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FRACTURE RESISTANCE PARAMETERS FOR CEMENTITIOUS MATERIALS AND THEIR EXPERIMENTAL DETERMINATIONS

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1 INTRODUCTION

The need for characterization of fracture resistance in metals has long been recognized. Spurred on by the development of turbine engines and the aviation industry, advances in linear and non-linear fracture mechanics have rapidly established standardized testing techniques to rank materials and to aid in the design against fracture failure in many metal structures. Unfortunately, the same has not happened in the concrete industry. The ACI code, for example, does not embody concepts from fracture mechanics. It has been said that the code is a "low-tech" one and at least part of the reason may be attributed to our lack of understanding of the fracture behavior in concrete and the proper application of fracture mechanics in concrete structure design. Also, our inability to characterize fracture resistance in concrete almost certainly has an effect on prohibiting the rapid development of new cementitious composites with improved strength and ductility. The need for a rational basis of concrete structure design with regard to public safety and economy, and the increasing demand in load carrying capability of concrete structures under severe environments are forcing us to reconsider our past strategy. Adequate effort must be invested in the research on the mechanical behavior of advanced cementitious composites and their application to concrete structures. Fracture mechanics provides a convenient tool to describe material and structural behavior, particularly when cracking or severe localization of deformation is involved.

A very basic need is defining what we mean by fracture resistance. This appears to be easier than it really is. If fracture resistance is a material property, it should not depend on the details of how we perform a test, i.e. it must not depend on the size of the specimen, the geometry of the specimen, or how the load is applied. In addition to theoretical considerations, there are also practical constraints. The parameter we choose to describe fracture resistance must be readily measured in the laboratory. This implies that the testing procedure must be simple enough to be standardized and repeated in different laboratories and the specimen must have size small enough to be easily handled and without exceeding the loading capacity of regular testing machines. These considerations expediate the general adoption of the testing method which benefits the users of the material being tested. In this report, we shall focus particularly on the theoretical aspects of specimen size and geometry effects of some of the fracture resistance parameters discussed in this conference (Application of Fracture Mechanics to Cementitious Composites) and particularly in the session on Experimental Methods of Determining Fracture Parameters.

In discussing the fracture resistance, it is useful to have some understanding of the physical processes leading to fracture failure of cementitious composites. In particular, concrete is a brittle material, but is quite different from glass in its fracture behavior because of its heterogeneity on the microstructural scale. The brittleness in concrete makes it a much weaker material under tension than in compression, because of the formation of microcracks and their extension and coalescence to form macrocracks. It has been observed that for a given specimen geometry and loading configuration, the tensile deformation localizes onto an eventual fracture plane. The mechanics of such localization is still not fully understood, but it is probably related to the presence of local defects such as concentration of voids. Research in damage mechanics should help in acquiring a handle on this aspect of concrete failure and in explaining the non-linear inelastic behavior prior to peak load in a tensile stress-strain curve (see, e.g. the session on "Damage and Continuum Modeling" of this volume). Once a macroscopic crack has formed, the crack tip can experience high tensile stress. Microcracking in the cement paste and in the cement-aggregate interface as well as pull-out of the aggregate from the cement matrix dissipate the energy which drives the propagation of the macrocrack. Observation under the microscope (see e.g., the session on "Fracture Processes in Cement Composites: Experimental Observations" of this volume) reveals interaction of microcracks and voids in the region ahead of the macrocrack tip. This zone of inelastic deformation, often known as the process zone in concrete, appears to be rather planar on the macroscopic scale. This is in contrast to the more ductile behavior exhibited by metal, where inelastic deformation occurs by generation of dislocations, and where the plastic zone does not in general lie on the plane of fracture.

If the inelastic deformation at the crack tip is confined to a small region (compared to crack length, distance to boundary of structure, distance to nearest rebar etc.), then it is known as small scale yielding (ssy). This terminology is originally intended to describe the plastic yielding zone in metal, but we should have no problem in using it to describe concrete if it is understood that "yielding" here really means the inelastic processes as described in the previous paragraph. Further, if the material outside the process zone is behaving elastically, then the application of linear elastic fracture mechanics (LEFM) is suitable. In particular, Irwin's stress intensity factor K would be useful to describe the intensity of stress and the fracture toughness K_{IC} could be used to rank such material as to its resistance to tensile fracture. However, it is very doubtful whether cracking in concrete observes the assumptions used in LEFM. In particular the process zone size has been observed to be large relative to normal specimen sizes so that the assumption of ssy is not valid in most cases. For example, Hillerborg (1) shows that for 3-point bend specimens, beam depths of less than 1-2m would give invalid K_{IC} values. Ingraffea and Gerstle (2) also give a detailed discussion of this problem, with special reference to the length and development of the process zone. Hence it is not surprising that most measurements of K_{IC} for concrete and mortar reported in the literature show some kind of size dependence, with larger values of K_{IC} obtained for large size specimens. Except for K_{IC} measured with very large specimens (see e.g. Sok (3)) most reported K_{IC} values for concrete are therefore not true material property. It should be pointed out, however, that although K_{IC}-testing is probably not valid for laboratory specimens, it does not necessarily imply that LEFM cannot be used to describe fracture behavior in real structures. That is, even if the process zone is on the order of a meter, it may still be a relatively small size in a large concrete structure so that ssy is still observed. In the following paragraphs, we shall discuss a modified K_{IC} procedure as proposed by Jeng and Shah (4), a G_F-test method recommended by a RILEM Committee (5) based on work by Petersson (6), R-curves for concrete, and also briefly discuss a test technique under development at MIT, which allows one to obtain the tension softening curve and the energy release rate. In the discussion, we shall take the approach of asking two very basic questions: Is what one measures truly a material parameter that describes fracture resistance? If so, how could it be used in the analysis and design of concrete structures?

