HEAVY FLAVOR RESONANCES AND QED RADIATIVE CORRECTIONS*

J. ALEXANDER, G. BONVICINI, P. DRELL, R. FREY AND V. LUTH

Lawrence Berkeley Laboratory, University of California, Berkeley CA 94720

University of Michigan, Ann Arbor, MI 48109

Stanford Linear Accelerator Center, Stanford University, Stanford CA 94309

ABSTRACT

An application of high precision QED against experimental data is presented. When the corrections to ψ and Υ families are improved according to the method described below, the masses and widths of the resonances below open flavor trheshold change by up to three standard deviations from presently accepted experimental values.

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In this paper we examine in detail the QED corrections to very narrow resonances such as the ψ and Υ particles. We focus on the determination of their mass M, their total width Γ , and their partial width to electrons Γ_e^0 . Our objective is to review experimental results that were obtained in analyses with incorrect radiative corrections. Our analysis shows in fact that the errors incurred are sometimes bigger than the uncertainties quoted for the current world averages.^[1]

In the presence of a very narrow resonance, the radiative corrections depend critically on the exact treatment of the infrared region, and differences in the formulae used to fit the data have an impact on many resonance parameters. A basic understanding of the infrared divergences associated with the vanishing photon mass was first achieved by Bloch and Nordsieck in 1937. They stated that in charged particle scattering the number of photons emitted is undetermined, and that the cross section for the emission of zero energy and no photons is exactly zero. Many treatments of radiative corrections that exist in the literature violate this theorem by containing terms that correspond to a finite, non-zero probability for the emission of no photons. In particular, a result from truncated perturbation expansion alone produces elastic (i.e., with no photon emitted) terms which violate the Bloch-Nordsieck theorem. All the results which were shown to produce errors in the extraction of the resonance mass and width at the Z^0 include such terms.

1. Initial State Radiative Corrections to Narrow Resonances

In e^+e^- collisions, the nominal collision energy, $\sqrt{s}=2E$, is set by E, the energy of the incident beams. The actual c.m. energy available for the annihilation is reduced by Bremsstrahlung to $\sqrt{s(1-k)}$, where kE is the total energy of the emitted photons. The observed cross section, $\sigma_{obs}(s)$ at the nominal energy \sqrt{s} , can be written as a convolution of the cross section $\sigma_0(s(1-k))$ and a dimensionless sampling function f(k,s), [3]

$$\sigma_{obs}(s) = \int f(k,s)\sigma_0(s(1-k)) dk. \tag{1.1}$$

and

$$\sigma_0 = \sigma_{nonres} + \sigma_{peak} \frac{s \Gamma^2}{(s - M^2)^2 + s \Gamma^2}, \tag{1.2}$$

where M is the mass and Γ is the total width of the resonance.

Expressions for f(k, s) which attain the required precision of 1% have been obtained by several authors in the literature. It is well known that f(k, s) is dominated by inital state effects. Effects of final state radiation on the cross section are usually ignored at the fraction of a percent level.

We employ the following expression for f(k,s), truncated to first order in the hard photon terms, and to second order in the vertex terms:

$$f(k,s) = (1+\delta_{vp})(1+K)[\beta k^{\beta-1}(1+\delta_1+\delta_2) - \beta(1-\frac{k}{2})].$$
 (1.3)

 β is the electron equivalent radiator thickness,

$$\beta = \frac{2\alpha}{\pi} (\log \frac{s}{m_e^2} - 1). \tag{1.4}$$

The δ_n terms arise from the leading parts of the vertex correction diagrams of order n, δ_{vp} is the vacuum polarization contribution and K is the K-factor. All these terms are reported in the Appendix.

In the past, most experimenters have fit the narrow resonances of the ψ and Υ families using a different expression for f(k,s), based on the classic work of Jackson and Scharre, [9]

$$f'(k,s) = \delta_{tot}\delta(k) + \beta k^{\beta-1} - \beta(1-\frac{k}{2}), \qquad \delta_{tot} = \delta_1 + \delta'_{vp} + K. \tag{1.5}$$

 $\delta(k)$ refers to the Dirac function. This expression has been obtained from a first order perturbative calculation with the inclusion of exponentiation.

