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FRACTURE CHARACTERIZATION OF RANDOM SHORT FIBER REINFORCED THERMOSET RESIN COMPOSITES

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Abstract—The fracture toughness characterization of random fiber reinforced polymer composites has been investigated by several research groups in recent years. This paper discusses the methods and results of some of these investigations with regard to the applicability of classical linear elastic fracture mechanics approaches to such materials. In polymers randomly reinforced with short fibers (0.5 mm) the region of inelastic behavior ahead of crack tips is sufficiently small that LEFM toughness tests are valid with standard specimen sizes. However, calculations suggest that during fracture of composites with "long" fibers (24 or 50 mm), inelasticities occur to such an extent that the small scale yielding requirements of LEFM are not satisfied. An alternative approach based on the material tension-softening curve may be more appropriate to characterize fracture toughness in fiber reinforced composites.

1. INTRODUCTION

A MAJOR advantage of fiber reinforced composites (FRC) is the enhanced fracture toughness which these materials display. Though the individual constituents of FRC are typically quite brittle in nature, fibre/matrix interface mechanisms and the bridging of fibers across crack faces result in materials with extended load-carrying capabilities even with the presence of material defects and cracks. The application of FRC to technically sophisticated structures which are loaded nearly to design capacity necessitates the ability to properly characterize the fracture toughness of FRC.

Much research has been performed since the early 1970s to determine the applicability of linear elastic fracture mechanics (LEFM) in fracture toughness determination of FRC. Many researchers have noted considerable variations in their experimental results as a function of crack size, specimen loading or statistical variations in the composition of the specimens studied. The goal of the following discussion is to relate the results of independent research with specific emphasis upon correlating fracture characterization of similar types of randomly oriented fiber reinforced resins and plastics. Deficiencies in fracture toughness predictability based on LEFM will be underscored, and a plausible explanation for discrepancies between data offered. An alternative methodology based on non-linear fracture mechanics for the determination of fracture parameters in FRC is proposed.

2. APPLICABILITY OF LEFM

2.1. *Background*

A basic assumption in classical linear elastic fracture mechanics is that the material behaves linear elastically everywhere away from the crack in question, except for a 'small' region of inelastic deformation at the crack tip. The extent of this region may be addressed in the context of an explicit constitutive relation for the inelastic behavior of the material. In metals, this is usually the transition from elastic to plastic deformation, and may be described by, for example, the Von Mises yield criterion. For FRC, the assumption of plasticity is invalid and a different constitutive relation (an example is described later) is required to describe the inelastic deformation at crack tips.

According to LEFM, if a material is observed to produce a $1/\sqrt{r}$ stress singularity at a notch

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tip then it is proper to describe the weighting of this stress field by the factor K. A solution of the linear equations of elasticity (see, for example ref. [1]) allows K be written in the form

$$K_I = \sigma\sqrt{\pi a}f, \quad (1)$$

where a is the initial crack length, f is a dimensionless function appropriate for the specimen geometry and loading configuration, and the subscript "I" refers to an opening fracture mode where the displacement is normal to the crack faces.

The critical stress intensity factor K_{Ic} may be determined by measuring the magnitude of load which causes failure of a specimen with an initial crack of known length. The critical load value and the crack length may then be substituted into eq. (1), yielding a candidate critical stress intensity factor, K_Q. Another method utilizes the concept of the elastic energy release rate, G. The change in specimen compliance as the crack length increases may be related to G by the following expression[2]:

$$G = \frac{(P^2)\,\mathrm{d}C}{2B\,\mathrm{d}a}, \quad (2)$$

where P is the load, B is the specimen thickness and C is the specimen compliance. The specimen compliance is usually determined from the unloading stiffness at various stages of loading and crack growth. If one assumes that LEFM is applicable to the specimen material and geometry, then a candidate value of stress intensity may be found from the relation[3]

$$K^2 = \frac{GE}{(1-\nu^2)}, \quad (3)$$

where E is the elastic Young's modulus and the term $(1-\nu^2)$ is included to account for the state of plane strain. The critical value of K is then obtained from the asymptotic value of G, which is the critical energy release rate, G_c.

2.2 *Specimen sizing*

With either experimental procedure, the specimen size must be chosen such that the $1/\sqrt{r}$ stress field near the crack tip is actually produced. If a large inelastic zone develops ahead of the crack tip then the conditions of "small scale yielding" (SSY) for LEFM applicability are not met. For materials which deform elastic–plastically such as metals, the criterion for SSY, as recommended by the ASTM testing procedure ASTM E399, is that the smallest specimen dimension, d—either crack length, ligament length or specimen thickness—be greater than 15 times the approximate radius of the plastic zone. This criterion may be expressed in terms of materials properties by the condition[4]

$$d \geqslant \frac{15}{2\pi}\left(\frac{K_Q}{\sigma_y}\right)^2, \quad (4)$$

where σ_y is the yield stress value.

