Complementarity of the Maldacena and Karch-Randall Pictures [1]

M. J. Duff, James T. Liu and H. Sati

Michigan Center for Theoretical Physics
Randall Laboratory, Department of Physics, University of Michigan
Ann Arbor, MI 48109–1120, USA

Abstract. We perform a one-loop test of the holographic interpretation of the Karch-Randall model, whereby a massive graviton appears on an AdS_4 brane in an AdS_5 bulk. Within the AdS/CFT framework, we examine the quantum corrections to the graviton propagator on the brane, and demonstrate that they induce a graviton mass in exact agreement with the Karch-Randall result. Interestingly enough, at one loop order, the spin 0, spin $1/2$ and spin 1 loops contribute to the dynamically generated $(\text{mass})^2$ in the same $1:3:12$ ratio as enters the Weyl anomaly and the $1/r^3$ corrections to the Newtonian gravitational potential.

1. INTRODUCTION

An old question is whether the graviton could have a small but non-zero rest mass. If so, it is unlikely to be described by the explicit breaking of general covariance that results from the addition of a Pauli-Fierz mass term to the Einstein Lagrangian. This gives rise to the well-known Van Dam-Veltman-Zakharov [1, 2] discontinuity problems in the massless limit, that come about by jumping from five degrees of freedom to two. Moreover, recent attempts [3, 4] to circumvent the discontinuity in the presence of a non-zero cosmological constant work only at tree level and the discontinuity re-surfaces[2] at one loop [6]. On the other hand, in analogy with spontaneously broken gauge theories, one might therefore prefer a dynamical breaking of general covariance, which would be expected to yield a smooth limit. However, a conventional Higgs mechanism, in which a scalar field acquires a non-zero expectation value, does not yield a mass for the graviton. The remaining possibility is that the graviton acquires a mass dynamically and that the would-be Goldstone boson is a *spin one bound state*. Just such a possibility was suggested in 1975 [7].

Interestingly enough, the idea of a massive graviton arising from a spin one bound state Goldstone boson has recently been revived by Porrati [8] in the context of the Karch-Randall brane-world [9] whereby our universe is an AdS_4 brane embedded in an

[1] Talk presented by M. J. Duff
[2] A similar quantum discontinuity arises in the "partially massless" limit as a result of jumping from five degrees of freedom to four[5].

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AdS_5 bulk. This model predicts a small but finite four-dimensional graviton mass

$$M^2 = \frac{3L_5^2}{2L_4^4}, \tag{1}$$

in the limit $L_4 \to \infty$, where L_4 and L_5 are the 'radii' of AdS_4 and AdS_5, respectively. From the Karch-Randall point of view, the massive graviton bound to the brane arises from solving the classical $D = 5$ linearized gravity equations in the brane background [9]. Furthermore, holography of the Karch-Randall model [10, 11] consistently predicts an identical graviton mass.

In a previous paper [12], the complementarity between the Maldacena AdS/CFT correspondence [13, 14, 15] and the Randall-Sundrum [16] Minkowski braneworld picture was put to the test by calculating the $1/r^3$ corrections to the Newtonian gravitational potential arising from the CFT loop corrections to the graviton propagator. At one loop we have [17]

$$V(r) = \frac{G_4 m_1 m_2}{r}\left(1 + \frac{\alpha G_4}{r^2}\right), \tag{2}$$

where G_4 is the four-dimensional Newton's constant,

$$\alpha = \frac{1}{45\pi}(12n_1 + 3n_{1/2} + n_0), \tag{3}$$

and where n_0, $n_{1/2}$ and n_1 count the number of (real) scalars, (Majorana) spinors and vectors in the multiplet. The coefficient α is the same one that determines that part of the Weyl anomaly involving the square of the Weyl tensor [18]. The fields on the brane are given by $\mathcal{N} = 4$ supergravity coupled to a $\mathcal{N} = 4$ super-Yang-Mills CFT with gauge group $U(N)$, for which $(n_1, n_{1/2}, n_0) = (N^2, 4N^2, 6N^2)$. Using both the AdS/CFT relation, $N^2 = \pi {L_5}^3/2G_5$, and the brane world relation, $G_4 = 2G_5/L_5$, we find

$$G_4\alpha = \frac{G_4 {L_5}^3}{3G_5} = \frac{2{L_5}^2}{3}, \tag{4}$$

where G_5 is the five-dimensional Newton's constant. Hence

$$V(r) = \frac{G_4 m_1 m_2}{r}\left(1 + \frac{2{L_5}^2}{3r^2}\right), \tag{5}$$

which agrees exactly with the Randall-Sundrum bulk result.