There are essential differences between the distribution functions f(k,s) and f'(k,s) and the way they get convoluted according to Eq. (1.1). The differences occur in second order in α . First, in the formulation by Jackson and Scharre, the photon vacuum polarization δ'_{vp} is approximated by the electron loop δ_e only, excluding contributions from hadrons, muons, and τ leptons, δ_h , δ_μ , and δ_τ . Secondly, the vertex correction, $(1 + \delta_1)$, should multiply the Bremsstrahlung term $k^{\beta-1}$, at least to first order, and hence should enter as an overall multiplicative factor to the soft term, as in our definition of f(k,s). The factorization of the virtual terms arises naturally from those semi-classical formalisms which are based on factorization principles. This factorization of the virtual corrections can be checked to first order by doing an explicit second order calculation. A second order calculation does not, however, determine unambiguously that the δ_2 term factorizes, though it is a natural choice and it agrees with the Bloch-Nordsieck theorem. In the definition of f'(k,s), the virtual corrections were not properly

separated and the $\delta(k)$ term gives a finite probability for the electron and positron to annihilate without soft photon emission, in direct disagreement with the Bloch-Nordsieck theorem. This locally distorts the cross section by a fraction $\delta_{tot} \approx 14\%$ at 10 GeV and $\delta_{tot} \approx 10\%$ at 3 GeV.

The convolution integral of a Breit-Wigner resonance cross section with f(k,s) can be solved analytically. We use the expression given in the Appendix, which was derived by Cahn^[5] for the Z^0 resonance. We have added the photon vacuum polarization and δ_2 terms. We also account for the energy spread of the incident beams (which can be two orders of magnitude bigger than the resonance width, therefore forbidding direct observation of the resonance structure) by further convoluting the cross section with a Gaussian resolution function of the appropriate width σ_E . The error associated with Eq.(1.3)is of order 1%, mostly in the normalization due to vacuum polarization uncertainties.

2. Distortion of the Resonance Shape and Analysis Method

A resonance is described by its mass, M, and two of the following three parameters: the total width, Γ , the cross section integral A, and the cross section at the peak, σ_{peak} . These three parameters are related by the equation

$$A = \frac{\pi}{2} \Gamma \sigma_{peak}. \tag{2.1}$$

A can be related to the measured partial width to electrons, Γ_e^{exp} , and the branching ratio for this process, in our case B_{had} , by

$$A = \frac{\pi}{2} \Gamma \sigma_{peak} = \frac{6\pi^2}{M^2} \Gamma_e^{exp} B_{had} \quad \text{with} \quad B_{had} = \frac{\Gamma_{had}}{\Gamma}. \tag{2.2}$$

Under the assumption that the total width is the sum of the partial width to hadrons and charged lepton pairs and that the leptonic widths are all equal, we have

$$\Gamma = \Gamma_{had}^{exp} + m\Gamma_e^{exp}, \quad \text{and} \quad mB_e + B_{had} = 1.$$
 (2.3)

Here m stands for the number of partial widths into lepton pairs, m=2 for charmonium and m=3 for bottomonium states. The leptonic branching ratios are determined experimentally, and therefore the relations above can be used to measure the quantities Γ and Γ_e^{exp} .

We note explicitly the nature of Γ_e^{exp} , defined in Eq. (2.3), and draw the distinction with the quantity of theoretical interest, $\Gamma_e^{[12]}$ Γ_e^0 . Γ_e^{exp} is the physical coupling of the resonance to leptons through one photon, and is obtained from the data by making all radiative corrections except vacuum polarization corrections. This is the quantity which, divided by the measured branching ratio, gives the total width. The value of Γ_e^0 , on the other hand, is drawn from the data by making all radiative corrections including vacuum polarization. Thus Γ_e^0 reflects the coupling strength at tree level only. The quantity Γ_{had} , which couples to the resonance mostly through three gluons, does not have QED vacuum polarization corrections, and in this case $\Gamma_{had}^{exp} = \Gamma_{had}^0$.

Historically, experimenters have generally included some level of vacuum polarization in their corrections, and have therefore implicitly extracted Γ_e^0 . For the remainder of our discussion we follow this precedent, though at the end we include values for Γ_e^{exp} in summary tables. The relationship between the two quantities is

$$\Gamma_e^{exp} = (1 + \delta_{vp})\Gamma_e^0. \tag{2.4}$$

Since radiative effects in the final states are negligible, the branching ratios do not depend on radiative corrections. Thus differences in the formulation of the radiative corrections will cause changes in two parameters, the integral A and the partial width Γ_e^0 . They will scale proportionally, with a factor that depends on the branching ratio for the particular channel under study. If one studies simultaneously the resonance cross sections into hadrons, muon pairs, and electron pairs, the three integrals will change by the same fraction, giving approximately the same change to Γ_e^0 , while the ratio between the three integrals (which determines the branching ratios) remains unchanged.