2.3. *Inelastic deformations in FRC*

In materials which deform elastic–plastically, the inelastic region ahead of a crack tip is a three-dimensional, volumetric deformation. Fiber reinforced composites do not, in general, deform in an elastic–plastic manner. Instead, the inelastic deformation in the process zone at the crack tip in an FRC is associated with fiber debonding, pull-out and breakage. Such inelastic deformations in FRC, unlike those in plastic materials, are localized onto a plane directly ahead of the crack tip[5,6]. The extent of this behavior may be exemplified by material specimen load–displacement curves.

Figure 1 shows a notched specimen of sheet molded compound SMC-R50, a material composed of a polyester matrix with random glass fibers 25 mm in length. In the second photograph it is evident that much tearing and breaking of fibers occurred during the fracture process, and the fibers were long enough to cause bridging between the crack faces. A schematic of a typical load–displacement curve for this material is shown in Fig. 2, where the nonlinear and quasi-brittle behavior of the opening mode may be observed. Load–displacement plots for epoxy specimens,

Fig. 1. Fracture of SMC-R50 Fiber Composite[19].

both with and without 50 mm long glass fibers, are reproduced in Fig. 3. In contrast to the brittle epoxy which exhibits a sharp stress drop after reaching peak load, the FRC curves suggest a post-peak softening behavior. This is a strong indication that LEFM may not be valid.

2.4. *Tension-softening curve*

A plausible description of the localized inelastic process in FRC is the tension softening curve [7], which is a constitutive relation between the tensile stress, σ, and the separation distance, δ, in the deformed region at the crack tip referred to as the process zone. A schematic of the crack tip region and an accompanying stress distribution are shown in Fig. 4. The process zone begins at the end of the traction-free crack, where the critical separation distance, δ_c, occurs. The material ahead of the process zone is in a state of stress which rises from the far-field value, at the specimen boundary, to the ultimate tensile strength at the edge of the process zone. Within the process zone the stress decays from a peak value (at the perimeter) equal to the ultimate strength to zero at the crack tip, in a manner dictated by the material's tension softening curve. As the stress ahead of the crack tip exceeds the ultimate strength, new material becomes a part of the process zone. Simultaneously the physical crack length may increase as the fibers at the crack tip are pulled out or broken. The process zone thus grows in size and eventually translates as loading continues.

This description of decreasing traction with increasing material separation at the crack tip is similar in concept to the cohesion model of Barenblatt[8]. Such a model was also applied to concrete and fiber-reinforced concrete by Hillerborg *et al.*[9] and by Li and Liang[7]. Rice[10] and Li[11] have shown that the path-independent J-integral over a contour surrounding the process zone may be reduced to

$$J = \int_0^{\delta} \sigma(\delta) d\delta. \tag{5}$$

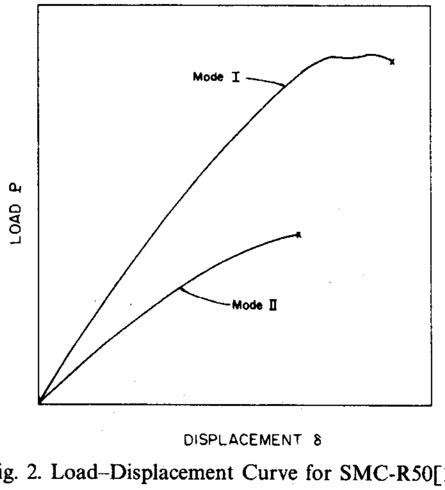

Fig. 2. Load–Displacement Curve for SMC-R50[19].

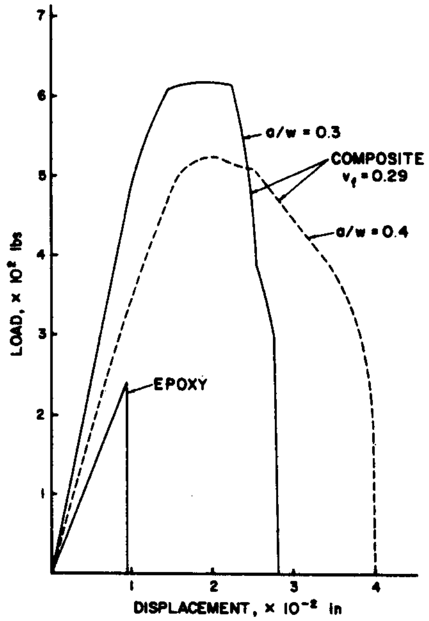

Fig. 3. Load–Displacement Curve for epoxy composites[23].

