Introduction

In recent years, advanced and well-developed numerical models for the analysis of cracked structures have been introduced to model dynamic cracks in concrete. These models are based on the assumption that the crack propagation is a function of the stress intensity factor and the crack opening displacement. The models are typically used to analyze the behavior of cracked structures under dynamic loading conditions.

The propagation of dynamic cracks in concrete is a complex phenomenon that involves the interaction of various factors, including the material properties, the geometry of the crack, and the applied load. The models used to analyze these cracks are based on the principles of fracture mechanics and are designed to predict the behavior of the crack under dynamic loading conditions.

In this paper, the application of a discrete element method is proposed for the analysis of dynamic cracks in concrete. The method is based on the principle of energy conservation and is used to predict the propagation of cracks under dynamic loading conditions.

Conclusions

The proposed method is shown to be effective in predicting the behavior of dynamic cracks in concrete under various loading conditions. The results obtained using the method are in agreement with experimental data, indicating the validity of the approach.

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References

Hybrid Method

An efficient approach to solving the crack problem with regards to the crack size and orientation is the hybrid method. This method combines the advantages of the finite element method and the boundary element method. It is particularly useful when dealing with complex crack geometries or problems where the stress distribution is not uniform.

![Diagram of Hybrid Method](image)

Figure 1. Numerical Models for Fraction Transfer in Cracks: (a) Model Proposed by Hilfer and Moodie; (b) Model Proposed by Kestin and Lax; (c) Model Proposed by Hilfer and Moodie and Kestin and Lax; (d) Model Proposed by Kestin and Lax; (e) Model Proposed by Kestin and Lax and Moodie; (f) Model Proposed by Kestin and Lax and Moodie and Hilfer; (g) Model Proposed by Kestin and Lax and Moodie and Hilfer and Moodie; (h) Model Proposed by Kestin and Lax and Moodie and Hilfer and Moodie and Kestin and Lax.

Many researchers have investigated the applicability of the hybrid method in concrete.
\[ \mathbf{U} = \mathbf{N} ( \mathbf{L} - \mathbf{C} \mathbf{P} ) + \mathbf{C} \mathbf{P} \]

The different matrices for the crack are set up by using Cauchy stress:

\[ \mathbf{S} = \mathbf{C} \mathbf{F} \]

The global generalized stress matrix can be written as

\[ \mathbf{S} = \mathbf{S} + \mathbf{C} \mathbf{F} \]

where

\[ \mathbf{S} = \mathbf{S} \mathbf{C} \mathbf{F} \]

and

\[ \mathbf{F} = \mathbf{F} \mathbf{C} \mathbf{F} \]

The total applied load vector \( \mathbf{P} \) is

\[ \mathbf{P} = \mathbf{F} \mathbf{C} \mathbf{F} \]

Thus, the reactions due to the crack are

\[ \mathbf{R} = \mathbf{F} \mathbf{C} \mathbf{F} \]

Similarly, the reactions along the crack will be

\[ \mathbf{R} = \mathbf{F} \mathbf{C} \mathbf{F} \]

These reactions are in terms of the crack density amplitudes, which act on the infinite body. The corresponding influence functions are

\[ \mathbf{W} = \mathbf{W} \mathbf{C} \mathbf{F} \]

The total crack due to the crack are

\[ \mathbf{U} = \mathbf{F} \mathbf{C} \mathbf{F} \]

Consider also a body without a crack which is represented by three

\[ \mathbf{U} = \mathbf{F} \mathbf{C} \mathbf{F} \]

The displacements at the nodes in terms of the crack density

\[ \mathbf{U} = \mathbf{F} \mathbf{C} \mathbf{F} \]

The displacement of the nodes is represented by

\[ \mathbf{U} = \mathbf{F} \mathbf{C} \mathbf{F} \]

The stress function is a function of the cracks and the size of the body.

\[ \mathbf{S} = \mathbf{S} \mathbf{C} \mathbf{F} \]

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\[ \mathbf{S} = \mathbf{S} \mathbf{C} \mathbf{F} \]
Fig. 3—Check formation from pressurization configuration.

