(a) Brief History of the Working Group

This is the second meeting at PMENA of this RMT working group. The idea of this working group emerged during a series of three-day conferences on representations of mathematics teaching held in Ann Arbor, Michigan in 2009 and 2010, organized by ThEMaT (Thought Experiments in Mathematics Teaching), an NSF-funded research and development project directed by Herbst and Chazan. ThEMaT originally created animated representations of teaching using cartoon characters to be used for research, specifically to prompt experienced teachers to relay the rationale they draw upon to justify or indict actions in teaching. The original workshops were conceived to begin disseminating those animations to be used in teacher development. The (Representations of Mathematics Teaching) RMT conferences in 2009 and 2010 gathered developers and users of all kinds of representations of teaching to present their work and discuss issues that might be common to them. These conferences included users of video, written cases, dialogues, photographs, comic strips, and animations. An outcome of the 2009 RMT conference was a special issue (Volume 43, issue 1, 2011) of the journal ZDM—The International Journal of Mathematics Education, guest edited by Herbst and Chazan. Outcomes of the 2010 Conference included two sessions at the 2011 NCTM Research Presession. The 2011 Conference (June 13-15) will also work toward the goal of creating events related to the use of representations of mathematics teaching in other conferences. In proposing a continuation of the working group for PMENA 2011 we are interested in continuing the discussion and work we had in Columbus during PMENA 2010 around the elaboration and investigation of a pedagogical framework for teacher development that makes use of representations of teaching and work toward an edited book on the subject.

(b) Issues in the Psychology of Mathematics Education that Will Be the Focus of the Work

Review of Existing Work Relating the Theme of the Working Group to the Field

The use of representations of mathematics teaching, particularly those that are maintained in a digital form, calls for specialized pedagogical practices from teacher developers. They also open new areas for investigation of how future professionals learn to practice and the role that various technologies play in scaffolding that learning. In the 2010 PMENA discussion paper, Herbst, Bieda, Chazan, and González (2010) briefly reviewed the literature on the use of video records and written cases in teacher education. We also noted that classroom scenarios sketched as cartoon animations have begun to be utilized for those purposes and argued that they have affordances that are distinct from those of video and written cases (see also Herbst, Chazan, Chen, Chieu, & Weiss, 2011). We also noted existing literature on the use of written and video cases in teacher education and cited examples that concern mostly face-to-face facilitation. We argued that the increased capabilities of information technologies for creating, manipulating, and collaborating over multimedia point to a promising future for teacher development assisted by representations of practice. In the special issue of ZDM referenced above, several new articles
have added to this literature. In particular Ghousseini and Sleep (2011) and Nachlieli (2011) describe the facilitation of face-to-face discussions around representations of practice and provide two views on what makes these effective for studying practice. Yet the features of novel media and their use with digital technologies may require other pedagogical strategies for teacher education that have not been sufficiently identified and explored.

In this document we complement the previous year’s review by briefly accounting for three areas of emerging scholarship: (1) information technologies that support teachers’ learning from representations of practice; (2) the particular challenge of helping prospective teachers understand students’ thinking; and (3) research and theory about what is important or possible to achieve in having prospective teachers look at or work with representations of teaching. 

**Information Technology and Teacher Education**

The field of teacher education is becoming more adept at using technology in teacher education classes. Communication technology is connecting teacher education students, university faculty, and mentor teachers through shared access to classroom videos (Price & Chen, 2003; Whipp, 2003), as well as providing another (sometimes more effective) mode of communication between teacher education students and university faculty (Derry, Seigel & Stampen, 2002; Reasons, Valadares & Slavkin, 2005). Teacher education is also leveraging communication technology to support the development of teacher communities (Farooq et al, 2007; Gomez et al, 2008). These communities increase teacher candidates’ access to resources and interaction with peers.

