

# Reduced-Order Aerothermoelastic Framework for Hypersonic Vehicle Control Simulation

Nathan J. Falkiewicz\* and Carlos E. S. Cesnik†  
University of Michigan, Ann Arbor, Michigan 48109-2140

and  
Andrew R. Crowell‡ and Jack J. McNamara§  
Ohio State University, Columbus, Ohio 43210-1226

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**Hypersonic vehicle control system design and simulation require models that contain a low number of states. Modeling of hypersonic vehicles is complicated due to complex interactions between aerodynamic heating, heat transfer, structural dynamics, and aerodynamics. Although there exist techniques for analyzing the effects of each of the various disciplines, these methods often require solution of large systems of equations, which is infeasible within a control design and evaluation environment. This work presents an aerothermoelastic framework with reduced-order aerothermal, heat transfer, and structural dynamic models for time-domain simulation of hypersonic vehicles. Details of the reduced-order models are given, and a representative hypersonic vehicle control surface used for the study is described. The methodology is applied to a representative structure to provide insight into the importance of aerothermoelastic effects on vehicle performance. The effect of aerothermoelasticity on total lift and drag is found to result in up to an 8% change in lift and a 21% change in drag with respect to a rigid control surface for the four trajectories considered. An iterative routine is used to determine the angle of attack needed to match the lift of the deformed control surface to that of a rigid one at successive time instants. Application of the routine to different cruise trajectories shows a maximum departure from the initial angle of attack of 8%.**

## Nomenclature

$A$	=	snapshot matrix	$k$	=	generalized thermal conductivity matrix of reduced system in modal space
$a_i$	=	$i$ th snapshot corresponding to $i$ th column of $A$	$K_G$	=	geometric stiffness matrix
$C$	=	correlation matrix	$K_s$	=	structural stiffness matrix
$c$	=	modal coordinate of thermal proper orthogonal decomposition basis vector	$k_s$	=	generalized stiffness matrix of reduced system in modal space
$C(x)$	=	correlation model for kriging	$k_T$	=	thermal conductivity of material
$c_p$	=	specific heat at constant pressure	$K_s^*$	=	modified structural stiffness matrix
$d$	=	structural modal coordinates, kriging sample point	$L$	=	aerodynamic lift, length
$E$	=	modulus of elasticity	$M$	=	thermal capacitance matrix of full system in physical space, Mach number
$F$	=	thermal load vector of full system in physical space	$m$	=	generalized thermal capacitance matrix of reduced system in modal space
$f$	=	generalized thermal load vector of reduced system in modal space	$M_s$	=	structural mass matrix of full system in physical space
$F_s$	=	structural load vector of full system in physical space	$m_s$	=	generalized mass matrix of reduced system in modal space
$f_s$	=	generalized structural load vector of reduced system in modal space	$n$	=	number of proper orthogonal decomposition snapshots
$H_i$	=	coefficient matrices for integration of equations of motion	$n_e$	=	number of snapshots for kriging evaluation cases
$h_i$	=	thickness of $i$ th layer of thermal protection system	$n_k$	=	number of kriging snapshots
$K$	=	thermal conductivity matrix of full system in physical space	$n_s$	=	number of structural parameters in reduced-order aerothermodynamic model
			$n_t$	=	number of thermal parameters in reduced-order aerothermodynamic model
			$\dot{q}$	=	heat flow rate vector
			$\mathcal{R}$	=	residue
			$r$	=	number of degrees of freedom of reduced system in modal space after modal truncation
			$R(x)$	=	regression model for kriging
			$s$	=	number of degrees of freedom of full-order thermal system
			$St$	=	Stanton number
			$T$	=	vector of discrete nodal temperatures
			$t$	=	time
			$U$	=	matrix containing left singular vectors of $A$
			$u_i$	=	$i$ th left singular vector of $A$
			$V$	=	matrix containing right singular vectors of $A$ , velocity
			$v_i$	=	$i$ th right singular vector of $A$
			$w$	=	structural displacement in $z$ direction

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\*Ph.D. Candidate, Department of Aerospace Engineering, 1320 Beal Avenue. Student Member AIAA.

†Professor of Aerospace Engineering, Department of Aerospace Engineering, 1320 Beal Avenue. Associate Fellow AIAA.

‡Ph.D. Candidate, Department of Mechanical and Aerospace Engineering, 2036 Neil Avenue. Student Member AIAA.

§Assistant Professor of Aerospace Engineering, Department of Mechanical and Aerospace Engineering, 201 W. 19th Avenue. Senior Member AIAA.