2 A MODIFIED K_{IC} PROPOSED BY JENQ AND SHAH

Motivated by the lack of constancy of K_{IC} measured values published in the literature, Jenq and Shah (4) proposed the measurement of a modified K_{IC}, which they claim to be a material constant. The modification comes about by explicitly accounting for the inelastic process zone. Thus the energy release rate is partitioned into energy absorbed into the linear elastic K-field and energy absorbed into the planar process zone:

$$G_c = K_{IC}^2/E' + \int_0^{w_c} \sigma(w)\,dw \tag{1}$$

where σ is the tensile stress acting across the process zone and w is the material separation in the process zone. Their procedure of determining K_{IC} involves the use of a predetermined tension softening relation $\sigma(w)$, and the length of the process zone was determined by matching the numerically predicted and experimentally measured crack mouth opening displacement in a 3-point bending test. They found that the K_{IC} computed in this manner remains a constant for different specimen size and for different crack depths. The K_{IC} value they obtained for concrete is on the order of 1000 psi$\sqrt{\text{in}}$..

Is the partitioning of the energy release rate physically meaningful? Afterall, when Dugdale (7) and Barenblatt (8) introduced their strip yield model, their idea was to introduce a cohesive zone to cancel the stress singularity which would otherwise exist. So why would one still insist on the presence of the singular K-field ahead of the process zone? To answer this question, one should recall that concrete is a much more heterogeneous material than metal on the size scale of mm. The fracture process in concrete involves microcracking and aggregate pull-out as explained earlier. It is possible that the aggregate pull out occurring in the process zone acts as a separate mechanism from the microcracking at the tip of the process zone. Of course in order that the microcracking maintains a K-field, it has to satisfy the ssy condition, i.e. the size of the microcracking zone must be small in relation to the crack length and the ligament size of the specimen. Furthermore, since the stress in this zone can be larger than the tensile strength, the size of this zone should be smaller than the continuum scale. These ideas are illustrated in Figure 1.

Figure 2 shows pictorially the partitioning of the energy release rate as in equation 1. In general a complete tension softening curve as illustrated in Figure 2c would adequately describe everything about the fracture resistance, but the delta-function

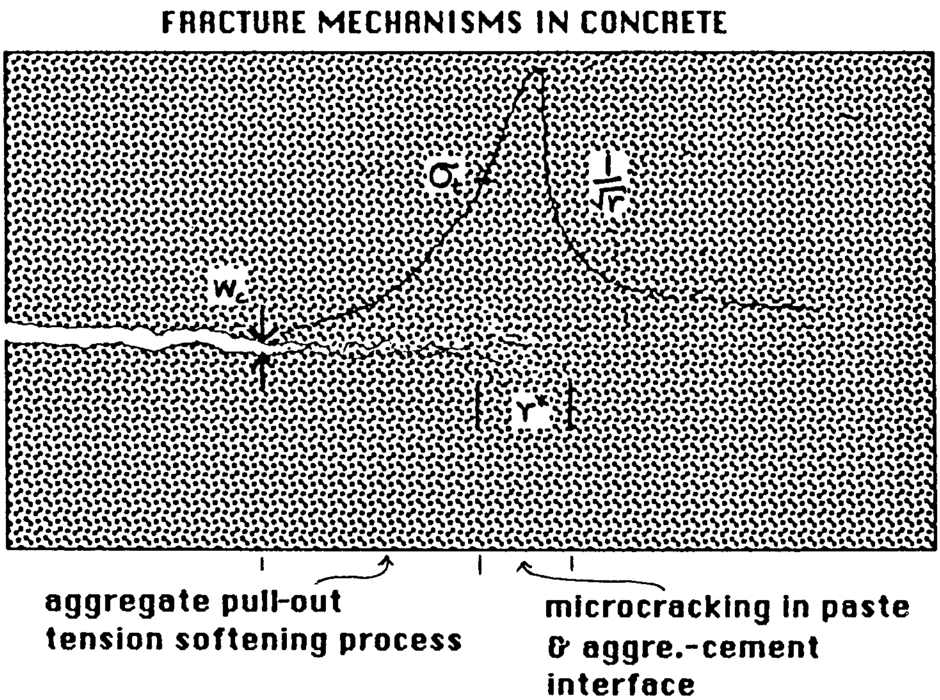


Figure 1: Physical Interpretation of Jenq and Shah's Model

like stress jump may occur with very small material separation w, making it impractical to be measured within experimental accuracy.