Digital video technology allows for prospective teachers to interact with practice in more meaningful ways than in the past. In addition to watching videos, teacher education students can now annotate and edit video quickly and easily (Chieu, Herbst, & Weiss, 2011; Pea, Mills, Rosen & Dauber, 2004; Rich & Hannafin, 2008, 2009). These technologies allow for faster communication as well as discussions that focus on specific moments of an instructional episode and therefore on the particulars of teaching practice. Video and animations have also been used effectively to illustrate to prospective teachers how the theoretical principles that they learn in university courses can be put into practice in the classroom (Moreno & Ortegano-Layne, 2008).

The availability of new technologies to represent teaching for its use in teacher education offers an interesting challenge for research on the pedagogy of teacher preparation and teacher candidates’ learning to teach: These issues are of importance to the PMENA community. Scholars have noted, for example, that in addition to the functionalities afforded by various technologies, it is paramount to consider the tasks that teacher candidates engage with, and the support that teacher education students receive from the course instructor (Lockhorst, Admiraal & Pilot, 2002). Other scholars have looked more in depth at technology-enabled instructional interventions and expanded upon features that could be situated into Lockhorst et al’s list of important features. Chieu, et al. (2011) looked at the use of online forums containing embedded animations of classroom episodes and argued that this access allows for richer conversations among teacher candidates. Llinares & Valls (2009, 2010) focus on the tasks posed to teacher candidates and the effect of these tasks on learning These studies all highlight the role of pedagogy in the use of new technologies in teacher education. As technology develops, teacher educators need to continue to develop their pedagogies to make the best use of the technology.

**The Challenge to Help Teacher Candidates Understand Students’ Thinking**

There continues to be a need for improving the connection between research on students’ thinking and learning and the work of teacher development. What kind of cognitive research on
students is useful for teaching and teacher education? How might teachers come to learn it and use it? How can new media assist in this work? Researchers have addressed those questions by 1) using multimedia to show examples of actual student responses to research-based tasks (e.g. Franke et al., 1998); 2) drawing upon research to develop representations of student thinking (e.g. Balacheff, 1988); and 3) using such research to analyze pedagogical moves in classroom dialogues created by teacher candidates (Crespo, Oslund, & Parks, 2011).

Professional development connected to the research on Cognitively Guided Instruction (CGI) pioneered the use of records of student work to engage teachers with research on students’ thinking (Carpenter, Fennema & Franke, 1996). By showing teachers video of students solving mathematical problems, as well as providing students’ written responses for further analysis, the CGI project aimed to build upon teachers’ existing knowledge of student thinking by systematizing and enriching this knowledge. The responses from participants of that program highlighted the value of having teachers analyze student work as a way to build knowledge that they could draw upon to make instructional decisions.

Beyond records of students’ work, research on students’ thinking has also been produced other representations of students’ work, though less is known about the effects of their use in teacher education. Balacheff (1988) provides an early example, using a comic strip to represent responses provided by two students as they worked on the prompt: “provide a means of calculating the number of diagonals of a polygon when you know the number of vertices it has” (p. 220). This representation, included in a book for teachers, suggested a new way to acquaint teachers with student thinking in story form. Students’ work has also been represented in narrative cases embedded within cases of teaching episodes (e.g. Stein et al., 2000). It is important to continue to look for ways of making students thinking accessible to teacher candidates and to find out what teacher candidates learn from these different representations of students thinking.

What is Possible to Achieve with Representations in Teacher Education

The combination of the use of representations of teaching and new technologies has facilitated the goals of teacher education in several areas. In particular, teacher educators have more diverse access to several types of representations of teaching (Grossman et al, 2009) and are able to use these in more flexible ways to support the learning of teaching “in, from, and for practice” (Lampert, 2010). Two important goals of teacher education are to increase teachers’ ability to notice aspects and events in the instructional environment (Rosaen et al, 2008; Sherin & Han, 2004; Sherin & van Es, 2009) and to afford teachers the skills and opportunities to reflect on their practice (Santagata & Angelici, 2010; Stockero, 2008). Two related goals are to enable teachers to learn from their own practice (Borko et al, 2011; Santagata & Guarino, 2011) and to alter or improve their teaching practices (Lampert et al, 2010; Polly & Hannafin, 2010, 2011; Van Zoest & Stockero, 2008).

