WORKSHOP ON
SIBERIAN SNAKES AND DEPOLARIZING RESONANCES CORRECTING TECHNIQUES

Organizer: T. Roser (Michigan)

Program:

Saturday, September 10, 1988

9:00  Existing and ongoing efforts
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      T. Roser (Michigan)  Introduction
      K. M. Terwilliger (Michigan)  Resonance strengths
      S. Hiramatsu (KEK)  Resonance jumping and correcting
      D. G. Undervood (ANL)  Spin precession Snake at Fermilab
      A. D. Krisch (Michigan)  Snake demonstration at IUCF Bloomington

      Discussion and/or working groups

2:00  Compact Snake design
      -----------------------
      T. Roser (Michigan)  Partially excited Snake
      E. Courant (BNL)  Continuous helical Snakes
      S. Mane (Fermilab)  Multiturn Siberian Snakes
      U. Wienands (TRIUMF)  Discrete helical Snakes

      Discussion and/or working groups

Sunday, September 11, 1988

9:00  Resonance strength with Snakes
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      S. Tepikian (BNL)  Snake resonances
      K. Yokoya (KEK)  Depolarization in rings with multiple Snakes
      K. Steffen (DESY)  Strong spin-matching with Snakes

      Discussion and/or working groups

2:00  Future machines
      ---------------
      Yu. Shatunov (Novosibirsk)  Comparison of electron and proton machines
      U. Wienands (TRIUMF)  Polarized protons in TRIUMF II
      S. Y. Lee (BNL)  Polarized protons in RHIC
      A. Vasiliev (Serpukhov)  Polarized protons in UNK
      K. Yokoya (KEK)  Polarized protons in SSC

      Discussion and/or working groups
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INTRODUCTION

This first workshop was exclusively devoted to the techniques aimed at preserving the polarization of protons in existing and future accelerators. It was held during September 9 and 10, 1988, in Minneapolis immediately preceding the 8th International Symposium on High Energy Spin Physics.

The two main topics of this workshop were compact Siberian Snake design for lower energy accelerators and the study of depolarization in very high energy machines like the SSC. In both areas recent developments were presented and discussed. In fact with the Siberian Snake demonstration project being completed at this time at the Indiana University Cyclotron Facility1 many of these ideas can soon be studied experimentally for the first time.

DEPOLARIZING RESONANCES

In a circular accelerator the spin of the protons precess around the direction of the main bending field. The number of precessions per revolution (also called 'spin tune') increases with energy according to the relation

\[ \nu_s = G \gamma \]  

(1)

\( G \) is the anomalous magnetic moment of the proton and \( \gamma = E/m \). The spin component in the direction of the precession axis is conserved and thus vertical polarization is not affected by circular accelerators.

Horizontal fields, however, can not be avoided, especially in strong focusing synchotrons. Horizontal fields will drive the polarization away from the vertical direction. If they occur with the same frequency as the spin tune fast resonant depolarization will result. There are three important types of depolarizing resonances:

- **Intrinsic Resonances:** The horizontal fields are the focusing fields in the quadrupoles. They are experienced by the protons with the frequency of the vertical betatron oscillation. The resonance condition is

\[ \nu_s = nP \pm \nu_y \]

(2)

\( n : \) integer

\( P : \) superperiodicity

\( \nu_y : \) vertical betatron tune
Since, on resonance, there are many coherent kicks from horizontal fields per turn (at most as many as there are quadrupoles) intrinsic resonances are strong but there are only few of them.

- **'Weak' Imperfection Resonances:** In this case horizontal fields are imperfection fields mainly from magnet misalignments. Since the fields are experienced by the protons only once per turn the resonance condition is:

\[
\nu_s = n
\]

These resonances are comparatively weak but there are many of them.

- **'Strong' Imperfection Resonances:** Even small imperfection fields can excite large closed orbit oscillation if their frequency is close to the betatron tune. Due to these orbit modulations the protons pass through the horizontal fields of the quadrupoles. The resonance condition is:

\[
\nu_s = nP \pm m \quad m,n: \text{integers}, \quad m \approx \nu_y
\]

Like intrinsic resonances these resonances are strong but there are only few of them.

Depolarization can be reduced or avoided either by reducing the horizontal driving fields or by reducing the number of coherent kicks experienced by the protons during passage through the resonance condition.

The first method involves precision survey and correction of the closed orbit to reduce the strength of the imperfection resonances and minimizing the emittance for the intrinsic resonances. However, all this is also required for optimal performance of the accelerator and thus in most cases is already as good as possible.

The second method was used successfully with weak intrinsic resonances at several machines (AGS, ZGS, KEK). By rapidly changing the vertical betatron tune with fast quadrupoles the length of time the protons are in the resonance condition can be reduced to less than one revolution. An other example is the Siberian Snake concept where the coherence of consecutive kicks is broken by introducing a phase shift in the spin precession.

