

**PREDICTION OF BRAKE TEMPERATURES ON  
URBAN BUS ROUTES**

**Daniel Videla  
Paul Fancher**

**June 1990**

1. Report No.		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle <b>PREDICTION OF BRAKE TEMPERATURES ON URBAN BUS ROUTES</b>				5. Report Date <b>June 1990</b>	
				6. Performing Organization Code	
7. Author(s) <b>Dan Videla and Paul Fancher</b>				8. Performing Organization Report No. <b>UMTRI-90-32</b>	
9. Performing Organization Name and Address <b>The University of Michigan Transportation Research Institute 2901 Baxter Road, Ann Arbor, Michigan 48109</b>				10. Work Unit No. (TRAIS)	
				11. Contract or Grant No. <b>DTOS-88-G-0005</b>	
12. Sponsoring Agency Name and Address <b>Great Lakes Cneter for Truck Transportation Research 201 UMTRI Building, 2901 Baxter Road Ann Arbor, MI 48109-2150</b>				13. Type of Report and Period Covered <b>Final</b>	
				14. Sponsoring Agency Code	
15. Supplementary Notes <b>Sponsored by a grant from the U.S. Department of Transportation, University Transportation Centers Program</b>					
16. Abstract <b>The purpose of this research is to see if a simple thermodynamic model developed for studying truck braking could be utilized in predicting variations in brake temperatures on urban bus routes. A long term goal of this work is to provide a computerized model that can be used as a tool for studying the influences of bus routing and operating practices on brake temperatures and hence on brake wear.</b>  <b>The results of the research provide evidence indicating that the program is able to approximate brake temperatures with a fair amount of accuracy. The computer program could serve as an engineering tool to aid in investigating where problems may exist in routes and schedules that contribute disproportionately to brake wear and maintenance costs.</b>  <b>The report discusses the tests, measurements, and analyses performed to (a) obtain data describing pertinent mechanical properties and (b) perdict brake temperature variations for a typical urban bus. In a practical sense, the main contribution of this report may well be the presentation of methods for obtaining the parametric data needed to predict brake temperature. With regard to bus operations, the stopping strategy employed by the driver was found to be a major factor influencing brake temperature and thereby influencing brake wear to a greater extent than expected from considerations of braking effort alone.</b>					
17. Key Words <b>buses, brake temperatures, cooling rates</b>			18. Distribution Statement		
19. Security Classif. (of this report)		20. Security Classif. (of this page)		21. No. of Pages <b>19</b>	22. Price

## **PREFACE**

The University of Michigan Transportation Research Institute (UMTRI) has had a long interest in the study of truck braking. Brake temperatures and the influences of those temperatures have been subjects that we have examined in several research studies. Specifically with regard to this report, Mr. Daniel Videla worked on the bus brake temperature project as part of the Summer Research Opportunity Program at the University of Michigan.

In the past year, his previous work has been extended and used to improve capabilities for simulating stop-and-go vehicle operations. That effort and this report have been supported by the Great Lakes Center for Truck Transportation Research, which is a member of the U.S. DOT's University Transportation Centers Program. In addition, Mr. Dan Hodges of the Ann Arbor Transportation Authority (AATA) provided services in kind including buses and drivers for use in this study. As a result of the knowledge gained from this study, we have developed a "computerized simplified model" for predicting brake temperatures encountered in various types of service for trucks and buses. This model, entitled, "The Brake Temperature Model", is one of several heavy vehicle dynamics models available from UMTRI.

Paul Fancher

UMTRI

June 1990

## CONTENTS

<i>SECTION</i>	<i>PAGE</i>
Preface .....	i
Abstract and Introduction .....	1
Coast Down Testing .....	2
Cool Down Testing .....	3
Brake Proportioning .....	4
Velocity Profile .....	4
Validation of Predicted Temperatures .....	5
Study of Variations in Brake Proportioning .....	8
Concluding Remarks .....	8
References .....	9
 <i>APPENDICES</i>	
Appendix A: Glossary of Descriptive Parameters .....	A1
Appendix B: Calculations .....	B1

## ABSTRACT AND INTRODUCTION

Brake wear on urban city buses is a source of high maintenance expenditures. The rate of wear increases greatly with operating temperature. This study uses a computer program that had been developed previously for studying speed-control of trucks operating on long downhill grades [1]. (Reference [1] now contains revisions that make this program suitable for studying "stop and go" operations applicable to buses.) The purpose of the research is to see if the simple thermodynamic model developed for studying truck braking could be utilized in predicting variations in brake temperatures on urban bus routes. A long term goal of this work is to provide a computerized model that can be used as a tool for studying the influences of bus routing and operating practices on brake temperatures and hence on brake wear.

