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REACTOR NOISE ANALYSIS FROM OBSERVATIONS ON THE HIGH ENERGY RADIATION FROM THE REACTOR CORE

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ABSTRACT

Reactor noise analyses are conventionally performed by using neutron detectors to make direct observations on the fluctuations in the neutron distribution within a reactor core. The purpose of this paper is to present a theory for the interpretation of a noise experiment performed instead by using a photon detector to measure the fluctuations in the high energy radiation distribution from the reactor core. In practice one whould choose to detect the high energy ($E_0 \geq 5$ MeV) radiation, because in many instances there is a negligible fraction of delayed photons at these energies. The many groups of low energy delayed photons only complicate an otherwise practical and direct interpretation of such an experiment.

To deal theoretically with fluctuation phenomena a deductive approach is employed in which the Liouville equation is used to generate a coupled set of transport equations for the first and second moments of the appropriate numbers (in fact densities) of particles and photons that are adequate to describe the entire system of interest. The type of system that is considered here is one in which a photon detector (and its associated discriminating, counting, and recording equipment) is placed outside a reactor core. It is likely that a photon detector may be positioned outside the core proper, in contrast to conventional noise experiments in which neutron detectors are usually placed in-core, because the mean free path

of photons in the core is very much greater than that of neutrons in the core.

Particular attention is devoted to the theoretical description of the observables of an experiment. Two commonly used measures of fluctuations are a variance and a power spectral density; our analysis is accordingly couched in these terms. Applying consistent P-1 approximation procedures to the neutron distributions, the set of transport equations is solved for the type of system described above and spatially dependent expressions for the power spectral density of detected particles are obtained. Upon comparison with the corresponding expressions from a conventional neutron noise analysis, it is found that the same neutron correlation information is obtainable in principle by a photon detecting noise analysis as by conventional techniques.

INTRODUCTION

In this paper we present a theoretical interpretation of a reactor noise analysis based upon observations on the high energy radiation distribution emitted from the reactor core. This is in contrast to the more conventional means of performing reactor noise experiments in which neutron detectors make observations directly upon the neutron distributions within the core. It will be shown that striking similarities exist in the results obtained for these two approaches. In fact, that information which can be obtained from studies of fluctuations and correlations in the neutron distributions by observations on the neutrons themselves will be shown to be equally accessible, in principle, from observations on fluctuations in the high energy photon distribution.

The class of systems that we consider in this paper are nuclear reactor systems in which the radiation of interest is generated by fission and (n,γ) interactions. It can be shown that photons produced by (n,γ) interactions behave as prompt gammas accompanying fission events. In the type of experiments that we discuss (those in which the power spectral density is measured for the charged particles arising from detection events) considerable convenience of interpretation is achieved by restricting attention to just the prompt radiation of the system. So far as the delayed photons are concerned it is expected that only those delayed photons emitted in the time interval between 10^{-4} seconds and 1 second after a fission event would adversely affect the interpretation of a measurement. Therefore those photons emitted in time intervals shorter than 10^{-4} seconds following a fission event are taken to behave as prompt. Maienschein et al.[1] indicate that delayed gamma radiation arising from the fission products in the time interval 10^{-4} seconds to 1 second after fission is a negligible fraction of the total gamma energy emitted per fission event. Of course very long-lived delayed photons present problems also. However, Chapman et al.[2] show for the Bulk Shielding Reactor II that the delayed gamma radiation with energies greater than 5 MeV is considerably less than that arising promptly (within 10^{-4} seconds) from fission and (n,γ) interactions. Thus to achieve a practical, directly interpretable experiment we have suggested that high energy ($E_0 \geq 5$ MeV) prompt photons be observed.

It is recognized that present discriminating techniques (to allow the observation of only high energy photons) cannot avoid the adverse effect caused by the pile-up of low energy photons. Furthermore, shielding against these low energy gammas due to long lived fission product decay can be achieved only at the expense of detector efficiency. It therefore appears that photon detecting noise experiments will be limited to cold, clean reactor cores for the present as are neutron detecting noise experiments.

It might then be asked, "Why consider a photon-detecting noise experiment?" There are at least two good reasons for this consideration. First, it may be possible to perform noise analyses with a photon detector placed outside the reactor core proper. The fact that the mean free path of high energy photons in the core is much greater than that of neutrons allows this possibility. Thus the perturbing effect of a detector placed in-core, such as is the case presently in neutron detecting noise experiments, may be eliminated. This is especially desirable in low power experiments when the total number of neutrons in the core is small. Also since a photon detector is capable of effectively "seeing" an appreciable volume of the reactor core, it may be possible to smooth out or perhaps in some instances to remove spatially-dependent effects. Of course this would be a disadvantage if one is in fact trying to observe space-dependent effects. We therefore retain spatial dependence in the results we obtain so that their importance may be assessed in given instances.

With these qualitative remarks in mind on the practical aspects of the problem the remainder of this paper deals mainly with a more detailed theoretical analysis of the problem of obtaining neutron correlation information by the observation of fluctuations in the high energy radiation distribution from the reactor core. Since a connection between the theory of fluctuations and their measurement is conveniently established through the variance of the particles arising from detection events or through the power spectral density of their detection rates, this investigation proceeds from the appropriate coupled set of balance equations for the relevant particle and photon densities that describe the system to the order that the observations are performed. We obtain these equations by a deductive quantum mechanical approach in which the joint probability density for the system satisfies the Liouville equation. The joint probability density referred to above is one which describes the probability for all "particles" (this term is extended here to include photons as well as ordinary particles) of the system to be jointly distributed in phase space in a given manner as a function of time. The set of transport equations is then reduced to a form consistent with the conditions imposed by the model of the system we have discussed above. Finally a solution for the power spectral density of detected particles is obtained and analyzed in terms of corresponding results from a conventional neutron detecting noise analysis.

DEDUCTION OF THE SET OF WORKING EQUATIONS

To discuss appropriately fluctuations about a mean density (singlet density) it is necessary to consider second order stochastic quantities which we will refer to as doublet densities. In the quantum formalism the singlet density, for instance, of a given type of "particle" is expressed theoretically as the expectation value of the appropriate number operator for that type of "particle". This is just the first moment of the joint probability density with the number operator. Doublet densities are in turn just expectation values of second order monomials of appropriate number operators. A measure of the fluctuations of a given distribution of particles or photons is obtained through a variance, where the variance is defined as the difference between the doublet density of interest and the product of the corresponding singlet densities. It can now be noted that the techniques which we employ to deduce the appropriate set of transport equations have been presented in sufficient detail previously in applications to reactor systems[3], neutral gases[4], and plasmas[5,6]. Therefore we will attempt here to present just the essence of the principles involved and to abbreviate the calculational detail in light of the references given above that deal specifically with these details.

