# The Effect of Mortgage Distress on Retirement Savings: Evidence from the PSID

Spencer Smith

# Outcome-Independent Payoffs in Strategic Voting

Daniel Russo

Spencer Smith

A thesis comprising two essays submitted in partial fulfillment of the requirements for the degree of

*Bachelor of Science*

Department of Economics

University of Michigan

April 20, 2011*

*awarded Highest Honors April 29, 2011

# Contents

# The Effect of Mortgage Distress on Retirement Savings: Evidence from the PSID*

Spencer Smith†

**Abstract**

What effect might recent mortgage distress have on retirement savings? Using data from the Panel Study of Income Dynamics (PSID), I find that families with mortgage distress are more likely to report cashing in part of a pension, annuity, or IRA. Among measures of mortgage distress, falling behind on mortgage payments and expecting to fall behind are the strongest predictors of whether families report cashing in. A large number of Americans are behind on payments or expect to fall behind, making this relationship particularly momentous. Those who cash in early will face withdrawal penalties and decreased wealth at retirement. Households strapped for cash should instead consider borrowing against their retirement savings in the form of 401(k) loans or similar debt. However, tax incentives and employer matching can make IRAs, 401(k)s, and other retirement accounts superior forms of all-purpose saving. In these cases, families do well to use retirement accounts for buffer stock saving, even if they expect to cash in early.

*I am grateful to Frank Stafford for his outstanding guidance and support.

†University of Michigan, spenceds@umich.edu

# 1 Introduction

In the last decade, the United States experienced an unsustainable housing bubble, a financial crisis, and an economic recession with lingering unemployment. These events continue to affect families and their assets in dramatic and, in some cases, irreversible ways. Anecdotal evidence suggests mortgage distress, in particular, affected many areas of families' balance sheets, including liquid savings, short-term investments, and even long-term investments, such as pensions and retirement savings.

The changing structure of pensions and other retirement saving instruments makes these forms of saving particularly important to household responses to mortgage distress and other aspects of the recent economic crisis. Over the last thirty years, many retirement savings plans underwent dramatic shifts from traditional, defined benefit formulas to less structured retirement accounts. Workers today are much more likely to rely on 401(k) plans, IRAs, and annuities than the kind of conventional pensions that support older Americans. According to the U.S. Department of Labor (2010), defined contribution plans covered approximately 82 million Americans in 2008, an increase of 66 million from the 16 million covered in 1978. Defined benefit plans, on the other hand, covered 42 million in 2008, an increase of only 6 million from the 36 million covered in 1978. Poterba *et al.* (2007) predict these changes will actually increase wealth at retirement for future retirees. Whether this is true or not, new forms of retirement saving certainly offer greater *flexibility* to future retirees. For better or worse, workers with IRAs and annuities are able to withdraw what they want (and when they want it) much easier than those with traditional pensions.

Facing shrinking home equity and perhaps even foreclosure, are Americans likely to dip into their retirement savings for help? If so, these decisions have important consequences. Families experiencing mortgage distress may jeopardize their financial future in order to satisfy the immediate demands of lenders.

In this paper, I explore the relationship between mortgage distress and retirement savings decisions using data from the Panel Study of Income Dynamics (PSID). The paper proceeds as follows: First, I describe the data I use to study the topic. I then outline and estimate several econometric models, highlighting the strengths and weaknesses of each. I present and discuss the results of each estimation, and I conclude by exploring alternative explanations, including a discussion of tax incentives at play.

## 2 Data

To estimate the effects of mortgage distress on retirement savings decisions, I rely on data from the Panel Study of Income Dynamics (PSID), a national, longitudinal study of nearly 9,000 American families. The families are considered a fairly representative sample of the United States. PSID researchers began collecting economic and demographic data in 1968 and continue to do so today.

In 1999, PSID interviewers began to ask respondents whether they or someone in their family cashed in any part of a pension, private annuity, or IRA since their last interview (typically two years prior). Respondents also reported the amount they cashed in. Interviewers continued to collect this information in the 2001, 2003, 2005, and 2007 waves. The question of whether a respondent or family member cashed in any part of a pension, private annuity, or IRA will serve as the primary variable of interest for my analysis. I seek to identify and isolate the causes of this decision and measure the relative importance, if any, of mortgage distress.

From the 2007 wave, I consider family-level data on pensions and retirement savings, housing and mortgages, family characteristics, education, employment, liquid savings, debt, and expenses.

In April 2010, the Institute for Social Research (ISR) released key data on housing and wealth from the 2009 wave of the PSID. This release contains timely information about the recent economic crisis and includes data from new questions on foreclosure activity, falling behind on mortgage payments, expectations of falling behind on payments in the next 12 months, and more. This release also includes the usual housing and mortgage data as well as data on liquid savings, debt, and retirement savings (including whether respondents have an

IRA or annuity, whether they or someone in their family cashed in any part of a pension, private annuity, or IRA in the past two years, and how much). However, the limited release lacks some basic data on pensions and pension structure, family characteristics, education, employment, and expenses. Thus, much of my analysis will rely on data from both the 2007 and 2009 waves. The complete 2009 data set will become available in mid-2011.

# 3 Cross-sectional Analysis

I begin by considering the 2009 data in isolation. To estimate the effect of mortgage distress on the decision to cash in retirement savings, I split econometric models into five model specifications. Each specification uses a different measure of mortgage distress. The five model specifications are listed below.

| Model Specification | Mortgage Distress Variable |
|---|---|
| I | second mortgage |
| II | behind on mortgage |
| III | expect to be behind |
| IV | underwater |
| V | foreclosure |

The mortgage distress variables are binary: each takes on 1 or 0 for "yes" or "no" values, respectively. The variables "second mortgage," "behind on mortgage," and "expect to be behind" correspond to specific questions from the 2009 questionnaire. (The variable "expect to be behind" refers to whether respondents expect to be behind on their mortgage payments *in the next 12 months*.) The variable "underwater" was generated using a ratio of mortgage balances to home value (the loan-to-value, or LTV, ratio). Families with LTV ratios greater than 1 (that is, housing debt that exceeds home value) are said to be "underwater." Finally, the variable "foreclosure" reflects whether a family underwent foreclosure between 2006 and 2009. The procedure I used to generate the “underwater” and “foreclosure” variables was borrowed from Stafford and Gouskova (2010).

Let us first consider a linear model of the form

$$cashed\_in_i = \alpha + \mathbf{x}_i'\boldsymbol{\beta} + \mathbf{y}_i'\boldsymbol{\gamma} + \delta \cdot mortgage\_distress_i + \varepsilon_i$$

and estimate the coefficients using ordinary least squares. Here, $cashed\_in_i$ is the variable of interest: a binary dependent variable that takes on the value 1 for families who report cashing in any part of a pension, private annuity, or IRA since January 2007 and 0 otherwise. The vector $\mathbf{x}_i$ contains age and retirement savings dummy variables relevant to the decision to cash in, and the vector $\mathbf{y}_i$ contains two other relevant pieces of information from the 2009 wave: (1) “liquid savings,” which represents the value of all checking accounts, savings accounts, CDs, and other liquid assets, and (2) “total debt” excluding mortgage debt. Together, these variables represent net worth (excluding home equity). The variable $mortgage\_distress_i$ takes on the mortgage distress variable determined by the model specification.

The coefficient estimates from an ordinary least squares regression are listed in Table 1.

**Table 1—Ordinary Least Squares**

| Independent Variable | Model Specification I | II | III | IV | V |
|---|---|---|---|---|---|
| age 30-39 | .006<br>(.008) | .006<br>(.008) | .006<br>(.008) | .008<br>(.008) | .008<br>(.008) |
| age 40-49 | .0004<br>(.0078) | –.002<br>(.008) | –.004<br>(.008) | .002<br>(.008) | .002<br>(.008) |
| age 50-59 | .008<br>(.009) | .007<br>(.009) | .006<br>(.009) | .011<br>(.009) | .010<br>(.009) |
| age 60 or older | .103***<br>(.011) | .101***<br>(.011) | .102***<br>(.011) | .104***<br>(.011) | .103***<br>(.011) |
| IRA/private annuity | .088***<br>(.010) | .094***<br>(.010) | .093***<br>(.010) | .091***<br>(.010) | .092***<br>(.010) |
| liquid savings (in $10,000s) | –.0001<br>(.0001) | –.0001<br>(.0001) | –.0001<br>(.0001) | –.0001<br>(.0001) | –.0001<br>(.0001) |
| total debt (in $10,000s) | .001<br>(.002) | .003<br>(.001) | .003<br>(.001) | .003<br>(.001) | .003<br>(.001) |
| second mortgage | .037*<br>(.016) | | | | |
| behind on mortgage | | .137***<br>(.036) | | | |
| expect to be behind | | | .070***<br>(.017) | | |
| underwater | | | | .050*<br>(.020) | |
| foreclosure | | | | | .073*<br>(.030) |
| constant | .004 | .003 | .002 | .003 | .003 |
| *F*-statistic | 19.1*** | 19.9*** | 20.5*** | 19.3*** | 18.8*** |
| *N* | 7,331 | 7,336 | 7,318 | 7,338 | 7,338 |

*Source: Panel Study of Income Dynamics (PSID)*
*Note:* Standard errors are in parentheses.
* significant at the .05 level
** significant at the .01 level
*** significant at the .001 level

Because $cashed\_in_i$ is a binary dependent variable, the OLS model is a linear probability model. Linear probability models are easy to estimate and the coefficients have useful interpretations. However, the model does not restrict the values of the binary dependent variable to [0,1], so predictions may erroneously fall outside this range.

A popular binary response model that corrects for this flaw is the probit model. The probit model assumes

$$\Pr\left(y_i = 1|\mathbf{x}_i\right) = \mathbf{\Phi}\left(\mathbf{x}_i'\boldsymbol{\beta}\right)$$

where $y_i$ is a binary dependent variable and $\mathbf{x}_i$ is a vector of independent variables. To corroborate the results of the OLS model, I estimate a probit model using the same regressand and regressors as before. The results are shown in Table 2.

Both models suggest a strong relationship between mortgage distress and the decision to cash in retirement savings. Coefficients on mortgage distress are statistically significant for all five model specifications under both OLS and probit. Moreover, each model has overall significance (see $F$ and $\chi^2$ statistics near the bottom of each table). According to OLS, families who are falling behind on mortgage payments or expect to do so are especially more likely to report cashing in a pension, private annuity, or IRA. Those who are falling behind are 13.7% more likely to report cashing in and those who *expect* to fall behind are 7% more likely to report cashing in.

The coefficients on liquid savings and total debt have the expected sign (those with greater liquid savings are *less* likely to report cashing in and those with greater debt are *more* likely to report cashing in), but neither is significant under OLS. Under probit, the coefficient on total debt is significant under model specifications II, III, IV, and V.

**Table 2—Probit**

| Independent Variable | Model Specification I | II | III | IV | V |
|---|---|---|---|---|---|
| age 30-39 | .193 | .216 | .202 | .220 | .221 |
| | (.126) | (.126) | (.126) | (.125) | (.125) |
| age 40-49 | .109 | .109 | .069 | .140 | .139 |
| | (.127) | (.129) | (.129) | (.128) | (.128) |
| age 50-59 | .226 | .233 | .221 | .266* | .253* |
| | (.127) | (.127) | (.127) | (.125) | (.126) |
| age 60 or older | .829*** | .835*** | .836*** | .855*** | .837*** |
| | (.113) | (.114) | (.114) | (.112) | (.113) |
| IRA/private annuity | .600*** | .652*** | .644*** | .621*** | .630*** |
| | (.063) | (.064) | (.064) | (.063) | (.063) |
| liquid savings (in $10,000s) | –.003 | –.003 | –.003 | –.003 | –.003 |
| | (.002) | (.003) | (.003) | (.003) | (.003) |
| total debt (in $10,000s) | .011 | .012* | .012* | .013* | .013* |
| | (.063) | (.057) | (.058) | (.061) | (.060) |
| second mortgage | .293** | | | | |
| | (.094) | | | | |
| behind on mortgage | | .875*** | | | |
| | | (.149) | | | |
| expect to be behind | | | .527*** | | |
| | | | (.098) | | |
| underwater | | | | .428*** | |
| | | | | (.123) | |
| foreclosure | | | | | .547*** |
| | | | | | (.169) |
| constant | –2.198 | –2.228 | –2.232 | –2.225 | –2.213 |
| $X^2$-statistic | 216.3*** | 226.3*** | 233.2*** | 216.3*** | 213.7*** |
| *N* | 7,331 | 7,336 | 7,318 | 7,338 | 7,338 |

*Source: Panel Study of Income Dynamics (PSID)*
*Note:* Standard errors are in parentheses.
* significant at the .05 level
** significant at the .01 level
*** significant at the .001 level

# 4 Panel Analysis

Panel data have several advantages relative to cross-sectional data. First, panel data allow researchers to explore important temporal effects. For example, I can consider the effect of income, employment, or mortgage distress in one year on retirement savings decisions in another year. Second, researchers can explore the data using more sophisticated econometric models, such as the fixed effects model discussed later in this section. Finally, using previous waves of PSID data, I can include important information missing from the limited release of 2009 data. For example, I can consider the effect family structure, pension structure, income, education, employment, and expenses on the decision to cash in part of a pension, IRA, or private annuity.

For my panel analysis, I consider a balanced panel of families using data from 2007 and 2009. Because the 2007 wave includes data on marital status, I separate married couples from families with single heads of household. Why? Married couples face significantly different retirement decisions, and their decision to cash in a pension, private annuity, or IRA will involve extra considerations, such as the employment status or retirement savings of a spouse.

I begin by estimating linear and probit models similar to the ones described in Section 3, but I add independent variables related to pension structure, education, income, employment, and school expenses. The five model specifications remain the same, but the models now relate a number of 2007 variables to 2009 outcomes.

Results from ordinary least squares and probit regressions for single individuals can be found in Table 3 and Table 4. Table 5 and Table 6 describe results from OLS and probit regressions for married couples. All models were estimated using 2007 cross-sectional p-weights.

