# DETERMINATION OF $\eta^{\prime}$ PARAMETERS FROM THE REACTION 

$\mathrm{K}^{-} \mathrm{p} \rightarrow \Lambda \eta^{\prime} \mathrm{AT} 2.2 \mathrm{GeV} / \mathrm{c}^{*}$
Jerome S. Danburg, Samuel R. Borenstein, George R. Kalbfleisch, Richard C. Strand, and Vance D. VanderBurg Physics Department, Brookhaven National Laboratory;

Upton, New York 11973

J. W. Chapman and Richard K. Kiang Department of Physics, University of Michigan;<br>Ann Arbor, Michigan 48104

## ABSTRACT

We present new data on the $\eta^{\prime}$ based on a sample of events obtained from 870,000 pictures of the BNL 31 -inch hydrogen bubble chamber exposed to a $2.2 \mathrm{GeV} / \mathrm{c}^{-}$beam. We have measured the $\eta^{\prime}$ mass, width, and branching ratios, and we present strong evidence that the quantum numbers of the $\eta^{\prime}$ are $J^{P C}=0^{+}$.

## INTRODUCTION

We have studied the reaction $K^{-} \mathrm{p}^{\rightarrow} \Lambda \eta^{\prime}, \eta^{\prime}$ decaying into 1) $\pi^{+} \pi^{-} \eta_{\mathrm{N}}$, 2) $\pi^{+} \pi^{-} \eta_{c}$, 3) $\pi^{+} \pi^{-} \gamma$, and 4) all neutrals. This report is based on 870,000 pictures (about 35 events per $\mu$ barn) of the hydrogen-filled (with 4 mole-percent neon) 31 -inch BNL bubble chamber to a $2.2 \mathrm{GeV} / \mathrm{c}$ $K^{-}$beam at the AGS. The results given here are not final, because most, but not all of the data selection and fitting procedures are final, and because the data sample will soon be increased by about $15 \%$. We feel, however, that most of the final results will not differ significantly from those given here.
$\eta^{\prime}$ MASS, WIDTH, AND BRANCHING RATIOS
Figs. 1, 2, 3, and 4 show, respectively, the mass spectra of $\pi^{+} \pi^{-} \eta_{\mathrm{N}}, \pi^{+} \pi^{+} \pi^{-} \Pi^{-} \pi^{0}$ (or $\gamma$ ), $\pi^{+} \pi^{-} \gamma$, and neutrals recoiling against a visible $\Lambda$. $\eta_{\mathbb{N}}$ means a missing neutral constrained to have the mass of the $\eta(549 \mathrm{MeV})$. The peak in the $\pi^{+} \pi^{+} \pi^{-} \pi^{-} \pi^{0}$ (or $\gamma$ ) spectrum is associated with the decay sequence $\eta^{\prime} \rightarrow \pi^{+} \pi^{-} \eta_{C}, \gamma_{C} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ (or $\gamma$ ). The choice of $\pi^{0}$ or $\gamma$ is determined by a missing-mass selection.

From an analysis of these four $\eta^{\prime}$ decay modes we have determined the $\eta^{\prime}$ mass, width, and branching ratios given in Table I.
*Work supported by the United States Atomic Energy Commission


Fig. 1. Mass of $\pi^{+} \pi^{-} \eta$ in GeV (10 MeV bins).


Fig. 2. Mass of $\pi^{+} \pi^{+} \pi^{-} \pi^{-} \pi^{0}$ (or $\gamma$ ) in GeV ( 5 MeV bins)


Fig. 3. Mass of $\pi^{+} \pi^{-} \gamma$ in GeV ( 5 MeV bins)


Fig. 4. Mass-squared of neutrals in $\mathrm{GeV}^{2}$ ( $\mathrm{N}_{\mathrm{MeV}} \mathrm{Ma}^{2}$ bins)

TABLE I $\quad \eta^{\prime}$ MASS, WIDTH, BRANCHING RATIOS

Fits to mass spectra of $\pi^{+} \pi^{-} \eta_{\mathrm{N}}, \pi^{+} \pi^{-} \eta_{\mathrm{C}}, \pi^{+} \pi^{-} \mathrm{Y}$ yield:

$$
\begin{gathered}
M\left(\eta^{\prime}\right)=958.4 \pm 0.3 \mathrm{MeV} \\
\Gamma\left(\eta^{\prime}\right)<3.8 \mathrm{MeV}(90 \% \mathrm{conf} .)
\end{gathered}
$$