In order to calibrate the program, field tests were conducted in cooperation with the Ann Arbor Transit Authority. The experimental data included measurement of brake temperatures along actual bus routes, coast down tests, and brake cooling tests. In addition to pertinent details on the operating cycles, information was also obtained on the mechanical properties of the buses and their brakes. The data were then compiled and used as input to the program to determine its ability to predict the temperatures which were experimentally observed.

The results of the research provide evidence indicating that the program is able to approximate brake temperatures with a fair amount of accuracy. The computer program could serve as an engineering tool to aid in investigating where problems may exist in routes and schedules that contribute disproportionately to brake wear and maintenance costs.

The report discusses the tests, measurements, and analyses performed to (a) obtain data describing pertinent mechanical properties and (b) predict brake temperature variations for a typical urban bus. In a practical sense, the main contribution of this report may well be the presentation of methods for obtaining the parametric data needed to predict brake temperature. With regard to bus operations, the stopping strategy employed by the driver was found to be a major factor influencing brake temperature and thereby influencing brake wear to a greater extent than expected from considerations of braking effort alone.

## COAST DOWN TESTING

In order to utilize the computer program, a number of vehicle parameters must be obtained. Simple tests and/or calculations are necessary to find some of these parameters. A glossary of these descriptive parameters can be found in *Appendix A*.

Coast down tests were conducted on an Ann Arbor Transit Authority bus equipped with radial tires and wedge brakes. The tests involved coasting down from 35 to 25 mph on a relatively flat road. Tests were conducted with the transmission in drive and with the transmission in neutral. This way the retardation due to engine drag could be separated from the "natural" retardation due to rolling resistance and aerodynamic drag.

The data gathered from the coast down tests were used to determine values for the Auxiliary Retarding Power table employed in the computer program. Since the velocity profile is entered into the program as a function of distance and not time, the distances involved in coasting were calculated using the equations for uniformly accelerated motion (*see Appendix B, CALCULATIONS, ROLLING RESISTANCE*).

Once the distances were obtained, they were entered into the velocity profile in the computer program. First the coast down test with the transmission in neutral was simulated on the computer. The program indicated a net of 15 HP to the brakes in order to decelerate the bus at the same rate as was experimentally achieved in the coast down tests with no brake application. Therefore the parameters which affect the natural drag of the vehicle (frontal area, air drag coefficient, and road surface coefficient) had to be reevaluated. It was found that frontal area and air drag coefficient do not significantly affect the natural drag of the vehicle at the low speeds tested. The road surface coefficient, however, proved to be a significant factor used in representing the necessary horsepower to decelerate the bus at a rate comparable to that experimentally observed with no brake application.

An experimentally determined value for the road surface coefficient allowed the program to accurately simulate the deceleration obtained in the coast down test with the transmission in neutral with no brake application. In order to evaluate the program's ability to predict brake temperatures, a value of 2.1 for the road surface coefficient was found to be acceptable for representing natural retardation of the bus.

By applying the data from the coast down test with the transmission in drive, a value for the retarding power of the engine was obtained. It was found that at 35 MPH the engine contributed approximately half of the drag necessary to decelerate the bus at the same rate as was observed in the test with no braking. The remaining half of the drag was attributed to other sources. (Note that there were only two values entered on the Auxiliary Retarding Power table, although the program is capable of accepting more values.)

The next set of inputs to the program are the brake parameters. These include: 1) initial temperatures of the brakes ( $^{\circ}\text{F}$ ), 2) thermal capacity of brakes (HP-Hr. per  $^{\circ}\text{F}$ ), 3) cooling coefficients K1 and K2 (HP per  $^{\circ}\text{F}$  and HP per  $^{\circ}\text{F}$ -MPH, respectively), where the coefficient K2 represents the influence of velocity on cooling, and 4) the proportioning of the braking effort between the brakes.

