INFLUENCE OF REINFORCING BARS ON SHRINKAGE STRESSES IN CONCRETE SLABS

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ABSTRACT: It is well known that cracking in concrete slabs significantly influences their service life. Concrete shrinkage may be the principle reason for the initial crack formation in the slabs. This paper presents an attempt to provide an analytical tool for the prediction of stress distribution in reinforced concrete slabs after undergoing matrix shrinking restrained by the reinforcing bars. The model incorporates the material parameters of the reinforcing bar and the concrete matrix, and it enables prediction of the stress development in concrete with time.

INTRODUCTION

As a kind of structural element, continuous reinforced concrete slabs have been widely used in modern transportation engineering, such as highway and airport pavements and bridge decks. The average life of a concrete slab is determined by many factors including initial design, material properties, traffic, environment, salt application, presence and effectiveness of protective systems, maintenance practices, etc. All of these factors influence the development of cracks in slabs during service. Cracking in slabs reduces the load capacity and speeds up fatigue failure (Matsui 1997; Perdikaris and Beim 1988; Perdikaris et al. 1989; Kumar and GangaRao 1998). In addition, cracks allow water and other chemical agents, such as deicing salt, to go through the cover layer to come into contact with the reinforcements, leading to reinforcement corrosion and rupture.

Concrete shrinkage may be the principal reason for the initial crack formation in slabs because, in general, slabs have a much larger surface area compared to other kinds of structural members, such as beams and columns. As a result, shrinkage-induced cracking in the slabs becomes more critical. Concrete shrinks as the cement paste hardens. The magnitude of shrinkage can be reduced by using concrete with the smallest possible amount of water and cement compatible with other requirements, such as strength and workability, and by moist-curing of sufficient duration. However, no matter what precautions are taken, a certain amount of shrinkage is usually unavoidable. If a slab of moderate dimensions rests freely on its supports, it can contract to accommodate the shortening of its length produced by shrinkage. Usually, however, slabs and other members are jointed rigidly to other parts of the structure and cannot contract freely. The reinforcements in the slab will also serve as a kind of restraint source to prevent concrete shrinking. This results in tensile stresses known as shrinkage stresses. A change in temperature may have an effect similar to shrinkage, which also results in stresses in the slab, with increasing temperature results in compressive stresses and decreasing temperature results in tensile stresses. As the tensile stresses produced by temperature and shrinkage attains the tensile strength of the concrete, cracking occurs in the slab. In this article, the effect of reinforcement on the stresses induced by matrix shrinkage in slabs will be modeled and analyzed.

PROBLEM FORMULATION

To simulate the stress distributions in the matrix and reinforcement due to matrix shrinkage, a representative volume element containing one reinforcing bar with width B (bar spacing), height H, length L, and bar diameter $2r_s$ is modeled. The representative volume element is further simplified into an equivalent cylinder with a reinforcing bar embedded along the longitudinal direction, as shown in Fig. 1, where r is the direction perpendicular to the reinforcing bar axis, x is the direction parallel to the reinforcing bar axis, and both ends of the reinforcing bar are located at $x = 0$ and $x = L$, respectively. By setting the same cross section area between the rectangle and a circle, the corresponding equivalent outer radius R is given by

$$R = \sqrt{\frac{HB}{\pi}} \tag{1}$$

In calculating the stress field developed due to shrinkage deformation, several simplifying assumptions are made: (1) The matrix and steel reinforcement are both elastic materials; (2) the interface between matrix and steel reinforcement is infinitesimally thin; (3) there is no slip between the reinforcement and the matrix at the interface; and (4) the shrinkage strain in the matrix ε_m, at a distance R from the reinforcement, is equal to the free shrinkage of the matrix.

When the matrix subjects a shrinkage strain ε_m in the direc-

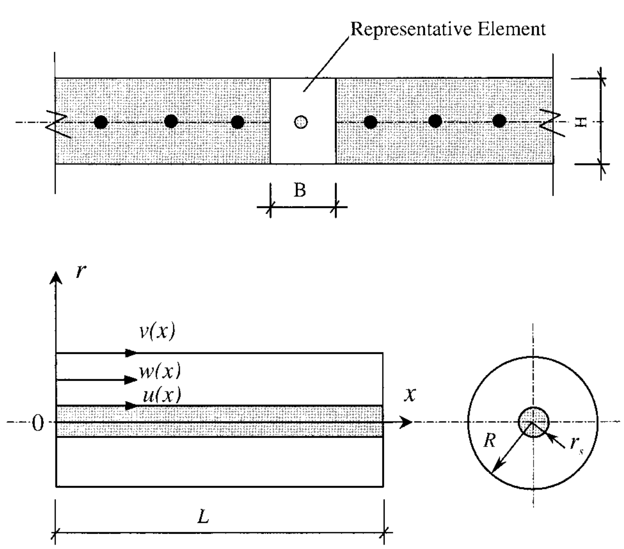

FIG. 1. Schematic Diagrams Showing Representative Element in Typical Steel-Reinforced Concrete Slab and Equivalent Model Used in Analyses

