RESEARCH CHALLENGES IN TOUGHNESS DEVELOPMENT OF FIBER REINFORCED CEMENTITIOUS COMPOSITES

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ABSTRACT. This paper presents a discussion of the research challenges in fiber reinforced cementitious composites within the context of the Performance-Processing-Property-Structure framework. Specific examples on the inter-relationships between two types of quasi-brittleness and the material structure are described, and their links to pseudo-ductility and processing conditions are briefly mentioned. The need to bring the disciplines of micromechanics and materials processing closer together underlines a fundamental challenge to the research community in advancing the engineering design of high performance fiber reinforced cementitious composites.

1. Introduction

Research in fiber reinforced cementitious composites (FRCC) has been ongoing for at least two decades. Yet the use of this material at the present time, while growing, is still relatively limited. Although FRCC has been repeatedly demonstrated to be an improved structural material over plain concrete, its present commercial 'push' is more on 'reducing shrinkage cracks'. In other words, after curing, the presence of fibers in concrete is not expected to contribute to the properties of this material. While this usage of fibers is indeed important, especially when structural durability is considered, the structural properties of FRCC appear to be underutilized at present.

The present design of FRCC is governed mainly by the need of a workable mix and by economics. Both of these considerations leads to very low fiber loading, typically less than one volume percent. In the long run, it seems prudent to design FRCC on the basis of long term performance on a unit cost basis. This price/performance approach would require knowledge in (1) the required performance for a given structure and its relation to the

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unique properties of FRCC, and (2) route of optimization of this performance per unit volume fraction of fibers.

In recent years, there appears to be a rising interest in FRCC, partly because of the commercial need of finding substitute materials for asbestos cement which has been found to be carcinogenic and is being phased out in most industrialized countries, and partly because of the increasing availability of a wide variety of fiber types and geometries for use as reinforcements. In addition, the increasing ease of making high strength concrete and the recognition of the brittleness and difficulty in quality control of this material is begging for a solution in the form of fiber reinforcement. These developments present an opportunity for realizing the structural utilization of fiber reinforced cementitious composites. Despite advances in processing and in property studies, there remains a large gap in our knowledge of the linkage between performance, property, processing and material structure of FRCC. These linkages are the key to successful optimization and hence the achievement of advanced composites suitable for a price sensitive construction industry and market. Although it is not the intention of this paper to review the complete picture of performance-property-process-structure relationships, it is still useful to have a brief overview of these relationships to place the more detailed discussions to follow in the context of this framework.

2. The Performance-Property-Process-Structure Relationship

Performance, property, process and material structure form the apexes of a tetrahedron schematically shown in Figure 1. Their relationship to one another was proposed as a general framework for the study of modern engineered materials by the National Research Council (1990). For our present purpose, we shall restrict the use of this framework to FRCC.

The **performance** of FRCC may include pseudo-ductility, durability, reliability and safety, manufacturability, seismic resistance (energy absorption capacity), flexural and shear capacity and other desirable features of the specific structure which utilize this material. These are elements which the end user and the structural engineer normally would like to see improvements in. In certain circumstances, trade-off in performances may necessitate a critical evaluation of required performance of a structure under given loading and environmental conditions.

The important **properties** of FRCC may include stiffness, strength, fracture toughness and others. In the past, most construction codes are concerned with stiffness and (usually compressive) strength. However, the effectiveness of fiber reinforcement in improving these properties are usually insignificant. This may be one of the reasons for the lack of appreciation for the unique structural property which FRCC offers. These include for example, the tensile first and post-cracking strength and fracture energy which have a direct bearing on several of the performance parameters such as reliability, seismic resistance, and shear and flexural capacities.

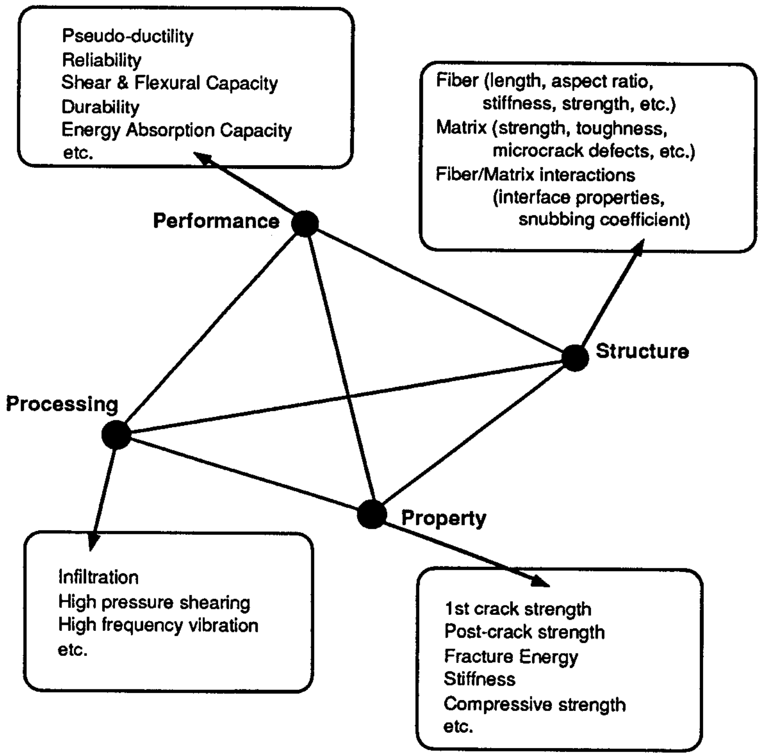


Fig. 1: The Performance-Processing-Property-Structure Tetrahedron for FRCC

Until recently, both **processing** and **material structure** received little attention from researchers. For FRCC, the use of pultrusion, intense shear rolling, infiltration, high frequency vibration processing techniques, and controlled curing processes have been attempted with varying successes. They are also responsible for some of the more high performance modern FRCC materials such as SIFCON and reinforced MDF and DSP cement. The material structure of FRCC includes the fibers and their orientation, the cement or concrete matrix and the pores in them, and the fiber/matrix interface.