The thermal capacity of the brakes was determined from the product,  $m c_p$ , where  $m$  is the mass of the brake and  $c_p$  is the specific heat of the brake drums. The front brake drums were weighed (125 lbs. each) and the weights of the rear brake drums were approximated according to their dimensions (175 lbs. each). (*See Appendix B, CALCULATIONS, THERMAL CAPACITY.*) A standard value of  $8.13\text{E}(-5)$  was used for the specific heat of the brake drums. The final values for thermal capacity were 0.01016 HP-HR per  $^{\circ}\text{F}$  for the front brakes and 0.01422 HP-HR per  $^{\circ}\text{F}$  for the rear brakes.

### COOL DOWN TESTING

Cooling tests were also conducted in cooperation with the AATA. These tests were necessary to determine the cooling coefficients K1 and K2. Two different tests were conducted, a static cooling test and a constant velocity cooling test. Since the pyrometer proved to be inaccurate due to the hub configuration on the bus, the temperatures were measured with a temperature probe applied directly to the brake drum. The data from these tests along with the applicable thermodynamic relation were used to calculate K1 and K2. (*See Appendix B, CALCULATIONS, COOLING COEFFICIENTS.*) It was found that K1 was 0.01364 HP/ $^{\circ}\text{F}$  for both front and rear brakes, and that K2 was 0.0005314 HP/ $^{\circ}\text{F}$ -MPH for the front brakes and 0.000797 HP/ $^{\circ}\text{F}$ -MPH for the rear brakes. One might have expected the value of K1 for the rear brakes to have been larger than the front brakes due to the greater mass of the rear brake drum, but it was observed from the static cooling test that the rear brakes cooled at the same rate as the front brakes. This could be attributed to the difference between conduction to the hub for the two sets of brakes.

To simulate the static cooling test on the computer a special velocity profile had to be entered. This was necessary because the program calculated temperatures in relation to changes in the distance, elevation, and velocity of the vehicle. The route entered into the computer caused the bus to travel 0.001 miles (5.28 ft) at .03 MPH (.044 ft/s). This yielded a simulated time of 240 seconds while the actual static cooling test time was 257 seconds. So the velocity profile was a fair representation of the static condition in the actual test. The differences between the simulated cooling temperatures and the actual values were off by less than 2  $^{\circ}\text{F}$ . This small deviation from the actual value seems to indicate that the derived value for K1 was fairly accurate. (The program in [1] has been modified now so that the bus can remain stationary and there is no need to let it "creep" in

order to approximate static cooling. These modifications allow the vehicle to remain stopped and the brakes to cool as they would if the bus were at a bus stop.)

Next, the K2 value was tested by entering the constant velocity test route into the program's velocity profile. Once again, the simulated and experimental values did not differ by more than 2 °F. This also indicated that the derived value for K2 was fairly accurate.

## BRAKE PROPORTIONING

The proportioning of the braking effort from brake to brake is another parameter which must be entered into the program. Since the exact proportioning of the brakes is difficult to measure, data involving the total effective air chamber area were used to estimate the values. (The wedge angles for all of the brakes were 10°.) (*See Appendix B, CALCULATIONS, PROPORTIONING.*) It was assumed in the calculations that the brakes were proportioned equally from side to side. Although that may not always be the case, sampling of all four brake temperatures on the bus used in this study revealed only a few degrees difference between sides. The results from the calculations indicated that 42.9% of the total braking would be handled by the front brakes and 57.1% would be done by the rear brakes.

## VELOCITY PROFILE

Finally, the velocity profile of the route traveled must be entered into the computer. The program allows the user to input the distance (miles), elevation (ft), and speed (MPH). In order to accurately represent the bus route, certain approximations and calculations were necessary. For the purposes of this research it was assumed that the elevation was zero since there were no significant increases or decreases in elevation on the routes studied.

The first concern was accurately representing the acceleration/deceleration of the bus between stops. After running the program with different values for acceleration/deceleration rates, it became apparent that the rate of deceleration was an important determinant in the brake temperatures reached. The same conclusion was reached during testing when temperatures reached after gradual stops and sudden stops were compared. This might be explained by the use of coasting down. If the driver maintained speed right up to the bus stop and then decelerated rapidly, the temperatures reached were much higher than if the driver "coasted" without using much braking and slowed down gradually to the stop. More tests were conducted to determine the average deceleration rate so that useful values could be entered into the velocity profile.