When the process zone is small in comparison to all planar dimensions of the specimen, they further showed that J coincides with the energy release rate G. In this case the area under the tension-softening curve may be interpreted as the critical energy release rate [12], i.e.

$$G_c = \int_0^{\delta_c} \sigma(\delta) d\delta. \tag{6}$$

2.5. *FRC specimen sizing*

It was noted earlier that in materials which deform plastically, one criterion for the applicability of LEFM as a means of toughness characterization was the satisfaction of a specimen size requirement based on the approximate size of the plastic zone. An analogous criterion for materials with a process zone may be formulated using the tension-softening concept. Assuming a linear decay of stress within the process zone, the steady state process zone size, W, is estimated to be[7]

$$W = \frac{9\pi}{32} \frac{E}{(1-\nu^2)\sigma_t} \frac{l_f}{4}. \tag{7}$$

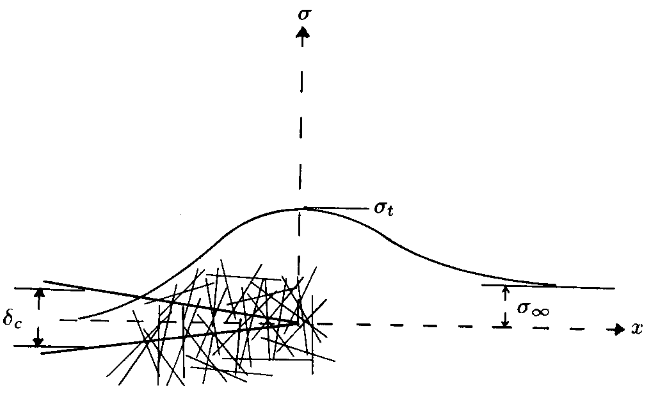

Fig. 4. Schematic of the region near a crack tip, showing the stress distribution in and ahead of the process zone.

In this equation it has been assumed that the critical separation distance, δ_c, in the tension-softening curve is equal to half of the fiber length, l_f, when the fibers are randomly distributed. This assumption has also been utilized by Shah and Wecharatanta[13]. The linear decay of stress within the process zone is approximately consistent with materials which exhibit a linear tension-softening behavior[14] such as that shown in Model 1 (Fig. 5). For materials with extended non-linear tension-softening curves similar to Model 2 (Fig. 6), eq. (7) is inappropriate and underestimates the extent of the process zone. In a manner analogous to plastic SSY criteria (see eq. 4), for LEFM approaches to be valid for the characterization of fracture toughness, the smallest specimen in-plane dimension should be larger than the steady state process zone size, W.

2.6. *Peak load prediction*

For materials which may properly be characterized using LEFM procedures (i.e. brittle materials), peak loading carried by a specimen will correspond to a critical stress intensity factor equal to the fracture toughness. One may therefore obtain the peak load, σ_p, for a given material as a function of K_{Ic} and a_0, the initial crack length, through eq. (1). A relationship may be written between the peak load normalized by the ultimate strength and the crack length, as follows:

$$\frac{\sigma_p}{\sigma_t} = \frac{1}{f}\left(\frac{l_{ch.}}{\pi a_0}\right)^{1/2}, \tag{8}$$

where a material characteristic length, $l_{ch.}$, is defined to be

$$l_{ch.} = EG_c/\sigma_t^2 = (K_{Ic}/\sigma_t)^2. \tag{9}$$

This equation has been plotted in Fig. 7, using an appropriate polynomial expression for the function f derived for single edge-notched (SEN) specimens[15]. It may be seen that f does not differ significantly from unity, which is the value appropriate for a center-cracked panel specimen. Also plotted in Fig. 7 are the predictions of peak load for center-cracked panels with process zone sizes controlled by the tension-softening curves of Models 1 and 2. These curves are determined from the numerical solution of a singular integral equation governing the non-linear fracture process, described in detail in Li and Liang[7]. Data from toughness tests performed on specimens which satisfy the LEFM requirements should, when plotted in non-dimensional form on Fig. 7, fall on or near the solid line derived from LEFM assumptions. Data which plot below the LEFM line indicate that LEFM characterization procedures for the specimen size and material type may be invalid; either a much larger specimen size or a non-linear approach based on the tension-softening concept is required for these materials.

