Renormalons and $1/Q^2$ corrections

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Abstract.
   We argue that the appearance of the Landau pole in the running coupling of QCD introduces $1/Q^2$ power corrections in current correlator functions. These terms are not accounted for by the standard operator product expansion and is the price to be paid for the lack of a unique definition of the running coupling at the $1/Q^2$ level. We review also possible phenomenological implications of the $1/Q^2$ terms in an alternative language of ultraviolet renormalon.

   1. Renormalons by construction are a part of the dynamics of the standard model since they are simply a set of perturbative graphs existing within, say, QED or QCD [1] (for a review see, e.g., Ref. [2]). Nevertheless, there are links of renormalons to the physics beyond the standard model. For example, one may consider renormalons within theories going beyond the standard model, in particular, supersymmetric gauge theories [2]. In this part of the talk we shall choose another route and concentrate on hints to the non-standard dynamics revealed by renormalons. More specifically, we will consider $1/Q^2$ terms which are absent from the standard operator product expansion [3] but are indicated by the ultraviolet (UV) renormalons [4–8]. The material we are presenting is to a great extent of a review nature. There are also some new points, in particular, the relation of the $1/Q^2$ terms to the Landau ghost has not been emphasized so far.

   2. To explain, why the $1/Q^2$ terms could signal a kind of non-standard dynamics let us first outline schematically the standard picture. For our purposes, it is most coveniently formulated in terms of the QCD sum rules. The basic features of the sum rules is the emphasis on the power-like corrections. In a simplidfied form, one derives the sum rules of the type:

$$\int exp(-s/M^2)R(s)ds \approx (parton\ model)\left(1 + \frac{\alpha_s(M^2)}{\pi} + c_G\frac{<0|\alpha_s(G^a_{\mu\nu})^2|0>}{M^4} + ...\right)$$

(1)

where $R(s)$ is the total cross section of e^+e^- annihilation into hadrons in the

standard units, M^2 is a large mass parameter and $< 0|\alpha_s(G^a_{\mu\nu})^2|0 >$ is the so called gluon condensate, c_G is a coefficient calculable perturbatively.

Moreover, in analyzing the sum rules one assumes that it is the power-like corrections of order M^{-4} which signal breaking of asymptotic freedom at moderate M^2 of order $\leq (GeV)^2$. Phenomenologically, this breaking is due to appearances of resonances. Note the absence of $1/M^2$ corrections from Eq. (1). This is a direct consequence of the OPE since the first gauge invariant operator, that is $\alpha_s(G^a_{\mu\nu})^2$ has dimension $d = 4$.

It is important to emphasize that the matrix element $< 0|\alpha_s(G^a_{\mu\nu})^2|0 >$ is saturated by infrared contributions. In particular, the $1/M^4$ corrections can be traced by means of infrared renormalons [9]. Within the renormalon approach the gluon condensate manifests itself as $n!$ growth of the coefficients a_n of perturbative expansions in the running coupling $\alpha_s(Q^2)$:

$$(a_n)_{IR} \sim \int_0^{\sim Q^2} k^2 dk^2 (b_0)^n ln(Q^2/k^2)^n \sim b_0^n 2^{-n} n!. \qquad (2)$$

Here n is the order of perturbative expansion considered to be large, b_0 is the first coefficient of the β-function which is positive in QCD, k^2 is virtual mometum flowing through gluon line and Q is an external momentum such that $Q^2 \gg \Lambda^2_{QCD}$. The large numerical value of a_n is due to the contribution of characteristic momenta of order

$$k^2_{char} \sim e^{-n/2} Q^2. \qquad (3)$$

Independent of whether one is using the general OPE or IR renormalons, the resulting picture can be described in a very simple way: the presence of low-lying resonances on the phenomenological side is signaled by power corrections of infrared narure derivable within fundamental QCD.