**Table 3—Ordinary Least Squares (Single Individuals)**

| Independent Variable | Model Specification I | II | III | IV | V |
|---|---|---|---|---|---|
| age 50-59 | .020 (.015) | .020 (.014) | .021 (.014) | .023 (.014) | .022 (.014) |
| age 60 or older | .063 (.017)*** | .070 (.016)*** | .071 (.016) | .070 (.016)*** | .070 (.016)*** |
| db pension (current employer) | –.022 (.017) | –.022 (.016) | –.023 (.017) | –.023 (.017) | –.023 (.017) |
| retirement account (current employer) | .012 (.026) | .010 (.025) | .014 (.026) | .013 (.026) | .014 (.026) |
| both (current employer) | .023 (.029) | .023 (.029) | .022 (.029) | .023 (.029) | .023 (.029) |
| db pension (previous employer) | .079 (.032)* | .078 (.032)* | .078 (.032)* | .077 (.032)* | .078 (.032)* |
| retirement account (previous employer) | .027 (.029) | .033 (.028) | .032 (.028) | .032 (.028) | .032 (.028) |
| both (previous employer) | .199 (.131) | .184 (.127) | .187 (.131) | .195 (.132) | .189 (.129) |
| IRA/private annuity | .082 (.021)*** | .084 (.021)*** | .081 (.021)*** | .081 (.021)*** | .083 (.021)*** |
| college degree | –.020 (.015) | –.016 (.015) | –.018 (.015) | –.019 (.015) | –.018 (.015) |
| total income (‘06) (in $10,000s) | .002 (.002) | .002 (.002) | .002 (.002) | .002 (.002) | .0002 (.0002) |
| unemployed (‘07) | .004 (.018) | .009 (.018) | .008 (.018) | .006 (.018) | .007 (.018) |
| education expenses (‘07) (in $1,000s) | .003 (.002) | .003 (.002) | .003 (.002) | .003 (.002) | .003 (.002) |
| liquid savings (‘07) (in $10,000s) | –.0009 (.001) | –.0009 (.001) | –.0009 (.001) | –.0009 (.001) | –.0009 (.001) |
| total debt (‘07) (in $10,000s) | .006 (.004) | .006 (.003) | .006 (.004) | .006 (.004) | .005 (.003) |
| second mortgage (‘07) | .039 (.039) | | | | |
| behind on mortgage | | .184 (.070)** | | | |
| expect to be behind | | | .049 (.029) | | |
| underwater (‘07) | | | | –.026 (.026) | |
| foreclosure | | | | | .099 (.008) |
| constant | .002 | –.007 | –.005 | –.003 | –.005 |
| *F*-statistic | 4.14*** | 4.47*** | 4.33*** | 4.05*** | 4.21*** |
| *N* | 3,206 | 3,213 | 3,208 | 3,214 | 3,214 |

*Source: Panel Study of Income Dynamics (PSID)*
*Note:* Standard errors are in parentheses.
* significant at the .05 level
** significant at the .01 level
*** significant at the .001 level

**Table 4—Probit (Single Individuals)**

| Independent Variable | Model Specification I | II | III | IV | V |
|---|---|---|---|---|---|
| age 50-59 | .259 (.183) | .277 (.145) | .287 (.145)* | .297 (.143)* | .290 (.143)* |
| age 60 or older | .608 (.173)*** | .662 (.133)*** | .667 (.133)*** | .641 (.132)*** | .654 (.133)*** |
| db pension (current employer) | –.175 (.209) | –.175 (.206) | –.171 (.206) | –.174 (.207) | –.183 (.206) |
| retirement account (current employer) | .150 (.192) | .088 (.192) | .160 (.193) | .147 (.192) | .165 (.192) |
| both (current employer) | .241 (.224) | .267 (.225) | .241 (.224) | .248 (.223) | .258 (.223) |
| db pension (previous employer) | .452 (.155)** | .480 (.157)** | .467 (.155)** | .459 (.154)** | .468 (.155)** |
| retirement account (previous employer) | .301 (.191) | .339 (.186) | .314 (.186) | .316 (.188) | .322 (.186) |
| both (previous employer) | .906 (.396)* | .837 (.398)* | .838 (.405)* | .908 (.400)* | .855 (.397)* |
| IRA/private annuity | .557 (.123)*** | .591 (.125)*** | .563 (.124)*** | .561 (.123)*** | .579 (.124)*** |
| college degree | –.123 (.130) | –.085 (.132) | –.104 (.131) | –.108 (130) | –.109 (.131) |
| total income (‘06) (in $10,000s) | .015 (.010) | .016 (.011) | .017 (.010) | .017 (.010) | .017 (.010) |
| unemployed (‘07) | .058 (.232) | .084 (.232) | .064 (.232) | .036 (.233) | .059 (.231) |
| education expenses (‘07) (in $1,000s) | .010 (.005)* | .011 (.005)* | .011 (.005)* | .011 (.005)* | .011 (.005)* |
| liquid savings (‘07) (in $10,000s) | –.006 (.009) | –.007 (.010) | –.007 (.010) | –.007 (.010) | –.007 (.010) |
| total debt (‘07) (in $10,000s) | .043 (.020)* | .043 (.020)* | .043 (.020)* | .042 (.020)* | .040 (.019)* |
| second mortgage (‘07) | .284 (.220) | | | | |
| behind on mortgage | | 1.06 (.247)*** | | | |
| expect to be behind | | | .429 (.182)* | | |
| underwater (‘07) | | | | –.447 (.449) | |
| foreclosure | | | | | .728 (.291)* |
| constant | –2.236 | –2.328 | –2.305 | –2.268 | –2.295 |
| $X^2$-statistic | 122.5*** | 134.6*** | 125.9*** | 121.0*** | 127.0*** |
| *N* | 3,206 | 3,213 | 3,208 | 3,214 | 3,214 |

*Source: Panel Study of Income Dynamics (PSID)*
*Note:* Standard errors are in parentheses.
* significant at the .05 level
** significant at the .01 level
*** significant at the .001 level

**Table 5—Ordinary Least Squares (Married Couples)**

| Independent Variable | Model Specification I | II | III | IV | V |
|---|---|---|---|---|---|
| **head** age 50-59 | –.017 (.010) | –.017 (.010) | –.016 (.010) | –.017 (.010) | –.018 (.010) |
| **head** age 60 or older | .109 (.017)*** | .108 (.017)*** | .112 (.016)*** | .106 (.017)*** | .106 (.017) |
| db pension—**head** (current employer) | –.018 (.014) | –.014 (.014) | –.012 (.014) | –.015 (.014) | –.015 (.015) |
| retirement account—**head** (current employer) | –.014 (.014) | –.010 (.014) | –.009 (.014) | –.012 (.014) | –.011 (.014) |
| both—**head** (current employer) | –.030 (.018) | –.028 (.018) | –.026 (.018) | –.029 (.018) | –.028 (.018) |
| db pension—**head** (previous employer) | –.015 (.023) | –.013 (.023) | –.011 (.023) | –.014 (.023) | –.014 (.023) |
| retirement account—**head** (previous employer) | .015 (.024) | .018 (.024) | .018 (.025) | .018 (.024) | .018 (.024) |
| both—**head** (previous employer) | .059 (.068) | .057 (.068) | .058 (.068) | .058 (.068) | .058 (.068) |
| db pension—**wife** (current employer) | –.014 (.016) | –.011 (.016) | –.016 (.015) | –.012 (.016) | –.012 (.016) |
| retirement account—**wife** (current employer) | –.009 (.017) | –.008 (.017) | –.011 (.016) | –.008 (.017) | –.007 (.017) |
| both—**wife** (current employer) | –.002 (.024) | .004 (.025) | .004 (.025) | .002 (.024) | .002 (.025) |
| db pension—**wife** (previous employer) | .028 (.030) | .028 (.030) | .028 (.030) | .028 (.030) | .028 (.030) |
| retirement account—**wife** (previous employer) | –.027 (.021) | –.026 (.021) | –.025 (.021) | –.026 (.021) | –.026 (.021) |
| both—**wife** (previous employer) | .036 (.076) | .042 (.077) | .043 (.077) | .040 (.077) | .041 (.077) |
| IRA/private annuity | .092 (.014)*** | .094 (.014)*** | .094 (.014)*** | .094 (.014)*** | .094 (.014) |
| college degree—**head** | .020 (.016) | .024 (.015) | .025 (.015) | .023 (.015) | .023 (.014) |
| college degree—**wife** | –.003 (.014) | –.002 (.014) | .001 (.014) | –.002 (.014) | –.002 (.014) |
| total income (‘06) (in $10,000s) | –.0009 (.0004)* | –.0009 (.0004)* | –.0009 (.0004)* | –.0009 (.0004)* | –.0009 (.0004) |
| unemployed—**head** (‘07) | .065 (.041) | .060 (.040) | .060 (.040) | .063 (.041) | .061 (.041) |
| unemployed—**wife** (‘07) | .032 (.034) | .030 (.034) | .032 (.034) | .031 (.035) | .029 (.034) |
| education expenses (‘07) (in $1,000s) | .0002 (.0008) | .0003 (.0008) | .0003 (.0008) | .0003 (.0008) | .0003 (.0008) |
| liquid savings (‘07) (in $10,000s) | .001 (.001) | .0009 (.0009) | .001 (.0009) | .0009 (.0009) | .0009 (.0009) |
| total debt (‘07) (in $10,000s) | –.0005 (.0004) | –.0005 (.0004) | –.0005 (.0004) | –.0005 (.0004) | –.0005 (.0004) |
| second mortgage (‘07) | .050 (.018)** | | | | |
| behind on mortgage | | .084 (.032)** | | | |
| expect to be behind | | | .074 (.020)*** | | |
| underwater (‘07) | | | | .041 (.044) | |
| foreclosure | | | | | .033 (.029) |
| constant | .016 | .015 | .008 | .019 | .019 |
| *F*-statistic | 4.95*** | 4.86*** | 5.12*** | 4.65*** | 469*** |
| *N* | 3,909 | 3,910 | 3,902 | 3,912 | 3,912 |

*Source: Panel Study of Income Dynamics (PSID)*
*Note:* Standard errors are in parentheses.
* significant at the .05 level
** significant at the .01 level
*** significant at the .001 level

**Table 6—Probit (Married Couples)**

| Independent Variable | Model Specification I | II | III | IV | V |
|---|---|---|---|---|---|
| **head** age 50-59 | –.131 (.113) | –.129 (.114) | –.116 (.114) | –.135 (.113) | –.122 (.113) |
| **head** age 60 or older | .663 (.094)*** | .651 (.094)*** | .690 (.094)*** | .628 (.093)*** | .632 (.093)*** |
| db pension—**head** (current employer) | –.090 (.124) | –.047 (.124) | –.042 (.123) | –.065 (.123) | –.058 (.123) |
| retirement account—**head** (current employer) | –.050 (.124) | –.017 (.124) | –.015 (.126) | –.032 (.124) | –.028 (.124) |
| both—**head** (current employer) | –.176 (.167) | –.146 (.165) | –.141 (.166) | –.159 (.165) | –.155 (.165) |
| db pension—**head** (previous employer) | –.093 (.122) | –.082 (.122) | –.064 (.123) | –.091 (.122) | –.090 (.123) |
| retirement account—**head** (previous employer) | .188 (.144) | .209 (.146) | .228 (.146) | .198 (.145) | .200 (.145) |
| both—**head** (previous employer) | .255 (.268) | .255 (.269) | .269 (.271) | .253 (.269) | .249 (.269) |
| db pension—**wife** (current employer) | –.061 (.135) | –.041 (.135) | –.090 (.135) | –.046 (.134) | –.045 (.134) |
| retirement account—**wife** (current employer) | –.054 (.136) | –.055 (.135) | –.096 (.136) | –.043 (.135) | –.038 (.135) |
| both—**wife** (current employer) | –.019 (.177) | .030 (.176) | .031 (.178) | .019 (.175) | .016 (.176) |
| db pension—**wife** (previous employer) | .166 (.142) | .162 (.144) | .156 (.145) | .166 (.14) | .167 (.144) |
| retirement account—**wife** (previous employer) | –.171 (.157) | –.173 (.159) | –.171 (.160) | –.161 (.158) | –.167 (.159) |
| both—**wife** (previous employer) | .240 (.354) | .300 (.354) | .323 (.359) | .285 (.352) | .296 (.352) |
| IRA/private annuity | .630 (.088)*** | .645 (.089)*** | .652 (.090)*** | .642 (.088)*** | .644 (.088)*** |
| college degree—**head** | .135 (.097) | .151 (.098) | .165 (.099) | .142 (.097) | .145 (.097) |
| college degree—**wife** | –.011 (.097) | –.010 (.098) | .007 (.098) | –.012 (.097) | –.012 (.097) |
| total income (‘06) (in $10,000s) | –.009 (.005) | –.008 (.005) | –.008 (.005) | –.008 (.005) | –.008 (.005) |
| unemployed—**head** (‘07) | .446 (.221)* | .403 (.221) | .402 (.223) | .423 (.220) | .417 (.220) |
| unemployed—**wife** (‘07) | .264 (.248) | .242 (.250) | .250 (.253) | .269 (.247) | .245 (.248) |
| education expenses (‘07) (in $1,000s) | .001 (.005) | .002 (.005) | .002 (.005) | .002 (.005) | .002 (.005) |
| liquid savings (‘07) (in $10,000s) | .004 (.004) | .004 (.004) | .004 (.004) | .004 (.004) | .004 (.004) |
| total debt (‘07) (in $10,000s) | –.002 (.002) | –.002 (.002) | –.002 (.002) | –.002 (.002) | –.002 (.002) |
| second mortgage (‘07) | .371 (.107)*** | | | | |
| behind on mortgage | | .582 (.173)*** | | | |
| expect to be behind | | | .543 (.118)*** | | |
| underwater (‘07) | | | | .284 (.277) | |
| foreclosure | | | | | .270 (.202) |
| constant | –2.002 | –2.002 | –2.077 | –1.962 | –1.971 |
| $X^2$-statistic | 169.5*** | 165.8*** | 177.4*** | 159.2*** | 161.1*** |
| *N* | 3,909 | 3,910 | 3,902 | 3,912 | 3,912 |

*Source: Panel Study of Income Dynamics (PSID)*
*Note:* Standard errors are in parentheses.
* significant at the .05 level
** significant at the .01 level
*** significant at the .001 level

Overall, these panel results are similar to the cross-sectional results. Notice mortgage distress appears to more strongly affect retirement savings decisions made by married couples than those made by individuals. The probit models indicate stronger significance of mortgage distress variables than OLS, but in none of the models is being underwater statistically significant. Undergoing foreclosure, too, is rarely statistically significant. The OLS coefficients on foreclosure, however, are quite large. Thus, although the foreclosure variable is, in general, not *statistically* significant, foreclosure still appears to significantly affect the decision to cash in some part of a pension, IRA, or private annuity. For example, single individuals who report undergoing foreclosure since 2006 are approximately 10% more likely to report cashing in between 2007 and 2009.