This computation is initiated by dividing the six dimensional phase space into hypercells of volume $(2\pi)^3$. (This is just coarse graining in phase space.) Configuration space is divided into non-overlapping cells of volume L^3 and $\underline{K}$ space into cells of volume $\frac{(2\pi)^3}{L^3}$. Here $\underline{K}$ is the wave vector associated with a momentum $\hbar\underline{K}$. The center of a hypercell is located by co-ordinates $(\underline{X},\underline{K})$, and all "particles" (recall that photons are included in this set as well as ordinary particles) in the cell are located by giving these co-ordinates. It is noted that the hypercells are of sufficient volume that the uncertainty principle is not violated.

The singlet densities for particles of kind "A" are defined by

$$F_1^A(\underline{X},\ \underline{K},\ a,\ t) \equiv \frac{1}{(2\pi)^3}\,\mathrm{Tr}\ \rho^A(\underline{X},\ \underline{K},\ a)D(t), \tag{1}$$

and the doublet densities for particles of kinds "A" and "B" are defined by

$$F_2^{AB}(\underline{X},\underline{K},a;\ \underline{X}',\underline{K}',b,t) \equiv \frac{1}{(2\pi)^6}\,\mathrm{Tr}\ \rho^A(\underline{X},\underline{K},a)\ \rho^B(\underline{X}'\underline{K}'b)D(t)\ . \tag{2}$$

The operators $\rho^A(\underline{X},\underline{K},a)$ are number operators whose eigenvalues in a diagonalizing representation represent the possible numbers of particles of kind "A" in the phase-space hypercell centered at the point $(\underline{X},\underline{K})$. The lables "a" (and "b") specify the quantum numbers necessary to complete the description of the particle's state. They designate such things as polarization, spins, and internal states. All particles in a given phase cell are assigned the coordinates of the center of the cell, their momenta being given by $\underline{P} = \hbar\ \underline{K}$. Evidently these phase points are discretely distributed. However, whenever appropriate, they will be assumed to be sufficiently dense to be regarded as a continuum. The quantity D is the density operator for the system.

The density operator is assumed to satisfy the Liouville equation given by

$$\frac{\partial D(t)}{\partial t} = \frac{i}{\hbar}\,[D,H] \ , \tag{3}$$

where H is the Hamiltonian of the system. It is useful to write H as

$$H = \sum_A H^A + V \ . \tag{4}$$

In this case H^A describes the kinetic energy of the "A"-type particle (this is the contribution from the free photon field when "A" refers to photons), and V represents all other contributions to the energy of the system.

The first step in the present derivation of a transport equation is to display (generically)

$$\frac{F(t+\tau)-F(t)}{\tau} = \frac{\partial F}{\partial t}\left[1 + \frac{\tau}{2}\,\frac{\partial^2 F/\partial t^2}{\partial F/\partial t} + \ldots\right] \simeq \frac{\partial F}{\partial t} \ , \tag{5}$$

for sufficiently small τ and for densities which do not vary too rapidly in time. For example, if $F(t) \sim e^{t/T}$, then the above approximation implies the neglect of a series of terms, the largest of which is $\theta(\tau/T)$ for $\frac{\tau}{T} < 1$. An obvious lower limit for τ is interaction times; which, in the case that we will consider, will not likely exceed 10^{-10} seconds. It is to be noted that this approximation (coarse-graining in time) is a necessity and not merely a calculational trick, since it is meaningless to compare densities at two instants closer together than an interaction time. It can then be shown[3,7] that the equation for the singlet density of "A"-type particles can be written as:

$$\left[\frac{\partial}{\partial t} + \underline{v}^A\cdot\underline{\nabla}\right]F_1^A(\underline{X},\underline{K},a;t) = \frac{1}{(2\pi)^3}\sum_{nn'}\left[\rho^A_{n'n'}(\underline{X},\underline{K},a) - \rho^A_{nn}(\underline{X},\underline{K},a)\right]$$
$$+ \text{ terms off-diagonal in } D(t) \ . \tag{6}$$

We have introduced $\underline{v}^A$ to represent the velocity of the "A"-type particles.

Note that $\underline{v}^A \equiv h\underline{K}/m_A$ when "A" applies to an ordinary particle, and m_A is the mass of the particle. When "A" applies to photons, $\underline{v}^A \equiv c\underline{\Omega}$ and c = 3×10^{10} cm/sec because the medium is non-dispersive for the radiation of interest. The direction vector for the photons is given by $\underline{\Omega}$. $T_{n'n}$ stands for the probability per unit time for a transition to occur between an initial state designated by n and a final state designated by n'. The summation over n and n' includes all initial and final states. Expressions for $T_{n'n}$ can be obtained by conventional perturbation techniques. Many approximations are required to go from equations (1), (3), and (5) to equation (6). All of these approximations have been displayed elsewhere explicitly, some interpreted qualitatively, but few estimated quantitatively. Many of these considerations have been dealt with specifically in references 3, 6, 7, which are directed more toward the actual development of a transport theory. We therefore proceed at this point to writing the generic doublet equation (neglecting terms proportional to off-diagonal elements of D(t) henceforth) as:

$$[\frac{\partial}{\partial t} + \underline{v}^A\cdot\underline{\nabla}+\underline{v}^B\cdot\underline{\nabla}']F_2^{AB}\,(\underline{X},\underline{K},a;\underline{A}',\underline{K}',b,t) = \frac{1}{(2\pi)^6}\sum_{nn'} T_{n'n}(x)$$
$$(x)[\rho^A_{n'n'}(\underline{X},\underline{K},a)\rho^B_{n'n'}(\underline{X}',\underline{K}',b)-\rho^A_{nn}(\underline{X},\underline{K},a)\rho^B_{nn}(\underline{X}',\underline{K}',b)]D_{nn}(t)\ . \tag{7}$$

Note that the gradients which appear in the equations at this point are symbolic and have the meaning of a finite difference in the density of interest at two adjacent cells in configuration space, divided by the linear dimension of the cells. They will take on the usual meaning of gradients when we pass to the continuum for densely spaced points.

SPECIALIZATION OF THE TRANSPORT EQUATIONS

Before finally specializing the set of working equations to their most useful form for our purposes, it may be helpful to further define the model of the system with which we are dealing. Recall that a judicious choice of energy threshold for observing prompt photons is $E_0 \geq 5$ MeV, because only a small tail of the distribution of delayed gammas occurs above 5 MeV in comparison with the number of prompt gammas in this energy range. It is also reasonable to assume that only a very small fraction of the photons in this high-energy range have previously undergone a scattering interaction and maintained a final energy greater than 5 MeV[2]. Also bremsstrahlung (mean energy of .5 MeV) is a negligible photon source for $E_0 \geq 5$ MeV. For the sake of discussion it is assumed that a scintillator is used to detect photons, and the photoelectrons that result from the scintillations interacting with the photocathode material are simply recorded and accumulate in time. Photofission is neglected, and photons are assumed not to interact with other photons. The only photon interactions that are considered to be relevant in this model are those events by which a photon appears to be absorbed. Detection processes, photoelectric absorption, and pair production are the obvious photon absorbing interactions. Photon scattering is also taken to behave as a means of absorption here, because we assumed that a photon is removed from the energy range of interest by a scattering interaction. The emission of photons and particles by neutron interactions is taken to be isotropic, and extraneous neutron and photon sources are assumed to be isotropic and constant in time. Finally we neglect delayed neutrons and delayed photons for expediency of calculation.

