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and Operational Efficiency

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Complexity-Based Triage: A Tool for Improving Patient Safety and Operational Efficiency

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Most hospital Emergency Departments (ED's) use triage systems that classify and prioritize patients almost exclusively on the basis of urgency. We propose a new triage system that incorporates patient complexity as well as urgency. Using a combination of analytic and simulation models, we demonstrate that complexity-based triage can substantially improve both patient safety (i.e., reduce risk of adverse events) and operational efficiency (i.e., shorten average length of stay). Our analysis indicates that ED's with a higher resource (physician and/or examination room) utilization, higher heterogeneity between the average treatment time of simple and complex patients, and a closer to equal split between simple and complex patients benefit more from the proposed complexity-based triage system. Finally, while misclassification of a complex patient as simple is slightly more harmful than vice versa, complexity-based triage is robust to misclassification error rates that are on the order of 5% to 25%.

Key words: Healthcare Operations Management; Emergency Department; Triage; Priority Queues; Patient prioritization.

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1. Introduction

Overcrowding and lapses in patient safety are prevalent problems in Emergency Departments (ED's) in the U.S. and around the world. In one study, 91% of U.S. ED's responding to a national survey reported that overcrowding was a problem, and almost 40% of them reported overcrowding as a daily occurrence (American Hospital Association (2002)). In addition to causing long wait times, many research studies have linked the delays due to overcrowding to elevated risks of errors and adverse events (see, e.g., Thomas et al. (2000), Gordon et al. (2001), and Trzeciak and Rivers (2003)). This situation prompted the Institute of Medicine's Committee on Future of Emergency Care in the United States Health System to recommend that *"hospital chief executive officers adopt enterprisewide operations management and related strategies to improve the quality and efficiency of emergency care"* (Institute of Medicine (2007)). The triage process is a natural place to introduce operations management (OM) into the ED.

Triage (a word derived from the French verb "trier," meaning "to sort") refers to the process of sorting and prioritizing patients for care. While all ED's perform triage, the processes and personnel used to do this vary considerably between hospitals. For instance, most ED's in Australia use the

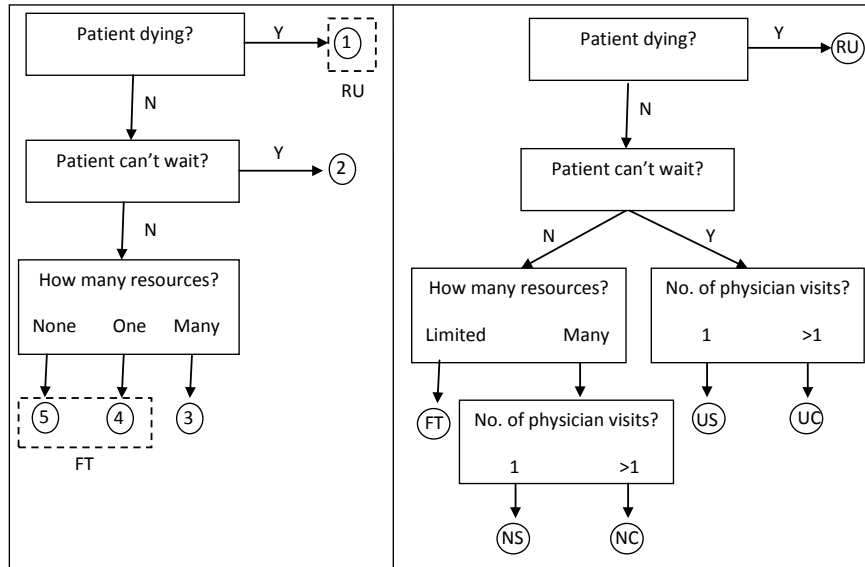


Figure 1 Left: Current practice of triage (Emergency Severity Index (ESI) algorithm version 4); Right: proposed complexity-based triage (RU: Resuscitation Unit, FT: Fast Track, NS: Non-urgent Simple, NC: Non-urgent Complex, US: Urgent Simple, UC: Urgent Complex).

Australasian Triage Scale (ATS), the Manchester Triage Scale (MTS) is prevalent in the U.K., and ED's in Canada generally use the Canadian Triage Acuity Scale (CTAS). While they differ in their details, all of these systems classify patients strictly in terms of urgency. In the U.S., many ED's continue to use a traditional 3-level triage scale based on urgency (e.g., emergent, urgent and non-urgent). But U.S. hospitals are increasingly adopting the 5-level Emergency Severity Index (ESI) system (see Fernandes et al. (2005)), which combines urgency with an estimate of resources (e.g., tests) required. In the ESI system, urgent patients who cannot wait are classified as ESI-1 and 2, while non-urgent patients who can wait are classified as ESI-3, 4, and 5, as shown in Figure 1 (left). ESI-4 and 5 patients are usually directed to a fast track (FT) area, while ESI-1 patients are immediately moved to a resuscitation unit (RU). ESI-2 and 3 patients are served in the main area of the ED. Figure 1 (right) captures our proposed complexity-based triage system, which will be discussed and analyzed in this paper.

The emphasis on urgency in triage systems is understandable, given the critical role of speed in life-and-death situations. The Center for Disease Control and Prevention (CDC) estimates 379,000 deaths occurred in U.S. ED's in 2000 (McCaig and Ly (2002)). A portion of these deaths in ED's can be attributed to long waiting times and ineffective triage. For example, on June 19, 2008, a 49-year-old woman named Esmin Green died in the waiting room of an ED in Brooklyn, New York after waiting for over 24 hours (Zhou (2010)). A 70-year-old woman, Mary Tate, who had a history of cardiac disease, died in the waiting area of a St. Louis emergency room on May 17, 2001, after hospital staff ignored her son's cries for help (Zhou (2010)). In 2006, 49-year-old Beatrice Vance

died in the waiting room of the ED of Vista Medical Center East in Waukegan, IL, after she was triaged as non-urgent (SoRelle (2006)). In 2009, 63-year-old Latino musician Joaquin Rivera died of a heart attack while waiting in an ED in Philadelphia (Zhou (2010)). Cases like these illustrate the wrenching human consequences of the research conclusion that ED delays are dangerous to patient safety (see, e.g., Trzeciak and Rivers (2003)).

Despite the importance of recognizing urgency, this is not the only role of triage. FitzGerald et al. (2010) argue that there are two main purposes for triage: “[1] to ensure that the patient receives the level and quality of care appropriate to clinical need (clinical justice) and [2] that departmental resources are most usefully applied (efficiency) to this end.” (see Moskop and Ierson (2007) for further discussion of the underlying principles and goals of triage). While current urgency-based systems address the first (clinical justice) purpose of triage, the second (efficiency) purpose has been largely overlooked. Only the ESI system, which is often used to send low acuity patients (ESI-4 and 5) patients to a fast track, makes some attempt to address resource usage via triage. All other systems consider only patient urgency. But even the 5-level ESI system does not help with effective allocation of resources to patients once they have been assigned to tracks.

Developing a triage system that achieves both the clinical justice and the resource efficiency objectives poses two challenges: (a) deciding what information should be collected at the time of triage, and (b) determining how this information should be used to assign patients to tracks and prioritize them within tracks. Saghafian et al. (2010) proposed that ED’s can improve performance by asking triage nurses to predict the final disposition (admit or discharge) of patients in addition to assigning an ESI level. Assigning patients to separate admit and discharge streams can reduce average time to first treatment for admit patients and average length of stay for discharge patients. But this research also indicated that the performance of the streaming policy improves as the difference between the average treatment times of admit and discharge patients becomes larger. This suggests that classifying patients according to complexity may be even more useful than classifying them according to ultimate disposition.

There is ample evidence from the OM literature that giving priority to patients with shorter service times (e.g., by following a Shortest Processing Time (SPT) priority rule) can indeed result in better use of resources, and thereby reduce the average waiting time among all patients. Furthermore, empirical studies from the emergency medicine literature suggest that patients can be effectively classified by complexity at the time of triage. Specifically, Vance and Spirvulis (2005) defined complex patients as those requiring at least two procedures, investigations, or consultations and concluded that “*Triage nurses are able to make valid and reliable estimates of patient complexity. This information might be used to guide ED work flow and ED casemix system analysis.*” Using the number of (treatment related) interactions with the physician (which correlates

directly with expected treatment duration) as an indicator of patient complexity, we propose and investigate the benefit of the new complexity-based triage process depicted in Figure 1 (right).

To evaluate the effectiveness of complexity-based triage with respect to both clinical justice and efficiency, we consider ED performance in terms of both risk of adverse events (capturing patient safety) and average length of stay (capturing operational efficiency). Specifically, we make use of a combination of analytic and simulation models to examine the following:

- 1. Prioritization:** *How should ED's use complexity-based triage information to prioritize patients?*
- 2. Magnitude:** *How much benefit does complexity-based triage (which also includes urgency) offer relative to urgency-based triage?*
- 3. Sensitivity:** *How sensitive are the benefits of complexity-based triage to misclassification errors and other characteristics that may vary across ED's?*

The remainder of the paper is organized as follows. Section 2 summarizes previous OM and medical research relevant to the above questions. Section 3 describes our performance metrics and analytical modeling approach. For modeling purposes, we divide the patient experience in the ED into Phase 1 (from arrival until assignment to an examination room) and Phase 2 (from assignment to an examination room until discharge/admission to the hospital). Section 4 focuses on Phase 1 and uses analytical queueing models to compare performance under urgency-based and complexity-based triage systems. Section 5 considers Phase 2 by developing and analyzing a Markov Decision Process model. Section 6 uses a high-fidelity simulation model of the full ED to validate the insights obtained through our analytical models and to refine our estimates of the magnitude of performance improvement possible with complexity-based triage. We conclude in Section 7.

2. Literature Review

In this section, we review studies related to our work from both the operations research/management literature and the medical literature.

2.1. Operations Research/Management Studies

The effect of assigning priorities in queueing systems has been studied in the operations research literature for a long time. One of the first works to rigorously analyze such systems under perfect classification was Cobham (1954). Assuming perfect customer classification, Cobham (1954) and Cobham (1955) showed that the expected waiting time among all customers can be reduced by assigning priorities. van der Zee and Theil (1961) extended Cobham's results to the case with

imperfect classification for a two-priority single-channel system. They recommended creating a “mixed” group for customers who cannot be classified with certainty into either group 1 or 2, and assigning priorities probabilistically within this group. Further analysis of priority queueing systems can be found in Cox and Smith (1961), Jaiswal (1968), and Wolf (1989).

Under an average holding cost objective, perfect classification, Poisson arrivals, and a non-preemptive non-idling single server model, Cox and Smith (1961) used an interchange argument to show that the well-known $c\mu$ rule is optimal among priority rules. Kakalik and Little (1971) extended this result and used a semi-Markov decision process to show that the $c\mu$ rule remains optimal even among the larger class of state-dependent policies with or without the option of idling the server. The $c\mu$ rule has been shown to be optimal in many other queueing frameworks; see, e.g., Buyukkoc et al. (1985), Van Mieghem (1995), Veatch (2010), Saghafian et al. (2011), and references therein.