$X$	=	kriging sample points
$x$	=	structural degrees of freedom in physical space
$y(x)$	=	kriging prediction at point $x$
$\alpha$	=	angle of attack
$\alpha_T$	=	coefficient of thermal expansion
$\Delta t$	=	time-step size
$\varepsilon$	=	absolute error in approximation of snapshots by their projection onto basis
$\varepsilon_{\text{rel}}$	=	relative error in approximation of snapshots by their projection onto basis
$\varepsilon_T$	=	relative error tolerance
$\lambda$	=	eigenvalue of correlation matrix
$\nu$	=	Poisson's ratio
$\rho$	=	density of material
$\Sigma$	=	diagonal matrix containing singular values of $A$
$\sigma_i$	=	$i$ th singular value of $A$
$\Phi$	=	modal matrix of full set of proper orthogonal decomposition basis vectors
$\Phi_{\text{ref}}$	=	modal matrix of structural reference modes
$\varphi_i$	=	$i$ th proper orthogonal decomposition basis vector

#### Subscripts

AE	=	aeroelastic
aero	=	heat flux component due to aerodynamic heating
AT	=	aerothermal
$E$	=	elastic
HT	=	heat transfer
max	=	maximum application temperature
min	=	angle of attack minimizing $\mathcal{R}$
$R$	=	rigid
rad	=	component due to radiation
$t$	=	total or stagnation quantity
$w$	=	wall
0	=	initial
$\infty$	=	freestream

#### Superscripts

$T$	=	matrix transpose
$(\cdot)$	=	truncated

## I. Introduction

DESIGN and simulation of hypersonic vehicle (HSV) control systems requires consideration of a variety of disciplines due to the highly coupled nature of the flight regime [1–4]. To capture all of the potential effects on vehicle controllability, one must consider the aerodynamics, aerodynamic heating, heat transfer, and elastic airframe, as well as the interactions between these disciplines. The problem is further complicated by the high computational expense involved in capturing all of these effects and their interactions in a full-order sense. Aerothermoelastic modeling is particularly challenging for control system design and evaluation due to the need for models with a low number of states for this type of analysis. While high-fidelity modeling techniques exist for each of these disciplines, the use of such techniques is computationally infeasible in a control design and simulation setting for such a highly coupled problem. Early in the design stage, many iterations of analyses may need to be carried out as the vehicle design matures, thus requiring quick analysis turnaround time. Additionally, the number of states and number of degrees of freedom used in the analyses must be small enough to allow for efficient control simulation and design. As a result, alternative approaches must be considered for vehicle simulations. There are two methodologies that can be used in the generation of low-order models. The first approach is to apply simplifying assumptions that enable the use of first-principles models. These models are characterized by their low-order form and they can often be solved analytically, thus preventing the need to time march the solution. While these models are useful in generating a low-order representation of the physics, the simplifying assumptions made to

employ these models often preclude the ability to model detailed geometries or complex physics. This work will therefore make use of an alternative approach that involves the use of reduced-order models (ROMs) that are derived from high-fidelity analysis tools. Use of high-fidelity tools alone is infeasible due to their high order and long run time. Thus, this work seeks to go beyond simply coupling existing high-fidelity codes and routines, and it instead proposes to use the output of these tools along with model reduction techniques to generate computationally tractable systems of governing equations. While aerothermoelastic analysis for control system design has been performed previously, many simplifying assumptions have been used to reduce the size of the problem. The objective of the current work is to make aerothermoelastic simulation of complex geometries feasible for control design without the need for a priori assumptions regarding the physics of the problem. As opposed to a first-principles modeling approach, by first modeling as much of the physics as possible and then systematically reducing the order of the system, one can control and quantify the error incurred through model reduction. This also allows for tailoring of the number of states and degrees of freedom as different levels of fidelity may be required for control system design and evaluation.

This work focuses on the coupling of the aerodynamics, aerodynamic heating, heat transfer, elastic airframe, and vehicle dynamics and control. The overall goal of this work is to provide a reduced-order modeling framework that can be used to assess the robustness of the HSV control effector to aerothermoelastic effects in an efficient manner. To do so, careful consideration of the coupling mechanisms between the various disciplines is essential. A flowchart illustrating the coupling of these disciplines is shown in Fig. 1. Because of the high speed involved in hypersonic flight, stagnation effects and the turbulent boundary layer lead to the existence of an aerodynamic heat flux at the surface of the vehicle. To calculate the spatially and temporally varying heat flux, it is necessary to know the aerodynamic flow properties over the vehicle. Note the two-way coupling between the aerodynamic heat flux and the transient temperature distribution. The upward-pointing arrow illustrates the fact that the aerodynamic heat flux represents a thermal boundary condition (BC) for the heat transfer problem. The downward-pointing arrow indicates the fact that the heat flux is dependent on the wall temperature of the structure. Additionally, thermal radiation between the outer surface of the structure and the environment must be included. A two-way coupling also exists between the radiation and temperature distribution. Once the aerodynamic problem is solved and the heat flux and radiation flux are known, the boundary conditions for the heat transfer problem are generated. The transient thermal problem is then solved, resulting in a spatially varying temperature distribution. The loads on the structure will have two components: thermal loads resulting from differential thermal expansion of the structure and aerodynamic pressure loads. Additionally,