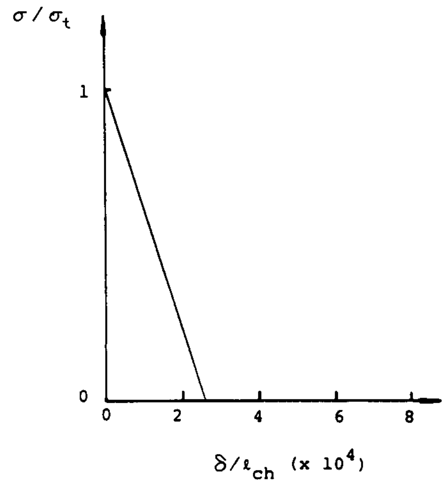

Fig. 5. Tension-softening curve with linear behavior, Model 1[7].

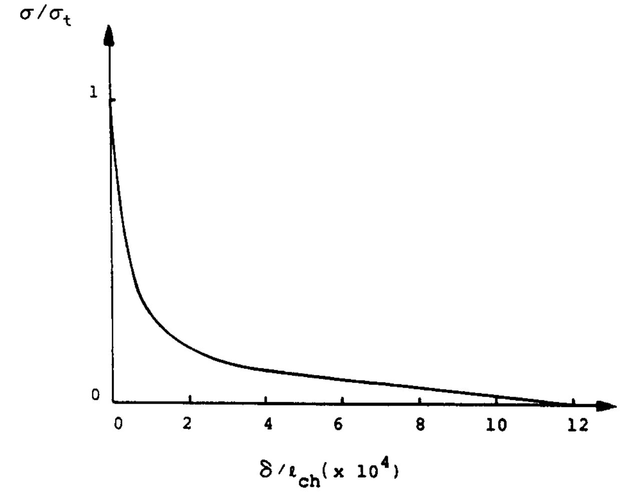

Fig. 6. Tension-softening curve with non-linear behavior, Model 2[7].

3. EXPERIMENTAL INVESTIGATIONS OF LEFM APPLICABILITY

3.1. *Introduction*

Common approaches to determining the validity of LEFM in FRC simply measure the observed K_Q for a variety of testing configurations for each material specimen. If little variation with specimen size, crack aspect ratio a/w, or method of loading is observed then it is assumed that LEFM is a legitimate approach to characterizing toughness; the measured K_Q is then considered to be a material property, K_c. Much experimental work has been performed during the past decade investigating the applicability of LEFM to fiber reinforced composites. The focus of this paper is on polymers reinforced with randomly oriented fibers. The results of several groups of researchers are discussed in this section with particular emphasis placed on the question of the validity of LEFM based fracture toughness testing for the materials and specimen sizes used. Material properties, specimen parameters and experimental results for each of the groups are tabulated in Table 1. Data are reproduced in non-dimensional form on the plot in Fig. 7, which provides additional insight as to whether the materials and testing procedures fall within the realm of a linear or a non-linear analysis.

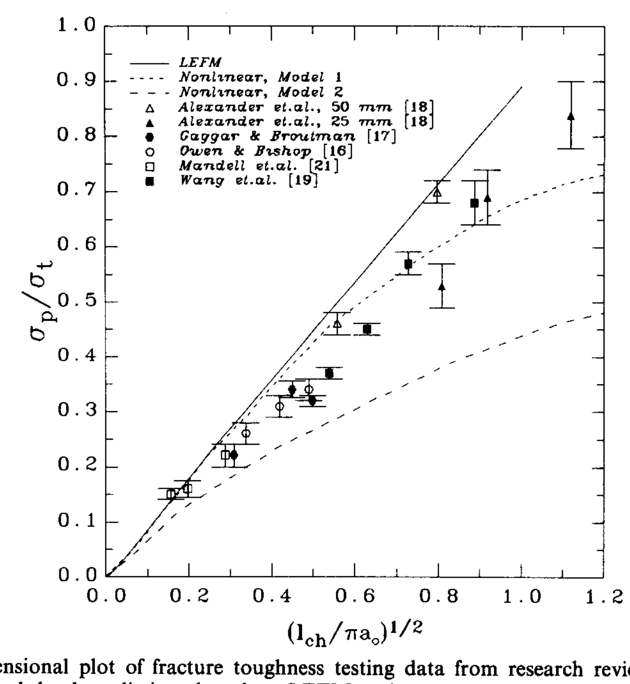

Fig. 7. Non-dimensional plot of fracture toughness testing data from research reviewed in this paper superimposed on peak load predictions based on LEFM and on a non-linear analysis by Li and Liang[7].