Research related to our work that analyzes the performance of ED’s from an operations perspective is not vast. Saghafian et al. (2010) considered streaming of ED patients based on triage estimations of the final disposition (admit or discharge) and found that an appropriate “virtual streaming” policy can improve performance with respect to the operational characteristics of average length of stay and time to first treatment. Siddharathan et al. (1996) considered the impact of non-emergency patients on ED delays using urgency-based triage, and proposed a simple priority queueing model to reduce average waiting times. Wang (2004) considered a queue of heterogeneous high risk patients, for which treatment times are exponential, and patient classification is perfect, and concluded that patients should be prioritized into as many urgency classes as possible in order to maximize survival. Argon and Ziya (2009) used the average waiting time as the performance metric in a service system with two classes of customers, in which customer classification is imperfect, and showed that prioritizing customers according to the probability of being from the class that should have a higher priority when classification is perfect outperforms any finite-class priority policy.

Although the above studies suggest that separating patients according to complexity can reduce waiting times through a better resource allocation, we note that they (a) lack insights into clinical justice/safety issues that are vital in ED’s, and (b) are limited to simple/stylized queueing models with features (e.g., one-stage service, fixed number of customers all available at time zero or time stationary arrivals, availability of the customers at any time during service, no bound on the number of customers that can be assigned to a server, no change in the conditions of customers after they begin service, perfect customer classification, etc.) that do not capture the reality of ED’s. In this paper, we seek to address both safety and efficiency, and to account for the key features that define ED environments.

2.2. Medical Studies

Our research approach was informed by empirical studies of ED's and triage processes. Gilboy et al. (2005), FitzGerald et al. (2010), and Ierson and Moskop (2007) provide good reviews of the history of the triage process and its development over time. Most studies attribute the first formal battlefield triage system to the distinguished French military surgeon Baron Dominique-Jean Larrey who recognized a need to evaluate and categorize wounded soldiers. He recommended treating and evacuating those requiring the most urgent medical attention, rather than waiting hours or days for the battle to end before treating patients, as had been done in previous wars (Ierson and Moskop (2007)). Since that time, triage in medicine has been mainly based on urgency. However, the idea of considering the complexity of patients goes back to World War I triage recommendations: "*A single case, even if it urgently requires attention, –if this will absorb a long time,– may have to wait, for in that same time a dozen others, almost equally exigent, but requiring less time, might be cared for. The greatest good of the greatest number must be the rule.*" (Keen (1917)). The ESI triage system shown in Figure 1 (left) is the most serious effort to date at introducing complexity into the triage process. However, because (a) the number of resources required does not necessarily correlate with the physician time required by the patient, and (b) ED's do not use ESI information in a consistent manner to prioritize patients, the ESI system falls short of true complexity-based triage.

Looking forward to more direct complexity-based triage, Vance and Spirvulis (2005) found that triage nurses are able to make reliable estimates of patient complexity, and suggested that this type of information could be used to improve patient flow in ED's (although they did not specify how). Another way to collect patient complexity data is to use physicians at triage. However, Han et al. (2010) and Russ et al. (2010) empirically investigated the benefit of putting a physician at triage (under the current practice of triage) and found that it is not an effective method for reducing total length of stay, although it may reduce the average time spent in an ED bed.

Finally, several studies have been published in medical journals that aim at investigating and/or validating the ESI triage system which are relevant to our work. For this stream of research, we refer interested readers to Fernandes et al. (2005), which summarized the findings and recommendations of a task force from the American College of Emergency Physicians (ACEP) and the Emergency Nurses Association (ENA) appointed in 2003 to analyze the literature and make recommendations regarding use of 5-level triage systems in the United States. While this committee found the 5-level ESI system to be a good option compared to other available methods, they encouraged further in-depth research for improving the triage system.

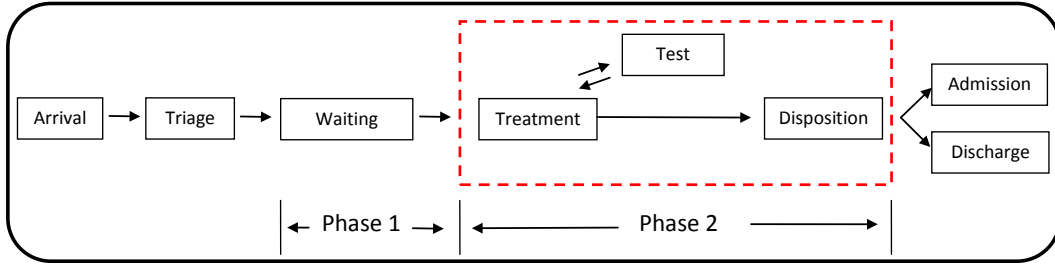


Figure 2 General flow of patients in the main ED.

3. Modeling the ED

To answer the three questions we posed in Section 1 (i.e., prioritization, magnitude, and sensitivity), we need to model patient flow through the ED. A high level schematic of this flow is presented in Figure 2. A patient’s path through the ED begins with *arrival*, which occurs in a non-stationary stochastic manner. After arrival, the patient goes to the *triage*, where s/he is classified according to a predefined process (based on urgency and/or complexity), which inevitably involves some misclassification errors. If an examination room is not immediately available, s/he goes to the ED *waiting* area until s/he is called and brought to an examination room. There s/he goes through some stochastic number of interactions with a physician. The physician-patient interactions, which we label as *treatment* stages, are stochastic in duration. These treatment stages are followed by testing (MRI, CT scan, etc.) or preparation/processing activities that do not involve the physician. We refer to these activities collectively as *test* stages; during them, the patient is unavailable to the physician. The final processing stage after the last physician interaction is *disposition*, in which the patient is either *discharged* to go home or *admitted* to the hospital.

We refer to the time a patient spends after s/he is triaged and before s/he is stabilized in an examination room as “Phase 1,” and label the remainder of time in the ED as “Phase 2.” Because they are under observation and care, patients have a lower risk of adverse events during Phase 2 than during Phase 1. Patients are taken from Phase 1 to Phase 2 by a charge nurse based on a Phase 1 priority rule that can make use of the patient classification performed at triage. Similarly, in Phase 2, physicians can use a priority rule to choose which patient to see next.

To gain insights into appropriate triage and priority rules, we first focus on the risk of adverse events and average waiting times in Phase 1 by considering the dashed area in Figure 2 (i.e., Phase 2) as a single-stage service node with a single super server. Since ED’s rarely send a patient back to the waiting area of Phase 1 once s/he has begun service, we assume a non-preemptive service protocol. We also approximate the non-stationary arrival process by a stationary Poisson process. These simplifications allow us to gain insights into suitable Phase 1 priority rules using a multi-class non-preemptive priority M/G/1 queueing model. We refer to this model as the *simplified single-stage ED model*.

After this, we focus on the risk of adverse events and average waiting times in Phase 2. To do this, we note that physicians can preempt their current interaction with a patient to visit another patient with a higher priority (e.g., a severely acute patient), and hence, we allow for preemption in Phase 2. Again approximating arrivals with a stationary Poisson process arrival stream, we can represent the multi-stage service process in Phase 2 as a Markov Decision Process model, which we label the *simplified multi-stage ED model*. We use this model to get insights into appropriate Phase 2 priority rules that physicians can implement when choosing their next patient. Finally, we test the insights from both analytic models under realistic conditions with a high fidelity simulation model of the full ED calibrated with a year of data from University of Michigan Hospital ED as well as time study data from the literature.

4. Phase 1: A Simplified Single-Stage ED Model

To formalize the Phase 1 sequencing problem, we define a patient to be of type ij if his/her urgency level is $i \in \mathcal{U}$ and his/her complexity type is $j \in \mathcal{C}$, where $\mathcal{U} = \{U(\text{Urgent}), N(\text{Non-urgent})\}$ and $\mathcal{C} = \{C(\text{Complex}), S(\text{simple})\}$. We suppose patients of type $ij \in \mathcal{U} \times \mathcal{C}$ arrive according to a Poisson process with rate λ_{ij} and have service times (i.e., total time spent in Phase 2) that follow a distribution, $F_{ij}(s)$ with first moment $1/\mu_{ij}$ (where $\mu_{iC} \leq \mu_{iS}$ for all $i \in \mathcal{U}$) and a finite second moment. We assume patients of type ij are subject to adverse events which occur according to a Poisson process with rate θ_{ij} , where $\theta_{Uj} \geq \theta_{Nj}$ for all $j \in \mathcal{C}$. Notice that since adverse events only rarely result in death, we assume that the process continues, so that it is possible for a patient to experience more than one adverse event.

Assuming $R_\pi^\Omega(t)$ represents the counting process that, under patient classification (i.e., triage) policy Ω and priority rule π , counts the total number of adverse events (for all patients) until time t , we consider $R_\pi^\Omega = \lim_{t \rightarrow \infty} R_\pi^\Omega(t)/t$ (when the limit exists) as our metric and refer to it as the *risk of adverse events (ROAE)*. However, if $\theta_{ij} = 1$ for all $i \in \mathcal{U}$ and $j \in \mathcal{C}$, then $R_\pi^\Omega / \sum_{i \in \mathcal{U}} \sum_{j \in \mathcal{C}} \lambda_{ij}$ represents the average *length of stay (LOS)*. (Notice that the sample path costs of LOS and adverse events with unit risk rates divided by total arrival rate will be different, but they are equal in expectation.) Hence, we can use our metric to characterize performance with respect to both safety and efficiency.

4.1. Urgency-Based Triage - Phase 1

We first consider current practice in most ED's, in which patients are classified solely based on urgency and use our simplified single-stage model to focus on Phase 1 sequencing decisions. We start with the case where classification based on urgency level is perfect and then consider the case where such classification is subject to errors.

When patients can be perfectly classified as either urgent (U) or non-urgent (N), the arrival rates for U's and N's are $\lambda_U = \sum_{j \in \mathcal{C}} \lambda_{Uj}$ and $\lambda_N = \sum_{j \in \mathcal{C}} \lambda_{Nj}$, respectively. Similarly, the average service times for U's and N's are $1/\mu_U = \sum_{j \in \mathcal{C}} (\lambda_{Uj}/\lambda_U)(1/\mu_{Uj})$ and $1/\mu_N = \sum_{j \in \mathcal{C}} (\lambda_{Nj}/\lambda_N)(1/\mu_{Nj})$, respectively. Furthermore, from known results for the performance of non-preemptive priority queues (see, Cobham (1954), van der Zee and Theil (1961), Section 3.3 of Cox and Smith (1961), or Section 10.2 of Wolf (1989)), the average waiting (queue) time of the k th priority class is

$$W_k = \frac{\lambda \mathbb{E}(s^2)}{2(1 - \sum_{l < k} \rho_l)(1 - \sum_{l \leq k} \rho_l)}, \quad (1)$$

where $\rho_l = \lambda_l/\mu_l$ for class l . Hence, if U's are prioritized over N's, then the average waiting time is $W_U = \lambda \mathbb{E}(s^2)/2(1 - \rho_U)$ for U's and $W_N = \lambda \mathbb{E}(s^2)/2(1 - \rho_U)(1 - \rho)$ for N's. Furthermore, the average rate of adverse events for U's is $\theta_U = (\lambda_{US}/\lambda_U)\theta_{US} + (\lambda_{UC}/\lambda_U)\theta_{UC}$ and for N's is $\theta_N = (\lambda_{NS}/\lambda_N)\theta_{NS} + (\lambda_{NC}/\lambda_N)\theta_{NC}$. With these, the ROAE under an urgency-based triage policy (i.e., patient classification with respect to set \mathcal{U}) that gives priority to U's is

$$R_U^{\mathcal{U}} = \theta_U \lambda_U (\lambda \mathbb{E}(s^2)/2(1 - \rho_U)) + \theta_N \lambda_N (\lambda \mathbb{E}(s^2)/2(1 - \rho_U)(1 - \rho)). \quad (2)$$

Similarly, we can obtain the ROAE under an urgency-based triage policy that gives priority to N's:

$$R_N^{\mathcal{U}} = \theta_N \lambda_N (\lambda \mathbb{E}(s^2)/2(1 - \rho_N)) + \theta_U \lambda_U (\lambda \mathbb{E}(s^2)/2(1 - \rho_N)(1 - \rho)). \quad (3)$$

Comparing these reveals that, without misclassification errors, the best priority rule is to prioritize U's (N's) if, and only if, $\theta_U \mu_U \geq (\leq) \theta_N \mu_N$. Given the criteria used to classify a patient as urgent, we expect θ_U and θ_N be such that $\theta_U \mu_U > \theta_N \mu_N$, meaning that U's will be given priority.

We now introduce the possibility of misclassification errors into our model of urgency-based triage and prioritization. Let γ_U and γ_N denote the misclassification probabilities for urgent and non-urgent patients, respectively. The arrival rates for patients classified (correctly or erroneously) as U and N are $\lambda'_U = \lambda_U(1 - \gamma_U) + \lambda_N \gamma_N$ and $\lambda'_N = \lambda_N(1 - \gamma_N) + \lambda_U \gamma_U$, respectively. Similarly, the mean service times for patients classified as U and N are $1/\mu'_U = [\lambda_U(1 - \gamma_U)(1/\mu_U) + \lambda_N \gamma_N(1/\mu_N)]/\lambda'_U$ and $1/\mu'_N = [\lambda_N(1 - \gamma_N)(1/\mu_N) + \lambda_U \gamma_U(1/\mu_U)]/\lambda'_N$, respectively. Finally, the ROAE for patients classified as U and N are $\theta'_U = [\lambda_U(1 - \gamma_U)\theta_U + \lambda_N \gamma_N \theta_N]/\lambda'_U$ and $\theta'_N = [\lambda_N(1 - \gamma_N)\theta_N + \lambda_U \gamma_U \theta_U]/\lambda'_N$, respectively.

Using (2) with these new "error impacted" rates shows that when priority is given to U's, the ROAE under imperfect classification is:

$$R_U^{\mathcal{U}'} = \theta'_U \lambda'_U (\lambda \mathbb{E}(s^2)/2(1 - \rho'_U)) + \theta'_N \lambda'_N (\lambda \mathbb{E}(s^2)/2(1 - \rho'_U)(1 - \rho)), \quad (4)$$

where $\rho'_U = \lambda'_U/\mu'_U$. Similarly, using (3) shows that when priority is given to N's:

$$R_N^{\mathcal{U}'} = \theta'_N \lambda'_N (\lambda \mathbb{E}(s^2)/2(1 - \rho'_N)) + \theta'_U \lambda'_U (\lambda \mathbb{E}(s^2)/2(1 - \rho'_N)(1 - \rho)), \quad (5)$$

where $\rho'_N = \lambda'_N / \mu'_N$.

The above results enable us to state:

PROPOSITION 1 (Phase 1 Prioritization - Urgency-Based Triage). *In the simplified single-stage ED model with imperfect urgency-based classification:*

(i) *The best priority rule is to prioritize U patients if $\theta'_U \mu'_U \geq \theta'_N \mu'_N$; otherwise, prioritize N patients.*

(ii) *The best priority rule is the same as that for the case without misclassification error if $\gamma_N + \gamma_U \leq 1$; otherwise, the best priority ordering is reversed.*

Empirical studies have observed misclassification levels γ_N and γ_U to be in the range 9-15% depending on the level of triage nurse experience (Hay et al. (2001)). Thus, if, as we expect, prioritizing urgent patients is optimal when there is no misclassification error, the above proposition implies that doing so remains optimal even under realistic levels of misclassification errors.

4.2. Complexity-Based Triage - Phase 1

We now consider the complexity-based triage policy shown in Figure 1 (right), and compare its performance with respect to that of urgency-based triage. By doing this we seek to gain insights into the prioritization, magnitude, and sensitivity questions posed in the Introduction.

To evaluate the performance of complexity-based triage when classification is imperfect, we let γ_U and γ_N denote the misclassification error rates with respect to set \mathbf{U} . That is, γ_U and γ_N denote the probabilities of classifying a U patient as an N , and an N patient as a U , respectively. Similarly, let γ_C and γ_S denote the misclassification error rates with respect to set \mathbf{C} ; γ_C denotes the probability that a C patient is classified as an S , and γ_S denotes the probability that an S patient is classified as a C . We assume the misclassification probabilities with respect to sets \mathbf{U} and \mathbf{C} are independent. As noted earlier, misclassification error rates in terms of urgency have been observed to be in the range of 9-15% (Hay et al. (2001)). Vance and Spirvulis (2005) have tested the ability of triage nurses to evaluate patient complexity (where complexity is defined as requiring two or more procedures, investigations, or consultation) and observed a misclassification rate of 17%.

Similar to what we did in Section 4.1, we need to calculate the error impacted rates λ'_{ij} , θ'_{ij} , and μ'_{ij} . Let $\underline{\lambda} = (\lambda_{US}, \lambda_{UC}, \lambda_{NS}, \lambda_{NC})$ and $\underline{\lambda}' = (\lambda'_{US}, \lambda'_{UC}, \lambda'_{NS}, \lambda'_{NC})$. Then $\underline{\lambda}'$ can be obtained through a linear transformation of $\underline{\lambda}$; $\underline{\lambda}'^T = A \underline{\lambda}^T$, where A is a (known) *misclassification error matrix*, and is defined as

$$A = \begin{pmatrix} (1 - \gamma_U)(1 - \gamma_S) & (1 - \gamma_U)\gamma_C & \gamma_N(1 - \gamma_S) & \gamma_N\gamma_C \\ (1 - \gamma_U)\gamma_S & (1 - \gamma_U)(1 - \gamma_C) & \gamma_N\gamma_S & \gamma_N(1 - \gamma_C) \\ \gamma_U(1 - \gamma_S) & \gamma_U\gamma_C & (1 - \gamma_N)(1 - \gamma_S) & (1 - \gamma_N)\gamma_C \\ \gamma_U\gamma_S & \gamma_U(1 - \gamma_C) & (1 - \gamma_N)\gamma_S & (1 - \gamma_N)(1 - \gamma_C) \end{pmatrix}. \quad (6)$$

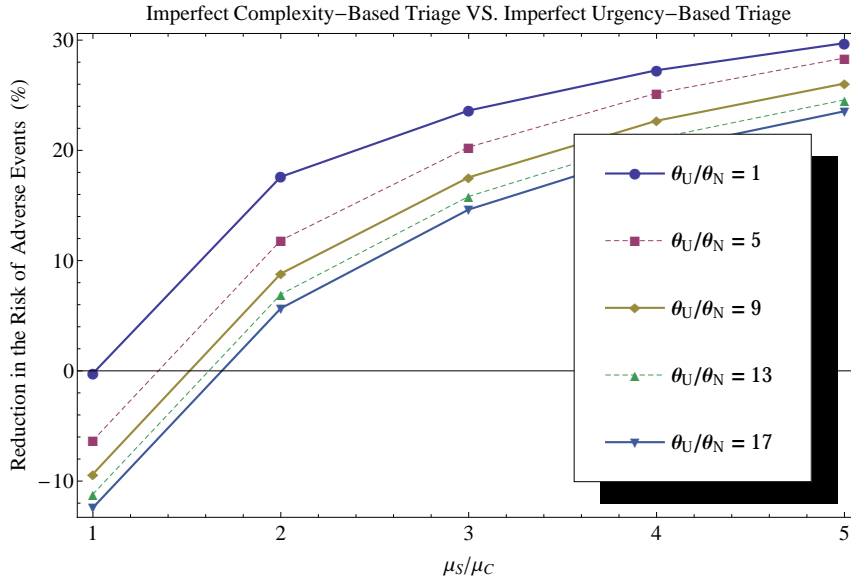


Figure 3 Benefit of complexity-based triage over urgency-based triage with practical misclassification rates ($\gamma_U = \gamma_N = 10\%$, $\gamma_S = \gamma_C = 17\%$).

Similarly, if $\underline{\theta}'$ and $\underline{\mu}'$ denote the vector of error impacted adverse event and service rates, we have $\underline{\theta}'^T = (A(\underline{\lambda} \times \underline{\theta})^T)/\underline{\lambda}'$ and $(\underline{1}/\underline{\mu}')^T = (A(\underline{\lambda}/\underline{\mu})^T)/\underline{\lambda}'$, where $\underline{1} = (1, 1, 1, 1)$ and operators “ \times ” and “/” are componentwise multiplier and division, respectively.

With these, the waiting times for each customer class under an imperfect $\mathcal{U} \cup \mathcal{C}$ classification can be computed using (1) with rates replaced with their error impacted counterparts. This model permits us to show the following.

PROPOSITION 2 (Phase 1 Prioritization - Complexity-Based Triage). *In the simplified single-stage ED model with imperfect urgency and complexity classifications:*

- (i) *The best priority rule is to prioritize patients in decreasing order of $\theta'_{ij} \mu'_{ij}$ values.*
- (ii) *$R_*^{\mathcal{U}' \cup \mathcal{C}'} \leq R_*^{\mathcal{U}'}$. That is, even with misclassification errors, implementing the best priority rule for complexity-based triage is always (weakly) better than the optimal priority rule for urgency-based triage.*
- (iii) *The best priority rule of part (i) is optimal even among the larger class of all non-anticipative (state or history dependent, idling or non-idling, etc.) policies.*

Proposition 2 (i) addresses the prioritization question by suggesting a simple way (a modified version of the well-known “ $c\mu$ ” rule) to incorporate complexity information into Phase 1 sequencing. Proposition 2 (ii) begins to address the magnitude question by suggesting that complexity-based triage outperforms urgency-based triage. With errors of 50%, the two are equal but with the indices computed with the errors taken into account, the complexity information is useful. While priority

rules are greedy and usually suboptimal, part (iii) confirms that they are optimal in this setting. The surprise is that it is never optimal to idle when only low priority patients are available, even though the system is working under a non-preemptive setting.

Figure 3 provides additional insights into the magnitude question by illustrating the amount of improvement for a numerical example with $\mu_{UC} = \mu_{NC} = \mu_C = 1$, $\mu_{US} = \mu_{NS} = \mu_S$ varying from 1 to 5, $\lambda_{US} = (1/5)\mu_{US}$, $\lambda_{UC} = 1/4$, $\lambda_{NS} = (1/3)\mu_{NS}$, $\lambda_{NC} = 1/6$, $\mathbb{E}(s^2) = 4$, $\theta_{NS} = \theta_{NC} = \theta_N = 1$, $\theta_{US} = \theta_{UC} = \theta_U$. Note that (1) the amount of improvement is depicted both in terms of average length of stay and risk of adverse events (since when $\theta_U/\theta_N = 1$, the percentage improvement in risk of adverse events and length of stay are equal), and (2) reduction in the length of stay results in reduction in congestion (by Little's Law), which can serve as a potent remedy for the prevalently observed phenomenon of ED overcrowding. Figure 3 suggests that, if the average service time of complex patients is about 3 – 4 times larger than that of simple patients, then complexity-based triage can reduce the risk of adverse events (ROAE) and average length of stay (LOS) by about 15 – 20% and 21 – 25%, respectively. Finally, we can address the sensitivity question by using our model to determine the environmental factors that favor complexity-based triage.

PROPOSITION 3 (Attractiveness of Complexity-Based Triage). *Under the simplified single-stage ED model, complexity-based triage is more beneficial in ED's with (i) higher utilization, (ii) higher heterogeneity in the average service time of simple and complex patients, (iii) a more equal fraction of simple and complex patients, and (iv) lower error rates in classifying simple and complex patients.*

5. Phase 2: A Multi-Stage ED Model

The analysis of the previous section was limited to patient waiting and risk of adverse events prior to entry into an examination room. But, as illustrated in Figure 4, a great deal of ED activity takes place after this point, which contributes to both patient length of stay and risk of adverse events. Since triage classification can be used to sequence patients within the ED, as well as in the waiting room, it is important to consider Phase 2 sequencing as part of our evaluation of complexity-based triage.

To do this, consider the multi-stage service process illustrated in Figure 4 and suppose patients of class $ij \in \mathcal{U} \times \mathcal{C}$ arrive according to a Poisson process with rate λ_{ij} . Further, suppose the rate of adverse events in Phase 2 is denoted by the vector $\hat{\theta} = (\hat{\theta}_{ij})_{ij \in \mathcal{U} \times \mathcal{C}}$ (which is usually less than the risk of adverse events in Phase 1, θ , because patients are monitored and treated in the examination rooms). As they enter examination rooms, patients are assigned to physicians who treat them, often with multiple visits, until their discharge or admission to the hospital. Since an individual physician may be assigned to several patients s/he often has a choice about who to see next

among his/her available patients. (We call patients who have completed tests and have results and are ready for a physician visit “available,” and patients being tested, prepared, or waiting for results “unavailable.”) Suppose each interaction with a patient of class ij takes an exponentially distributed amount of time with rate $\hat{\mu}_{ij}$ and assume (for tractability) that the physician can preempt an interaction to see a patient of a different class. When a physician returns to a preempted interaction, we assume s/he must repeat the process (e.g., review vital signs, lab results, etc.), and so we assume a preempt-repeat protocol.¹

After each completed interaction, a patient of class ij may be disposed (discharged home or admitted to the hospital) with probability $p_{ij} > 0$, or with probability $1 - p_{ij}$ requires another round of test and treatment. We model the test time, which includes any preparation and wait times associated with the test, as a $\cdot/M/\infty$ queueing system with average service time of η^{-1} . Because we aggregate test times, waiting times for the test results, and preparations for tests into a single “test” stage, and also aggregate these for all possible types of tests, the long-run average time spent for a generic “test,” denoted by η^{-1} , can be assumed to be roughly similar among different patient classes (for more detailed data on test turnaround times see Steindel and Howanitz (1997) and Holland et al. (2005)).

To model the physician decision of who to see next, we let $\underline{x} = (x_{ij})_{ij \in \mathbf{u} \times \mathbf{c}}$ (respectively $\underline{y} = (y_{ij})_{ij \in \mathbf{u} \times \mathbf{c}}$) represent the number of patients of each class available (not available) for a physician visit. With these, we can define the state of the system at any point of time, t , by the vector $(\underline{x}(t), \underline{y}(t)) \in \mathbb{Z}_+^4 \times \mathbb{Z}_+^4$, and model the process $\{(\underline{x}(t), \underline{y}(t)) : t \geq 0\}$ as a Continuous Time Markov Chain (CTMC). We assume the parameters of the system are such that this CTMC is stabilizable; i.e., there exists at list one policy under which the risk of adverse events is finite (otherwise, the problem does not represent a real ED). However, notice that since the transition rates are not bounded, we cannot use uniformization in the spirit of Lippman (1975) to formulate a discrete time equivalent of the CTMC where the times between consecutive events are i.i.d. (for all states). However, in what follows, we consider a sequence of CTMC’s with an increasing but bounded sequence of (maximum) transition rates converging to the original CTMC. We do this by replacing the $\cdot/M/\infty$ test stage with four parallel $\cdot/M/k$ systems (one devoted to each patient class), index the underlying CTMC with k , and let $k \rightarrow \infty$. The advantage of having four parallel $\cdot/M/k$ queues (instead of one $\cdot/M/k$) is that the order of jobs in each queue becomes irrelevant, and hence, does not need to be captured in the system’s state. Another novel aspect of our approach is that we truncate the transition rates as opposed to the state space, thereby avoiding the artificial boundary

¹ In practice, emergency physicians can, and sometimes do, preempt patients to deal with emergencies. But for fairness and efficiency reasons, they do this rarely. Hence, we test our conclusions under the assumption of non-preemption in Phase 2 in Section 6 using a realistic simulation model.

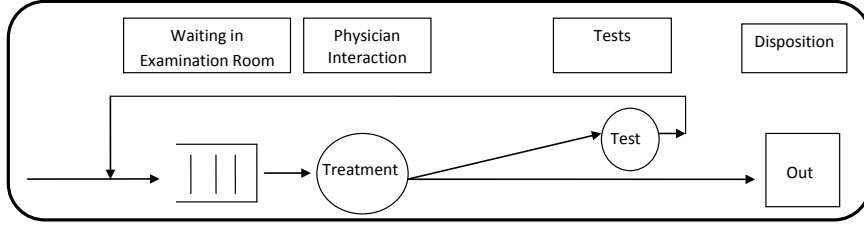


Figure 4 Patient flow after a patient is moved to an examination room/bed (Phase 2 sequencing).

effects that usually impact the policy. Since the transition rates in the CTMC indexed by k (for all k) are bounded by $\psi_k = \max_{ij \in \mathbf{U} \times \mathbf{C}} \hat{\mu}_{ij} + 4k\eta + \sum_{ij \in \mathbf{U} \times \mathbf{C}} \lambda_{ij} < \infty$, we can use the standard uniformization technique to derive the optimal policy for each CTMC. We then use a convergence argument (taking the limit as $k \rightarrow \infty$) to derive the optimal policy for the original problem. It should be noted that we can always start with a sufficiently large k such that the stability of the underlying system is not affected (since the original system is stable by assumption).

For the system indexed by k , the optimal rate of adverse events, R^{k*} , and the optimal physician behavior can be derived from the following average cost optimality equation:

$$\begin{aligned}
 J^k(\underline{x}, \underline{y}) + R^{k*} = & \frac{1}{\psi_k} \left[\hat{\theta}(\underline{x} + \underline{y})^T + \sum_{ij \in \mathbf{U} \times \mathbf{C}} [\lambda_{ij} J^k(\underline{x} + \underline{e}_{ij}, \underline{y}) + (y_{ij} \wedge k) \eta J^k(\underline{x} + \underline{e}_{ij}, \underline{y} - \underline{e}_{ij})] \right. \\
 & + \min_{a \in \mathcal{A}(\underline{x})} \left\{ \sum_{ij \in \mathbf{U} \times \mathbf{C}} \mathbb{1}_{\{a=ij\}} \hat{\mu}_{ij} [p_{ij} J^k(\underline{x} - \underline{e}_{ij}, \underline{y}) + (1 - p_{ij}) J^k(\underline{x} - \underline{e}_{ij}, \underline{y} + \underline{e}_{ij})] \right. \\
 & \left. \left. + (\psi_k - \sum_{ij \in \mathbf{U} \times \mathbf{C}} [\lambda_{ij} + (y_{ij} \wedge k) \eta + \mathbb{1}_{\{a=ij\}} \hat{\mu}_{ij}]) J^k(\underline{x}, \underline{y}) \right\} \right], \quad (7)
 \end{aligned}$$

where $J^k(\underline{x}, \underline{y})$ is a relative cost function (defined as the difference between the total expected cost of starting from state $(\underline{x}, \underline{y})$ and that from an arbitrary state such as $(\underline{0}, \underline{0})$), $a \wedge b = \min\{a, b\}$, \underline{e}_{ij} is a vector with the same size as \underline{x} with a 1 in position ij and zeroes elsewhere, a is an action determining which patient class to serve, and $\mathcal{A}(\underline{x}) = \{ij \in \mathbf{U} \times \mathbf{C} : x_{ij} > 0\} \cup \{0\}$ is the set of feasible actions (class 0 represents the idling action) when the number of patients of each class in the examination rooms is \underline{x} .

The optimal behavior of the physician is an appealing and simple index rule as follows.

THEOREM 1 (Phase 2 Prioritization). *The physician should not idle when there is a patient available in an exam room. Furthermore, regardless of the number and class of available and unavailable patients, the physician should prioritize available patients in decreasing order of $p_{ij} \hat{\theta}_{ij} \hat{\mu}_{ij}$.*

Theorem 1 provides a simple prioritization index for physicians computed as the probability that the visit will be the final interaction with the patient (p_{ij}) times the estimated risk of adverse events ($\hat{\theta}_{ij}$) divided by the average duration of each visit ($1/\hat{\mu}_{ij}$). Such a policy is easy to implement, since (a) the physician does not need to consider the number and class of patients available

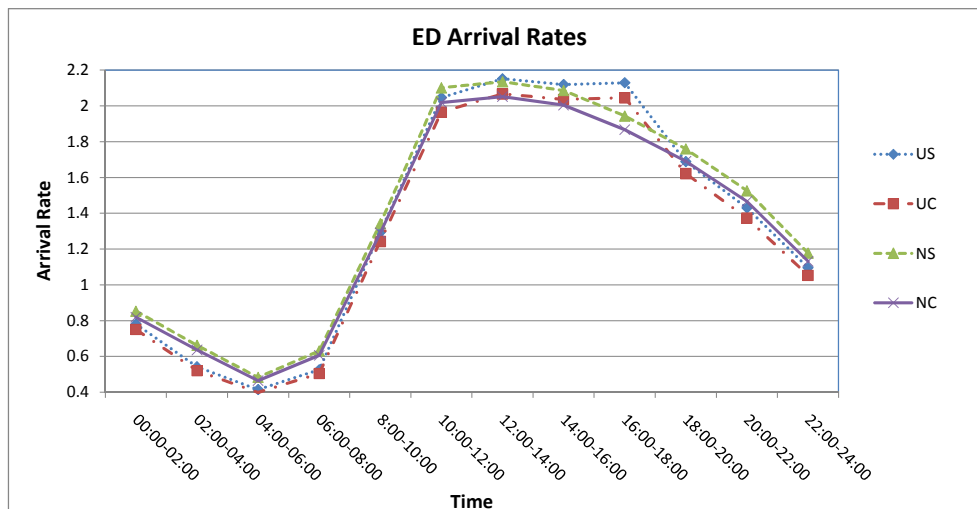


Figure 5 Class dependent arrival rates to the ED for an average day (obtained from a year of data in UMED).

in the examination rooms or under tests, and (b) the physician (or a decision support system) can dynamically estimate the required quantities. The authors have developed a smart phone application that can be used by physicians to facilitate collection of required data and computation of patient priorities.

6. A Realistic Simulation Analysis of Complexity-Based Triage

In this section, we test the conjectures suggested by our analytic models by means of a detailed ED simulation model. This simulation incorporates many realistic features of the University of Michigan ED (UMED) that are representative of most ED's in large research hospitals, including dynamic non-stationary arrivals, multi-stage service, multiple physicians and exam rooms, inaccuracy in triage classifications (both in terms of urgency and complexity), and limits on the number of patients physicians handle simultaneously. Our base case model uses a year of data from UMED plus time study data from the literature. We first describe the main features of our simulation framework, and then describe the test cases and our conclusions from them.

Patient Classes. At the time of triage, patients are classified according to both urgency (urgent or non-urgent) and complexity (simple or complex). For modeling purposes, we omit the resuscitation unit (RU) and fast track (FT) classifications, shown in Figure 1 (right), since these patients are typically tracked separately from the main ED. Following the definition of complex patients in Vance and Spirvulis (2005), we define S patients as those who only require one interaction and C patients as those requiring two or more interactions. With ESI-4 and 5 patients omitted, we can equate U patients with ESI-2 patients, and N patients with ESI-3 patients for our purposes. Both urgency and complexity classifications at the point of triage are subject to errors with different error rates. We assume the true type of a patient is not known until the final disposition decision

is made. Consistent with the empirical findings of (Hay et al. (2001)) and (Vance and Spirvulis (2005)), we assume urgency and complexity classifications are subject to 10% and 17% error rates, respectively. For simplicity, we also assume urgency-based and complexity-based misclassification rates are independent and symmetric (i.e., triage nurses are equally likely to classify U (C) patients as N (S) as they are to classify N (S) patients as U (C), respectively).

Arrival Process. Class-based patient arrivals are modeled using non-stationary Poisson processes that approximate our data. The non-stationary arrival rates for different classes are depicted in Figure 5. These arrival rates were obtained from a year of UMEM data using the ESI levels based on two-hour intervals of the day. Furthermore, (since patients are not currently triaged based on complexity), we used the empirical results of Vance and Spirvulis (2005) (who found that about 49% of patients are complex) to obtain these arrival rates using a (stationary) splitting mechanism. The general pattern illustrated in Figure 5 is similar to those reported in other studies (e.g., Green et al. (2006)). A “thinning” mechanism (see Lewis and Shedler (1979a) and Lewis and Shedler (1979b)) is used to simulate the non-stationary Poisson process arrivals for each class of patients (with rates depicted in Figure 5) in our base case. But we consider asymmetric errors in our sensitivity analysis.

Service Process. The ED service process is multi-stage and similar to the schematic in Figure 4. Each patient goes through one or more phases of patient-physician interactions followed by test/preparation/wait activities during which the physician cannot have a direct interaction with the patient (all such stages are labeled as Test in Figure 4). We also consider the initial and final preparations by a nurse. The initial preparation happens when the patient is moved to an exam room for the first time (before the first interaction with the physician) and the final preparation happens after the final visit by the physician and before the patient is discharged home or admitted to the hospital. The duration of each physician interaction is random and its average may depend on the class of the patient and the number of previous interactions. Our data suggest that the first and last interactions are typically longer than the intermediate interactions. As mentioned before and illustrated in Figure 1 (right), S patients are defined to be those who have only one (treatment related) interaction. For C patients, we can estimate the distribution of the total number of physician interactions per patient as shown in Figure 6 using data from a detailed time study (see Table 3 of Graff et al. (1993)) (normalized to represent our NC and UC patient classes). The simulated service process is considered to be non-collaborative, since an ED physician rarely transfers his/her patients to another physician, and also non-preemptive.

Physician-Patient Assignments and Priorities. As mentioned earlier, the process of connecting patients with physicians involves two phases. In Phase 1, patients are brought back from the waiting area to exam rooms whenever a room becomes available based on a Phase 1 sequencing

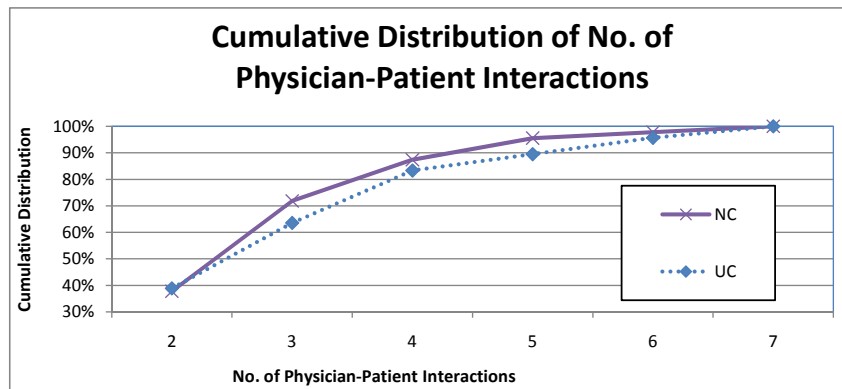


Figure 6 Cumulative number of class-based physician-patient interactions for complex patients (those requiring more than one interaction).

priority. Phase 1 is usually performed by a charge nurse. In Phase 2, whenever a physician becomes available, and if s/he has fewer than his/her maximum number of patients that s/he can handle simultaneously (7 seems to be typical), s/he chooses the next patient from those available based on a Phase 2 sequencing rule, which will depend on the type of triage being used. For urgency-based triage, we assume U patients get priority over N patients in both Phase 1 and Phase 2. For complexity-based triage, patients are prioritized in both Phase 1 and Phase 2 according to the strict priority ordering US, UC, NS, NC (ranked from high to low priority) which we found to be optimal in the simplified ED models discussed previously (see Proposition 2). When a patient is brought back to an examination room, we assume that s/he is assigned to the physician with the lowest number of patients. If all physicians are handling more than 7 patients, the patient must wait. Due to misclassification errors, Phase 1 and Phase 2 priority decisions can only be made based on the estimated class of the patient, but adverse events occur based on the true class of the patient.

ED Resources. We consider 20 beds and 4 physicians in our base case scenario. We then perform a sensitivity analysis to understand the effect of number of both beds and physicians on the benefit of complexity-based triage. For simplicity, we do not consider end of shift effects and/or variations in the level of staff available. Furthermore, we consider test facilities (ancillary services) as exogenous resources (i.e., test times are independent of the volume of ED patients) because these facilities often handle many other patients besides those from the ED.

Adverse Events. Adverse events are simulated using Poisson processes with rates that depend on the class of patients, as well as the phase of service. Specifically, we assume that U patients have a higher rate of adverse events than N patients, and that after patients enter an exam room (Phase 2 of service), their rate of adverse events decreases by 60% (in our base case) relative to their rates in the waiting area (Phase 1 of service). As in our previous models, we do not consider

fatal events that would terminate the adverse events counting process, since the impact of these rare events on our objective function is extremely small.

Runs. The simulation was written in a C++ framework and makes use of a cyclo-stationary model with a period of a week. Each data point was obtained for 5000 replications of one week, where each replication was preceded by a warm-up period of one week (which was observed to be sufficient because correlations in the ED flow are very small for spans of two or more days). The number of replications (5000) was chosen so that the confidence intervals are tight enough that (1) the sample averages are reliable, and (2) we can omit these very tight intervals from our data presentations.

In the following sections, we describe how we used our simulation model to analyze the benefit of complexity-based triage over urgency-based triage.

6.1. Performance of Complexity-Based Triage

We start by comparing complexity-based triage to urgency-based triage in our base case model, under the assumption that both types of triage make use of their respective priority rules for sequencing patients in both Phase 1 and Phases 2. This leads to the following:

Observation 1. *In the base case, implementing complexity-based triage improves ROAE and LOS by 18.0% and 21.3%, respectively.*

To consider the case where Phase 2 sequencing cannot follow the optimal rule due to a lack of data, patient discomfort, or other factors, we also compare complexity-based triage with urgency-based triage when Phase 2 sequencing uses a service-in-random-order (SIRO) rule. This leads to improvements of 17.1% and 20.7% in ROAE and LOS, respectively. Hence, it appears that the benefits of complexity-based triage are robust to the policy used in Phase 2. At least in our base case, it is the refined sequencing in Phase 1 that drives the majority of the improvement. We note that our simplified-single stage ED model predicted similar improvements in terms of magnitude of improvement in Phase 1 (see Figure 3).

The smaller effect of Phase 2 sequencing compared to that of Phase 1 prioritization is mainly due to the fact that, under the conditions of our base case, physicians in Phase 2 often do not have many available patients from which to choose. This is because patients are unavailable for a considerable amount of time while being tested and waiting for test results. However, in ED's with shorter test times (e.g., more test facilities dedicated to the ED, or more responsive central test facilities), larger case loads (patients per physician), and enough examination rooms/beds to accommodate patients, there will be more choices among in-process patients, and hence more improvement from an effective Phase 2 sequencing policy. To test this, we consider an ED with test rates 70% faster than the base case values, 40 beds, 3 physicians, and a maximum number of 10 patients per physician. Under these conditions, if Phase 2 sequencing is done according to SIRO

for both the urgency-based and complexity-based triage systems, then complexity-based triage achieves improvements of 8.7% and 7.7% in ROAE and LOS, respectively, relative to urgency-based triage. In contrast, if the urgency-based triage system prioritizes patients in Phase 2 by urgency ($U > N$) and the complexity-based triage system prioritizes patients in Phase 2 by complexity and urgency ($US > UC > NS > NC$), then complexity-based triage achieves improvements of 10.3% and 10.8% in ROAE and LOS, respectively, relative to urgency-based triage. This leads us to the following:

Observation 2. *In ED's where physicians have more choice about what patient to see next, using complexity information to prioritize patients in Phase 2 becomes more valuable.*

6.2. How to Define Complex Patients?

In the previous section, we investigated the benefit of complexity-based triage using the approach of Vance and Spirvulis (2005) to define complex patients as those requiring at least two (treatment related) interactions with a physician². This results in a nearly even split between complex and simple patients (49% C vs. 51% S) as well as substantial heterogeneity between their treatment time (both of which were predicted in Proposition 3 to be factors that improve the performance of complexity-based triage). But we could use other definitions of patient complexity. In Figure 7, we illustrate the impact of complexity-based triage on ROAE and LOS when complex patients are defined to be as those with more than one (resulting in 49% C patients), more than two (resulting in 39% C patients), and more than three (resulting in 30% C patients) interactions. From this we conclude:

Observation 3. *If the number of (treatment related) interactions is used as the metric for patient complexity, the benefit of complexity-based triage is greatest when complex patients are defined to be those requiring at least two interactions.*

The reason for this is that increasing the number of interactions required for a patient to be considered complex decreases the fraction of complex patients substantially, but only slightly increases the difference in treatment times between complex and simple patients. Thus, as predicted by Proposition 3, the benefit of complexity-based triage declines.

6.3. The Effect of ED Resource Levels

Another factor predicted by Proposition 3 to favor complexity-based triage is resource utilization. In that proposition, resources refer to physicians and examination rooms (which are indistinguishable in the single-stage simplified ED model). Hence, we expect higher utilization of either physicians or examination rooms to increase the benefit of complexity-based triage. Figure 8 illustrates the

²To clarify, we would still classify it as a simple case if the physician were to order an X-ray (without spending significant time on examination) and after receiving the results, spend one visit prior to discharge

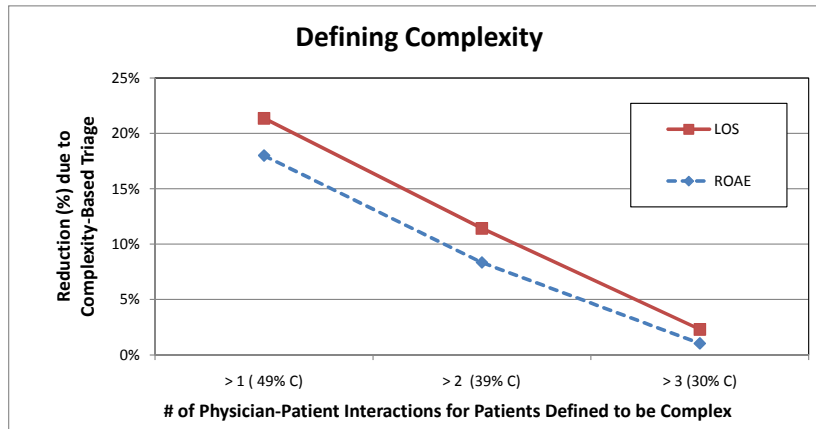


Figure 7 Performance of complexity-based triage when defining complex patients to be those having more than one, more than two, and more than three physician-patient interactions.

percentage improvement of complexity-based triage over urgency-based triage for varying numbers of examination rooms and physicians. From this figure we observe the following:

Observation 4. *The benefit of complexity-based triage is greater in ED's with a higher bed and/or physician utilization.*

As we observed in the Introduction, most ED's are overcrowded, so high utilization is a common situation. Hence, results from our analytic and simulation models suggest that complexity-based triage is most effective precisely in ED's most in need of improvement.

6.4. The Effect of Misclassification

Finally, we investigate the impact of complexity-based misclassification errors, which are inevitable in any triage system. Figure 9 (left) shows the benefits (in ROAE and LOS) of complexity-based triage over urgency-based triage for variations of the base case, in which complexity misclassification error rates range from 5% to 25%. Figure 9 (left) assumes these errors to be symmetric; that is, the chance of classifying an S patient as C is equal to the chance of classifying a C patient as S.

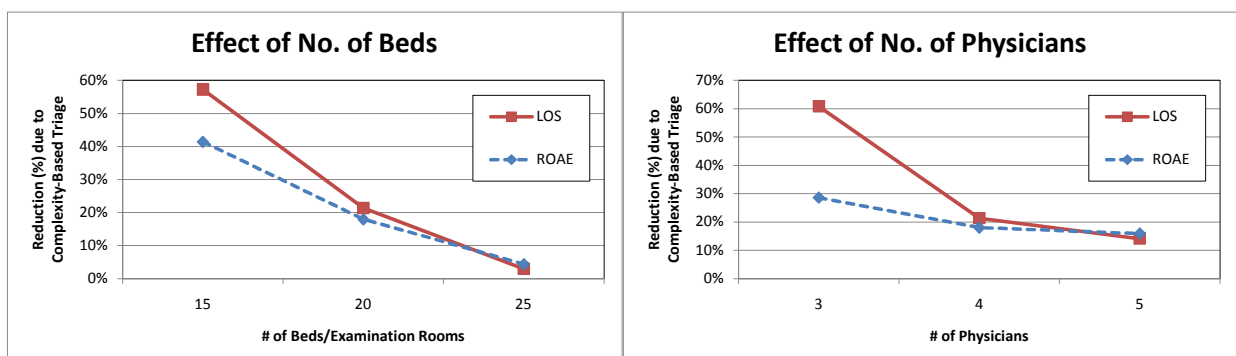


Figure 8 The effect of resources (beds and physicians) on the benefit of complexity-based triage over the current practice of urgency-based triage [Left: the effect of beds (4 physicians); Right: the effect of physicians (20 beds)].

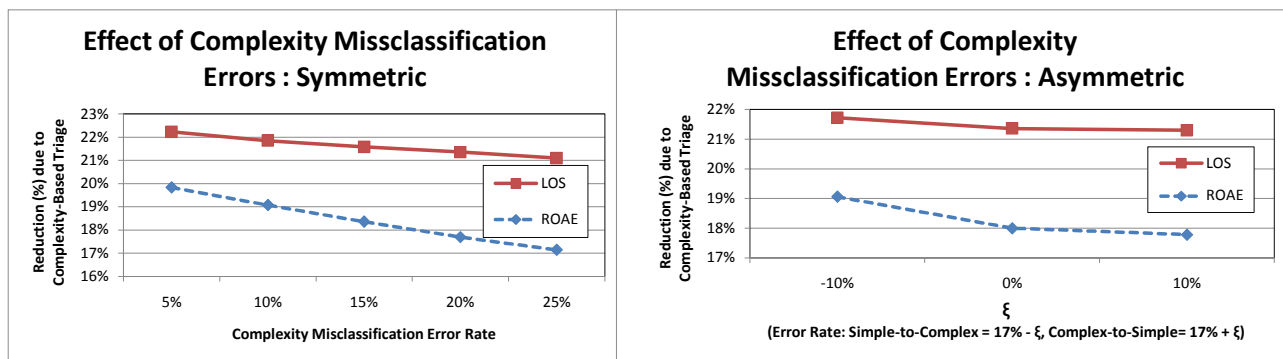


Figure 9 The effect of complexity misclassification error rates on the benefit of a complexity-based triage (compared to an urgency-based only) [Left: symmetric misclassification; Right: asymmetric misclassification].

Figure 9 (right) considers asymmetric error rates while keeping the average misclassification rate constant and equal to the base-case value of 17%. From these figures, we observe the following:

Observation 5. *The benefit of complexity-based triage is robust to complexity misclassification errors. However, complex-to-simple misclassifications are slightly more harmful than simple-to-complex misclassifications.*

The intuition behind the second part of this observation is that a complex-to-simple misclassification error moves a complex patient up in the queue, potentially delaying many other patients. In contrast, a simple-to-complex misclassification error moves a simple patient back in the queue, delaying only that patient. So, it is slightly better to err on the side of classifying ambiguous patients as complex rather than simple.

7. Conclusion

In this paper, we propose a new triage system for ED's in which patients are classified on the basis of complexity, as well as urgency. Our results suggest that complexity-based triage can significantly improve ED performance in terms of both patient safety (ROAE) and operational efficiency (LOS), even if patient classification is subject to error. A simple classification scheme, which defines patients to be simple if they require only a single procedure (and complex otherwise) works very well as the basis for complexity-based triage as it results in (1) a nearly even split between simple and complex patients, and (2) a substantial difference between average treatment time of complex and simple patients. This classification scheme has been shown empirically to be feasible for nurses to implement at triage with reasonable accuracy, and hence, appears to be a promising enhancement of the triage process.

While complexity-based triage can yield substantial safety and efficiency improvements even if complexity information is used to prioritize patients only up to the point where they enter examination rooms (Phase 1), we also find that in ED's where physicians have a significant amount of choice about what patient to see next within examination rooms (Phase 2), complexity information

gathered at triage can yield additional benefits by facilitating internal sequencing decisions. For both Phase 1 and Phase 2, the benefit of complexity-based triage is greatest in ED's with high physician and/or examination room utilization. Since ED's are widely overcrowded, our results suggest that complexity-based triage is an effective way for ED's to improve safety and reduce congestion without adding expensive human or physical capacity.

Appendix

Proof of Proposition 1: The proof of part (i) follows directly from comparing (4) and (5). To show part (ii), notice that, using the result of part (i) for a special case where there is no misclassification error, prioritizing U (N) patients is optimal if, and only if, $\theta_U \mu_U \geq (\leq) \theta_N \mu_N$. Next, observe that $\theta'_U \mu'_U - \theta'_N \mu'_N = [\lambda_N \lambda_U \mu_N \mu_U (\theta_U \mu_U - \theta_N \mu_N) (1 - \gamma_N - \gamma_U)] / [(\lambda_N \mu_U \gamma_N + \lambda_U \mu_N (1 - \gamma_U)) (\lambda_N \mu_U (1 - \gamma_N) + \lambda_U \mu_N \gamma_U)]$. Combining these two results completes the proof of part (ii), as the sign of the numerator changes when the sum of errors exceeds 1. \square

LEMMA 1 (Perfect Classification - Prioritization). *In the simplified single-stage ED model under perfect urgency and complexity based classification:*

(i) *The best priority rule is to prioritize patients in decreasing order of $\theta\mu$ values. Hence, if $\theta_{UC}\mu_{UC} \geq \theta_{NS}\mu_{NS}$, then the best priority rule is to follow the ordering: US, UC, NS, NC. Otherwise, the ED should follow the priority ordering: US, NS, UC, NC.*

(ii) $R_*^{\mathcal{U}\cup\mathcal{C}} \leq R_*^{\mathcal{U}}$. *That is, the risk of adverse events under the optimal priority rule using both complexity and urgency information is (weakly) smaller than that under the optimal apriority rule using only urgency information.*

(iii) *The best priority rule of part (i) is optimal even among the larger class of all non-anticipative policies (state or history dependent, idling or non-idling, etc.).*

Proof of Lemma 1: Notice that, using (1), we can compute the average waiting time of each class of patients under any (static) priority rule. Furthermore, under priority rule π , we have

$$R_\pi^{\mathcal{U}\cup\mathcal{C}} = \sum_{i \in \mathcal{U}} \sum_{j \in \mathcal{C}} \theta_{ij} \lambda_{ij} W_{ij}^\pi, \quad (8)$$

where W_{ij}^π is the average waiting of class ij under priority rule π . The proof of part (i) then follows from Cox and Smith (1961) (see pages 83-84), where an interchange argument is used (when the number of customer classes is at least 3) to show that the best rule (among the priority policies) to minimize the holding cost in a non-preemptive M/G/1 is to follow the $c\mu$ rule. Replacing holding cost values (c) with adverse event rates (θ), and noticing that the patient class US (NC) has the

highest (lowest) $\theta\mu$ value complete the proof of part (i). Next, using the result of part (i) together with (1) and (8), when $\theta_{UC}\mu_{UC} \geq \theta_{NS}\mu_{NS}$, we have:

$$R_*^{\mathcal{U}\cup\mathcal{C}} = \lambda\mathbb{E}(s^2) \left[\frac{\lambda_{US}\theta_{US}}{2(1-\rho_{US})} + \frac{\lambda_{UC}\theta_{UC}}{2(1-\rho_{US})(1-\rho_{US}-\rho_{UC})} + \frac{\lambda_{NS}\theta_{NS}}{2(1-\rho_{US}-\rho_{UC})(1-\rho_{US}-\rho_{UC}-\rho_{NS})} \right. \\ \left. + \frac{\lambda_{NC}\theta_{NC}}{2(1-\rho_{US}-\rho_{UC}-\rho_{NS})(1-\rho_{US}-\rho_{UC}-\rho_{NS}-\rho_{NC})} \right] \quad (9) \\ \leq \min\{R_U^{\mathcal{U}}, R_N^{\mathcal{U}}\} = R_*^{\mathcal{U}},$$

where the inequality follows from (2) and (3) together with the result of part (i) of Proposition 1 (for the special case where there is no misclassification error).

When $\theta_{UC}\mu_{UC} < \theta_{NS}\mu_{NS}$, we have:

$$R_*^{\mathcal{U}\cup\mathcal{C}} = \lambda\mathbb{E}(s^2) \left[\frac{\lambda_{US}\theta_{US}}{2(1-\rho_{US})} + \frac{\lambda_{NS}\theta_{NS}}{2(1-\rho_{US})(1-\rho_{US}-\rho_{NS})} + \frac{\lambda_{UC}\theta_{UC}}{2(1-\rho_{US}-\rho_{NS})(1-\rho_{US}-\rho_{NS}-\rho_{UC})} \right. \\ \left. + \frac{\lambda_{NC}\theta_{NC}}{2(1-\rho_{US}-\rho_{UC}-\rho_{NS})(1-\rho_{US}-\rho_{UC}-\rho_{NS}-\rho_{NC})} \right], \quad (10)$$

and similar to the previous case, it can be easily seen that $R_*^{\mathcal{U}\cup\mathcal{C}} \leq R_*^{\mathcal{U}}$. The proof of part (iii) follows from Kakalik and Little (1971) (after replacing holding cost with adverse event rates) who (for the average holding cost objective) showed that the $c\mu$ policy of Cox and Smith (1961) remains optimal even when inserting idleness is allowed and/or when the priority rule is dynamic (i.e., state-dependent). \square

LEMMA 2 (Perfect Classification - Attractiveness). *In the simplified single-stage ED model, perfect complexity-based triage yields a larger improvement over perfect urgency-based triage when (i) ED utilization is higher, (ii) heterogeneity in the average service time of simple vs. complex patients is larger, and/or (iii) the fraction of simple and complex patients are closer to equal.*

Proof of Lemma 2: To show the result, first consider the case where under the $\mathcal{U}\cup\mathcal{C}$ classification it is optimal to follow the priority order US, UC, NS, NC, and under the \mathcal{U} classification, it is optimal to follow the priority order U, N (i.e., prioritizing urgent patients first). Let $f = R_*^{\mathcal{U}\cup\mathcal{C}} - R_*^{\mathcal{U}}$, and notice that with $\mu_{i\mathcal{C}} = \mu_{\mathcal{C}}$ and $\mu_{i\mathcal{S}} = \mu_{\mathcal{S}}$ ($\forall i \in \mathcal{U}$), and $\theta_{Uj} = \theta_U$ and $\theta_{Nj} = \theta_N$ ($\forall j \in \mathcal{C}$) (i.e., when complexity is based only on set \mathcal{C} and urgency is based only on set \mathcal{U}), from (9) and (2) we have:

$$f = - \left[\frac{\theta_U \lambda_{US} \lambda_{UC} (1/\mu_{\mathcal{C}} - 1/\mu_{\mathcal{S}})}{2(1-\rho_U)} + \frac{\theta_N \lambda_{NC} \lambda_{NS} (1/\mu_{\mathcal{C}} - 1/\mu_{\mathcal{S}})}{2(1-\rho_U)(1-\rho)} \right]. \quad (11)$$

Then, a careful treatment of utilization (realizing that $\rho_U = \lambda_U/\mu_U$ and $\rho = \rho_U + \rho_N$) shows that f is non-increasing in utilization, ρ . To prove part (ii), it then can be seen that f is non-increasing in $1/\mu_{\mathcal{C}} - 1/\mu_{\mathcal{S}}$ (keeping utilization and other factors the same). To see part (iii), let $\alpha \in [0, 1]$

denote the fraction of patients that are complex, and $(1 - \alpha)$ denote the fraction of patients that are simple, so $\lambda_{US} = (1 - \alpha)\lambda_U$, $\lambda_{UC} = \alpha\lambda_U$, $\lambda_{NC} = \alpha\lambda_N$, and $\lambda_{NS} = (1 - \alpha)\lambda_N$. Replacing these in (11), it follows that f , as a function of α , can be written as $f = -[\alpha(1 - \alpha)]k$, for some constant $k \geq 0$. Thus, $\alpha = 0.5$ yields the maximum benefit. The proof for other cases (i.e., when other priority rules are optimal) follows a similar argument after computing f using either (9) or (10), and either (2) or (3), depending on the optimal priority rule under $\mathbf{U} \cup \mathbf{C}$ and \mathbf{U} classifications, respectively. \square

Proof of Proposition 2: The proof of part (i) follows directly from the proof of part (i) of Lemma 1, since all rates are replaced with their error impacted counterparts. That is, the same interchange method of Cox and Smith (1961) (see pages 83-84) after replacing all rates with their error impacted counterparts proves that the best priority rule is to give priority based on a decreasing order of $\theta\mu$ values. The proof of part (ii) follows from the proof of Lemma 1 (found earlier in this appendix) part (ii) after replacing parameters with their error impacted counterparts. The proof of part (iii) follows from the result of Kakalik and Little (1971), after replacing holding cost with the error impacted rate of adverse events, and all the other rates with their error impacted counterparts. \square

Proof of Proposition 3: The proof of parts (i) - (iii) follows mainly from the proof of Lemma 2 (found earlier in this appendix). First, consider the case where under the $\mathbf{U}' \cup \mathbf{C}'$ (i.e., imperfect urgency and complexity) classification it is optimal to follow the priority order US, UC, NS, NC, and under the \mathbf{U}' (i.e., imperfect urgency) classification, it is optimal to follow the priority order U, N (i.e., prioritizing urgent patients over non-urgent patients). With $f = R_*^{\mathbf{U}' \cup \mathbf{C}'} - R_*^{\mathbf{U}'}$, and after replacing rates with their error impacted counterparts in (11) we have:

$$f = -\left[\frac{\theta'_U \lambda'_{US} \lambda'_{UC} (1/\mu'_C - 1/\mu'_S)}{2(1 - \rho'_U)} + \frac{\theta'_N \lambda'_{NC} \lambda'_{NS} (1/\mu'_C - 1/\mu'_S)}{2(1 - \rho'_U)(1 - \rho')}\right]. \quad (12)$$

Next, notice that $\rho' = \rho$ (i.e., the total utilizations with and without misclassifications are the same). Hence, similar to the proof of part (i) of Lemma 2, it can be seen that f is non-increasing in ρ . Moreover, it can be seen that f is non-increasing in $1/\mu'_C - 1/\mu'_S$. Next, notice that $(\underline{1}/\underline{\mu}')^T = (A(\underline{\lambda}/\underline{\mu})^T)/\underline{\lambda}'$, where A is defined in (6). Thus, similar to the proof of part (ii) of Lemma 2, it can be seen that f is non-increasing in $1/\mu_C - 1/\mu_S$, which proves part (ii). Furthermore, similarly to the proof of part (iii) of Lemma 2, let $\lambda_{US} = (1 - \alpha)\lambda_U$, $\lambda_{UC} = \alpha\lambda_U$, $\lambda_{NC} = \alpha\lambda_N$, and $\lambda_{NS} = (1 - \alpha)\lambda_N$. It can be seen that f as a function of α is minimized at $\alpha = 0.5$, which proves part (iii). It can also be seen that f is non-decreasing in complexity misclassification error rates,

γ_S and γ_C , which proves part (iv). The proof for other cases (i.e., when other priority rules are optimal) follows a similar line of argument after computing f . \square

Proof of Theorem 1: To show the result, we use an *interchange* argument; we show that if classes $uc \in \mathbf{U} \times \mathbf{C}$ and $sl \in \mathbf{U} \times \mathbf{C}$ are such that $p_{uc} \hat{\theta}_{uc} \hat{\mu}_{uc} \geq p_{sl} \hat{\theta}_{sl} \hat{\mu}_{sl}$, then it is (weakly) better to serve class uc than class sl when in state $(\underline{x}, \underline{y})$ with $x_{uc}, x_{sl} > 0$. This will also prove that the optimal policy will not idle the physician when there are one or more patients available in the rooms, since idling can be thought of serving an additional class, class 0, with $\hat{\theta}_0 = \hat{\mu}_0 = p_0 = 0$ (see, for instance, Buyukkoc et al. (1985)). To show that it is (weakly) better to serve class uc than class sl , we first consider the problem in an N -period discounted cost setting with four parallel (one for each class of patients) $\cdot/M/k$ systems (to guarantee bounded transition rates for the purpose of uniformization) in place of the $\cdot/M/\infty$ test stage, and show that the results hold for any number of periods to go $n \in 1, 2, \dots, N$. (Notice that using four parallel $\cdot/M/k$ systems removes the need for considering the sequence and the type of patients within the common queue.) Using a convergence argument, as $n \rightarrow \infty$, it then follows that the result is true for an infinite-horizon (and hence, average cost) scenario with the four k -server test system. Next, taking limit as $k \rightarrow \infty$, it follows that the result is true even when transition rates are not bounded due to the existence of the $\cdot/M/\infty$ stage.

Now consider the finite horizon discounted cost version of (7). With β denoting the discount factor, the optimal discounted cost when there are $n + 1$ (uniformized) periods to go is

$$\begin{aligned} V_{n+1}^k(\underline{x}, \underline{y}) = & \frac{1}{\psi_k} \left[\hat{\theta}(\underline{x} + \underline{y})^T + \beta \left[\sum_{ij \in \mathbf{U} \times \mathbf{C}} [\lambda_{ij} V_n^k(\underline{x} + \underline{e}_{ij}, \underline{y}) + (y_{ij} \wedge k) \eta V_n^k(\underline{x} + \underline{e}_{ij}, \underline{y} - \underline{e}_{ij})] \right. \right. \\ & + \min_{a \in \mathcal{A}(\underline{x})} \left\{ \sum_{ij \in \mathbf{U} \times \mathbf{C}} \mathbb{1}_{\{a=ij\}} \hat{\mu}_{ij} [p_{ij} V_n^k(\underline{x} - \underline{e}_{ij}, \underline{y}) + (1 - p_{ij}) V_n^k(\underline{x} - \underline{e}_{ij}, \underline{y} + \underline{e}_{ij})] \right. \\ & \left. \left. + (\psi_k - \sum_{ij \in \mathbf{U} \times \mathbf{C}} [\lambda_{ij} + (y_{ij} \wedge k) \eta + \mathbb{1}_{\{a=ij\}} \hat{\mu}_{ij}]) V_n^k(\underline{x}, \underline{y}) \right\} \right], \end{aligned} \quad (13)$$

or equivalently (grouping the terms related to control in the minimization and self-loop)