In this system we are interested only in charged particles from detection events, neutrons, and photons. All other distributions in the system are assumed to be known. To deduce the interaction terms on the right hand side of the balance equations, it is well to restrict our attention to the dominant interactions that affect the system. In the present case the dominant processes are neutron fission, capture, and scattering; the production of gamma radiation by fission and (n,γ) events; the absorption of gammas by the medium; and the detection process. Neutrons scattered by neutrons are neglected. It has also been found[3,5] that about all the information that is needed regarding transition probabilities is their dependence upon occupation numbers. In the present application ρ is neglected compared to unity, because measurements on this system will be insensitive to quantum statistical effects.

The final reduction of equations (6) and (7) is carried out in the same manner as employed previously in the literature[3,7]. Upon performing that task and passing to the continuum, where one assumes that the discrete points in phase space are sufficiently closely spaced that they can be treated as continuous, we can write out the set of working equations completely for this system, starting with the doublet density of detected particles, as

$$\begin{aligned}
&\left[\frac{\partial}{\partial t} + \frac{\hbar}{m_\beta}\left(\underline{K}\cdot\underline{\nabla}+\underline{K}'\cdot\underline{\nabla}'\right)\right] f_2^{\beta\beta'}(\underline{X},\underline{K};\underline{X}'\underline{K}';t) = \int d^3\kappa''\, r_D(\kappa'')(x) \\
&(x)\left[\mathcal{J}^D(\underline{\kappa}''\rightarrow\underline{K}) f_2^{\gamma\beta'}(\underline{X},\underline{\kappa}'';\underline{X}',\underline{K}';t) + \mathcal{J}^D(\underline{\kappa}''\rightarrow\underline{K}') f_2^{\beta\gamma}{}'(\underline{X},\underline{K};\underline{X}'\underline{\kappa}'';t)\right](x) \qquad (8) \\
&(x)H(t) + \delta(\underline{X}-\underline{X}')\delta(\underline{K}-\underline{K}') \int d^3\kappa''\, r_D(\kappa'')\mathcal{J}^D(\underline{\kappa}''\rightarrow\underline{K}) f_1^{\gamma}(\underline{X},\underline{\kappa}'',t)H(t),
\end{aligned}$$

where $\mathcal{J}^D(\underline{\kappa}'\rightarrow\underline{K})d^3K$ is the probability that a photon with wave vector $\underline{\kappa}'$ will, upon detection by a photoelectric detector, produce a photelectron with wave vector $\underline{K}\varepsilon d^3K$, and $H(t)$ is the unit step function.

The equation for the singlet density of detected particles is written as:

$$[\frac{\partial}{\partial t} + \frac{h}{m_\beta} \underline{K}\cdot\underline{\nabla}]f_1^\beta(\underline{X},\underline{K},t) = \int d^3\kappa'' r_D(\kappa'')\mathcal{P}^D(\underline{\kappa}''\rightarrow\underline{K})f_1^\gamma(\underline{X},\underline{\kappa}'',t)H(T). \quad (9)$$

The equation for the cross doublet density of charged particles and photons is written:

$$\begin{aligned}
&[\frac{\partial}{\partial t} + c\underline{\Omega}'\cdot\underline{\nabla}'+ \frac{\hbar}{m_\beta} \underline{K}\cdot\underline{\nabla}+r_a(\kappa')]f_2^{\beta\gamma'}(\underline{X},\underline{K};\underline{X}',\underline{\kappa};t)\\
&\quad = \int d^3\kappa'' r_D(\kappa'')\mathcal{P}^D(\underline{\kappa}''\rightarrow\underline{K})f_2'(\underline{X},\underline{\kappa}'';\underline{X}',\underline{\kappa}';t)H(t)\\
&\quad + \int d^3k'' r_f(k'')f_2^{\beta N'}(\underline{X},\underline{K};\underline{X}',\underline{k}'';t)\sum_{Q,\eta}\eta G_\eta^Q(\underline{k}'',\underline{\kappa}') \qquad (10)\\
&\quad + f_1^\beta(\underline{X},\underline{K},t)\Gamma(\underline{X}',\underline{\kappa}')-\delta(\underline{X}-\underline{X}')r_D(\kappa'\mathcal{P}^D(\underline{\kappa}'\ \underline{K})f_1^\gamma(\underline{X},\underline{\kappa}',t)H(t),
\end{aligned}$$

and $f_2^{\gamma\beta\ '}(\underline{X},\underline{\kappa};\underline{X}',\underline{K}';t)$ is obtained by interchanging arguments. The quality $G_\eta^Q(\underline{k}'',\underline{\kappa}')d^3\kappa'$ is defined as the probability that a fission event induced by a neutron at $\underline{k}''$ will produce exactly Q photons, η of which have $\underline{\kappa}'\epsilon\ d^3\kappa'$, and $\Gamma(\underline{X}',\underline{\kappa}')d^3\underline{X}'d^3\kappa'$ is the expected number of photons produced per second in d^3X' about $\underline{X}'$ and in $d^3\kappa'$ about $\underline{\kappa}'$ by extraneous photon sources.

The equation for the singlet density of photons is:

$$\begin{aligned}
&[\frac{\partial}{\partial t} + c\underline{\Omega}\cdot\underline{\nabla}+r_a(\kappa)]f_1^\gamma(\underline{X},\underline{\kappa},t) = \int d^3k'' r_f(k'')f_1^N(\underline{X},\underline{k}'',t)(x) \qquad (11)\\
&\quad (x)\sum_{Q,\eta}\eta G^Q(\underline{k}'',\underline{\kappa})+\Gamma(\underline{X},\underline{\kappa}),
\end{aligned}$$

and the photon doublet equation is written as:

$$\begin{aligned}
&[\frac{\partial}{\partial t} + c(\underline{\Omega}\cdot\underline{\nabla}+\underline{\Omega}'\cdot\underline{\nabla}')+r_a(\kappa)+r_a(\kappa')]f_2^{\gamma\gamma'}(\underline{X},\underline{\kappa};\underline{X}',\underline{\kappa}';t)\\
&\quad = f_1^\gamma(\underline{X},\underline{\kappa},t)\Gamma(\underline{X}',\underline{\kappa}')+\Gamma(\underline{X},\underline{\kappa})f_1^{\gamma'}(\underline{X}',\underline{\kappa}',t)+\int d^3k'' r_f(k'')(x) \qquad (12)\\
&(x)\sum_{Q,\eta}\eta\left\{G_\eta^Q(\underline{k}'',\underline{\kappa})f_2^{N\gamma'}(\underline{X},\underline{k}'';\underline{X}',\underline{\kappa}';t)+G_\eta^Q(\underline{k}'',\underline{\kappa}')f_2^{\gamma N'}(\underline{X},\underline{\kappa};\underline{X}',\underline{k}'';t)\right\}\\
&\quad + \delta(\underline{X}'-\underline{X}')\int d^3k'' r_f(k'')f_1^N(\underline{X},\underline{k}'',t)\sum_{Q,\eta,\nu}\eta\nu G_{\eta\nu}^Q(\underline{k}''|\underline{\kappa},\underline{\kappa}')+\delta(\underline{X}-\underline{X}')(x)
\end{aligned}$$

$$(x)\delta(\underline{\kappa}-\underline{\kappa}')[r_a(\kappa)f_1^{\gamma}(\underline{X},\underline{\kappa},t)+\Gamma(\underline{X},\underline{\kappa})] \ . \qquad (12)$$