3. This picture, which seems perfectly selfconsistent, is challenged by UV renormalons. Ultraviolet renormalons are known [1] to dominate perturbative expansions at large orders of perturbation theory. The expansion coefficient at large n is proportional to:

$$(a_n)_{UV} \sim \int_{\sim Q^2}^\infty (b_0)^n \frac{dk^2}{k^4} (lnQ^2/k^2)^n \sim (-1)^n b_0^n n!. \qquad (4)$$

Note that we will use a one-term β-function for simplicity. In fact, evaluation of the UV renormalon can be pursued much further [5,10] but the improvements are not crucial for our purposes.

Thus, there are two basic features of the UV renormalons which are important for our discussion:

(i) the UV renormalon is Borel summable because of the sign oscillations, $(-1)^n$,

(ii) UV renormalons are related to very large virtual momenta:

$$k_{char}^2 \sim e^n Q^2. \tag{5}$$

Both features seem to meet our intuition: because of asymptotic freedom the large momenta in QCD should represent no major prblems and, as a reflection of this, the contribution of the UV renormalons can be summed up.

However, a paradox arises [4] once it is realized that the UV renormalons is Borel summable to a $1/Q^2$ piece:

$$\sum_{n_{crit}}^{\infty} b_0^n (-1)^n n! (\alpha_s(Q^2))^n \rightarrow c_{UV} \frac{\Lambda_{QCD}^2}{Q^2} \tag{6}$$

where n_{crit} is the critical value of the order of the perturbative expansion starting from which the perturbative contributions start to rise as function of n because the factor $n!$ prevails over the factor $(\alpha_s(Q^2))^n$ and c_{UV} is a constant, while the arrow means that the standard Borel summation is applied to the asymptotical perturbative expansion.

What is surprising about the Eq. (6) is that it defies the standard picture outlined above. Indeed, phenomenologically the Λ_{QCD}^2/Q^2 piece would be associated with the contributions of low-lying resonances while the UV renormalon is associated with large virtual momenta $k^2 \gg Q^2 \gg \Lambda_{QCD}^2$. Thus, it appears that physics in the infrared should match physics in ultraviolet as far as power like corrections are involved.

Another important point about the UV renormalons is that the Borel summation is not the only way to deal with the divergence (4) of the perturbative expansions and the results of alternative procedures is not obviously the same. Namely, one can utilize either a conformal mapping [11] or expansion in the coupling normalized at a high scale μ^2, $\mu^2 \gg Q^2$ [12] to avoid the divergence due to the UV renormalon. In particular, if one uses the expansion in $\alpha_s(\mu^2)$ then the uncertainty of the perturbative expansion due to its asymptotical nature caused by the UV renormalons is of order [12]:

$$\Delta_{UV}(Q^2) \sim c_{UV} \frac{\Lambda_{QCD}^2}{Q^2} \frac{Q^4}{\mu^4} \tag{7}$$

and can be made arbitrarily small by choosing $\mu^2 \gg Q^2$. Although this trick might appear to solve the problem of the UV renormalon it rather brings new problems in fact. Indeed, if one detects the presence of $1/Q^2$ correction in one formulation of the perturbative expansion and looses track of it while using another formulation then this might imply either the inconsistency of the whole approach or existence of further consistency conditions to be yet established.

4. The paradoxes are resolved, to our mind, by the simple observation that perturbatively the coupling is not well defined at the $1/Q^2$ level because of the Landau pole. Indeed, if we use the running coupling in the standard form:

$$\alpha_s(Q^2) = \frac{1}{b_0 ln Q^2/\Lambda_{QCD}^2} \tag{8}$$

then we introduce a pole at $Q^2 = \Lambda^2_{QCD}$ which is a fake singularity in the sence that all the perturbative graphs have only cuts which start from $s = 0$. The emergence of the Landau pole is an artifact of the summation procedure which leads to (8). This summation procedure is well known to be justified in the leading log approximation and does not introduce any inconsistency at the logarithmic level. However, as far as the power like correction $1/Q^2$ is concerned it is in fact not fixed by perturbation theory itself. Note that our simplifying use of the one-term β-function is not crucial at this point, at least as far as the use of a more realistic β-function does not remove the Landau pole.

