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Technical Report

EFFECTS OF WEAK COLLISIONS ON DRIFT WAVE INSTABILITIES

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I. INTRODUCTION

1.1 Three distinct waves at frequencies lower than the ion cyclotron frequency can propagate in a uniform plasma in a strong magnetic field (10). These are the electrostatic ion-acoustic wave, the electromagnetic transverse Alfven wave, and the compressional Alfven wave. If the ratio of the plasma pressure to the magnetic pressure is low ($\beta \ll 1$) the Alfven waves will propagate at a much greater speed than the ion-acoustic wave. It follows that in a low-β plasma the electrostatic ion acoustic wave branch is well separated from the other two branches which involve magnetic field line bending and compression.

In an inhomogeneous plasma with a density gradient across the magnetic field these three branches are distorted for waves with small parallel wave numbers and become the so-called drift waves. This situation may occur if the direction of propagation is almost perpendicular to the field, or if the wavelength is sufficiently long. The presence of the inhomogeneity introduces a preferred direction across the field, resulting in the degeneracy of each of the three branches into a pair of waves, corresponding to propagation in each of the two distinguishable directions perpendicular to the magnetic field.

Microinstabilities of scale lengths of the order of an ion Larmor radius may feed on currents or inhomogeneities in the plasma, causing the low frequency electrostatic waves to become unstable. These instabilities occur because an inhomogeneous plasma is necessarily non-Maxwellian and is hence not in kinetic equilibrium. Such a plasma contains free energy[1] which may be accessible to drive the instabilities.

The simplest approach to the problem of determining the characteristics of these instabilities has been the use of the electrostatic Vlasov system of equations, namely the collisionless Boltzmann equation with the electric field given self-consistently by the Poisson equation. Using this system a large number of microinstabilities has been found. Prominent among these are the two-stream instability of the ion-acoustic waves (9), the density and temperature gradient instabilities of the drift waves (27), and the drift-cyclotron instability when the drift wave frequency is near that of the ion cyclotron frequency (17). In this thesis the effect of multiparticle collisions on these instabilities is investigated.

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[1] For example, in the case of a plasma with density gradient, the free energy is from the kinetic energy of the diamagnetic currents which exist in a non-uniform plasma due to the uneven distribution of Larmor centers.

1.2 The Vlasov system neglects the Coulomb scattering among the particles but allows for the influence on the motion of each particle by the averaged electric field of all the other particles.

As is well known, although the Coulomb force has a long range, the effective force field of a charged particle in a plasma does not reach much beyond one Debye radius. Thus a fundamental criterion for the validity of the Vlasov equations is that there is a large number of charged particles within a Debye sphere[2] so that the averaged or self-consistent electrostatic field in the Vlasov system represents a good approximation. In fact, the quantity $g \equiv n^{-1} \lambda_D^{-3}$, where n is the particle number density and λ_D is the Debye length, is the basic parameter for expansion in the BBGKY hierachy in plasma kinetic theory (19), the lowest order with $g = 0$ being the Vlasov system. Since $g = 6 \times 10^{-4} n^{1/2} T^{-3/2}$, a necessary condition for the Vlasov system to be valid kinetically is that the plasma is hot and rarefied.

If $g \neq 0$, the effect of discrete collisions becomes important. Due to the effectiveness of the Coulomb force within a Debye sphere, these collisions are necessarily multiparticle rather than binary interactions.

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[2]Strictly speaking, it is only necessary that the charges within the Debye sphere be uniformly distributed spatially.

The effect of collisions on low frequency wave propagation depends on the relative magnitude of the characteristic time and length scales of the plasma and the waves. If the wave frequencies are much greater than the effective collision frequencies, or if the wavelengths are much smaller than the mean free path, it is customary to neglect collisions in considering these waves and their stability. However, even if these conditions are satisfied, the growth or damping rates, which are often smaller than the corresponding wave frequencies, may be of the same order as or even less than the collision frequencies. Thus the growth rates and stability conditions predicted by the collisionless theory may be incorrect. It is furthermore expected that by modifying the growth rates of the unstable waves, collisions may constitute a competitive process to the quasi-linear and nonlinear saturation or enhancement of the instabilities (13, 29). Finally, collisions may themselves cause the so-called drift-dissipative instabilities which are non-existent in a collisionless plasma (11). It is therefore important to consider the effect of collisions on the stability of low frequency plasma waves.

Since simple fluid-type macroscopic equations describing a collision dominated plasma do not predict wave-particle resonant interactions from which most of the aforementioned instabilities arise, the use of kinetic equations is essential in considering the effect of collisions on these instabilities.

1.3 Interparticle collisions in a fully ionized gas are described by a Fokker-Planck (FP) type kinetic equation (19). The FP collision integral depicts the effect of multiparticle small-angle Coulomb interaction as phase-space diffusion and friction. As mentioned previously, the self-consistent electric field in the Vlasov system already includes the average influence of all the particles on any test particle in the plasma, so that the collision term discussed here is to account for the error in neglecting the discreteness of the charges within the Debye sphere.

Current investigations on the effect of collisions on microinstabilities in a plasma proceed generally in two directions. One is to apply simplifying physical assumptions and mathematical techniques to solve the full FP kinetic equation (28). This procedure in principle would describe the actual situation accurately. Unfortunately, due to the complexity of the full FP collision operator, only the simplest problems can be handled in this way. Furthermore, simple iterative methods used in solving the FP equation have been shown to be often divergent (2) for low collision frequencies, rendering the corresponding solutions questionable. Thus this procedure is rather impractical in considering more complicated problems. On the other hand one may introduce model equations which are mathematically simpler to solve while retaining some of the important features of the original collision operator. In this manner although one scarifices some of the features of the full FP operator, but more complicated problems can be considered. Another significance of the model equations is that exact integrals of these partial differential equations generally exist, so that it is possible to consider intermediate collision frequencies whose magnitudes are comparable to the wave frequencies, although usually one has to resort to numerical integration due to the complexity of these integrals.

1.4 The model collision operator most frequently used in considering wave propagation is the Bhatnagar-Gross-Krook (BGK) simple relaxation model (1, 18). In this model the time rate of change of the particle distribution function $f(\vec{x}, \vec{v}, t)$ due to collisions is assumed to be of the form

$$\left(\frac{\partial f}{\partial t}\right)_{Coll} = -\nu \left(f - f_{Max}\right) \tag{1.1}$$

where f_{Max} is the locally Maxwellian distribution

$$f_{Max}(\vec{x}, \vec{v}) = n \left(\frac{m}{2\pi T}\right)^{3/2} e^{-\frac{m(\vec{v}-\vec{u})^2}{2T}} . \tag{1.2}$$

with $n(\vec{x})$, $u(\vec{x})$, $T(\vec{x})$ defined as follows

$$n(\vec{x}) = \int f \, d^3v \tag{1.3}$$

$$\vec{u}(\vec{x}) = \frac{1}{n} \int f \, \vec{v} \, d^3v \tag{1.4}$$

$$T(\vec{x}) = \frac{1}{n} \int f \, \frac{1}{2} (\vec{v} - \vec{u})^2 \, d^3v. \tag{1.5}$$

Thus, when the plasma is disturbed, it will relax through collisions to the locally Maxwellian state in a time of order ν^{-1}.

Many authors have carried out investigations of wave propagation problems in an inhomogeneous plasma using the BGK model (18). It turns out that the method of characteristics, or integration over the particle trajectory (26) widely used to solve the Vlasov equation, is still applicable for this model, thus simplifying the mathematics involved.

However, the BGK solutions are often misleading in predicting the effect of Coulomb interactions, especially when the perpendicular wave length is shorter than the ion Larmor radius (22). In fact, it was shown that the effective collision frequency obtained for the full FP operator differs from that of the BGK model by a multiplicative factor $b \equiv \frac{1}{2} K_{\perp}^{2} R_{i}^{2}$, where $K_{\perp}$ is the perpendicular wave number and R_i is the ion Larmor radius. This finding led Kadomtsev, et al (12) to replace the factor ν in the BGK model by $b\nu$. Such an intuitive approach is clearly not dependable, as may also be shown by comparing their results in the appropriate limits to those obtained from the approximate calculations using the full FP equation (28).

1.5 The form of the model FP equation which will be used in this thesis has its origin in the study of the Brownian motion of minute particles suspended in a fluid (3). The theory has recently been extended into various forms by several authors (16, 21) for application in plasma theory. It is not surprising that the theory of Brownian motion should have so much resemblance to that of a plasma, since in both cases the instantaneous motion of a particle is governed by its interaction with many other particles; in the former case due to the finite size of the test particle, while in the latter case due to the long range behavior of the Coulomb field.

In this model the collision operator has the basic form

$$\left(\frac{\partial f}{\partial t}\right)_{Coll} = \nu \frac{\partial}{\partial \vec{v}} \cdot \left[\frac{T}{m} \frac{\partial f}{\partial \vec{v}} + (\vec{v} - \vec{u}) f \right] \tag{1.6}$$

where ν is an effective collision frequency and is usually assumed to be constant. For simplicity, in Eq. (1.6) we have omitted some details related to the conservation properties of the plasma. These details shall be introduced when the full theory is presented in the next section. It suffices here to note that the form of the collision term resembles that of the FP operator. The first term depicts diffusion in the velocity space with diffusion coefficient $\nu T/m$, and the second term depicts friction in the velocity space with the coefficient of friction $\nu(\vec{v} - \vec{u})$. Unlike the FP equation for a plasma, the collision frequency here is not given by the theory and is taken to be arbitrary. Thus the details of the collision mechanism is not explicitly included.

