APPLICABILITY OF J-INTEGRAL TO TENSION-SOFTENING MATERIALS

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ABSTRACT: This brief paper addresses the application of the J-integral to fracture analysis and toughness characterization of quasi-brittle materials, with specific focus on the tension-softening behavior in the fracture process zone. Concern is often raised on the validity of the J-integral when unloading occurs in the material points along which the J-integral contour path traverses, particularly during process-zone growth. This paper clarifies that this concern is misplaced. Whereas the J-integral contour wraps around the tension-softening line springs of the bridging elements in the process zone, the material points on the J-integral contour undergoes elastic loading and unloading. Therefore application of the J-integral to quasi-brittle materials does not violate the basic tenet of the assumption of nonlinear elastic material behavior underlying the J-integral theoretical formulation.

INTRODUCTION

The J-integral was developed as a means of characterizing the fracture toughness of ductile metals (Rice 1968). As a fracture parameter serving both as a stress intensity characterization and as an energy supply characterization, its use in theoretical treatment of crack analysis and in experimental toughness determination of ductile metal have been accepted.

Over the last two decades, the J-integral has been suggested as an appropriate means to characterize and to measure the toughness of quasi-brittle material such as rock, concrete, ceramics and fiber reinforced cement, mortar, concrete, and ceramics (e.g., Mindess et al. 1977; Halvorsen 1980; Li et al. 1987; Li and Ward 1989; Rokugo et al. 1989; Teramura et al. 1990; Hashida 1990; Sato et al. 1994; Hashida et al. 1994; Budianksy 1995). The need for the J-integral instead of the simpler K_{IC} fracture parameter is associated with the often large (millimeters to tens of centimeters scale) fracture process zone in which the material undergoes tension-softening [see, for example, Hillerborg (1983)]. The physical processes of tension-softening is specific to the microstructure of a particular material. For example, in concrete, microcrack branching, aggregate bridging and other micromechanisms have been identified for being responsible for the tension-softening process [see Li and Maalej (1996) for a review]. In fiber-reinforced brittle-matrix composites, fiber bridging involving debonding (of fiber/matrix interface), fiber sliding, yielding, fracture, and other fiber/matrix interactions such as snubbing and spalling, are micromechanisms giving rise to tension-softening in such composites.

TENSION-SOFTENING IN FRACTURE PROCESS ZONE AND J-INTEGRAL

Despite the extensive theoretical and experimental validation of the J-integral for toughness characterization of quasi-brittle materials, there has been a persistent apprehension of the application of the J-integral in materials that exhibit tension-softening behavior. The J-integral has been defined for nonlinear elastic behavior, such that unloading is required to follow the same stress-strain curve as loading.

For tension-softening material, the elastic behavior of the crack-tip material prior to reaching peak load is shown schematically in Fig. 1. Under increasing ambient load, such material may undergo tension-softening, thus creating and growing a fracture process zone linking the crack flanks (Fig. 2). A simple representation of the behavior of the fracture process-zone material is a line of springs that continuously softens as it opens. This line-spring representation is especially suitable for materials that have damage localized onto a fracture plane, as in most quasi-brittle materials. Mathematically, the process-zone material can be represented by a line of springs located at $y = 0$. These springs have a constitutive relation that can be described in terms of a tensile stress versus spring stretching. Hence the spring stress captures the tensile traction across the crack line, while the spring stretch represents the opening of the crack. Obviously, the crack opening and therefore the spring stretch is a function of position x along the process zone.

The confusion of the applicability of the J-integral to tension-softening materials occurs when the J-integral is evaluated on a contour wrapped around the process zone. The concern arises in whether unloading of the tension-softening material violates the nonlinear elastic assumption behind the J-integral formulation. This issue can be clarified by considering first the J-integral contour Γ_{remote}, far removed from the fracture process zone (Fig. 2). Based on the path-independent property of the J-integral, the J-integral contour and the contour Γ_{pz} wrapped around the process zone Γ_{pz}, shrunk around the fracture process zone, has the same value as that for Γ_{remote}. When the material in the process zone (represented by the line springs) unloads inelastically (i.e. decreasing tensile traction σ and increasing crack opening δ on the line $y = 0$, [inset (a) in Fig. 2], the material points on Γ_{pz} (at $y = 0^+$, and $y = 0^-$) must also unload, by equilibrium consideration. However, this unloading is elastic and follows the elastic loading stress-strain relationship [inset (b) in Fig. 2]. Thus the basic tenet of nonlinear elastic behavior is in fact observed in the application of the J-integral to tension-softening quasi-brittle materials, even when the process zone grows in length.