As a rough estimate, the size of the microcracking zone may be obtained by using the asymptotic stress field and r* may be calculated as follows:

$$\sigma = \frac{K_{IC}}{\sqrt{2\pi r}} \rightarrow r^* = \frac{1}{2\pi}(K_{IC}/\sigma_t)^2$$

$$\begin{aligned} \text{Typically,} \quad K_{IC} &\simeq 1000 \text{ psi}\sqrt{\text{in}} \\ \sigma_t &\simeq 500 \text{ psi} \\ \rightarrow r^* &\simeq 0.6 \text{ in} \end{aligned}$$

$$\mathcal{G}_c = K_{IC}/E' + \int_0^{\delta_c} \sigma(\delta)\, d\delta$$

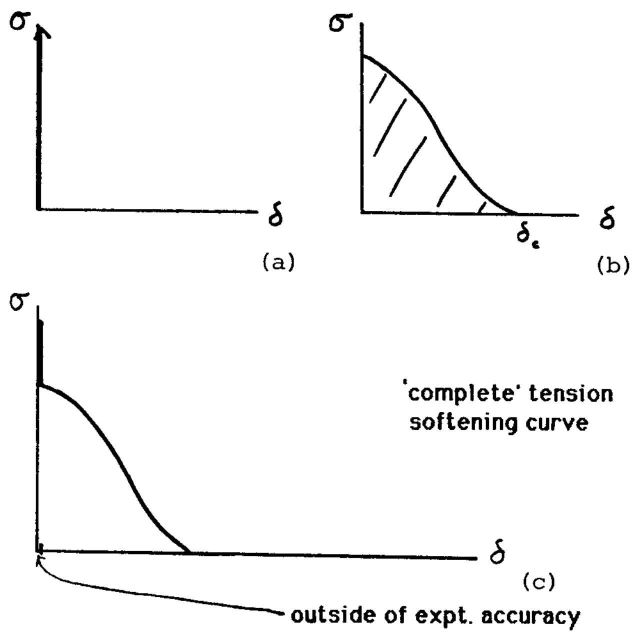


Figure 2: Partitioning of Energy Release Rate

If we assume the continuum scale to be on the order of several times the maximum aggregate size (0.75 in. used in the concrete mixture in the specimens of Jenq and Shah), then r* as calculated is certainly less than the continuum scale. Hence the model proposed by Jenq and Shah does have a physical basis and their successful application to experimental results by several researchers makes it all the more impressive. It would be of interest to check if the same procedure would derive the same K_{IC} values for specimens of geometries other than notched beams.

It should be pointed out, however, that the K_{IC} as obtained in this manner does not carry the same meaning as Irwin's K_{IC}. First the energy absorption is not fully described by this modified K_{IC}, and hence it is not a true fracture resistance parameter. Second, when the stress at the crack tip rises in response to increase loading, and when K_I reaches K_{IC}, unstable crack propagation do not necessarily occur, as would be the case in the ideally brittle material. These differences of the modified K_{IC} from Irwin's K_{IC} are of course a result of the presence of the process zone. Further, it should be mentioned that the computation of

