PROGRESS IN M-THEORY*

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After reviewing how M-theory subsumes string theory, we report on some new and interesting developments, focusing on the "brane-world".

1. M-theory and dualities

Not so long ago it was widely believed that there were five different superstring theories each competing for the title of "Theory of everything," that all-embracing theory that describes all physical phenomena. See Table 1.

Moreover, on the (d, D) "branescan" of supersymmetric extended objects with d worldvolume dimensions moving in a spacetime of D dimensions, all these theories occupied the same $(d = 2, D = 10)$ slot. See table 2. The orthodox wisdom was that while $(d = 2, D = 10)$ was the Theory of Everything, the other branes on the scan were Theories of Nothing. All that has now changed. We now know that there are not five different theories at all but, together with $D = 11$ supergravity, they form merely six different corners of a deeper, unique and more profound theory called "M-theory" where M stand for Magic, Mystery or Membrane. M-theory involves all of the other branes on the branescan, in particular the eleven-dimensional membrane $(d = 3, D = 11)$ and eleven-dimensional fivebrane $(d = 6, D = 11)$, thus resolving the mystery of why strings stop at ten dimensions while supersymmetry allows eleven[2].

Although we can glimpse various corners of M-theory, the big picture still eludes us. Uncompactified M-theory has no dimensionless parameters, which is good from the uniqueness point of view but makes ordinary perturbation theory impossible since there are no small coupling constants to provide the expansion parameters. A low energy, E, expansion is possible in powers of E/M_P, with M_P the Planck mass, and leads to the familiar $D = 11$ supergravity plus corrections of higher powers in the curvature. Figuring out what governs these corrections would go a long way in pinning down what M-theory really is.

Why, therefore, do we place so much trust in a theory we cannot even define? First we know that its equations (though not in general its vacua) have the maximal number of 32 supersymmetry charges. This is a powerful constraint and provides many "What else can it be?" arguments in guessing what the theory looks like when

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	Gauge Group	Chiral?	Supersymmetry charges
Type I	$SO(32)$	yes	16
Type IIA	$U(1)$	no	32
Type IIB	–	yes	32
Heterotic	$E_8 \times E_8$	yes	16
Heterotic	$SO(32)$	yes	16

Table 1: The Five Superstring Theories

compactified to $D < 11$ dimensions. For example, when M-theory is compactified on a circle S^1 of radius R_{11}, it leads to the Type IIA string, with string coupling constant g_s given by

$$g_s = {R_{11}}^{3/2} \tag{1}$$

We recover the weak coupling regime only when $R_{11} \to 0$, which explains the earlier illusion that the theory is defined in $D = 10$. Similarly, if we compactify on a line segment (known technically as S^1/Z_2) we recover the $E_8 \times E_8$ heterotic string. Moreover, although the corners of M-theory we understand best correspond to the weakly coupled, perturbative, regimes where the theory can be approximated by a string theory, they are related to one another by a web of dualities, some of which are rigorously established and some of which are still conjectural but eminently plausible. For example, if we further compactify Type IIA string on a circle of radius R, we can show rigorously that it is equivalent to the Type IIB string compactified on a circle or radius $1/R$. If we do the same thing for the $E_8 \times E_8$ heterotic string we recover the $SO(32)$ heterotic string. These well-established relationships which remain within the weak coupling regimes are called *T-dualities.* The name *S-dualities* refers to the less well-established strong/weak coupling relationships. For example, the $SO(32)$ heterotic string is believed to be S-dual to the $SO(32)$ Type I string, and the Type IIB string to be self-S-dual. If we compactify more dimensions, other dualities can appear. For example, the heterotic string compactified on a six-dimensional torus T^6 is also believed to be self-S-dual. There is also the phenomenon of *duality of dualities* by which the T-duality of one theory is the S-duality of another. When M-theory is compactified on T^n, these S and T dualities are combined into what are termed U-dualities. All the consistency checks we have been able to think of (and after 5 years there dozens of them) have worked and convinced us that all these dualities are in fact valid. Of course we can compactify M-theory on more complicated manifolds such as the four-dimensional $K3$ or the six-dimensional Calabi-Yau spaces and these lead to a bewildering array of other dualities. For example: the heterotic string on T^4 is dual to the Type II string on $K3$; the heterotic string on T^6 is dual to the the Type II string on Calabi-Yau; the Type IIA string on a Calabi-Yau manifold is dual to the Type IIB string on the mirror Calabi-Yau manifold. These more complicated compactifications lead to many more parameters in the theory, known to the mathematicians as *moduli*, but