(con't.)

In equation (12) we have introduced $G^Q_{\eta\nu}(\underline{k}''|\underline{\kappa},\underline{\kappa}')d^3\kappa d^3\kappa'$ to be the probability that a fission event induced by a neutron at $\underline{k}''$ will produce Q photons, η of which have $\underline{\kappa}\varepsilon d^3\kappa$ and ν of which have $\underline{\kappa}'\varepsilon d^3\kappa'$. When $\underline{\kappa}=\underline{\kappa}'$,

$$G^Q_{\eta\nu}(\underline{k}''|\underline{\kappa},\underline{\kappa}')\rightarrow\delta_{\eta\nu}\delta(\underline{\kappa}-\underline{\kappa}')G^Q_{\eta}(\underline{k}'',\underline{\kappa}) \ .$$

The neutron singlet equation is:

$$[\frac{\partial}{\partial t}+\frac{\hbar}{m_N}\underline{k}\cdot\underline{\nabla}+L(\underline{k})]f_1^N(\underline{X},\underline{k},t) = S(\underline{X},\underline{k}) \ , \qquad (13)$$

where

$$L(\underline{k})f_1^N(\underline{X},\underline{k},t) = r_t(k)f_1^N(\underline{X},\underline{k},t)-\int d^3k''[r_s(k'')\mathscr{J}^{SC}(\underline{k}''\rightarrow\underline{k})$$

$$+r_f(k'')\sum_{J,\alpha}\alpha B^J_{\alpha}(\underline{k}'',\underline{k})]f_1^N(\underline{X},\underline{k}'',t) \ . \qquad (14)$$

The quantity $S(\underline{X},\underline{k})d^3Xd^3k$ is the expected number of neutrons produced per second in d^3X about $\underline{X}$ and in d^3k about $\underline{k}$ by means other than the fission process; $B^J_{\alpha}(\underline{k}'',\underline{k})d^3k$ is the probability that a fission induced by a neutron with wave vector $\underline{k}''$ will produce J prompt neutrons of which α have $\underline{k}\varepsilon d^3k$; and $\mathscr{J}^{SC}(\underline{k}''\rightarrow\underline{k})d^3k$ is the probability that a neutron with an initial wave vector $\underline{k}''$ will be scattered into $\underline{k}\varepsilon d^3k$. The equation for the neutron doublet density is given by:

$$[\frac{\partial}{\partial t}+\frac{\hbar}{m_N}(\underline{k}\cdot\underline{\nabla}+\underline{k}'\cdot\underline{\nabla}')+L(\underline{k})+L(\underline{k}')]f_2^{NN'}(\underline{X},\underline{k};\underline{X}',\underline{k}';t)$$

$$= f_1^N(\underline{X},\underline{k},t)S(\underline{X}',\underline{k}')+S(\underline{X},\underline{k})f_1^{N'}(\underline{X}',\underline{k}',t)+\delta(\underline{X}-\underline{X}')(x) \qquad (15)$$

$$(x)\delta(\underline{k}-\underline{k}')S(\underline{X},\underline{k})+\delta(\underline{X}-\underline{X}')\Lambda(\underline{X};\underline{k},\underline{k}';t) \ ,$$

where

$$\Lambda(\underline{X};\underline{k},\underline{k}';t) = \delta(\underline{k}-\underline{k}')\left\{r_t(k)f_1^N(\underline{X},\underline{k},t)+\int d^3k''r_s(k'')\mathscr{J}^{SC}(\underline{k}''\rightarrow\underline{k})f_1^N(\underline{X},\underline{k}'',t)\right\}$$

$$-[r_s(k)\mathscr{J}^{SC}(\underline{k}\rightarrow\underline{k}')+r_f(k)\sum_{J,\alpha}\alpha B^J_{\alpha}(\underline{k},\underline{k}')]f_1^N(\underline{X},\underline{k},t) \qquad (16)$$

$$-[r_s(k')\mathcal{J}^{SC}(\underline{k}'\to\underline{k})+r_f(k')\sum_{J,\alpha}\alpha B_\alpha^J(\underline{k}',\underline{k})]f_1^N(\underline{X}',\underline{k}',t)$$

$$+\int d^3k''r_f(k'')\sum_{J,\alpha,\beta}\alpha\beta B_{\alpha\beta}^J(\underline{k}''|\underline{k},\underline{k};)f_1^N(\underline{X},\underline{k}'',t) \; . \tag{16 (con't.)}$$

In this equation we have introduced $B_{\alpha\beta}^J(\underline{k}''|\underline{k},\underline{k}')d^3kd^3k'$ as the probability that a fission induced by a neutron at $\underline{k}''$ produces J prompt neutrons of which α have $\underline{k}\varepsilon d^3k$ and β have $\underline{k}'\varepsilon d^3k'$. When $\underline{k}=\underline{k}'$, $B_{\alpha\beta}^J(\underline{k}''|\underline{k},\underline{k}')\to\delta_{\alpha\beta}\delta(\underline{k}-\underline{k}')$ $B_\alpha^J(\underline{k}'',\underline{k})$.