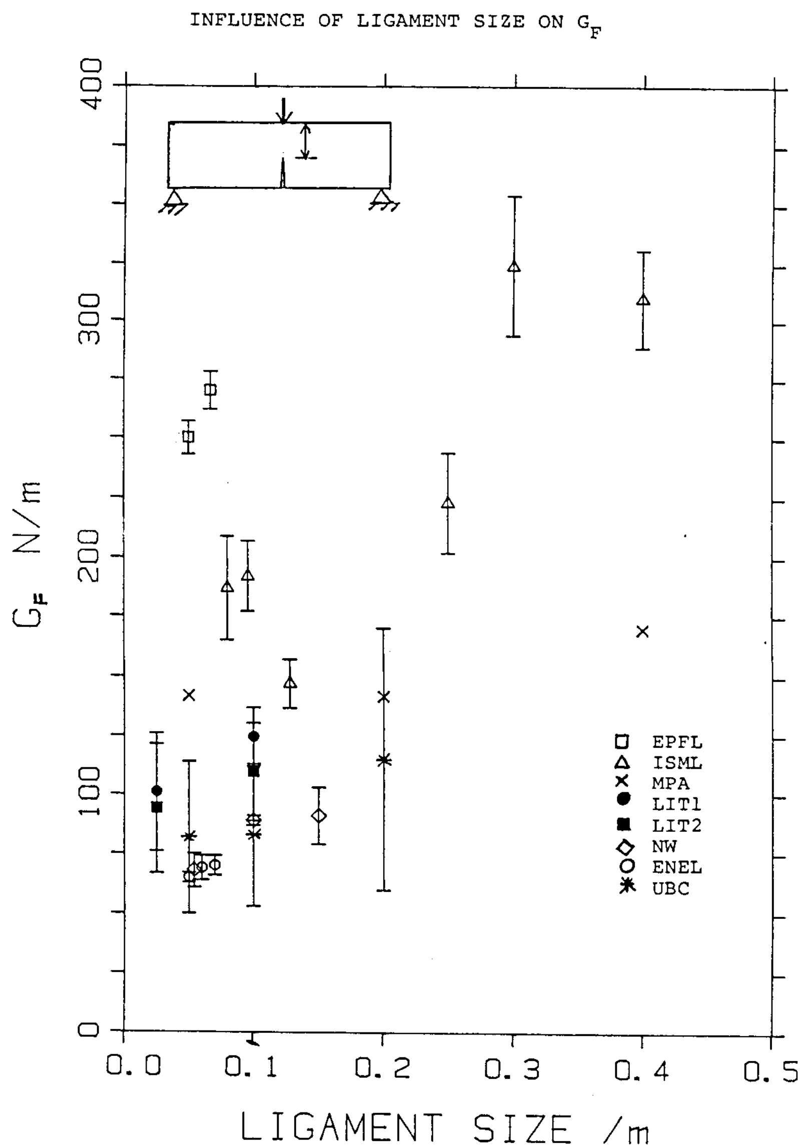
K_{IC} requires some knowledge of the tension softening curve. These considerations unfortunately make the modified K_{IC} less useful as a material parameter characterizing fracture resistance of concrete.

3 G_F-TEST AND SPECIMEN SIZE DEPENDENCE

The G_F-test has been under study by a RILEM-committee on Fracture of Concrete as a standard test for measuring fracture resistance of concrete (5). The test involves a 3-point bend test on a concrete beam loaded to complete failure. G_F is measured as the area under the load-deformation curve (with a correction for the weight of the beam) divided by the area of the fracture plane. The obvious advantage of this test method is the simplicity of the test procedure.

An assumption of this testing technique is that the work provided by the applied load all goes into the creation of the fracture plane. Care is used in ensuring that no energy is lost in crushing at the loading point and at the supports. However, it appears that G_F is still a specimen size-dependent parameter. This is illustrated in Figure 3 where G_F as measured by seven different laboratories are plotted as a function of ligament size. These data are based on reports (11), (6), (4) and (10). In each set of data, the w:c ratio, the maximum aggregate size, the span to depth ratio, and the concrete curing time have been kept constant. The data suggests a general increase of G_F with the ligament size. It is possible that prior to localization of deformation onto the eventual fracture plane, diffuse inelastic deformation or damage is occurring in the concrete beam. Thus part of the area under the load vs. load point displacement curve should have been attributed to this inelastic energy dissipation. Without properly accounting for this, the G_F value may be expected to be inflated and could only serve as an upperbound to the true fracture energy. The increase in G_F with beam size is probably associated with the large volume of material damaged during the loading process. This confirms earlier findings of G_F size-dependence by other authors (4), (13), (12).

The above discussion suggests that smaller beams may produce more accurate results. Indeed Figure 3 shows that the smaller beams in general seem to have less size dependence. More recently, the RILEM committee has initiated a second round-robin test involving 7 laboratories, with special attention paid to size effect (13). These tests are all performed with small size beams with ligament sizes of .0005m, .001m and .0015m. Even these small beams show a definite size effect. For example, the .001m beams show a 20% increase in G_F and the .0015m beams show a 30% increase in G_F over that for the small .0005m beams. Even



Constant { Age, D_{max}, Span/Depth, w:c ratio } for each set

Figure 3: Influence of Ligament Size on G_F

though these deviations may not be large when compared to size effects of other parameters such as cylinder compressive strength commonly adopted for concrete, it should be pointed out that the 20% and 30% deviations are that above the .0005m beams. It is not clear how much the G_F value measured from the .0005m beams differ from the true value such as that measured as the area under an accurate tension softening curve. It is also worrisome that the G_F as measured, being an upper bound value, is unconservative for engineering design purposes.

4 R-CURVE FOR CONCRETE

Many measurements of resistance curves for concrete and for fibre-reinforced concrete (see, e.g. Wecharatana & Shah (14), Mai (15)) have been published in the literature. In the following we discuss whether an R-curve calculated from laboratory experiments could be used to characterize fracture resistance uniquely.