$D \uparrow$												
11	.			S			T					
10	.	V	S/V	V	V	V	S/V	V	V	V	V	
9	.	S				S						
8	.				S							
7	.			S			T					
6	.	V	S/V	V	S/V	V	V					
5	.	S		S								
4	.	V	S/V	S/V	V							
3	.	S/V	S/V	V								
2	.	S										
1	.											
0	.	.	.	.	.	.	.	.	.	.	.	.
	0	1	2	3	4	5	6	7	8	9	10	11 $d \rightarrow$

Table 2: The branescan, where S, V and T denote scalar, vector and antisymmetric tensor multiplets.

in physical uncompactified spacetime have the interpretation as expectation values of scalar fields. Within string perturbation theory, these scalar fields have flat potentials and their expectation values are arbitrary. So deciding which topology Nature actually chooses and the values of the moduli within that topology is known as the *vacuum degeneracy problem.*

2. Branes

In the previous section we outlined how M-theory makes contact with and relates the previously known superstring theories, but as its name suggest, M-theory also relies heavily on membranes or more generally p-branes, extended objects with $p = d - 1$ spatial dimensions (so a particle is a 0-brane, a string is a 1-brane, a membrane is a 2-brane and so on). In $D = 4$, a charged 0-brane couples naturally to an Maxwell vector potential A_μ, with field strength $F_{\mu\nu}$ and carries an electric charge

$$Q \sim \int_{S^2} * F_2 \tag{2}$$

and magnetic charge

$$P \sim \int_{S^2} F_2 \tag{3}$$

where F_2 is the Maxwell 2-form field strength, $*F_2$ is its 2-form dual and S^2 is a 2-sphere surrounding the charge. This idea may be generalized to p-branes in D dimensions. A p-brane couples to $(p+1)$-form potential $A_{\mu_1 \mu_2 \ldots \mu_{p+1}}$ with $(p+2)$-form field strength $F_{\mu_1 \mu_2 \ldots \mu_{p+2}}$ and carries an "electric" charge per unit p-volume

$$Q \sim \int_{S^{D-p-2}} * F_{D-p-2} \tag{4}$$

and "magnetic" charge per unit p-volume

$$P \sim \int_{S^{p+2}} F_{p+2} \quad (5)$$

where F_{p+2} is the $(p+2)$-form field strength, $*F_{D-p-2}$ its $(D-p-2)$-form dual and S^n is an n-sphere surrounding the brane. A special role is played in M-theory by the BPS (Bogomolnyi-Prasad-Sommerfield) branes whose mass per unit p-volume, or tension T, is equal to the charge per unit p-volume

$$T \sim Q \quad (6)$$

This formula may be generalized to the cases where the branes carry several electric and magnetic charges. The supersymmetric branes shown on the branescan are always BPS, although the converse is not true. M-theory also makes use of non-BPS and non-supersymmetric branes not shown on the branescan, but the supersymmetric ones do play a special role because they are guaranteed to be stable.

The letters S, V, and T on the branescan refer to scalar, vector and antisymmetric tensor supermultiplets of fields that propagate on the worldvolume of the brane. Historically, these points on the branescan were discovered in three different ways. The S branes were classified by writing down spacetime supersymmetric worldvolume actions that generalize the Green-Schwarz actions on the superstring worldsheet[5]. By contrast, the V and T branes were shown to arise as soliton solutions[a] of the underlying supergravity theories[6]. However, the solitonic V branes found this way were bound by $p \leq 7$. The 8-brane and 9-brane slots were included on the scan only because they were allowed by supersymmetry[6]. Subsequently, all the V-branes were given a new interpretation as Dirichlet p-branes, called D-branes, surfaces of dimension p on which open strings can end and which carry R-R (Ramond-Ramond) charge[9]. The IIA theory has D-branes with $p = 0, 2, 4, 6, 8$ and the IIB theory has D-branes with $p = 1, 3, 5, 7, 9$. They are related to one another by T-duality. In terms of how their tensions depend on the string coupling g_s, the D-branes are mid-way between the fundamental (F) strings and the solitonic (S) fivebranes:

$$T_{F1} \sim {m_s}^2, \quad T_{Dp} \sim \frac{{m_s}^{p+1}}{g_s}, \quad T_{S5} \sim \frac{{m_s}^6}{{g_s}^2} \quad (7)$$