Observational studies of interface microstructure (see e.g. Mindess and Shah, 1988) have received much attention in recent years, but quantitative links between interface observations and bond properties are rare.

The performance-processing-property-structure relationship suggests a useful framework offering a wide variety of challenges to the research community. The links between these four cornerstones affords significant opportunities for materials engineering of FRCC which may lead to advanced cement based structures overcoming many of the problems we currently experience in our infrastructures.

3. Two types of Quasi-Brittleness

Quasi-brittleness in FRCC is a desirable property imparted by the inclusions of fibers in a brittle cementitious matrix. Depending on the volume fraction and fiber aspect ratios (among other parameters), two very different types of quasi-brittleness can be achieved. For high fiber loading, and usually in the form of continuous fibers, a type of pseudo-strain-hardening can be accomplished. This pseudo strain-hardening can be in the form of a high critical strain or first cracking strength property of the FRCC, or it can be in the form of multiple cracking. In some cases, both of these forms are exhibited in the same composite. High critical strain or high first crack strength is a direct result of microcrack stabilization by fibers. That is, the microcracks in the matrix propagate against an increasing closing pressure exhibited by the stretched fibers, thus allowing a large number of microcracks to grow stably (volumetrically) resulting in large strain capacity, before any one microcrack grows unstably to form a macrocrack. If the material is unloaded prior to formation of this macrocrack, the microcracks can probably close with no visible signs of material damage. This type of pseudo-strain hardening effect is somewhat analogous with the stabilization of dislocation growth by pinning at grain-boundaries in metals. An example of such a FRCC is a cement composite reinforced with 12% volume fraction of continuous Polypropylene fibers (Figure 2, from Krenchel and Stang, 1988) produced by a pultrusion process. This composite has a first crack strength of about 15 MPa (about five times normal strength concrete) and a critical strain up to .001 (about ten times normal strength concrete). It also exhibits multiple cracking to as much as 2% strain. Another example of a high performance FRCC is a so called Compact Reinforced Composite (Figure 3, from Bache, 1987) produced with more than 6% discontinuous fine steel fibers and then heavily reinforced with continuous steel bars. For this composite within a composite, a critical strain of .003 (about 30 times normal strength concrete) and an increased flexural strength of 150 MPa (about 5 to 10 times that of normal steel reinforced concrete). This particular composite shows significant deformability but does not appear to fail with any sign of multiple cracking. Although multiple cracking allows even higher strain capacity, the resulting damage is permanent, with sub-parallel visible macrocracks.

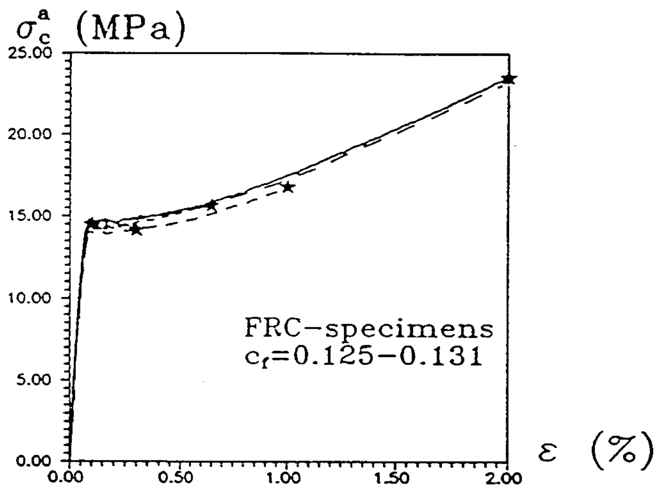


Fig. 2: A Tensile Stress-strain Relationship for a Continuous Polypropylene FRCC (Krenchel and Stang, 1988)

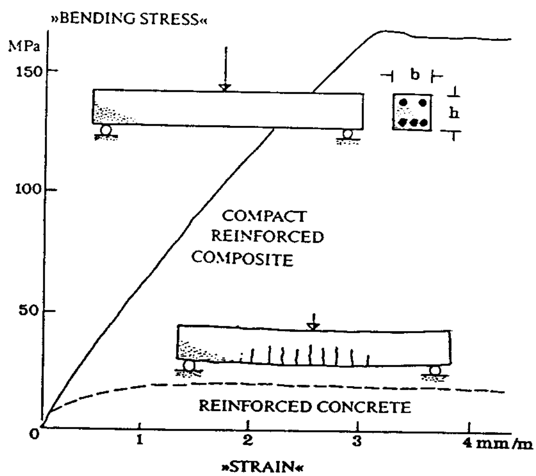
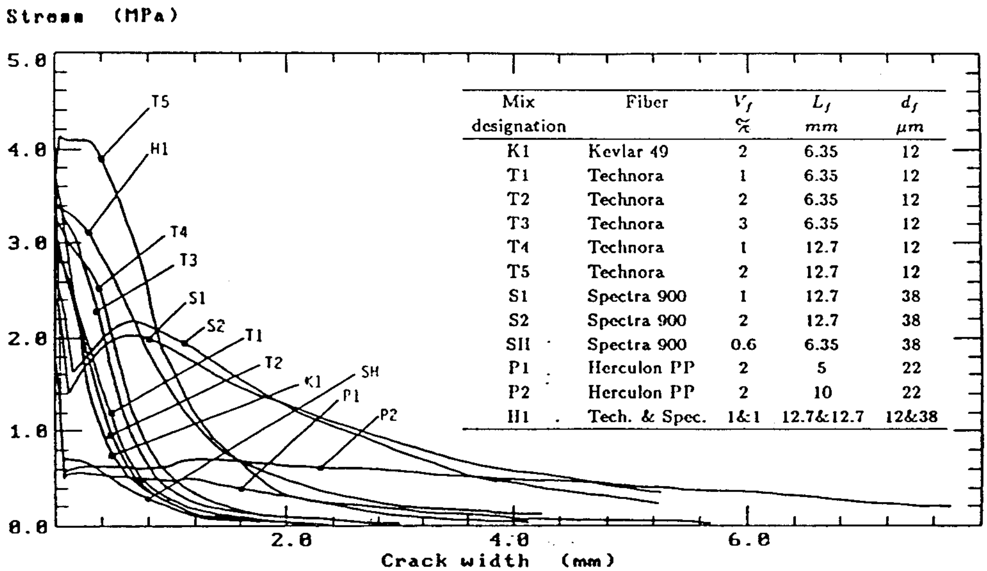