The R-curve concept is originally used in metal, where it is found that the thickness of specimens plays a strong influence on the fracture resistance. This is due to the development of the size of the plastic zone which is sensitive to the stress state at the crack tip. In particular, the plane stress state allows growth of a large plastic zone in comparison to the plane strain situation where material constraints inhibit plastic zone growth, resulting in a lower fracture toughness. Thus in moderately thin metal sheets, fracture resistance increases with stable crack growth accompanied by plastic zone growth. This also happens under conditions approaching plane strain in highly ductile materials. In concrete, it has been found that plane stress or plain strain does not play an important role in determining the fracture resistance. In contrast to metal, where the inelastic behavior at the crack tip owes itself to true plasticity (dislocations), the inelastic behavior in concrete is due to microcracking and aggregate pull-out, which is most sensitive to straining normal to the crack plane (in mode I) but not so much to the stress in the perpendicular direction. Rather, the increase in fracture resistance in concrete with crack growth is associated with the development of the process zone. Ingraffea and Gerstle (2), e.g., shows that the process zone increases in length with applied load before reaching a steady value by means of finite element calculation and the assumption of tension softening on the crack line. They also show that the length of the process zone is not a material parameter, but rather depends on the specimen geometry and loading configuration. These considerations imply that the R-curve in general could not possibly be a unique material property. It is possible, however, to produce R-curves which look rather similar (e.g. Wecharatana and Shah (14), Bazant et al (15)). The similarity may be attributed to similar loading conditions producing similar stress fields which control the crack tip processes. For example the R-curves calculated by Bazant et al (15) were based on two 3-point bend tests and a compact tension test. Both test configurations produce a compressive zone at the outer fibres of the ligament and the resulting stress field may be expected to be rather similar.

Indeed, the applicability of the R-curve to describe fracture resistance in a real structure made of the same material (as the test specimen) depends on the presence of a dominant stress field in the test specimen and in the real structure. As an example, if the crack in a real structure experiences a dominant K-field, then the laboratory calculated K_R-curve is applicable provided the test has been performed under a dominant K-field. Such requirements are probably difficult to achieve for concrete.

Suppose one somehow obtains a unique R-curve, another concern is how one might use it. In general the R-curve is useful in determining stability of crack growth if one knows the increase of energy release rate with crack length. For concrete, if the process zone is large, this last relation is not easily determined so that the usefulness of the R-curve seems to be rather limited in practice.

5 INDIRECT MEASUREMENT OF TENSION SOFTENING CURVE

It appears that one of the most basic properties of concrete fracture is the relation between the tensile stress and the separation distance of material in the process zone, generally known as the tension softening curve. The tension softening curve has been used successfully as a constitutive relation on the crack line in several numerical schemes to describe behavior of structures containing cracks (see, e.g., Hillerborg et al (17), Ingraffea and Gerstle (2), Bazant and Oh, (18). Assuming no singular field exists at the crack tip, the area under the tension softening curve can be shown to be the critical energy release rate G_c. Several attempts at measuring the tension softening curve have been made (see e.g. Evans and Marathe (19), Petersson (6), Gopalaratnam and Shah (21), Reinhardt (22)). An inherent difficulty of this kind of testing is that the deformation is unstable and some kind of stiffening of the testing machine is necessary to perform a valid test. Various kinds of stiffening mechanisms have been used, including parallel steel bars in the direction of loading and closed loop feedback systems. Some of these tests have performed quite well, although the testing procedure can hardly become a standard one because of the need of rather intricate modifications on the loading machines. In the following paragraphs, we shall describe an indirect technique which has the following characteristics:

a) simple testing procedure
b) simple testing machine
c) small specimens

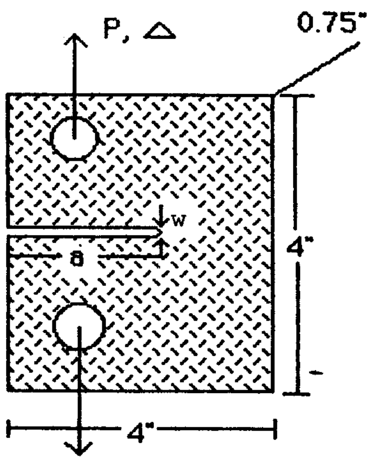
These characteristics are important if the technique is going to be widely adopted. They are also consistent with economy of testing, especially when a large number of specimens are to be tested.

We first describe the theoretical basis of this testing technique. For a cohesive type model of fracture process, it may be shown that the J-integral (see, e.g. Rice (23),(24)) when taken on a contour surrounding the process zone, may be written in the form

$$J = \int_{0}^{w_c} \sigma(w)\, dw \tag{2}$$

Taking the derivative with respect to w of equation (2) gives

$$\partial J/\partial w = \sigma(w) \tag{3}$$



a= 1.55 and 1.75 in.
cement:water:mortar sand: no. 8 sand= 2:1:2:2
curing: 7 days under water
cross head displ. rate= 0.002 in/min.