The tests showed that on average the bus decelerated from 35 to 0 MPH in approximately 8.75 seconds. This figured to be a deceleration rate of approximately 0.2



g's. (See Appendix B, CALCULATIONS, DECELERATION.) Although these data were used in testing the accuracy of the program, it should be noted that the deceleration rate used is subject to considerable error since it varies for each individual driver and the characteristics of our test driver were not necessarily the same as those of the actual driver for the route in question.

To reduce the tediousness of entering values into the road profile, a general "formula" was employed to approximate the actual operation cycle of the bus. Thus, the velocity profile of the bus between stops involved the following steps:

- 1) acceleration to the speed limit (35 MPH) in 0.1 miles
- 2) in most cases, dependent upon distance between stops, constant velocity travel at speed limit (35 MPH) until
- 3) deceleration from speed at which brakes are applied (30 MPH) to 0 MPH, initiated 0.043 miles before the stop

Note that braking deceleration occurs from 30 to 0 MPH, because it was observed that "on average" the driver coasted from 35 to 30 MPH before applying the brakes.

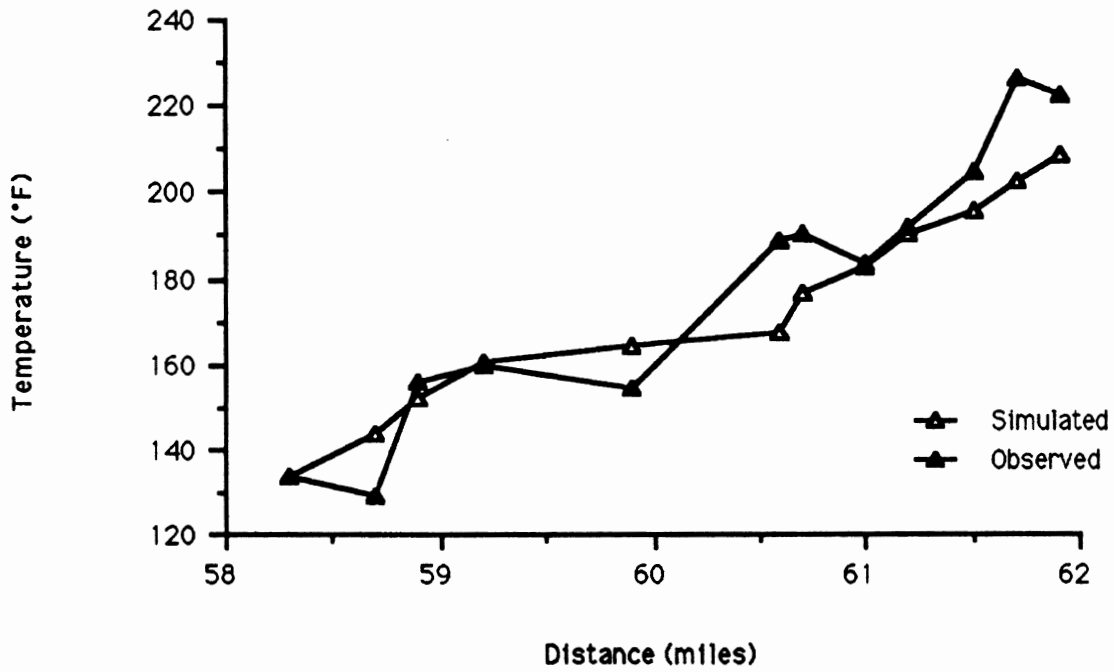
The next consideration, for which the program was not originally designed but now handles properly, involved the static cooling which takes place when the bus is loading/unloading passengers. To insure that this cooling would be accounted for, a data point was added to the velocity profile. The bus was allowed to travel 0.001 miles (5.28 feet) at 0.12 MPH (0.176 ft/s). This resulted in a 60 second time period which was also the average time required to obtain the temperature readings on the test route. It should be noted that on an actual route a bus may stop for as long as 5 minutes, or it may not stop at all.