Being behind on mortgage payments is statistically significant under both OLS and probit and for both individuals and married couples. The OLS coefficients are sizable. Single individuals and married couples who report being behind on their mortgage payments are 18.4% and 8.4% more likely to report cashing in, respectively. The statistical significance and magnitude of these coefficients have important implications. In 2009, 15% of PSID respondents with mortgages report they are behind on their mortgage payments or expect to be behind in the next twelve months. This means a whopping 15% of respondents are between 8% and 18% more likely to cash in their retirement savings. The large number of Americans falling behind on mortgage payments and the strong effect of falling behind may very well combine to drain retirement savings accounts in the United States.

These results suggest mortgage distress affects the decision to cash in retirement savings, but the most powerful predictors of whether families cash in a pension, annuity, or IRA are still age and the structure of retirement savings (such as whether someone in the family holds flexible retirement accounts). Note the strong effect of having both a defined benefit pension *and* retirement account with a previous employer. Single individuals with this characteristic are nearly 20% more likely to report cashing in. Married couples with heads who have both a defined benefit pension and retirement account with a previous employer are nearly 6% more likely to report cashing in.

Families with school expenses and families with an unemployed head or wife in 2007 are consistently more likely to report cashing in a pension, annuity, or IRA in 2009, but these regressors are rarely statistically significant. The other regressors do not exhibit consistent results.

### Fixed Effects Estimation

We can also examine the panel data using a fixed effects model. The fixed effects model and other panel-specific models are advantageous because they use repeated observations to account for unobserved heterogeneity. Given some simple assumptions, the fixed effects model can even yield unbiased and consistent estimates of coefficients of endogenous variables.

For my analysis, however, panel methods have some drawbacks. The fixed effects model can only be used to estimate the effects of *time-varying* regressors. Thus, a fixed effects model cannot account for pension structure, income, employment, education, school expenses, or other variables found in the 2007 data but not the 2009 data. Moreover, a fixed effects model cannot estimate the effects of falling behind on mortgage payments, expectations of falling behind, foreclosure, or other variables found only in the 2009 data. Thus, we can only use the fixed effects method to estimate model specifications I and IV (which measure the effect of having a second mortgage and the effect of being underwater, respectively).

I consider a fixed effects model of the form

$$cashed\_in_{it} = \alpha + \mathbf{x}_{it}'\boldsymbol{\beta} + \mathbf{y}_{it}'\boldsymbol{\gamma} + \delta \cdot mortgage\_distress_{it} + \mu_i + \varepsilon_{it}$$

where $i = 1, \dots, N$ and $t = 2007, 2009$. Here, $\mathbf{x}_{it}$ is a vector of age and retirement savings dummy variables, $\mathbf{y}_{it}$ is a vector of independent variables on liquid savings and total debt, $mortgage\_distress_{it}$ takes on the mortgage distress variable determined by the model specification, $\mu_i$ represents unobserved heterogeneity, and $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are vectors of coefficient parameters. I estimate the model using 2007 cross-sectional p-weights. Again, single individuals and married couples are considered separately. The results of these regressions are reported in Table 10 and

Table 11, which can be found in the appendix. The coefficients reported are within-estimators.

As Table 10 and Table 11 show, the fixed effects model does not support my hypothesis. Neither of the mortgage distress variables are significant predictors of cashing in, nor does the coefficient on having a second mortgage even have the expected sign. That said, the models themselves are also not significant (see $F$ statistics near the bottom of the tables). That is, given the available data, the fixed effects framework seems unsuitable to answer the question at hand. Thus, the model really neither strengthens nor weakens my prediction that mortgage distress influences Americans' decisions to cash in their retirement savings.

# 5 Discussion

Why would someone cash in their retirement to make mortgage payments? Is this an optimal decision? The answer is probably "No," particularly in the case of young families. The fact that—among other mortgage distress variables—falling behind on mortgage payments and expecting to fall behind are the most significant and consistent predictors of cashing in retirement savings says a lot about the nature of the problem. This fact suggests that decisions to cash in early have more to do with cash flow limitations than with decreases in total wealth. Indeed, Lu and Mitchell (2010) find that people who are liquidity-constrained are more likely to "borrow from themselves" using 401(k) loans.

A 401(k) loan allows the owner of a 401(k) to borrow against his plan balance. As Li and Smith (2008) describe, these loans offer several advantages relative to other forms of borrowing. First, 401(k) loans have low transaction costs. Second, borrowers repay principal and interest into their own account rather than to a private lender. Thus, the cost of a 401(k) loan is essentially the foregone return on borrowed funds (Li and Smith, 2008). For 401(k) owners who need cash now, borrowing against 401(k) plan balances seems more sensible than simply cashing in, which involves an early withdrawal fee. According to VanDerhei et al. (2009), 88% of 401(k) participants were in plans that offered loans in 2008.

So why are folks cashing in? Do they simply have no concept of present value? Perhaps they suffer from hyperbolic discounting as described by Laibson (1997). Hyperbolic discounters develop dynamically inconsistent preferences. That is, their "today selves" and "tomorrow selves" disagree about optimal decisions. Perhaps savers' today selves want to cash in their retirement savings to make mortgage payments even though their retiree selves would rather not.

### Tax Incentives

Another possible explanation has to do with the tax incentives of 401(k)s, IRAs, and other retirement accounts. Before I explore these incentives, let me first describe important characteristics of the accounts, beginning with 401(k)s:

Employers set up 401(k) retirement plans for their employees (these are called 403(b) plans in the non-profit sector). Employees set aside part of their salary for retirement, and employers sometimes match a percentage of that contribution. The amount saved for retirement is not taxed as ordinary income, and the 401(k) accrues interest tax-free. When a retiree withdraws from a 401(k), the withdrawal is then taxed as ordinary income.

IRAs are similar to 401(k)s. These are optional retirement accounts for individuals without employer-provided 401(k)s. Like 401(k)s, contributions to IRAs are not taxed, and interest on the investment accrues tax-free. Funds are taxed upon withdrawal. Another variation, Keogh accounts, offers the same tax incentives to individuals who are self-employed.

The tax benefits of 401(k)s, IRAs, and similar retirement accounts create strong incentives to save. Moreover, these retirement plans are almost always preferable to other forms of private retirement saving. To see why, consider the following simple example taken from Hyman (2008).

**Table 7—The Tax Advantage of IRA Savings**

| Account Type | Earnings | Tax on Earnings ($\tau$ = .25) | Initial Deposit | Earned Interest ($r$ = .10) | Taxed Paid Upon Withdrawal | Total Amount Withdrawn |
|---|---|---|---|---|---|---|
| regular | $100 | $25 | $75 | $7.50 | 0.25 * $7.50 = $1.88 | $75 + $7.50 – $1.88 = $80.62 |
| IRA | $100 | $0 | $100 | $10 | 0.25 * $110 = $27.50 | $100 + $10 – $27.50 = $82.50 |

Source: *Public Finance: A Contemporary Application of Theory to Policy* by Hyman (2008)

As you can see, tax incentives exist in just one period. As interest compounds over multiple periods, the incentives become even stronger. Plus, employers sometimes match contributions, in which case 401(k)s and IRAs are unbeatable ways to save for retirement.

With these incentives in mind, might someone simply use a 401(k), IRA, or another retirement account for their general saving rather than (or in addition to) their retirement saving? As it turns out, a number of mechanisms exist to prevent this. First, there are limits to employee and employer contributions. For traditional IRAs, employee contributions are limited to $5,000 per year and total contributions are limited to $10,000 per year. For 401(k)s, annual employee contributions are limited to $16,500. Second, any withdrawal made before the age of $59\frac{1}{2}$ years incurs a hefty penalty (typically 10%). In the simple example above, the employee would be better off to save privately than to save using an IRA, withdraw after one year, and pay an additional 10% of the withdrawal.

But is this still the case over a longer time period? And with larger employee contributions? And what happens when an employer partially or fully matches employee contributions? Suppose an individual contributes $C$ each year to a traditional IRA. The IRA accumulates interest with a rate of interest $r$. The individual faces a marginal tax rate $\tau$ and an early withdrawal penalty $\rho$. Then an individual who invests for $T$ years and withdraws before age $59\frac{1}{2}$ will walk away with

$$(1-\tau-\rho)\sum_{t=1}^{T} C\left(1+r\right)^{t}.$$

An individual who invests the same amount privately each year, with the same rate of interest and the same marginal tax rate, will have

$$\sum_{t=1}^{T} C(1-\tau)(1+r(1-\tau))^t$$

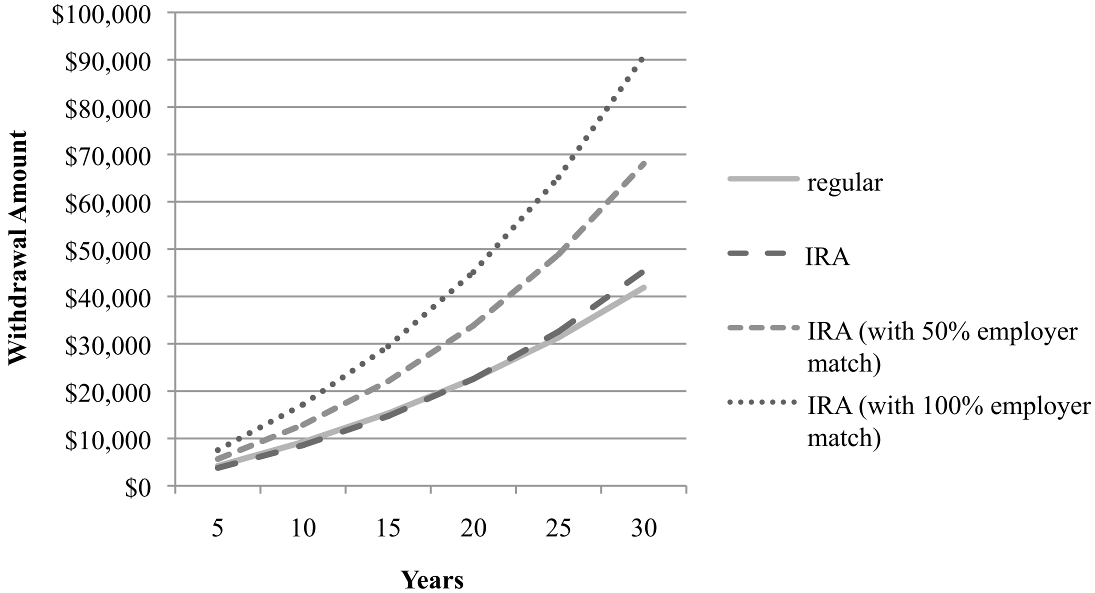
after $T$ years. Consider a simple example where an employee wants to save \$1,000 each year and can choose between a traditional IRA and private saving. Both accounts accrue interest at a rate of 5%. The marginal tax rate is 25%, and the IRA has an early withdrawal penalty of 10%. Table 8 displays the total amount available after 5, 10, 15, 20, 25, and 30 years with and without employer matching.

**Table 8—Savings Over Time**

| Account Type | Annual Employee Contribution | Amount after *T* years (*r* = .05, τ = .25, ρ = .10) | | | | | |
|---|---|---|---|---|---|---|---|
| | | *T* = 5 | *T* = 10 | *T* = 15 | *T* = 20 | *T* = 25 | *T* = 30 |
| regular | $1,000 | $4,194 | $9,235 | $15,295 | $22,579 | $31,336 | $41,863 |
| IRA | $1,000 | $3,771 | $8,584 | $14,727 | $22,568 | $32,574 | $45,345 |
| IRA (with %50 employer match) | $1,000 | $5,657 | $12,876 | $22,091 | $33,852 | $48,861 | $68,018 |
| IRA (with 100% employer match) | $1,000 | $7,542 | $17,168 | $29,454 | $45,136 | $65,148 | $90,690 |

In this example, the IRA with no employer matching and a 10% early withdrawal penalty becomes more profitable between 20 and 25 years of investment. An IRA with at least 50% employer matching, however, is more profitable than private saving given any time horizon. These results are displayed graphically in Figure 1. The results also hold for other levels of annual employee contributions (up to the \$5,000 limit). Those scenarios are displayed graphically in Figures 3-6, which can be found in the appendix.

**Figure 1—Savings Over Time**
**($1,000 annual contribution)**




The numbers reported in the preceding examples are in nominal, not real, terms. With an inflation rate $\pi$, an individual who contributes $C$ to a traditional IRA each year for $T$ years at a rate of interest $r$, with a marginal tax rate $\tau$, and with an early-withdrawal penalty $\rho$ will walk away with

$$(1-\tau-\rho)\sum_{t=1}^{T} C(1-\pi)^{T-t}(1+r-\pi)^t.$$

An individual who opts to save the same amount for the same number of years at the same level of interest, but without the tax benefits of an IRA, will walk away with

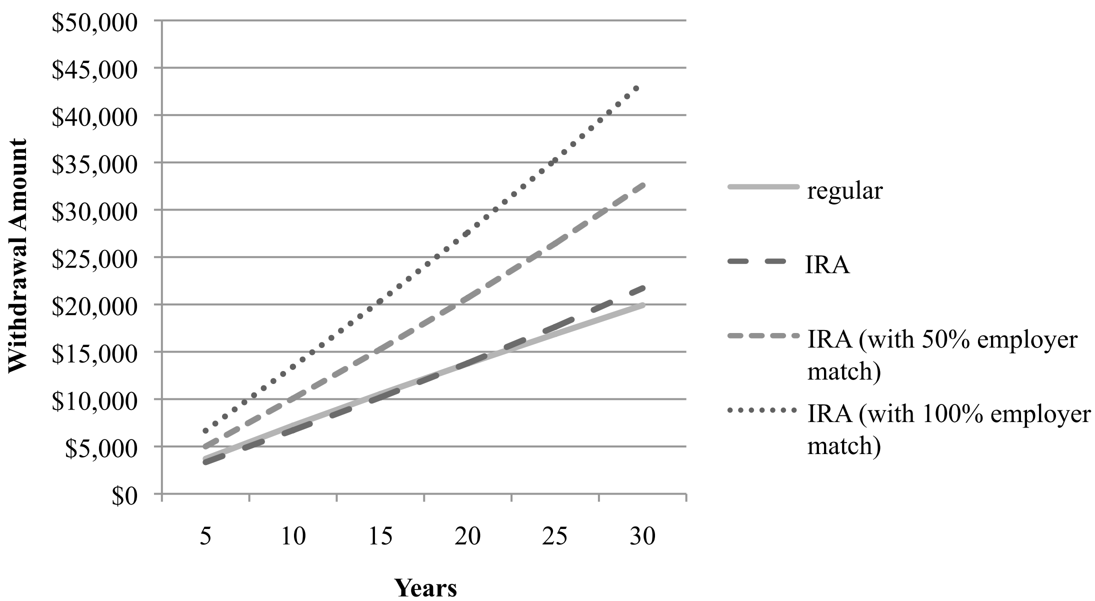
$$\sum_{t=1}^{T} C(1-\tau)(1-\pi)^{T-t}(1+r(1-\tau)-\pi)^t.$$

We can repeat the simulation where $C = \$1,000$, $r = .05$, $\tau = .25$, and $\rho = .10$ and account for inflation. Let $\pi = .025$ (2.5% inflation). The results are displayed in Table 9 and Figure 2.