Table 1. Comparison of fracture specimen material parameters

Reference	[16]	[19]	[18]	[17]	[21]
Matrix	polyester	polyester	polyester	epoxy	polyphenylene
Fibers	E-glass	glass	glass	glass	glass
Composite σ_t(MPa)	137	130	130	188	181
Modulus, E (GPa)	9.9	13.3	13.3	12	11
Poisson ratio, ν	0.3	0.32	0.32	0.3	0.3
Fiber length, l_f (mm)	50	24	24	50	0.5
Ligament length (mm)	50–100	25–40	35–46	15–21	6–30
Crack length, a_0 (mm)	12–25	10–25	2–25	4–10	4–20
Process zone, W (mm)	860	600	600	900	6
Exp. K_{Ic} (MNm$^{-3/2}$)	13	20.7	16.5	10.6	6.6
Exp. K_{Ic} variation (%)	10–15	5–10	5–10	10–15	10–30
$l_{ch} = (K_{Ic}/\sigma_t)^2$ (mm)	9	25	16	3.2	1.3
Range of $(l_{ch}/\pi a_0)^{1/2}$	0.3–0.5	0.5–0.9	0.6–1.1	0.3–0.5	0.1–0.3

3.2. *Owen and Bishop* [16]

A logical prediction is that the applicability of LEFM to fiber reinforced composites may well be dependent upon the length and orientation of fibers. To test this hypothesis, Owen and Bishop measured K_I using double edge-notched tensile specimens for several fiber orientations in glass FRC[16]. Candidate stress intensity factors were calculated from peak load measurements using an equation derived for the specimen geometry based on the assumption that a linear elastic analysis is valid. The only material which exhibited a constant K_Q, for various crack lengths and specimen widths, was one which had aligned fibers and was tested with a crack propagating parallel to the fibers. This result is not surprising because the test simply measured the toughness of the homogeneous polyester matrix itself, which was quite brittle and most likely satisfied LEFM criteria. Materials with randomly oriented fibers and those with varying degrees of anisotropy produced K_Q values which increased with increasing specimen width. (A portion of Owen and Bishop's data is reproduced here in Fig. 8.) The authors modified the stress intensity factor calculations by using crack length corrections based on effective yield stresses to produce invariant values of K_{Ic} for each material.

Parameters for Owen and Bishop's specimens with randomly oriented fibers (chopped-strand mat) are listed in Table 1. We have computed (using eq. 7) the steady state process zone size for this material to be on the order of 860 mm, which far exceeds the specimen crack and ligament lengths of 12–25 mm and 50–100 mm. Therefore, we would expect the presence of the process zone to have a significant influence on the apparent toughness of the material. When the experimental data from Owen and Bishop is plotted on Fig. 7, it is evident that a strict LEFM characterization of this material is inappropriate.

3.3. *Gaggar and Broutman* [17]

Studying epoxy resins reinforced with randomly oriented glass fibers, Gaggar and Broutman used both single and double edge notched tensile tests and notched bend tests to measure critical

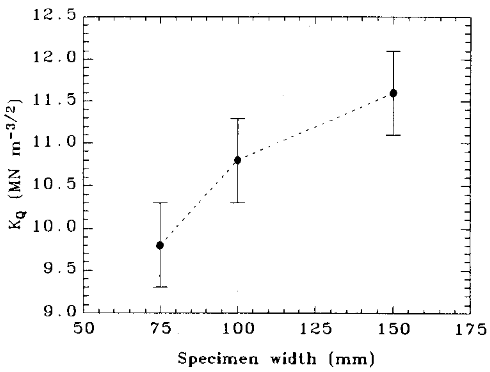

Fig. 8. Data showing K_Q increasing with width for a random, short glass fiber, polyester composite[16].

stress intensity factors[17]. Both compliance (eq. 2) and peak load measurements were used with LEFM assumptions to determine critical stress intensity factors. The SEN tests consistently produced higher K_Q values, and an increase in K_Q was observed as the crack length was increased in all of the tests. (A portion of Gaggar and Broutman's data is reproduced here in Fig. 9.) Based on the notion that fiber debonding is the principal energy absorber in random fiber composites and incorporating the observation of debonding at about 65% of the failure stress, they calculated an effective K_D associated with debonding. This parameter proved to be entirely independent of specimen geometry and was suggested for use as a design criterion since the onset of brittle fracture is very rapid subsequent to the initiation of debonding.