Fig. 3: Flexural Stress-strain Relationship for a CRC Beam (Bache, 1987)

A second type of quasi-brittleness is achieved with fiber pull-out as a macrocrack opens. The energy absorbed by frictional work on interface sliding can be significant, sometimes up to several orders of magnitude in comparison to the fracture energy of the unreinforced material. This type of FRCC exhibits a softening stress-displacement curve as shown in Figure 4 for a mortar matrix reinforced with various types of polymeric fibers (processed with a vacuum equipped bladeless Omni-mixer, Wang et al, 1989). The current commercial FRCC (using usually steel or polypropylene fibers) is of this type. The low volume percentage makes processing relatively easy. However, the large gain in fracture energy is sometimes deceptive since to access the full amount, the crack opening would have to be of the order of half the fiber length which can be unacceptable in engineering practice. Even so, there is no doubt that such material property can be utilized in energy absorption members of earthquake resistant structures or structures which need to withstand impact loads.



Mix designation	Fiber	V_f %	L_f mm	d_f μm
K1	Kevlar 49	2	6.35	12
T1	Technora	1	6.35	12
T2	Technora	2	6.35	12
T3	Technora	3	6.35	12
T4	Technora	1	12.7	12
T5	Technora	2	12.7	12
S1	Spectra 900	1	12.7	38
S2	Spectra 900	2	12.7	38
SH	Spectra 900	0.6	6.35	38
P1	Herculon PP	2	5	22
P2	Herculon PP	2	10	22
H1	Tech. & Spec.	1&1	12.7&12.7	12&38

Fig. 4: Stress-Displacement Relationship for Low V_f FRCC (Wang et al, 1989)

4. Concept of Bridging Stress-Crack Width Relation

The concept of bridging stress-crack width relation may be illustrated with the help of Figure 5. In Figure 5c, the matrix has fully cracked. For this configuration, if the specimen is loaded from zero load, the complete bridging stress-crack width relation (σ_B-δ) of Figure 6a can be measured. In reality, figure 5c follows the two previous stages of loading in Figure 5a which shows a microcrack being stabilized, and Figure 5b which shows a microcrack just becoming unstable and spread across the full cross-section of the specimen. The loading at this stage corresponds to the first crack strength (Figure 5b and Figure 6b). Between these two stages (A and B), distributed microcracking (not shown) can be expected. After stage B, multiple cracking (not shown) can occur, depending on the relative magnitude of the first crack strength σ_{fc} and on the post-crack strength σ_{pc} (Li and Leung, 1990). Subsequently, the load bearing capacity is fully due to the bridging fibers and the load-deformation behavior follows that of the softening σ_B-δ curve (Figure 6b). This discussion reveals that the pre-peak σ_B-δ relation plays a dominant role in determining the presence or absence of the first type of quasi-brittleness, i.e. the pseudo strain-hardening discussed earlier, whereas the post-peak σ_B-δ curve controls the second type of quasi-brittleness, i.e. the composite fracture energy. Theoretical and experimental studies of the σ_B-δ curve has traditionally focused on the post-peak part. Indeed, the pre-peak part is generally not accessible experimentally, although it can be studied theoretically.

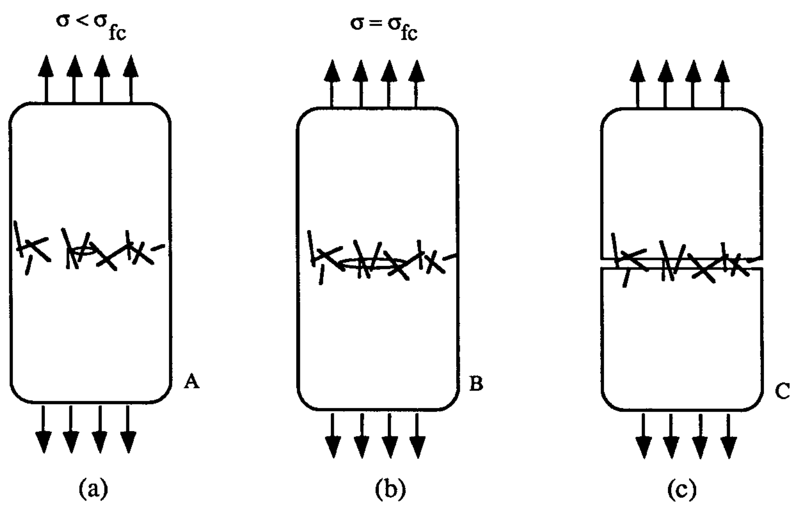


Fig. 5: Three Stages of Deformation Under Uniaxial Loading of a FRCC, (a) Prior to First Crack, (b) at First Crack Load, (c) Post-Peak