We note that in this model the friction coefficient is proportional to the particle velocity relative to the average velocity. This assumption is valid for most of the particles, the important exception being the fast particles in the high energy tail of the distribution. For these fast particles the friction coefficient should be proportional to v^{-3} (Ref. 19). Thus, problems associated with these high energy particles, such as the runaway phenomenon (7), cannot be predicted by this model. Furthermore we note that the diffusion coefficient is isotropic and independent of velocity. Since the perturbation in the particle velocity due to diffusion arises from those particles within the Debye sphere of the particle, the above property implies that the spectrum of the electric field fluctuation is isotropic at the scale length of the Debye radius. Hence the model is valid when the Debye length is small compared to the Larmor radius of a thermal particle (4).

Equation (1.6) may be derived by assuming that a particle in the plasma is subjected to simultaneous grazing collisions with many other particles such that its progress in velocity space becomes a random walk (3). In this case ν^{-1} becomes the time required for the test particle to change its direction appreciably. Alternatively, one expects that the model operator ought to be derivable from more realistic kinetic equations under appropriate assumptions as a limiting case. Recently, Doughherty and Watson (6) succeeded in obtaining this model operator for a nearly uniform plasma based on an expansion of the full FP equation, with the diffusion and friction coefficients chosen to give the correct rates of transfer of momentum and energy among the thermal particles. However, such derivations are not unique and further research on the foundations of this model is still needed.

II. DERIVATION OF THE DISPERSION RELATION

2.1 Several forms of the proposed model FP equation exist in the literature. Lenard and Bernstein (16) considered a model with a simplified friction term which omits the average velocity $\vec{u}$ in Eq. (1.6) and investigated the collisional damping of waves near the plasma frequency. Zakharov and Karpman (32) showed that the effect of collisions in the Landau wave-particle resonance region can be represented by a term similar to that of Lenard and Bernstein and obtained the corresponding effective collision frequency. They used this equation to investigate the relative significance of the collisional and quasi-linear effects (8) on small amplitude plasma waves. Karpman (13) and others (32) later used the same equation to consider the effect of collisions on the initial value or ballistic terms in the solution of Landau's problem. Dougherty (5) introduced a formal method of solving the model FP equation for an electron plasma in a magnetic field and investigated the effect of collisions on cyclotron waves. Later, Oppenheim (21) considered a similar problem but included the motion of ions. The later considerably complicates the solution.

2.2 The model kinetic equation which shall be used here is

$$\frac{\partial f_j}{\partial t} + \vec{v}\cdot\frac{\partial f_j}{\partial \vec{x}} + \frac{\vec{F}_j}{m_j}\cdot\frac{\partial f_j}{\partial \vec{v}} = \sum J_{j\ell}(f_j) \tag{2.1}$$

$$J_{j\ell}(f_j) = \nu_{j\ell}\,\frac{\partial}{\partial \vec{v}}\cdot\left[\frac{1}{2\alpha_j}\frac{\partial}{\partial \vec{v}} + (\vec{v}-\vec{u}_{j\ell})\right] f_j \tag{2.2}$$

$$j = i, e, \qquad \ell = i, e$$

$$\vec{u}_{jj} = \vec{u}_j, \qquad \alpha_j = \frac{m_j}{2T_j} \tag{2.3}$$

where F_j is the force on species j and $\nu_{j\ell}$ is the effective collision frequency between species j and ℓ. The quantity $\vec{u}_{j\ell}$ $(\ell \neq j)$ is to be defined so that the following conservation equations are satisfied.

$$\int \begin{bmatrix} 1 \\ \vec{v} \\ (\vec{v}-\vec{u})^2 \end{bmatrix} J_{jj}\; d^3v = 0 \tag{2.4}$$

$$\int J_{j\ell}\; d^3v = 0, \qquad j \neq \ell \tag{2.5}$$

$$\int (m\, J_{ei} + M\, J_{ie})\, \vec{v}\; d^3v = 0 \tag{2.6}$$

where m and M are the electron and ion masses respectively. Equation (2.4) describes conservation of number density, momentum and energy in like particle interactions. Equations (2.5) and (2.6) represent conservation of total number and total momentum is unlike particle interactions. Conservation of total energy in unlike particle interactions is not imposed here, so that problems involving overall energy dissipation by the plasma may be considered. Because of this relaxation, the quantities $\vec{u}_{ie}$ and $\vec{u}_{ei}$ are no longer unique, instead they shall be chosen to satisfy other constraints such as energy conservation of the entire plasma-external field system. Thus we let

$$\begin{aligned} \vec{u}_{ei} &= \frac{M}{m} \frac{\nu_{ie}}{\nu_{ei}} \vec{u}_i \\ \vec{u}_{ie} &= \frac{m}{M} \frac{\nu_{ei}}{\nu_{ie}} \vec{u}_e \end{aligned} \tag{2.7}$$

for which the rate of energy loss by the plasma due to unlike particle collisions is

$$\begin{aligned} &-n \int \left(m\, J_{ei} + M\, J_{ie} \right) \frac{v^2}{2}\, d^3v \\ &= n\, m\, \nu_{ei}\, u_e^2 \end{aligned} \tag{2.8}$$

in the coordinate system with $\vec{u}_i = 0$. We shall show in the next section that this loss of energy is consistent with the maintenance of a steady state for a plasma in an external electric field.

Like the full FP collision terms, the model collision terms also vanish when the ions and electrons are in the same local Maxwellian state. Furthermore it can be shown that an H theorem is satisfied by the model equation (6, 21).

2.3 In this section the model equations are linearized for considering low frequency waves in an inhomogeneous plasma in constant magnetic and electric fields. The usual slab configuration (Fig. 1) is assumed.

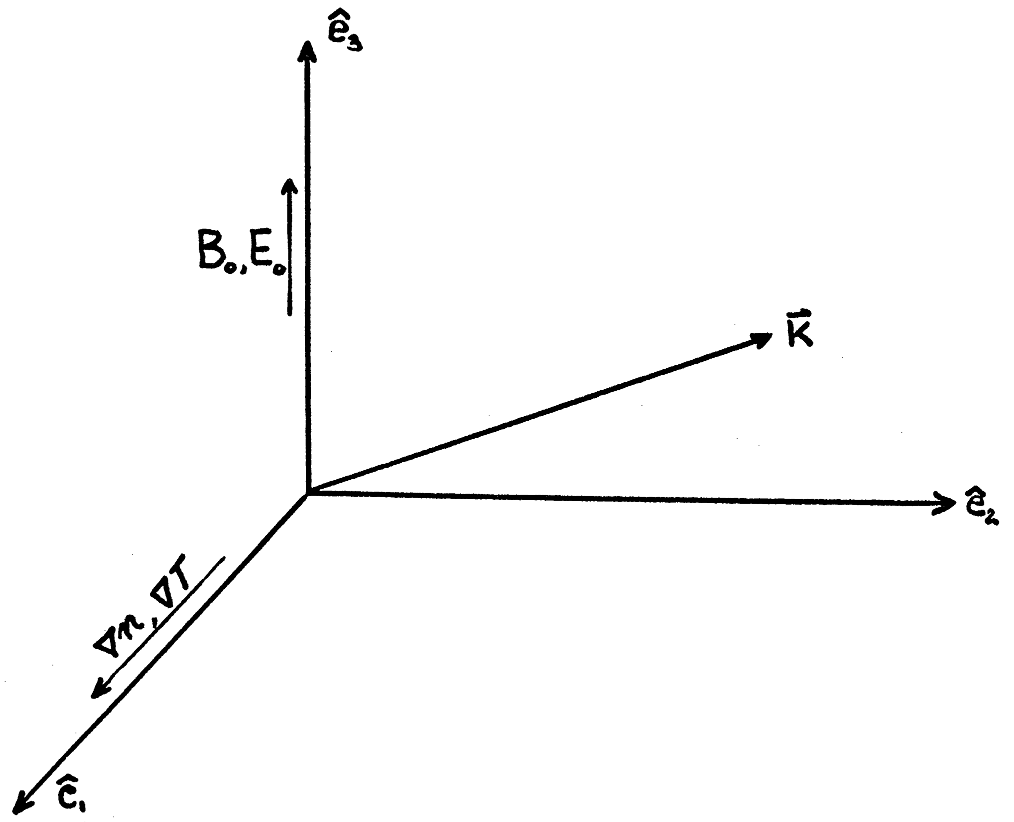

Fig. 1. The Slab Configuration.

In Fig. 1, $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ are unit vectors defining the coordinate directions, $\vec{E}_o$ and $\vec{B}_o$ are the external electric and magnetic fields, and $\vec{K}$ is the wave vector.

The kinetic equations describing this system are

$$\frac{\partial f_j}{\partial t} + \vec{v} \cdot \frac{\partial f_j}{\partial \vec{x}} + \frac{e_j}{m_j}\left[\vec{E} + \frac{\vec{v} \times \vec{B}}{c}\right] \cdot \frac{\partial f_j}{\partial \vec{v}}$$

$$= \nu_j \frac{\partial}{\partial \vec{v}} \cdot \left[\frac{1}{2\alpha_j} \frac{\partial f_j}{\partial \vec{v}} + (\vec{v} - \vec{u}_j^*) f_j\right], \qquad j = i, e \tag{2.9}$$

where

$$\alpha_j = \frac{m_j}{2T_j} = \frac{1}{v_{T_j}^2}$$

$$\nu_j = \nu_{je} + \nu_{ji}$$

$$\vec{u}_e^* = \frac{\nu_{ee} \vec{u}_e + \nu_{ei} \vec{u}_{ei}}{\nu_e}$$

$$\vec{u}_i^* = \frac{\nu_{ii} \vec{u}_i + \nu_{ie} \vec{u}_{ie}}{\nu_i} .$$

In the above we have combined the like and unlike particle interaction terms.