Table 1 indicates the process zone size for Gaggar and Broutman's specimens to be about 900 mm, which again is far greater than the specimens crack and ligament lengths; this implies that SSY requirements for LEFM were not met by the specimens used. The observed increase in toughness with increased crack length is consistent with the process zone size being larger than the specimen dimensions. The higher K_Q values produced by the SEN tests is due to the fact that more extensive crack tip damage was permitted with this method. The data as plotted on Fig. 7 indicate that the material exhibits nonlinear behavior and would be better characterized by a nonlinear analysis than by LEFM.

3.4. *Alexander* et al. [18]

Alexander *et al.* measured fracture toughness of sheet moulded compound (SMC-50, polyester resin with randomly oriented 25 mm long E-glass fibers) as a function of crack length and specimen width using DEN tensile tests[18]. The researchers used peak load measurements to determine candidate stress intensity factors. They reported considerable scatter in data without dependency upon crack length or loading rate. The average K_Q was reported to be 14.6 MPa $\sqrt{m}$ for the 25 mm wide specimens and 16.5 MPa $\sqrt{m}$ for the 50 mm wide specimens. It was noted that the smaller specimens produced much scatter in data, which the authors felt was due to randomness in flaw size in the material tested. The 50 mm wide specimens exhibited K_Q values which were relatively constant; variations were judged by the investigators to be caused by experimental variations. Because the 50 mm wide specimens appeared to have minimized the size effect on toughness characterization, the authors suggested that LEFM approaches might be applicable and they planned to test still larger specimens.

The steady state process zone size for SMC-R50 may be read from Table 1 to be about 600 mm. Because the process zone size is far larger than any of the specimen dimensions, it could be expected that, as Alexander *et al.* observed, the measured toughness would increase as the specimen size was increased from 25 to 50 mm in width. We would expect that, to measure a stress intensity factor which represented a true material property, a specimen of width greater than 600 mm would have to be used. Fig. 7 indicates that Alexander's smaller specimens produced data requiring a nonlinear analysis, while the larger specimens approached the region of appropriate LEFM procedures.

3.5. *Wang* et al. [19]

Other investigations regarding the toughness of sheet molded compound have been performed

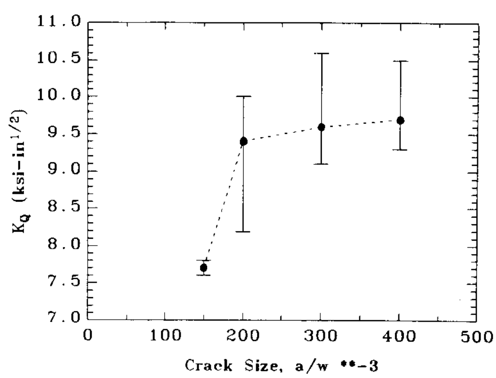

Fig. 9. Data showing increasing K_Q with crack length for a random, short glass fiber, epoxy composite[17].

due to its growing use in the automotive and aircraft industries. Using SMC-R50 as the composite material, Wang *et al.* performed toughness tests using single edge-notched specimens for mode I fracture and double edge-notched specimens for mode II fracture[19]. To investigate the size effect upon the validity of an LEFM analysis the researchers held the specimen width, w, constant at 50 mm while varying the relative crack size, a/w. Stress intensity factors were calculated from eq. (1) using a function f given by Pook for the specimen geometry used by Wang *et al.*[20] They concluded that fracture toughness, K_Q, could be used to characterize the SMC random composite in both mode I and II fracture modes because "the value of K_Q is relatively independent of crack length." An average value of K_{Ic} equal to 20.7 MPa $\sqrt{m}$ was reported for SMC-R50.

We note again that the process zone size, W for SMC-R50 is on the order of 600 mm and argue that to obtain a true toughness parameter using a linear elastic analysis, testing specimens used should have all in-plane dimensions greater than 600 mm. The impact on K_Q of the extent to which the specimen contains the process zone is shown by the increase in K_Q from the test results of Alexander (16.5 MPa $\sqrt{m}$) to those of Wang *et al.* (20.7 MPa $\sqrt{m}$) for the same material (glass fiber, polyester matrix). Though Wang *et al.* reported fairly constant K_Q data and suggested that an LEFM approach for characterization of Mode I toughness is suitable for SMC-R50, Fig. 7 indicates that a nonlinear analysis would be more appropriate.