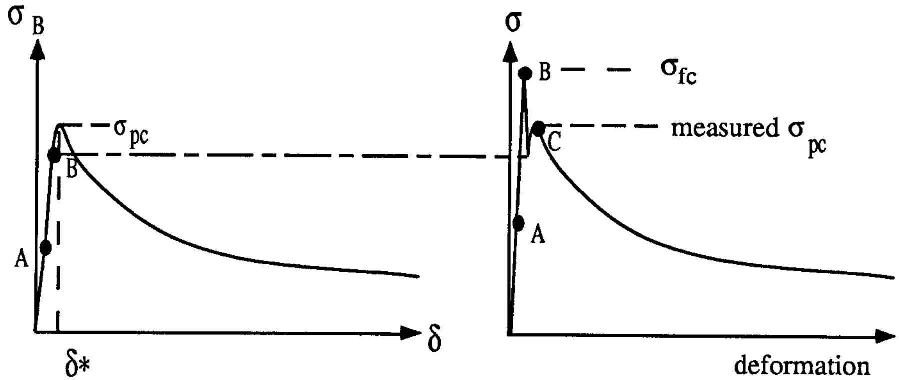


Fig. 6 (a): The Complete σ_B-δ curve and (b) the Corresponding Uniaxial Load-Deformation Curve

5. A Micromechanical Model of σ_B-δ Curve

In the following, we review briefly a micromechanical model of the σ_B-δ curve proposed by Li and co-workers. This model focuses on flexible fibers such as most steel and polymeric fibers used in reinforcing cement matrixes. The material structure of FRCC consists of fibers (volume fraction V_f, length L_f, diameter d_f, stiffness E_f and strength σ_f), matrix and interface. For discontinuous fibers, they are typically randomly oriented, although processing condition or structural boundaries may bias the distribution (Stroeven, 1988). The σ_B-δ relation may be derived from the single fiber bridging force $P(\delta)$ through

$$\sigma_B(\delta) = \frac{V_f}{\pi d_f^2/4} \int_{\phi=0}^{\pi} \int_{z=0}^{(L_f/2)\cos\phi} P(\delta) p(\phi) p(z) \, dz \, d\phi \qquad (1)$$

where $p(\phi)$ and $p(z)$ are probability density functions of the orientation angle (with respect to the tensile loading direction) and centroidal distance of fibers from the crack plane. For uniform random distributions, $p(\phi) = \sin\phi$, and $p(z) = 2/L_f$ (Li et al, (1990a)). The upper integration limit for z in (1) is intended to discount those fibers not bridging across the matrix crack in their initial position. That is, only fibers located or oriented in such a direction as to have a positive initial embedded length would contribute to the composite bridging stress. In particular, all fibers lying parallel to the matrix crack plane (at ϕ=90°) are excluded in (1).

Eqn. (1) has been written in a modular form where the mechanisms of bridging (contained in the P(δ) term) and the distribution of fiber location and orientation are separated. This modularity enhances the flexibility of including a variety of mechanisms of bridging, such as fracture based fiber debonding (Gao et al, 1988) and/or fiber rupture (Li et al, 1990a).

To estimate the single fiber bridging force, mechanisms of fiber/matrix interaction has to be considered (Figure 7). The simplest model would be a frictional interface, with debonding and slippage resisted by a frictional bond strength of τ. Thus fiber bridging force would be a function of crack opening δ, embedded length ℓ and the bond strength τ. Experimental test of Nylon and Polypropylene fibers, however, indicate that the pull-out load can be a strong function of the angle of inclination ϕ of the fiber to the loading axis (Figure 8). Morton and Grove (1976) and Li et al (1990) suggested that a flexible fiber being pulled from a matrix is similar to a rope pulled over a friction pulley. Thus the fiber bridging force may be modelled as

$$P(\delta)=\begin{cases}\pi\sqrt{E_f d_f^3 \tau\delta/2}\,e^{f\phi} & \text{for} \quad \delta\le\delta_o \\ \pi\tau\ell d_f\left[1-(\delta-\delta_o)/\ell\right]e^{f\phi} & \text{for} \quad \ell\ge\delta>\delta_o\end{cases} \tag{2}$$

where $\delta_o \equiv \dfrac{2\ell^2\tau}{E_f d_f}$ corresponds to the crack width at which debonding of a fiber with

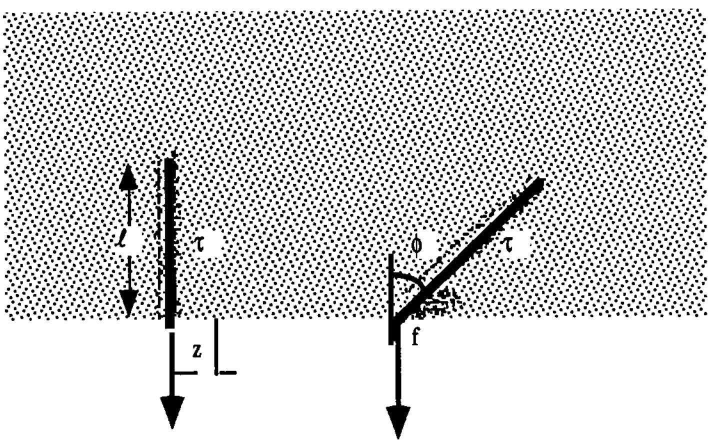


Fig. 7: Fiber Bridging Force and Mechanisms; **P** (δ; ℓ (L_f, z, ϕ), d_f, E_f, τ, f, ϕ)

