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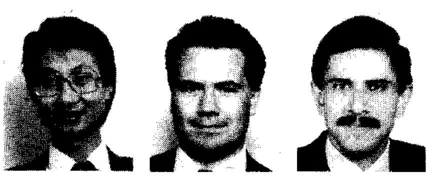
Steel and Synthetic Fibers as Shear Reinforcement



by Victor C. Li, Robert Ward, and Ali M. Hamza

The ultimate shear strength of longitudinally reinforced fiber mortar and concrete beams without shear stirrups is examined by testing beams under center-point bending. All beams without fibers failed by diagonal shear cracking. Increases in ultimate shear strength up to 183 percent were recorded due to random reinforcement with volume fractions up to 2 percent of short fibers. In some cases, shear failure was prevented and ultimate failure was in flexure, with yielding of the longitudinal tensile steel. The testing program employed four fiber types (steel, acrylic, aramid, and a high-strength polyethylene); shear span-effective depth ratios a/d *ranging from 1.0 to 4.25; reinforcement ratios* ρ *of 1.1, 2.2, and 3.3 percent; and beam depths* d *of 102 and 204 mm. It is shown that correlations exist between the shear strength and a parameter that involves the flexural and splitting tensile strength, the* a/d *and* ρ *ratios, and the beam depth. The relationships suggest simple means of predicting the shear strength of axially reinforced mortar and concrete beams containing fibers. The results are quite versatile for a wide range of fiber types. This work demonstrates that tensile property improvements through fiber reinforcements can be translated into shear capacity improvements.*

Keywords: mortars (material); reinforced concrete; **shear strength; synthetic fibers;** tensile properties.

Shear failure of a concrete beam reinforced with longitudinal steel only occurs when the principal tensile stresses within the shear span exceed the concrete strength and a diagonal crack propagates through the beam web. This failure is usually very sudden due to the brittle behavior of plain concrete in tension. Conventional design procedure has been to provide vertical or inclined stirrup reinforcements in the web at intervals throughout the beam length, which act to arrest diagonal cracks and substantially increase the shear capacity of the beam. Sufficient stirrups are usually provided to insure that, in the case of accidental overload, ultimate failure of the beam is due to yielding of the longitudinal steel, resulting in large cracks and deflections before final failure, which provide adequate warning of imminent collapse.

The main purpose of the tests reported in this paper was to examine the influence of fiber reinforcement on both the strength and failure mode of longitudinally reinforced beams without stirrups, subjected to moment and shear. There are several reasons why it may be preferable to use fibers rather than stirrups to resist shear forces in reinforced concrete structures. They are randomly distributed throughout the concrete volume at relatively small spacings and thus provide equal resistance to stresses in all directions. This may be particularly beneficial in structures designed to resist shear forces due to earthquake and wind loading. Secondly, fibers increase the concrete's resistance to crack formation and propagation. The resulting reduction in crack size and beam deflection under service load conditions may be critical to the success of using high-strength reinforcing steel and ultimate limit state design without being restricted by service load performance. Also, the increased resistance of the concrete cover to spalling and cracking helps to protect steel from corrosion in adverse environments and, hence, improve structural durability. Thirdly, since conventional stirrups require relatively high labor input to bend and fix in place, fiber reinforcement may significantly reduce construction time and costs, especially in an era of high labor costs and possibly even labor shortages. Fiber concrete can also be easily placed in thin or irregularly shaped sections such as architectural panels, where it may be very difficult to place stirrups.

Many reports published over the past 2 decades, which conform the effectiveness of fibers as shear reinforcement, focus exclusively on steel fibers.[1-5] In this paper, the effectiveness of both steel and synthetic fibers of various kinds is examined, and a method of predicting the ultimate shear strength of fiber reinforced beams is proposed, which takes account of the flexural and tensile strengths of the material, the shear span-effective depth ratio *a/d,* the amount of longitudinal reinforcement ρ, and the effective depth of the beam *d.*

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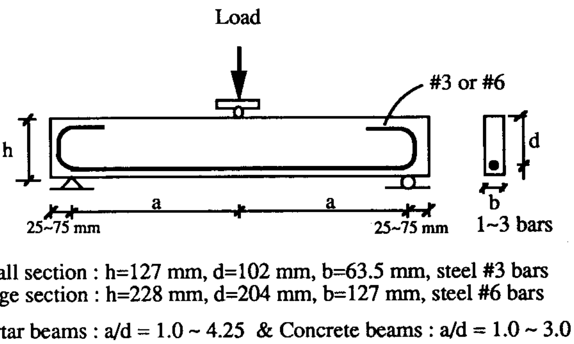

Fig. 1 — Loading configuration and specimen details of shear beams

Table 1 — Properties of fibers

Fiber type	Fiber length, mm	Aspect ratio, l/d	Tensile strength, MPa	Elastic modulus, GPa	Specific density, g/cc	Surface type
Steel 25	25	28.5	1000	200	7.9	Crimped
Steel 50	50	57	1000	200	7.9	Crimped
Aramid	6.4	530	2800	130	1.45	Straight
Acrylic	6.4	470	400	6	1.15	Crimped
Polyethylene	12.7	334	2000	100	0.97	Straight
Steel 30*	30	60	1172	200	7.85	Hooked
Steel 50*	50	100	1172	200	7.85	Hooked

*Used only for concrete specimens.

RESEARCH SIGNIFICANCE

The aim of this research is to illustrate the effectiveness of both steel and synthetic fibers as shear reinforcement in longitudinally reinforced mortar and concrete beams. Experimental results suggest a simple means of predicting the shear strength of mortar and concrete beams containing fibers based on the flexural and splitting tensile strength, the span-depth ratio, reinforcement ratio, and beam depth. This work demonstrates experimentally the dependence of shear structural properties on material tensile properties which can be efficiently modified by short random fiber reinforcement.

TEST PROGRAM

The behavior of fiber reinforced mortar subjected to combined shear and flexural stresses was examined by testing 252 longitudinally reinforced mortar beams and 60 similarly reinforced concrete beams (in sets of three indentical tests) under center-point loading, as shown in Fig. 1. Plain mortar/concrete as well as mortar/concrete reinforced with various volume fractions of aramid, polyethylene, acrylic, and steel fibers were tested. Reinforcement ratios ($\rho = A_s/bd$) of 1.1, 2.2, and 3.3 percent, effective depths of 102 and 204 mm, and a range of shear span-effective depth ratios between 1.0 and 4.25 were employed in the testing program.