**Table 9—Savings Over Time (with 2.5% inflation)**

| Account Type | Annual Employee Contribution | Amount after $T$ years ($r$ = .05, τ = .25, ρ = .10, π = .025) | | | | | |
|---|---|---|---|---|---|---|---|
| | | $T$ = 5 | $T$ = 10 | $T$ = 15 | $T$ = 20 | $T$ = 25 | $T$ = 30 |
| regular | $1,000 | $3,705 | $7,208 | $10,546 | $13,757 | $16,872 | $19,921 |
| IRA | $1,000 | $3,335 | $6,713 | $10,184 | $13,804 | $17,628 | $21,716 |
| IRA (with %50 employer match) | $1,000 | $5,003 | $10,070 | $15,276 | $20,706 | $26,442 | $32,574 |
| IRA (with 100% employer match) | $1,000 | $6,670 | $13,426 | $20,368 | $27,608 | $35,256 | $43,432 |

**Figure 2—Savings Over Time with 2.5% Inflation**
**($1,000 annual contribution)**




Here, we see that an IRA with no employer matching and a 10% early withdrawal penalty becomes more profitable slightly earlier: between 15 and 20 years of investment. Again, employer matching of at least 50% makes an IRA an unambiguously better choice, even with a hefty 10% withdrawal penalty. And again, these results hold for other levels of annual employee contributions up to $5,000 (Figures 7-10 in the appendix).

Is it possible that those who invest in a 401(k), IRA, or another retirement account with plans to cash in early are saving optimally? According to these results, yes. Tax incentives and employer matching may outweigh early withdrawal penalties, making IRAs and other retirement accounts preferred outlets for all-purpose saving. Indeed, these plans "can be seen not only as a vehicle for retirement saving but also as a medium for precautionary saving that can help protect against income or consumption shocks" (Lu and Mitchell, 2010, p. 19). Savers can benefit from generous employer matching, avoid taxes upon deposit, and accrue interest tax-free. If they need to withdraw early to catch up on mortgage payments or to avoid foreclosure, the penalty may be well worth it. Perhaps this explains the behavior of some individuals and families who cash in their retirement savings earlier than expected. These folks are simply using retirement accounts for buffer stock saving. Of course, there are caveats. Without employer matching, tax-deferred retirement saving takes approximately 20 years to trump private saving. Moreover, early withdrawal will invariably reduce the amount of funds available at retirement.

# 6 Conclusion

Given the changing landscape of pension coverage, are American families likely to dip into retirement savings in response to recent mortgage distress? I explore this question using data from the Panel Study of Income Dynamics (PSID). Data from 2007 and 2009 suggest families with mortgage distress are more likely to report cashing in part of a pension, annuity, or IRA. Among a number of mortgage distress variables, falling behind on mortgage payments and expecting to fall behind are the strongest predictors of whether families cash in. The large number of Americans who report falling behind on mortgage payments makes this relationship particularly momentous.

Although mortgage distress appears to influence retirement savings decisions, age and the structure of retirement savings are still the strongest predictors of whether families report cashing in. Liquid savings, debt, school-related expenses, and unemployment also appear to play some role, but to a lesser degree.

Are those who cash in early acting optimally? Possibly. Early withdrawal will reduce wealth at retirement, and to reinforce patience, many retirement savings accounts have hefty early withdrawal penalties. However, tax incentives and employer matching may make IRAs and 401(k)s superior forms of all-purpose saving.

# 7 Appendix

**Table 10—Fixed Effects (Single Individuals)**

| | Model Specification | | | | |
|---|---|---|---|---|---|
| Independent Variable | I | II | III | IV | V |
| age 50-59 | .006 (.022) | | | .006 (.022) | |
| age 60 or older | .054 (.038) | | | .052 (.038) | |
| IRA/private annuity | .030 (.014)* | | | .033 (.014)* | |
| liquid savings (in $10,000s) | –.0004 (.0004) | | | –.0004 (.0004) | |
| total debt (in $10,000s) | .0002 (.0009) | | | .0002 (.0009) | |
| second mortgage | –.063 (.026) | | | | |
| underwater | | | | .050 (.027) | |
| constant | .027 | | | .025 | |
| *F*-statistic | 2.41* | | | 1.98 | |
| *N* | 6,513 | | | 6,525 | |

*Source: Panel Study of Income Dynamics (PSID)*
*Note:* Standard errors are in parentheses.
* significant at the .05 level
** significant at the .01 level
*** significant at the .001 level

**Table 11—Fixed Effects (Married Couples)**

| | Model Specification | | | | |
|---|---|---|---|---|---|
| Independent Variable | I | II | III | IV | V |
| age 50-59 | .024 (.023) | | | .023 (.023) | |
| age 60 or older | .077 (.033)* | | | .076 (.033)* | |
| IRA/private annuity | .022 (.011) | | | .022 (.011) | |
| liquid savings (in $10,000s) | –.00005 (.0003) | | | –.00005 (.0003) | |
| total debt (in $10,000s) | .0005 (.0005) | | | .0005 (.0005) | |
| second mortgage | –.009 (.015) | | | | |
| underwater | | | | .011 (.018) | |
| constant | .035 | | | .034 | |
| *F*-statistic | 1.82 | | | 1.81 | |
| *N* | 8,023 | | | 8,028 | |

*Source: Panel Study of Income Dynamics (PSID)*
*Note:* Standard errors are in parentheses.
* significant at the .05 level
** significant at the .01 level
*** significant at the .001 level

