# ANTIPROTON ANNIHILATION DYNAMICS IN THE GASDYNAMIC FUSION ROCKET

Terry Kammash and Myoung-Jae Lee
Department of Nuclear Engineering
The University of Michigan
Ann Arbor, MI 48109
(313) 764-0205

## Abstract

The use of antiprotons to initiate the fusion reactions in the Gasdynamic Fusion Rocket (GDFR) is examined as potential replacement of the neutral beam injection system often cited in connection with fusion power reactors. The effectiveness of this approach depends critically, however, on the ability of the antiprotons to penetrate the plasma and reach the center of the engine without undergoing many annihilation reactions along the way. Using expressions for the annihilation rate per unit distance and the stopping power of antiprotons in a fully ionized hydrogenous plasma we calculate the annihilation distribution and the fraction of antiprotons that reach the central region in a relatively cold deuterium-tritium plasma. We apply these results to a rocket engine 16 m in length and containing plasma with $10^{16}$ $cm^{-3}$ density, and we find that well over 90% of the annihilations take place within a few centimeters from the midplane of the engine when the initial plasma temperature is 20 eV. Under these conditions we find that about $10^{-5}$ grams per second of antiprotons injected at an energy of about 4 MeV are required to ignite the plasma in this rocket engine.

## INTRODUCTION

One of the most promising propulsion systems that could be utilized in interplanetary travel is the gasdynamic fusion rocket (GDFR) (Kammash and Lee 1995a). It is based on the magnetic mirror confinement concept in which a hot plasma is confined for a sufficiently long time to produce the needed energy while allowing enough particles to escape through the end to generate the desired thrust. Unlike the conventional mirror where the plasma may be viewed as collisionless, the plasma in the GDFR is of sufficient density to be collisional and hence behave much like a continuous medium — a fluid. Under these conditions the confinement properties change to the extent that the plasma confinement time becomes length dependent, and its dependence on the mirror ratio becomes linear instead of logarithmic. It has been shown that such a device, when used as a propulsion engine, is capable of producing specific impulse well in excess of $10^5$ seconds and thrusts of tens of kilonewtons. It has also been shown that for this device to be an effective thruster it should operate at $Q \geq 1$ where $Q$ is the gain factor defined as the ratio of fusion power to injected power. For many of the missions for which GDFR is particularly suited the injected power would be sufficiently large to require somewhat massive power supplies and associated equipment. This in turn will add significantly to the dry mass of the vehicle and make mission travel times perhaps unacceptably long and economically unattractive.

To circumvent this problem we explore in this paper the potential use of antimatter annihilation to drive the system. It is well known that antimatter is the most energetic on-board energy source for spacecraft propulsion. The question considered here involves the localization of the annihilation region within this plasma-core engine, and the dynamics of the antiprotons that are injected from a vacuum into such a medium. The charged annihilation products which would be combined by the axial magnetic field will heat the background cold plasma to thermonuclear temperatures to initiate the fusion reactions, and some of the energetic ions will then emerge through the mirrors to generate the thrust. As a result, it is necessary to determine the injection energy that will center the annihilation region within the engine and the size of the annihilation region. This size must be small compared to engine size so that all annihilations occur within the engine, and to further minimize the leakage of the annihilation products.

## ANALYSIS

The desired position and spread of the annihilation region can be ascertained by calculating the quantity $F$ which represents the fraction of antiprotons that have annihilated after penetrating a given distance into the engine. A desirable profile for $F$ would be one where it remains small until the penetration distance is near the center and then rise rapidly to unity. In this case a minimum amount of the annihilation products, which carry the annihilation energy will escape from the engine before depositing their energy in the plasma. Calculation of $F$ requires knowing $dp/dx$, the annihilation rate of antiprotons per unit distance. An estimate of this quantity for antiprotons annihilating in a fully ionized hydrogenous plasma for MeV antiproton energies and below is given by (Morgan 1988)

$$\frac{dp}{dx} = 0.19\pi r_0^2 n\left(\frac{\gamma}{1-e^{-\gamma}}\right)\frac{c}{v}, \tag{1}$$