Two different mortar mixes were used. Mix A had a cement:sand:water ratio of 1:1:0.5 and Mix B a ratio of 1:1:0.4. For the concrete mix, the cement:sand:aggregate:water ratio was 1:1.5:2.5:0.45. Limestone aggregate and river sand were used for the concrete mix. For mortar, sand passed through a No. 8 sieve was used. Type III rapid-hardening cement was used for both mortar and concrete mixes. The properties of the various fibers used are listed in Table 1. Grade 60 deformed reinforcing bars with yield strength in the range of 440 to 460 MPa and ultimate strength between 670 and 725 MPa were used as longitudinal steel. No. 3 bars (diameter = 9.53 mm) were used in beams with 102-mm effective depth, and No. 6 bars (diameter = 19.1 mm) in beams with an effective depth of 204 mm. One, two, or three bars were used in each beam, in a single row, depending on the required reinforcement ratio. A mixer in which random movement of particles is induced by a wobbling flexible drum bottom was used for mixing. The absence of blades in the mixer helped to insure good fiber distribution. All the materials were added to the mixer initially and mixing was performed for about 3 min. Superplasticizer was used with the synthetic fiber mixes. All beams were cast perpendicular to the testing direction. Specimens were covered with plastic for approximately 20 hr after casting and were then removed from the molds and stored in air at 20 C until testing at between 14 and 16 days of age.

Testing for the mortar beams was carried out using an 890-kN capacity displacement-controlled testing machine. The cross-head speed was set so that the maximum load was reached after about 3 to 5 min. The load was applied to the 102-mm deep beams through a 12.5-mm wide steel bar across the full width of the specimen. A 25-mm wide bar was used for the 204-mm deep beams. The midspan deflection was measured with a linear variable differential transformer (LVDT). A similar loading system and instrumentation were employed for the concrete beams. The load at the first visible shear crack, the maximum load, and the crack propagation patterns were noted in each test. First crack shear stress and maximum shear strength are listed in Table 2. In Table 2, all data represent average results of three identical tests. Shear stress values were calculated by dividing the shear force (half the applied load) by the product of the beam's width and effective depth. A shear crack is defined as an inclined crack extending above the mid-depth of the beam.

Nonreinforced beams and cylinders were also cast from each mix for flexural, splitting tension, and compression tests. Details of these tests are reported elsewhere.[6,7] Values for flexural, splitting tensile, and com-

Table 2(a) — First shear crack and ultimate strengths for mortar beams

Fiber properties: Type	l, mm	V_f, percent	Beam properties: d, mm	a/d	ρ, percent	First shear crack strength f_{vc}, MPa	Ultimate shear strength f_v, MPa
Mix Type A							
		0	102	3.00	2.2	1.20	1.20
		0	204	3.00	2.2	1.02	1.02
Aramid	6.4	2	102	3.00	2.2	2.48	3.40
		2	204	3.00	2.2	2.22	2.68
Steel	25	1	102	3.00	2.2	1.96	2.55
		1	204	3.00	2.2	1.46	1.95
		2	102	3.00	2.2	2.41	3.21
		2	204	3.00	2.2	1.92	2.56
Mix Type B							
		0	102	3.00	2.2	1.61	1.61
		0	204	3.00	2.2	1.33	1.33
Acrylic	6.4	1	102	3.00	2.2	2.07	2.07
		1	204	3.00	2.2	1.80	1.80
		2	102	3.00	2.2	1.95	2.17
		2	204	3.00	2.2	2.06	2.15
Steel	25	1	102	3.00	2.2	2.41	2.74
		1	204	3.00	2.2	2.11	2.37
		2	102	3.00	2.2	3.11	3.79*
Steel	50	1	102	3.00	2.2	2.95	3.55*
		1	204	3.00	2.2	2.83	3.45*
		2	204	3.00	2.2	2.90	3.63*
		0	102	1.00	2.2	3.88	5.17
		0	102	1.50	2.2	2.26	3.37
		0	102	2.00	2.2	1.92	2.31
		0	102	2.25	2.2	1.58	2.04
		0	102	2.50	2.2	1.52	1.52
		0	102	2.75	2.2	1.71	1.71
		0	102	3.75	2.2	1.69	1.69
		0	102	4.25	2.2	1.56	1.56
Aramid	6.4	1	102	1.00	2.2	6.75	7.76
		1	102	1.50	2.2	4.94	5.25
		1	102	1.75	2.2	3.55	3.99
		1	102	2.00	2.2	3.20	3.68
		1	102	2.25	2.2	3.06	3.15
		1	102	2.50	2.2	2.92	3.48
		1	102	2.75	2.2	3.03	3.19
		1	102	3.00	2.2	2.94	3.03
		1	102	3.75	2.2	2.99	2.99
		1	102	4.25	2.2	—	2.75*
Steel	25	1.0	102	1.00	2.2	5.24	7.82
		1.0	102	1.50	2.2	4.02	5.15
		1.0	102	1.75	2.2	2.92	4.64
		1.0	102	2.00	2.2	2.59	3.99

*Flexural failure.

pressive strengths are listed in Table 3. Fig. 2 shows some typical flexural load-deflection curves for each fiber type in the mortar beams.

OBSERVED FAILURE MODES

Beam action, $a/d \geqslant 2.5$

In all beams with an a/d of 2.5 or greater, failure occurred suddenly when the first diagonal shear crack appeared. In each case, the diagonal crack propagated along the compressive stress path toward the load point and also along the reinforcement toward the support. Some flexural cracks formed in the beams before failure, with more cracking being observed for higher a/d values and lower reinforcement ratios.