## VALIDATION OF PREDICTED TEMPERATURES

After all the necessary data had been entered into the computer, tests were conducted to evaluate the program's ability to predict the temperature along a route consisting of eleven stops. The computer was usually able to predict the temperatures within 10 °F of the measured values. (See Figure 1. *Simulated vs. Observed Temperatures.*)

There were a few instances where the values varied by as much as 20 °F. This variation can be attributed to several factors. The deviations may be due to the difficulties incurred while measuring the temperatures along the route. Since the test was held on a sunny day around noon, the LED display on the temperature probe proved difficult to read. Another influencing factor was the strong breeze which caused a fluctuation in the temperature readings from the probe. An additional indicator which points to the measurements as the source of the problem is the "inconsistency" of some values which do

### Front Brake Temperatures: Simulated vs. Observed



### Rear Brake Temperatures: Simulated vs. Observed



Figure 1. Simulated vs Observed Temperatures

### PROPORTIONING: FRONT vs. REAR

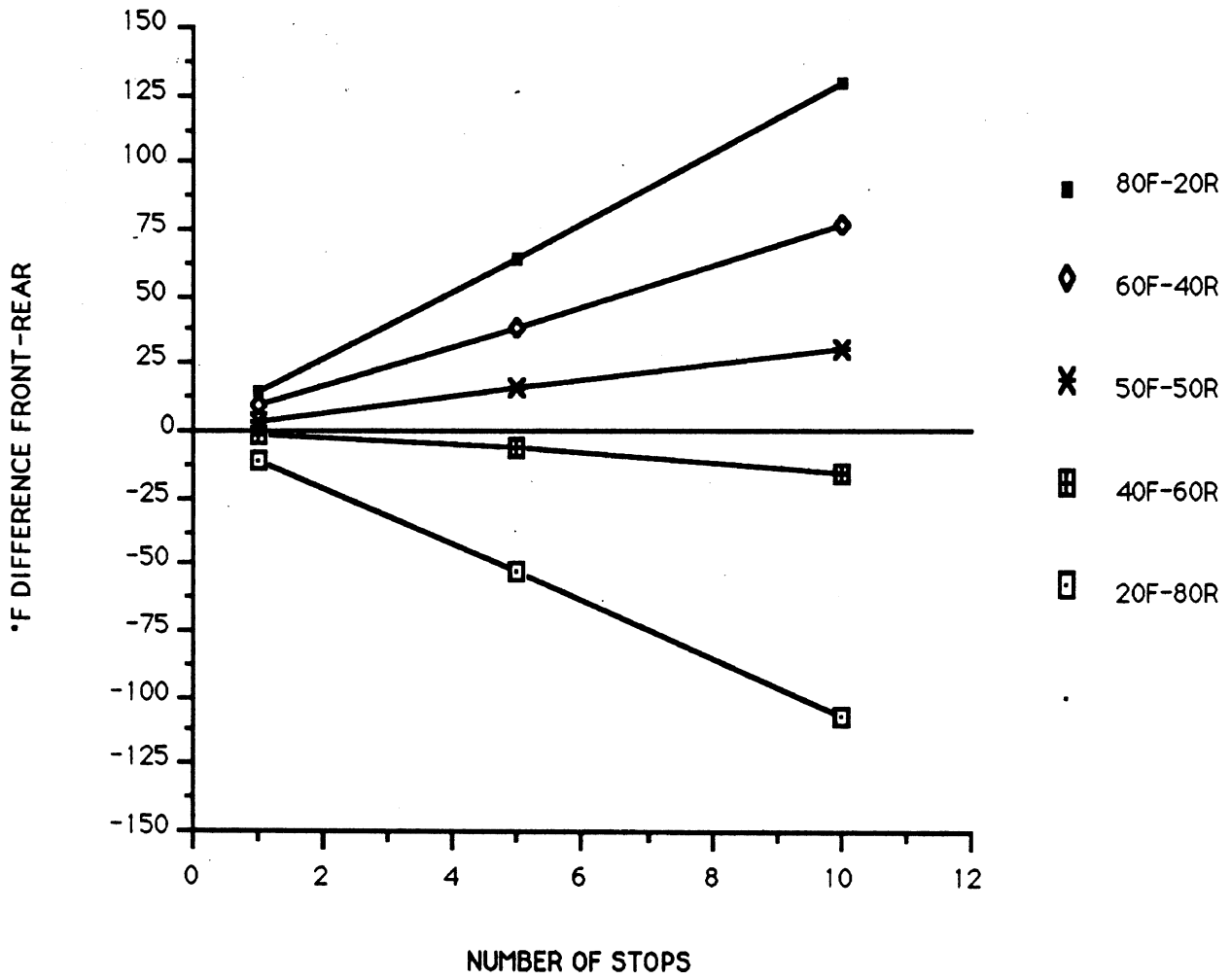


Figure 2. Influence of Brake Proportioning

not follow the temperature build-up curve. With these factors in consideration, the program predicted the temperatures with a more than fair amount of accuracy. Apparently, the highest influential determinant of the operating temperatures reached was the deceleration rate and stopping frequency.

## STUDY OF VARIATIONS IN BRAKE PROPORTIONING

Once it was found that the program could accurately predict the temperature build-up on buses, the brake proportioning was varied to determine its significance in relation to the output obtained. The variations involved the proportioning between front and rear brakes, keeping side to side adjustments equal. The simulated test involved proportioning of 20-80%, 40-60%, and 50-50% between front and rear brakes and vice versa. The brakes started with an initial temperature of 150 °F (ambient temperature of 90 °F) and were tested on a route of 10 stops with all other data remaining the same as in the other tests.

In the base test where each brake contributes 25% to the total braking (50-50% proportioning), the front brakes were 3.9° hotter than the rears at the first stop and 30.6° hotter by the tenth stop. The 40-60% front to rear proportioning showed the least difference between front and rear brake temperatures after ten stops. In the 40-60% front to rear case, the front brakes were 1.0° lower than the rear brakes at the first stop and 15.2° lower after the tenth stop. The effects of proportioning between front and rear brakes on temperature can be best illustrated by graphical example. (*See Figure 2. The Influence of Proportioning.*) These results show that the current selection of brake chamber sizes and wedge angles on the test vehicle result in very good proportioning of the bus's brakes with respect to a temperature balance.