where,

$$\gamma = 2\pi\alpha c/v.$$

In these equations $r_0$ is the classical electron radius ($2.82 \times 10^{-13}$ cm), $n$ the plasma density, $c$ the speed of light, $v$ the antiproton speed and $\alpha$ the fine structure constant (1/137). The above equation gives $dp/dx$ as a function of the antiproton kinetic energy $E$ (through the presence of $v$) rather than as a function of $x$, the distance of the antiproton penetration into the engine. Thus to obtain $F$ it is also necessary to know $dE/dx$, the energy loss rate of antiprotons in the medium which provides the needed relation between $E$ and $x$. Assuming that the antiprotons slow down primarily on the electrons of the plasma the energy loss per unit time can be expressed by (Kammash 1975)

$$\frac{dE}{dt} = v\frac{dE}{dx} = -\frac{16\sqrt{\pi}}{3\sqrt{2}}ne^4\frac{\sqrt{m_e}\,E}{m_{\bar{p}}T_e^{3/2}}\ell n\Lambda = -c_1 E, \tag{2}$$

where $e$ is the electron charge, $T_e$ the electron temperature, $m_{\bar{p}}$ the mass of antiproton, and $\ell n\Lambda$ the familiar Coulomb logarithm. The above equation can be cast in the simpler form

$$\frac{dE}{dx} = -c_1\sqrt{\frac{m_{\bar{p}}}{2}}E^{1/2}, \tag{3}$$

which upon integration yields

$$E^{1/2} = E_0^{1/2} - c_1\sqrt{\frac{m_{\bar{p}}}{8}}\,x, \tag{4}$$

where we have used the fact that $x = 0$ at $E = E_0$, the initial antiproton energy. If we further note that the energy of the antiproton at the end of its path, *i.e.*, at the center of the engine is given by $E_c$, then we can calculate the range $x_c$ directly from Eq. (4), and rewrite it in the form

$$E(x) = \left[E_c^{1/2} + c_1\sqrt{\frac{m_{\bar{p}}}{8}}\left(x_c - x\right)\right]^2. \tag{5}$$

Combining Eqs. (1) and (3) allows us to calculate $p$ as a function of $E$, as:

$$\frac{dp}{dE} = \frac{dp}{dx}\left(\frac{dE}{dx}\right)^{-1}, \tag{6}$$

or

$$\frac{dp}{dE} = \frac{0.19\pi^2 r_0^2 n\alpha c^2 \sqrt{2m_{\bar{p}}}\, E^{-3/2}}{-c_1\left(1 - e^{-2\pi\alpha c\sqrt{m_{\bar{p}}}/\sqrt{2E}}\right)}, \tag{7}$$

which upon integration becomes

$$p = \int_0^p dp = \frac{0.19\pi^2 r_0^2 n\alpha c^2 \sqrt{2m_{\bar{p}}}}{-c_1} \int_{E_0}^{E} \frac{E^{-3/2}\, dE}{1 - e^{-2\pi\alpha c\sqrt{m_{\bar{p}}}/\sqrt{2E}}}. \tag{8}$$

It should be noted in applying these results that the antiproton is assumed to come to a sudden stop at the end of its range and immediately annihilate. The integration shown in Eq. (8) is done numerically, and upon evaluating $p$ then it can be substituted in

$$F = 1 - e^{-p}, \tag{9}$$

to calculate $F$. The value of the initial antiproton energy $E_0$ is adjusted so that $F = 1$ occurs near the center of the engine. The value $F_{1/2}$ of $F$ at the point halfway to the center may be taken as a measure of the spread of the annihilation region and of the fraction of annihilations that occur at possibly undesirable locations. Its value can be calculated from Eqs. (8) and (9) with the value of $E$ obtained from Eq. (5) with $x$ set equal to $x_c/2$.