Infinite Body

Crack

\[ \text{Finite body} \]

\[ \begin{align*}
\text{(a)} & \quad T = T' + T'' \\
\text{(b)} & \quad T = T' + T'' = 0 \\
\text{(c)} & \quad T = T' + T'' = 0
\end{align*} \]

The crack surface traction can then be written in terms of the crack.

\[ \text{Displacement discontinuity multiple:} \]

\[ u = \frac{D^2 T^2}{\pi} \]

\[ \text{Directional derivative across the plane of the crack:} \]

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\text{(c)} & \quad T = T' + T'' = 0
\end{align*} \]

\[ \text{The surface displacement functions are then given by the potential functions of the crack.} \]

\[ \text{The strain energy, which expresses the stress energy stored in the crack, is given by the potential functions of the crack.} \]

\[ \text{Another consideration is the use of these functions. The particular coordinates, point, and of the crack, will be determined by the particular eigenfunctions and eigenvalues, which are then differential.} \]

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\[ \text{Although the constitutive relations for the two components of the crack, the stress energy stored in the crack, is given by the potential functions of the crack.} \]
The behavior of concrete changes significantly once cracks form a network. A tensile stress is defined as the stress that causes the material to elongate. The majority of observations determined the tensile strength of concrete by taking the square root of the mean stress in tension. The most common method is to express the stress intensity factors in terms of the square root of the mean stress. This method can be used to determine the crack initiation and propagation. The stress intensity factors can be expressed as:

\[
\Delta K = \sqrt{\frac{\pi a}{2}} \left( \frac{C}{\Delta P} \right)
\]

where \( \Delta K \) is the stress intensity factor, \( C \) is a material constant, and \( \Delta P \) is the load change.

The fracture transfer across cracks in concrete can be expressed as:

\[
\frac{\Delta P}{\phi} = (\sigma)^\Phi = C
\]

where \( \Delta P/\phi \) is the effective tangential stiffness of the crack, \( \sigma \) is the stress, and \( \Phi \) is a material constant.

The stress transfer across cracks can be expressed as:

\[
\phi = \frac{\Delta P}{C}
\]

This expression is used to determine the effective stress transfer across cracks.

The final equation can be used to determine the effective stress transfer across cracks:

\[
\phi = \frac{\Delta P}{C}
\]

where \( \phi \) is the effective stress transfer across cracks, \( \Delta P \) is the load change, and \( C \) is a material constant.
Experimental Results of Tensions in Cracks under Tensile Test (22)
be expected to remain small along the path of the yield of the specimen (peak detected substantially, and K is expected to remain small along the path of the yield.) Since the shear stresses vary in the original stress field, it can be expected that any further expansion will proceed as shown in Figure 4.2.5, with the yield stress from the peak point of the crack. The path of the crack in the plane of the crack. A short additional crack length (short) was added at the end of the crack, prior to the final crack spread. The crack path was determined by initially specifying the saw cut notch.

The path method described before was incorporated into the finite element analysis code.

**Simulation of Experimental Results**

The crack under plane strain conditions. Crack growth is modeled using a characteristic length, which describes the basic characteristics of the crack. The experimental results qualitatively match the predictions.

The crack path is determined by specifying the saw cut notch. The path method described before was incorporated into the finite element analysis code.

**Constitutive Relations for Crack—The Expression of the Proposed Model**

The constitutive relations for the crack between stress and strain. The proposed relation between stress and strain is described by the following equation:

\[ \sigma = E \varepsilon + \tau \]

where \( \sigma \) is the stress, \( E \) is the Young's modulus, \( \varepsilon \) is the strain, and \( \tau \) is the shear stress. The proposed relation between stress and strain is expressed by the following equation:

\[ \tau = G \varepsilon_n \]

where \( G \) is the shear modulus, and \( \varepsilon_n \) is the normal strain.
Conclusions

A method is proposed for the analysis of discrete cracks in concrete and steel. The method is based on the approximation of the crack path by a series of discrete points, each representing a crack tip. The crack path is determined by solving equations that relate the crack tip stresses to the applied loads. The method is verified by comparing the results with experimental data.

References


Fig. 10—Calculated Applied Loads

Fig. 9—Calculated Crack Path

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The loads corresponding to each crack length were obtained by calculating the modulus of elasticity of 3,000 ksi (21 MPa) and the initial modulus of elasticity of 3,000 ksi (21 MPa). The applied loads were determined by solving equations that relate the crack tip stresses to the applied loads.
Appendix II — Nomenclature

Superscripts

\( ^{s} \) = surface integral
\( ^{f} \) = fringe element
\( ^{g} \) = ground effect

Subscripts

\( i \) = internal
\( j \) = external
\( n \) = normal
\( t \) = tangential
\( e \) = effective

The following symbols are used in this paper: