Optimization of Discontinuous Fiber Composites

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Abstract: Discontinuous Random Fiber Reinforced Composites (DRFRC) exhibit different behaviors depending on their microstructural properties. Small fraction of fiber rupture can have a beneficial effect on the tensile and fracture energy properties of the composite. This paper briefly summarizes the results of an analytical model that predicts the pre-peak and post-peak bridging stress-COD relationship which accounts for fiber pull-out and rupture. The model can be used to optimize the design of DRFRC in terms of composite tensile strength and fracture energy.

1. Introduction

After first cracking, a composite may still be able to resist higher levels of loading if adequately reinforced. In this case the composite tensile strength is purely controlled by the bridging fibers. After the peak-stress has been reached, the composite behavior becomes dominated by fiber pull-out and/or rupture. The post-peak stress-displacement relationship controls the composite fracture energy. Both the composite tensile strength and fracture energy can be optimized if the composite bridging stress-COD (σ_c-δ) relationship is known in terms of the composite microstructural properties. These properties dictate whether or not fiber rupture will take place in the composite.

The σ_c-δ relationship associated with fiber pull-out has been studied, (Fiber Pull-Out Model) and a complete closed form analytic solution is available for discontinuous random fiber reinforced brittle matrix composites [Li, 1991]. Fracture energy associated with fiber bridging has also been studied taking into account the effect of fiber rupture for the same type of composites [Li et al, 1991]. This paper extends the Fiber Pull-Out Model by explicitly accounting for potential fiber rupture. The new model is then used to perform a parametric study which evaluates the effect of each microstructural property on the composite tensile strength and fracture energy. The results of the parametric study can be used to design composites for optimum properties.

2. Fiber Pull-Out Model (FPM)

By using a simple fiber stress-displacement relationship, based on a purely frictional matrix-fiber interface, Li et al [1991] predicted the composite σ_c-δ relationship by integrating over the contributions of the individual fibers which bridges a matrix crack plane:

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$$\sigma_c = V_f \int_{\phi=0}^{\pi/2} \int_{z=0}^{(L_f/2)\cos\phi} \sigma_b(\delta,\phi,z)p(\phi)p(z)dzd\phi \qquad (1)$$

$p(\phi)$ and $p(z)$ are probability density functions of the orientation angle and centroidal distance of fibers from the crack plane defined as follow:

$$p(\phi) = \sin(\phi) \quad \text{for } 0 \le \phi \le \pi/2 \quad \text{(for 3-D randomness)} \qquad (2)$$

$$p(z) = 2/L_f \quad \text{for } 0 \le z \le (L_f/2)\cos(\phi) \qquad (3)$$

$\sigma_b(\delta,\phi,z)$ defines the stress-displacement relationship for a single fiber oriented at an angle ϕ with respect to the normal direction of the crack plane and having a centroidal distance z from the crack plane.

$$\sigma_b(\delta,\phi,z) = \sigma_d e^{f\phi} \quad \text{for } 0 \le \delta \le \delta_o \qquad (4)$$

$$\sigma_b(\delta,\phi,z) = \sigma_p e^{f\phi} \quad \text{for } \delta_o \le \delta \le \ell \qquad (5)$$

where $\sigma_d = \sqrt{4(1+\eta)\tau E_f\delta/d_f}$, $\eta = V_fE_f/(V_mE_m)$, $\sigma_p = 4\tau(\ell+\delta_o-\delta)/d_f$, $\ell = L_f/2 - z/\cos\phi$, and $\delta_o = 4\tau\ell^2/[E_fd_f(1+\eta)]$. The coefficient f is an interface material parameter called the snubbing friction coefficient. Integration of eqn. (1) yields:

$$\sigma_c = \sigma_o\, g[2(u/u^*)^{1/2} - (u/u^*)] \quad \text{for } u \le u^* \qquad (6)$$

$$\sigma_c = \sigma_o\, g(1-u)^2 \quad \text{for } u^* \le u \le 1 \qquad (7)$$

where $\sigma_o = V_f\tau L_f/(2d_f)$, $g = 2(1+e^{f\pi/2})/(4+f^2)$, $u = \delta/(L_f/2)$, and $u^* = 2\tau L_f/[E_fd_f(1+\eta)]$.

In this model, all fibers were assumed to pull-out after complete debonding. This model has provided a good prediction for the peak composite stress and the post-peak tension-softening behavior for a number of composites where the fibers did not rupture [Li, 1991]. However, discrepancies were observed between the predictions of this model and some experimental measurements which suggest the occurrence of fiber rupture [Li and Wu, 1991]. Fiber rupture has also been experimentally observed in carbon, glass, and SiC DRFRC.