CONCRETE SPECIMEN

Figure 4: Specifications of Compact Tension Specimen

Hence the tension softening curve may be obtained from a knowledge of the change of J with respect to the separation w. For the calculation of J, we choose the compliance test on compact tension specimens. To fix ideas, two identical single edge notched specimens as shown in Figure 4 are loaded in tension. The only difference in the specimens are the crack lengths, which are cut by

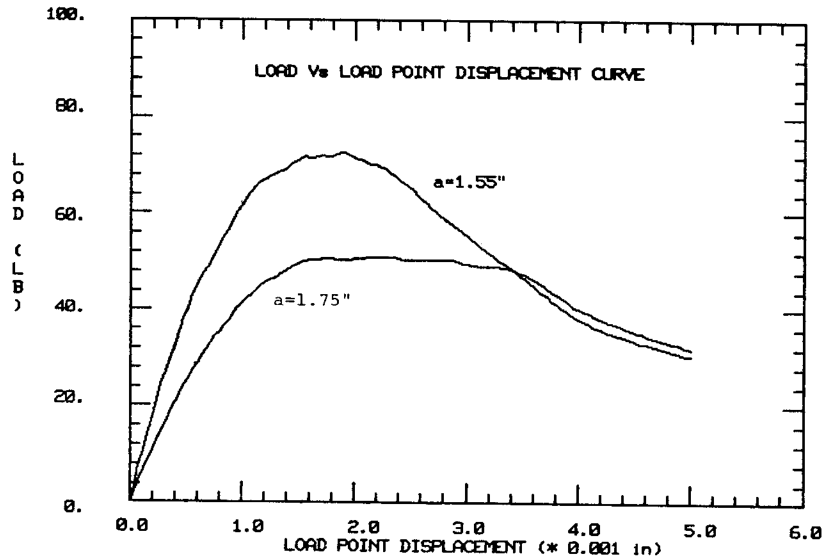


Figure 5a: Load vs. Load Point Displacement Curves from Experiment

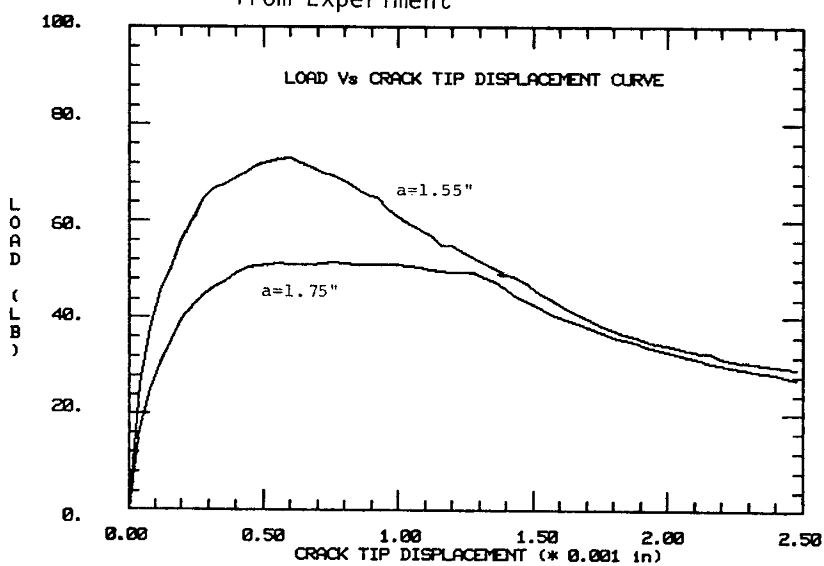


Figure 5b: Load vs. Crack Tip Displacement Curves from Experiment

diamond saws. During the test the load vs. load point displacement and the load vs. crack tip opening is continuously recorded by means of extensometers, as shown in Figures 5a,b. The area between the load vs. load point displacement curves give the value of $J(a_1-a_2)B$ where B is the thickness of the specimens, since an alternative definition of J is

$$J(w) = \int_o^w (\partial P/\partial a)_w \, dw \tag{4}$$

where P is the load magnitude.

As the crack length difference (a_1-a_2) is known a priori, J may be readily calculated. Once J(w) is known, its slope then gives the desired tension softening curve.