Since they are BPS, there is a no-force condition between the branes that allows us to have many branes of the same charge parallel to one another. The gauge group on a single D-brane is an abelian $U(1)$. If we stack N such branes on top of one another, the gauge group is the non-abelian $U(N)$. As we separate them this decomposes into its subgroups, so in fact there is a Higgs mechanism at work whereby the vacuum expectation values of the Higgs fields are related to the separation of the branes. For example the theory that lives on a stack of N Type IIB $D3$ branes is a four-dimensional $U(N)$ $n = 4$ super Yang-Mills theory. In the limit of large N

[a]The 3-brane soliton of Type IIB supergravity was an early candidate for a 'brane-world', firstly because of its dimensionality[7,8] and secondly because gauge fields propagate on its worldvolume[8].

the geometry of this configuration tends to the product of five dimensional anti-de Sitter space and a five dimensional sphere, $AdS_5 \times S^5$.

In $D = 11$, M-theory has two BPS branes, an electric 2-brane and its magnetic dual which is a 5-brane. Their tensions are related to each other and the Planck mass by

$$T_2^3 \sim T_5 \sim M_P^6 \tag{8}$$

if we stack N such branes on top of one another, the M2-brane geometry tends in the large N limit to $AdS_4 \times S^7$ and the M5-brane geometry to $AdS_7 \times S^4$. In addition there are two other objects in $D = 11$, the plane wave and the Kaluza-Klein monopole, which though not branes are still BPS. When spacetime is compactified a p-brane may remain a p-brane or else become a $(p-k)$-brane if it wraps around k of the compactified directions. For example, the Type IIA fundamental string emerges by wrapping the M2-brane around S^1 and shrinking its radius to zero, and the Type IIA 4-brane emerges in a similar way from the $M5$-brane.

3. Spin-offs of M-theory

What do we now know with M-theory that we did not know with old-fashioned string theory? Here are a few examples, references to which may be found in Ref. 2.

1) Electric-magnetic (strong/weak coupling) duality in $D = 4$ is a consequence of string/string duality in $D = 6$ which in turn is a consequence of membrane/fivebrane duality in $D = 11$.

2) *Exact* electric-magnetic duality, first proposed for the maximally supersymmetric conformally invariant $n = 4$ super Yang-Mills theory, has been extended to *effective* duality by Seiberg and Witten to non-conformal $n = 2$ theories: the so-called Seiberg-Witten theory. This has been very successful in providing the first proofs of quark confinement (albeit in the as-yet-unphysical super QCD) and in generating new pure mathematics on the topology of four-manifolds. Seiberg-Witten theory and other $n = 1$ dualities of Seiberg may, in their turn, be derived from M-theory.

3) Indeed, it seems likely that all supersymmetric quantum field theories with any gauge group, and their spontaneous symmetry breaking, admit a geometrical interpretation within M-theory as the worldvolume fields that propagate on the common intersection of stacks of p-branes wrapped around various cycles of the compactified dimensions, with the Higgs expectation values given by the brane separations.

4) In perturbative string theory, the vacuum degeneracy problems arises because there are billions of Calabi-Yau vacua which are distinct according to classical topology. Like higher-dimensional Swiss cheeses, each can have different number of p-dimensional holes. This results in many different kinds of four-dimensional gauge theories with different gauge groups, numbers of families and different choices of quark and lepton representations. Moreover, M-theory introduces new non-perturbative effects which allow many more possibilities, making the degeneracy problem apparently even worse. However, most (if not all) of these manifolds are

in fact smoothly connected in M-theory by shrinking the p-branes that can wrap around the p-dimensional holes in the manifold and which appear as black holes in spacetime. As the wrapped-brane volume shrinks to zero, the black holes become massless and effect a smooth transition from one Calabi-Yau manifold to another. Although this does not yet cure the vacuum degeneracy problem, it puts it in a different light. The question is no longer why we live in one topology rather than another but why we live in one particular corner of the unique topology. This may well have a dynamical explanation.