Finally the equation for the cross doublet density of detected particles and neutrons is

$$[\frac{\partial}{\partial t} + \hbar\,(\frac{\underline{K}\cdot\underline{\nabla}}{m_\beta} + \frac{\underline{k}'\cdot\underline{\nabla}'}{m_N}) + L(\underline{k}')]f_2^{\beta N'}(\underline{X},\underline{K};\underline{X}',\underline{k}';t)$$

$$=\int d^3\kappa''r_D(\kappa'')\mathcal{J}^D(\underline{\kappa}''\to\underline{K})f_2^{\gamma N'}(\underline{X},\underline{\kappa}'';\underline{X}',\underline{k}';t)H(t) \tag{17}$$

$$+\; f_1^\beta(\underline{X},\underline{K},t)S(\underline{X}',\underline{k}') \; ,$$

and the cross doublet equation for photons and neutrons is given by:

$$[\frac{\partial}{\partial t} + c\underline{\Omega}\cdot\underline{\nabla}+ \frac{h}{m_N}\,\underline{k}'\cdot\underline{\nabla}'+r_a(\kappa)+L(\underline{k}')]f_2^{\gamma N'}(\underline{X},\underline{\kappa};\underline{X}'\underline{k}';t)$$

$$= \Gamma(\underline{X},\underline{\kappa})f_1^{N'}(\underline{X}'\underline{k}'t)+f_1^\gamma(\underline{X},\underline{\kappa},t)S(\underline{X}',\underline{k}')+\int d^3k''r_f(k'')(x)$$

$$(x)f_2^{NN'}(\underline{X},\underline{k}'';\underline{X}',\underline{k}';t)\sum_{Q,\eta}\eta G_\eta^Q(\underline{k}'',\underline{\kappa})-\delta(\underline{X}-\underline{X}')r_f(k')(x) \tag{18}$$

$$(x)f_1^N(\underline{X},\underline{k}',t)\sum_{Q,\eta}\eta G_\eta^Q(\underline{k}',\underline{\kappa}) \; .$$

To this point the multiplet densities are general for all real times. However, experiments will be interpreted in terms of accumulations of charged particles in view of the fact that a physical measurement is to run positively in time starting from, say, t=0 to time t. Thus the step function H(t) is employed above in expressions describing the detection of photons. It is seen that the charged particle multiplet densities are continuous functions in time and represent accumulations of charged particles. The rates of accumulation, i.e., the first time derivatives of charged particle multiplets are discontinuous, having been zero at times prior to t=0 and non zero for $t \geq 0$ due to the observation starting at t=0. It is then noted that the second time derivatives of charged particle multiplets are singular at t=0. Thus we have seen that the consequences of this restricted validity are conveniently accounted for by interpreting the counting rate per photon, r_D, as proportional to the step function, H(t), which has the properties:

$$H(t) = 0 \qquad t < 0 \tag{19}$$

$$H(t) = 1 \qquad t \geq 0, \tag{20}$$

and

$$\frac{dH(t)}{dt} = \delta(t) \ . \tag{21}$$

The following list defines the quantities appearing in the above equations that were heretofore undefined:

a) The superscripts on the densities designate the type of "particle" referred to as:

$N \rightarrow$ neutrons

$\beta \rightarrow$ photoelectrons (are the detected particles)

$\gamma \rightarrow$ photons

b) $\Sigma_S(k)$ is the probability per unit path for small paths that a neutron with momentum of magnitude $\hbar$ k will be scattered. The interaction rates expressed by r's are just the product of the macroscopic cross sections and the mean speeds of the relevant particles. The subscripts for interactions of interest are:

$S \rightarrow$ neutron scattering

$f \rightarrow$ fission

$a_N \rightarrow$ neutron absorption

t → total neutron events

c → capture of neutrons

a → photon absorption

D → detection of photons by photoelectric material.

Our final reduction of the above set of transport equations is accomplished by integrating the equations over photoelectron energies and over the relevant photon energies ($E_0 \geq 5$ MeV). Streaming of the photoelectrons is neglected, and consistent P-1 approximation procedures will be applied to the set of all neutron densities and cross densities. Neutron quantities are integrated over all neutron energies, and energy averaged system parameters, such as interaction rates, are taken to be independent of the density that is used as the weight function in the average taking process. The emission of photons and neutrons by neutron events has been assumed to be isotropic, and the emission of neutrons by external sources is taken to be isotropic, uniform, and constant in time. Photon and neutron densities and neutron currents are assumed to be stationary and the extraneous gamma source, Γ, is neglected from here on for expediency. Performing this last set of reduction steps the set of working equations is written finally as:

$$\frac{\partial f_1^{\beta}(\underline{X},t)}{\partial t} = \Delta f_1^{\gamma}(\underline{X})H(t) \tag{22}$$

$$\frac{\partial f_2^{\beta\beta'}(\underline{X},\underline{X}',t)}{\partial t} = \left\{\Delta f_2^{\gamma\beta'}(\underline{X},\underline{X}'t)+\Delta' f_2^{\beta\gamma'}(\underline{X},\underline{X}',t)\right\}H(t) + \delta(\underline{X}-\underline{X}')\Delta f_1^{\gamma}(\underline{X})H(t) \tag{23}$$

$$[\frac{\partial}{\partial t} + B_{\gamma'}]f_2^{\beta\gamma'}(\underline{X};\underline{X}',\underline{\Omega}';t) = \Delta f_2^{\gamma\gamma'}(\underline{X};\underline{X}',\underline{\Omega}')H(t)+ \frac{P_\gamma}{4\pi} f_2^{\beta N'}(\underline{X},\underline{X}',t) - \delta(\underline{X}-\underline{X}')r_D f_1^{\gamma}(\underline{X},\underline{\Omega}')H(t) \tag{24}$$

$$B_\gamma f_1^\gamma(\underline{X},\underline{\Omega}) = \frac{P_\gamma}{4\pi} f_1^N(\underline{X}) \tag{25}$$

$$[B_\gamma+B_{\gamma'}]f_2^{\gamma\gamma'}(\underline{X},\underline{\Omega};\underline{X}',\underline{\Omega}') = \frac{P_\gamma}{4\pi}\left\{f_2^{N\gamma'}(\underline{X};\underline{X}',\underline{\Omega}')+f_2^{\gamma N'}(\underline{X},\underline{\Omega};\underline{X}')\right\}$$
$$+ \delta(\underline{X}-\underline{X}')\left\{\frac{P_{\gamma\gamma'}}{(4\pi)^2} f_1^N(\underline{X})+\delta(\underline{\Omega}-\underline{\Omega}')r_a f_1^\gamma(\underline{X},\underline{\Omega})\right\}. \tag{26}$$

The following new notation has been introduced:

$$\Delta f^\gamma(\underline{X}) \equiv \int d^3\kappa r_D(\kappa) f^\gamma(\underline{X},\underline{\kappa}) . \tag{27}$$