## CONCLUDING REMARKS

The purpose of this research was to determine whether or not the Mountain Descent Brake Temperature Program could accurately predict brake temperatures on urban bus routes. Given data from appropriate tests, calculations, and approximations required to obtain the necessary inputs to the program, the results show that the program is capable of accurately predicting brake temperatures on bus routes.

Whether the brake temperature program will be used by bus maintenance and driver training personnel remains to be determined. The capabilities for predicting bus brake temperatures were presented to the Eastern Bus Maintenance Conference on May 16, 1990. Perhaps this exposure to the transit community may instigate questions and interest in the subject, particularly if the connection between temperature and brake wear is understood. Possibly, a future study on brake wear is needed to quantify the connection between wear

and temperature. If, at some future time, a bus driver training simulator were to be developed, it could include temperature calculations as a means for evaluating operator performance in a manner that could be easily related to physically understandable targets for brake temperatures.

## **REFERENCES**

- [1] A Brake Temperature Model, SIMPLIFIED MODELS OF TRUCK BRAKING AND HANDLING, UMTRI, Engineering Research Division, 1990.

## **APPENDIX**

## APPENDIX A

### *Glossary of Descriptive Parameters for the Brake Temperature Model*

This section contains an alphabetical listing of all of the descriptive parameters (of the vehicle and of the "maneuvers") which are required as input by the *Brake Temperature Model*. Terminology in brackets ([ ]) applies to the English version of the programs.

**Air drag coefficient:**

This is the coefficient for the aerodynamic drag of the vehicle. The value of this parameter must lie between 0.0 and 1.0. Suggested values are 0.80 for a truck not equipped with aerodynamic aids and 0.64 for a truck equipped with aerodynamic aids.

**Ambient temperature [°F]:**

The temperature of the surroundings in which the vehicle is operating.

**Brake proportioning:**

This is the decimal percentage of the total horsepower taken in by each brake to decelerate the vehicle. A value between 0.0 and 1.0 must be entered for each individual brake so that the sum of these values equals 1.0.