embedded length ℓ is completed, and f is a snubbing coefficient. Apart from Nylon and Polypropylene fiber in normal strength cement data, steel wire pulled out at two different angles from a epoxy matrix was reported by Morton and Grove (1976), Figure 8. However, at the higher angle (40°), they reported that matrix yielding limited any increase in bridging force. In addition, Naaman and Shah (1976) reported inclined fiber pull out test in normal strength cement, but found that the peak load wavered about a medium value for ϕ between 0° and 75°. These data are perhaps indicative of matrix strength limiting the snubbing effect. The limit must be set by the fiber diameter and stiffness, as well as by the matrix spall resistant strength. Even for the polymer fibers which data follows the snubbing friction concept reasonably well, the data for high angle testing reveals significant amount of standard deviation associated with matrix surface spall. Matrix spalling can be observed under the SEM (Figure 9).

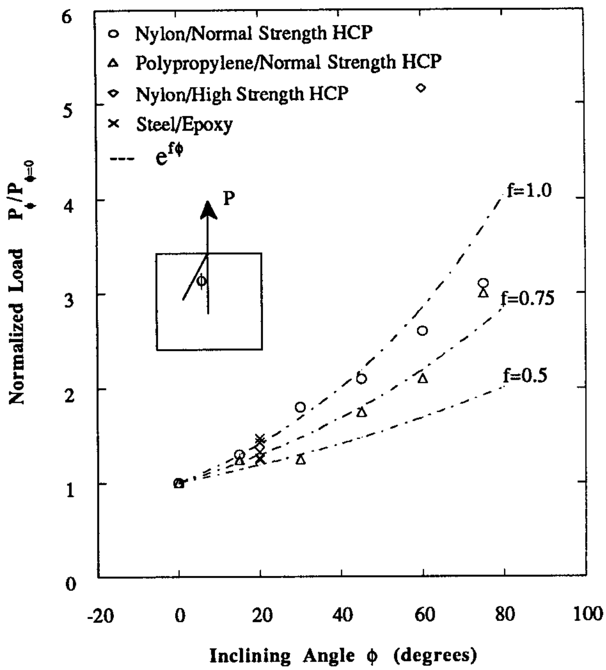
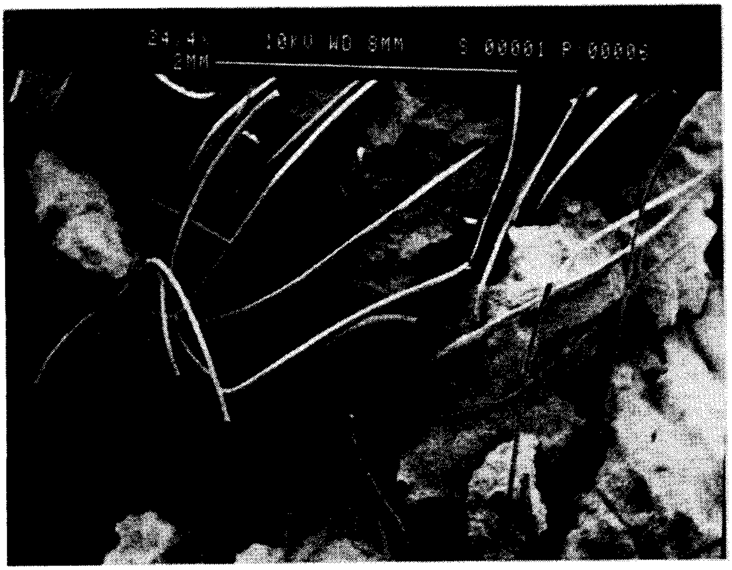


Fig. 8: Maximum Pull-out Load as a Function of Inclining Angle



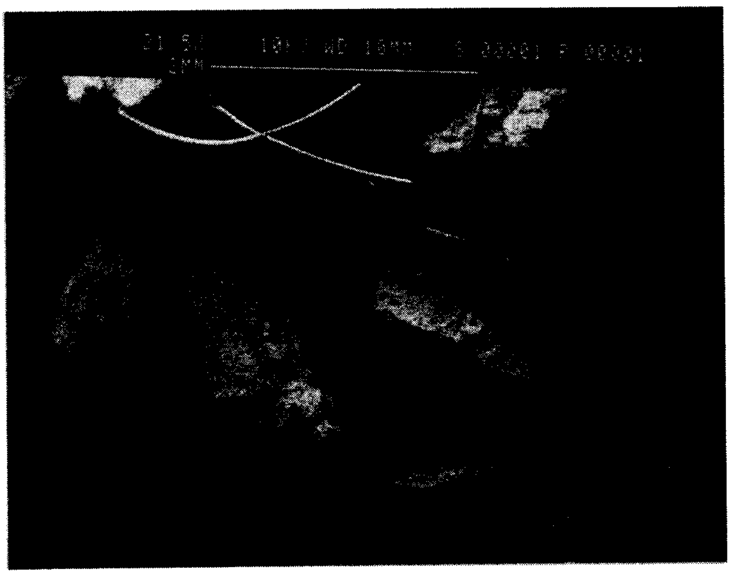


Fig 9: SEM of Surface Spalls from the Fracture Surface of Spectra FRCC

With (1) and (2), Li (1990) showed that the pre-peak and post-peak σ_B-δ can be expressed as:

$$\tilde{\sigma}_B(\tilde{\delta}) = \begin{cases} g\left[2(\tilde{\delta}/\tilde{\delta}^*)^{1/2} - \tilde{\delta}/\tilde{\delta}^*\right] & for\ \tilde{\delta} \le \tilde{\delta}^* \\ g(1-\tilde{\delta})^2 & for\ 1 > \tilde{\delta} > \tilde{\delta}^* \end{cases} \tag{3}$$

where $\tilde{\sigma}_B \equiv \sigma_B/\sigma_o$ and $\sigma_o \equiv V_f \tau (L_f/d_f)/2$, and $\tilde{\delta} \equiv \delta/(L_f/2)$. $\tilde{\delta}^* \equiv (\tau/E_f)(L_f/d_f)$ corresponds to the maximum attainable (normalized by $L_f/2$) value of δ_o for the fiber with the longest embedment length of $L_f/2$. The snubbing factor is defined as

$$g \equiv \frac{2}{4+f^2}\left(1+e^{\pi f/2}\right) \tag{4}$$

For the range of f between 0 and 1, g range from 1 to 2.32. Predicted pre-peak and post-peak σ_B-δ curves are shown in Figure 10 for three values of the snubbing coefficient f. A similar form of the post-peak $(1-\tilde{\delta})^2$ dependence was derived by Cotterell and Mai (1988) although their approach would not be able to account for the snubbing effect.

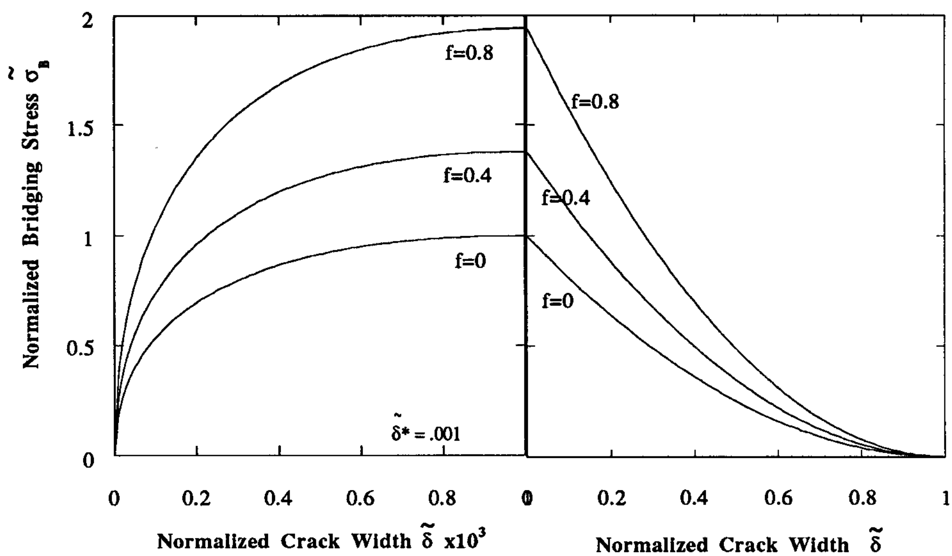


Fig. 10: Predicted Prepeak and Post-peak σ_B-δ curves

The pre-peak part of the σ_B-δ curve cannot be directly verified since there is no experimental data of this type. There is, however, plenty of data for the post-peak part of the σ_B-δ curve, for various FRCCs. Unfortunately, the values of g are typically not reported. Even so, it can be estimated from the peak value of the σ_B-δ curve and by noting that this peak value, often known as the post-crack strength σ_{pc}, is given (from (3)) by

$$\sigma_{pc} = \frac{1}{2} g\tau V_f \left(\frac{L_f}{d_f} \right) \tag{5}$$

By plotting σ_{pc}/τ versus the reinforcement index 1/2 V_f (L_f/d_f) (Li, 1990), data from Visalvanich and Naaman (1982) suggests a value of f ~1 for steel FRCC (a bond strength of 4 MPa is assumed) whereas data from Wang et al (1989) suggests a value of f~0.55 and f~0.77 for Spectra fibers (a high modulus polyethylene) in normal strength and high strength mortar, respectively (a measured bond strength of 1MPa is used, Li et al , 1990b). The well known effect of fiber plastic bending (Morton and Groves, 1974) is not accounted for in this calculation. For polymeric fibers, the bending stiffness is insignificant. Even in steel fibers, Morgan and Groves (1976) suggested that the bending effect is probably small compared to the snubbing effect based on limited pull-out test of steel fibers embedded in an epoxy matrix. However, contradictory data have been obtained by Naaman and Shah (1976).

Figure 11 shows the derived post peak σ_B-δ curve in normalized form (dashed line), and five sets of steel FRCC data (open symbols, Visalvanich and Naaman, 1982) made up of different fiber volume fractions, lengths and aspect ratios. Figure 12 shows a similar plot by Wecharantana and Shah (1983) of a different set of experimental data (except for the one labeled Visalvanich). They used a curve fitting procedure and obtained the same $(1-\tilde{\delta})^2$ dependence as theoretically derived (3). The post-peak σ_B-δ data of Spectra FRCC data (solid symbols, Wang et al, 1989) is also shown in Figure 11 . These plots suggest a definite universality of the shape of the post-peak σ_B-δ curve for FRCCs where fiber bridging composed of debonding and pull-out mechanisms controls the post-cracking behavior of the composite. When experimental data of concrete is examined, it is remarkable to find that a similar $(1-\tilde{\delta})^2$ dependence describes the data reasonably well. For example, Figure 13 shows a composite curve (Karihaloo and Nallathambi, 1990) and a $(1-\tilde{\delta})^2$ plot for the post-peak σ_B-δ of concrete. This consistency is indicative that in a material like concrete, the post-peak behavior may be governed by a pull-out mechanism of aggregates and /or unbroken ligaments bridging across the crack surface similar to the fiber bridging mechansim in FRCC.