A goal still ahead is to improve the connection between theories and conceptualizations of mathematics teaching and specifications of the curriculum of mathematics teacher education: Can we expect theories of teaching to determine the curriculum of teacher education? For example, in the context of the Teacher Education Initiative at the University of Michigan an innovative program for elementary teacher preparation has been emerging with a practice-centered curriculum. This curriculum prescribes teacher candidates’ learning of specific practices of teaching that are deemed high leverage—these include “leading whole-class discussions of content” and “recognizing and identifying common patterns of student thinking in a content
domain” (Curriculum Group, 2009). This curriculum gathers those practices in domains such as “assessing students” and “enacting instruction” (Curriculum Group, 2008). While it is possible to tie each of these practices and domains to the existing literature, it is not evident how one could argue that they constitute all the domains and practices that teacher candidates need to learn. A theory of practice would be useful to systematically generate all the domains and practices, to provide an underlying coherence to these domains and practices, and eventually to limit or at least modularize expansions of the curriculum.

An example of a theory of teaching that could provide such systematization is our theory of instructional exchanges in which we conceptualize the work of teaching as centered on the problem of exchanging students’ mathematical work for claims on their knowledge of the mathematics at stake within classroom environments that respond to four obligations (to the discipline, the individual students, the groups of students, and the school institution; see Herbst, 2010). But while this theory provides a conceptual basis for subject-specific descriptive models of the regulations of the work of teaching (with constructs such as situations, norms, and dispositions) it has not yet operationalized the work of teaching in ways that support professional education or development. There is a pervasive need in the field to articulate prescriptive theories of teacher education with descriptive theories of teaching—the latter can give completeness and validity to the former while the former can operationalize the latter.

**Toward a Pedagogy for Teacher Development Assisted by Representations of Practice**

Building on the proposal from last year, the working group’s purpose is somewhat ambitious: to design a pedagogy for teacher development that meets the goal of helping teacher candidates learn teaching in, from, and for practice by taking advantage of representations of practice and new technologies. This development includes conceptual developments, for example in articulating connections between descriptive theories of teaching and the prescription of a curriculum for teacher education. Our work will include asking questions such as, is it possible to further develop the descriptive norms (e.g. when a teacher expects students to do a proof she will provide them with ‘givens’ and the ‘prove’) into an operational specification of what a teacher candidate needs to learn to do, say, not only to be able to proficiently comply with such norms but also to negotiate tasks that depart from the norm? The working group will discuss the articulation between theories of teaching practice and practices of teacher education.

The convenors of this working group are also particularly interested in exploring how cartoon-based representations of practice facilitate teacher learning. Our thinking of how to make such representations usable for teacher education has been influenced by Lampert’s (2010) notion of learning teaching in, for, and from practice and by Grossman et al. (2009) account of professional education’s use of representations, decompositions, and approximations of practice. We surmise that cartoon-representations of teaching are sufficiently malleable to create not only representations of practice (that prospective professionals can view and annotate) but also to create decompositions of practice that prospective professionals can study from as well as approximations of practice in which prospective professionals can practice their skills. In this sense, cartoons can fashion virtual settings for teacher learning in, for, and from practice before the teacher candidates are ready to learn in real settings.

The LessonSketch environment (www.lessonsketch.org) is one example of a set of tools and resources that can exemplify what such a virtual setting for teacher development could be like. We are interested in having the RMT working group use LessonSketch to engage concretely in applying the elements of a pedagogical framework for teacher education assisted by representations. We expect that such work will help improve the framework and further develop
This revised pedagogical framework considers the need for a larger library of representations of practice. While in the past we had only included representations of lessons in which teaching and learning mathematics were integrated in scenarios of classroom instruction, the notion of decomposition of practice proposed by Grossman et al. (2009) suggests the need for two more sets of representations. One of those sets of representations consists of representations of students’ work: Depictions of how students solve problems, indexed in a database that permits searches by problems, operators, representation systems, and controls (Balacheff & Gaudin, 2010). The other new set of representations consists of representations of practices, strategies, tactics, and techniques in teaching, depicted through commented scenarios of instruction and also indexed in a database. We surmise that the cartoon medium can be useful for researchers on students’ cognition in the PME-NA community to reach teachers with representations of how students think about specific conceptions; likewise the cartoon medium can be useful to support the learning of teaching skills by prospective teachers.