3.6 *Mandell* et al.[21]

Researchers have also investigated the toughness of polymers reinforced with randomly oriented very short fibers, as used in typical injection-molding manufacturing processes. Mandell, Darwish and McGarry have performed tests on nylon, polycarbonate and polyphenylene matrices reinforced with fibers of glass and carbon[21]. The composites had fibers of length equal to or less than 0.5 mm; other parameters are listed in Table 1. Using SEN specimens they held the ratio of crack length to specimen width, a/w constant at 0.40 and varied the width from 0.6 cm to 4.8 cm to investigate the size effect on apparent toughness. Stress intensity factors were calculated using LEFM assumptions and with a geometry factor, f, appropriate for their specimens[22]. They reported an increase in K_Q for all of the composites when the specimen width is increased from 0.6 to 1.2 cm. Further increasing the width had little effect on observed toughness; therefore Mandell *et al.* suggested that small scale yielding requirements were satisfied for specimen widths greater than 1.2 cm and linear elastic approaches were valid for the composites which they tested containing very short fibers. They further concluded that the toughness of short-fiber random reinforced polymers may be characterized in the following manner[21]:

$$K_Q = (\sigma_t)\,(2\pi l_f^*)^{1/2}, \tag{10}$$

where l_f^* is the length of the longest fibers and σ_t is the ultimate tensile strength of the composite material.

We shall discuss here just one of Mandell's composite materials, which yielded data exhibiting trends similar to their other composites, and which is most similar to the other composite materials investigated by other researchers (the most notable difference being the short fibers of Mandell *et al.*). Table 1 indicates that we have calculated the steady process zone size to be about 6 mm in length. Consequently, with the exception of the case with a 4 mm initial crack length, the process zone length was smaller than all in-plane specimen dimensions indicating that an LEFM analysis could be fully valid. Figure 7 shows two data points from Mandell's research which fall nearly onto the line corresponding to LEFM behavior; the third data point which lies in the nonlinear region resulted from the specimen with the 4 mm crack length. Thus we would agree that LEFM approaches may be fully valid for the short-fiber randomly reinforced composites.

In the case when the process zone is small, one may show that eq. (10) is consistent with the tension-softening concept. Assuming a linear tension-softening relationship and taking the critical separation distance, δ_c, as half of the fiber length, l_f, we may approximate that $G_c = 1/4\sigma_t l_f$. From eq. (3) we may also write that $G = K^2(1 - \nu^2)/E$. If we further assume the ultimate tensile strength, σ_t, to be proportional to the elastic modulus ($\sigma_t = C_1 E$), we may hypothetically write

$$K_c = \sigma_t \sqrt{\frac{1}{8\pi C_1(1 - \nu^2)}}\,(2\pi l_f)^{1/2} = C_2\sigma_t(2\pi l_f)^{1/2} \tag{11}$$

where C_1 and C_2 are proportionality constants.

Thus, fracture toughness prediction using an assumed linear tension-softening curve qualitatively agrees with eq. (10).

Alternatively we may assume the expression given by Mandell *et al* (eq. 10) to be correct ($C_2 = 1$ in eq. 11) and gain some insight into the tension-softening behavior of one of their specimens. From the data in Table 1 we find that $\sigma_t \simeq 0.016E$. Taking the area beneath the tension-softening curve as $G_c = C\sigma_t l_f/2$ and combining with eq. (3) yields a value $C \simeq 0.23$. A material with linear tension-softening would have have a value of $C = 0.5$; hence the tension-softening curve for the specimens of Mandell *et al.* probably falls below a straight line between $\sigma = \sigma_t$ and $\delta = l_f$.

4. CONCLUSION

The fracture toughness characterization of random fiber reinforced polymer composites has been investigated by several research groups in recent years. The methods and results of some of these investigations were discussed in this paper with regard to the applicability of classical linear elastic fracture mechanics approaches to such materials. Our calculations suggest that in composites with "long" fibers (24 or 50 mm), inelastic behavior ahead of crack tips in the "process zone" occur to such an extent that the small scale yielding requirements of LEFM are not satisfied. The steady state process zone in long-fiber composites may exceed a meter in length, thus necessitating enormously large fracture specimens to satisfy the "small scale yielding" requirements in LEFM procedures. In polymers randomly reinforced with short fibers (0.5 mm) the process zone is sufficiently small that LEFM toughness tests are valid with standard specimen sizes (e.g. compact tensile specimens).

An alternative approach to toughness characterization based on the material tension-softening curve has been suggested for use with fiber reinforced composites. Once the tension-softening curve has been obtained, the area under this curve yields the critical energy release rate (eq. 6). The most direct manner of measuring the tension-softening curve is by means of an uniaxial tensile specimen instrumented with strain gages on the specimen surface. In practice this experiment has proven difficult to carry out, particularly because of the post-peak instability associated with the tension-softening process. A stable experimental technique, based on the J-integral and eq. (5) has been proposed by Li *et al.*[24] for extracting fracture parameters from quasi-brittle materials which exhibit the tension-softening behavior. Because the functional behavior of the tension-softening curve is a material property, toughness testing based on this relation will be independent of specimen geometry, size and loading configurations.