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tion of a reinforcing bar, based on the above assumptions, similar to the treatment by Cox (1952), the axial force equilibrium for a reinforcing bar of length dx in the presence of the axial stress σ_s in the reinforcing bar and the matrix/reinforcing bar interfacial shear stress τ_0 requires

$$\frac{\partial \sigma_s}{\partial x} + \frac{2}{r_s}\tau_0 = 0 \quad (2)$$

Further differentiation of (2) with respect to x results in

$$\frac{\partial^2 \sigma_s}{\partial^2 x} + \frac{2}{r_s}\frac{\partial \tau_0}{\partial x} = 0 \quad (3)$$

From the shear lag theory (Cox 1952), one has

$$\tau_0 = \frac{E_m}{2(1+\nu_m)}\frac{v-u}{r_s \log(R/r_s)} \quad (4)$$

where E_m and ν_m = Young's modulus and Poisson's ratio of matrix, respectively; and v and u = displacement field at $r = R$ and $r = r_s$, respectively (Fig. 1). Then one has

$$\frac{\partial \tau_0}{\partial x} = \frac{E_m}{2(1+\nu_m)r_s \log(R/r_s)}\left(\varepsilon_m - \frac{\sigma_s}{E_s}\right) \quad (5)$$

Replacing $\partial\tau_0/\partial x$ with (5) in (3), the general equation governing the axial stress distribution in the reinforcing bar σ_s is

$$\frac{\partial^2 \sigma_s}{\partial^2 x} + \frac{E_m}{r_s^2(1+\nu_m)}\frac{1}{\log(R/r_s)}\left(\varepsilon_m - \frac{\sigma_s}{E_s}\right) = 0 \quad (6)$$

This differential equation with boundary conditions that $\sigma_s = 0$ at $x = 0$ and $x = L$, yields

$$\sigma_s = E_s\varepsilon_m\left[1 - \frac{\cosh \beta\left(\frac{L}{2} - x\right)}{\cosh \beta \frac{L}{2}}\right] \quad (7)$$

where

$$\beta = \sqrt{\frac{E_m}{(1+\nu_m)r_s^2 E_s \log(R/r_s)}}$$

Also, the mechanical equilibrium of the extent load and internal stress distributions in the matrix-reinforcing bar composite cylinder at any location x requires

$$\sigma_s A_s + 2\pi\int_{r_s}^{R}\sigma_m r\, dr = 0 \quad (8)$$

where σ_m = axial stress in the matrix. Introducing an average stress in the matrix σ_{ma}, which is defined by

$$\sigma_{ma} = \frac{2\pi\int_{r_s}^{R}\sigma_m r\, dr}{A_m} \quad (9)$$

Then one has

$$\sigma_s A_s + \sigma_{ma} A_m = 0 \quad (10)$$

where A_s and A_m = cross-sectional areas of the reinforcing bar and matrix, respectively, and

$$\sigma_{ma} = -\phi E_s\varepsilon_m\left[1 - \frac{\cosh \beta\left(\frac{L}{2} - x\right)}{\cosh \beta \frac{L}{2}}\right] \quad (11)$$

where $\phi = A_s/A_m$ = reinforcing degree. The negative sign in (11) indicates that the stress in the concrete matrix is opposite to that in the steel bar. The volume shrinkage of concrete imposes a compressive stress on the reinforcing bars and a tensile stress on the concrete matrix itself. When $d\sigma_{ma}/dx = 0$, one obtains $x = l/2$, and so the maximum average stress in the matrix σ_{ma0} can be given by

$$\sigma_{ma0} = -\phi E_s\varepsilon_m\left[1 - \frac{1}{\cosh \beta \frac{L}{2}}\right] \quad (12)$$

Eq. (12) provides the general relationship between maximum matrix stress and matrix shrinkage for a given slab geometry and reinforcing degree. From this relationship, it is noted that for a given matrix shrinkage level ε_m, the average maximum tensile strength σ_{ma0} increases with slab length L. As σ_{ma0} attains the tensile strength of the matrix material, cracking will occur. It is noted also that there is an asymptotic limit of (12) for a given shrinkage strain (i.e., as $L \to \infty$, $\sigma_{ma0} \approx -\phi E_s\varepsilon_m$, which only depends on the reinforcing degree, matrix shrinkage strain, and elastic modulus of reinforcing bar).