The subscript has intentionally been deleted, because this notation applies for all photon densities and cross densities. Also

$$\Delta' f^{\gamma'}(\underline{X}') \equiv \int d^3\kappa' r_D(\kappa') f^{\gamma'}(\underline{X}',\underline{\kappa}') \tag{28}$$

$$B_\gamma \equiv c\underline{\Omega}\cdot\underline{\nabla}+r_a \tag{29}$$

$$B_{\gamma'} \equiv c\underline{\Omega}'\cdot\underline{\nabla}'+r_a \tag{30}$$

$$\frac{P_\gamma}{4\pi} f^N(\underline{X}) = P_\gamma(\underline{\Omega})f^N(\underline{X}) \equiv \int d^3k'' r_f(k'')\langle\eta(\underline{\Omega})\rangle f^N(\underline{X},\underline{k}'') = \frac{\langle\eta\rangle r_f}{4\pi} f^N(\underline{X}), \tag{31}$$

where

$$\langle\eta(\underline{\Omega})\rangle \equiv \text{expected number of photons that are born into } d\Omega \text{ about } \underline{\Omega} \text{ by a fission event.} \tag{32}$$

$$\frac{P_{\gamma\gamma'}}{(4\pi)^2} f^N(\underline{X}) = P_{\gamma\gamma'}(\underline{\Omega},\underline{\Omega}')f^N(\underline{X}) \equiv \int d^3k'' r_f(k'')\langle\eta(\underline{\Omega})\nu(\underline{\Omega}')\rangle f^N(\underline{X},\underline{k}'')$$
$$= \frac{\langle\eta\nu\rangle}{(4\pi)^2} r_f f^N(\underline{X}) \tag{33}$$

where

$\langle\eta(\underline{\Omega})\nu(\underline{\Omega}')\rangle d\Omega d\Omega' \equiv$ expected product of the number of photons, η, born into $d\Omega$ about $\underline{\Omega}$ jointly with ν born into $d\Omega'$ about $\underline{\Omega}'$ by a fission event. (34)

It has been shown by Akcasu[8] that the class of distributions to which $\langle\eta(\underline{\Omega})\nu(\Omega')\rangle$ belongs has the property:

$$\int d\Omega \int d\underline{\Omega}' \langle\eta(\underline{\Omega})\nu(\underline{\Omega}')\rangle = \langle\eta'^2\rangle . \tag{35}$$

SOLUTIONS OF THE EQUATIONS

It is worthwhile at this point to devote further attention to the observables of a noise experiment. So let us consider the power spectral density of the detected particles. The power spectral density, $\Phi(\omega)$, is the cosine transform of the autocorrelation function $\phi(\tau)$.

The autocorrelation function relevant to the measurements with which we are concerned is the autocorrelation function of the output current, $I(t)$, from the photon detector. The computation of $\Phi(\omega)$ can be performed analytically by processing the raw data from an FM tape, for instance, which might be used to record the output current from the detector; or suitable electronic equipment utilizing a succession of filters among other circuitry can be used to give $\Phi(\omega)$ directly from the output current. The autocorrelation function is given for stationary currents by:

$$\phi(\tau) = \langle I(t)I(t+\tau)\rangle \ . \tag{36}$$

It can be shown[9] that

$$\phi(\tau) = \frac{1}{2}\,\frac{\partial^2 f_2^{\beta\beta}(\tau)}{\partial\tau^2} \tag{37}$$

which then gives the power spectral density as:

$$\Phi(\omega) = \frac{1}{2}\int_{-\infty}^{\infty} d\tau \cos\omega\tau \, \frac{\partial^2 f_2^{\beta\beta}(\tau)}{\partial\tau^2} \ . \tag{38}$$

It is apparent that the connection between the experimental quantity $\phi(\tau)$ in (36) and the theoretical $\phi(\tau)$ in (37) is through the observable quantities $f_2^{\beta\beta}(\tau)$ and $f_1^{\beta}(\tau)$. These are seen to be:

$$f_1^{\beta}(\tau) = \int_{D.V.} d^2X \, f_1^{\beta}(\underline{X},\tau) \tag{39}$$

and

$$f_2^{\beta\beta}(\tau) = \int_{D.V.} d^3X \int_{D.V.} d^3X' f_2^{\beta\beta'}(\underline{X},\underline{X}',\tau) \ , \tag{40}$$

where

$$\text{D.V.} \equiv \text{detector volume.} \tag{41}$$

As was indicated previously, $f_1^{\beta}(\tau)$ and $f_2^{\beta\beta}(\tau)$ represent accumulations over the interval τ because of the initial conditions that were applied to all β densities.

These initial conditions are:

$$f_1^{\beta}(0) = 0 \tag{42}$$

$$f_2^{\beta\beta}(0) = 0 \tag{43}$$

$$f_2^{\beta\gamma}(0) = f_2^{\gamma\beta}(0) = 0 \tag{44}$$

$$f_2{}^{\beta N}(0) = f_2^{N\beta}(0) = 0 \ . \tag{45}$$

The power spectral density may also be written (for the sake of calculational convenience) in terms of the detected particle variance, $V^{\beta\beta'}(t)$, (for stationary photoelectron currents) as:

$$\Phi(\omega) = \frac{1}{2}\int_{-\infty}^{\infty} dt \cos \omega t \, \frac{\partial^2 V^{\beta\beta'}(t)}{\partial t^2} \tag{46}$$

(valid for non-zero frequencies), where the photoelectron variance, $V^{\beta\beta'}(t)$, is defined as

$$V^{\beta\beta'}(t) \equiv \int_{(\text{D.V.})_1} d^3X \int_{(\text{D.V.})_2} d^3X' \{f_2^{\beta\beta'}(\underline{X},\underline{X}',t) - f_1^{\beta}(\underline{X},t) f_1^{\beta}(\underline{X}',t)\} \tag{47}$$

The equations for the variances that are needed are obtained directly from our previously reduced multiplet equations. They are

$$\frac{\partial V^{\beta\beta'}(\underline{X},\underline{X}',t)}{\partial t} = \Big\{\Delta V^{\gamma\beta'}(\underline{X},\underline{X}',t) + \Delta' V^{\beta\gamma'}(\underline{X},\underline{X}',t) + \delta(\underline{X}-\underline{X}')\Delta f_1^{\gamma}(\underline{X})\Big\} H(t) \ , \tag{48}$$

and by differentiating

$$\frac{\partial^2 V^{\beta\beta'}(\underline{X},\underline{X}',t)}{\partial t^2} = \left\{\Delta \frac{\partial V^{\gamma\beta'}(\underline{X},\underline{X}',t)}{\partial t} + \Delta' \frac{\partial V^{\beta\gamma'}(\underline{X},\underline{X}',t)}{\partial t}\right\} H(t) \tag{49}$$

$$+ \delta(\underline{X}-\underline{X}')\Delta f_1^\gamma(\underline{X})\delta(t) \ . \tag{49 con't.}$$

To get this equation we have used the properties of the step function and the initial conditions given as

$$V^{\beta\beta'}(0) = V^{\beta\gamma'}(0) = V^{\gamma\beta'}(0) = V^{\beta N'}(0) = V^{N\beta'}(0) = 0 \ . \tag{50}$$