**Cooling coefficient constants: (K1: hp / °F)  
(K2: hp / (°F-mph))**

The cooling coefficient constants, K1 and K2, represent static cooling and cooling as a function of velocity in the brakes. Both constants must have a value between 0.0 and 99.99 for every individual brake. Suggested values for cooling coefficient constants on a two axle vehicle are  $1.364 \times 10^{-2}$  hp / °F for K1 for both the front and rear brakes, and  $5.314 \times 10^{-4}$  hp / (°F-mph) and  $7.97 \times 10^{-4}$  hp / (°F-mph) for K2 on the front and rear brakes, respectively.

**Distance (longitudinal) [miles]:**

These values are simply the odometer readings of the vehicle at every point in the road profile. They are used to determine the longitudinal distance between the points in the road profile.

**Elevation [ft]:**

These values describe the changes in elevation of the vehicle as it travels. These values are relative to the starting elevation of the vehicle which may be taken as zero, and need not refer specifically to feet above sea level.

**Frontal area of vehicle [ft<sup>2</sup>]:**

This is the total area of the front of the vehicle. Suggested values are 75 ft<sup>2</sup> for conventional tankers, 85 ft<sup>2</sup> for cab-over tankers, 84 ft<sup>2</sup> for transit buses, and 64 ft<sup>2</sup> for school buses.

**Heat capacity of brakes [(hp-hr) / °F]:**

The thermal capacity of a brake is determined from the product of the mass and specific heat of the brake drum. A heat capacity value must be entered for each individual brake and must lie between 0.0 and 1000.00. Suggested values for cast iron brake drums are 60 lbs - 175 lbs for weight and  $8.13 \times 10^{-5}$  (hp-hr) / (lb-°F) for the specific heat.

**Initial brake temperatures [°F]:**

The initial temperature of each individual brake, T0, ( 2 per axle) must be entered as a brake parameter.

**Number of points in the auxilliary retarding table (max. 10):**

This value, which must be between 2 and 10, indicates the number of data points that will be used to describe the retardation capabilities of the vehicle. Values for velocity and retarding power [hp] must be entered for each data point in the auxilliary retarding table.

**Number of points in the road profile (max. 100):**

The entry here indicates the number of data points that are going to be used to describe the velocity and elevation profiles of the vehicle and its route. For every point in the road profile, a value for distance, elevation, and velocity must be entered.

**Number of stops (max. 99):**

This value prescribes the number of instances which the vehicle is at rest and must coincide with the number of stops entered into the road profile. A "stop" is recognized by the program as two lines of identical data with zero velocities occurring back to back in the road profile. A prompt will appear after the road profile requesting the time spent at each stop.

**Retarding power [hp]:**

This value serves as input to the auxilliary retarding table and includes engine, driveline, and trailer axle retarding power.

**Road surface coefficient:**

This coefficient reflects the type and condition of surface on which the vehicle is operating. The value of this parameter must lie between 0.0 and 99.99. Suggested values are 1.0 for smooth concrete, 1.5 for worn concrete, brick, and cold blacktop, and 2.0 for hot blacktop.

**Stop times [minutes]:**

The period of time at which the vehicle is at rest. A value between 0.0 and 99.99 minutes must be entered for every stop made.

**Total number of axles on the vehicle (max. 13):**

This is the total number of axles on the vehicle. For each axle specified, data pertaining to both brakes is required.

**Total weight [lbs] {kg}:**

The total weight of the vehicle and its load.



**Type of tire: (1- Bias ply)  
(2 - Radial)**

**This parameter indicates the type of tires the vehicle is equipped with and is used in determining rolling resistance.**

**Velocity [mph]:**

**These values describe the velocity profile of the vehicle throughout its route in the Brake Temperature Model. In order to signify a "stop", two zero velocities must be entered back to back in the road profile along with identical values for distance and elevation.**

## APPENDIX B

### CALCULATIONS

#### ROLLING RESISTANCE

##### *Transmission in drive:*

$v_f = 25$  mph,  $v_o = 35$  mph,  $t = 14$  s  
 $(v_f - v_o)/t =$  acceleration,  $a = -0.0325$  g's  
 $(v_f^2 - v_o^2)/2a =$  distance,  $d = 0.117$  miles

##### *Transmission in neutral:*

$v_f = 25$  mph,  $v_o = 35$  mph,  $t = 28$  s  
acceleration,  $a = -0.0163$  g's  
distance,  $d = 0.233$  miles

##### Comments:

The third test with the transmission in drive matched the first test almost perfectly and is therefore not represented in the calculations above.