To linearize the equations for considering small amplitude waves, we let

In the coordinate system where $\vec{u}_{io} = 0$ we obtain

$$\vec{u}_{eo} = \frac{-e\vec{E}_o}{m\,\nu_{ei}} \,. \tag{2.14}$$

The rate of energy loss due to unlike particle collisions given by Eq. (2.8) is then $n_o\, e^2\, E_o^{\,2}/m\,\nu_{ei}$. This equals in magnitude the ohmic heat $n_o\, e\, u_{eo}\, E_o$ which would be transferred to the plasma by the applied electric field. If the total energy of unlike particle interactions were conserved, the plasma temperature would rise and there can be no steady state (31). Nevertheless a steady state exists here, since the energy input to the plasma is exactly equal to the energy loss. In the present problem the energy loss may be attributed to radiation.

Equation (2.9) in the first order is

$$\begin{aligned} &-(i\omega - i\vec{K}\cdot\vec{v} + 3\nu)f_1 + \frac{e}{m}\Big[\vec{E}_o + \frac{\vec{v}\times\vec{B}_o}{c} \\ &\quad - \nu(\vec{v} - \vec{u}_o^{\,*})\Big]\cdot\frac{\partial f_1}{\partial \vec{v}} - \frac{\nu}{2\alpha}\,\frac{\partial^2 f_1}{\partial \vec{v}^2} \\ &= -\Big(\frac{e}{m}\vec{E}_1 + \nu\,\vec{u}_1^{\,*}\Big)\cdot\frac{\partial f_o}{\partial \vec{v}} \end{aligned} \tag{2.15}$$

where we have assumed that there is no perturbation in temperature.

In the next section we shall solve Eq. (2.15) for the case of small collision frequencies.

$$
\begin{aligned}
f(\vec{x},\vec{v},t) &= f_0(x_1,\vec{v}) + f_1(\vec{v})\, e^{-i\omega t + i\vec{K}\cdot\vec{x}} \\
\vec{B} &= \vec{B}_0 = B_0\,\hat{e}_3 \\
\vec{E} &= E_0\,\hat{e}_3 + \vec{E}_1\, e^{-i\omega t + i\vec{K}\cdot\vec{x}} \\
\vec{u}^* &= \vec{u}_0^* + \vec{u}_1^*\, e^{-i\omega t + i\vec{K}\cdot\vec{x}} \, .
\end{aligned}
\tag{2.10}
$$

Equation (2.9) to the lowest order is

$$
\begin{aligned}
&v_1 \frac{\partial f_0}{\partial x_1} + \frac{e}{m}\left[\vec{E}_0 + \frac{\vec{v}\times\vec{B}_0}{c}\right]\cdot\frac{\partial f_0}{\partial \vec{v}} \\
&= \nu\, \frac{\partial}{\partial \vec{v}}\cdot\left[\frac{1}{2\alpha}\frac{\partial f_0}{\partial \vec{v}} + (\vec{v}-\vec{u}_0^*)\, f_0\right] .
\end{aligned}
\tag{2.11}
$$

Here the specie subscript j has been suppressed. For $\nu/\Omega \ll 1$, where Ω is the Larmor frequency. An approximate solution of Eq. (2.11) is the near Maxwellian distribution (14)

$$
f_0 = n_0 \left(\frac{\alpha}{\pi}\right)^{3/2} e^{-\alpha(\vec{v}-\vec{u}_0)^2}
\tag{2.12}
$$

where n_0, α may be functions of the constants of motion v^2, $|v_3 - u_0|$ and $x_1 + v_2/\Omega$.

The velocity u_0 is given by the equation

$$
\frac{e\vec{E}_0}{m} = \nu\left(\vec{u}_0 - \vec{u}_0^*\right) .
\tag{2.13}
$$

2.4 If the gradients are weak, the lowest order distribution f_o may be expanded, say near the point $x_1 = 0$ of maximum density gradient,

$$f_o(x_1, \vec{v}) \approx \left\{ 1 + \left(x_1 + \frac{v_2}{\Omega}\right)\left[\epsilon + \frac{3\delta}{2} - \delta\alpha\,(\vec{v} - \vec{u}_o)^2\right] \right\} f_{Max} \tag{2.16}$$

where

$$\epsilon = \frac{1}{n_o} \frac{d n_o}{d x_1}\bigg|_{x_1=0}, \qquad \delta = -\frac{1}{T}\frac{dT}{dx_1}\bigg|_{x_1=0}$$

and

$$f_{Max} = n_o(0)\left(\frac{\alpha(0)}{\pi}\right)^{3/2} e^{-\alpha(0)(\vec{v} - \vec{u}_o)^2} \tag{2.17}$$

is the Doppler shifted Maxwell-Boltzmann distribution.

To solve Eq. (2.15) we first introduce a Fourier transformation in the velocity space such that

$$G(\vec{\sigma}) = \int_{-\infty}^{\infty} e^{-i\,\vec{v}\cdot\vec{\sigma}} f_1(\vec{v})\, d^3v\,. \tag{2.18}$$

Equation (2.15) then becomes

$$a(\vec{\sigma})\,G(\vec{\sigma}) + \vec{b}(\vec{\sigma})\cdot\frac{\partial G(\vec{\sigma})}{\partial\vec{\sigma}} = c(\vec{\sigma}) \tag{2.19}$$

where

$$a(\vec{\sigma}) = -i\omega + \frac{\nu\sigma^2}{2\alpha} + i\nu u_0\sigma_3$$

$$\vec{b}(\vec{\sigma}) = -\vec{K} + \nu\vec{\sigma} + \vec{\sigma}\times\vec{\Omega}$$

$$c(\vec{\sigma}) = -n_0\left(\frac{e}{m}\vec{E}_1 + \nu\vec{u}_1^*\right)\left[i\vec{\sigma} + \frac{\hat{e}_2\epsilon}{\Omega} + \frac{\hat{e}_2\delta\sigma^2}{4\alpha\Omega}\right]\exp\left(-i\sigma_3 u_0 - \frac{\sigma^2}{4\alpha}\right). \tag{2.20}$$

The quantities $\vec{E}_1$ and $\vec{u}_1{}^*$ may be expressed in terms of the electron and ion number densities by using the Poisson and continuity equations respectively,

$$\frac{\partial}{\partial\vec{x}}\cdot\vec{E} = 4\pi\sum_j e_j n_j \tag{2.21}$$

$$\frac{\partial}{\partial t}n_j + \frac{\partial}{\partial\vec{x}}\cdot(n_j\vec{u}_j) = 0. \tag{2.22}$$

From these one obtains the relations

$$\vec{E}_1 = -\frac{4\pi i\vec{K}}{K^2}\sum_j e_j n_{j1} \tag{2.23}$$

$$\vec{u}^{*}_{j1} = -\frac{i\,(\epsilon\hat{e}_1 + i\vec{K})}{n_0 K^2 \nu_j}\Big[\nu_{jj}(\omega - K_3 u_{j0})\, n_{j1}$$

$$+ \nu_{\ell j}\frac{m_\ell}{m_j}(\omega - K_3 u_\ell)\, n_{\ell 1}\Big], \qquad \ell \neq j\,. \tag{2.24}$$

Equation (2.19) is a first order partial differential equation and may be solved by the method of characteristics. Introducing the parametric variable t, we obtain the characteristic equations

$$\frac{d\vec{\sigma}(t)}{dt} = \vec{f}\,[\vec{\sigma}(t)] \tag{2.25}$$

$$a[\vec{\sigma}(t)]\, G[\vec{\sigma}(t)] + \frac{d}{dt} G[\vec{\sigma}(t)] = c\,[\vec{\sigma}(t)] \tag{2.26}$$

where $\vec{\sigma}\,(t = 0) = \vec{\sigma}$.

The above set of first order ordinary differential equations may be readily integrated. Equation (2.25) has solutions

$$\sigma_1(t) = \frac{-K_2\Omega}{\Omega^2+\nu^2} + e^{\nu t}\Big[\Big(\sigma_1 + \frac{K_2\Omega}{\Omega^2+\nu^2}\Big)\cos\Omega t + \Big(\sigma_2 - \frac{K_2\nu}{\Omega^2+\nu^2}\Big)\sin\Omega t\Big]$$

$$\sigma_2(t) = \frac{K_2\Omega}{\Omega^2+\nu^2} + e^{\nu t}\Big[\Big(\sigma_2 - \frac{K_2\nu}{\Omega^2+\nu^2}\Big)\cos\Omega t - \Big(\sigma_1 + \frac{K_2\Omega}{\Omega^2+\nu^2}\Big)\sin\Omega t\Big] \tag{2.27}$$

$$\sigma_3(t) = \frac{K_3}{\nu} + \Big(\sigma_3 - \frac{K_3}{\nu}\Big)e^{\nu t}.$$

The integral of Eq. (2.26) is

$$G[\vec{\sigma}(t)] = \int_{-\infty}^{t} \exp\left\{ \int_{t}^{t'} a[\sigma(t'')]\, dt'' \right\} \cdot c\left[\vec{\sigma}(t)\right] dt' \tag{2.28}$$

where we have assumed $G[\vec{\sigma}(t=-\infty)] = 0$.

The integral (2.28) is extremely complicated to evaluate. Fortunately since we are only interested in the perturbed number density n_1 where

$$n_1 = \int f_1\, d^3v = G[\vec{\sigma}(0) = 0], \tag{2.29}$$

the matter is much simplified.