The preceding comments are applicable to quasi-brittle materials with either a cohesive crack or a bridging crack (Cox and Marshall 1994). In the cohesive crack model, the physical mechanisms responsible for the spring traction in the process zone is the same as that which causes advance of the physical

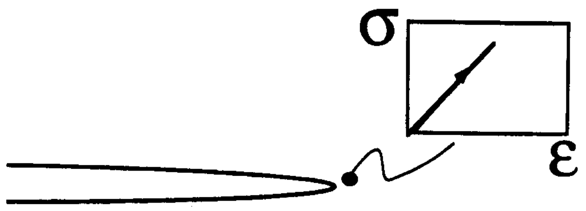

FIG. 1. Elastic Loading of Crack Tip Material before Growth of Process Zone

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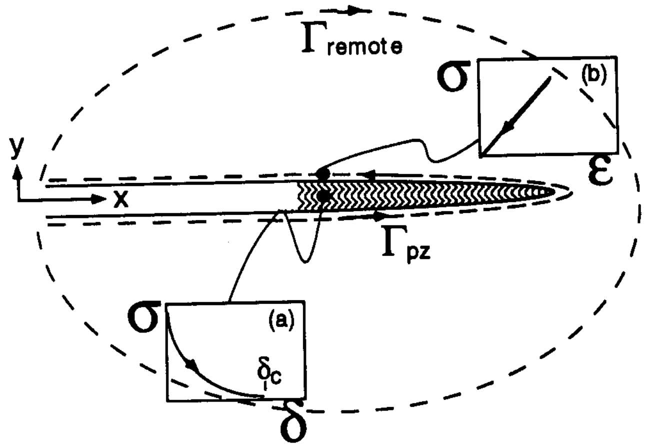

FIG. 2. Elastic Unloading [Inset (b)] of Material Point on Contour Γ_{pz} Wrapped around Process Zone in which Material Undergoes Inelastic Tension-Softening [Inset (a)], during Process Zone Growth

crack tip. When the process zone is taken into account explicitly, the stress profile is continuous across the crack front into the process zone, and no crack-tip singularity exists. In the bridged crack model, the advance of the matrix crack is accompanied by fiber bridging (as a common bridging element). The physical mechanism of matrix crack extension and fiber bridging are distinct, and may result in a tension-softening fiber-bridging process zone growing behind a crack tip with stress-singularity (Li et al. 1994).

An application of the J-integral using a remote contour Γ_{remote} combined with a contour Γ_{pz} wrapped around the process zone is used to determine the balance between energy supplied and energy consumed in the fracture process of tension-softening materials. Evaluations of the J-integral for the cohesive crack model and the bridging model are identical for the contours along both sides of the crack flanks. For the bridged crack model, the presence of crack tip singularity will result in the usual K_{tip} term when the additional contour wrapping around the crack tip shrinks to zero radius. Evaluation of the J-integral for cohesive cracks has been shown by Rice (1968). Eq. (1) reproduces this evaluation result for the bridging crack model with the additional crack tip term.

$$J = \frac{K_{tip}^2(1 - \nu^2)}{E} + \int_0^{\delta_t} \sigma(\delta) d\delta \qquad (1)$$

where the $\sigma(\delta)$ function describes the tension-softening relation of the process zone material. K_{tip}, E and ν = fracture toughness, elastic modulus, and the Poisson's Ratio of the crack tip material, respectively. δ_t is the crack opening at the end of the fracture process zone and is equal to the δ-value at which the bridging σ falls to zero in the $\sigma(\delta)$ function. Eq. (1) can be interpreted as the balance of crack-driving force J supplied by remote loading with energy consumed partly by the crack-tip singularity, and partly by the inelastic material breakdown process of the bridging elements [second term in (1)]. The critical value of J (i.e. $J = J_c$) occurs when the traction-free crack extends as δ_t reaches δ_c corresponding to $\sigma_c = 0$ [see Fig. 2, inset (a)].

Numerically, the energy consumed in the process zone is usually much higher than that consumed by crack front advance. Adopting the bridged crack model for concrete, the first term involving K_{tip} in (1) can be considered the cement-paste toughness that has been measured [e.g., Mindess (1983)] at about 0.01 kJ/m^2 or less, depending on the water/cement (w/c) ratio. The $\sigma(\delta)$ for a variety of concrete has been carefully measured in uniaxial tension specimens by Stang and Aare (1992) and shown to follow the relationship

$$\sigma(\delta) = \frac{\sigma_m^u}{1 + \left(\frac{\delta}{\delta_o}\right)^p} \qquad (2)$$

where σ_m^u, δ_o, and p are empirically determined. Integration of the second term in (1) using (2) up to δ_c (depending on maximum aggregate size) generally places the process zone energy consumption at around 0.1 kJ/m^2. For fiber-reinforced concrete, the $\sigma(\delta)$ curve has been measured by numerous researchers and found to be representable (Li 1992) in most cases by

$$\sigma(\tilde{\delta}) = \sigma_o (1 - \tilde{\delta})^2 \qquad (3)$$

where σ_o is a maximum bridging stress at zero crack opening, and δ have been normalized by fiber and interface parameters. Integration of the second term in (1) using (3) up to δ_c (equals half-fiber length) generally places the bridging zone energy consumption at around 1–10 kJ/m^2. Further details of fracture mechanisms and process zones in cementitious materials can be found in Li and Maalej (1996).

CONCLUSION

The application of the J-integral in tension-softening quasi-brittle materials is fundamentally sound, even in the presence of an advancing process zone.

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