3. Fiber Rupture Model (FRM)

In this model we make the same assumptions adopted by the FPM, except that we allow the fiber axial stress $\sigma_b(\delta,\phi,z)$ to reach the fiber strength σ_{fu}. In addition, we assume that all fibers have a uniform tensile strength along their lengths, so that rupture always occurs at the matrix crack plane. In this case, when a fiber breaks, it no longer contributes to the composite bridging stress.

The FRM assumes that fibers having an embedment length ℓ less than $L_ce^{-f\phi}$ are pulled-out subsequent to complete debonding, and those with an embedment length ℓ greater than $L_ce^{-f\phi}$ rupture after incomplete debonding. The stress-displacement relationship for the group of fibers that eventually rupture is defined by a step function as follow:

$$\sigma_b(\delta,\phi,z) = \sigma_d\, U(\delta_c e^{-2f\phi} - \delta)e^{f\phi} \qquad (8)$$

where $\delta_c = \sigma_{fu}^2 d_f/[4E_f\tau(1+\eta)]$, and $L_c = \sigma_{fu}d_f/(4\tau)$.

"Pre-Peak" Bridging Stress-Displacement Curve: This refers to initial portion of the σ_c-δ curve ($\delta \le \delta^*$) that ends when all intact fibers are pulling-out of the matrix. Using equations (2)-(5) and (8) in (1), we obtain:

$$\sigma_c = \sigma_o\, g[2(u/u^*)^{1/2} - (u/u^*)] \quad \text{for } u \le u_ce^{-f\pi} \qquad (9)$$

$$\sigma_c = \sigma_o\left[g(\phi_c)[(u/u^*)^{1/2} - 0.5(u/u^*)] + a(\phi_c,-f)L^2\right] \quad \text{for } u_ce^{-f\pi} \le u \le u^* \qquad (10)$$

where $u_c = \delta_c/(L_f/2) = L^2u^*$, $L = L_c/(L_f/2)$, $g(x) = 2[\{f\sin(2x) - 2\cos(2x)\}e^{fx} + 2]/(4+f^2)$, $a(x,y) = [\{2\cos(2x) - y\sin(2x)\}e^{yx} + 2e^{y\pi/2}]/(4+y^2)$, and $\phi_c = -\ln(u/u_c)/(2f)$.

Post-Peak Bridging Stress-Displacement Curve:

$$\sigma_c = \sigma_o L^2\left[0.5g(\phi_b)[1/L-(u/L)]^2+a(\phi_b,-f) - 2a(\phi_b,0)(u/L)+a(\phi_b,f)(u/L)^2\right] \quad \text{for } u^* \le u \le Le^{-f\pi/2} \qquad (11)$$

$$\sigma_c=\sigma_o L^2\left[0.5g(\phi_b)[1/L-(u/L)]^2+b(\phi_a,\phi_b,-f)-2b(\phi_a,\phi_b,0)(u/L)+b(\phi_a,\phi_b,f)(u/L)^2\right] \quad \text{for } Le^{-f\pi/2} \le u \le 1 \qquad (12)$$

where $b(x,y,z) = [\{z \sin(2x)-2 \cos(2x)\}e^{zx} -\{z \sin(2y)-2 \cos(2y)\}e^{zy}]/(4+z^2)$, $\phi_a= -\ln(u/L)/f$, and $\phi_b= \ln(L)/f$. Figures 1 and 2 show the pre-peak and post-peak σ_c-δ curves for composites with different fiber lengths. Figure 3 shows a comparison between the prediction of the FPM and that of the FRM for the pre-peak σ_c-δ curve. Thus, the FPM overestimates the peak bridging stress when fiber rupture occurs.