In like manner we have

$$[\frac{\partial}{\partial t} + B_\gamma]V^{\gamma\beta'}(\underline{X},\underline{\Omega};\underline{X}';t) = \Big\{\Delta' V^{\gamma\gamma'}(\underline{X},\underline{\Omega};\underline{X}') \tag{51}$$

$$+ \frac{P_\gamma}{4\pi} V^{N\beta'}(\underline{X},\underline{X}',t) - \delta(\underline{X}-\underline{X}')r_D f_1^\gamma(\underline{X},\underline{\Omega})\Big\} H(t),$$

and $V^{\beta\gamma'}(\underline{X};\underline{X}',\underline{\Omega}';t)$ is obtained by interchanging arguments. We also have

$$[\frac{\partial}{\partial t} + B_N]V^{N\beta'}(\underline{X},\underline{X}',t) = \Delta' V^{N\gamma'}(\underline{X},\underline{X}')H(t) \ . \tag{52}$$

Applying consistent P-1 procedures to reduce the neutron equations, we can write the neutron Boltzmann operator, B_N, as

$$B_N f^N(\underline{X}) = [a_1 - D_1\nabla^2]f^N(\underline{X}) \ , \tag{53}$$

where

$$D_1 \equiv \langle v\rangle^2/3b_1 \tag{54}$$

$$\langle v\rangle \equiv \text{mean speed of neutrons} \tag{55}$$

$$b_1 \equiv r_{aN} + [1-\langle\mu\rangle]r_s = r_t - \langle\mu\rangle r_s \tag{56}$$

$$\langle\mu\rangle \equiv \text{first angular moment of the neutron scattering frequency} \tag{57}$$

$$a_1 \equiv r_{aN} - \langle J\rangle r_f = r_{aN}(1-k\infty) \ . \tag{58}$$

The details concerning the reduction of the neutron equations have been given careful attention by Osborn and Natelson[3] and others[10], and will not be belabored by us at this point. The equation for $V^{\beta N'}(\underline{X},\underline{X}',t)$ is obtained by interchanging arguments of equation (52).

The solutions for these variances can be obtained by Fourier-Laplace transformation. In transform space we have

$$V^{\gamma\beta'}(\underline{k},\underline{\Omega};\underline{X}';s) - \frac{\Delta' V^{\gamma\gamma'}(\underline{k},\underline{\Omega};\underline{X}') - e^{-i\underline{k}\cdot\underline{X}'} f_1^{\gamma}(\underline{X}',\underline{\Omega}) r_D}{ic\underline{k}\cdot\underline{\Omega}+r_a}$$

$$(x)\left[\frac{1}{s} - \frac{1}{s+ic\underline{k}\cdot\underline{\Omega}+r_a}\right] + \frac{P_\gamma}{4\pi}\frac{V^{N\beta'}(\underline{k};\underline{X}';s)}{s+ic\underline{k}\cdot\underline{\Omega}+r_a} , \tag{59}$$

where it is noted that $\underline{k}$ is the Fourier variable from here on, and s is the Laplace variable.

If we now transform equation (52) and let

$$b \equiv a_1 + D_1 k^2 , \tag{60}$$

we get:

$$V^{N\beta'}(\underline{k},\underline{X}',s) = \frac{\Delta' V^{N\gamma'}(\underline{k},\underline{X}')}{s(s+b)} . \tag{61}$$

Then going back to equation (59) and taking the inverse Laplace transformation, it can be shown that upon inversion the first term which is proportional to $[\frac{1}{s} - \frac{1}{s+ic\underline{k}\cdot\underline{\Omega}+r_a}]$ in equation (59) will not contribute to the power spectral density at observable frequencies and can therefore be neglected. This is because the first term in brackets gives something proportional to $\delta(\omega)$, and the second term will give something proportional to $\frac{1}{\omega^2+r_a^2}$ which is constant and small over the observable frequency range of $\Phi(\omega)$. Substituting (61) into the remaining expression for $V^{\gamma\beta'}(\underline{k},\underline{\Omega};\underline{X}';s)$, taking the inverse transforms, and using $V^{\gamma\beta'}(\underline{X},\underline{\Omega};\underline{X}';t=0) = 0$, we get

$$\frac{\partial V^{\gamma\beta'}(\underline{X},\underline{\Omega};\underline{X}';t)}{\partial t} = \frac{P_\gamma \Delta' e^{-bt}}{4\pi}\int_{R=0}^{\infty}\frac{dR}{c}\, e^{-R/\lambda_a}\, V^{N\gamma'}(\underline{X}-R\underline{\Omega},\underline{X}') . \tag{62}$$

We have neglected terms proportional to $e^{-r_a t}$ and b compared to r_a. We can also now make use of the solution for $V^{N\gamma'}(\underline{X}-R\underline{\Omega};\underline{X}',\underline{\Omega}')$ which can readily be found to be

$$V^{N\gamma'}(\underline{X}-R\underline{\Omega};\underline{X}',\underline{\Omega}) = \int_{R'=0}^{\infty} \frac{dR'}{c}\, e^{-R'/\lambda_a} \left\{ \frac{P'_\gamma}{4\pi}\, V^{NN'}(\underline{X}-R\underline{\Omega}';\underline{X}'-R'\underline{\Omega}') \right. \tag{63}$$

$$\left. - \delta(\underline{X}-R\underline{\Omega}-(\underline{X}'-R'\underline{\Omega}'))\, \frac{P'_\gamma}{4\pi}\, f_1^N(\underline{X}-R\underline{\Omega}) \right\} .$$

However the neutron variance and the neutron singlet density have been worked out in detail in the work of Natelson, Osborn, and Shure[10]. They found:

$$V^{NN'}(\underline{X}-R\underline{\Omega};\underline{X}'-R'\underline{\Omega}') = \sum_{n,m,i} c_{inm} \frac{\langle J(J-1)\rangle r_f A_i^N \psi_n(\underline{X}-R\underline{\Omega})\psi_m(\underline{X}'-R'\underline{\Omega}')}{2a_1+D_1(B_n^2+B_m^2)}$$

$$+ \delta(\underline{X}-R\underline{\Omega}-(\underline{X}'-R'\underline{\Omega}))f_1^N(\underline{X}-R\underline{\Omega}) \ , \tag{64}$$

where

$$c_{inm} = \int_{R.V.} d^3X \psi_i(\underline{X})\psi_n(\underline{X})\psi_m^+(\underline{X}) \tag{65}$$

and

$$f_1^N(\underline{X},t) = \sum_n^\infty A_n(t)\psi_n(\underline{X}) \ . \tag{66}$$

By using (62), (63), and (64) in (49) we get

$$\frac{\partial^2 V^{\beta\beta'}(t)}{\partial t^2} = \int_{(D.V.)_1} d^3X \int_{(D.V.)_2} d^3X' \left\{ \frac{P_\gamma P'_\gamma}{(4\pi)^2}\, \Delta\Delta'\, e^{-bt} \int_{R=0}^{\infty} \frac{dR}{c} \int_{R'=0}^{\infty} \frac{dR'}{c} \right. (x)$$

$$(x) e^{-R/\lambda_a} e^{-R'/\lambda_a} \left[2 \sum_{n,m,i} c_{inm} \frac{\langle J(J-1)\rangle r_f A_i^N \psi_n(\underline{X}-R\underline{\Omega})\psi_m(\underline{X}'-R'\underline{\Omega}')}{2a_1+D_1(B_n^2+B_m^2)} \right] H(t)$$

$$\left. + \delta(\underline{X}-\underline{X}')\, \frac{\Delta P_\gamma}{4\pi} \int_{R=0}^{\infty} \frac{dR}{c}\, e^{-R/\lambda_a} f_1^N(\underline{X}-R\underline{\Omega})\delta(t) \right\} . \tag{67}$$

To evaluate this expression further, consider a general case with detectors 1 and 2 placed outside the reactor core. The set of points $\{\underline{X}_1\}$ define the volume occupied by detector 1, and the set of points $\{\underline{X}_2\}$ correspondingly obtain for detector 2. Detectors 1 and 2 will be able to "see" photons from within some given volume, V_{s1} and V_{s2} respectively, of the reactor core.