Branching ratios:

$$
\begin{aligned}
& \frac{\Gamma\left(\pi^{+} \pi^{-} \gamma\right)}{\Gamma\left(\pi^{+} \pi^{-} \eta_{N}\right)}=0.81 \pm 0.09 \\
& \frac{\Gamma(p \gamma)}{\Gamma\left(\pi^{+} \pi^{-} \gamma\right)}=1.1 \pm 0.1 \\
& \frac{\Gamma(\pi \pi \eta)}{\Gamma(a l l)}=0.71 \pm 0.03 \\
& \frac{\Gamma\left(\pi^{+} \pi^{-} \gamma\right)}{\Gamma(a l l)}=0.27 \pm 0.03
\end{aligned}
$$

$$
\frac{\Gamma(\gamma y)}{\Gamma(\text { all })}=0.02 \text { (input) }
$$

The width upper limit of 3.8 MeV is the least definite of the parameters given, and we feel that the final value for the width will be even smaller. The branching ratios displayed are based only on the decays into $\pi^{+} \pi^{-} \eta_{N}$ and $\pi^{+} \pi^{-} \gamma$, since these final states are subject to the same scanning biases. The numbers of events found in the other decay modes are consistent with those in the two modes listed. The $2 \% ~ \gamma Y$ decay mode is the result of recent counter experiments, as compiled by the Particle Data Group.

## $\eta$ ' QUANTUM NUMBERS: DALITZ PLOTS

To determine the $\eta^{\prime}$ quantum numbers we have first fitted the Dalitz plots for the decays $\eta^{\prime} \rightarrow \pi^{+} \pi^{-} \eta_{\mathrm{N}}, \pi^{+} \pi^{-} \eta_{C}, \pi^{+} \pi^{-} \gamma$ to the predictions of various quantum number assignments for $s$ pin values of 0 , 1 , and 2. The $\pi T \eta$ decays are characterized by the orbital angular momentum values $\ell_{\pi \pi}$ (the relative orbital angular momentum of the two pions) and $\ell_{\eta}$ (the relative orbital angular momentum of the $\eta$ with respect to the $\pi \pi$ system) ; for most of the $\pi \pi \eta$ decays only a particular choice is possible. For the $J^{P C}=2^{+}$hypothesis, however, two choices are possible: $\ell_{\eta}=0, \ell_{\pi \pi}=2$; or $\ell_{\eta}=2, \ell_{\pi \pi}=$ 0 . The $J^{\text {PC }}=2^{+}$fits have been done for each choice separately,
and for real and complex mixtures of the two possible matrix elements. For the $\mathrm{J}^{\mathrm{PC}}=0^{+\boldsymbol{+}}$ assignment we have also fitted the $\pi \pi$ decays to the simplest matrix element multiplied by the term ( $1+$ वY), where $Y$ is the $y$-coordinate of the "triangular" Dalitz plot. The explicit form of all the matrix elements is given in the Ph.D. thesis of Rittenberg. ${ }^{2}$

Fig. 5 shows the $\eta^{\prime} \rightarrow \pi^{+} \pi^{-} \eta_{\mathrm{N}}$ Dalitz plot, containing 543
events, about 40 of which are estimated to be background. The nearly uniform population of points on the plot is characteristic of $\mathrm{J}^{\mathrm{P}}=0^{-}$(but such a distribution can also be approximated by a suitable mixture of $J^{P}=2^{-}$matrix elements). Fig. 6 is the $\pi \pi$ mass spectrum of the events in Fig. 5, and Fig. 7 is the distribution of the cosine of the angle between the $\pi^{+}$and the $\eta_{N}$ in the $\pi \pi$
 Fig. 5. $M^{2}\left(\pi^{+}, \eta_{N}\right)$ vs. $M^{2}\left(\pi^{-}, \eta_{N}\right)\left(i n \operatorname{GeV}^{2}\right)$ for $543 \eta^{\prime} \rightarrow \pi^{+} \pi^{-} \eta_{N}$ events


Fig. 6. $\mathrm{M}\left(\pi^{+} \pi^{-}\right)$in GeV for $543 \eta^{\prime} \rightarrow \pi^{+} \pi^{-} \eta$ events ( 5 MeV bins). The curve is the phase space ( $\mathrm{J}^{\mathrm{PC}}=\mathrm{O}^{+}$) prediction.