It is then apparently possible to evaluate the integral numerically and obtain solutions for all values of the collision frequencies. However, we shall consider only the case of small collision frequencies. For this purpose the evaluation of the integrals becomes quite straightforward and analytical results may be obtained.

Neglecting terms of orders $(\nu/\Omega)^2$ and $(\nu/K_3 v_T)^2$, and using the formula

$$\exp(b \cos \Omega t) = \sum_{-\infty}^{\infty} I_l(b)\, e^{il\Omega t}$$

where I_ℓ is the modified Bessel function of order ℓ, we find the useful relation

$$\exp\left[-i\sigma_3 u_0 - \frac{\sigma^2}{4\alpha} + \int_0^t a\,dt\right]$$

$$\approx \sum_{-\infty}^{\infty}\left[1 + b\nu t\cos\Omega t - \frac{2\nu b}{\Omega}\sin\Omega t\right.$$

$$\left. - \frac{K_3^2\nu t^3}{12\alpha}\right] I(b)\exp\left[-i\omega_\ell t - \frac{K_3^2 t^2}{4\alpha} - b\right]$$

where

$$\omega_\ell = \omega - K_3 u_0 + i b\nu - \ell\Omega$$

$$b = \frac{K_2^2}{2\alpha\Omega^2} = \frac{1}{2}K_2^2 R^2 .$$

Substituting Eq. (2. 30) into Eq. (2. 29) and integrating, one obtains after some manipulation

$$n_1 = -\frac{n_0 e\phi}{m} e^{-b}\sum\left[\frac{\alpha^{1/2}}{K_3\Omega}P_1 + \frac{i\alpha\nu}{K_3^2\Omega}P_2\right.$$

$$\left. - \frac{i b\alpha^{3/2}\nu}{K_3}P_3\right] + \frac{i n_0\alpha\nu}{K_3}e^{-b}P_4 . \qquad (2.\ 31)$$

In obtaining the above result we have used the relation

$$\int_{-\infty}^{0} t^m e^{-i\omega t - \beta^2 t^2} dt = \frac{i^{m-1}}{(2\beta)^{m+1}} Z^{[m]}\left(\frac{\omega}{2\beta}\right) \tag{2.32}$$

where

$$Z(\varsigma) = \pi^{-1/2} \int_{-\infty}^{\infty} \frac{e^{-x^2}}{x-\varsigma} dx \tag{2.33}$$

is the plasma dispersion function and the bracketed superscript denotes derivative with respect to the argument.

The functions P_1 to P_4 are defined as follows

$$P_1 = 2\ell\alpha\Omega^2 Z_\ell I_\ell - K_3\alpha^{1/2} Z_\ell' I_\ell - K_2 \epsilon Z_\ell I_\ell + K_2 b \delta Z_\ell (I_\ell' - I_\ell) - \frac{1}{4} K_2 \delta Z_\ell'' I_\ell \tag{2.34}$$

$$P_2 = \frac{1}{12}\ell\alpha\Omega^2 Z_\ell''' I_\ell + \frac{1}{2} K_3\alpha^{1/2}\Omega Z_\ell'' I_\ell + \ell K_2^2 Z_\ell' I_\ell' + 2K_3\alpha^{1/2}\Omega\left(\ell^2 Z_\ell - \frac{1}{24} Z_\ell^{IV}\right) I_\ell - 2\ell K_3^2 Z_\ell' I_\ell - \frac{1}{12} K_2 \epsilon I_\ell Z_\ell''' + \frac{2\ell K_2 K_3}{\alpha^{1/2}\Omega} Z_\ell I_\ell \tag{2.35}$$

$$P_3 = Z_\ell I_\ell + Z_\ell I_\ell' + \frac{K_2 \epsilon}{K_3 \alpha^{1/2} \Omega} Z_\ell' I_\ell' \tag{2.36}$$

$$P_4 = \sum_{-\infty}^{\infty} \Bigg[\frac{i k_2 u_{11}^*}{\alpha^{1/2} \Omega} Z_\ell (I_\ell' - I_\ell) - \frac{2\ell}{k_2} \alpha^{1/2} \Omega \, u_{12}^* Z_\ell I_\ell$$

$$+ u_{13}^* Z_\ell' I_\ell \Bigg] \qquad (2.37)$$

where

$$Z_\ell = Z\left(\frac{\omega_\ell}{k_3 v_T}\right).$$

The second subscripts on u_1* denote coordinate components.

The dispersion relation may now be derived by combining Eq. (2. 23), (2. 24) and (2. 31).

III. THE ION-ACOUSTIC WAVE INSTABILITY

3.1 In this chapter we consider the effect of collisions on the ion-acoustic wave instability (9). This instability may occur when the electrons and ions have different mean velocities, which in our case are produced by the external electric field. To minimize unnecessary complication, we let $K_2 = 0$, so that only wave propagation parallel to the magnetic field is considered. This assumption greatly simplifies the problem since the effect of inhomogeneity is thereby eliminated.

Many authors have considered this problem (see for example Reiter (24) and the references therein) using different collision models and assumptions. It was generally found that the presence of even a small amount of collisions can significantly alter the stability condition as well as the growth rate of the instability. We shall here reconsider this problem and compare our results with those of other authors.

3.2 For $K_2 = 0$, Eq. (2.23), (2.24), and (2.31), from which the dispersion relation is to be derived, are greatly simplified. Assuming further that $\omega \ll \Omega$, these equations become respectively

$$K^2 \phi = 4\pi \sum e_j n_{j1} \tag{3.1}$$

$$u_{j1}^* = \frac{1}{n_0 K \nu_j}\Big[\nu_{jj}(\omega - K_3 u_{j0}) n_{j1} + \nu_{\ell j}\frac{m_\ell}{m_j}(\omega - K_3 u_{\ell 0}) n_{\ell 1}\Big] \tag{3.2}$$

$$n_{j1} = \frac{2\alpha_j n_0 e_j \Phi}{m_j}\Big[\Big(1 + \frac{i\nu_j \alpha_j \omega_j}{K^2} - \frac{2i\nu_j \alpha_j^2 \omega_j^3}{3K^4}\Big) W_j + \frac{i\nu_j \alpha_j \omega_j}{3K^2}\Big] + \frac{2i\alpha_j \nu_j n_0}{K} u_{j1}^* W_j \tag{3.3}$$

where $\vec{K} = K_3\hat{e}_3$, $\vec{E}_1$ and $\vec{u}_j{}^*$ are parallel to the magnetic field, and

$$W_j = \frac{1}{2} Z'\Big(\frac{\omega_j}{K v_{Tj}}\Big) \tag{3.4}$$

$$\omega_j = \omega - u_{j0}, \qquad -\nabla\Phi = \vec{E}.$$

We note that the effect of collisions does not enter directly in the plasma dispersion function as is the case for the BGK model (24).

Eq. (3. 1), (3. 2) and (3. 3) constitute a set of five algebraic equations with five unknowns n_e, n_i, $\vec{u}_e^*$, $\vec{u}_i^*$ and Φ. The dispersion relation is obtained by requiring the determinant of the coefficients of the unknowns to vanish, thus we obtain

$$K^2 = 4\pi \sum e_j \bar{n}_{j1} \tag{3.5}$$

where

$$\bar{n}_{j1} = \frac{2 n_0 \alpha_j e_j}{m_j}\Big[\Big(1 + \frac{i \nu_j \alpha_j \omega_j}{K^2} + \frac{2 i \nu_j \alpha_j^2 \omega_j^3}{3 K^4}\Big) W_j$$

$$+ \frac{2 i \alpha_j}{3K^2}\Big(\nu_{jj}\, \omega_j + \nu_{\ell j} \frac{m_\ell}{m_j}\, \omega_\ell\Big) W_j^2 + \frac{i \nu_j \alpha_j \omega_j}{3 K^2}\Big] \qquad (3.6)$$

$$- \frac{i \alpha_j}{2\pi e_j} \frac{m_\ell}{m_j}\, \omega_\ell\, W_j\,, \qquad \ell \neq j\,.$$

3.3 To consider the ion-acoustic wave instability we assume $v_{Ti} < (\omega/K) < v_{Te}$ (27), so that the Landau damping effect due to ions is insignificant. The limits of the W functions are given by

$$\lim_{\varrho \to 0} W(\varrho) \to -1 + 2\varrho^2 - i\sqrt{\pi}\, \varrho\, e^{-\varrho^2} \qquad (3.7)$$

$$\lim_{\varrho \to \infty} W(\varrho) \to \frac{1}{2\varrho^2} - i\sqrt{\pi}\, \varrho\, e^{-\varrho^2} \qquad (3.8)$$

Using these expansions, Eq. (3.6) written for electrons and ions are

$$\bar{n}_{e1} = \frac{-2 n_0 \alpha_e e}{m}\Big[-1 - \frac{2 i \nu_e \alpha_e \omega_e}{3 K^2} + \frac{2 \omega_e^2}{K^2 v_{Te}}$$

$$- i\sqrt{\pi}\, \frac{\omega_e}{K v_{Te}} + \frac{2 i \alpha_e}{3K^2}\Big(\nu_{ee}\, \omega_e + \nu_{ie} \frac{M}{m}\omega\Big)\Big(1 - 2 i\sqrt{\pi}\, \frac{\omega}{K v_{Te}}\Big)\Big] \qquad (3.9)$$

$$- \frac{i \alpha_e}{2\pi e} \frac{M}{m}\, \nu_{ie}\, \omega\Big(1 + i\sqrt{\pi}\, \frac{\omega}{K v_{Te}}\Big)$$

$$\bar{n}_{i1} = \frac{2 n_0 \alpha_i e}{M}\left(\frac{K^2}{2\alpha_i\omega^2} + \frac{i\nu_i}{2\omega}\right) + \frac{iK^2 m}{4\pi e M}\nu_{ei}\frac{\omega_e}{\omega^2} \tag{3.10}$$

where we have assumed $u_{io} = 0$. Furthermore, terms of order $\omega_e^3/(K^3V_{Te}^3)$, $(K^4V_{Ti}^4)/\omega^4$ and above are neglected.