The difference between our treatment of the radiative corrections and the formulation by Jackson and Scharre is illustrated in Figure 1. We plot the difference between the cross section for the $\Upsilon(9460)$ calculated with f(k,s) and f'(k,s) using the same input parameters. f'(k,s) overestimates the cross section on the resonance and below it, and underestimates the cross section above the resonance. We illustrate in this figure both the case where the vacuum polarization in f'(k,s) includes all terms, $\delta'_{vp} = \delta_e + \delta_\mu + \delta_\tau + \delta_h$, and where it is reduced to the electron loop, $\delta'_{vp} = \delta_e$. This latter case is, in fact, the formulation that most previous experiments had used to fit narrow resonances. It is evident that the use of the electron loop alone in the vacuum polarization reduces the difference in the predicted cross section at the peak resulting from the incorrect treatment of the virtual terms in f'(k,s).

The magnitudes of the shifts in the parameters obtained by the fit to the resonance enhancement will depend on details that will vary from experiment to experiment, such as the ratio of resonant to non-resonant cross section, the amount

of integrated luminosity taken on the peak, and the energy spread of the machine.

To correctly reproduce the complicated interplay of the fit parameters and to study the dependence and correlations among them, we resort to a technique of simulating the data obtained by various experiments to measure the ψ and Υ resonances. We generate data points by calculating the cross section at a given energy \sqrt{s} using our definition f(k,s) and errors proportional to $\sqrt{\sigma_{obs}}$. Subsequently, the generated data points are fit by functions based on both f(k,s) and f'(k,s). We study the changes to the fitted resonance parameters using the hadronic cross sections only. The four free parameters of the fit are M, Γ , σ_E and σ_{nonres} . B_e , the branching ratio into electrons, is fixed at the world average value.^[1]

For a compact presentation of the results in the following section we find it convenient to introduce the ratio

$$C = \frac{\delta_{tot}}{\delta_1 + \delta_{vv} + \delta_2 + K},\tag{2.5}$$

with the term δ_{tot} as defined in f'(k,s). The denominator is the factor multiplying the soft term in f(k,s) when we expand the product with σ_0 and assume the virtual terms are small. In the denominator, we take $\delta_{vp} = \delta_e + \delta_\mu + \delta_\tau + \delta_h$, while the value of δ'_{vp} implicitly contained in δ_{tot} may be reduced to δ_e as in the Jackson and Scharre ansatz. Using this ansatz we obtain C = 0.85 and C = 0.70 at 3.1 GeV and 10 GeV, respectively. Using full vacuum polarization in δ_{tot} we obtain C = 1.03.

3. Analysis of Simulated Data

In this section, we show how we apply corrections to published experimental results on the parameters of narrow resonances based on fits to our simulated data. We deliberately consider only experiments listed in the 1986 Review of Particle Properties. In changing values of the resonance parameters derived from previously applied radiative corrections to new values derived with our definition of the sampling function f(k, s), we strictly use information contained in the original experimental and theoretical f(k, s) papers.

In correcting published values of the resonance parameters, we take account of the fact that experiments differ from one another in several significant ways. First, e^+e^- storage rings differ in their energy resolution. Second, in different experiments, the percentage of the total luminosity collected on the resonance peak, as compared to below or above the peak, can vary substantially. These effects introduce small ($\approx .5 - 1\%$) differences in the corrections to different experiments which have been taken into account. Finally, most of the measurements have been radiatively corrected based on the prescription by Jackson and Scharre. For those, we typically derive changes in Γ_e^0 of 2% at the $\Upsilon(9460)$ by fits to simulated data. Adding the full and correct vacuum polarization to f'(k,s), results in a large correction to Γ_e^0 of $\approx 9\%$. Other experiments derived resonance parameters using algorithms ^{[4][14]} which are identical to ours, except for the vacuum polarization terms, and the δ_2 term.

In summary, the fact that the changes to the resonance parameters vary from experiment to experiment is almost completely due to the differences in the radiative corrections (which in turn differ either in normalization or in the value of C).