The remaining elements of the framework expand on what we offered last year, taking advantage of the materials we brought to the meetings of the working group and the feedback received from participants. Last year we proposed that a pedagogy of teacher preparation assisted by representations of practice needed at least four categories of elements: open ended expressions, activity types, problem types, and technology tools or screens.

The first element of the framework we call open-ended expressions. These are terms and expressions that can be used in transactions between teacher developers and their clients without needing to be completely defined; tokens that teacher developers and their clients may be able to take as shared so as to negotiate activities, problems, and representations. One such expression is “mathematical action” which we have observed being used in a geometry class for future teachers and in the context of having the students watch an animation of geometry instruction. Other open-ended expressions that can serve comparable purposes are “student thinking,” “teaching move,” “instructional goal,” “resources,” etc. We surmise that these boundary objects may be useful to mobilize the work with representations—enabling explorations of practice that may lead to shaping more precise meanings.

A second element of the framework proposed in last year’s document is a taxonomy of activity structures or activity types for mathematics teacher education—behavioral configurations that describe the formal division of labor between instructor and students. Some of these activity types could be the same as those found in K-12 classrooms, such as mini-lecture, homework review, etc. (see Lemke, 1990). But there are activity types that are particular to the work of mathematics teacher education assisted by representations of teaching. A quite common activity type could be described as “working on the math:” Quite often, mathematics teacher developers who intend to show a video that displays students working on a mathematical problem will first have their clients work on the mathematics problem that will be featured in the video. Another activity type we have used in teacher education is a form of review of homework in which clients enact scripts of action that they conceive outside of class in response to practical problems of teaching such as how to explain a step in a procedure. Usually those enactments give some clients practice enacting teaching moves they had planned; other clients give feedback, and the event supports raising more general questions about the task of teaching being
learned. In general different activity structures involve clients in interactions with (manipulate, annotate, etc.) the representations being used and for that reason they become quite important in the design of technological tools and interfaces. A partial list of activity types, which we brought to the meeting of the working group last year, is shown in Figure 1.

<table>
<thead>
<tr>
<th>Annotating a representation with free written comments</th>
<th>Creating a mix or mash of existing representations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discussing a representation with peers in class, chat, or forum</td>
<td>Searching for a representation that meets some conditions</td>
</tr>
<tr>
<td>Responding individually or in groups to a question about a representation</td>
<td>Scripting an event or lesson</td>
</tr>
<tr>
<td>Using a rubric to comment on a representation</td>
<td>Enacting scenarios or scripts of action publicly</td>
</tr>
<tr>
<td>Providing a verbal rejoinder to a representation</td>
<td>Rehearsing a practice, strategy, tactic, or technique given a script and a rubric</td>
</tr>
<tr>
<td>Viewing (or reading) a representation</td>
<td>Enacting scenarios or scripts in a chat room</td>
</tr>
<tr>
<td>Comparing two or more representations</td>
<td>Enacting a response to students at a specific moment</td>
</tr>
<tr>
<td>Introducing or framing a representation</td>
<td>Creating examples of actions that illustrate a practice, strategy, tactic, or technique given a rubric</td>
</tr>
<tr>
<td>Creating a new representation given an existing representation one (transcript, video)</td>
<td>Tagging a representation with elements of a rubric or coding scheme</td>
</tr>
<tr>
<td>Adding to a representation events that might come before, after, or in between</td>
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</tbody>
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**Figure 1. Activity types**