**THE SIBERIAN SNAKE**

The Siberian Snake idea was first introduced by Derbenev and Kondratenko\(^2\). It consists of a 180° rotation of the spin around a horizontal direction (Siberian Snake precession axis) without affecting the momentum of the particles. It is normally realized by a sequence of alternating horizontal and vertical bends causing the particle to wiggle as it passes through the spin rotator. It is for this reason and its place of invention that E. Courant introduced the name 'Siberian Snake'\(^3\).

Let us first examine the case where two Siberian Snakes are inserted at two locations around the ring that are opposite of each other. One Siberian
Snake rotates the spin around the tangential direction and the other rotates around the radial direction. They are called type I and type II Siberian Snakes, respectively. As a result the polarization will be up in one half of the ring and down an the other half. More important, however, horizontal components of the spin change sign on every turn around the ring. Thus kicks from horizontal fields cannot add up coherently over subsequent turns. Even more, they cancel exactly over two consecutive turns. In the case of intrinsic and strong imperfection resonances with several coherent kicks in one turn it is possible to use several pairs of Siberian Snakes to achieve cancellation already within a section of the ring.

The Siberian Snake concept can also be explained by its effect on the spin tune. It becomes one-half independent of energy and thus none of the resonance conditions can be met as long as the vertical betatron tune is different from a half integer. To ensure that the spin tune is one-half it is sufficient to require that the axes of the Sibirian Snake pair lie in the horizontal plane and are orthogonal. K. Steffen found a whole family of Siberian Snake designs that allow for an arbitrary axis.

In the presence of a (isolated) resonance, however, the spin tune will deviate from one-half according to the formula:

\[
\cos \nu \pi = -\frac{\text{Im}(\epsilon^2)}{\epsilon^2} \sin^2 \left( \frac{\pi |\epsilon|}{2} \right)
\]

We see that when the strength \( \epsilon \) approaches unity the spin tune can become one and depolarization is possible again. Therefore \( \epsilon = 1 \) is the limiting strength for a single Siberian Snake pair.

COMPACT SIBERIAN SNAKES

In low energy machines excursions inside a standard Siberian Snake design become excessively big. In addition existing accelerators usually don’t have long enough straight sections for the typically 10 m long Siberian Snakes. For this reason several people tried to find compact Siberian Snake designs that minimize both excursions and length.

- **Partially Excited Siberian Snake:** The resonance strength in low energy machines (< 50 GeV) is typically less than \( 10^{-2} \). This value is well below the limit for the Siberian Snake. It is thus conceivable to use only a partially excited Siberian Snake - a spin rotator of less than 180°. In this case the kicks on the spin vector are cancelled only after several turns but this should be sufficient particularly for ‘weak’ imperfection resonances. L.Ratner presented a design for a 9° spin rotator that is under consideration at Brookhaven for the AGS.

- **Helical Siberian Snake:** Instead of using discrete horizontal and vertical bends it is possible to use a continuous helical magnet as proposed by E. Courant. This reduces the size of excursions inside the Snake especially...
as one increases the number of twists that the beam goes through inside the Snake.

The concept of the helical Snake can also be realized by discrete skewed bending magnets although not quite as efficiently. U. Wienands presented a family of solutions for a range of discretization degrees and twist numbers. His preferred solution consists of three twists each realized by four magnets.

- **Multi-Turn Steffen Snakes:** S. Mane showed that the family of Snake designs with arbitrary precession axis found by K. Steffen can be generalized to multiple twists. However, it is noted that only designs with tangential Snake axis (type I) show a reduction of the excursions inside the Snake.

The designs of compact Snakes have been studied quite thoroughly. However, it turns out that compact designs could only be found for type I Snakes. It is possible to operate an accelerator with only one type I Snake but in this case the spin direction has to be in the horizontal plane. This would require additional spin rotators in the injection and extraction lines. Unless a compact type II Snake can be found the partially excited Snake offers an attractive solution since even with only one partial Snake the spin direction remains very close to the vertical direction.

**HIGH ENERGY MACHINES**

The strength of depolarizing resonances increases with energy. This results from the fact that the spin rotation associated with a given deflection increases linearly with energy. At some energy the strengths will be bigger than what can be handled by one Siberian Snake pair. This will be the case in machines like the SSC or UNK. The question of how to extend the Siberian Snake concept and/or reduce the strength of the resonances was the focus of many contributions and discussions.