The dependence of the correction to M and Γ_e^0 versus C will generate corrections to the leptonic width and mass as displayed in Figures 2 and 3. The shift in the mass ΔM is normalized to the energy resolution σ_E , because we find empirically that for a fixed ratio C the mass shift is proportional to σ_E . This behaviour can be understood, because the equivalent radiator thickness β is the same at the ψ and Υ to within 10%. These curves can be used to correct experimental results which are not listed here.

Table 1 lists the values previously measured along with our refitted ones for the experiments that determine the mass and widths of the ψ and Υ resonances and that are referenced in the 1986 Review of Particle Properties. We would like to point out that our method is one of simulation; it shows fluctuations of typically 2-3% in the fitted parameters when cross sections are assigned errors that are comparable to those in published experiments. The overall error of our method, based on much smaller point to point errors, is conservatively estimated to be 1%.

Using the corrections to Γ_e^0 , we have derived the corrections to Γ taking into account the error on the branching ratio. We decouple the measurement of Γ_e^0 and Γ by consistently using the world average branching ratio, and not the particular value as measured by a given experiment.

Table 2 contains the summary of our results, presented in the form of new world averages for the resonance parameters that change significantly with our new analysis. Quantities which do not change the world average by at least 50% of a

standard deviation are not listed here. The corrections to resonances above open flavor threshold resemble the corrections discussed for the $Z^{0,[3]}$ and they are small.

4. Conclusions and Discussion

In conclusion, we have reviewed prescriptions for QED radiative corrections to resonance production by e^+e^- annihilation, and present a formula for QED corrections to narrow resonances (convoluted with a Gaussian resolution function to account for the spread in the beam energy) which has an estimated uncertainty of 1.0% in the 3-10 GeV energy region.

Recently two other papers [16] [17] have dealt with the subject of radiative corrections to narrow resonances. Both use a formulation that is consistent with f(k,s). Buchmüller and Cooper [17] rescale the results for the Υ states using only the peaks of the resonances, thereby obtaining changes to world averages which are slightly larger than ours. The correction method of Königsmann [16] gives results for the Υ resonances which are nearly identical to those of Ref. 17. However, his results for the ψ states differ substantially from ours, and we believe that this is because our method of simulating cross section data correctly accounts for the various nontrivial effects arising from a resonance fit. As discussed in Section 2, the J/ψ and ψ' resonance data are more sensitive to these effects than are the Υ data.

We have applied our prescription to correct existing measurements of the mass, total width, and electron partial width for the ψ and Υ resonances. The observed shifts are small, but when we combine the new values for all experiments and form new world averages, the changes are significant. The values of several quantities

change as a result of our reevaluation of the radiative corrections by up to one standard deviation. The implications of our analysis for quarkonium potential models have been discussed elsewhere.^{[16][17]}

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APPENDIX

We use in our analysis the form of the distribution function f(k, s) convoluted with a Gaussian beam energy spread according to Cahn^[5]. We add, however, the δ_2 term:

$$g(s) = \sigma_{peak}(1+\delta_1+\delta_2)\frac{\Gamma^2}{\Gamma^2+M^2} \left[\frac{s}{M^2} a^{\beta-2} \Phi(\cos\theta,\beta) - a^{\beta-1} \frac{\beta}{1+\beta} \Phi(\cos\theta,1+\beta) \right] - \sigma_{peak} \beta \frac{\Gamma}{\sqrt{s}} \left[\tan^{-1} \frac{2M}{\Gamma} - \tan^{-1} \frac{2(M-\sqrt{s})}{\Gamma} \right].$$

The quantities a, $\cos \theta$, and $\Phi(\cos \theta, \beta)$ are defined as follows:

$$a^{2} = \frac{b^{2} + c^{2}}{d}, \cos \theta = -\frac{Mb + \Gamma c}{ad}, \Phi(\cos \theta, \beta) = \frac{\pi \beta \sin((1 - \beta)\theta)}{\sin \pi \beta \sin \theta}$$

where $b = M(s/M^2 - 1)$, $c = \Gamma s/M$ and $d = \Gamma^2 + M^2$. The terms δ_1 , δ_2 are

$$\delta_1 = \frac{3}{4}\beta, \qquad \delta_2 = -\frac{\beta^2}{24}(\frac{1}{3}\log\frac{s}{m_e^2} + 2\pi^2 - \frac{37}{4}).$$