If we let the set of directional vectors extending from points $\{\underline{X}_1\}$ into V_{s1} be denoted by $\{\underline{\Omega}_1\}$, the detection rate has the following property:

$$r_{D1}(\underline{X}) = r_{D1} \text{ for } \underline{X} \text{ in } \{\underline{X}_1\} \text{ and for photons along vectors } \{\underline{\Omega}_1\}$$
$$= 0 \qquad \text{otherwise} \tag{68}$$

If we consider the detection rate to be effectively constant over the photon energies of interest, we can write for detector 1, using (27):

$$\Delta_1 f^\lambda(\underline{X}) = \int d^3\kappa \; r_{D1}(\underline{X}) f_1^\lambda(\underline{X},\underline{\kappa}) \tag{69a}$$

or

$$\Delta_1 f^\lambda(\underline{X}) = r_{D1}(\underline{X}) \int_{\{\underline{\Omega}_1\}} d\Omega \; f^\lambda(\underline{X},\underline{\Omega}) \tag{69b}$$

and likewise for detector 2

$$\Delta_2 f^\lambda(\underline{X}') = r_{D2}(\underline{X}') \int_{\{\underline{\Omega}_2\}} d\Omega' \; f^\lambda(\underline{X}',\underline{\Omega}'). \tag{70}$$

From here on we let $r_{D1} = r_{D2} = r_D$. The power spectral density can then be written as:

$$\Phi(\omega) = \frac{1}{2} \int_{(D.V.)_1} d^3X \int_{(D.V.)_2} d^3X' \; \frac{r_D(\underline{X})\langle\eta\rangle r_f}{c} \left\{ \frac{2r_D(\underline{X}')\langle\eta\rangle r_f}{c} \int_{R=0}^{\infty} dR \; (x) \right.$$

$$\int_{\{\underline{\Omega}_1\}} d\Omega \; \frac{e^{-R/\lambda_a}}{4\pi} \int_{R'=0}^{\infty} dR' \int_{\{\underline{\Omega}_2\}} d\Omega' \; \frac{e^{-R'/\lambda_a}}{4\pi} \sum_{n,m,i} \frac{c_{inm}\langle J(J-1)\rangle r_f A_i^N \psi_n(\underline{X}-R\underline{\Omega})\psi_m(\underline{X}'-R'\underline{\Omega}')}{2a_1+D_1(B_n^2+B_m^2)} (x)$$

$$\left. (x) \; \frac{b}{\omega^2+b^2} + \delta(\underline{X}-\underline{X}') \int_{R=0}^{\infty} dR \int_{\{\underline{\Omega}_1\}} d\Omega \; \frac{e^{-R/\lambda_a}}{4\pi} \; f_1^N(\underline{X}-R\underline{\Omega}) \right\}. \tag{71}$$

SUMMARY AND CONCLUSIONS

It can first be observed that in an oversimplified model of a homogeneous infinite reactor and infinite detector model, in which the photon detector is assumed to be uniformly distributed over the entire reactor, equation (71) can be reduced to the following form:

$$\Phi(\omega) = \frac{(C.R.)_\gamma}{2}\left\{\frac{\alpha(C.R.)_\gamma}{P}\,\frac{<J(J-1)>k_\infty^2}{<J>^2(1-k_\infty)^2}\,\frac{a_1^2}{\omega^2+a_1^2}+1\right\}, \tag{72}$$

where

$$\alpha \equiv 3\text{x}10^{-11} \text{ watts per fission/sec} \tag{73}$$

$$P \equiv \text{reactor power, watts} \tag{74}$$

$$(C.R.)_\gamma \equiv \text{the count rate of gamma detection events.} \tag{75}$$

The corresponding expression for the power spectral density from a neutron detecting experiment is

$$\Phi(\omega) = \frac{(C.R.)_N}{2}\left\{\frac{\alpha(C.R.)_N}{P}\,\frac{<J(J-1)>k_\infty^2}{<J>^2(1-k_\infty)^2}\,\frac{a_1^2}{\omega^2+a_1^2}+1\right\}, \tag{76}$$

where $(C.R.)_N$ is the count rate of neutron detection events. Thus in this idealized case the degree of observability of information from power spectral density measurements by a photon detecting experiment and by a neutron detecting experiment on a given reactor operating at a given power will compare as the ratio of the count rates that can be obtained by each technique. It is clear that there is no difference in principle in the type of information that is available by these noise measurements.

Then going back to equation (71) which applies for more realistic situations, it is again found by comparison with the results of Natelson, Osborn, and Shure[10] for neutron detecting experiments that the same basic information relevant to dynamic reactor parameters is obtainable in principle from a photon detecting experiment. Of course the degree of observability is largely dependent upon the volume of the reactor core that the photon detector is capable of "seeing". This factor may be evaluated for individual cases by computing the photon streaming integrals over R and R', the "solid angle" integrals over $\underline{\Omega}$ and $\underline{\Omega}'$, and the integration of $\underline{X}$ and $\underline{X}'$ over detector volumes in equation (71). In general numerical techniques would be necessary for such a calculation. However strictly qualitative considerations indicate that counting rates of a photon detector outside the core will be maximized by positioning the photon detector such that there is not too large a thickness of moderator or some other material interposed between the detector and the reactor core. What we consider large here are thicknesses on the order of a mean free path of a photon in the interposed medium. The "solid angle factor" that is involved here may be maximized by positioning the largest practical detector as near the core as possible (and still avoid large perturbing effects on the neutron distribution as well as avoiding damage to the detector). That is, one simply wishes to maximize within practical limits both the solid angle subtended by the detector as "seen" by the reactor core and the solid angle subtended by the reactor core as "seen" by the detector.

Since neutron detectors "see" only that volume of the core which they physically occupy, whereas a photon detector may effectively "see" a much larger volume of the core (due to the longer mean free path of photons), it is conceivable that in some instances photon detecting noise experiments may be as efficient as neutron detecting experiments for determining information relevant to the reactor. It has been demonstrated that the same stochastic information pertinent to the system resides in the photon distribution of the reactor as in the neutron distribution.

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