The dispersion relation (3. 5) then becomes

$$\begin{aligned} 0 = & -1 - K^2\lambda_e^2 - \frac{2i\nu_e\alpha_e\omega_e}{3K^2} + \frac{2\alpha_e\omega_e^2}{K^2} - \frac{i\sqrt{\pi}\,\omega_e}{K v_{Te}} \\ & + \frac{2i\alpha_e}{3K^2}\left(\nu_{ee}\omega_e + \nu_{ie}\frac{M}{m}\omega\right) \\ & + \frac{T_e}{T_i}\left(\frac{K^2}{2\alpha_i\omega^2} + i\frac{\nu_i}{\omega}\right) + iK^2\lambda_e^2\nu_{ei}\frac{m}{M}\frac{\omega_e}{\omega^2} \\ & + O\left(\frac{\nu_{ie}\omega}{\omega_{pe}}, \frac{\omega_e^2\nu_e}{K_3^3 v_{Te}^3}, \frac{\omega_e^3}{K_3^3 v_{Te}^3}, \frac{K^4 v_{Ti}^4}{\omega^4}\right) \end{aligned} \tag{3.11}$$

where $\lambda_e = \left[\frac{T_e}{4\pi n_0 e^2}\right]^{1/2}$ is the electron Debye length.

Assuming $\omega = \omega_0 + i\gamma$ where $\gamma \ll \omega_0$, the real part of the dispersion relation is

$$K^2(1+K^2\lambda_e^2) = 2\alpha_e(\omega_0 - K u_{e0})^2 + \frac{K^4}{2\alpha_i\omega^2}\frac{T_e}{T_i}\,. \tag{3.12}$$

Since we are interested in the case where $\omega_0 \approx K u_{eo}$, the approximate solution of Eq. (3. 12) is

$$\frac{\omega_o}{K} = \left[\frac{T_e}{M(1+K^2\lambda_e^2)}\right]^{1/2} \tag{3.13}$$

which gives the phase velocity of the ion-acoustic wave.

The imaginary part of Eq. (3.11) gives

$$\gamma = -\left[\frac{T_e}{T_i}\frac{1}{\omega_o^3} - 4\frac{m}{M}\frac{T_i}{T_e}(\omega_o - u_{eo})\right]^{-1}\left\{\left[\left(\pi\frac{mT_i}{MT_e}\right)^{1/2}\right.\right.$$
$$\left.\left. + \frac{2}{3}\frac{m}{M}\frac{T_i}{T_e}\nu_{ei}\right](\omega_o - u_{eo}) + \frac{T_e}{T_i}\frac{\nu_i}{\omega_o} + 2\omega_o\nu_{ie}\frac{T_i}{T_e}\right\} \tag{3.14}$$

where all the frequencies have been non-dimensionalized with respect to Kv_{Ti}, and u_{eo} non-dimensionalized with respect to v_{Ti}. The term containing $\sqrt{\pi}$ gives the usual collisionless growth rate due to resonant interaction between electrons and waves. From the remaining terms we see that electron-ion collisions in general enhance the instability, while other collisions damp it. This result is in agreement with that of the collision dominated theory (11) in which resistivity due to electron-ion collisions is the cause of the instability.

The marginal electron current necessary for the onset of the instability is given by

$$u_{eo} \approx \omega_o - \left(\frac{MT_e}{\pi m T_i}\right)^{1/2}\left(\frac{T_e}{T_i}\frac{\nu_i}{\omega_o} + 2\nu_{ie}\omega_o\frac{T_i}{T_e}\right) \tag{3.15}$$

which we note is independent of the electron collision frequencies.

In an earlier work (20) a more detailed calculation including higher order terms in the approximation has been performed[3]. The resulting stability diagram is given in Fig. 2. In obtaining this diagram we have assumed a hot dilute plasma with $\nu_{ei} \approx 10^{-7}\ \omega_{pe}$ and have used the following relation between the collision frequencies

$$\nu_{ee} : \nu_{ei} : \nu_{ii} : \nu_{ie} = 1 : 1 : \left(\frac{m}{M}\right)^{1/2}\left(\frac{T_e}{T_i}\right)^{3/2} : \frac{m}{M}\left(\frac{T_e}{T_i}\right)^{3/2} \tag{3.16}$$

3.4 From Fig. 2 we see that the effect of collisions is to reduce the critical current for the onset of ion-acoustic wave instabilities. Using an expansion of the full FP equation, Kulsrud and Shen (5) obtained a similar conclusion for a plasma with $(T_e/T_i) < 20$, and $K\lambda_e = 0$. However, due to their infinite wavelength assumption, their estimated critical current is not necessarily the smallest current required to excite the ion-acoustic wave instability.

From the diagram we also note that the adverse effect of weak collisions reduces as T_e/T_i decreases. This is in agreement

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[3] In that paper we used a different and less straightforward approach in deriving the dispersion relation, but the principles and results are the same as given here.

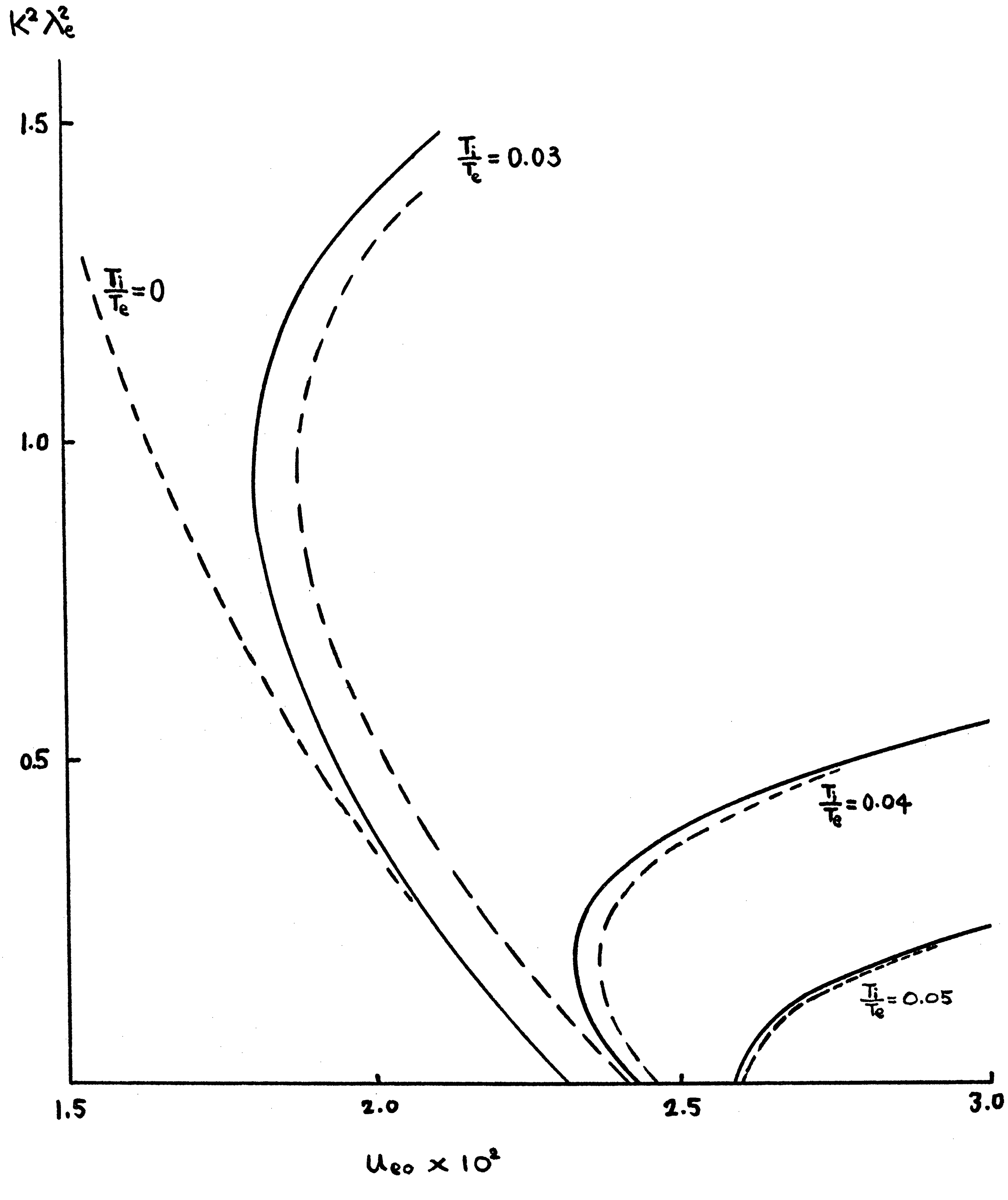

Fig. 2. Marginal Stability Curves. The solid curves represent collisional results, the dashed curves collisionless.

with the results of Reiter (24), who used the BGK model with $T_e = T_i$ and predicted only collisional damping, i. e., an increase in the critical current.

Finally we note that as $K\lambda_e$ increases, the collisional effects become less and less significant.

IV. THE DENSITY AND TEMPERATURE GRADIENT INSTABILITIES

4.1 In an inhomogeneous plasma drift waves can arise as a result of the combined effects of density gradient, transverse ion inertia and parallel electron motion. The stability of drift waves in a collisionless plasma has been investigated by many authors (see, for example, Krall and Rosenbluth (14) and references therein). Many different modes of instability are possible, due to the different relationships between the phase velocity of the wave and the thermal velocities of the charged particles, in other words the Landau wave-particle resonance effect.