The K-factor and δ_{vp} terms are

$$K = \frac{\alpha}{\pi} (\frac{\pi^2}{3} - \frac{1}{2}), \delta_{vp} = \delta_l + \delta_h \quad \text{with} \quad \delta_l = \delta_e + \delta_\mu + \delta_\tau.$$

The vacuum polarization contribution of charged leptons of mass m_i is

$$\delta_l = -\sum_i \frac{2\alpha}{\pi} (\frac{5}{9} + \frac{1}{3} \log \frac{m_i^2}{s}).$$

The hadronic part of the vacuum polarization, δ_h , is calculated numerically ^[18] and is $\delta_h = 1.1 \pm 0.5\%$ at 3 GeV and $3.4 \pm 1.0\%$ at 10 GeV. The quoted uncertainties are our estimates.

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TABLE CAPTIONS

- 1: Summary of the corrections to the parameters of ψ and Υ resonances, by experiment, as listed in Ref. 1, Meson Full Listing.
- 2: New world averages for those resonance parameters which change by more than 50% of a standard deviation. Also given are the percentage change in the experimental quantities, and the statistical significance of the change in units of overall experimental error.

FIGURE CAPTIONS

- 1) The difference between the cross section for the Υ (9460) calculated with f(k,s) from Eq. (1.3) and with f'(k,s) from Eq. (1.5) using the same input parameters. The solid line represents the difference for the full vacuum polarization terms in f'(k,s), while the broken line gives the difference for only the electron contribution the vacuum polarization. C is defined later in the text.
- 2) Corrections to Γ_e^0 versus C for the five narrow resonances of the ψ and Υ families. The corrections to the $\Upsilon(10023)$, are roughly equal for $\sigma_E=8$ MeV and for $\sigma_E=4$ MeV.
- 3) Corrections to the mass M as a function of the ratio C. ΔM is given in units of the machine resolution σ_E .

Table 1

Quantity	Reference	New value Γ_e^0	New value Γ_e^{exp}	Old value
$\Gamma_e,J/\psi(3097)$	Boyarski	4.6 keV	4.8 keV	4.8 keV
$\Gamma_e, J/\psi(3097)$	Baldini	4.5 keV	4.7 keV	4.6 keV
$\Gamma_e, J/\psi(3097)$	Esposito	4.5 keV	4.7 keV	4.6 keV
$\Gamma_e, J/\psi(3097)$	Brandelik	4.5 keV	4.6 keV	4.4 keV
$\Gamma_e, \psi(3685)$	Lüth	2.0 keV	2.1 keV	2.1 keV
$\Gamma_e, \psi(3685)$	Brandelik	2.1 keV	$2.2~{ m keV}$	$2.0~{ m keV}$
$\Gamma_e, \Upsilon(9460)$	Berger	1.36 keV	1.46 keV	1.33 keV
$\Gamma_e, \Upsilon(9460)$	Bock	1.10 keV	1.18 keV	1.08 keV
$\Gamma_e, \Upsilon(9460)$	Albrecht	1.25 keV	1.34 keV	1.23 keV
$\Gamma_e, \Upsilon(9460)$	Niczyporuk	1.15 keV	1.24 keV	1.13 keV
$\Gamma_e, \Upsilon(9460)$	Tuts	1.18 keV	1.27 keV	1.15 keV
$\Gamma_e, \Upsilon(9460)$	Giles	1.42 keV	1.53 keV	1.30 keV
Γ_e , $\Upsilon(10023)$	Bock	0.40 keV	.43 keV	0.39 keV
$\Gamma_{\epsilon}, \Upsilon(10023)$	Niczyporuk	0.58 keV	.62 keV	0.56 keV
$\Gamma_e, \Upsilon(10023)$	Albrecht	0.60 keV	.65 keV	0.58 keV
$\Gamma_e, \Upsilon(10023)$	Tuts	0.58 keV	.62 keV	0.56 keV
Γ_e , $\Upsilon(10023)$	Giles	0.57 keV	.61 keV	0.52 keV
$\Gamma_e, \Upsilon(10355)$	Tuts	0.40 keV	.43 keV	0.39 keV
$\Gamma_e, \Upsilon(10355)$	Giles	0.46 keV	.49 keV	0.42 keV
$M, \Upsilon(9460)$	Artamonov	9460.5 MeV	-	9460.6 MeV
$M, \Upsilon(9460)$	Mac Kay	9459.87 MeV	-	9459.97 MeV