**Figure 3—Savings Over Time**
**($2,000 annual contribution)**

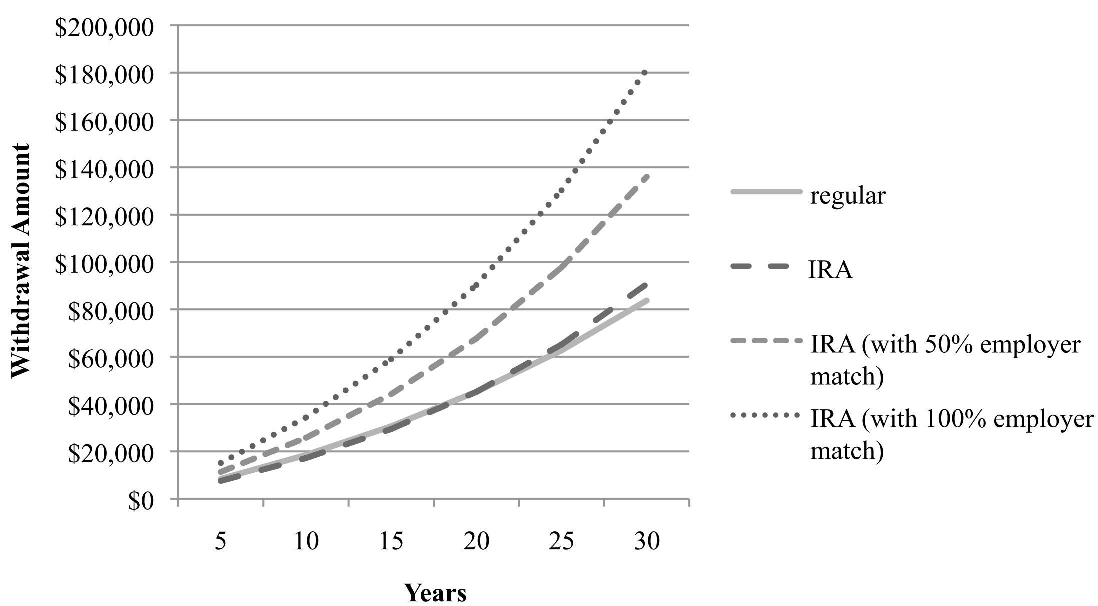


**Figure 4—Savings Over Time**
**($3,000 annual contribution)**

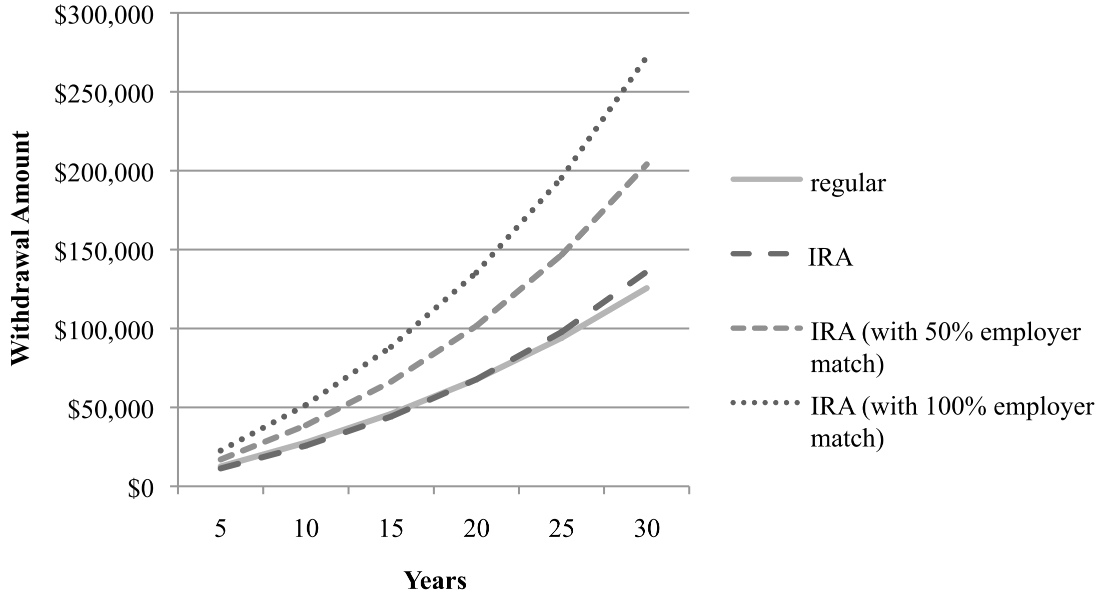



**Figure 5—Savings Over Time**
**($4,000 annual contribution)**

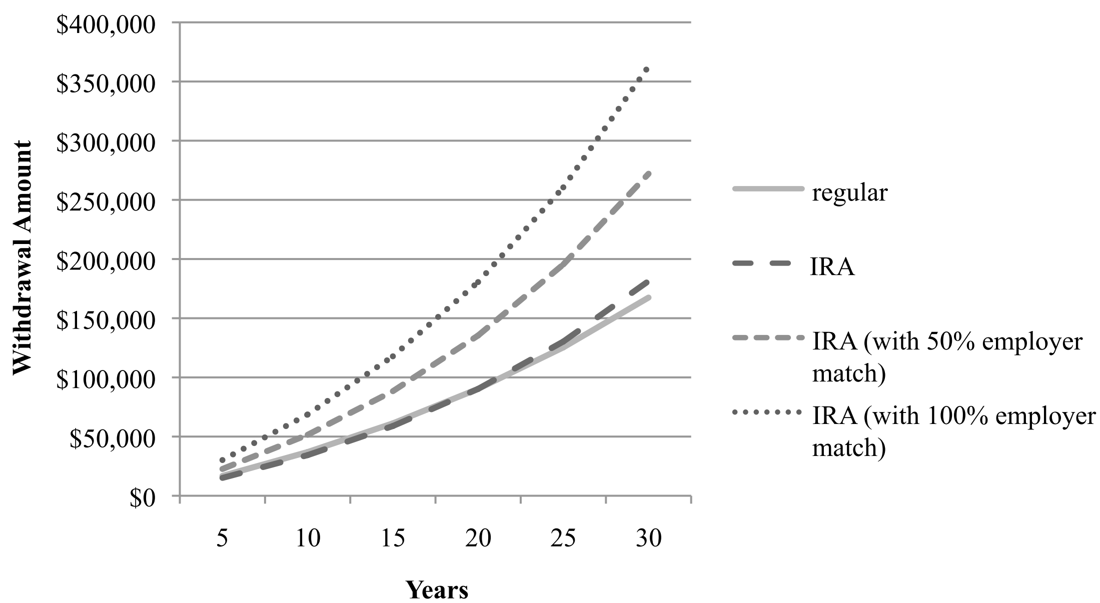


**Figure 6—Savings Over Time**
**($5,000 annual contribution)**

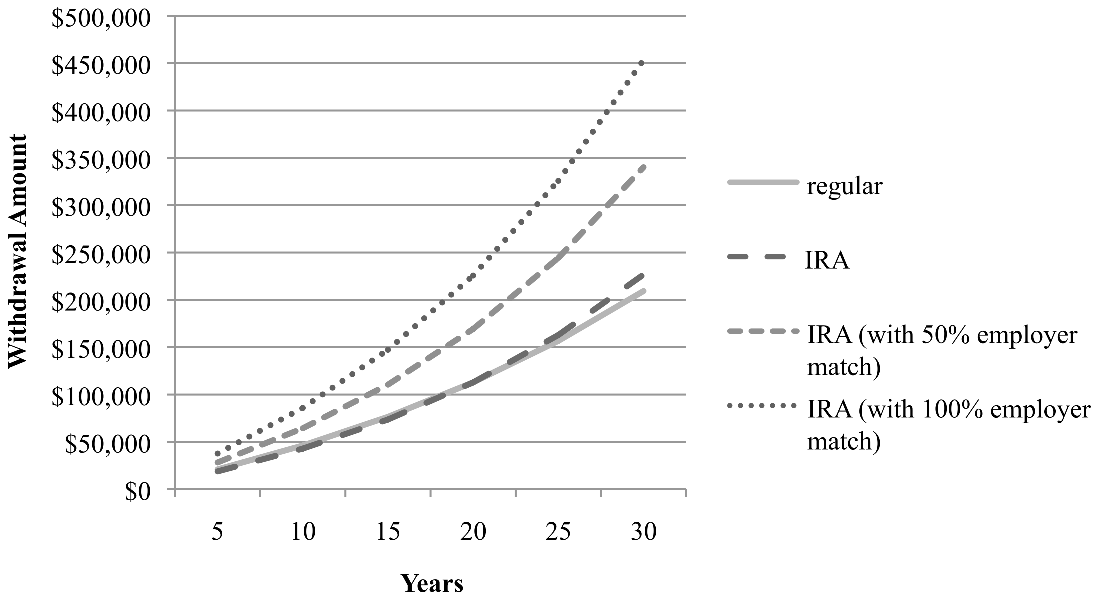



**Figure 7—Savings Over Time with 2.5% Inflation**
**($2,000 annual contribution)**

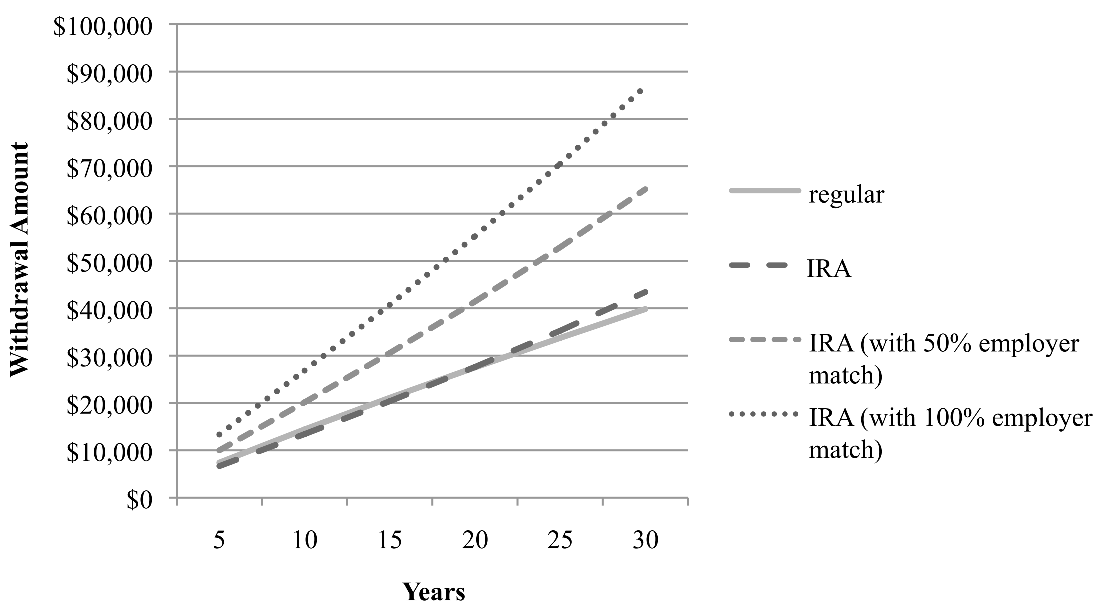


**Figure 8—Savings Over Time with 2.5% Inflation ($3,000 annual contribution)**

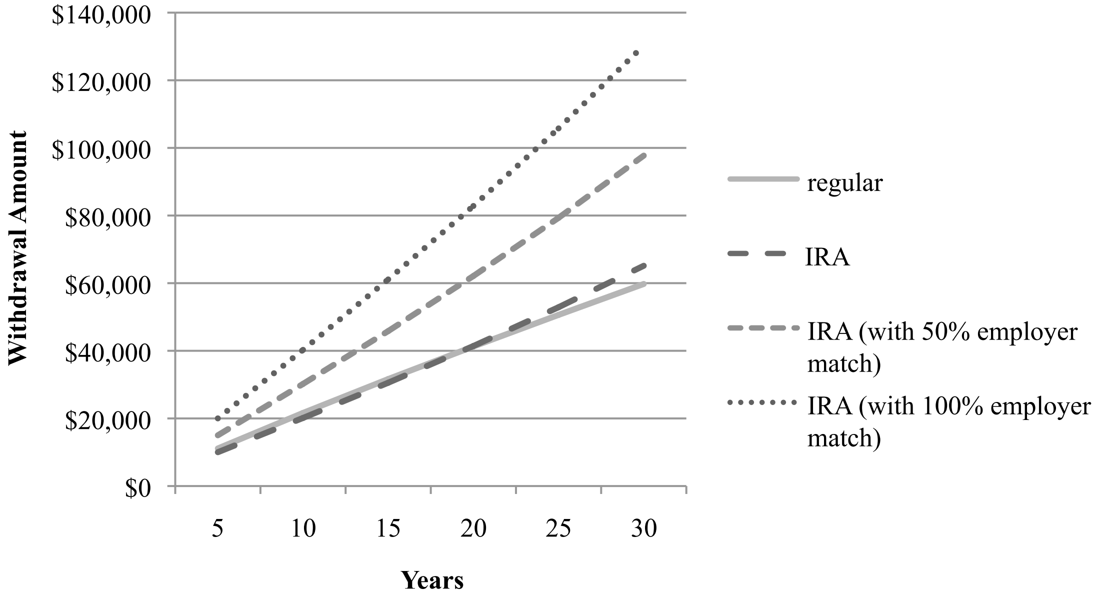



**Figure 9—Savings Over Time with 2.5% Inflation ($4,000 annual contribution)**

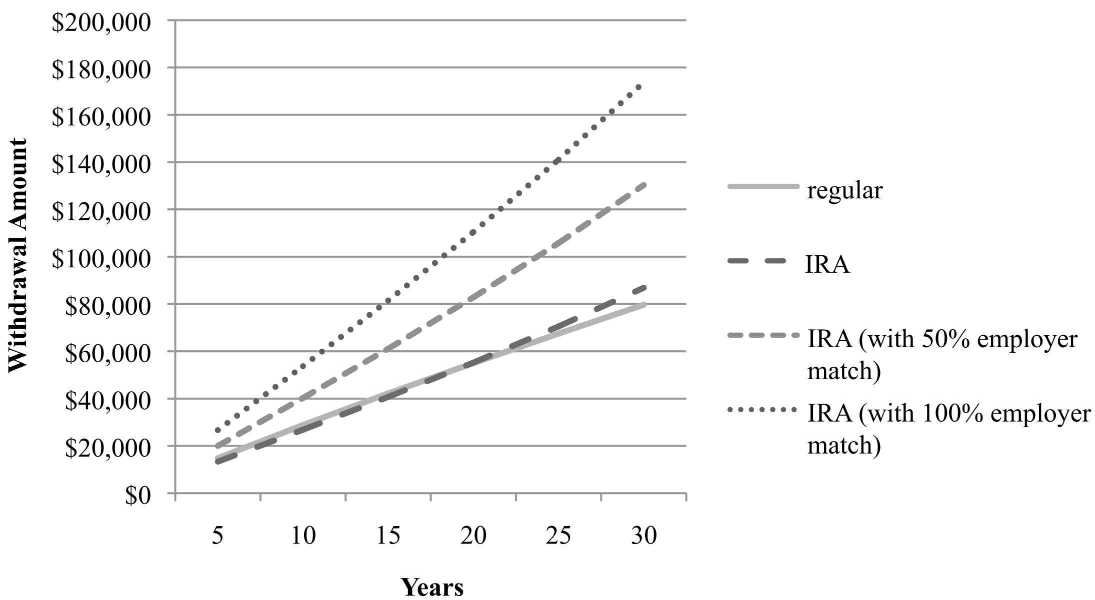
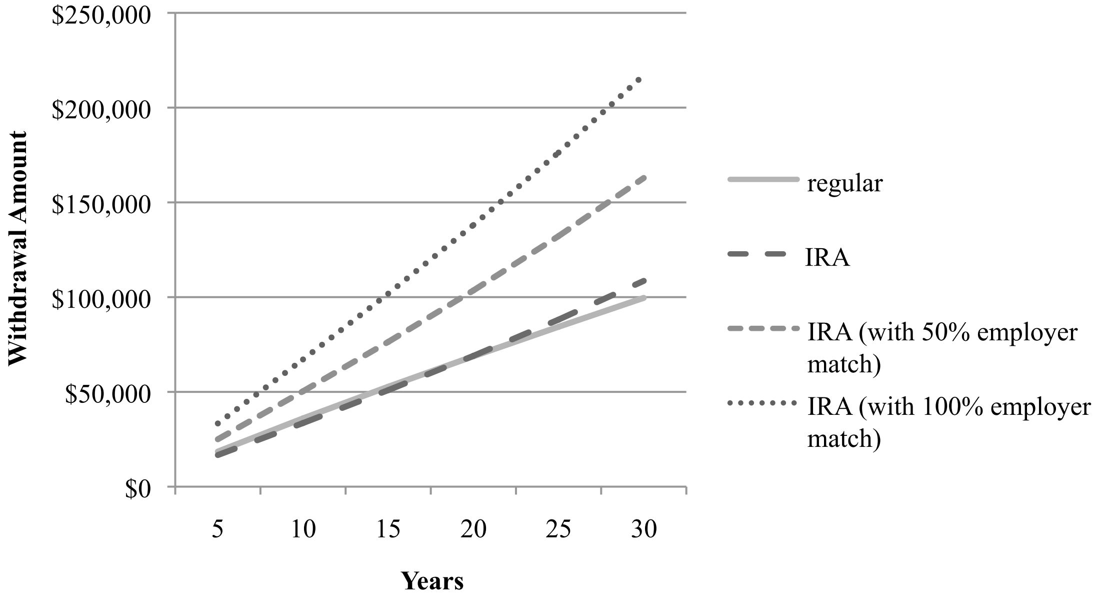


Figure 10—Savings Over Time with 2.5% Inflation
($5,000 annual contribution)




# References


[1] Hyman, D. N. (2008). *Public Finance: A Contemporary Application of Theory to Policy.* Thomson South-Western.

[2] Laibson, D. (1997). Golden Eggs and Hyperbolic Discounting. *The Quarterly Journal of Economics*, 112(2): 443-477.

[3] Li, G. and Smith, P. A. (2008). Borrowing from Yourself: 401(k) Loans and Household Balance Sheets. Federal Reserve Board Finance and Economics Discussion Series 2008-42.

[4] Lu, J. and Mitchell, O. S. (2010). Borrowing from Yourself: The Determinants of 401(k) Loan Patterns. Michigan Retirement Research Center 2010-221.

[5] Poterba, J., Wise, D. A. and Venti, S. (2007). The Changing Landscape of Pensions in the United States. NBER Working Paper 13381.

[6] Stafford, F. P. and Gouskova, E. (2010). Mortgage Contract Decisions and Mortgage Distress: Family and Financial Life Cycle Factors.

[7] U.S. Department of Labor (2010). Private Pension Plan Bulletin: Historical Tables and Graphs.

[8] VanDerhei, J., Holden, S. and Alonso, L. (2009). 401(k) Plan Asset Allocation, Account Balances, and Loan Activity in 2008. *EBRI Issue Brief*, no. 335.

# Outcome-Independent Payoffs in Strategic Voting*

Daniel Russo[†]

Spencer Smith[‡]

**Abstract**

We consider a game-theoretic model of voting in which players have an outcome-independent component to their preferences. This outcome-independent component—even if arbitrarily small—can dramatically affect the set of Nash equilibria in voting games because it determines how voters behave when they are not pivotal. Given incomplete information, some weak restrictions on voter preferences, and a sufficiently large number of voters, there is a unique Bayesian Nash equilibrium in which every player votes according to the outcome-independent component of his preferences. Our model helps explain (1) why people vote when participation is optional and voting is costly, and (2) why public and secret voting may lead to different outcomes.

*We thank Stephen Salant for his devoted attention, careful guidance, and invaluable suggestions. We also thank Lones Smith, Scott Page, and Tilman Börgers for their helpful comments.

[†]University of Michigan, djruss@umich.edu

[‡]University of Michigan, spenceds@umich.edu

# 1 Introduction

Many game-theoretic models of voting assume individuals vote in order to influence the outcome of an election. There is clear empirical evidence, however, that an individual voter rarely casts a deciding vote. For example, Mulligan and Hunter (2001) estimate the frequency of a pivotal vote to be about $2/v$, where $v$ is the total number of votes cast in an election. For elections with 1,000 votes cast, this puts the frequency of a pivotal vote at about 0.2%. For elections with at least 100,000 votes, which include United States presidential elections, congressional elections, and elections for statewide office, the frequency of a pivotal vote is less than 0.002%. If an individual voter has no power to influence the outcome of an election, why does he take the trouble to vote?

This paradox of voter participation motivates our model. Why vote? Riker and Ordeshook (1968) suggest voters compare the cost of voting to their "expected benefit," which they describe as the difference in benefit (between outcomes) multiplied by the probability the voter induces his desired outcome.[1] What if individuals have some payoff from how they vote that is independent of the outcome their vote induces? Adding an "outcome-independent" component to voter preferences allows us to study interesting scenarios. For example, imagine a committee of voters where members care about not only the outcome of the vote, but also about the way in which their vote is perceived by other members, or perhaps by a committee chair. These outcome-independent payoffs may override committee members' preferences over outcomes. Similarly, politicians have incentives to vote in a way that pleases their constituency, or they may respond to outcome-independent payoffs associated with vote trading or lobbying.

Our goal is to develop a game-theoretic model of voting behavior that accounts for these payoffs. (Riker and Ordeshook consider these payoffs, but their model is decision-theoretic.) We begin by decomposing agents' utility functions into outcome-dependent and

[1] This model was pioneered by Downs (1957). For a review of these approaches and others, see Aldrich (1993) and Dhillon and Peralta (2002).

outcome-independent components. This decomposition permits a simple but interesting analysis of voter behavior. We show that an outcome-independent component dramatically affects the set of pure strategy Nash equilibria of a voting game. The intuition is quite simple: each non-pivotal voter cannot influence the vote's outcome and must therefore vote in order to maximize the outcome-independent component of his payoff. We show that the only equilibrium in which some individual does not vote in accordance with the outcome-independent component of his preferences is the knife-edge case in which there exist pivotal voters.

We then consider a game with incomplete information. Motivated by empirical evidence that voters are rarely pivotal in large elections, we study the case where the number of voters is large. We show that if voters are not certain to vote in any particular way, the probability any voter is pivotal tends to zero as the number of voters tends to infinity. This result and a few additional restrictions allow us to construct a voting game with incomplete information and to show that—given enough voters—there exists a unique Bayesian Nash equilibrium in which everyone votes according to the outcome-independent component of his preferences. We then use our model to help explain (1) why people vote when participation is optional and voting is costly, and (2) why public and secret voting may lead to different outcomes. We conclude by discussing some applications.

Our results demonstrate a lack of robustness in classical voting theory. In many contexts, one can reasonably expect voters to have an outcome-independent component to their preferences. Yet, even arbitrarily small outcome-independent payoffs can lead to large and systematic deviations from the predictions of common models of voting. In this paper, we present a simple and stylized model to emphasize the intuition behind this lack of robustness and demonstrate its potential to change the way we view strategic voting.