## RESULTS AND DISCUSSION

A plot of $F$ as a function of the distance traversed by the injected antiprotons is shown in Fig. 1 for different values of the final energy assuming that they become in thermal equilibrium with the target electrons at the end of their motion, *i.e.*, $E_c = T_0$ where $T_0$ is the initial electron temperature. We have applied these calculations to a GDFR engine (Kammash and Lee 1995b) whose length is 16 m so that $x_c = 8$ m represents the center of the device.

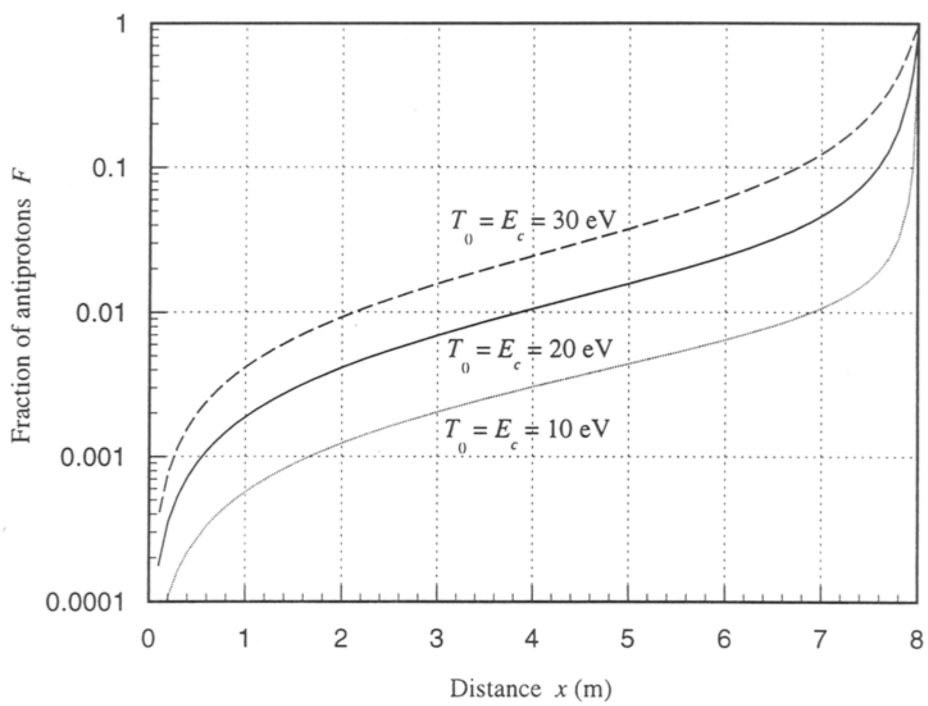


FIGURE 1. Fraction of Annihilation as a Function of Distance Traveled.