Fiber reinforced beams with a/d greater than 2.5 usually exhibited flexural-shear cracking, with diagonal shear cracks forming as an extension of a flexural crack. In many cases, a number of shear cracks formed along the beam span before ultimate load. As shear cracks propagated and bent over to follow the compressive stress trajectory, some cracks began to propagate along the reinforcement. The improved shear capacity of the fiber reinforced beams may be qualitatively explained as follows. Before any cracks form in the beam, a parabolic shear stress distribution may be expected across the beam web. When flexural cracks develop, the variation in steel stress between one crack and the next tends to bend and shear the concrete "teeth" that lie between successive cracks. As the cracks propagate, they become inclined under combined flexural and shear stresses. Interface shear and direct tensile stresses across the crack, as well as dowel forces in the reinforcement, tend to resist the crack propagation. As the crack length and relative slippage across crack faces increase, it may be expected that the shear-resisting forces across the crack faces decrease and an increasing proportion of the shear force is resisted by direct shear in the compression zone above the crack and by the dowel action in the reinforcing bars. Ultimate failure occurs by a breakdown of dowel ac-

Table 2(a) (Continued) — First shear crack and ultimate strengths for mortar beams

Fiber properties			Beam properties			First shear crack strength f_{vc}, MPa	Ultimate shear strength f_v, MPa
Type	l, mm	V_f, percent	d, mm	a/d	ρ, percent		
Steel	25	1	102	2.25	2.2	2.58	3.62
		1	102	2.50	2.2	2.50	3.17
		1	102	2.75	2.2	2.48	2.75
Polyethylene	12.7	1	102	1.00	2.2	7.45	7.78
		1	102	2.00	2.2	3.56	5.31
		1	102	3.00	2.2	3.48	3.71*
		1	102	3.75	2.2	3.02	2.97*
		1	102	4.25	2.2	2.45	2.61*
		0	102	1.00	1.1	3.46	4.75
		0	102	2.00	1.1	1.84	2.10
		0	102	2.50	1.1	1.35	1.35
		0	102	3.00	1.1	1.32	1.32
		0	102	3.75	1.1	1.31	1.31
		0	102	4.25	1.1	1.19	1.19
Aramid	6.4	1	102	1.00	1.1	5.91	5.91
		1	102	2.00	1.1	3.02	3.21
		1	102	2.50	1.1	2.50	2.50
		1	102	3.00	1.1	—	2.48*
		1	102	3.75	1.1	—	1.91*
		1	102	4.25	1.1	—	1.51*
Steel	25	1	102	3.00	1.1	1.98	1.98
Polyethylene	12.7	1	102	1.00	1.1	5.50	5.70
		1	102	2.00	1.1	3.15	3.87
		1	102	3.00	1.1	2.33	2.48*
		1	102	3.75	1.1	2.03	2.03*
		1	102	4.25	1.1	—	1.64*
		0	102	1.00	3.3	4.10	6.15
		0	102	2.00	3.3	2.04	2.57
		0	102	3.00	3.3	1.61	1.61
		0	102	3.75	3.3	1.63	1.63
		0	102	4.25	3.3	1.54	1.54
Aramid	6.4	1	102	1.00	3.3	7.40	8.40
		1	102	2.00	3.3	3.42	3.82
		1	102	3.00	3.3	3.05	3.14
		1	102	3.75	3.3	3.13	3.13
		1	102	4.25	3.3	2.74	2.74
Steel	25	1	102	3.00	3.3	2.50	2.75
Polyethylene	12.7	1	102	2.00	3.3	3.45	5.65
		1	102	3.00	3.3	3.73	3.77
		1	102	3.75	3.3	2.86	3.72
		1	102	4.25	3.3	2.95	3.26*
Steel	50	1	102	3.00	3.3	3.24	3.90

*Flexural failure.

Table 2(b) — First shear crack and ultimate strengths for concrete beams

Fiber properties			Beam properties			First shear crack strength f_{vc}, MPa	Ultimate shear strength f_v, MPa
Type	l, mm	V_f, percent	d, mm	a/d	ρ, percent		
		0	204	3	2.2	1.56	1.63
		0	102	1	1.1	3.99	4.79*
		0	102	2	1.1	2.31	3.14
		0	102	3	1.1	1.32	1.55
		0	102	1	2.2	4.33	4.54*
		0	102	2	2.2	2.19	2.64
		0	102	3	2.2	1.59	1.65
		0	102	1.5	1.1	3.92	4.46
Polyethylene	12.7	1	102	1	1.1	5.07	5.74*
	12.7	1	102	2	1.1	3.60	4.17
	12.7	1	102	3	1.1	2.08	2.48
	12.7	1	102	2	2.2	3.79	4.37
	12.7	1	102	3	2.2	2.36	2.82
	12.7	1	102	1.5	1.1	3.93	4.74
Steel	30	1	204	3	2.2	2.27	3.05
	30	1	102	3	2.2	2.78	3.16
	30	1	102	3	1.1	2.14	2.43
	30	1	102	1.5	1.1	4.72	5.64
Steel	50	1	204	3	2.2	2.11	3.05
	50	1	102	3	2.2	2.99	3.55

*Bearing failure.

tion due to excessive cracking along the reinforcing bars or by failure in the compression zone under combined shear and compressive stresses. It is well recognized that fiber reinforcement greatly improves the resistance to crack propagation and also gives much greater tensile stress capacity across an existing crack. Swamy and Bahia[8] found that dowel forces increased almost linearly with the material flexural strength, which is greatly improved by fibers. Thus, for beams with $a/d \geqslant 2.5$, the major contribution of fibers to increased shear capacity comes through increased tensile capacity across the shear crack and the development of larger dowel forces in the reinforcement.