# 2 The Model

We consider the case where $N$ agents simultaneously submit votes over a finite set of alternatives, $\mathcal{V}$. We say that there is an outcome function $O : \mathcal{V}^N \to \sigma(\mathcal{V})$, where $\sigma(\mathcal{V})$ is a set of probability distributions over $\mathcal{V}$. We eventually focus on the case where the outcome is decided by 'plurality rule'. That is, if one outcome receives strictly more votes than any other, it is selected with probability 1 (ties are broken randomly and with equal probability). For now, however, we allow the outcome function to have a more general form. For example, any bill passed by Congress may be vetoed by the president, so that the outcome of a vote in Congress is the resulting probability the bill in question becomes law. This probability may be some complicated function of the voting profile.

Suppose each voter has utility function $u_i : \mathcal{V}^N \to \mathbb{R}$ that can be decomposed into the sum of two functions, $u_i^O$ and $f_i$. That is,

$$u_i(\bar{v}) = u_i^O(O(\bar{v})) + f_i(v_i)$$

where $v_i$ is player $i$'s vote and $\bar{v}$ is the voting profile. Here, $u_i^O$ represents the component of voter $i$'s utility that depends on the outcome, and $f_i$ represents the component of $i$'s utility that is outcome-independent.

What motivates these preferences? Voters may respond to a number of economic and social payoffs. Individuals may vote for a policy or a candidate whose success directly or indirectly benefits them. This would generate preferences over votes *dependent* on the outcome. However, individuals may also vote to express their personal beliefs (Brennan and Lomasky 1993; Schuessler 2000). Or perhaps individuals vote in response to bribes or threats that do not depend on the vote's outcome (Dal Bó 2007). These incentives generate preferences over votes *independent* of the outcome.

Note that outcome-dependent preferences and outcome-independent preferences do not necessarily conflict. In fact, we expect them to agree in many scenarios. However, decomposing the utility function this way permits a simple but interesting analysis of voter behavior.

To start, we make the following simple assumptions:

**Assumption 1.** There is complete information. That is, each player knows the payoffs and strategies available to other players.[2]

**Assumption 2.** Each agent $i$ is required to vote.[3]

**Assumption 3.** Let $v_{-i}$ be the profile of votes cast by all players except player $i$. For any player $i$ and any fixed voting profile $v_{-i}$, the preferences over $\mathcal{V}$ induced by $u_i$ are strict. That is, we assume voters are never completely indifferent between voting for two alternatives. Also assume that the preferences over $\mathcal{V}$ induced by $u_i^O$ and $f_i$ are strict.

**Definition 1.** Given an outcome mapping $O : \mathcal{V}^n \to \sigma(\mathcal{V})$ and a voting profile $\bar{v}$, we say that an agent is pivotal if his vote can alter the outcome. That is, $i$ is pivotal if $O(v_{-i}, v_i) \neq O(v_{-i}, v_i')$ for some $v_i, v_i' \in \mathcal{V}$.

**Proposition 1.** *In any pure strategy Nash equilibrium, if some agent $i$ is not pivotal, then he must vote in order to maximize $f_i$.*

*Proof.* If agent $i$ is not pivotal, $u_i^O(v_{-i}, v_i)$ is the same for all $v_i$. Therefore, for any $v_i$, $v_i' \in \mathcal{V}$, $u_i(v_i') - u_i(v_i) = f_i(v_i') - f_i(v_i)$ and $v$ maximizes $u_i$ if and only if $v$ maximizes $f_i$. □

**Corollary 1.** *If when each agent votes to maximize $f_i$, there is a non-pivotal victory, then this outcome is a Nash equilibrium in pure strategies.*

**Corollary 2.** *If the outcome is decided by plurality rule, any equilibrium in which some agent $i$ does not vote according to $f_i$ has the following form: A group of pivotal agents vote for some victorious outcome and each non-pivotal agent votes according to $f_i$.*

[2]We relax Assumption 1 in Section 3.
[3]We relax Assumption 2 in Section 4.

**Example 1.** Consider the following voting scenario with five voters. Suppose $\mathcal{V} = \{a, b, c\}$ and that each individual $i$ has a von Neumann-Morgenstern utility function $u_i = u_i^O + f_i$ where $u_i^O(a) = u_i^O(b) + 1 = u_i^O(c) + 2$ and $f_i(a) = f_i(b) - \epsilon = f_i(c) - 2\epsilon$, for arbitrarily small $\epsilon > 0$. In this case, each voter prefers $a$ to $b$ and $b$ to $c$ with respect to $u^O$, but each voter prefers $c$ to $b$ and $b$ to $a$ with respect to $f$. Let the outcome be decided by plurality rule and let ties be broken by assigning an equal probability of winning to each tied alternative. Then there are three pure strategy Nash equilibria[4]:

1. All five voters vote for $c$.

2. Three voters vote for $a$ and two vote for $c$.

3. Three voters vote for $b$ and two vote for $c$.

Note that in the first equilibrium, $c$ is the least preferred outcome from the viewpoint of every voter. Yet, $c$ wins unanimously. Every voter is rational and no voter is confused about the voting system. No voter is pivotal, so each simply responds to the outcome-independent component of his payoff. In the third equilibrium, like the first, no one votes for the outcome everyone most prefers.

Example 1 resembles an extreme case of the provision of a public good. Every voter is strictly better off if at least three individuals vote for alternative $a$. However, unless *exactly* two other individuals do so (so that a vote for alternative $a$ by any of the other three voters would seal its victory), each player prefers to vote for alternative $b$ or $c$.

**Example 2.** Suppose there are 100 voters and two alternatives ($a$ and $b$), and suppose outcomes are decided by majority rule (ties are broken randomly and with equal probability). Assume exactly 50 of the 100 voters prefer to vote for $a$ if they are pivotal (i.e. $u_i^O(a) > u_i^O(b)$) and 50 prefer to vote for $b$ if they are pivotal. Also assume that among these 100 voters, $K > 51$ individuals prefer to vote for $a$ if they are not pivotal (i.e. $f_i(a) > f_i(b)$) and $100 - K$

[4]To be precise, there are three *types* of pure strategy Nash equilibria (2 and 3 can have different permutations of agents voting for $a$, $b$, or $c$).

prefer to vote for $b$ if they are not pivotal. Then there is a pure strategy Nash equilibrium in which $K$ individuals vote for $a$, and $a$ wins. This equilibrium, however, is not necessarily unique. We can easily specify agents' preferences to generate a second equilibrium in which exactly 50 people vote for each alternative (and so everyone is pivotal).[5]

Suppose we add a voter who prefers $a$ according to both $u^O$ and $f$. Now there are 51 individuals who prefer to vote for $a$ if they are pivotal and $K+1 > 52$ who prefer to vote for $a$ if they are not pivotal. In this scenario, there is a *unique* pure strategy Nash equilibrium in which $K+1$ people vote for $a$.

Remember that without the additional voter, the remaining 100 voters may split evenly between $a$ and $b$. With the additional voter, however, alternative $a$ wins with $K+1 > 52$ votes (possibly unanimously). The knife-edge case where 51 vote for $a$ and 50 vote for $b$ is no longer an equilibrium. Why? No one who votes for $b$ is pivotal, and $K+1 > 52$ individuals prefer to vote for $a$ given they are not pivotal. Thus, at least $K - 50 > 1$ agents have an incentive to deviate, so this cannot be a Nash equilibrium.

Examples 1 and 2 demonstrate the important effect of $f$ on the set of Nash equilibria in voting games. Even though $f$ may have an arbitrarily small effect on each individual's overall utility function, its existence affects voter behavior in a surprisingly powerful manner. This is because the effect of an individual's vote on his payoff is highly contingent on the voting profile of other agents. In particular, whether or not voter $i$ is *pivotal* profoundly affects his voting behavior. While, in a loose sense, individuals' preferences over outcomes may be much stronger, the outcome-independent component of preferences ultimately dictates how non-pivotal voters behave.

[5]For this to be an equilibrium, the following inequalities must hold: $\frac{1}{2}\left(u_i^O(a) - u_i^O(b)\right) \geq (f_i(b) - f_i(a))$ for each agent $i$ who votes for $a$ and $\frac{1}{2}\left(u_i^O(b) - u_i^O(a)\right) \geq (f_i(a) - f_i(b))$ for each agent $i$ who votes for $b$.

# 3 Incomplete Information

Our main result demonstrates that individual voter behavior is highly contingent on the voting profile of other agents. In particular, the way each agent votes hinges on whether he is pivotal. This invites the question: when are voters pivotal? Our intuition is that voters are unlikely to be pivotal when the total number of voters is large.

For example, individual voters are very unlikely to be pivotal in large elections, such as presidential elections in the United States. Indeed, voters (correctly) rarely see themselves as pivotal in these elections. We often hear voters say, "My vote won't make a difference," especially in states that historically lean strongly toward one party. Even "swing states" are often decided by thousands, tens of thousands, or even hundreds of thousands of votes. In these cases, voters are very unlikely to be pivotal, and their realization of this fact may affect their voting behavior.

In this section, we (1) formalize the notion that voters are unlikely to be pivotal when the total number of voters is large and (2) consider implications for voting games with incomplete information.

**Proposition 2.** *Suppose outcomes are decided by plurality rule and that given any voter $i$ and any alternative $a_k$, the probability voter $i$ votes for alternative $a_k$ is at most $\bar{p}_k < 1$. Then the probability that any voter is pivotal tends to zero as the number of voters tends to infinity.*

*Proof.* See Appendix.

The proof of Proposition 2 requires a lemma about the distribution of the sum of $n$ independent Bernoulli random variables. We state and prove this lemma in the appendix. To prove Proposition 2, we bound the probability that voter $i$ is pivotal by considering the conditions necessary (but not sufficient) for voter $i$ to be pivotal between any two outcomes. We show that the probability that these conditions hold tends to zero as $N$ tends to infinity. See the appendix for the full proof.

Proposition 2 allows us to comment on the expected outcome of voting games with incomplete information when the number of voters is large.

Let us relax Assumption 1 and consider a standard game of incomplete information in which $N \geq 2$ voters decide between $M \geq 2$ alternatives. In the first stage, Nature assigns each voter a pair of functions, $u_i^O$ and $f_i$. These types are drawn independently from some probability distribution. Voters' types are their private information. In the second stage, players cast their votes simultaneously. Assume outcomes are decided by plurality rule and that ties are broken randomly and with equal probability.[6]

Our result requires the following conditions:

**Condition 1.** For each alternative $k$, each voter has probability at least $\underline{p_k} > 0$ of being assigned preferences such that voting for $k$ is a strictly dominated strategy.

**Condition 2.** For any distinct alternatives $a_k$ and $a_j$, there exist $\epsilon_{k,j} > 0$ and $R_{k,j} < \infty$ such that for each voter $i$, $|f_i(a_k) - f_i(a_j)| \geq \epsilon_{k,j}$ and $\left|u_i^O(a_k) - u_i^O(a_j)\right| \leq R_{k,j}$.

These conditions are not as restrictive as they might initially appear. Condition 1 requires that for each alternative there is *some* probability, perhaps arbitrarily small, that voting for that alternative is a strictly dominated strategy. Condition 2 is only a weak extension of Assumption 1, which requires that the preferences induced by $f_i$ and $u^O$ are strict. Assumption 1 guarantees that both $f_i(a_k) - f_i(a_j)$ and $u_i^O(a_k) - u_i^O(a_j)$ are nonzero, and, in particular, that $|f_i(a_k) - f_i(a_j)| \geq \epsilon_{k,j}$ holds for any finite set of voters. Condition 2, then, simply guarantees that as $N \to \infty$, $(f_N(a_k) - f_N(a_j)) \nrightarrow 0$ and $\left(u_N^O(a_k) - u_N^O(a_j)\right) \nrightarrow \infty$. (In fact, it guarantees this for any subsequence.)

**Proposition 3.** *Given a sufficiently large number of voters, there is a unique pure strategy Bayesian Nash equilibrium in which each player $i$ votes according to $f_i$.*

*Proof.* Proposition 2 allows us to establish this result without studying the properties of the set of Bayesian Nash equilibria in depth. By Condition 1, we have that each agent votes for

[6] Our result also holds for majority rule or for any supermajority rule.

each alternative $a_k$ with probability at most $1-\underline{p_k} < 1$. Applying Proposition 2, we see that *in any* pure strategy Bayesian Nash equilibrium, the probability that any voter is pivotal becomes arbitrarily small as the number of voters grows. Let $p_{j,k}$ be the probability that voter $i$ is pivotal between outcomes $a_j$ and $a_k$. Then the expected benefit to voter $i$ from voting for alternative $a_j$ over $a_k$ can be expressed as

$$\mathrm{E}[u_i(a_j) - u_i(a_k)] = p_{j,k}\left(u_i^O(a_j) - u_i^O(a_k)\right) + f_i(a_j) - f_i(a_k).$$

By Condition 2, $|u_i^O(a_j) - u_i^O(a_k)| \leq R_{k,j}$ and $|f_i(a_j) - f_i(a_k)| \geq \epsilon_{k,j}$, so that if $p_{j,k} < \frac{\epsilon_{k,j}}{R_{k,j}}$, the expected benefit to voter $i$ is higher from voting for $a_j$ if and only if $f_i(a_j) > f_i(a_k)$. As $N \to \infty$, $p_{j,k} \to 0$, so this must be the case for sufficiently large $N$. Since $i$, $a_j$, and $a_k$ are arbitrary, this establishes that there is a unique pure strategy Bayesian Nash equilibrium in which each player $i$ votes for the alternative that maximizes $f_i$. □

Proposition 3 suggests that for large enough $N$, rational players do not vote to affect outcomes. Instead, they cast votes according to the outcome-independent component of their preferences (as reflected in $f$).[7] Again, $f$ may represent bribes, threats, desire for social standing, payoffs associated with conformity, or a host of other factors unrelated to voters' preferences over outcomes.

Some voters denounce other voters who "waste" their vote on obscure third party candidates, when they could "make a difference" by voting for one of two major candidates. Our result suggests the logic works in reverse: the probability that these voters affect the outcome (that is, "make a difference") is extremely small, so they simply vote according to $f$.

[7]Of course, it may be the case that the outcome-dependent and outcome-independent component of their preferences agree.