REFERENCES

[1] A. P. Parker, *The Mechanics of Fracture and Fatigue, An Introduction*. E & F. N. Spon, London (1981).

[2] D. Broek, *Elementary Engineering Fracture Mechanics*. p. 116, Martinus Nijhoff, Netherlands (1982).

[3] D. Broek, *Elementary Engineering Fracture Mechanics*. p. 17, Martinus Nijhoff, Netherlands (1982).

[4] E-399 Plane Strain Fracture Toughness of Metallic Materials. *Annual Book of ASTM Standards*. ASTM, Philadelphia (1981).

[5] B. Z. Jang and Y. K. Lieu, Fracture behavior of short fiber reinforced thermoplastics I. Crack propagation mode and fracture toughness. *Appl. Polym. Sci.* **30,** 3925–3942 (1985).

[6] J. F. Mandell, D. D. Huang and F. J. McGarry, Crack propagation modes in injection molded fiber reinforced thermoplastics. *Short Fiber Reinforced Composite Materials, ASTM STP* **772** 3–32 (1982).

[7] V. C. Li and E. Liang, Fracture processes in concrete and fiber reinforced cementitious composites. *J. Engng Mech.* **122,** 566–586 (1986).

[8] G. I. Barenblatt, The mathematical theory of equilibrium cracks in brittle fracture. *Advances in Applied Mechanics*. Vol. 7, pp. 55–125. Academic Press, London (1962).

[9] A. Hillerborg, M. Modeer and P. E. Peterson, Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. *Cem. Concr. Res.* **6,** 773–782, (1976).

[10] J. R. Rice, Mathematical analysis in the mechanics of fracture, in *Fracture: An Advanced Treatise*. Vol. 2, pp. 191–311. Academic Press, (1968).

[11] V. C. Li, Mechanics of shear rupture applied to earthquake zones, in *Rock Fracture Mechanics* (Edited by B. Atkinson). Academic Press, London (1986).

[12] J. R. Rice, A path independent integral and the approximate analysis of strain concentrations by notches and cracks. *J. Appl. Mech.* **35,** 379–386 (1968).

[13] M. Wecharatana and S. P. Shah, A model for predicting fracture resistance of fiber reinforced concrete. *Cem. Concr. Res.* **13,** 819–829 (1983).

[14] A. C. Palmer and J. R. Rice, The growth of slip surfaces in the progressive failure of over-consolidated clay. *Proc. R. Soc.* **A.332,** 527–548 (1973).

[15] D. Broek, *Elementary Engineering Fracture Mechanics*, p. 76. Martinus Nijhoff, Netherlands (1982).
[16] M. J. Owen and P. T. Bishop, Critical stress intensity factors applied to glass reinforced polyester resins. *J. compos. Mater.* **7,** 146–159 (1973).
[17] S. Gaggar and L. J. Broutman, Fracture toughness of random glass fiber epoxy composites: an experimental investigation. *Flaw growth and fracture, ASTM STP* **631,** 310–330 (1977).
[18] R. M. Alexander, R. A. Schapery, K. L. Jerina and B. A. Sanders, Fracture characterization of a random fiber composite material. *Short fiber reinforced composite materials, ASTM STP* **772,** 208–224 (1982).
[19] S. S. Wang, E. S.-M. Chim, T. P. Yu and D. P. Goetz, Fracture of random short-fiber SMC composite. *J. compos. Mater.* **17,** 299–315 (1983).
[20] L. P. Pook, The effect of friction on pin jointed single edge notched fracture toughness test specimens. *Int. J. Fracture* **4,** 295–297 (1968).
[21] J. F. Mandell, A. Y. Darwish and F. J. McGarry, Testing of injection-molded glass and carbon fiber-reinforced thermoplastics. *Test methods and design allowables for fibrous composites, ASTM STP* **734,** 73–90 (1981).
[22] P. C. Paris and G. C. Sih, Stress analysis of cracks. *Fracture toughness testing and applications, ASTM STP* **381,** 30 (1965).
[23] S. Gaggar and L. J. Broutman, Crack growth resistance of random fiber composites. *J. compos. Mater.* **9,** 218 (1975).
[24] V. C. Li, C. M. Chan and C. Leung, Determination of the tension-softening curve in comentitious composites. *Cem. Concr. Res.* (accepted for publication).