Thus, when one demonstrates the absence of the $1/Q^2$ corrections by applying the expansion in $\alpha_s(\mu^2), \mu^2 \to \infty$ (see discussion above) one heavily uses in fact the analyticity properties of the perturbative graphs. On the other hand, the use of the running coupling $\alpha_s(Q^2)$ to demonstrate the $1/Q^2$ uncertainty due to the UV renormalon relies on a resummation procedure which does not observe these analyticuty properties on the level of the $1/Q^2$ terms. As far as the problem of the Landau pole is not solved within the the perturbation theory there is no way to decide which derivation is preferable.

One may introduced a running coupling with the Landau ghost removed:

$$\tilde{\alpha}_s(Q^2) = \frac{1}{b_0 ln Q^2/\Lambda^2_{QCD}} + \frac{\Lambda^2_{QCD}}{b_0(\Lambda^2_{QCD} - Q^2)} \tag{9}$$

which at large Q^2 clearly differs from the "standard" coupling (8) by a $1/Q^2$ correction. This kind of redefinition of the coupling goes back to the fifties [13] and was reviewed very recently in the context of QCD in Ref. [14] where further references can also be found. One can of course introduce further modifications which in turn remove the $1/Q^2$ term:

$$\tilde{\alpha}'_s(Q^2) = \frac{1}{b_0 ln Q^2/\Lambda^2_{QCD}} + \frac{\Lambda^2_{QCD}}{b_0(\Lambda^2_{QCD} - Q^2)} + \frac{\Lambda^2_{QCD}}{b_0 Q^2} \tag{10}$$

According to ref. [14] the advantage of this form is that the pole is shifted now from unphysical Q^2 to $Q^2 = 0$ which is the beginning of the physical cut. Moreover, the coupling (10) differs from the standard definition (8) only by $1/Q^4$ terms.

5. Thus, the $1/Q^2$ terms are viewed now as arising from the uncertainties of the perturbative definition of α_s. Although the OPE says that there is no $1/Q^2$ correction this should be understood rather as a statement about the difference of the full answer for the polarization operator $\Pi(Q^2)$ (or its imaginary part proportional to $R(s)$) and its perturbative expansion (in finite orders). Since the perturbative expansion itself is in fact not defined at the $1/Q^2$ level, the prediction on the absence of the $1/Q^2$ terms is not well formulated yet. An extra hypothesis is to be made concerning the precise definition of α_s which

avoids $1/Q^2$ corrections in the polarization operator. [1] Also, the interplay between low and high momenta revealed first by the UV renormalons (see above) becomes less surprising. Indeed, it is non-perturbative effects which presumably settle the problem of the Landau pole in the infrared. The corresponding ultraviolet "tail"" in the coupling behaves itself as $1/Q^2$.

It is worth emphasizing that various definitions of the coupling are not simply related by adding or removing $1/Q^2$ in a universal way from all terms of different order in α_s. Consider as an example a term of second order in the running coupling $\sim \alpha_s^2(Q^2)$. Imagine furthermore that we would like to remove an unphysical singularity due to the Landau pole from the dispersive representation of $\alpha_s(Q^2)$, i.e. to work out an expression similar to Eq. (9). To this end we should remove a single pole from:

$$\frac{1}{ln^2 Q^2/\Lambda_{QCD}^2} \rightarrow \frac{1}{ln^2 Q^2/\Lambda_{QCD}^2} - \frac{\Lambda_{QCD}^2}{Q^2 - \Lambda_{QCD}^2}. \tag{11}$$

Thus, we readily see that as far as $1/Q^2$ terms are concerned the uncertainty in α_s^2 is no less than in α_s so that the whole perturbative series collapses for the power correction. A similar phenomenon occurs in fact in case of the $1/Q$ correction due to IR renormalons in event shapes (see, e.g., Ref. [2]).

From the phenomenological point of view this collapse of the perturbative expansion in the power-like corrections implies that the value of the $1/Q^2$ correction is to be considered as am independent fit parameter. In particular, relation between $1/Q^2$ contributions to various observables can be established only at the price of new model dependant assumptions.

