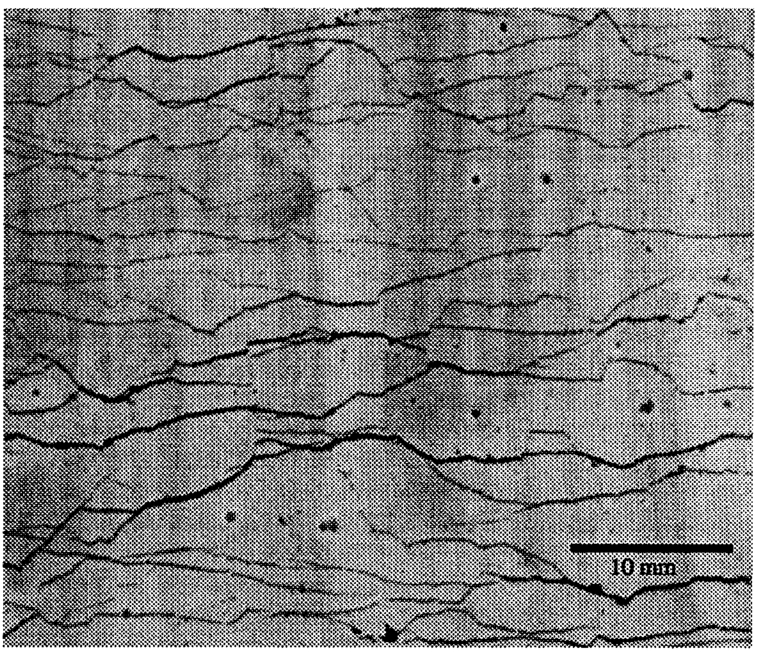

Figure 13. Computer scanned image of a spectra fiber reinforced mortar (Vf = 3%) showing multiple cracking.

pseudo strain-hardening properties that has been achieved so far. We suggest that micromechanically based models may be used to predict these properties in terms of fiber, matrix and interface parameters, and that such models may be used for composite engineering design purposes. However, critical issues faced uniquely by SR systems must be dealt with in order to overcome mechanisms which may limit achievable pseudo strain-hardening. Some directions for addressing these issues are mentioned.

7 Acknowledgements

Research at the ACE-MRL have been supported by grants from NSF (Program manager: Dr. K. Chong) and AFOSR (Program manager: Dr. S. Wu) to the Univ. of Michigan, Ann Arbor. Helpful discussions with T. Hashida and Y.W. Chan are gratefully acknowledged.

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