This complementarity can be generalized to the Karch-Randall AdS braneworld picture. From an AdS/CFT point of view, one may equally well foliate a Poincaré patch of AdS_5 in AdS_4 slices. The Karch-Randall brane is then such a slice that cuts off the AdS_5 bulk. However, unlike for the Minkowski braneworld, this cutoff is not complete, and part of the original AdS_5 boundary remains [9, 11]. Starting with a maximally supersymmetric gauged $\mathcal{N} = 8$ supergravity in the five dimensional bulk, the result is a gauged $\mathcal{N} = 4$ supergravity on the brane coupled to a $\mathcal{N} = 4$ super-Yang-Mills CFT with gauge group $U(N)$, however with unusual boundary conditions on the CFT fields [10, 11, 19, 8, 20].

As was demonstrated in Ref. [8], the CFT on AdS_4 provides a natural origin for the bound state Goldstone boson which turns out to correspond to a *massive* representation of $SO(3,2)$. However, while Ref. [8] considers the case of coupling to a single conformal scalar, in this letter we provide a crucial test of the complementarity by computing the dynamically generated graviton mass induced by a complete $\mathcal{N}=4$ super-Yang-Mills CFT on the brane and showing that this quantum computation correctly reproduces the Karch-Randall result, (1).

We begin by providing a general framework for the dynamical generation of graviton mass. We are mainly interested in the properties of the one-loop graviton self-energy, $\Sigma_{\mu\nu,\alpha\beta}(x,y)$. As emphasized in Refs. [7, 8], mass generation is compatible with the gravitational Ward identity arising from diffeomorphism invariance. Thus the self-energy remains transverse, $\nabla_x^{\mu}\Sigma_{\mu\nu,\alpha\beta}=\nabla_y^{\alpha}\Sigma_{\mu\nu,\alpha\beta}=0$. One is then able to write Σ as a non-local expression evaluated at point x^{μ}, compatible with transversality

$$\Sigma_{\mu\nu,\alpha\beta}(x)=\beta(\Delta)\Pi_{\mu\nu,\alpha\beta}(\Delta)+\gamma(\Delta)K_{\mu\nu,\alpha\beta}(\Delta), \tag{6}$$

where [8]

$$\begin{aligned}\Pi_{\mu\nu}{}^{\alpha\beta} &= \delta_{\mu}^{\alpha}\delta_{\nu}^{\beta}-\frac{1}{3}g_{\mu\nu}g^{\alpha\beta}+2\nabla_{\mu}\left(\frac{\delta_{\nu}^{\beta}+\nabla_{\nu}\nabla^{\beta}/2\Lambda}{\Delta-2\Lambda}\right)\nabla^{\alpha}\\ &\quad -\frac{\Lambda}{3}(g_{\mu\nu}+\frac{3}{\Lambda}\nabla_{\mu}\nabla_{\nu})\frac{1}{3\Delta-4\Lambda}(g^{\alpha\beta}+\frac{3}{\Lambda}\nabla^{\alpha}\nabla^{\beta})\end{aligned} \tag{7}$$

is the transverse-traceless projection and

$$K_{\mu\nu}{}^{\alpha\beta}=\frac{\Delta-\Lambda}{3\Delta-4\Lambda}d_{\mu\nu}d^{\alpha\beta};\quad d_{\mu\nu}=g_{\mu\nu}+\frac{1}{\Delta-\Lambda}\nabla_{\mu}\nabla_{\nu} \tag{8}$$

is the transverse but trace projection. Here, $\Lambda=-3/L_4^2$ is the four-dimensional cosmological constant and Δ is the general Lichnerowicz operator which commutes with covariant derivatives. Symmetrization on $(\mu\nu)$ and $(\alpha\beta)$ is implied throughout.

In Feynman gauge, the tree-level massless graviton propagator in AdS takes the form

$$D_{\mu\nu}{}^{\alpha\beta}=\frac{1}{\Delta-2\Lambda}(\delta_{\mu}^{\alpha}\delta_{\nu}^{\beta}-\tfrac{1}{2}g_{\mu\nu}g^{\alpha\beta}). \tag{9}$$

Using the self-energy written in the form (6), the quantum corrected propagator may be summed to yield

$$\begin{aligned}\widetilde{D}_{\mu\nu}{}^{\alpha\beta} &= \frac{1}{\Delta-2\Lambda-\beta}\left(\delta_{\mu}^{\alpha}\delta_{\nu}^{\beta}-\frac{\Delta-\Lambda}{3\Delta-4\Lambda}g_{\mu\nu}g^{\alpha\beta}\right)\\ &\quad -\frac{1}{\Delta-\Lambda+\gamma/2}\left(\frac{1}{2}\frac{\Delta-\Lambda}{3\Delta-4\Lambda}g_{\mu\nu}g^{\alpha\beta}\right)\end{aligned} \tag{10}$$