6. The lack of guidance as to which model is to be selected has hindered the progress in the phenomenology of $1/Q^2$ corrections and we conclude this note with a mini review of the attempts to develop the phenomenology of $1/Q^2$ corrections made so far.

In fact these attempts were formulated mostly in terms of the UV renormalons. On the other hand, one may notice that the very existence of the $1/Q^2$ can be guessed simply on the basis of existence of the Landau pole. We do not think, however, that this change of the language is indeed very significant. The central problem is working out a reasonable model and any approach to the $1/Q^2$ terms would be the equally good as far as the model turns out to be adequate in describing the data.

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[1] One of the present authors (V.Z.) discussed the Landau pole as an origin of the $1/Q^2$ terms in the coupling constant with M. Beneke and V. Braun in 1994. A *critique* of this point of view can be found in Ref. [15] where it is claimed that the Landau pole does not introduce any uncertainty of order $1/Q^2$ in theoretical predictions for physical quantities. It is our understanding that this statement is based in fact on a tacit assumption that $1/Q^2$ terms do not enter OPE provided that the standard running coupling or its modification a la (10)is used. Because of the inconsistency of the perturbation theory revealed by the Landau pole this assumption cannot be proven, to our mind.

The most frequently discussed channel is the e^+e^- annihilation into hadrons, see in particular Refs. [6,7,16], since the data are most rich in this channel. Estimates of $1/Q^2$ terms in the tau-decays which arise from using various types of dispesion relations can be found in Ref. [7]. On the other hand. one can invert the problem and get constraints on possible $1/Q^2$ terms from the data [16]. The bounds turn to be quite stringent.

Relatively large $1/Q^2$ would be welcome on phenomenological grounds in the pseudoscalar (pion) channel [6]. Moreover, the description of the $1/Q^2$ corrections in terms of the UV renormalon can match in this case the description of low-energy physics in terms of the Nambu-Jona-Lasinio model. While this hypothesis is far from being firmly established let us mention developments on the theoretical side which do favour this possibility. First, it turns out that the UV renormalon is dominated by contribution of the same four-quark operators which are postulated within the NJL model [5]. This is not a trivial statement since these operators emerge first only on the level of two renormalon chains, not a single renormalon chain. This dominance of the four-quark operators in the UV renormalon is true for various further observables as well [10].

Although it is attractive to assume that a $1/Q^2$ correction is actually the leading power-like correction in the pseudoscalar channel, at first sight this picture cannot recoinciled with the bounds on the $1/Q^2$ in the vector channel (see discussion above). It seems indeed remarkable therefore that the direct evaluation of the two renormalon chains in the vector and pseudoscalar cahnnels indeed indicates a substantial numerical disparity of these contributions [8]. More specifically it turns out that the contribution of the UV renormalon in these channels is related as [8]:

$$(UV\ renormalon)_{PS}\ =\ 18\ (UV\ renormalon)_V \tag{12}$$

where the large numerical factor emerges pure numerically.

7. To summarize, the very existence of the Landau pole induces, generally speaking, $1/Q^2$ corrections at large Q^2. Alternatively, these corrections can be ascribed to the UV renormalons. Although the phenomenology of such corrections is still in its infancy there are some reasons to expect that the pion is in fact dual to the $1/Q^2$ corrections of the fundamental QCD. [2] If this interpretation turns to be true it would provide with a new insight into the dynamics of hadrons and interplay of short and large distances in QCD which might be useful for understanding of the physics beyond the standard model as well.

We would like to acknowledge useful discussions with V. Braun and A.I. Vainshtein. Very recently, similar remarks on the role of the Landau pole in generating $1/Q^2$ terms were made by G. Marchesini and G. Grunberg and we are thankful to G. Marchesini for private communications on these matter. Finally, after this note was written there appeared a paper by G. Grunberg [17] devoted to the role of the Landau singularity in generating $1/Q^2$ corrections.

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[2] A possible connection to the Adler anomaly is worthy of investigation.

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