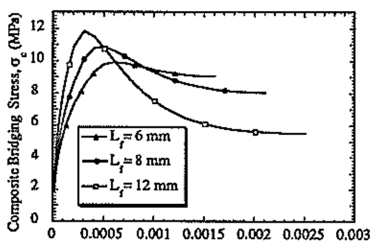

Figure 1 Model-Predicted Pre-Peak σ_c-δ Curve for Different Fiber Lengths

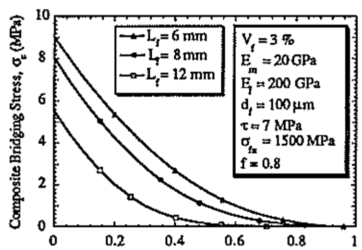

Figure 2 Model-Predicted Post-Peak σ_c-δ Curve for Different Fiber Lengths

Composite Fracture Energy: The composite fracture energy G can be computed by integrating the area under the σ_c-δ curve. The contribution of the pre-peak portion is negligibly small. Therefore, G was computed by using the post-peak σ_c-δ relationship. Due to the complexity of the integration, no exact analytic solution was obtained. Numerically, however, G was in perfect agreement with the fracture energy computed earlier by Li et al (1991). G computed by Li et al was given by an expression which can be reduced to the following form:

$$G = G_o[(1/6)\, g(\phi_b)\, L^{-2}+(1/3)\, a(\phi_b,-2f)\, L] \qquad (13)$$

where $G_o = V\tau L_c^2/d_f$. This expression was then used in the parametric study. It should be noted that equations (9)-(13) are only valid for values of ϕ_b between zero and $\pi/2$. For ϕ_b greater than $\pi/2$, the fiber pull-out model is valid and can be used. For ϕ_b less than zero (i.e. $L_f \ge 2L_c$), σ_c and G can be computed by taking the limits of equations (9)-(13) at ϕ_b=0. In this case, the pre-peak portion of the σ_c-δ curve ends at $\delta = \delta_c$, and the pull-out process ends at $\delta = L_c$.

4. Parametric Study

The purpose of the parametric study is to evaluate the effect of each microstructural property ($L_f,d_f,\sigma_{fu},\tau,f$) on the composite tensile strength and fracture energy. Figure 4 shows the effect of the fiber length on peak bridging stress and fracture energy. It can be deduced from this figure that there is an optimum fiber length that falls slightly below the critical fiber length $2L_c$ and beyond which an increase in the fiber length slightly increases the peak stress but significantly reduces the fracture energy. Table 1 shows the results of the parametric study. In this table σ_{cu} and G refer to the peak bridging stress and fracture energy corresponding to the optimum fiber length. This table shows, in particular, that the composite tensile strength at the optimum fiber length is unaffected when the fiber diameter or the bond strength are changed. Figure 4 and Table 1 can be used to design composites for optimum performance.

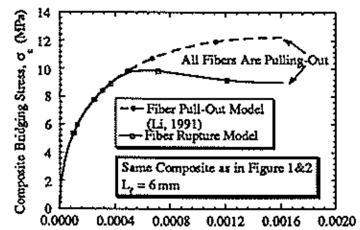

Figure 3 Comparison Between the Prediction of the FPM and the FRM for the Pre-Peak Composite Bridging Stress

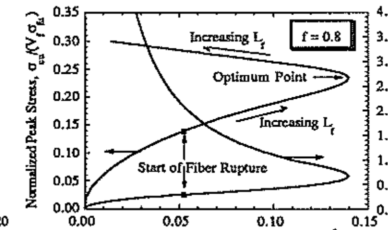

Figure 4 Effect of Fiber Length on Peak Stress and Fracture Energy

Table 1 Summary of Parametric Study

Parameter	Peak Stress	Fracture Energy
Fiber diameter : d_f ↑	σ_{cu} Unaffected	G ↑
Bond strength: τ ↑	σ_{cu} Unaffected	G ↓
Fiber strength: σ_{fu} ↑	σ_{cu} ↑	G ↑
Snubbing friction: f ↑	σ_{cu} ↓	G ↓

5. Conclusion

The FRM has produced a tool for designing DRFRC for optimum performance. Using this model, the behavior of the composite can be controlled through the microstructural properties. Therefore, the composite can be designed to achieve (1) the highest fracture energy, (2) the highest tensile strength, (3) the highest flexural strength/tensile strength ratio, (4) a compromise between the preceding three properties that fits a particular engineering application. However, there are practical and theoretical limitations to this model. First, fibers are generally supplied at discrete sizes. Second, the processing technique might restrict the fiber length to a certain maximum limit so that fibers with optimum length become difficult if not impossible to handle. Third, this model is more applicable to fibers having a high Weibull modulus (i.e. steel fibers m≈100). Fibers with low Weibull modulus (i.e. carbon fibers m≈10) will not necessarily rupture at the matrix crack plane. For these fibers, therefore, the strength distribution should be accounted for in the model. The experimental validation of this model, the relationship between the flexural strength and tensile strength, and the effect of fiber strength distribution on the σ_c-δ relationship are the subject of future research.

References

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