The problem concerning the effect of collisions on the stability of drift waves has been considered by several authors using different collision models and approximations. Mikhailovskii and Pogutze (18) used the BGK model and found that electron-ion collisions generally increase the growth rate of the instability while ion-ion collisions reduce it. Bhadra (2) used an iterative procedure to solve the full FP equation for electrons while neglecting collisional effects on the ion motion equation. He found that the growth rates obtained from the FP and BGK models are quite different. It was suggested that the disagreement may be due to the velocity space diffusional nature of the FP collision operator, since the BGK model, being of a simple relaxation type, does not have this feature.

In this section we shall consider the problem in detail using the model FP collision operator.

4.2 To consider the drift wave instability, we assume $u_0 = 0$, $\omega \ll \Omega$ and $K_2, K_3 \neq 0$. Neglecting terms of order higher than ω/Ω, the perturbation number density given by Eq. (2.31) becomes

$$
\begin{aligned}
n_{j1} = \frac{-2 n_0 \alpha_j e_j \phi}{m_j} \Big\{ & 1 - (1+W_j) A_j \Big[1 - \frac{\omega_j^*}{\omega_j}\Big(1 - \frac{\eta_j}{2} \\
& + (C_j - 1) b_j \eta_j \Big) - \frac{i \nu_j b_j}{\omega_j} \Big] \\
& - W_j A_j \Big[\frac{K_2 \delta \omega_j}{2 K_3^2 \Omega_j} + \frac{i \nu_j \alpha_j \omega_j}{K_3} (1 - 2 b_j B_j) \\
& - \frac{2 i \nu_j \alpha_j^2 \omega_j^3}{3 K_3^4} \Big] - \frac{i \nu_j \alpha_j \omega_j A_j}{3 K_3^2} \\
& - \frac{i \nu_j \alpha_j \omega_j^*}{K_3^2} \Big[-2 b_j B_j W_j + \frac{1}{3} \Big(-1 + 3 W_j \\
& + \frac{2 \alpha_j \omega_j^2}{K_3^2} W_j \Big) A_j \Big] \Big\} \\
& + 2 i n_0 \alpha_j \nu_j \Big\{ \frac{i K_2 U_{j11}^*}{2 \alpha_j \Omega_j \omega_j} (A_j - B_j)(1 + W_j) \\
& - \frac{U_{j12}^*}{K_2} (1 - A_j) + \frac{U_{j13}^*}{K_3} A_j W_j \Big\}
\end{aligned}
\tag{4.1}
$$

where

$$A_j = I_0(b_j)\, e^{-b_j}, \qquad B_j = I_1(b_j)\, e^{-b_j}$$

$$C_j = I_1(b_j)/I_0(b_j)$$

$$\omega_j = \omega - i\, b_j \nu_j\,, \qquad \omega_j^* = \frac{k_2 \epsilon}{2\alpha_j \Omega_j}$$

$$\eta_j = -\frac{\delta_j}{\epsilon_j} = \frac{T_j'/T_j}{n_0'/n_0}\,.$$

We have assumed $\epsilon_i = \epsilon_e$ following the neutrality condition.

The dispersion relation may now be derived from Eq. (2.23), (2.24) and (4.1). After some algebra we obtain

$$K^2 = 4\pi \sum_j e_j \bar{n}_{j1} \tag{4.2}$$

$$\begin{aligned} \bar{n}_{j1} = -\frac{1}{4\pi e_j \lambda_j^2} \Big\{ 1 &- (1+W_j) A_j \Big[1 - \frac{\omega_j^*}{\omega_j} \Big(1 - \frac{\eta_j}{2} \\ &- (1 - C_j)\, b_j \eta_j \Big] + \eta_j \omega_j \omega_j^* W_j A_j \\ &+ i \nu_j Q_{j1} - 2 i \nu_j \omega_j^* \eta_j A_j Q_{j2} \Big\} \\ &+ \frac{i \nu_{jj} K_3^2 \omega_j}{2\pi e_j} Q_{j3} + \frac{i K_3^2 \omega_j}{2\pi e_j K^2 \lambda_j^2} \Big(\nu_{jj} + \nu_{ll} \frac{m_l}{m_j} \Big) Q_{j4} \end{aligned} \tag{4.3}$$

where all the frequencies have been non-dimensionalized with respect to $K_3 v_{T_j}$ and

$$Q_{j1} = \omega_j A_j \left[\frac{b_j}{\omega_j} - (1 - 2 b_j C_j)\omega_j + \frac{2}{3}\omega_j^3 \right] + \left(\frac{b_j}{\omega_j} - \frac{\omega_j}{3} \right) A_j + \omega_j^* A_j \left[\frac{1}{3} + \omega_j - 2 b_j \frac{B_j}{A_j} \omega_j - \frac{2}{3}\omega_j^2 W_j \right] \tag{4.4}$$

$$Q_{j2} = \frac{2}{3}(11 - 2 b_j + 5 b_j C_j) + (1 - A_j^{-1}) b_j \frac{K_3^2}{K_2^2} - \frac{1}{3}\omega_j^2 (14 - 2 b_j + 5 b_j C_j) + \left(8 - 2 b_j + \frac{1}{2} b_j^2 + b_j C_j - b_j^2 C_j\right) W_j - 24 \omega_j^2 W_j + 8 \omega_j^4 W_j \tag{4.5}$$

$$Q_{j3} = 1 - (1 + W_j) A_j - \frac{\omega_j^*}{\omega_j}(A_j - B_j)(1 + W_j) \tag{4.6}$$

$$Q_{j4} = Q_{j3} \left\{ 1 - (1 + W_j) A_j \left[1 - \frac{\omega_j^*}{\omega_j}\left(1 - b_j \eta_j (1 - C_j) + \frac{1}{2}\eta_j\right) \right] + \omega_j \omega_j^* \eta_j W_j A_j \right\} \tag{4.7}$$

In the following sections we shall investigate the stability of drift waves using the above dispersion relation.

4.3 If the temperature gradient is absent, the drift wave instability may occur if (11)

$$v_{T_i} < \frac{\omega}{K_3} < v_{T_e} \tag{3.1}$$

Assuming $\eta_j = 0$, and $\omega = \omega_o + i\gamma$, $\gamma \ll \omega_e$ the real part of the dispersion relation (4.2) yields

$$\frac{\omega_i^*}{\omega_o} = \frac{1 + \frac{T_i}{T_e} + K^2\lambda_i^2 - (1+W_i)A_i - \frac{T_i}{T_e}(1+W_e)}{(1+W_e) - (1+W_i)A_i} \tag{4.8}$$

where we have assumed $b_e = 0$ and all the frequencies have been normalized with respect to $K_3 v_{T_i}$. The subscript in b_i is dropped since no ambiguity results.

Under the conditions (3.1) and (3.16), and neglecting terms of order higher than m/M, the imaginary part of Eq. (4.2) yields

$$\gamma = \gamma_L + \gamma_e + \gamma_i \tag{4.9}$$

with γ_L, γ_e and γ_i given as follows,

$$\gamma_L = \left(\pi \frac{m}{M}\right)^{1/2} \frac{\omega_o^2}{\rho - A}\left[\frac{\rho - 2A}{A} - \left(\frac{M}{m}\right)^{1/2} \rho\, e^{-\omega_o^2}\right] \tag{4.10}$$

$$\gamma_e = -\frac{m}{M}\,\frac{\nu\,\omega_0^2}{\varrho - A}\Bigg\{-\frac{4}{3}\,\frac{2A-\varrho}{A} + 2\,\frac{K_3^2}{K^2} - 2\left[\frac{K_3^2}{K^2}(\varrho-1) - K_3^2\lambda_i^2\right](1+\varrho - 2A - \varrho C + B)\Bigg\} \tag{4.11}$$

$$\gamma_i = \left(\frac{m}{M}\right)^{1/2}\frac{\omega_0^2}{\varrho - A}\Bigg\{-\varrho\,\ell\,\frac{\nu}{\omega_0^2} + \frac{\nu}{2\omega_0^2}(A+2\ell B) + \frac{\ell\nu}{\omega_0^2}(\varrho - A)\,C - \frac{\nu}{2\omega_0^2 A}(\varrho - A) - \frac{2K_3^2}{K^2}\,\nu\,(\varrho-1)\left[1-\varrho-2A+B-\varrho C\right]\Bigg\} \tag{4.12}$$

$$\varrho = 2 + K^2\lambda_i^2 \tag{4.13}$$

where we have used the relation (3.16) and let $\nu = \nu_{ee}$. The two terms in γ_L gives respecitvely the electron and ion Landau resonance effects of a collisionless plasma (14) the damping effect of the latter is small, nevertheless it has been included for completeness. The term γ_e includes only the effect of electron-electron and electron-ion collisions, we note that it is of order $(m/M)^{1/2}$ smaller than γ_i, which gives the effect of ion-ion collisions. In view of the relations (3.16) and (4.2) we have also assumed $T_e = T_i$, with the understanding that the effect of ion-ion collisions would be enhanced if $T_e > T_i$.

4.4 In this section we shall analyze the above results in the limits of small and large b, which we recall is a measure of the combined effects of the ion Larmor radius and the transverse wavelength.