MATERIAL PARAMETERS

The free shrinkage stain ε_m, elastic modulus E_m, and tensile strength σ_t are time-dependent concrete material properties. Since cracking in the matrix is a combined result of these parameters, shrinkage crack development also becomes time-dependent. As an example, the following empirical expressions, which describe the time-dependent law of the above parameters, are used in the present analysis.

Long-term studies (Branson 1977) show that, for moisture-cured concrete at any time t (in days), the shrinkage strain can be predicted satisfactorily by the following:

$$\varepsilon_{m,t} = \frac{t}{35+t}\varepsilon_{m,u} \quad (13)$$

where $\varepsilon_{m,t}$ = free shrinkage strain at time t (in days); and $\varepsilon_{m,u}$ = ultimate value after a long period of time. The time-dependent law of the elastic modulus of the concrete matrix can be estimated by (Mosley and Bungey 1990)

$$E_{m,t} = E_{m,28}[0.52 + 0.15\log(t)] \quad \text{for} \quad t \le 28 \quad (14a)$$

$$E_{m,t} = 1.019E_{m,28} \quad \text{for} \quad t > 28 \quad (14b)$$

where $E_{m,t}$ and $E_{m,28}$ = elastic moduli at time t (in days) and 28 days, respectively.

MODEL RESULTS AND DISCUSSIONS

As an example of calculation, the related parameters used in the model are listed in Table 1. The average maximum concrete tensile stress σ_{ma0} as a function of time t since casting, with three steel bars with radii of 6, 10, and 15 mm, is shown in Figs. 2–4 for different slab lengths. First, it can be seen that the axial concrete stress increases with time. Before

TABLE 1. Related Parameters Used in Model Calculation

Parameter (1)	Value (2)
B (mm)	200
H (mm)	200
r_s (mm)	6, 10, 15
$\varepsilon_{m,u}$	8E10^{-4}
$E_{m,28}$ (GPa)	30
ν_m	0.2
E_s (GPa)	210

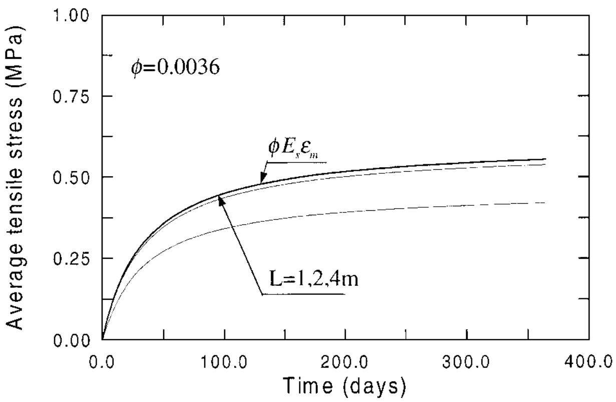

FIG. 2. Stress Development in Concrete due to Shrinkage in Terms of Stress-Time Curves, Showing Results of $\phi = 0.0036$ and Different Slab Lengths

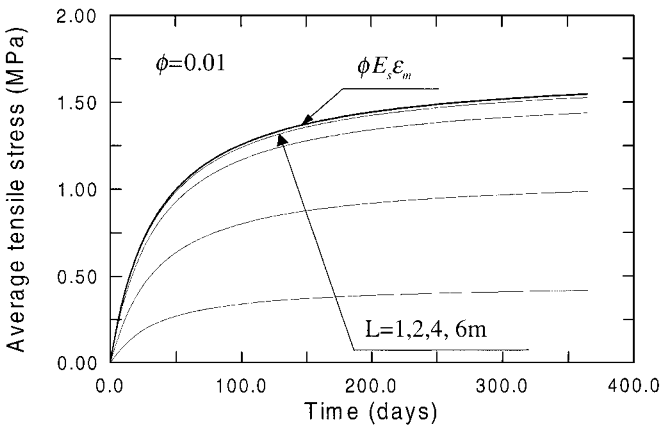

FIG. 3. Stress Development in Concrete due to Shrinkage in Terms of Stress-Time Curves, Showing Results of $\phi = 0.01$ and Different Slab Lengths