**Another Source of Uncertainty**

Voter uncertainty may not be due to incomplete information at all, but rather due to the difficulty of coordinating on a particular equilibrium. Consider a scenario similar to Example 2. Suppose there are 101 voters who have identical preferences: each prefers to vote 'yes' on some proposition based on the outcome-independent component of his preferences, but each prefers to vote 'no' if he is pivotal. Then if *any* subset of 51 voters vote 'no' while the others vote 'yes,' this forms a pure strategy Nash equilibrium. How did the voters decide which 51 voted 'no'? In many real world settings, such coordination is difficult, especially if the number of voters is large. This may lead to uncertainty about how other players will vote even with complete information.

# 4 Why Vote?

Let us relax Assumption 2 and suppose voting is optional. Our model of voting offers an answer to the classic question: why do people vote? If voters are unlikely to alter the outcome of a voting game, our model says $f$ will dictate whether or not they vote as well as which alternative they vote for. In particular, for large $N$, we take the Bayesian Nash equilibrium in which each agent $i$ votes according to $f_i$ as the best prediction for the voting game. That is, in large elections, rational voters do not cast votes in order to affect the outcome. Rather, they vote to make a statement, to impress friends, or to spite enemies, or perhaps they vote out of tradition, out of respect, or out of fear.[8] In short, people vote because of $f$.

If there is a cost to voting, then the decision to vote hinges on the magnitude of $f$. Suppose voter $i$ faces some cost to voting, $c_i$. For large enough $N$, the outcome does not depend on voter $i$'s individual vote. However, assume voter $i$ still benefits (or suffers) from

[8] Blais (2000) argues that a sense of civic duty motivates many to vote. For a discussion of group-based models, see Feddersen (2004).

the outcome whether or not he actually votes. In this case, voter $i$ solves

$$\max_{v_i \in \mathcal{V}, \lambda \in \{0,1\}} u_i(v_i) = u_i^O(O(\bar{v})) + \lambda(f_i(v_i) - c_i),$$

where $\lambda = 1$ if voter $i$ votes and $\lambda = 0$ otherwise. Because $u_i^O(O(\bar{v}))$ does not depend on $v_i$, voter $i$ votes if and only if $f_i(v_i) - c_i > 0$, where $v_i$ maximizes $f_i$.

In Section 2, we described how $f$ can affect the set of Nash equilibria in voting games, even if $f$ is arbitrarily small. In understanding why people vote, however, the size of $f$ is critical. In voting games with sufficiently large numbers of voters, the maximized $f_i$ must exceed $c_i$ for each agent $i$ who chooses to vote. That is, non-outcome-based incentives to vote must outweigh the cost to vote. There are many potential costs to voting, including time spent on registration, rearranging work schedules, transportation to and from the polls, and time spent researching issues or candidates. For each of the millions of citizens who participate in non-pivotal elections, the maximized $f_i$ must outweigh these costs. Those for whom the maximized $f_i$ does not exceed $c_i$ simply stay home. Thus, even though an arbitrarily small $f$ has important effects on the set of Nash equilibria in voting games, it is not necessarily the case that $f$ is negligible. On the contrary, it appears $f$ motivates millions of non-pivotal voters to cast votes in voluntary elections.

# 5 Public Versus Secret Ballot

Whether voting is public or secret can affect voter behavior and—as a result—voting outcomes. When votes are observed, each agent's vote may elicit some response. A voter may meet reward or retaliation for his actions, and this response can come from other voters or agents external to the voting game. For example, a congressman may receive favorable treatment for his district in an appropriations bill written by other legislators if he votes to confirm some federal judge. Or, the same congressman may receive campaign donations from an external lobbying firm if he votes to confirm the federal judge. Here, rewards come from agents internal and external to the voting game. The congressman may also face punishments for his vote in addition to rewards.

Our model accommodates the differences between public and secret voting quite well. Since agents' preferences over outcomes do not depend on whether voting is public or secret, $u^O$ is unaffected. If voting is public, each agent faces a response to his vote, $R_i(v_i)$. His payoff from this response is $g_i(R_i(v_i))$, where $g : \mathcal{R}_i \to \mathbb{R}$ is some function with domain $\mathcal{R}_i$, the set of all possible responses to votes cast by agent $i$. We write $f_i(v_i) = g_i(R_i(v_i)) + h_i(v_i)$, and we have

$$u_i(v_i) = u_i^O(O(v_{-i}, v_i)) + g_i(R_i(v_i)) + h_i(v_i),$$

where $h_i(v_i)$ is an outcome-independent component of agent $i$'s payoff unrelated to responses to his vote. Under secret ballot, $g(\cdot)$ still exists but there will be no response because $v_i$ is unobserved.

In Sections 2 and 3, we described how even arbitrarily small $f$ has important effects on the set of equilibria in voting games. If $g$ is a relatively large component of $f$, then we expect whether voting is public or secret to also have important effects on the outcomes of voting games. It may be the case that

$$f_i^{SECRET}(v) = g_i(\cdot) + h_i(v) > g_i(\cdot) + h_i(v') = f_i^{SECRET}(v')$$

but that

$$f_i^{PUBLIC}(v) = g_i(R_i(v)) + h_i(v) < g_i(R_i(v')) + h_i(v') = f_i^{PUBLIC}(v')$$

for some alternatives $v$ and $v'$. If these two are the only alternatives, then in any Nash equilibrium, each non-pivotal voter will vote for $v$ under secret voting but $v'$ under public voting. Thus, the Nash equilibria under public voting are almost entirely different from the equilibria under secret voting.[9]

[9]The only equilibria that may not change are the ones where there are an even number of voters and

# 6 Applications

What follows is a list of topics to which we can apply our model. This list is not exhaustive. We hope readers use it to understand and appreciate the flexibility of our model and to generate their own ideas for its application.

### Bribes and Threats

Consider an external player who attempts to influence the outcome of a voting game by either bribing or threatening voters. Suppose voting is public so that the external player can condition his bribes or threats on each agent's vote. Then by offering arbitrarily small bribes to these voters (and therefore affecting $f$), the external player can induce an equilibrium in which some arbitrary outcome $a$ wins, even if all voters strictly prefer an alternative outcome.

As in Dal Bó (2007), suppose this external player offers voters a bribe that is contingent on whether or not they are pivotal. That is, this external player offers to pay some voter $x_i^p$ if $i$ is pivotal and $x_i^n$ if $i$ is not pivotal. By offering a large bribe to pivotal voters and a smaller one to non-pivotal voters, the player can influence the voting game so that there is a unique equilibrium in which a particular alternative wins by a wide margin. Because the alternative wins non-pivotally, the external agent never needs to pay the larger bribe, $x_p$.

### Committees

Consider the difference between public and secret committee voting where the committee chair exerts some influence over voters. This may not be due to explicit bribes or threats, and the committee chair may not necessarily use his influence intentionally. However, if his influence outweighs the effect of $u_i^O$ for all $i$, then there is an equilibrium in which every member votes for the committee chair's most preferred alternative. Committee members may also exert some influence on each other, perhaps asymmetrically, and therefore affect voting outcomes.

exactly half of them vote for each alternative (that is, equilibria in which every voter is pivotal).

### Legislatures

Consider a politician voting between two alternatives, $a$ and $b$, where outcome $b$ is more politically feasible. Even if the politician is willing to sacrifice his chances of being re-elected in order to ensure that outcome $a$ is selected, he may vote for outcome $b$ if he does not believe that his vote will swing the outcome toward $a$. As this may be true for many legislators, it could be the case that outcome $b$ is selected in a landslide victory even though a majority of the legislature would prefer to vote for $a$ if they believed their votes to be pivotal.

### Altruism

Our model may seem like a somewhat depressing representation of reality: we use bribes, threats, corrupt influence, or other self-serving motives as examples of outcome-independent payoffs. Some would rather believe that voters—even non-pivotal ones—vote altruistically. Although public choice models often describe voters as selfish, Mansbridge (1990) and Caplan (2002) offer evidence to the contrary. Our model accounts for such behavior. Suppose an individual has an outcome-independent payoff from voting for a socially redistributive policy, even though he strictly prefers the status quo. If $\Delta f_i$ exceeds $\Delta u_i^O$, or if he thinks his vote is unlikely to be pivotal, he will vote altruistically.

### Stubborn Voters

Consider "stubborn" voters, who derive additional personal satisfaction (through $f$) by voting according to their outcome preferences (according to $u^O$). Then even if $u_i^O(a) > u_i^O(b) > u_i^O(c)$ and voter $i$ could be pivotal between $b$ and $c$, he may still vote for $a$ (even if his vote results in $c$, his least favorite outcome) because of the strength of $f_i$. When these stubborn voters are not pivotal, they will still, of course, vote according to $f_i$. In cases of external influence, it will be harder to affect the way these "stubborn" individuals vote.

### Mandates

In candidate elections, those who receive sizable majorities often claim a mandate; they say "the electorate has spoken." Are these candidates correct to assume they have the full faith and support of voters? According to our model, perhaps not. Consider Example 1. In two Nash equilibria, the outcome each voter most prefers—outcome $a$—receives zero votes. In fact, in one of these equilibria, the outcome each voter *least* prefers—outcome $c$—wins unanimously. In either case, talking heads would declare outcome $a$ dead: this outcome, perhaps a candidate, clearly carries no support among voters. The truth, of course, is that each voter prefers $a$ to either other outcome. Outcome-independent payoffs, however, direct non-pivotal voters away from $a$. Thus, the fact that a candidate receives an overwhelming number of votes does not imply that an overwhelming number of voters prefer the candidate. "Mandates" may not be mandates at all, but rather the result of non-pivotal voters responding to outcome-independent payoffs.

## 7 Conclusion

Motivated by our intuition that voters respond to incentives unrelated to the outcome of a vote, we develop a game-theoretic model of voting in which we decompose voter preferences into outcome-dependent and outcome-independent components. Outcome-independent components of preferences can dramatically affect the set of Nash equilibria because they dictate the behavior of non-pivotal voters. Why? Voters who are not pivotal cannot, by definition, vote to influence the outcome of the vote. They must therefore vote to maximize their outcome-independent payoff. The only equilibria in which at least one voter does not vote in accordance with the outcome-independent component of his preferences are equilibria in which there exist pivotal voters.

We then construct a game of incomplete information in which Nature first assigns to each voter a type, specifying the outcome-dependent and outcome-independent components of his preferences. Voters' types are their private information, and voters cast ballots simultaneously. Within this framework, we formalize the notion that voters are unlikely to be pivotal when the total number of voters is large. In particular, we prove that the probability any voter is pivotal tends to zero as the number of voters tends to infinity. With this result, we can show that given a sufficiently large number of voters, there exists a unique pure strategy Bayesian Nash equilibrium in which each player votes according to the outcome-independent component of his preferences. In voting games with incomplete information and many voters, outcome-independent payoffs reign supreme.

Our model offers perspective on the timeless question: why vote? Suppose voting is optional and participation is costly. Further suppose the number of voters is large, making the probability of a pivotal vote small. Then for the individuals who participate, outcome-independent payoffs must exceed the cost of voting. Thus, even though arbitrarily small outcome-independent payoffs can dramatically affect the set of Nash equilibria in voting games, it is not necessarily the case that these payoffs are negligible. After all, millions of non-pivotal voters bear very real costs to cast ballots in national elections.

Outcome-independent components of preferences are also relevant to the topic of public and secret voting. We expect outcome-independent payoffs to exert greater influence under public voting as they may include societal responses to observed votes. Consider, for example, committee members voting under the watchful eyes of a committee chair or legislators voting under the watchful eyes of their constituents. Can we be sure these actors vote for the outcome they most prefer? Or might they respond to outcome-independent payoffs in the form of bribes, threats, political pressure, or otherwise personal gain?

# 8 Appendix

In order to prove the lemma stated in this appendix, we use an adapted version of the Berry-Esseen theorem as developed by Batirov, Manevich, and Nagaev (1977). The Berry-

Esseen theorem provides a uniform bound on the rate of convergence of independent and identically distributed (i.i.d.) random variables to the standard normal distribution. Batirov, Manevich, and Nagaev consider a random sum of random variables that are not necessarily i.i.d. Below, we provide a simpler statement of the theorem that assumes the number of terms in the random sum is some fixed integer $N$. For a more straightforward statement of Batirov, Manevich, and Nagaev's result, see Chaidee and Tuntapthai (2009).

**Theorem** (adapted from Chaidee and Tuntapthai (2009))

Suppose $X_1, X_2, \ldots$ are independent but not necessarily identically distributed random variables with $\mathrm{E}[X_i] = 0$, $\mathrm{E}[X_i^2] = \sigma_i^2$, and $\mathrm{E}\left[|X_i|^3\right] = \gamma_i < \infty$. Define

$$S_N^2 = \sum_{i=1}^{N} \sigma_i^2, \qquad \beta_N = \sum_{i=1}^{N} \gamma_i, \qquad \text{and} \qquad Y_N = \frac{X_1 + ... + X_N}{S_N}$$

Then there exists a constant $C$ such that

$$\sup_{x \in \mathbb{R}} |\mathbb{P}\{Y_N \leq x\} - \Phi(x)| \leq C \left( \frac{\beta_N}{\left(S_N^2\right)^3} \right).$$

By this result, if $\frac{\beta_N}{\left(S_N^2\right)^3} \to 0$, then $Y_N$ not only converges in distribution to the standard normal distribution, it converges uniformly.

**Lemma.** *Let $Z_N = \sum_{i=1}^{N} z_i$ be the sum of $N$ independent random variables where $z_i \sim$ Bernoulli$(p_i)$ for $p_i \leq \bar{p} < 1$. Then for any fixed $M \geq 2$,*

$$\lim_{N \to \infty} \left( \max_{k \geq \lfloor \frac{N-1}{M} \rfloor} \mathbb{P}\{Z_N = k\} \right) = 0$$

*Proof.* Let $\{z_n\}_{n \in \mathbb{N}}$ be any sequence of independent Bernoulli random variables. That is $\mathbb{P}\{z_i = 1\} = p_i \leq \bar{p} < 1$ and $\mathbb{P}\{z_i = 0\} = 1 - p_i$. Define $X_i = z_i - \mathrm{E}[z_i]$ so that $\mathrm{E}[X_i] = 0$. Define $\sigma_i^2$, $S_N^2$, $\beta_N$, and $Y_N$ as in the theorem above. Let $S_N = \sqrt{S_N^2}$.

**Case 1**: Suppose $\lim_{N \to \infty} \frac{\sum_{i=1}^{N} \sigma_i^2}{N} \neq 0$.

We break down the proof into a series of steps.

*Step 1*: $\frac{\beta_N}{\left(S_N^2\right)^3} \to 0$ and $S_N \to \infty$.