Arch action, $a/d \leqslant 2.5$

In these beams, cracking usually initiated between the load point and the support just below the mid-depth of the beam. The crack then propagated toward the load point and the support in a manner similar to that observed in a splitting tension test. Fenwick and Paulay[9] postulated that appreciable arch action develops when the diagonal crack extends to the support, thereby separating the tension and compression zones of the shear span and allowing the relatively large translational displacement associated with arch action to occur. The plain mortar/concrete beams tended to fail by splitting along the line of the arch compressive force. Many of the fiber beams with much higher splitting strengths failed either by crushing of the mortar/concrete at the support or by sudden ejection of the upper part of the shear span due to failure of the compression zone under combined shear and compression, together with sliding along the diagonal crack faces. Some beams with larger a/d values failed due to the propagation of a flexural tension crack from the top of the beam down to meet the diagonal crack. This was caused by an eccentric line of thrust between the load point and the support. The polyethylene fiber beams failed by gradual shearing of the diagonal crack faces leading to gradual reduction in load capacity after the ultimate. This is due to the ability of these fibers to transfer relatively high tensile stresses even at very large crack openings.[10]

Table 3(a) — Flexural, splitting tensile, and compressive strengths for mortar beams

Mix type	Material: Fiber type	Material: l, mm	Material: V_f, percent	Flexural strength f_r,* MPa	Splitting strength f_t,† MPa	Compressive strength f'_c,‡ MPa
A			0	2.2	2.2	51.1
A	Aramid	6.4	2	6.7	4.0	37.7
A	Steel	25.0	1	4.8	3.4	53.0
A	Steel	25.0	2	6.1	4.1	50.2
B			0	2.6	2.9	57.0
B	Acrylic	6.4	1	4.0	4.3	45.3
B	Acrylic	6.4	2	3.7	3.6	33.0
B	Aramid	6.4	1	5.1	4.4	50.3
B	Polyethylene	12.7	1	8.9	3.2	45.7
B	Steel	25.0	1	5.4	3.9	62.6
B	Steel	25.0	2	7.0	4.8	57.0
B	Steel	50.0	1	7.4	4.3	54.1
B	Steel	50.0	2	8.9	5.6	—

*Flexural test (beam 114 x 114 x 342 mm).
†Splitting test (cylinder 77 x 154 mm).
‡Compressive test (cylinder 77 x 154 mm).

Table 3(b) — Flexural, splitting tensile, and compressive strengths for concrete beams

Material: Fiber type	Material: l, mm	Material: V_f, percent	Flexural strength f_r,* MPa	Splitting strength f_t,† MPa	Compression strength f'_c,‡ MPa
		0	4.8	3.0	17.8
Polyethylene	6.4	1	6.1	3.3	19.1
Steel	30	1	10.2	5.2	22.7
Steel	50	1	12.1	5.3	26.0

*Flexural test (beam 100 x 100 x 300 mm).
†Splitting test (cylinder 100 x 200 mm).
‡Compressive test (cylinder 100 x 200 mm).

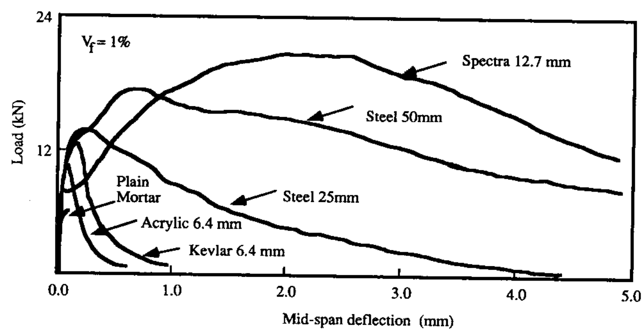

Fig. 2 — Typical flexural load-deflection curves for mortar reinforced beams with various fiber types (beam depth d = 114 mm; width w = 0.55 d; span l = 3 d; 1 mm = 0.0394 in.)

TEST RESULTS AND DISCUSSION

The test results are arranged in a manner that illustrates the influence of fiber type and volume fraction, shear span-effective depth ratio, amount of longitudinal steel, and beam depth on the shear strength.

First crack strength and ultimate strength

Fig. 3 shows the first crack and ultimate strengths of plain mortar/concrete beams and beams with a 1 percent volume fraction of each fiber type at a/d values of 2.0 and 3.0. Due to the development of arch action after cracking in the shorter beams, there is a greater difference between first crack and ultimate strengths for $a/d = 2.0$. The polyethylene and steel fiber beams show the greatest difference between first crack and ultimate strengths because of their greater ability to transfer stresses after cracking, as seen from the flexural load-deflection curves illustrated in Fig. 2. At $a/d = 3.0$, after the diagonal crack forms, some force is transferred to dowel action and shear compression. It may be expected that as the crack opens and propagates, the stress transfer capability of the short acrylic and aramid fibers falls off much more quickly than that of the polyethylene or steel fibers, thus causing a faster rate of force transfer to the other shear-resisting mechanisms. This type of explanation also applies when $a/d = 2.0$. In general, an increase in the reinforcement ratio leads to increased cracking strength due to delayed formation and subsequent propagation of flexural and inclined cracks.

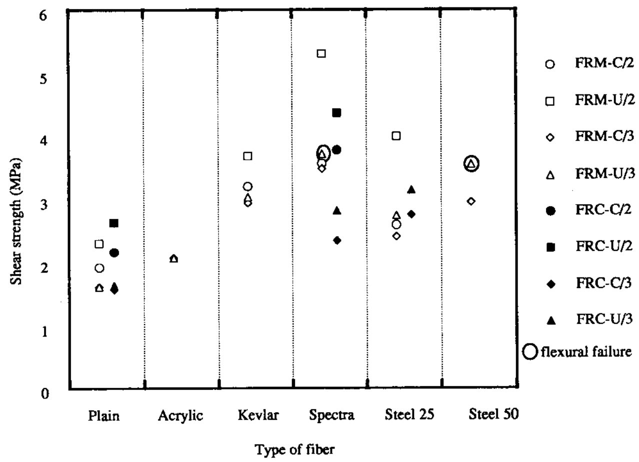

Fig. 3 — Cracking C *and ultimate* U *shear strength of various fiber reinforced mortar (FRM, open symbols) and fiber reinforced concrete (FRC, solid symbols) beams. Data are for* V_f *= 1.0 percent;* ρ *= 2.2 percent; and* a/d *= 2 or 3 (1 MPa = 145 psi)*