when evaluated between conserved sources. This indicates that a constant piece in the traceless self-energy, $\beta = -M^2$, will shift the spin-2 pole in the propagator, thus yielding a non-zero graviton mass. The second term, involving the trace, may combine with the scalar part of the first. However a potentially dangerous scalar ghost pole at $3\Delta = 4\Lambda$ may appear. This ghost is absent whenever the residue of the pole vanishes, *i.e.* provided $\gamma = \beta|_{4\Delta=3\Lambda}$. This is in fact the case, as may be seen by explicit computation below. Although the field theory is conformal, the presence of K is demanded by the Weyl anomaly [18]. However, this trace piece is entirely contained in the local part of Σ, and does not contribute directly to the mass. The net result is a pure massive spin-2 propagator

$$\widetilde{D}_{\mu\nu}{}^{\alpha\beta} = \frac{1}{\Delta - 2\Lambda + M^2}\left(\delta_\mu^\alpha \delta_\nu^\beta - \frac{1}{2}\left(\frac{2\Lambda - 2M^2}{2\Lambda - 3M^2}\right) g_{\mu\nu} g^{\alpha\beta}\right), \tag{11}$$

where we have taken $\beta = -M^2$.

The scalar loop contribution to the self energy was partially computed in Ref. [8]. There, the proper rôle of boundary conditions was emphasized. We find it convenient to work in homogeneous coordinates, which corresponds to the embedding of AdS_4 in R^5 with pseudo-Euclidean metric, $\eta_{MN} = \mathrm{diag}(-,+,+,+,-)$. AdS_4 is then given by the restriction to the hyperboloid $X^M X^N \eta_{MN} = -L_4^2$. Note that we denote homogeneous coordinates as $X^M, Y^M, \ldots$ $(M,N = 0,\ldots,4)$ and intrinsic coordinates as $x^\mu, y^\mu, \ldots$ $(\mu,\nu = 0,\ldots,3)$. Maximally symmetric scalar functions, $\phi(X,Y)$, are simple and can only depend on the invariant $|X-Y|^2/L_4^2 = -2(Z+1)$ where $Z = X\cdot Y/L_4^2$.

A normalized scalar propagator has short-distance behavior

$$\Delta_0(X,Y) \sim \frac{1}{8\pi^2 L_4^2}\frac{1}{Z+1} \sim -\frac{1}{4\pi^2}\frac{1}{|X-Y|^2}, \tag{12}$$

and reduces properly in the flat space limit. However, boundary conditions must still be satisfied by the addition of an appropriate solution to the homogeneous equation. For AdS energy $E_0 = 1$ or 2, and for mixed boundary conditions encoded by parameters α_+, α_-, the scalar propagator takes the form [21]

$$\Delta_0^{(\alpha)} = \frac{1}{8\pi^2 L_4^2}\left(\frac{\alpha_+}{Z+1} + \frac{\alpha_-}{Z-1}\right). \tag{13}$$

Although normalization demands $\alpha_+ = 1$, we nevertheless find it illuminating to keep α_+ arbitrary, as it highlights the symmetries in the latter expressions for the graviton self energy computation. Note that $\alpha_- = 0$ corresponds to transparent boundary conditions, while $\alpha = \pm 1$ corresponds to ordinary reflecting ones.

Using this general form of the scalar propagator, we compute the two-point function of the stress tensor to be [22]

$$\begin{aligned}\langle T_{MN}(X)T_{PQ}(Y)\rangle_0 &= \frac{1}{48\pi^4 L_4^8}\left[\frac{\alpha_+^2}{(Z+1)^4}\left(\frac{3Z^2+1}{4}T_1+T_2+ZT_3\right)\right.\\ &\quad +\frac{\alpha_-^2}{(Z-1)^4}\left(\frac{3Z^2+1}{4}T_1+T_2-ZT_3\right)\\ &\quad \left.+\frac{2}{3}\frac{\alpha_+\alpha_-}{(Z^2-1)^3}(5(3Z^2+1)T_1+(3Z^2-1)T_2-10Z^2T_3)\right] \qquad (14)\end{aligned}$$

(up to contact terms, which we drop). Here we have found it useful to define the three traceless combinations

$$\begin{aligned}T_1 &= \frac{1}{3(3Z^2+1)}[\mathcal{O}_1+16\mathcal{O}_2-4\mathcal{O}_4],\\ T_2 &= -\tfrac{1}{3}\mathcal{O}_1+\tfrac{2}{3}\mathcal{O}_2+\tfrac{1}{2}\mathcal{O}_3+\tfrac{1}{3}\mathcal{O}_4+\tfrac{1}{2}\mathcal{O}_5,\\ T_3 &= \frac{1}{2Z}[4\mathcal{O}_2+\mathcal{O}_5], \qquad (15)\end{aligned}$$

where the $\mathcal{O}_i$'s are a set of basis bi-tensors [23]

$$\begin{aligned}&\mathcal{O}_1=g_{MN}g_{PQ}, \qquad \mathcal{O}_2=n_Mn_Nn_Pn_Q, \qquad \mathcal{O}_3=2\hat{g}_M{}^{(P}\hat{g}_N{}^{Q)},\\ &\mathcal{O}_4=g_{MN}n_Pn_Q+n_Mn_Ng_{PQ}, \qquad \mathcal{O}_5=4\hat{g}_{(M}{}^{(P}n_{N)}n^{Q)}. \qquad (16)\end{aligned}$$

This follows the notation of Ref. [24], except that tensor quantities have been converted to homogeneous coordinates.