Fig. 7. Cosine of the $\pi^{+}, \eta_{\mathrm{N}}$ angle in the ${\pi^{+} \pi^{-}}^{-}$rest frame for 543 $\eta^{\prime} \rightarrow \pi^{+} \pi^{-} \eta_{\mathrm{N}}$ events. The straight line is the isotropic distribution predicted by phase space ( $\mathrm{J}^{\mathrm{PC}}=0^{-+}$).
rest frame. The $\eta^{\prime} \rightarrow \pi^{+} \pi^{-} \eta_{C}$ Dalitz plot (not shown) is very similar in appearance to Fig. 5, but it has only about $30 \%$ as many events as the $\pi^{+} \pi^{-} \eta_{\mathrm{N}}$ plot. The results of fitting the $\pi^{+} \pi^{-} \eta_{\mathrm{N}}$ and $\pi^{+} \pi^{-} \eta_{C}$ Dalitz plots to various spin-parity assignments are given in Tables II and III. The results for $\pi^{+} \pi^{-} \eta_{\mathrm{N}}$ and $\pi^{+} \pi^{-} \eta_{C}$ are consistent with each other. From Table II it is seen that only the $J^{P C}=$ $0^{+}$and $2^{+}$assignments are possible, with $0^{+}$strongly favored (but $2^{+}$not ruled out).

## TABLE II




Note: EXCLUDED means conf. level less than $10^{-6}$.
Turning now to the $\pi^{+} \pi^{-} \gamma$ decay mode, Fig. 8 shows the $\pi^{+} \pi^{-}$ mass spectrum for a background-subtracted sample of decays $\eta^{\prime} \rightarrow$ $\pi^{+} \pi^{-} \gamma$. This spectrum is well described by a $\rho$ resonance shape, and leads to the conclusion given in Table I that the $\pi^{+} \pi^{-} \gamma$ decay of the $\eta^{\prime}$ is consistent with being entirely $\eta^{\prime} \rightarrow \rho^{0} \gamma$. The background-subtracted (and folded) distribution of the cosine between the $\pi^{+}$and the $\gamma$ in the $\pi$ rest frame is shown in Fig. 9. Fitting the distributions of Figs. 8 and 9 to various hypotheses for the $\eta^{\prime}$ quantum numbers (for the explicit form of the $\pi^{+} \pi^{-} \gamma$ matrix elements, see

TABLE III



Note: EXCLUDED means conf. level less than $10^{-6}$.
ref. 2) gives the results displayed in Table IV. Again we find $\mathrm{J}^{\mathrm{PC}}=0^{+}$favored over $2^{+}$(which is, however, not excluded). A fit only to the angular distribution yields the result shown as the last entry in Table IV. A particular $J^{P}=2^{-}$matrix element recently proposed ${ }^{3}$ predicts an angular distribution of the form $1.48+\sin ^{2} \theta$. Our data exclude this prediction by more than two standard deviations.

From the Dalitz plot fits to three different decay modes we have seen that $\mathrm{J}^{\mathrm{PC}}=0^{-+}$is favored over $2^{+}$; now we go to a study of decay angular distributions for strong corroborative evidence.


Fig. 8. $\mathrm{M}\left(\pi^{+} \pi^{-}\right)$in GeV after background subtraction for $\eta^{\prime} \rightarrow \pi^{+} \pi^{-} \gamma$


Fig. 9. Folded distribution of the cosine of the $\pi^{+}, \gamma$ angle in the $\pi^{+} \pi^{-}$rest frame after background subtraction for $\eta^{\prime} \rightarrow$ $\pi^{+} \pi^{-} \gamma$. The curve is a sine-squared distribution.

## TABLE IV

$\eta^{\prime} \rightarrow \pi^{+} \pi^{-} y$


Fit to $\cos \left(\pi^{+}, Y\right)$ distribution only:
$f(\theta)=0.1_{-0.1}^{+0.6}+\sin ^{2} \theta$ 0.50

Note: EXCLUDED means conf. level less than $10^{-6}$.

## $\eta$ ' quantum numbers: decay angular distributions

To examine the decay angular distributions of the $\eta^{\prime}$ we work in the Jackson frame, which is the rest frame of the $\eta^{\prime}$, with the beam direction as the $z$-direction, the production normal as the $y$-direction, and the x-direction determined by the right-hand rule. The $\eta^{\prime}$ decay direction is the direction of the normal to the decay plane of the three decay particles ( $\pi \pi \eta$ or $\pi \pi \gamma$ ).