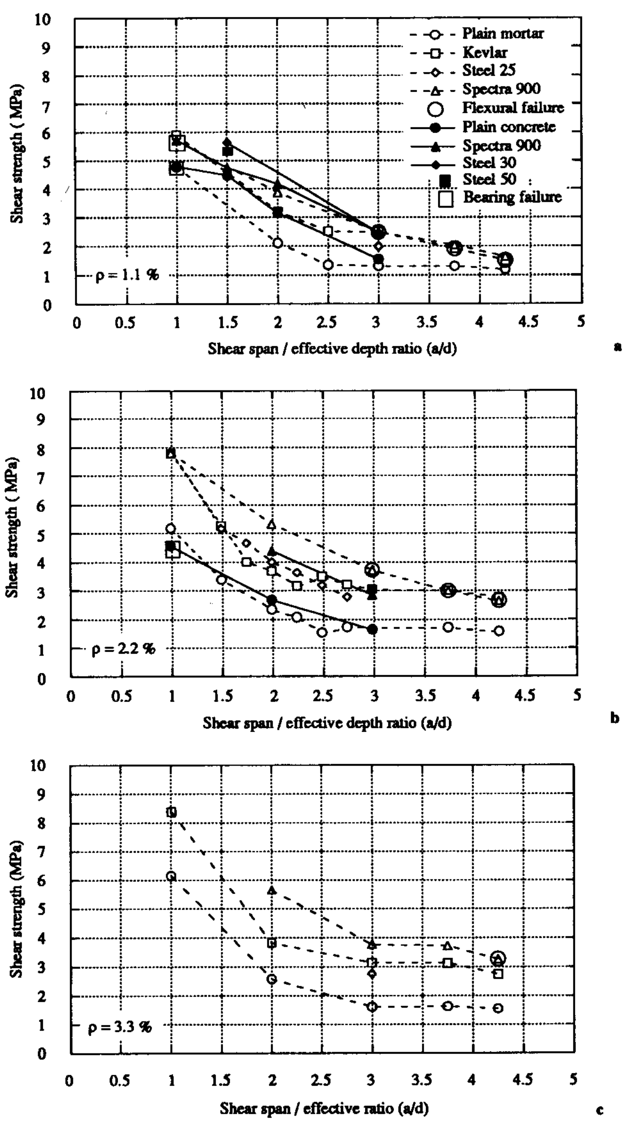

Fig. 4 — Influence of shear span-effective depth ratio on shear strength of FRM and FRC beams for (a) ρ *= 1.1 percent; (b)* ρ *= 2.2 percent; and (c)* ρ *= 3.3 percent (1 MPa = 145 psi)*

Shear span-effective depth ratio

Fig. 4 shows the effect of shear span-effective depth ratio *a/d* on the shear strength for various fiber types and for three different amounts of longitudinal reinforcement. All the curves have similar shapes with a distinct change in slope at an *a/d* value of approximately 2.5. (The concrete beams do not have enough data points to show this, but are expected to behave similarly.) The efficiency of fibers as shear reinforcement increases as the *a/d* ratio increases. This is because fiber reinforcement cannot improve arch action as effectively as it can improve beam action. Arch action depends on the development of compressive forces between the load point and support. The actual forces that can be developed depend on the fiber pullout behavior acting across diagonal cracks, the compressive strength of the material on each side of the crack, and the material behavior under combined shear and compression close to the loading point. It is most likely that fibers lead to significant increases only in the pullout forces across the crack, and may lead to decreased capacity in compression and shear compression when synthetic fibers are used. In these tests, fiber reinforcement prevented the splitting failures that occurred with the plain mortar/concrete beams, and allowed load increases until shear compression failure occurred. For *a/d* greater than 2.5, fiber reinforcement gives much larger tensile capacity across the diagonal crack and also much larger dowel forces in the reinforcing bar. These improvements can be directly related to the pullout performance of fiber concrete. Thus, the beam action failure mechanism is capable of using the improved material performance due to fiber reinforcement much more effectively than the arch action.

For a reinforcement ratio of 2.2 percent in the mortar beams with *a/d* equal to 3.0, the strength increases for a fiber volume fraction of 1 percent are 29 percent for the acrylic, 70 percent for the 25-mm steel, 88 percent for the aramid, 130 percent for the polyethylene, and 120 percent for the 50-mm steel fibers. The actual increase in shear capacity is somewhat greater than this for the polyethylene and 50-mm steel fibers because the beams failed in flexure. These fibers give shear strength increases of 134 and 142 percent, respectively, when the reinforcement ratio is 3.3 percent. At *a/d* = 1.0, reinforcement with aramid, polyethylene, or 25-mm steel fibers gave a maximum increase of 51 percent in the shear strength.

In general, the peak bending moment at beam failure is lowest for *a/d* between about 2.5 and 3.0. For greater *a/d* values, it increases toward the flexural strength. A number of fiber reinforced beams, particularly those with low reinforcement ratios and high *a/d* values, failed in a ductile manner in flexure with yielding of the longitudinal steel. Aramid fiber beams with reinforcement ratios of 1.1 and 2.2 percent failed in flexure at *a/d* values of 3.0 and 4.25, respectively. At *a/d* = 4.25, all the polyethylene fiber beams failed in flexure. All the polyethylene beams that failed in flexure showed very large amounts of cracking throughout much of the beam as the ultimate load approached. This type of behavior adds significantly to the energy-absorption capacity of these beams, before failure. Since none of the plain mortar beams failed in flexure, it is not known what effect, if any, fiber reinforcement had on the strength of those beams that failed in flexure. Swamy and Al-Ta'an[11] found that fibers do not significantly improve the flexural strength of reinforced concrete beams. Their test results showed a maximum increase of 10.5 percent with a steel fiber volume fraction of 1.0 percent. They concluded that fibers were effective in resisting deformation at all stages of loading from first crack to failure, and that the role of fibers in beams that fail in flexure (beams with stirrup reinforcement) is essentially to arrest any advancing cracks and increase the beam's post cracking stiffness.

The polyethylene fiber beams showed some crushing under the load point at a/d = 1.0 and a reinforcement ratio of 2.2 percent. Some extra tests with an 18-mm wide loading platen (instead of the usual 12.5-mm one) were carried out to obtain the ultimate shear strength.