A computation for spins 1/2 and 1 with mixed boundary conditions yields a similar result, except for overall factors and the fact that the mixed $\alpha_+\alpha_-$ term is not present. Specializing to the supersymmetric case, to preserve supersymmetry, the boundary conditions on all fields in the multiplet have to be chosen consistently [25]. This means a single set of α_+ (actually always 1) and α_- suffices for specifying the boundary conditions. Furthermore, for a complex scalar in a Wess-Zumino multiplet, the scalar and pseudoscalar transform with opposite boundary conditions (even when the parity condition is relaxed). Since this corresponds to opposite signs for α_- between the scalar and pseudoscalar, we see that the mixed term in (14) always drops out when considering pairs of spin-0 states as members of supermultiplets. As a result, we find a simple universal structure for the graviton self-energy

$$\begin{aligned}\Sigma_{MN,PQ}(X,Y) &= 8\pi G_4\langle T_{MN}(X)T_{PQ}(Y)\rangle\\ &= 8\pi G_4\frac{n_0+3n_{1/2}+12n_1}{48\pi^4L_4^8}\left[\frac{\alpha_+^2}{(Z+1)^4}\left(\frac{3Z^2+1}{4}T_1+T_2+ZT_3\right)\right.\\ &\quad \left.+\frac{\alpha_-^2}{(Z-1)^4}\left(\frac{3Z^2+1}{4}T_1+T_2-ZT_3\right)\right]. \qquad (17)\end{aligned}$$

We now extract the induced graviton mass from the long distance behavior of the self energy (17). We first note that the three terms of Π in Eq. (7) correspond to local tensor, non-local spin-1 and spin-0 exchange, respectively. The mass can be read off by identifying in Σ the spin-1 Goldstone boson exchange, given by the second term. Working in homogeneous coordinates, and using the explicit form of the Goldstone vector propagator, the spin-1 term in Π may be rewritten as a bi-local tensor

$$\Pi = -\frac{2Z}{3\pi^2 L_4^4 (Z^2-1)^3}[5(3Z^2+1)T_1 + 2T_2 - 5(Z^2+1)T_3]. \tag{18}$$

To read off the correctly induced graviton mass, we expand both expressions for large Z and match the asymptotic behavior. We find [22]

$$M^2 = 8\pi G_4 \frac{n_0 + 3n_{1/2} + 12n_1}{160\pi^2 L_4^4}(\alpha_+^2 - \alpha_-^2). \tag{19}$$

This expression is our main result, and generalizes that obtained in Ref. [8][3]. Note that the spin-0 term in Π has a different structure. However this term is canceled by the non-local part of K. The absence of spin-0 exchange in Σ is in agreement with the AdS Higgs mechanism [8], and yields the massive spin-2 propagator (11) without ghosts.

While we have focused on the dynamical breaking of general covariance, as evidenced by a mass for the graviton, in a supersymmetric Karch-Randall model, a dynamical breaking of local supersymmetry and local gauge invariance also occurs, as evidenced by a mass for the gravitinos and the gauge bosons.

For the Karch-Randall braneworld [9], where the CFT fields are that of $\mathcal{N} = 4$ $U(N)$ super-Yang-Mills, we substitute transparent boundary conditions ($\alpha_+ = 1$, $\alpha_- = 0$) into the expression for the graviton mass, (19), and find simply

$$M^2 = \frac{9G_4}{4L_4^4}\alpha, \tag{20}$$

which reproduces exactly the Karch-Randall result of Eq. (1) on using Eq. (4). Although we focused on the $\mathcal{N} = 4$ SCFT to relate the coefficient α to the central charge, the result (4) is universal, being independent of which particular CFT appears in the AdS/CFT correspondence. This suggests that α plays a universal rôle in both the Minkowski and AdS braneworlds, as indicated in (20) and (5), and that our result is robust at strong coupling. This presumably explains why our one-loop computation gives the exact Karch-Randall result. However, we do not know for certain whether this persists beyond one loop.

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[3] We note that this result differs by a factor of 160 from that of Ref. [8]. However we believe the procedure we have followed in extracting the appropriate long-range piece of Σ, which differs from that of [8], leads to the proper mass expression of (19).

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