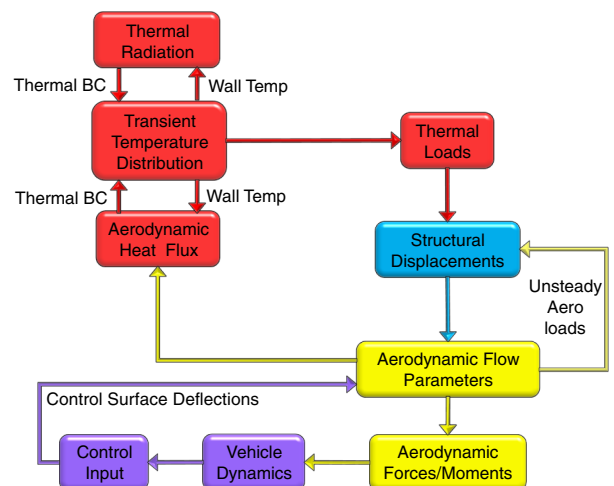


Fig. 1 Coupling between aerodynamic heating, heat transfer, elastic airframe, aerodynamics, and vehicle dynamics.

heating of the structure results in changing stiffness distribution due to temperature dependence of material properties such as Young's modulus. The development of thermal stresses results in a further change in the stiffness. As a result of the loads, the structure will displace relative to its undeformed configuration. Deformation of the structure leads to a modified aerodynamic profile, and the aerodynamic flow properties over the vehicle will change. The flow properties must then be recalculated, as they will affect the loads on the structure, the aerodynamic heat flux, and the aerodynamic forces and moments on the vehicle. Once the updated aerodynamic flow parameters are known, the pressures can be integrated over the vehicle to calculate the resultant forces and moments on the vehicle. Based on the forces and moments, the vehicle equations of motion are propagated and necessary control inputs (i.e., control surface deflections) are determined based on the commanded trajectory and vehicle performance. The control surface deflections in turn change the aerodynamic flow parameters. Once the control inputs are determined, the process is repeated at the next time step. Because the control surfaces are expected to provide a significant contribution to the aerodynamic lift, drag, and moments acting on the vehicle, thorough analysis of the major couplings involved with such a structure is required in order to accurately predict vehicle performance.

A variety of studies have focused on the aerothermoelastic response and stability of panels with various degrees of aerothermoelastic coupling. A review paper summarized various analytical methods for nonlinear panel flutter at supersonic and hypersonic speeds [5]. The methods are grouped into two main areas: 1) classical methods including Galerkin in conjunction with numerical integration, harmonic balance, and perturbation methods; and 2) finite element methods in the frequency or time domain. Two specific studies [6,7] considered nonlinear panel flutter for high-Mach-number flows under aerothermoelastic effects. These studies both made use of von Kármán plate theory for large displacements, and both include geometric nonlinearity due to compressive load. The panel temperature was taken to be equal to the instantaneous flow temperature and was assumed to be uniform to allow for a lumped-capacity approach to the solution of the heat transfer equations. Two related works used an explicit Taylor–Galerkin algorithm to solve the coupled fluid-thermal-structural equations to assess the impact of aerothermoelastic effects on leading edges [8] and panels [9]. These works employed an integrated finite element approach that solved the Navier–Stokes equations, energy equation, and quasi-static structural equations of motion in an integrated framework. Results from both works showed that structural deformation has a strong effect on flow properties and that severe aerodynamic heating can lead to large stresses due to steep temperature gradients. Another work [10] focused on the thermal analysis of thermal protection panels in hypersonic flow under an imposed deflection. The framework consisted of a finite volume Navier–Stokes CFD code coupled with a boundary element formulation of the transient heat transfer process. The ability of the framework to handle deflection was assessed by imposing a deformed structural configuration, calculating the thermal boundary conditions over the deformed panel, and solving for the transient temperature distribution.

Another research area in which aerothermoelasticity in high-speed flows has been studied widely is the area of functionally graded panels [11–16]. These works include aerothermoelastic effects to varying extents and are motivated largely by the need to assess the response of the structure as well as its aerothermoelastic stability. Praveen and Reddy [11] include thermal effects by imposing temperatures at the ceramic-rich and metal-rich surfaces and solving a simple steady-state heat transfer problem to obtain the through-thickness temperature distribution of the plate. Temperature was taken to be uniform in the plane of the plate. Structural equations of motion were based on a combination of first-order plate theory and von Kármán strains. Static and dynamic finite element analysis was performed, and the plate deflections and stresses were analyzed. Prakash and Ganapathi [16] also included thermal loads by imposing temperatures at the outer surfaces of the structure. This work investigated the influence of thermal loads on the supersonic flutter behavior of functionally graded flat panels. In contrast with [11], this