Table 2

Quantity	New world	New world	Fractional	Statistical
	average	average, $\Gamma_e = \Gamma_\mu$	change	change
$\Gamma_e^0, J/\psi(3097)$	4.57± 0.51 keV	$4.53\pm~0.35~\mathrm{keV}$	-4.0 %	0.5 σ
$\Gamma_e^0,\psi(3685)$	-	2.05 ± 0.21 keV	0	0
$\Gamma_e^0, \Upsilon(9460)$	-	$1.279 \pm 0.050 \text{ keV}$	4.5 %	1.1 σ
$\Gamma_e^0, \Upsilon(10023)$	-	$0.569 \pm 0.033 \; \mathrm{keV}$	6.0 %	1.0 σ
$\Gamma_e^0, \Upsilon(10355)$	-	$0.423 \pm 0.031 \text{ keV}$	5.2 %	0.7 σ
$\Gamma_e^{exp}, J/\psi(3097)$	4.77± 0.51 keV	$4.72\pm~0.35~\mathrm{keV}$	+0.4 %	0.1 σ
$\Gamma_e^{exp}, \psi(3685)$	-	$2.14 \pm 0.21 \text{ keV}$	4.4 %	.4 σ
$\Gamma_e^{exp}, \Upsilon(9460)$	-	$1.376 \pm 0.050 \text{ keV}$	12.4 %	3.0 σ
$\Gamma_e^{exp}, \Upsilon(10023)$	-	$0.612 \pm 0.033 \; \mathrm{keV}$	14.0 %	2.3 σ
$\Gamma_e^{exp}, \Upsilon(10355)$	-	$0.455 \pm 0.031 \text{ keV}$	13.2 %	1.7 σ
Γ, Υ(9460)	-	$48.5 \pm 3.2 \text{ keV}$	12.6 %	1.7 σ
Γ, Υ(10023)	-	$34.2 \pm 7.3 \text{ keV}$	14.0 %	0.6 σ
$M, \Upsilon(9460)$	9459.93±0.19 MeV	-	0.001 %	0.5σ

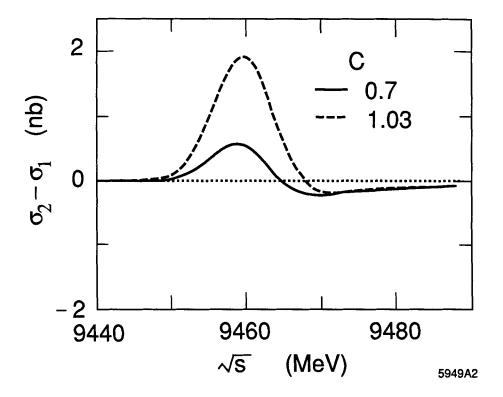
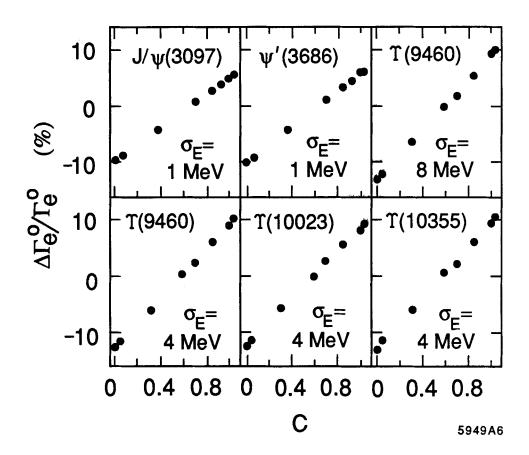


Fig. 1



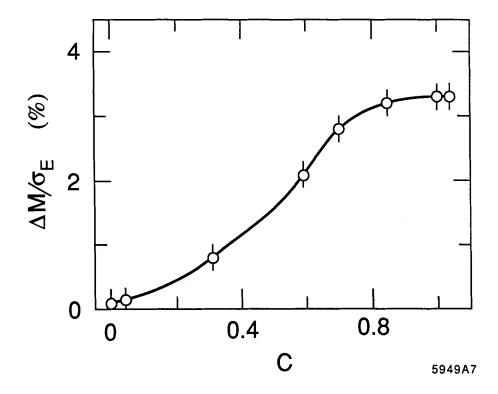


Fig. 3