Reinforcement ratio

Fig. 5 illustrates the effect of the amount of longitudinal steel on the shear strength of the mortar beams for three different a/d values. Strength generally increases as the reinforcement ratio increases. This trend is due to the reduced size of cracks and an increase in the total dowel force. Swamy and Bahia[8] related the dowel force to the width of the concrete layer at the reinforcement level as well as to the number of reinforcement bars. The spacing of the reinforcement bars is small at ρ = 3.3 percent, and this probably explains why the increase in shear strength between ρ = 2.2 percent and ρ = 3.3 percent is relatively small. Also, the fiber reinforced beams tend to show greater sensitivity to reinforcement ratio, especially at a/d = 3.0, probably because of their ability to develop more significant dowel forces prior to failure.

Beam depth

The general trend of shear strength of fiber reinforced mortar beams was found to decrease with beam size in the present set of tests in a manner consistent with test results from plain concrete beams[12-14] and also with theoretical work presented by Hillerborg.[15] Shioya et al.[14] found size effects even up to beam depths of 3 m, and proposed that strength was inversely proportional to the fourth root of the effective depth. Hillerborg[15] proposed a similar dependence using theoretical finite element analysis and a fracture mechanics approach. The test results (details of which can be found in Reference 6) show strength decreases approximately inversely proportional to some value between the fourth and the third root of the effective depth.

A limited number of tests by Williamson and Knab[16] on large fiber reinforced beams showed increasing shear strength with fiber volume fraction but not enough to prevent shear failure. This is a disadvantage of using fibers as shear reinforcement rather than stirrups, whose contribution to shear strength may be expected

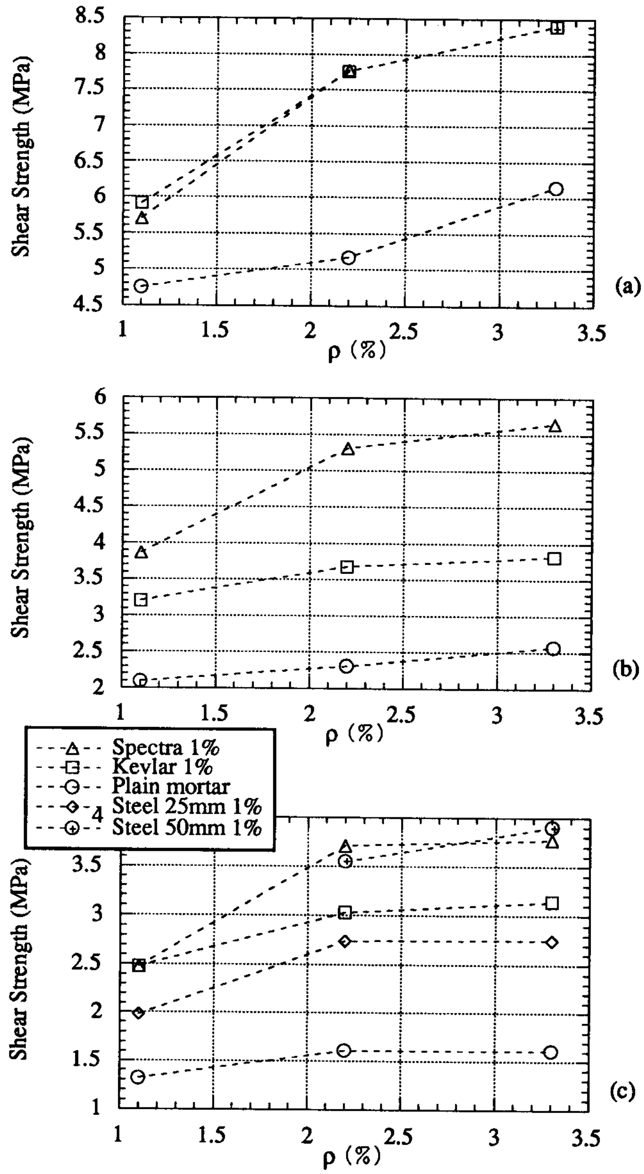

Fig. 5 — Influence of longitudinal reinforcement ratio on shear strength of FRM beams with (a) a/d = *1.0; (b)* a/d = *2.0; and (c)* a/d = *3.0 (1 MPa = 145 psi)*

to be size-independent. However, it should be remembered that even when using stirrups, the beam first cracking strength and the contribution of the concrete to ultimate strength are probably still size-dependent.

Plain Mortar Type B has a shear strength approximately 30 percent greater than that of Type A. Considering both beam sizes, the average increase in shear strength when Mortar A is reinforced with 1 percent of 25-mm steel fibers is 103 percent compared to an average increase of 74 percent when Mortar B is similarly reinforced. This finding of proportionally higher strength increases when fibers are used to reinforce weaker matrixes is consistent with results from shear beam tests as well as flexural and splitting tensile tests on two concrete qualitities presented by Niyogi and Dwarakanathan.[17] The reason for this may be that the contribution of fibers to composite strength, which mainly depends on fiber-matrix bond strength, does not increase as much (proportionally) as matrix strength when concrete quality is improved. Another perspective provided by crack mechanics[21] suggests that composite tensile strength improvement is dependent on the ratio between fiber-bridging toughness (related to energy in fiber debonding in the fracture process zone) and matrix toughness. For the same fiber reinforcement, a higher quality matrix will reduce this ratio and lessen the composite tensile strength improvement. For high-strength concrete reinforcements, therefore, higher fiber volume fraction or better interfacial bonds (among other options) will be necessary for effective enhancement of tensile and, therefore, beam shear strength. A volume fraction of 2 percent of aramid fibers gives an average increase of 174 percent in the shear strength of Mortar A. This is the largest increase, proportionally, recorded. However, it is believed that polyethylene or 50-mm steel fibers could increase the strength of this mortar even more. A 1 percent volume fraction of 50-mm steel fibers increased the strength of Mortar B sufficiently to cause a flexural failure in both beam sizes. The flexural strength of the large beam was about 3 percent less than that of the small beam. Increasing the volume fraction of 50-mm steel fibers from 1 to 2 percent led to an increase of 5 percent in the flexural strength of the large beams.