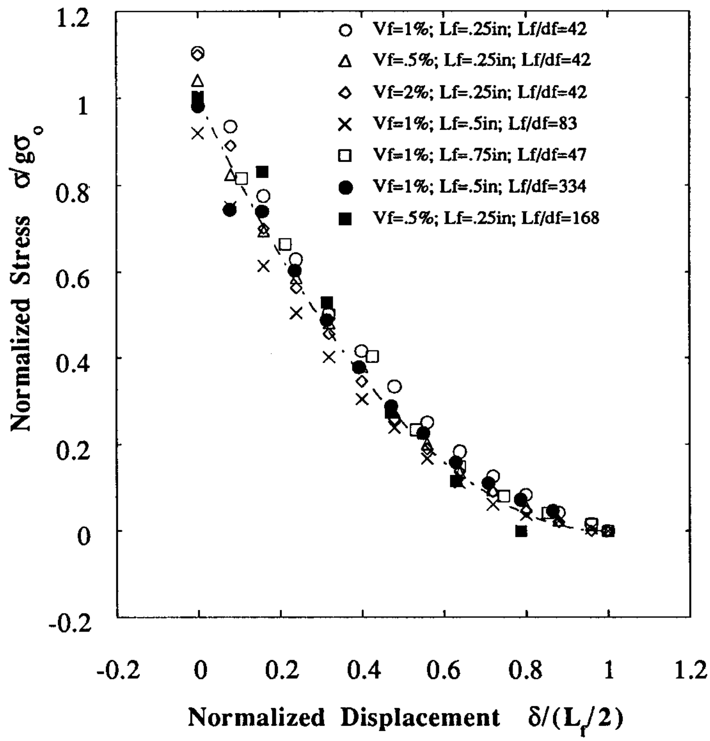


Fig. 11: Comparison Between Predicted (dashed line) and Experimentally Determined Post-peak σ_B-δ curve for Steel FRCC (open symbols) and for Spectra FRCC (solid symbols)

The fracture energy due to fiber pull-out can be derived from (3) and found to be (Li, 1990)

$$G_c = \frac{1}{12} g\tau V_f d_f \left(\frac{L_f}{d_f}\right)^2 \qquad (6)$$

Figure 14 shows a log-log plot of (6) with both the steel and Spectra FRCC data included.

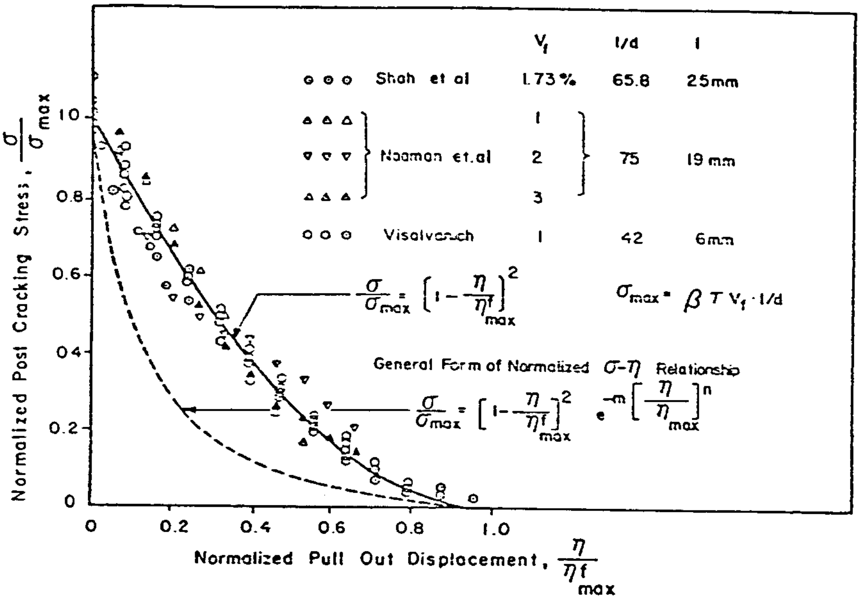


Fig. 12: Empirical Fit to Additional Steel FRCC Data by Wecharantana and Shah (1983)

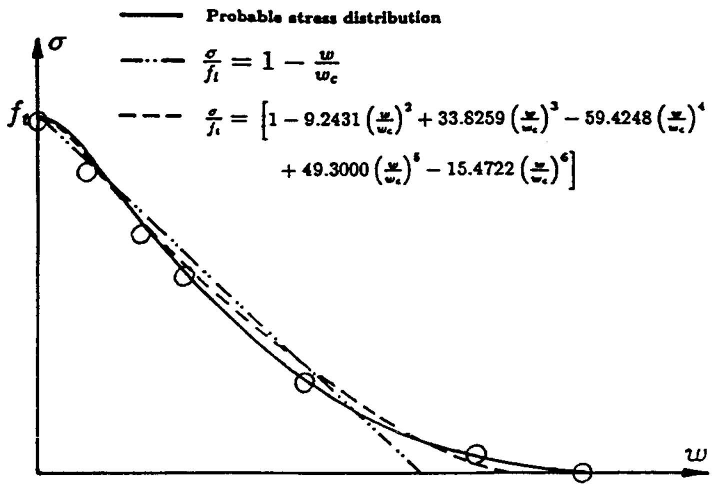


Fig. 13: Post-peak σ_B-δ curve for Concrete. Solid line represents "Probable Stress Distribution" (Karihaloo and Nallathambi, 1990). Circles denote that expressed by (3)

Eqn (5) and (6) suggest that the post-crack strength σ_{pc} and the fracture energy G_c scales with $g\tau$, V_f and $1/d_f$. To enhance σ_{pc} and G_c, therefore, it would be desirable to use a high loading of fibers with small diameters, good bonding and a high snubbing friction. This would be the case if the fibers do not rupture, in line with short fiber length for ease of processing. Otherwise the scaling laws of (5) and (6) no longer hold. Consideration of optimal fiber length for G_c with fiber rupture accounted for is given in Li et al (1990a).