**Intrinsic Resonances**

The strength of intrinsic resonances is expected to increase only with the square root of the energy since the amplitude of the betatron oscillations decreases and thus partially compensates for the increase in spin precession. Nevertheless

\[ \epsilon_{\text{max}}(\text{SSC}) \approx 8. \]  \hspace{1cm} (6)

As mentioned earlier many coherent kicks occur during one revolution. Consequently multiple Siberian Snake pairs should help to avoid depolarization. K. Yokoya found through computer simulation that for

\[ \frac{|\epsilon_{\text{max}}|}{M} < 0.5 \]  \hspace{1cm} (7)

depolarization is small. \( M \) is the number of Snake pairs. For the SSC this would result in the reasonable number of 16 Snake pairs.
Yokoya Terwilliger Derbenev and Kondratenko

\[
\begin{align*}
\v_{quad} & < 0.05 \text{ mm} \\
\phi_{bend} & < 0.1 \text{ mrad} \\
\phi_{bend} & + \leq 0.1 \text{ mrad} \\
\phi_{bend} & + \leq 5 \text{ mm} \\
\phi_{bend} & + \leq 10 \text{ mrad}
\end{align*}
\]

Table 1. Results from three contributions on the maximum size of magnet misalignments to avoid depolarization from 'weak' imperfection resonances.

However, condition (7) is only sufficient if the vertical betatron tune is kept at appropriate values. We have already seen earlier that the Siberian Snake only avoids intrinsic resonances if the betatron tune is different from a half-integer. Higher order extensions of the intrinsic resonance condition (2) lead to more fractional parts of the vertical betatron tune that have to be avoided. For an energy dependent spin tune the particles get accelerated through the resonances and weak higher order resonances are not important. With Siberian Snakes, however, the resonances can be felt at all energies and even weak higher order resonances become important. S. Y. Lee, S. Tepikian and K. Yokoya studied these so called 'Snake resonances' and found that under condition (7) wide enough gaps between Snake resonances exist to place the vertical betatron tune.

K. Steffen went a step further and proposed what he called 'strong spin matching'. He examined the conditions under which the effect of kicks in consecutive sections of the ring separated from each other by a Siberian Snake would cancel exactly. His conditions favor an integer value for the betatron tune of the two consecutive sections. This coincides with avoiding Snake resonances. In addition he obtains conditions for the Snake precession axes. Placing 16 Snake pairs in the SSC would leave 9 focusing-defocusing cells between Snakes. In this case strong spin matching would be achieved with all Snakes being identical. Clearly this would lead to impossible integer spin and betatron tunes for the whole ring. The necessary adjustments, however, could be made in the straight sections.

'Strong' Imperfection Resonances

After closed orbit corrections are applied to the point that the closed orbit excursions are smaller than the amplitudes of the betatron oscillations the strengths of the strong imperfection resonances are also smaller than the strengths of the intrinsic resonances. In addition multiple Snake pairs and strong spin matching are as effective as they are for intrinsic resonances.

'Weak' Imperfection Resonances

Although weak at low energy these resonances will present a problem at SSC energies (20 TeV). Since one can not assume any coherence of the kicks during one revolution multiple Snake pairs might not be more effective than a single pair. However, we did not achieve agreement on this important point during the workshop. Maybe a simulation is required to clarify this question.

Without multiple Snake pairs being effective the strengths of the weak
imperfection resonances have to be less than unity. This puts severe upper limits on the magnet misalignments for the 20 $TeV$ SSC. We had three independent contributions to the workshop that tried to estimate the maximum values for the two most critical alignment parameters: the vertical position of the quadrupoles ($y_{quad}$) and the roll angle of the bending magnets ($\phi_{bend}$). Table 1. summarizes the results. There is good agreement between K. Yokoya and K. M. Terwilliger but their values are difficult to realize. The limits from a contributed paper of Ya. S. Derbenev and A. M. Kondratenko, however, are much less severe. We were not able to resolve this discrepancy during the workshop but everybody agreed that this question needs to be answered preferably with a realistic simulation calculation.

CONCLUSIONS

In theory it has been firmly established that the insertion of Siberian Snakes into high energy accelerators prevents depolarization. We are eagerly awaiting the results from the first experimental test at IUCF.

Many designs were put forward for compact low energy Siberian Snakes although only of type I. Partially excited Siberian Snakes might offer a promising alternative for low energy accelerators.

New understanding but also new questions developed on the prospects of polarized protons in very high energy machines. Multiple Snake pairs and strong spin matching should allow to avoid depolarization from intrinsic and 'strong' imperfection resonances. However, it is not yet clear how to deal with the large number of 'weak' imperfection resonances each of which can be quite strong at very high energy.

I would like to thank all participants for their contributions which made this workshop a success. I would especially like to thank L. Ratner for initially suggesting the organization of this workshop.

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REFERENCES

1. A.D.Krisch, these proceedings.