To illustrate, we show some preliminary results based on testing mortar specimens, with material and preparation specifications as shown in Figure 4. Figures 6a and 6b show the J vs. w and the $\partial J/\partial w$ vs. w results obtained from the test data shown in Figures 5a,b already mentioned. The $\partial J/\partial w$ curve shows an initial rise to a peak before descending down to zero. We attribute this behavior to diffuse inelastic damage experienced by the specimen prior to localization of damage onto the plane of the eventual fracture. This observation is consistent with what we had suggested earlier for the beams of the G_F tests. Prior to localization the measurement of w reflects the diffuse inelastic deformation and do not represent any real material separation of the crack tip. The corrected tension softening curve is shown as the dashed curve in Figure 6b. Following the suggestion of Hillerborg we have reproduced the tension softening curve in normalized form, shown in Figure 7. The separation distance w has been normalized by a characteristic length $\ell_{ch} \equiv EG_c/f_t^2$, where the critical energy release rate G_c has been measured at 0.28ℓb/in., the tensile strength f_t at 380 psi. With an assumed value of 4.5×10^6 psi for the Young's modulus E, ℓ_{ch} is equal to 8.7 in. Our preliminary results reveal a tension softening curve which agrees qualitatively with that measured by direct tensile tests by Petersson (6), by Gopalaratnam and Shah (21), and by Reinhardt (22). As a rough check, we compare their experimentally determined values of f_t, G_c and the critical separation w_c with these obtained from our experiments, shown in table 1. Information concerning concrete mix and age, and specimen dimensions are given in table 2. Table 1 shows that our values are generally on the low side, perhaps due to the early age at testing and the small aggregate size used for our specimens. This is particularly true for w_c. Results from Petersson (6) and Gopalaratnam and Shah (21) showed a long tail and w_c in fact could not be determined in their experiments.

Table 1. Comparison of preliminary test results with test results from (6), (21), (22).

Reference	f_t (psi)	w_c (10^{-3} in.)	G_c (lb/in)
Preliminary Test (this study)	380	1.29	0.28
Petersson (6)	479	>4.3*	0.58
Gopalaratnam & Shah (21)	405	>2.4*	0.42
Reinhardt (22)	464	6.9	0.77

* the w_c values given by Petersson, and by Gopalaratnam and Shah are subjective numbers. Their G_c-values were calculated to the indicated w_c values. Beyond that limit contribution to G_c was neglected.

Table 2: Specimen and Material Compositions

Reference	Type of Test	Overall Dimension	Mix Proportion (c:s:A:w	Max. size of sand/ aggregate	Age at Testing (date)
Preliminary Test	Compact Tension	4"x4"x0.75"	1:1:1:0.5	sand:0.056" agg:0.093"	7
Petersson	Direct Tension	2"x2"x1.2"	1:0.6:2.3: 2.3	sand:0.157"	28
Gopalaratnam & Shah	Direct Tension	12'x3'x0.75"	1:0:2:0.5	agg:0.19"	28
Reinhardt	Direct Tension	9.8"x2.4"x 2.5"	1:0:3.4:0.5	agg:0.31"	55

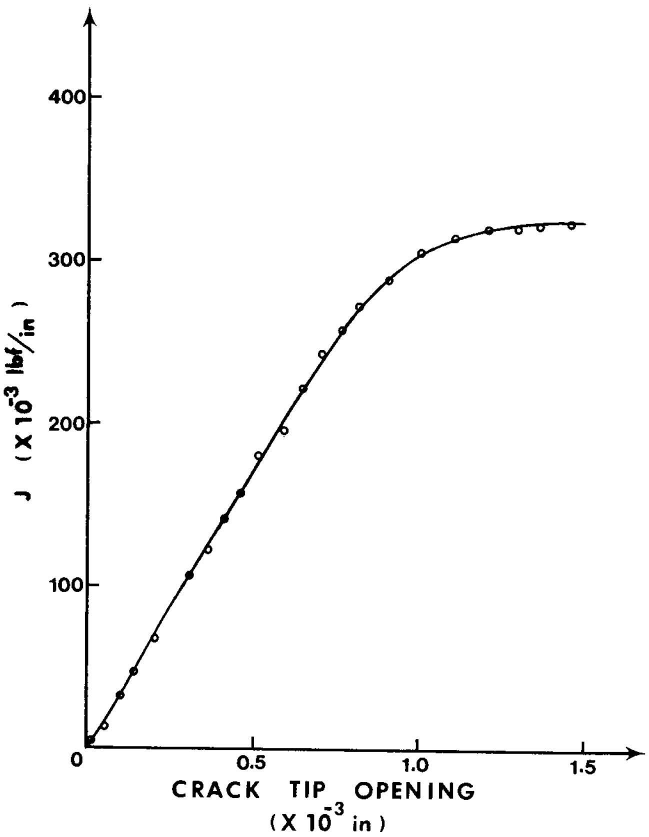


Figure 6a: J vs. Crack Tip Opening Displacement Deduced from Fig. 5a,b.