(a) For $b \ll 1$, Eq. (4.10), (4.11) and (4.12) become respectively

$$\gamma_L = \left(\pi \frac{m}{M}\right)^{1/2} \frac{\omega_0^2}{1+K^2\lambda_i^2+b}\left[\frac{K^2\lambda_i^2+2b}{1-b} - \left(\frac{M}{m}\right)^{1/2} \varsigma\, e^{-\omega_0^2}\right] \tag{4.14}$$

$$\gamma_e = \frac{m}{M}\frac{\nu\omega_0^2}{1+b+K^2\lambda_i^2}\left\{\frac{4}{3}\frac{K^2\lambda_i^2+2b}{1-b} - 2\frac{K_3^2}{K^2}\left[K^2\lambda_i^2 + \frac{1}{2}b\left(3-K^2\lambda_i^2\right)\right]\right\} \tag{4.15}$$

$$\gamma_i = \left(\frac{m}{M}\right)^{1/2}\frac{\omega_0^2}{1+K^2\lambda_i^2+b}\left\{-\frac{7\nu b}{2\omega_0^2} - \frac{\nu}{2\omega_0^2}K^2\lambda_i^2(1+3b) - 2\frac{K_3^2}{K^2}\nu\left(1+K^2\lambda_i^2\right)\left[1+K^2\lambda_i^2+\frac{1}{2}b\left(3-K^2\lambda_i^2\right)\right]\right\}. \tag{4.16}$$

The terms in γ_L are the familiar electron resonant growth and (weak) ion Landau damping respectively.

The first term in γ_e, which exerts a destabilizing effect, represents essentially the drift-dissipative instability (11). This instability is due to the reduction of electron mobility by collisional friction with ions, thus slowing down the redistribution of the electrons so that any density fluctuation will continue to grow. The stabilizing second term is the result of collisional coupling between the secondary currents and the self-consistent field. This term can reduce the instability, but not eliminate it. The above analysis seems to indicate that Bhadra's (2) result of a small electron-ion collisional damping at low collision frequencies has its origin other than the diffusional nature of the FP collision operator. It is probably due to the details of the collision mechanism, which we have not taken into account.

The first term in γ_i describes damping due to collisional diffusion. The remaining terms may be attributed to the coupling of collisions and collective phenomena.

(b) For $b > 1$, Eq. (4.10), (4.11) and (4.12) become respectively

$$\gamma_L = \left(\frac{m}{M}\right)^{1/2} \omega_0^2 \left[\pi (2b)^{1/2} - \left(\pi \frac{M}{m}\right)^{1/2} e^{-\omega_0^2} \right] \tag{4.17}$$

$$\gamma_e = \frac{4}{3} \frac{m}{M} \nu \left(2\pi b\right)^{1/2} \omega_0^2 \tag{4.18}$$

$$\gamma_i = -\frac{\nu}{\omega_0^2}\left(\frac{b}{2\pi}\right)^{1/2} \left(\pi + \rho^{-1}\right) - 2 \frac{K_3^2}{K^2} \nu \left(\frac{\rho - 1}{\rho}\right). \tag{4.19}$$

We note that the condition $b > 1$ is restricted by the requirement that $\beta < m/M$ such that the electrostatic assumption remains valid. Moreover K_2 may not be so large that the FP type collision operator ceases to be valid.

Equation (4.17) indicates that wave-electron resonance again causes instability.

Equation (4.18) shows that destabilization due to electron-ion collisions is proportional to $b^{1/2}$. This is remarkably different from that given by the BGK model, where (18)

$$\gamma_e = \frac{m}{M} \nu (2\pi b)^{-\frac{1}{2}} \omega_o^2 . \tag{4.20}$$

Equation (4.19) indicates that ion-ion collisions lead to a damping effect. This is however not true in general. For example, in the limit $\omega_o < K_3 v_{T_i}$ and $b \ll 1$ it can be shown that ion-ion collisions give rise to a growing term proportional to $1/3\nu$ (1 - 3b), although in that case ion Landau damping is dominant.

4.5 In the above discussion the principal cause of instability is the interaction of resonant electrons with the drift waves, or negative electron Landau damping. The ion Landau resonance term is stabilizing but small, due to the restriction $v_{T_i} < \omega/K_3$. However, if a temperature gradient is present, the same term can be destabilizing (11, 14). In this section we shall consider the effect of collisions on this instability. The limit $\omega/K_3 < v_{T_i} < v_{T_e}$ is assumed so that the relevant ion term dominates the imaginary part of ω.

For $T_i = T_e$, and $\omega < K_3 v_{T_i}$, $\gamma \ll \omega_o$ we obtain from the dispersion relation (4.2)

$$\omega_o = -\frac{\xi}{2\omega^* A[1-\eta-b\eta(1-C)]} \tag{4.21}$$

$$\gamma = \frac{1}{2[1-\eta-b\eta(1-C)]}\Big\{(\sqrt{\pi}-2b\nu_i)\Big[1-\frac{\eta}{2} - b\eta(1-C)\Big] - \nu_i\Big(\frac{2}{3}-2bC\Big) + 2\nu_i\eta\Big[-\frac{2}{3}+\frac{2b}{3}-\frac{b^2}{2} + \frac{7}{3}bC + b^2C + (1-A^{-1})b\frac{K_3^2}{K^2}\Big]\Big\}. \tag{4.22}$$

The stability condition may be shown to be (14)

$$\Big[1-\frac{\eta}{2}+\eta b(C-1)\Big]\eta > 0. \tag{4.23}$$

In the limit $b \ll 1$ we have

$$\gamma = \frac{1}{2(1-\eta-b\eta)}\Big[\sqrt{\pi}\Big(1-\frac{\eta}{2}-b\eta\Big)-\frac{2}{3}\nu_i(-1+2\eta+3b-5\eta b)\Big]. \tag{4.24}$$

For $b \approx 0$, Eq. (4.23) shows that the waves are stable if $0 < \eta < 2$. Equation (4.24) indicates that collisions enhance the growth rate except when $1/2 < \eta < 1$.

For b ≈ 1 we have from Eq. (4.22)

$$\gamma = \frac{1}{2\left(1-\frac{3}{2}\eta\right)}\left[\sqrt{\pi}\,(1-\eta) - \nu_i\left(\frac{5}{3} - \frac{2}{9}\eta - 3\eta\,\frac{K_3^2}{K^2}\right)\right]. \tag{4.25}$$

From Eq. (4.23) the stable region for this case is $0 < \eta < 1$. Equation (4.25) shows that collisions reduce the growth rate except when $2/3 < \eta < 15/2$.

For b > 1 we have from Eq. (4.22)

$$\gamma = \frac{1}{2(1-\eta)}\left[\sqrt{\pi}\left(1-\frac{\eta}{2}\right) + 2\eta\, b\, \nu_i\left(4 + \frac{b}{2} - (2\pi b)^{1/2}\,\frac{K_3^2}{K^2}\right)\right]. \tag{4.26}$$

The collisionless stability condition is $0 < \eta < 2$. Here collisional damping occurs except when $0 < \eta < 1$.

In the above discussion we have given the growth rate for all values of η. However due to the condition $\gamma \ll \omega_0 \ll K_3\, v_{T_i}$, the growth rate given by Eq. (4.22) is actually valid only for $|\eta| \gg 1$, otherwise the ion instability may be dominated by that of electrons (14). Thus we conclude that for large values of $|\eta|$ collisions in general tend to enhance the growth rate when $b \ll 1$ and reduce it when $b \geq 1$.

V. THE DRIFT-CYCLOTRON INSTABILITY

5.1 Mikhailovskii and Timofeev (17) investigated electrostatic wave propagation in a collisionless inhomogeneous plasma at frequencies near the harmonics of the ion cyclotron frequency. They found that such waves may be unstable if the transverse wavelengths are shorter than the ion Larmor radius ($K_{\perp} R_i > 1$). Since the intersection regions of the drift and ion-cyclotron oscillation branches are very narrow, it is expected that even rare collisions can greatly affect these instabilities.

Pogutse (23) considered the effect of collisions on the drift-cyclotron instability by using the BGK collision model. It was found that collisions tend to increase the region of the instability but decrease the growth rate. However, the BGK model has been shown to be questionable in describing short wavelength phenomena (22), so that it is necessary to use a more realistic model. Rukhadze and Silin (28) considered the problem by iteratively solving the linearized FP equation under the geometric optics approximation. The solution thus obtained has been shown by Bhadra (2) as being divergent.

In this section we reconsider the problem by using the model FP collision operator.