By definition, $\beta_N = \sum_{i=1}^N E\left[|X_i|^3\right] \leq N$, where the last inequality follows because $X_i$ only takes on values of 0 and 1. This implies:

$$\frac{\beta_N}{\left(S_N^2\right)^3} \quad \leq \quad \frac{N}{\left(\sum_{i=1}^N \sigma_i^2\right)^3} \leq \frac{1}{\left(\frac{N^2}{N^3}\right)\left(\sum_{i=1}^N \sigma_i^2\right)^3} = \frac{1}{N^2\left(\frac{\sum_{i=1}^N \sigma_i^2}{N}\right)^3} \to 0$$

where the last step follows because $\left(\frac{\sum_{i=1}^N \sigma_i^2}{N}\right) \nrightarrow 0$.

To see why $S_N \to \infty$, note that because $\lim_{N\to\infty} \frac{\sum_{i=1}^N \sigma_i^2}{N} \neq 0$, it must be that for some $i$, $\sigma_i \neq 0$, which implies $p_i > 0$. Therefore, we know $\beta_N \nrightarrow 0$ because $\beta_N \geq \mathrm{E}\left[|X_i|^3\right] = p_i^3$. Since $\beta_N \nrightarrow 0$, $\frac{\beta_N}{\left(S_N^2\right)^3} = \frac{\beta_N}{(S_N)^6} \to 0$ implies $S_N \to \infty$.

*Step 2*: We can rewrite $\mathbb{P}\{Z_N = k\}$ in the following way:

$$\begin{aligned}
\mathbb{P}\{Z_N = k\} &= \mathbb{P}\{Z_N \in (k-1, k+1)\} \\
&= \mathbb{P}\{z_1 + \cdots + z_N \in (k-1, k+1)\} \\
&= \mathbb{P}\left\{\frac{(z_1-\mu_1) + \cdots + (z_N - \mu_N)}{S_N} \in \left(\frac{k-1-\mu_1-\cdots-\mu_N}{S_N},\ \frac{k+1-\mu_1-\cdots-\mu_N}{S_N}\right)\right\} \\
&= \mathbb{P}\left\{\frac{X_1+\cdots+X_N}{S_N} \in \left(\frac{k-1-\mu_1-\cdots-\mu_N}{S_N},\ \frac{k+1-\mu_1-\cdots-\mu_N}{S_N}\right)\right\} \\
&= \mathbb{P}\left\{Y_N \in \left(\frac{k-1-\mu_1-\cdots-\mu_N}{S_N},\ \frac{k+1-\mu_1-\cdots-\mu_N}{S_N}\right)\right\},
\end{aligned}$$

where $Y_N = \frac{X_1+\ldots+X_N}{S_N}$ as in the theorem above.

*Step 3*: The necessary conditions for the theorem stated above are satisfied. This follows because $\frac{\beta_N}{\left(S_N^2\right)^3} \to 0$ and $\mathrm{E}\left[|X_i|^3\right]$ is obviously finite for all $i$.

*Step 4*: For sufficiently large $N$, the probability $Y_N$ is within any interval is arbitrarily close to the probability that a variable that follows the standard normal distribution falls within the same interval.

Fix $\epsilon > 0$. Pick $N_1 \in \mathbb{N}$ such that $N > N_1$ implies $C\left(\frac{\beta_N}{\left(S_N^2\right)^3}\right) < \frac{\epsilon}{2}$, where $C$ is the constant from the statement of the theorem. Then,

$$
\begin{aligned}
& \sup_{x_1, x_2 \in \mathbb{R}} \left|\left(\mathbb{P}\left\{Y_N \leq x_1\right\} - \mathbb{P}\left\{Y_N \leq x_2\right\}\right) - \left(\Phi(x_1) - \Phi\left(x_2\right)\right)\right| \\
\leq & \sup_{x_1 \in \mathbb{R}} \left|\mathbb{P}\left\{Y_N \leq x_1\right\} - \Phi(x_1)\right| + \sup_{x_2 \in \mathbb{R}} \left|\mathbb{P}\left\{Y_N \leq x_2\right\} - \Phi(x_2)\right| \\
\leq & 2C\left(\frac{\beta_N}{\left(S_N^2\right)^3}\right) \\
< & \epsilon
\end{aligned}
$$

*Step 5*: $\sup_{k \in \mathbb{R}} \left(\Phi\left(\frac{k+1-\mu_1-\ldots-\mu_N}{S_N}\right) - \Phi\left(\frac{k-1-\mu_1-\ldots-\mu_N}{S_N}\right)\right) \to 0$.

First, recall that $S_N \to \infty$. Therefore, the interval $\left(\frac{k-1-\mu_1-\ldots-\mu_N}{S_N}, \frac{k+1-\mu_1-\ldots-\mu_N}{S_N}\right)$, which has area $\frac{2}{S_N}$, converges to a single point as $N \to \infty$. The claim follows by the uniform continuity of the cumulative distribution function of the standard normal distribution.

*Step 6*: Combining steps 2, 4, and 5 proves that the lemma holds in this case.

Fix any $\epsilon > 0$. By Step 4, we can pick $N_1$ such that for any $N > N_1$

$$
\sup_{x_1, x_2 \in \mathbb{R}} \left|\left(\mathbb{P}\left\{Y_N \leq x_1\right\} - \mathbb{P}\left\{Y_N \leq x_2\right\}\right) - \left(\Phi(x_1) - \Phi\left(x_2\right)\right)\right| < \frac{\epsilon}{2}
$$

By Step 5, there exists $N_2$ such that $N > N_2$ implies

$$\sup_{k\in\mathbb{R}}\left(\Phi\left(\frac{k+1-\mu_1-...-\mu_N}{S_N}\right)-\Phi\left(\frac{k-1-\mu_1-...-\mu_N}{S_N}\right)\right)<\frac{\epsilon}{2}$$

Pick $N_3 = \max\{N_1, N_2\}$. Then, applying step 2, for any $N > N_3$,

$$\begin{aligned}\max_{k\geq\lfloor\frac{N-1}{M}\rfloor}\mathbb{P}\{Z_N=k\} &= \max_{k\geq\lfloor\frac{N-1}{M}\rfloor}\left(\mathbb{P}\left\{Y_N\leq\frac{k+1-\mu_1-...-\mu_N}{S_N}\right\}-\mathbb{P}\left\{Y_N\leq\frac{k-1-\mu_1-...-\mu_N}{S_N}\right\}\right)\\ &< \max_{k\in\left\{\lfloor\frac{N-1}{M}\rfloor,...,N\right\}}\left(\Phi\left(\frac{k+1-\mu_1-...-\mu_N}{S_N}\right)-\Phi\left(\frac{k-1-\mu_1-...-\mu_N}{S_N}\right)\right)+\frac{\epsilon}{2}\\ &< \epsilon\end{aligned}$$

**Case 2**: Suppose $\lim_{N\to\infty}\frac{\sum_{i=1}^N\sigma_i^2}{N}=0$.

We know that $\frac{\sum_{i=1}^N\sigma_i^2}{N}=\frac{\sum_{i=1}^N(1-p_i)p_i}{N}$. Because $p_i \leq \bar{p} < 1$ for each $i$, it must be that $\lim_{N\to\infty}\frac{\sum_{i=1}^N p_i}{N}=0$. But if the average parameter tends to zero, then $\mathbb{P}\left\{Z_N\geq\lfloor\frac{N-1}{M}\rfloor\right\}\leq\mathbb{P}\left\{\frac{z_1+...+z_N}{N}\geq\frac{1}{M}\right\}\to 0$. □

**Proof of Proposition 2.** Define $E_{a,b}^i$ to be the event that voter $i$ is pivotal between $a$ and $b$. For any voter $j$, denote $j$'s probability of voting for alternative $k\in\{a_1,..,a_M\}$ by $p_j^k$. We look at subsets of voters A, B, Z and Y as follows:

$$\begin{aligned}\mathrm{A_N} &:= \left\{j\in\{1,..i-1,i+1,...N\}\mid p_j^a>0 \textit{ and } p_j^b=0\right\}\\ \mathrm{B_N} &:= \left\{j\in\{1,..i-1,i+1,...N\}\mid p_j^a=0 \textit{ and } p_j^b>0\right\}\\ \mathrm{Z_N} &:= \left\{j\in\{1,..i-1,i+1,...N\}\mid p_j^a=0 \textit{ and } p_j^b=0\right\}\\ \mathrm{Y_N} &:= \left\{j\in\{1,..i-1,i+1,...N\}\mid p_j^a>0 \textit{ and } p_j^b>0\right\}\end{aligned}$$

Here, A is the set of voters other than $i$ who vote for $a$ with positive probability but who never vote for $b$, and B is defined analogously. The voters in Z never vote for $a$ or $b$, while those in Y vote for each $a$ and $b$ with positive probability.

Define the related random variables $X_a^N$, $X_b^N$, $Y_a^N$, and $Y_b^N$ as follows:

$$\begin{aligned} X_a^N &:= \#\{j \in \mathrm{A_N} \mid v_j = a\} \\ X_b^N &:= \#\{j \in \mathrm{B_N} \mid v_j = b\} \\ Y_a^N &:= \#\{j \in \mathrm{Y_N} \mid v_j = a\} \\ Y_b^N &:= \#\{j \in \mathrm{Y_N} \mid v_j = b\} \end{aligned}$$

In order for voter $i$ to be pivotal between $a$ and $b$, it must be that (1) the number of votes for $a$ is within one vote of the number of votes for $b$ and (2) the number of votes for $b$ (or $a$) is at least $\lfloor \frac{N-1}{M} \rfloor$. The second requirement follows because otherwise alternative $b$ could never win, so $i$ could not be pivotal.

These conditions allow us to bound the probability that voter $i$ is pivotal between $a$ and $b$.

$$\mathbb{P}\left\{E_{a,b}^i\right\} \leq \mathbb{P}\left\{X_a^N + Y_a^N = X_b^N + Y_b^N \geq \lfloor \frac{N-1}{M} \rfloor\right\} + \mathbb{P}\left\{X_a^N + Y_a^N = X_b^N + Y_b^N + 1 \geq \lfloor \frac{N-1}{M} \rfloor\right\}$$

Which implies:

$$\begin{aligned} \mathbb{P}\left\{E_{a,b}^i\right\} &\leq \sum_{k \geq \lfloor \frac{N-1}{M} \rfloor} \mathbb{P}\left\{X_a^N + Y_a^N = k\right\} \mathbb{P}\left\{X_b^N + Y_b^N \in \{k, k+1\} \mid X_a^N + Y_a^N = k\right\} \\ &\leq \max_{k \geq \lfloor \frac{N-1}{M} \rfloor} \mathbb{P}\left\{X_a^N + Y_a^N = k\right\} \sum_{k \geq \lfloor \frac{N-1}{M} \rfloor} \mathbb{P}\left\{X_b^N + Y_b^N \in \{k, k+1\} \mid X_a^N + Y_a^N = k\right\} \\ &\leq 2 \times \max_{k \geq \lfloor \frac{N-1}{M} \rfloor} \mathbb{P}\left\{X_a^N + Y_a^N = k\right\} \end{aligned}$$

Now, note that $X_a^N + Y_a^N$ is just a sum of independent Bernoulli random variables and that, by assumption, their parameters are bounded above by $\bar{p_a} < 1$. Furthermore, if $X_a^N + Y_a^N$ is the sum of fewer than $\lfloor \frac{N-1}{M} \rfloor$ Bernoulli random variables, $\max_{k \geq \lfloor \frac{N-1}{M} \rfloor} \mathbb{P}\left\{X_a^N + Y_a^N = k\right\} = 0$. If, on the other hand, $X_a^N + Y_a^N$ is the sum of at least $\lfloor \frac{N-1}{M} \rfloor$ Bernoulli random variables, picking arbitrarily large $N$ can guarantee that $X_a^N + Y_a^N$ is the sum of an arbitrarily large number of independent Bernoulli random variables. Therefore, by the lemma, we have that $\max_{k \geq \lfloor \frac{N-1}{M} \rfloor} \mathbb{P}\left\{X_a^N + Y_a^N = k\right\} \to 0$. Then, by the inequalities above,

$$\mathbb{P}\left\{E_{a,b}^i\right\} \leq 2 \times \max_{k \geq \lfloor \frac{N-1}{M} \rfloor} \mathbb{P}\left\{X_a^N + Y_a^N = k\right\} \to 0$$

Because the set of alternatives is finite, making a finite number of pairwise comparisons establishes that the probability voter $i$ is pivotal between any alternatives tends to zero as the number of voters tends to infinity. Since the selection of voter $i$ is arbitrary, this result holds for every voter. □

# References


[1] Aldrich, J. H. (1993). Rational Choice and Turnout. *American Journal of Political Science*, 37(1): 246-78.

[2] Batirov, K., Manevich, D. V. and Nagaev, S. V. (1977). The Esseen inequality for sums of a random number of differently distributed random variables. *Mat. Zametki*, 22(1): 143-6.

[3] Blais, A. (2000). *To Vote or Not to Vote? The Merits and Limits of Rational Choice.* Pittsburgh: University of Pittsburgh Press.

[4] Brennan, G. and Lomasky, L. (1993). *Democracy and Decision: The Pure Theory of Electoral Preference.* Cambridge: Cambridge University Press.

[5] Caplan, B. (2002). Sociotropes, Systematic Bias, and Political Failure: Reflections on the Survey of Americans and Economists on the Economy. *Social Science Quarterly*, 83(2): 416-35.

[6] Chaidee, N. and Tuntapthai, M. (2009). Berry-Esseen Bounds for Random Sums of Non-i.i.d. Random Variables. *International Mathematical Forum*, 4(26): 1281-8.

[7] Dal Bó, E. (2007). Bribing Voters. *American Journal of Political Science*, 51(4): 789-803.

[8] Dhillon, A. and Peralta, S. (2002). Economic Theories of Voter Turnout. *The Economic Journal*, 112 (June): 332-52.

[9] Downs, A. (1957). *An Economic Theory of Democracy.* New York: Harper.

[10] Feddersen, T. J. (2004). Rational Choice Theory and the Paradox of Not Voting. *Journal of Economic Perspectives*, 18(1): 99-112.

[11] Mansbridge, J. J. (1990). *Beyond Self-Interest.* Chicago: University of Chicago Press.

[12] Mulligan, C. B. and Hunter, C. G. (2003). The Empirical Frequency of a Pivotal Vote. *Public Choice*, 116(1-2): 31-54.

[13] Riker, W. H. and Ordeshook, P. C. (1968). A Theory of the Calculus of Voting. *The American Political Science Review*, 62(1): 25-42.

[14] Schuessler, A. A. (2000). *A logic of expressive choice.* Princeton: Princeton University Press.