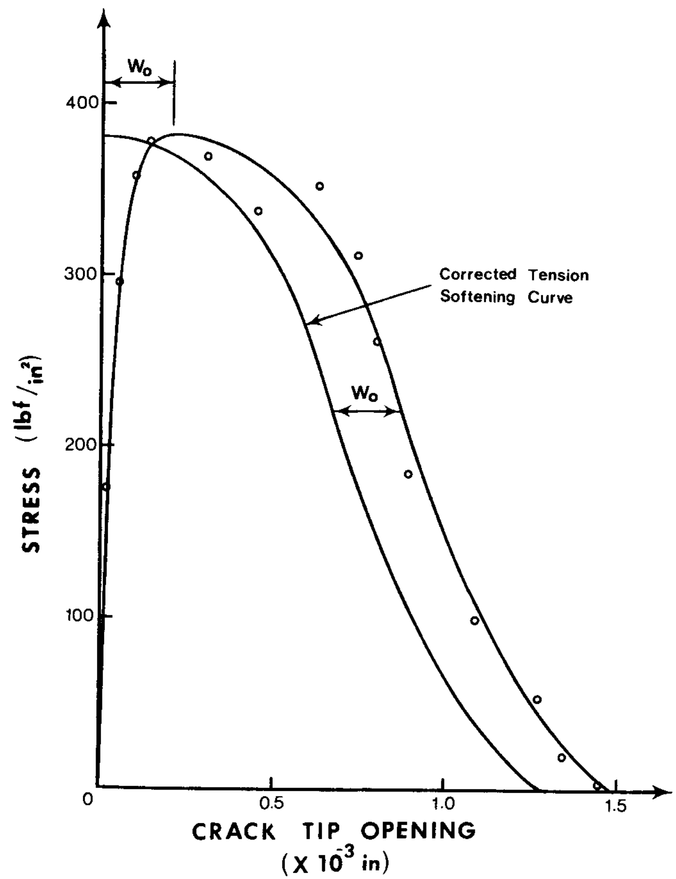


Figure 6b: Stress vs. Crack Tip Opening Deduced from Fig. 6a based on Equation (3)

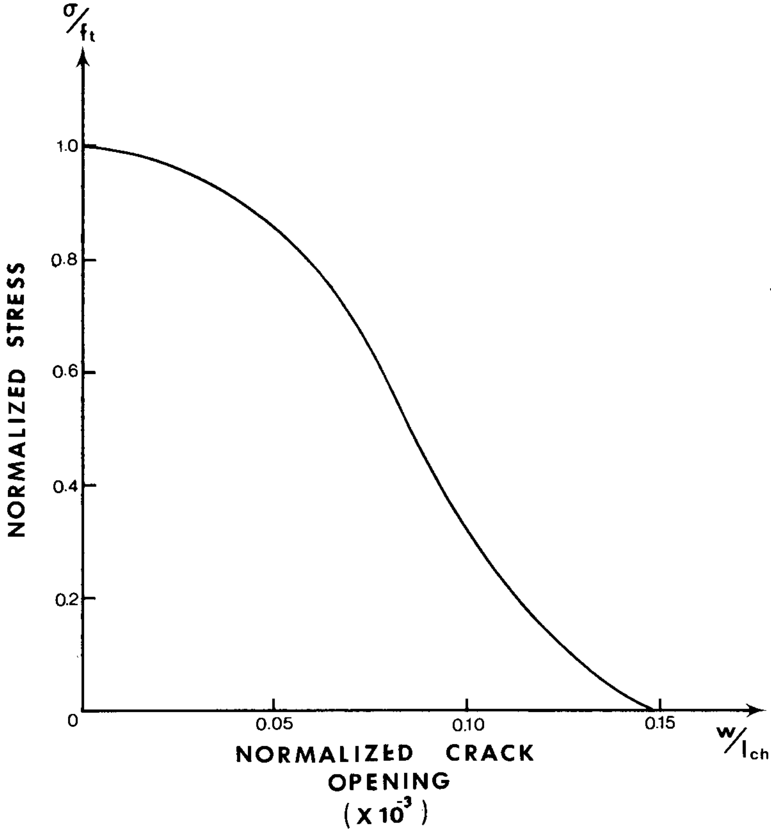


Figure 7: Normalized Tension-softening Curve

The qualitative agreements are encouraging. With further refinements in experimental details, the technique presented here may become a viable alternative to the direct tensile test. We expect that this experimental technique may also be applicable to other brittle materials with tension-softening behavior. Details of this testing technique and further experimental results are contained in a forthcoming report by Chan and Li (25).

6 CONCLUSIONS

We have discussed several fracture resistance parameters from the approach of getting at their physical basis. We have shown that some of these parameters are at best limited in their usefulness in describing concrete fracture resistance. A new technique of measuring tension-softening curves for brittle materials which overcomes problems of instability and is simple and economical to carry out is described and some preliminary results are presented.

7 ACKNOWLEDGEMENT

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