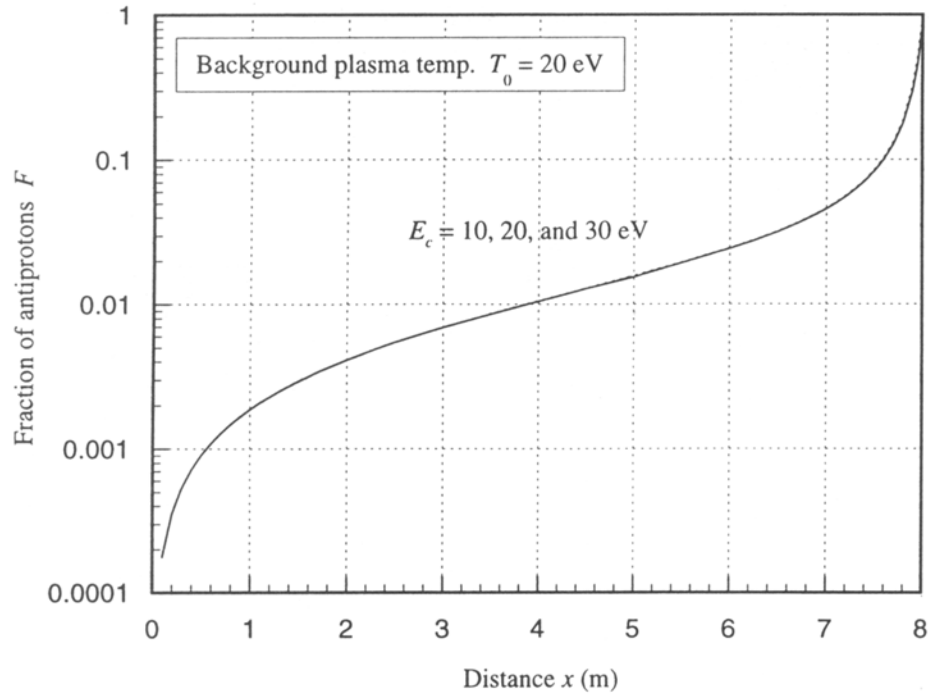


FIGURE 2. Fraction of Annihilation as a Function of Distance for a Fixed $T_0$.

It is shown that the fraction of annihilation is higher and closer to the center the colder the electron temperature. For $T_0$ = 10 eV, over 99% of the annihilations take place within a few centimeters from the center. At higher temperatures the results — though very good in their own sight — are not quite as good since the slowing down model becomes questionable as a result of ignoring the interactions with the ions of the plasma. Figure 2 gives $F$ as function of $x$ for a fixed electron temperature ($T_0$ = 20 eV) and three different final antiproton energies. The interesting result displayed by this plot is that $F$ is almost independent of $E_c$ (the three curves are practically coincident) so long as $E_c$ is significantly smaller than $E_0$ which in this case was about 4 MeV. Once again, it is readily seen that 90% of the annihilation occurs within about 40 cm from the center of the engine. For the engine to which we have applied these results, namely, one with a length of 16 m and a plasma density of $10^{16}$ cm$^{-3}$, the power needed to initiate the fusion reactions at a temperature of 15 keV is about 4700 MW. Using an annihilation energy of 1817 MeV we find that we need $1.6 \times 10^{19}$ antiprotons per second (or $2.7 \times 10^{-5}$ gm/sec) to ignite the plasma, and for a mission that requires a continuous burn a very small amount of antiprotons will be needed to accomplish such a mission particularly if most of the alpha particles generated by the fusion reactions remain in the system to sustain it.

## Acknowledgments

This work was carried out at the Nuclear Engineering Department of The University of Michigan.

## References


Kammash, T. (1975) *Fusion Reactor Physics — Principles and Technology*, Ann Arbor Science Publisher, Ann Arbor, MI: 81-129.

Kammash, T. and M. J. Lee (1995a) "Gasdynamic Fusion Propulsion System for Space Exploration," J. Propulsion and Power **11**: 544-553.

Kammash, T. and M. J. Lee (1995b) "A Near-Term High Performance Fusion Propulsion Concept," AIAA 95-2897, 31st Joint Propulsion Conference, July 10-12, 1995, San Diego, CA, American Institute of Aeronautics and Astronautics, 370 L'Enfant Promenade SW, Washington, DC 20024-2518

Morgan, D. L. Jr. (1988) "Annihilation Localization in Gas-Core and Plasma-Core Annihilation Rocket Engines," IAF-88-554, 39th Congress of the International Astronautical Federation, October 8-15, 1988, Bangalore, India.

## Nomenclature

| | | | |
|---|---|---|---|
| $c$: | Speed of light | $n$: | Plasma density |
| $e$: | Electron charge | $p$: | Number of antiproton annihilations |
| $E$: | Antiproton kinetic energy | $r_0$: | Classical electron radius ($= 2.82 \times 10^{-15}$ m) |
| $E_0$: | Initial antiproton energy | $T_e$: | Electron temperature |
| $E_c$: | Antiproton energy at the center of the engine | $v$: | Antiproton speed |
| $F$: | Fraction of antiprotons reaching center of engine | $x$: | Distance of antiproton penetration into the engine |
| $\ln\Lambda$: | Coulomb logarithm | $x_c$: | Position of engine center |
| $m_{\bar{p}}$: | Mass of antiproton | $\alpha$: | Fine structure constant (1/137) |