5) Ever since the 1970's, when Hawking used macroscopic arguments to predict that black holes have an entropy equal to one quarter the area of their event horizon, a microscopic explanation has been lacking. But treating black holes as wrapped p-branes, together with the realization that Type II branes have a dual interpretation as Dirichlet branes, allows the first microscopic prediction in complete agreement with Hawking. The fact that M-theory is clearing up some long standing problems in quantum gravity gives us confidence that we are on the right track.

6) It is known that the strengths of the four forces change with energy. In supersymmetric extensions of the standard model, one finds that the fine structure constants $\alpha_3, \alpha_2, \alpha_1$ associated with the $SU(3) \times SU(2) \times U(1)$ all meet at about 10^{16} GeV, entirely consistent with the idea of grand unification. The strength of the dimensionless number $\alpha_G = GE^2$, where G is Newton's constant and E is the energy, also almost meets the other three, but not quite. This near miss has been a source of great interest, but also frustration. However, in a universe of the kind envisioned by Witten, spacetime is approximately a narrow five dimensional layer bounded by four-dimensional walls. The particles of the standard model live on the walls but gravity lives in the five-dimensional bulk. As a result, it is possible to choose the size of this fifth dimension so that all four forces meet at this common scale. Note that this is much less than the Planck scale of 10^{19} GeV, so gravitational effects may be much closer in energy than we previously thought; a result that would have all kinds of cosmological consequences.

So what is M-theory?

There is still no definitive answer to this question, although several different proposals have been made. By far the most popular is M(atrix) theory[10]. The matrix models of M-theory are $U(N)$ supersymmetric gauge quantum mechanical models with 16 supersymmetries. Such models are also interpretable as the effective action of N coincident Dirichlet 0-branes.

The theory begins by compactifying the eleventh dimension on a circle of radius R, so that the longitudinal momentum is quantized in units of $1/R$ with total P_L N/R with $N \to \infty$. The theory is *holographic* in that it contains only degrees of freedom which carry the smallest unit of longitudinal momentum, other states being composites of these fundamental states. This is, of course entirely consistent with their identification with the Kaluza-Klein modes. It is convenient to describe these N degrees of freedom as $N \times N$ matrices. When these matrices commute, their simultaneous eigenvalues are the positions of the 0-branes in the conventional sense. That they will in general be non-commuting, however, suggests that to properly understand M-theory, we must entertain the idea of a fuzzy spacetime in which spacetime coordinates are described by non-commuting matrices. In any event, this matrix approach has had success in reproducing many of the expected properties of M-theory such as $D = 11$ Lorentz covariance, $D = 11$ supergravity as the low-energy limit, and the existence of membranes and fivebranes.

It was further proposed that when compactified on T^{d-1}, the quantum mechanical model should be replaced by an d-dimensional $U(N)$ Yang-Mills field theory defined on the dual torus $\tilde{T}^{d-1}$. Another test of this M(atrix) approach, then, is that it should explain the U-dualities. For $d = 4$, for example, this group is $SL(3, Z) \times SL(2, Z)$. The $SL(3, Z)$ just comes from the modular group of T^3 whereas the $SL(2, Z)$ is the electric/magnetic duality group of four-dimensional $n = 4$ Yang-Mills. For $d > 4$, however, this picture looks suspicious because the corresponding gauge theory becomes non-renormalizable and the full U-duality group has still escaped explanation. There have been speculations on what compactified M-theory might be, including a revival of the old proposal that it is really M(embrane)theory. In other words, perhaps $D = 11$ supergravity together with its BPS configurations: plane wave, membrane, fivebrane, KK monopole and the $D = 11$ embedding of the Type IIA eightbrane, are all there is to M-theory and that we need look no further for new degrees of freedom, but only for a new non-perturbative quantization scheme. At the time of writing this is still being hotly debated.

What seems certain, however, is that M-theory is not a string theory. It can be approximated by a string theory only in certain peculiar corners of parameter space. So "string phenomenology" will become an oxymoron unless, for some as yet unknown reason, our universe happens to occupy one of these corners.