$$\begin{aligned} V_{n+1}^k(\underline{x}, \underline{y}) = & \frac{1}{\psi_k} \left[\hat{\theta}(\underline{x} + \underline{y})^T + \beta \left[\sum_{ij \in \mathbf{U} \times \mathbf{C}} [\lambda_{ij} V_n^k(\underline{x} + \underline{e}_{ij}, \underline{y}) + (y_{ij} \wedge k) \eta V_n^k(\underline{x} + \underline{e}_{ij}, \underline{y} - \underline{e}_{ij})] \right. \right. \\ & - \max_{a \in \mathcal{A}(\underline{x})} \left\{ \sum_{ij \in \mathbf{U} \times \mathbf{C}} \mathbb{1}_{\{a=ij\}} \hat{\mu}_{ij} [p_{ij} \Delta_{ij}^y V_n^k(\underline{x} - \underline{e}_{ij}, \underline{y}) + \Delta_{ij}^{x,y} V_n^k(\underline{x} - \underline{e}_{ij}, \underline{y} + \underline{e}_{ij})] \right. \\ & \left. \left. + (\psi_k - \sum_{ij \in \mathbf{U} \times \mathbf{C}} [\lambda_{ij} + (y_{ij} \wedge k) \eta]) V_n^k(\underline{x}, \underline{y}) \right\} \right], \end{aligned} \quad (14)$$

where $\Delta_{ij}^y V_n^k(\underline{x}, \underline{y}) = V_n^k(\underline{x}, \underline{y} + \underline{e}_{ij}) - V_n^k(\underline{x}, \underline{y})$ and $\Delta_{ij}^{x,y} V_n^k(\underline{x}, \underline{y}) = V_n^k(\underline{x} + \underline{e}_{ij}, \underline{y} - \underline{e}_{ij}) - V_n^k(\underline{x}, \underline{y})$. Now let π ($\hat{\pi}$) be the policy that prescribes serving patients of class uc (sl) for every state $(\underline{x}, \underline{y})$ with $x_{uc}, x_{sl} > 0$ and in every period n . From (14), to show that π is (weakly) better than $\hat{\pi}$ in

every period, we need to show that the following property holds for every n and every state $(\underline{x}, \underline{y})$ with $x_{uc}, x_{sl} > 0$:

$$\begin{aligned} & \hat{\mu}_{uc} [p_{uc} \Delta_{uc}^y V_n^{k,\pi}(\underline{x} - \underline{e}_{uc}, \underline{y}) + \Delta_{uc}^{x,y} V_n^{k,\pi}(\underline{x} - \underline{e}_{uc}, \underline{y} + \underline{e}_{uc})] \\ & \geq \hat{\mu}_{sl} [p_{sl} \Delta_{sl}^y V_n^{k,\hat{\pi}}(\underline{x} - \underline{e}_{sl}, \underline{y}) + \Delta_{sl}^{x,y} V_n^{k,\hat{\pi}}(\underline{x} - \underline{e}_{sl}, \underline{y} + \underline{e}_{sl})]. \end{aligned} \quad (15)$$

To show property (15), we use induction on n . First, for $n = 0$, the property trivially holds since $V_0^\pi(\cdot, \cdot) = V_0^{\hat{\pi}}(\cdot, \cdot) = 0$. Next, suppose the property holds for n . We show that it will then also hold for $n + 1$. To do so, we need to consider different cases based on the state (i.e., partitions of the state space). First, consider the case where $x_{uc}, x_{sl} \geq 2$. Using action $a = uc$ (policy π) in both states $(\underline{x} - \underline{e}_{uc}, \underline{y})$ and $(\underline{x}, \underline{y})$ to compute $V_{n+1}^{k,\pi}(\underline{x} - \underline{e}_{uc}, \underline{y} + \underline{e}_{uc})$ and $V_{n+1}^{k,\pi}(\underline{x} - \underline{e}_{uc}, \underline{y})$ using (14), and subtracting the results we have $\Delta_{uc}^y V_{n+1}^{k,\pi}(\underline{x} - \underline{e}_{uc}, \underline{y}) =$

$$\begin{aligned} & \frac{1}{\psi_k} \left[\hat{\theta}_{uc} + \beta \left[\sum_{ij \in \mathbf{U} \times \mathbf{C}} [\lambda_{ij} \Delta_{uc}^y V_n^{k,\pi}(\underline{x}, \underline{y}) + (y_{ij} \wedge k) \eta \Delta_{uc}^y V_n^{k,\pi}(\underline{x} + \underline{e}_{ij} - \underline{e}_{uc}, \underline{y} - \underline{e}_{ij})] \right. \right. \\ & \quad \left. \left. + \mathbb{1}_{\{y_{uc} < k\}} \eta V_n^{k,\pi}(\underline{x}, \underline{y} - \underline{e}_{uc}) \right. \right. \\ & \quad \left. \left. - \hat{\mu}_{uc} [p_{uc} \Delta_{uc}^y \Delta_{uc}^y V_n^{k,\pi}(\underline{x} - 2\underline{e}_{uc}, \underline{y}) + \Delta_{uc}^y \Delta_{uc}^{x,y} V_n^{k,\pi}(\underline{x} - 2\underline{e}_{uc}, \underline{y} + \underline{e}_{ij})] \right. \right. \\ & \quad \left. \left. + (\psi_k - \sum_{ij \in \mathbf{U} \times \mathbf{C}} [\lambda_{ij} + (y_{ij} \wedge k) \eta]) \Delta_{uc}^y V_n^k(\underline{x} - \underline{e}_{uc}, \underline{y}) - \mathbb{1}_{\{y_{uc} < k\}} \eta V_n^{k,\pi}(\underline{x} - \underline{e}_{uc}, \underline{y}) \right] \right]. \end{aligned} \quad (16)$$

Similarly, we can derive $\Delta_{uc}^{x,y} V_{n+1}^{k,\pi}(\underline{x} - \underline{e}_{uc}, \underline{y} + \underline{e}_{uc})$ using (14) and action $a = uc$ (policy π) in both states $(\underline{x} - \underline{e}_{uc}, \underline{y} + \underline{e}_{uc})$ and $(\underline{x}, \underline{y})$ and subtracting the results. Doing so we have $\Delta_{uc}^{x,y} V_{n+1}^{k,\pi}(\underline{x} - \underline{e}_{uc}, \underline{y} + \underline{e}_{uc}) =$

$$\begin{aligned} & \frac{1}{\psi_k} \left[\beta \left[\sum_{ij \in \mathbf{U} \times \mathbf{C}} [\lambda_{ij} \Delta_{uc}^{x,y} V_n^{k,\pi}(\underline{x}, \underline{y} + \underline{e}_{uc}) + (y_{ij}^+ \wedge k) \eta \Delta_{uc}^{x,y} V_n^{k,\pi}(\underline{x} + \underline{e}_{ij} - \underline{e}_{uc}, \underline{y} - \underline{e}_{ij} + \underline{e}_{uc})] \right. \right. \\ & \quad \left. \left. - \mathbb{1}_{\{y_{uc} < k\}} \eta V_n^{k,\pi}(\underline{x}, \underline{y}) \right. \right. \\ & \quad \left. \left. - \hat{\mu}_{uc} [p_{uc} \Delta_{uc}^{x,y} \Delta_{uc}^y V_n^{k,\pi}(\underline{x} - 2\underline{e}_{uc}, \underline{y} + \underline{e}_{uc}) + \Delta_{uc}^{x,y} \Delta_{uc}^{x,y} V_n^{k,\pi}(\underline{x} - 2\underline{e}_{uc}, \underline{y} + \underline{e}_{ij} + \underline{e}_{uc})] \right. \right. \\ & \quad \left. \left. + (\psi_k - \sum_{ij \in \mathbf{U} \times \mathbf{C}} [\lambda_{ij} + (y_{ij}^+ \wedge k) \eta]) \Delta_{uc}^{x,y} V_n^k(\underline{x} - \underline{e}_{uc}, \underline{y} + \underline{e}_{uc}) + \mathbb{1}_{\{y_{uc} < k\}} \eta V_n^{k,\pi}(\underline{x} - \underline{e}_{uc}, \underline{y} + \underline{e}_{uc}) \right] \right], \end{aligned} \quad (17)$$

where $y_{ij}^+ = y_{ij}$ for all $ij \neq uc \in \mathbf{U} \times \mathbf{C}$, and $y_{uc}^+ = y_{uc} + 1$. In a similar way, and by using action $a = sl$ (policy $\hat{\pi}$) in (14) quantities $\Delta_{sl}^y V_{n+1}^{k,\hat{\pi}}(\underline{x} - \underline{e}_{sl}, \underline{y})$ and $\Delta_{sl}^{x,y} V_{n+1}^{k,\hat{\pi}}(\underline{x} - \underline{e}_{sl}, \underline{y} + \underline{e}_{sl})$ can be computed. Next, to check property (15) for $n + 1$, multiply (16) by $p_{uc} \hat{\mu}_{uc}$, and (17) by $\hat{\mu}_{uc}$ and add up the results. Similarly, multiply $\Delta_{sl}^y V_{n+1}^{k,\hat{\pi}}(\underline{x} - \underline{e}_{sl}, \underline{y})$ and $\Delta_{sl}^{x,y} V_{n+1}^{k,\hat{\pi}}(\underline{x} - \underline{e}_{sl}, \underline{y} + \underline{e}_{sl})$ by $p_{sl} \hat{\mu}_{sl}$ and $\hat{\mu}_{sl}$, respectively, and add up the results. Next, using the induction hypothesis and that $p_{uc} \hat{\theta}_{uc} \hat{\mu}_{uc} \geq p_{sl} \hat{\theta}_{sl} \hat{\mu}_{sl}$, after algebraic simplification it follows that

$$\begin{aligned} & \hat{\mu}_{uc} [p_{uc} \Delta_{uc}^y V_{n+1}^{k,\pi}(\underline{x} - \underline{e}_{uc}, \underline{y}) + \Delta_{uc}^{x,y} V_{n+1}^{k,\pi}(\underline{x} - \underline{e}_{uc}, \underline{y} + \underline{e}_{uc})] \\ & - \hat{\mu}_{sl} [p_{sl} \Delta_{sl}^y V_{n+1}^{k,\hat{\pi}}(\underline{x} - \underline{e}_{sl}, \underline{y}) + \Delta_{sl}^{x,y} V_{n+1}^{k,\hat{\pi}}(\underline{x} - \underline{e}_{sl}, \underline{y} + \underline{e}_{sl})] \geq 0, \end{aligned} \quad (18)$$

which establishes property (15) for $n + 1$ for the case where $x_{uc}, x_{sl} \geq 2$. In a similar way, this property can be established for other cases (i.e., the remaining partition of the state space). Hence, a non-idling strict priority rule is optimal for all n . Next, taking the limit as $n \rightarrow \infty$ it follows that the finite horizon problem converges to the infinite horizon one both in policy and cost (see Sennott (1999) Proposition 4.3.1). Furthermore, the convergence of the policy of the infinite-horizon discounted cost problem to that of average cost can easily be established (see Sennott (1999) Corollary 7.5.10). Therefore, the underlying non-idling strict priority policy is optimal under the average cost setting indexed by k (i.e., with $\cdot/M/k$'s in place of the $\cdot/M/\infty$) for any finite k . Since the result is true for any k , a convergence argument can be used to show that the result holds for the original problem with $k = \infty$. Notice that the existence of an optimal stationary policy for the original CTMS (i.e., when $k = \infty$) follows from the results of Guo and Liu (2001). \square

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