For a $J^{P}=0^{-} \eta^{\prime}$, the decay angular distributions of $\cos \theta$ (the decay cosine) and $\varphi$ (the decay azimuth) must be isotropic. For $J^{P}=2^{-}$it is conceivable that the angular distributions could be isotropic, but for nonzero-spin particles this is in general not the case. Fig. 10 is a plot of $\cos \theta$ vs. $\varphi$ for 543 decays $\eta^{\prime} \rightarrow$ $\pi^{+} \pi^{-} \eta \eta_{N}$. The isotropy of the plot is evident. The corresponding


Fig. 10. $\cos \theta$ vs. $\varphi$ (in degrees) for $543 \eta^{\prime} \rightarrow \pi^{+} \pi^{-} \eta_{N}$ events.
plots for $\eta^{\prime} \rightarrow \pi^{+} \pi^{-} \eta_{C}, \pi^{+} \pi^{-} \gamma$ (not shown) have a similar appearance. Furthermore, no $t$-cut or cut on the A-decay direction was found to produce a plot with any anisotropy. A measure of the isotropy of the decay angular distributions was provided by calculating the moments of $\mathrm{Y}_{l}^{\mathrm{m}}(\theta, \varphi)$ up to $l, \mathrm{~m}=5$ (35 moments) for events in the $\eta^{\prime}$ mass band of the mass spectra of a) the $\pi^{+} \pi^{-} \eta_{N}$, b) the $\pi^{+} \pi^{-} \eta_{C}$, c) the $\pi^{+} \pi^{-} \gamma$, and d) for events in the $\eta^{\prime}$ side bands (930-945 MeV and 975-990 MeV) for the $\pi^{+} \pi^{-} \gamma$ spectrum. The last sample was examined since the $\pi^{+} \pi^{-} \gamma$ sample studied had $37 \%$ background in the signal band, whereas the $\pi^{+} \Pi^{-} \eta_{\mathrm{N}, \mathrm{C}}$ samples included less than $10 \%$ background; it was felt that the $\pi^{+} \pi^{-} \gamma$ side bands would give a good indication of the behavior of the background events in the $\eta^{\prime}$ band. Fig. 11 summarizes the results of the moments calculation for these four samples. Each histogram is the distribution of moments versus statistical significance in standard deviations. For an isotropic angular distribution, each histogram should be a Gaussian curve of width $1 \sigma$, and the four distributions shown are consistent with this behavior. From the four plots containing 140 independent moments we expect about 7 moments more significant than $2 \sigma$. There are in fact a total of 5 moments of more than $2 \sigma$ significance, and no moment is significant (more than $2 \sigma$ ) in more than one sample. From this figure we conclude that all decay distributions (averaged over all t) are compatible with isotropy. The total number of $\eta^{\prime}$ events represented by Fig. 11 is about 1000; this is the largest sample of $\eta^{\prime}$ events examined for decay angular structure at a single energy.

Other experimenters studying smaller samples of $\eta^{\prime}$ decay angular distributions from $K^{-} p \rightarrow \Lambda \eta^{\prime}$ at $2.9 \mathrm{GeV} / \mathrm{c}$ (Brandeis-Maryland-Syracuse-Tufts collaboration) ${ }^{4}$ and at $3.9,4.6 \mathrm{GeV} / \mathrm{c}$ (BNL experiment) ${ }^{5}$ similarly observe no deviation from isotropy in the $\eta^{\prime}$ decay anguiar distributions. We consider it exceedingly unlikely that a production mechanism for a $J^{P}=2^{-} \eta^{\prime}$ could conspire to suppress the $\eta^{\prime}$ decay angular structure over a momentum range from 2 to 5 $\mathrm{GeV} / \mathrm{c}$.

## CONCLUSION

The Dalitz plot fits for the decays $\pi^{+} \pi^{-} \gamma, \pi^{+} \pi^{-} \eta_{C}$, and (especially) $\pi^{+} \pi^{-} \eta_{\mathrm{N}}$ strongly favor $\mathrm{J}^{\mathrm{PC}}=0^{-+}$over $2^{++}$for the $\eta^{\prime}$, but cannot by themselves entirely rule out $2^{-4}$. However, considering together the Dalitz plot results and the lack of $\eta^{\prime}$ decay angular structure for our large sample of $\eta^{\prime}$ events and for other samples at different beam energies provides, we believe, convincing evidence that the $\eta^{\prime}$ quantum numbers are $J^{P C}=0^{-+}$.


Fig. 11. Distribution of statistical significance of 35 moments of the $\eta^{\prime}$ decay angular distribution for four event samples.

1. Particle Data Group, Phys. Letters 39B, 1 (1972).
2. A. Rittenberg, Ph.D. thesis, UCRL 18863, 1969 (unpublished).
3. V.I. Ogievetsky, W. Tybor, and A.N. Zaslavsky, Phys. Letters 35B, 69 (1971).
4. M. Goldberg, private communication.
5. M. Aguilar-Benitez et al., BNL 16675, 1972 (Phys. Rev., to be published).