A third element of the framework consists of problem types. By this we mean specific intellectual work that participants do within an activity type involving representations—problem types specify the perspective or the goal with which clients confront a representation of practice. Different problem types may engage different kinds of thinking and doing on the part of clients and as a result different problem types might need to be chosen and articulated to promote different kinds of learning. A noticing problem might lead to developing the capacity to spot opportunities to probe student thinking, and a normativity problem might be used to engage the client in designing how to do so. Specific problem types are described in Figure 2.

| What else could [the teacher/a student] do at this moment? (Alternativity) |
| What would you call that [teaching move/mathematical action/etc.]? (Generality) |
| What should be done (what should the teacher/student do) in these circumstances? (Normativity) |
| Which alternative is more appropriate? (Normativity) |
| How would you have felt (what would you have done) if those were your students? (Projectiveness) |
| What do you think about this episode? (Reflectiveness) |
| How would you interpret this episode/action? (Reflectiveness) |
| At what moment would you say [such thing] happened? (Temporality) |
| What do you predict came before/after/in between? (Probability) |
| Which alternative would be more typical/probable? (Usuality/Probability) |
| Which alternative is more desirable according to [some standards]? (Desirability) |
| What did you see students/teacher doing? (Noticing) |

**Figure 2. Problem types**
The final element of this emerging framework addresses the technological affordances needed to realize this pedagogy of mathematics teacher education. Clearly one could do many of these activities having only a video projector and playing media off a single computer. But there are important pedagogical considerations associated with more technology-intensive environments. Chieu, et al. (2011) show evidence that clients’ comments in forum or chat benefitted from having an embedded screen for the animation being discussed, which they could access at the same time as they interacted with peers in a forum or chat. This media-enabled-forum is one of several functionalities available in LessonSketch. We expect the working group will be able to explore these functionalities, and the potential combinations that could be made with them; we also expect the working group to have suggestions of new functionalities to add. Figure 3 includes the tools and resources that we considered in last year’s meeting.

| Show media | Enable the viewing and commenting of others’ attached media |
| Show commented media | Enable the viewing and editing of others’ attached media |
| Show media and enable comments | Show media and enable pinning moments |
| Pose an open ended question | Enable the attachment of media to answers to questions |
| Pose a multiple choice question | Show media and enable marking intervals |
| Enable exporting comments, pins or intervals to forums or documents | Request a decision between representation based alternatives which direct users to different paths |
| Enable the revision of answers | Request the sketching of a scenario with Depict |
| Request an entry in a forum | Enable the creation of a multimedia document |
| Request a diagram or picture | Request an entry in a discussion thread with embedded media |
| Invite to a chat room | Enable the attachment of media to forum posting |
| Offer a text to read | Respond to an individual user’s comment or answer |

Figure 3. Technology tools and functionalities

We envision that instructional actions in a pedagogy of teacher education will integrate the elements of those categories in experiences in which clients use technology tools to work on problems (that resemble one problem type) involving boundary objects and in the context of an activity type supported by a particular representation of practice. These experiences are geared to help them learn about or learn to do a particular practice of teaching.

(c) Plan for active engagement of participants in productive reflection on the issues

The plan includes starting with a brief exposition by the authors of the structure and contents of the present framework. We’ll engage the audience in creating sketches on paper of sessions they’d like to engage their clients in. The idea is to use the collective planning of these sessions to enrich the framework by adding more items to the lists considered, and possibly also adding new categories of elements. The first meeting of the working group will involve a discussion of this document and the demonstration of the current framework. Participants will then form groups and spend the second half of the first session and the first half of the second session creating exemplars. Then the second half of the second session and the closing session will be dedicated to sharing these exemplars and improving the framework.

(d) Anticipated follow-up activities

By the time this working group meets we will have had the third conference on Representations of Mathematics Teaching in Ann Arbor (June 13-15, 2011). We will be proposing a session slot at the AMTE Annual Meeting in 2012 to continue this work. We plan to
use that slot to mirror the work done at the PMENA meeting and to engage in further work on (1) improving the exemplars and (2) using the exemplars to improve the taxonomies. We hope we will be able to use those products to continue this working group at next year’s PMENA.

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