#### THERMAL CAPACITY

mass of front brake,  $m_{fb} = 125$  lbs  
mass of rear brake,  $m_{rb} = 175$  lbs  
specific heat of brake drum,  $c_p = 8.13E-5$  hp-hr/°F-lb  
thermal capacity of front brake,  $m_{fb}c_p = 0.01016$  hp-hr/°F  
thermal capacity of rear brake,  $m_{rb}c_p = 0.01422$  hp-hr/°F

##### Comments:

An approximate value was used for the specific heat of the brake drums since the actual value was not measured.

#### COOLING COEFFICIENTS

cooling function:  $h(v) = K1 + K2*v$   
K1: static cooling coefficient  
K2: cooling coefficient as a function of velocity  
v: velocity

Solve for K1 using values from static cooling test:

$$\begin{aligned} -h(v)(T - T_a) &= m_b c_p dT/dt \\ -h(v) &= K1 + K2*v \text{ where } v=0 \text{ in static test} \\ -K1(T - T_a) &= m_b c_p dT/dt \end{aligned}$$

where:

$$\begin{aligned} dT/dt &= (-9.0 \text{ }^\circ\text{F} / 0.0714 \text{ hrs}) \\ T &= 158.9 \text{ }^\circ\text{F} \text{ where } T \text{ is the average temperature} \end{aligned}$$

$T_a = 65 \text{ }^\circ\text{F}$   
 $m_b = 125 \text{ lbs for front drum}$   
 $c_p = 8.13\text{E-}5 \text{ hp-hr / }^\circ\text{F-lb}$   
*answer:*  $K_1 = 0.01364 \text{ hp / }^\circ\text{F}$  for a front brake

Now solve for  $K_2$  using constant velocity cooling test:  
 $(K_1 + K_2 \cdot v)(T - T_a) = m_b c_p dT/dt$

where:

$dT/dt = (30.6 \text{ }^\circ\text{F} / 0.0786 \text{ hrs})$  at a front brake  
 $K_1 = 0.01364 \text{ hp / }^\circ\text{F}$   
 $T = 187.7 \text{ }^\circ\text{F}$  at a front brake  
 $v = 35 \text{ mph}$   
 all other values same as previous calculation  
*answer:*  $K_2 = 0.0005314 \text{ hp / }^\circ\text{F-mph}$

Solve for cooling coefficient  $K_2$  for rear brakes:  
 from static cooling test:  $K_{1\text{front}} = K_{1\text{rear}}$   
 approximate front cooling area:  $\pi \cdot \text{diameter} \cdot 8''$   
 approximate rear cooling area:  $\pi \cdot \text{diameter} \cdot 12''$   
 estimate that  $K_{2\text{rear}} = (12/8) K_{2\text{front}}$   
 $K_{2\text{rear}} = 0.000797 \text{ hp / }^\circ\text{F-mph}$

## PROPORTIONING

Total effective front air chamber area:  $2(12 \text{ in}^2)$   
 Total effective rear air chamber area:  $2(16 \text{ in}^2)$   
 % of braking done by front brakes:  $24 / (24+32) = 42.9\%$   
 % of braking done by rear brakes:  $32 / (24+32) = 57.1\%$

Comments:

We assumed that the right and left front, as well as the right and left rear, brakes were proportioned equally side to side.

The wedge angles of the front and rear brakes were  $10^\circ$  and hence they did not influence the level of proportioning.

## DECELERATION

Average deceleration rate from 35 to 0 mph:  
 ave. braking time from 35 to 0 mph: 8.75 s  
 equation:  $v_f = v_o + at$   
 $v_f = 0 \text{ mph}$   
 $v_o = 35 \text{ mph}$   
 $t = 8.75 \text{ s}$   
*answer:*  $a = -5.87 \text{ ft/s}^2 = 0.2 \text{ g's}$

Average stopping distance from 35 mph:  
 equation:  $d = (v_f^2 - v_o^2) / 2a$   
*answer:*  $d = 0.043 \text{ miles}$