5.2 In this section we shall derive the dispersion relation for drift wave propagation near the ion cyclotron frequency. For $\omega < K_3 v_{T_i} \ll K_3 v_{T_e}$, $b = b_i > 1$, $b_e \ll 1$ and $\omega \approx n\Omega_i$, Eq. (2.31) yields

$$n_{i1} = -\frac{2\alpha_i n_0 e\phi}{M}\left[\frac{\omega_i^* - \omega_i}{\omega_i - n\Omega_i} A + 1 + S_1 + S_2\right]$$

and

$$n_{e1} = \frac{2\alpha_e n_0 e\phi}{m}\left[b_e + \frac{\omega_e^*}{\omega} + S_3\right] \tag{5.2}$$

where

$$A \approx (2\pi b)^{1/2} \qquad \omega_i = \omega + i b \nu_i$$

$$S_1 = -\frac{i\nu_i A}{\omega_i - n\Omega_i}\left[2n^2(\omega_i - n\Omega_i) - b\omega_i\right]$$
$$-\frac{i\nu_i \omega_i^* A}{(\omega_i - n\Omega_i)^2}\left[b + \frac{n(\omega_i - n\Omega_i)}{\Omega_i}\right]$$
$$-\frac{2i\alpha_i \nu_i A^2\omega}{K^2}\frac{(\omega_i^* - n\Omega_i)(\omega_i^* - \omega_i)}{(\omega_i - n\Omega_i)^2} \tag{5.3}$$

$$S_2 = -\frac{K_3^2 A}{2\alpha_i}\frac{\omega_i - n\Omega_i - 2\omega_i^*}{(\omega_i - n\Omega_i)^3} + \frac{i\nu_i K_3^2 A}{4\alpha_i(\omega_i - n\Omega_i)^4}\Big[3b_i(\omega_i + n\Omega_i)$$

$$+ 2(\omega_i - 2n\Omega_i) - 4n^2(\omega_i - n\Omega_i)\Big]$$

$$+ i\nu_i \omega_i^* A \frac{K_3^2\ (3b+1)}{2\alpha_i\ (\omega_i - n\Omega_i)^4}$$

$$+ \frac{i\nu_i \omega A^2 K_3^2}{K^2}\left[1 - \frac{2(\omega_i^* - n\Omega_i)}{\omega_i - n\Omega_i}\right]\frac{\omega_i^* - \omega_i}{(\omega_i - n\Omega_i)^3} \tag{5.4}$$

$$S_3 = -\frac{K_3^2}{2\alpha_e\omega^2} - i\nu_e\frac{T_e}{T_i}\left(\frac{T_i}{T_e} + \frac{\omega_i^*}{\omega}\right)\frac{K_3^2}{2\alpha_e\omega^2} - \frac{i\nu_e K_3^2 T_e \omega_i^*}{K^2 T_e \omega^2}. \tag{5.5}$$

Substituting Eq. (5.1) and (5.2) into Eq. (3.1) we obtain the dispersion relation

$$K^2\lambda_i^2 = 1 + \frac{\omega_i^* - \omega_i}{\omega_i - n\Omega_i}A + \frac{T_i}{T_e}\Big(b_e$$

$$+ \frac{\omega_e^*}{\omega}\Big) + S_1 + S_2 + \frac{T_i}{T_e}S_3. \tag{5.6}$$

We note that due to the assumption $\omega \approx n\,\Omega_i$ the resulting dispersion relation is considerably different from those of the previously considered cases. Furthermore, the temperature gradient effects have been neglected.

5.3 In this section we briefly review the results of Mikhailovskii and Timofeev (17).

For $\nu_i = 0$, $\nu_e = 0$ and $K_3 = 0$, Eq. (5.6) reduces to

$$1 + \mu K_2^2 \lambda_i^2 + \frac{\omega_i^* - \omega}{\omega - n\Omega_i} A - \frac{\omega_i^*}{\omega} = 0 \tag{5.7}$$

where

$$\mu = 1 + \frac{\omega_{pe}^2}{\Omega_e^2} .$$

If the factor $\mu K_2^2 \lambda_i^2$ is neglected, the roots of the dispersion relation are real and they represent a drift wave $\omega = \omega_i^*$ and an ion cyclotron wave $\omega = n\,\Omega_i\,(1 - A)^{-1}$. However, if $\mu K_2^2 \lambda_i^2 \neq 0$, unstable complex roots may occur. To consider the latter case, we write Eq. (5.7) in the quadratic form

$$a\,\omega^2 + b\,\omega + c = 0 \tag{5.8}$$

where

$$a = 1 - A + \mu K_2^2 \lambda_i^2$$

$$b = -n\Omega_i(1 + \mu K_2^2 \lambda_i^2) - \omega_i^*(1 - A)$$

$$c = n\,\Omega_i\,\omega_i^* .$$

the solutions are

$$\omega = \frac{1}{2a}\left[-b \pm (b^2 - 4ac)^{1/2}\right]. \tag{5.9}$$

Following Mikhailovskii and Timofeev, we note that

$$b^2 - 4ac = \left[\omega_i^*(1-A) - n\Omega_i(1+\mu K_2^2\lambda_i^2)\right]^2$$

$$- 4\omega_i^* n\Omega_i A\mu K^2\lambda_i^2 , \tag{5.10}$$

$$\frac{-b}{2a} = \frac{n\Omega_i(1+\mu K_2^2\lambda_i^2) + \omega_i^*(1-A)}{2(1-A+\mu K_2^2\lambda_i^2)} . \tag{5.11}$$

Thus, complex roots appear at the intersection of the drift and the ion cyclotron wave dispersion curves, where

$$\omega_0 = R_e\,\omega = \frac{\omega_i^*}{1+\mu K_2^2\lambda_i^2} = \frac{n\Omega_i}{1-A} . \tag{5.12}$$

The growth or damping rate is

$$\gamma = I_m\,\omega = \pm n\Omega_i K_2\lambda_i\left(\frac{\mu A}{1+\mu K_2^2\lambda_i^2}\right)^{1/2}. \tag{5.13}$$

The growing solution in Eq. (5.13) corresponds to the drift-cyclotron instability, which we note is not originated from the Landau wave-resonant particle interaction, instead it may be attributed to the interaction of the ion cyclotron motion with drift waves.

It is also possible that the condition (5.12) cannot be satisfied for any real K_2, so that instability does not occur. This situation arises if (17)

$$\epsilon R_i \leq 2n\left(\frac{m}{M} + \frac{\Omega_i^2}{\omega_{pi}^2}\right)^{1/2}. \tag{5.14}$$

Thus the instability can occur only for sufficiently large density gradient ϵ.

5.4 The effects of collisions and finite K_3 shall be considered in this section. It is assumed that $\nu_i \ll \omega_o - n\,\Omega_i$ and $\nu_e \ll \omega_o$ so that the collisionless result obtained in the last section may be treated as the lowest order solution.

Assuming that the real part of the frequency is unchanged by collisions we may write

$$\omega = \omega_o + i\gamma_o + i\gamma_1, \qquad \gamma_1 \ll \gamma_o$$

where ω_o and γ_o are respectively given by Eq. (5.12) and (5.13). The dispersion relation then becomes

$$0 = i n \Omega_i \gamma_1 \left[\frac{-2K^2\lambda_i^2\mu\,\omega_o^2 A - 2iK^2\lambda_i^2\mu\,\omega_o\gamma_o A}{\gamma_o^2(\omega_o^2 + 2i\omega_o\gamma_o)}\right]$$

$$+ K_3^2\lambda_i^2 + S_1 + S_2 + \frac{T_i}{T_e}S_3 - \frac{\epsilon\nu_i A}{\gamma_o}\left(1 - \frac{\omega_i^* - \omega_o}{\omega_o - n\Omega_i}\right) \tag{5.15}$$

where ω in S_1, S_2, S_3 are to be replaced by $\omega_o + i\gamma_o$.

To simplify the calculations we shall neglect terms of order $(\gamma_o/\omega_o)^2 \sim K_2^2 \lambda_i^2 \mu A$, this is valid if $K_2 \lambda_i$ and $\lambda_i R_i^{-1}$ are sufficiently small. Then from the imaginary part of Eq. (5.15) we obtain

$$\gamma_1 = \frac{-2\gamma_o \frac{M}{m} K_3}{n^2 \Omega^2 \alpha_i (1 + K^2\lambda_i^2\mu)} - \frac{4\nu_i A(3n^2 + n^2K^2\lambda_i^2\mu + 2b)}{1 + K^2\lambda_i^2\mu}$$

$$- \frac{K_3^2 \nu_i}{2\alpha_i n^2 \Omega_i^2}\left[\frac{2}{\mu K^2\lambda_i^2}(3b - 4n^2) + \frac{3b(1+K^2\mu\lambda_i^2)(2+K^2\lambda_i^2\mu)}{K^4\lambda_i^4\mu^2 A}\right]$$

$$+ \nu_e \frac{K_3^2}{K^2} - \frac{\nu_e K_3^2}{2\alpha_i n^2 \Omega_i^2}\left(\frac{T_e}{T_i} + \frac{1}{1 + K^2\lambda_i^2\mu}\right). \tag{5.16}$$

The first term on the right hand side of Eq. (5.16) gives the effect of finite K_3, showing that it tends to reduce the growth rate given in the last section. The second term gives the damping effect of ion-ion collisions. The third term gives the coupling effect of collisions and finite K_3, it also contributes to damping. The fourth term ν_e (K_3^2/K^2) increases the growth rate and is attributed to the drift dissipative instability (11). The last term is from electron collisions and it reduces the instability.

5.5 Although our results are qualitatively similar to those of Pogutse (23) who used the BGK model, and Rukhadze and Silin (28) who used the full FP equation, it is essential to note that quantitatively our results agree well with those of the latter, but differ from those of Pogutse. Considering the effect of ion-ion collisions alone, Pogutse has $\gamma \propto -\nu_{ii}$, Rukhadze et al and we have $\gamma \propto -b^{1/2}\,\nu_{ii}$. As for the effect of electron-ion collisions, Pogutse has $\gamma \propto b\,\nu_{ei}$ while Rukhadze et al and we have $\gamma \propto -(\nu_{ei} K_3^{\,2})/(4\,\alpha_i\, n^2\, \Omega_i^{\,2})\,(T_e/T_i)$.

The above investigation seems to indicate that in the limit of large b_i, the model FP equation and the full FP equation give similar results.

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13. ABSTRACT

This paper is concerned with the effect of weak collisions on the stability of low frequency waves in an inhomogeneous plasma. A model collision operator of the Fokker-Planck type is used to describe interactions between charged particles. The effect of these interactions appear as velocity space diffusion and friction. An exact integral of the kinetic equation is obtained. In the limit of low collision frequencies, the effects of collisions on the ion-acoustic wave, the drift wave and the drift-cyclotron wave instabilities are considered and compared with those of the other models.

The present results indicate that this model differs considerably from that of Batnager, Gross and Krook, and is a more appropriate collision model for a plasma.

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