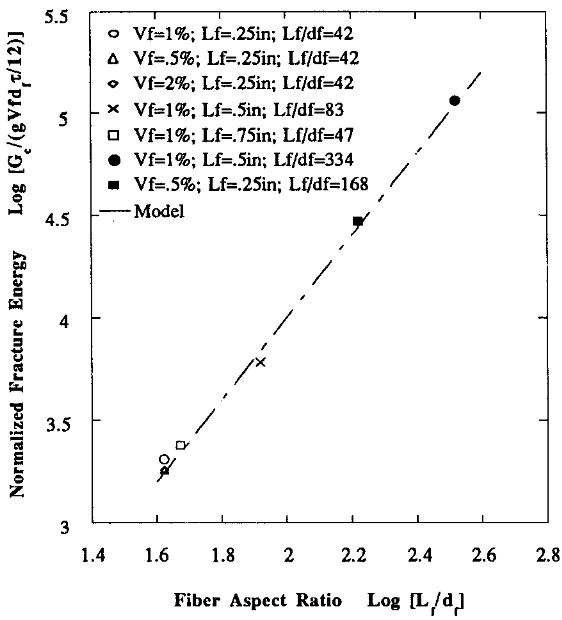


Fig. 14: Comparison of Predicted and Experimentally Determined Fracture Energy of Steel and Spectra FRCC

Because the snubbing friction term and the interface bonding appears as a product in (5) and (6), the snubbing friction has a multiplier effect rather than an additive effect. This implies that if the interface bond is doubled, say, due to surface finish treatment or due to mechanical deformation of the fiber, then σ_{pc} and G_c will quadruple if $f \sim 1$. There is, however, significant differences in the physical origin of the snubbing friction and the interface bond strength. For example, Table 1 (Li et al, 1989) shows the effect of matrix strength on pull-out load of Nylon monofilaments. In one sample, the fiber was pulled out normal to the matrix crack plane, whereas the other sample involved the fiber pulled out at an angle of 60°. For the normal pull-out case, the average of 8 tests for the higher strength (achieved with use of superplasticizer and microsilica together with a low w/c ratio) matrix shows a 6.3% increase over that for the normal strength matrix, but the standard deviation was large enough not to attach much significance to this increase. For the 60° pull-out, the average of 8 tests for the higher strength matrix shows a 127% increase over that for the normal strength matrix. This increase was significant in comparison to the standard deviation recorded in the test data, and suggests that while the interface bond strength may not have increased much, if at all, the snubbing effect appears to have been enhanced for

Table 1: Effect of Matrix Strength on Pull-out of Nylon Monofilament

	Pull-out Load (N) Normal strength HCP	High strength HCP
f = 0°	4.91 (CV = 5.6%)	5.22 (CV = 22.8%)
f = 60°	11.85 (CV = 27.1%)	26.94 (CV = 5.4%)
$P_{60°}/P_{0°}$ =	2.41	5.16

the fiber pulled out from the higher strength matrix, presumably due to the better spall resistance of the higher strength matrix. These observations are consistent with that of measured post-cracking strength for Spectra fiber reinforced normal and high strength concrete, discussed above. These preliminary results suggest plausibly different processing routes for the control of the snubbing friction and the interface bond strength.

Because the snubbing friction occurs only if the fiber is inclined to the loading direction, a high snubbing friction value would suggests a higher σ_{pc} and G_c than that of an aligned system. Again, this will be true only if no fiber rupture occurs. In an aligned fiber composite, more fibers tend to share the bridging load, whereas in a random fiber system, less number of fibers are sharing the bridging load, with each carrying a higher load. This implies a greater tendency for fiber rupture in the random case.

6. Conclusions

Although discussed only very briefly, and using extremely narrow and specific examples, the above presentation is meant to bring out the inter-dependencies between performance-properties-processing-structure for the development of cement based composites with pseudo-strain hardening and/or high fracture energy. It is shown that the fiber properties (distribution, length, aspect ratio, elastic stiffness and strength) and fiber/matrix interaction properties (interface bond strength, snubbing coefficient) directly govern the composite properties of post-crack strength and fracture energy which in turn controls the composite performance of pseudo strain-hardening and fracture energy. The need for novel processing routes of designing desirable bond strength, snubbing friction and fiber volume fraction is emphasized. Successful development of such an engineered material cannot be achieved without appreciating the inter-dependencies and constructing solutions which take advantage of the understanding of these inter-dependencies. In this process, it is recognized that micromechanics plays a significant role in quantifying the various links, and especially the link between material structure and properties.

The tetrahedron shown in Figure 1 also suggest an approach perhaps foreign to the development of construction materials -- the performance driven approach. If performance criteria (e.g. durability determined by the life expectancy of the structure; or energy absorption capacity determined by expected seismic load which may be imposed on the structure) can be specified, then the required material structure could be developed to achieve certain material properties using a certain processing route. In other words, a material could be engineered to satisfy the required performance of a given structure in a given environment. Certainly our current state of the art is far from this ideal, but that is precisely the challenge we must face in the research and development of high performance engineered FRCC.

7. Acknowledgement

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