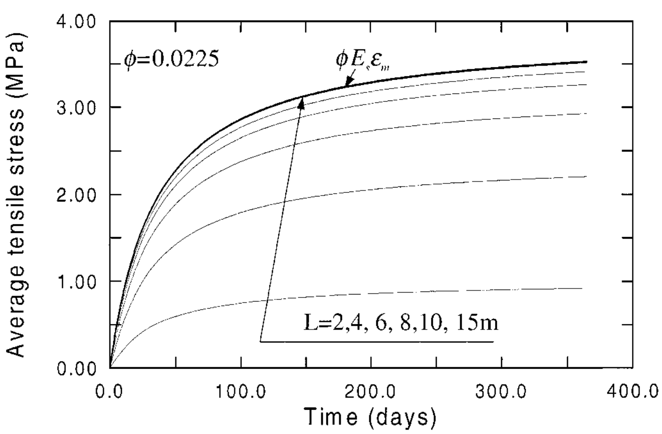

FIG. 4. Stress Development in Concrete due to Shrinkage in Terms of Stress-Time Curves, Showing Results of $\phi = 0.0225$ and Different Slab Lengths

around 50 days since casting, the stress increases with a high rate and later with a low rate. Second, the concrete tensile stress is also a function of slab length. The stress increases with increase in length L. However, the rate of increasing is gradually reduced and finally converges to a plateau of $\phi E_s \varepsilon_m$ (Fig. 5). A critical slab length L_c has been defined, such that at this critical slab length, the maximum average tensile stress in the concrete matrix is equal to 98% of that for an infinitely long slab under a given shrinkage strain; that is

$$L_c = \frac{2}{\beta} ar \cosh 50 \tag{15}$$

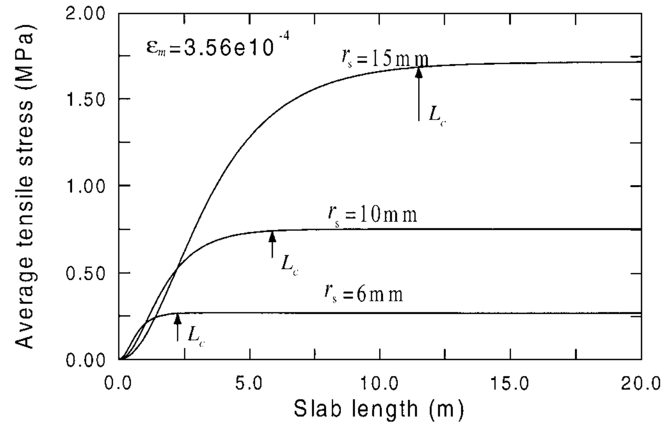

FIG. 5. Effect of Slab Length on Stress Developed in Concrete, for Three Typical Reinforcing Bars

In Fig. 5, it can be seen that L_c is a function of ϕ. For example, as ϕ changes from 0.0036 to 0.0225 (r_s changes from 6 to 15 mm), the L_c increases from 2.19 to 11.25 m. However, this does not mean that the higher reinforcing degree can delay the cracking in concrete because the cracking is controlled by the tensile strength of concrete. In fact, the higher reinforcing degree can raise the possibility of cracking due to the higher resulting tensile stress (Figs. 2–4). It is noted that the resulting concrete tensile stresses in a slab with a general slab length and reinforcing degree, such as 6 m and 1%, are lower than the tensile strength of normal concrete. However, as other shrinkage restraint factors exist, such as girders in bridge decks and subbase in concrete pavements, the total resulting tensile stresses might be large enough to crack the slab.

The present simple analysis considers only shrinkage restraint from the reinforcing bars. When other restraints such as a substrate supporting the slabs are present, they must also be accounted for in generating tensile shrinkage stresses in the slabs.

CONCLUSIONS

This paper presents an analytical model for predicting shrinkage stress in concrete due to the restraint of reinforcing bars in reinforced concrete slabs. The analytical solution indicates that as the matrix shrinks, compressive and tensile stresses are developed in the reinforcing bars and concrete matrix, respectively. The tensile stress in the concrete matrix is not only a function of shrinkage strain, but also a function of slab geometry, reinforcing degree, and elastic moduli of concrete and reinforcement. The model enables prediction of the stress development in concrete and reinforcement with time. The present analytical solution is applicable only to the case of restraint from reinforcing bars.

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