4. AdS/CFT and the brane-world

The year 1998 marked a renaissance in anti de-Sitter space (AdS) brought about by Maldacena's conjectured duality between physics in the bulk of AdS and a conformal field theory on its boundary[11]. For example, M-theory on $AdS_4 \times S^7$ is dual to a non-abelian $(n = 8, d = 3)$ superconformal theory, Type IIB string theory on $AdS_5 \times S^5$ is dual to a $(n = 4, d = 4)$ $U(N)$ super Yang-Mills theory and M-theory on $AdS_7 \times S^4$ is dual to a non-abelian $((n_+, n_-) = (2, 0), d = 6)$ conformal theory. In particular, as has been spelled out most clearly in the $d = 4$ $U(N)$ Yang-Mills case, there is seen to be a correspondence between the Kaluza-Klein mass spectrum in the bulk and the conformal dimension of operators on the boundary[12,13]. We note that, by choosing Poincaré coordinates on AdS_5, the metric may be written as

$$ds^2 = e^{-2y/L}(dx^\mu)^2 + dy^2, \tag{9}$$

where x^μ, $(\mu = 0, 1, 2, 3)$, are the four-dimensional brane coordinates. In this case the superconformal Yang-Mills theory is taken to reside at the boundary $y \to -\infty$.

The AdS length scale L is given by

$$L^4 = 4\pi\alpha'^2(g_{YM}^2 N) \tag{10}$$

The string coupling g_s and the Yang-Mills coupling g_{YM} are related by

$$g_s = g_{YM}{}^2 \tag{11}$$

The full quantum string theory on this spacetime is difficult to deal with, but we can approximate it by classical Type IIB supergravity provided

$$L^2 >> \alpha' \tag{12}$$

so that stringy correction to supergravity are small, and that $g_s << 1$ or

$$N \to \infty \tag{13}$$

so that loop corrections can be neglected. There is now overwhelming evidence in favor of this correspondence and it allows us to calculate previously uncalculable strong coupling effects in the gauge theory starting from classical supergravity. Models of this kind, where a bulk theory with gravity is equivalent to a boundary theory without gravity, have also been advocated by 't Hooft[14] and by Susskind[15] who call them *holographic* theories. Many theorists are understandably excited about the AdS/CFT correspondence because of what it can teach us about non-perturbative QCD. In my opinion, however, this is, in a sense, a diversion from the really fundamental question: What is M-theory? So my hope is that this will be a two-way process and that superconformal field theories will also teach us more about M-theory.

The Randall-Sundrum mechanism[16] also involves AdS but was originally motivated, not via the decoupling of gravity from D3-branes, but rather as a possible mechanism for evading Kaluza-Klein compactification by localizing gravity in the presence of an uncompactified extra dimension. This was accomplished by inserting a positive tension 3-brane (representing our spacetime) into AdS_5. In terms of the Poincaré patch of AdS_5 given above, this corresponds to removing the region $y < 0$, and either joining on a second partial copy of AdS_5, or leaving the brane at the end of a single patch of AdS_5. In either case the resulting Randall-Sundrum metric is given by

$$ds^2 = e^{-2|y|/L}(dx^\mu)^2 + dy^2, \tag{14}$$

where $y \in (-\infty, \infty)$ or $y \in [0, \infty)$ for a 'two-sided' or 'one-sided' Randall-Sundrum brane respectively.

The similarity of these two scenarios led to the notion that they are in fact closely tied together. To make this connection clear, consider the one-sided Randall-Sundrum brane. By introducing a boundary in AdS_5 at $y = 0$, this model is conjectured to be dual to a cutoff CFT coupled to gravity, with $y = 0$, the location of the Randall-Sundrum brane, providing the UV cutoff. This extended version of the Maldacena conjecture[17] then reduces to the standard AdS/CFT duality as the boundary is pushed off to $y \to -\infty$, whereupon the cutoff is removed and gravity becomes completely decoupled. Note in particular that this connection involves a single CFT at the boundary of a single patch of AdS_5. For the case of a brane sitting between two patches of AdS_5, one would instead require two copies of the CFT, one for each of the patches. A crucial test of this assumed complementarity of the Maldacena and Randall-Sundrum pictures is that both should yield the same corrections to Newton's law [18].

A third development in the brane-world has been the idea that the extra dimensions are compact but much larger than the conventional Planck sized dimensions in traditional Kaluza-Klein theories[19]. This is possible if the standard model fields are confined to the $d = 4$ brane with only gravity propagating in the $d > 4$ bulk[19]. We shall not pursue this possibility here.

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