PREDICTION OF ULTIMATE SHEAR STRENGTH

The development of a general simple formula to predict the shear strength of fiber reinforced beams is critical to the successful application of fibers as shear reinforcement in practice. Because of the number of parameters that affect the shear strength of longitudinally reinforced beams and the lack of a thorough understanding of the various force-resisting mechanisms acting in a beam prior to ultimate load, it has proved difficult to even develop a rational formula to predict the shear strength of plain concrete beams. Zsutty[18] developed the empirical relation in Eq. (1) to predict the shear strength of plain concrete beams with a/d greater than 2.5

$$f_v = 60(f_c' \rho\, d/a)^{1/3}, \text{ psi for } a/d \geqslant 2.5 \qquad (1)$$

where f_c' is the concrete compressive strength in psi. He later modified this relation[19] to account for arch action in short beams and proposed Eq. (2) for plain concrete beams with $a/d < 2.5$

$$f_v = 150(f_c' \rho)^{1/3}(d/a)^{4/3}, \text{ psi for } a/d < 2.5 \qquad (2)$$

Since fiber reinforcement does not significantly increase the compressive strength (10 percent increase for 1 percent of 25-mm steel fibers) and in many cases can cause a strength reduction, especially for synthetic fibers (20 percent reduction for 1 percent of 12.7-mm polyethylene fibers), it is not possible to use the compressive strength as a material parameter that reflects the strength in shear and flexure. Intuitively, it seems that the splitting tensile or the flexural strength should be much better indicators of the improved performance. Due to the distinct change in behavior at a/d = 2.5, it is proposed that two separate formulas should be developed, one for $a/d \geqslant 2.5$ and a second for $a/d \leqslant$ 2.5.

From the test results reported here, it appears that the important parameters influencing ultimate shear strength are the tensile and flexural strengths of the material, the shear span-effective depth ratio a/d, the reinforcement ratio ρ, and the beam depth. When examining the influence of each parameter on ultimate strength only, those beams that failed in shear before the flexural capacity was reached were included. For beams with $a/d \geqslant 2.5$, fibers are particularly effective at improving tensile capacity across the diagonal crack, and also at preventing dowel cracking, thereby allowing greater dowel forces to develop. The flexural strength of unreinforced fiber concrete beams, to some extent, reflects the post-cracking performance of the material. A high-stress capability across a cracked plane allows load increases even after a crack begins to propagate from the bottom of the beam in a flexural test. Thus, it may be expected that a material that gives high flexural strengths would also give high stresses across diagonal shear cracks. After diagonal cracks form in the beam, some shear force is transferred to dowel action in the reinforcing bars and to direct shear in the compression zone. The ability of the reinforcing bars to quickly react to this extra load depends on the stiffness of the load-deformation curve for dowel forces. It may be expected that a material with a high cracking tensile strength can effectively resist dowel cracking and thus has a stiff curve. This allows development of relatively high dowel forces before the shear forces across the diagonal crack drop significantly, and thus the overall shear capacity of the beam can continue to increase. From this qualitative discussion, it appears that both the flexural and splitting tensile strengths are important in predicting the ultimate shear strength. For simplicity, it was decided to attempt to relate the shear strength to some function of the product $f_f f_t$.

The test results presented in Fig. 4 and 5 show that the a/d and ρ ratios have similar influences on the strength of both plain mortar/concrete and fiber reinforced mortar/concrete beams. It was decided to try to relate ultimate shear strength to the product $(\rho\, d/a)^{1/3}$ for $a/d \geqslant 2.5$, similar to what Zsutty did. A size-effect term was also included that would be somewhat similar to those proposed by Shioya et al.[14] and Hillerborg.[15]

Fig. 6 shows good straight-line correlation between the ultimate shear strength and the product $(f_f f_t)^{3/4}$ $(\rho\, d/a)^{1/3}$ $(d)^{-1/3}$ for $a/d \geqslant 2.5$. The equation of the line drawn in Fig. 6 is given as follows

$$f_v = \alpha + \beta\, [(f_f f_t)^{3/4}(\rho\, d/a)^{1/3}(d)^{-1/3}] \quad \text{for } (a/d \geqslant 2.5) \qquad (3)$$

where f_v, f_f, and f_t are in MPa and d is in mm. For the mortar data, α = 0.53 and β = 5.47. For the concrete data, α = 1.25 and β = 4.68. Eq. (3) quantifies how increased tensile properties lead to improved shear

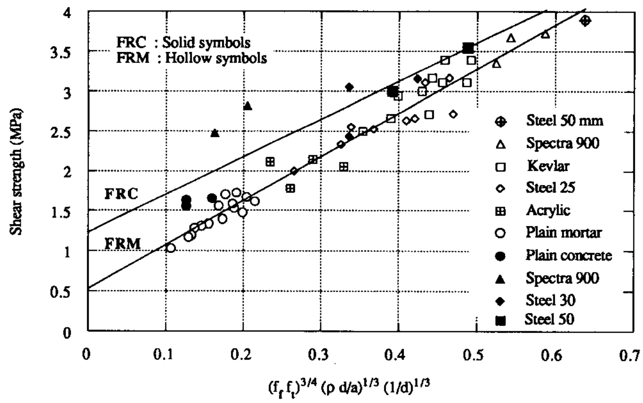

Fig. 6 — Semiempirical relationship between ultimate shear strength and material and geometrical properties for a/d $\geqslant$ *2.5 (units for horizontal axis are in* $N^{3/2}$ $mm^{-10/3}$*; 1 N = 0.225 lb-force; and 1 mm = 0.0349 in.)*

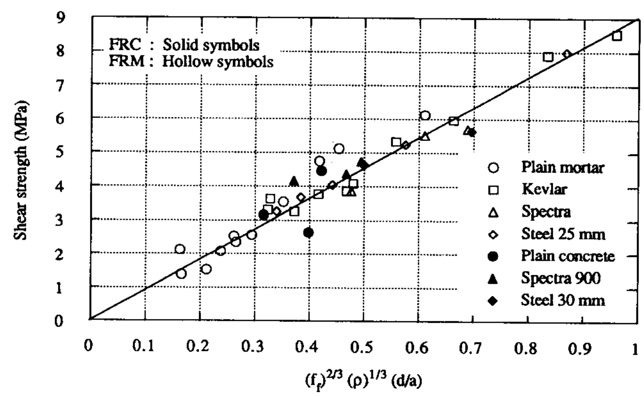

Fig. 7 — Semiempirical relationship between ultimate shear strength and material and geometrical properties for a/d $\leqslant$ *2.5 (units for horizontal axis are in* $N^{3/2}$ $mm^{-10/3}$*; 1 N = 0.225 lb-force; and 1 mm = 0.0349 in.)*

strength. The vertical shift in the lines in Fig. 6 (i.e., the difference in the α-value) between the mortar and concrete data probably accounts for the aggregate-interlock action associated with shear sliding on the curved diagonal cracks in the concrete specimens. For concrete mixes within practical range, contribution to the shear strength due to the aggregate-interlock mechanism is probably about constant. That is, the value of α in Eq. (3) may be expected to remain approximately 1.3 for all practical concrete mix types with various fibers. It can be seen that the plain concrete and plain mortar data groups lie to the lower left-hand corner, while the steel and polyethylene fibers data groups generally lie to the upper right-hand corner, suggesting the effectiveness of using fibers as shear reinforcement. It is interesting to note that beam depth can be cancelled out of the formula, giving dependence on only the material properties, the amount of reinforcement, and the shear span.

Eq. (3) gives conservative predictions of shear strength when a/d is less than 2.5. The particular increase in shear strength due to arch action is probably dependent on the width of the loading platen, with greater strength for a wider platen due to distribution of the compressive forces over a greater area. Zsutty[19] used the d/a ratio alone to account for strength increases due to arch action in plain concrete beams. However, because of the ability of fibers to more effectively improve beam action than arch action, it was considered appropriate to look at some material properties when considering a change from failure in long beams to failure in short beams. At lower a/d ratios, the differences between first crack and ultimate strength are much greater and, in many cases, a crack extends almost all the way from the support to the load point before ultimate load. Thus, it was considered reasonable to expect that cracking tensile strength has little influence on ultimate load and that post-cracking strength is the important material parameter. For this reason, it was decided to try to relate the ultimate shear strength to just the flexural strength. Since a relatively narrow loading platen was used in this test series, it may be expected that less arch action might develop than was the case in the tests analyzed by Zsutty. This would give less dependence on the d/a ratio. Fig. 7 shows good straight-line correlation between the ultimate shear strength and the product $(f_f)^{2/3}$ $(\rho)^{1/3}$ (d/a) for mortar and concrete beams with $a/d \leqslant 2.5$. (Data points for beams that suffer from bearing failures at the support points are not included in Fig. 7.) Comparison with Fig. 7 suggests that the aggregate-interlock mechanism may not be as significant in direct tension transfer as in shear transfer, which occurs mainly in the shear sliding of the diagonal cracks in beams with $a/d > 2.5$. The straight line shown through the origin is given by

$$f_v = 9.16[(f_f)^{2/3}(\rho)^{1/3}(d/a)] \text{ for } (a/d \leqslant 2.5) \quad (4)$$

with f_v and f_f expressed in MPa. It was not possible to include the effect of beam size in this formula because all tests for $a/d \leqslant 2.5$ were on a single beam size. However, computational fracture mechanics studies[20] seem to indicate similar size effect for all a/d values. If so, an additional factor indicating dependence on d must be included in Eq. (4).

There is usually some discontinuity in the strength-versus-span depth ratio curves predicted by Eq. (3) and (4) at $a/d = 2.5$. Problems with this can be avoided if it is realized that Eq. (3) is conservative for $a/d < 2.5$ and can be used instead of Eq. (4) for a/d values just less than 2.5, if it happens to give a higher strength value.

CONCLUSIONS

1. The cracking patterns that develop in longitudinally reinforced beams are similar for plain and fiber reinforced mortar/concrete. For a/d greater than 2.5, a crack forms and propagates suddenly at the maximum load in the plain mortar/concrete beams. In fiber reinforced beams, increased dowel capacity in the reinforcing bars due to a greater resistance to propagating dowel cracks along the steel, and increased capacity to resist tensile forces across the diagonal crack as the crack opens lead to significant increases in the beam

ultimate strength. Increased crack resistance also delays the onset of initial flexural and shear cracking, and thus leads to better performance of fiber reinforced beams under service load conditions.

2. Shear strength increases in the range of 100 to 200 percent were obtained using a volume fraction of 1 percent of polyethylene, aramid, or steel fibers. The percentage strength increases due to fiber reinforcement were usually greater for higher a/d values and weaker matrixes.

3. For various combinations of the reinforcement ratio ρ and the a/d ratio, it was possible to obtain ductile flexural failures with yielding of the main steel by reinforcing with polyethylene, aramid, or steel fibers. In general, the polyethylene and 50-mm steel fibers were most effective, the aramid and 25-mm steel fibers were somewhat less effective, and the acrylic fibers were least effective in increasing the shear strength.

4. Both the first shear crack stress and the ultimate shear strength were found to increase with the reinforcement ratio. More significant increases were found for the fiber reinforced beams, probably because of their increased resistance to the propagation of dowel cracks along the reinforcement, thus allowing larger dowel forces to develop.

5. A definite trend of decreasing shear strength as beam size increased was found. Plain mortar beams, beams reinforced with 2 percent aramid fibers, and beams reinforced with 2 percent 25-mm steel fibers showed decreases in shear strength of 15 percent, 21 percent, and 20 percent, respectively, as the beam depth increased from 102 to 204 mm with the a/d- and ρ-values remaining constant. Similar trends were observed in the concrete beams.

6. Two formulas, one for $a/d \geqslant 2.5$ and one for $a/d < 2.5$, are proposed for predicing the shear strength of longitudinally reinforced concrete beams with fibers. The splitting tensile and flexural strengths of the material, the shear span-effective depth ratio a/d, the reinforcement ratio ρ, and the beam depth are accounted for in the formulas. Good correlation between test results and each formula is observed.

7. The mortar and concrete beams behaved in similar manners in almost every respect. The only significant difference detected was the shear contribution of aggregate interlock in concrete for beams with $a/d \geqslant 2.5$. This contribution appears to disappear for beams of small span-depth ratio that fail in arch actions, such that crack face shearing is negligible.

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CONVERSION FACTORS

1 MPa = 145 psi
1 mm = 0.039 in.
1 kN = 0.225 kips

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