study included aerodynamic loads by considering the first-order high-Mach-number approximation to linear potential flow theory. The equations of motion were formulated using Lagrange's equations, and eigenvalue solutions were used to assess the critical aerodynamic pressure for flutter. A later work [15] extended the study of [16] by using piston theory to incorporate quasi-steady aerodynamic loads; however, this study did not include thermal effects. In a related work [14], thermal effects were incorporated along with the quasi-steady piston theory formulation. The temperature distribution was again obtained by considering temperature variation in the thickness direction only, specifying temperatures at the upper and lower surfaces and solving the steady-state heat transfer equation subject to these Dirichlet boundary conditions. Two other works [12,13] also used quasi-steady first-order piston theory and included thermal effects by imposing a uniform temperature change on the plate. While these works provide progress on aerothermoelastic modeling in high-speed flows, the solution of the heat transfer problem by assuming surface temperatures, using a steady-state thermal solution, or semiempirical methods is insufficient for the problem at hand. Additionally, the current work will use an unsteady aerodynamic solution as opposed to a quasi-steady one.

Recent research on aerothermoelastic stability of a HSV control surface used computational fluid dynamics (CFD) to compute the aerodynamic heating along with finite element thermal and structural models to assess its behavior in hypersonic flow [17]. In these studies, the aerodynamic heat flux was found by using rigid-body CFD solutions to compute the adiabatic wall temperature and heat transfer coefficient at the surface of the structure. The resulting thermal boundary conditions were applied to a transient thermal finite element analysis, and the resulting temperatures were applied to the structural dynamic finite element model. Free vibration frequencies and mode shapes of the heated structure were calculated under both material property degradation with temperature and thermal stresses at each desired point in time. The mode shapes were used in a modal representation of the equations of motion to obtain the aeroelastic response. Third-order piston theory and Euler aerodynamics were used to obtain the generalized loads. The use of CFD for solution of the aeroheating problem provides capability for capturing complex effects such as shock/boundary layer interaction and three-dimensional (3-D) flow effects. However, the computational cost associated with CFD is prohibitive in the control design and simulation setting, and the use of an aerothermal ROM is warranted. While [17] reduced the order of the equations of motion by applying a truncated set of free vibration mode shapes, an eigenvalue solution was still computed at each desired point in time to update the mode shapes. The current work extends that methodology by using a fixed basis throughout the simulation to avoid the need to solve an eigenvalue problem during the course of the simulation while still reducing the order of the equations of motion.

In two recent studies by Culler and McNamara [18,19], the flow-thermal-structural coupling of two-dimensional (2-D) and 3-D skin panels in hypersonic flow was performed. In [18], two-dimensional panel flutter analysis of a 2-D simply supported HSV skin panel was performed using von Kármán plate theory for the structural dynamic response, piston theory for the unsteady aerodynamics, and Eckert's reference enthalpy method for the aerodynamic heating. In [19], a similar study was carried out to assess the effect of aerothermoelasticity on a 3-D HSV skin panel. A 3-D finite element model was used to obtain the structural dynamic response as opposed to von Kármán plate theory. Loads on the structure consisted of aerodynamic pressure loads, fluctuating pressure loads due to sound pressure level in the turbulent boundary layer, and thermal loads due to temperature gradients.

The current work builds on the previously described literature in a number of areas. The thermoelastic portion of this study is a continuation of previous studies on reduced-order modeling of the heat transfer and structural dynamics problems [20–22]. The earliest of these [22] introduced the reduced-order thermoelastic modeling framework, which used proper orthogonal decomposition (POD) for reduction of the thermal problem and a modified modal method for reduction of the structural dynamics problem. The framework was

applied to a representative HSV structure and results demonstrated the accuracy of the POD formulation as well as the coupling of the equations of motion that results due to heating. A subsequent work [21] employed a quasi-steady aerothermoelastic time-marching procedure to assess the effect of thermal loads on the aerodynamic forces over a control surface. A more recent work [20] described an extension of the previous papers to unsteady form and specifically addressed the use of POD with time-dependent boundary conditions. The current effort makes use of the thermoelastic portion of the framework described in [20] along with a reduced-order aerothermal model.

The aerothermal model used for calculation of the aerodynamic heat flux at the surface of the structure is based on that developed in recent work [23], in which both a POD basis and a surrogate-based function were constructed using Latin hypercube parameter space sampling. The models were generated by performing full-order CFD solutions at various points within the identified parameter space. Results demonstrated the ability of the approach to enable an accurate, robust, and efficient method for incorporating CFD aerodynamic heating predictions into aerothermoelastic analysis. A similar approach to that of [23] is extended in the current work by incorporating the aerodynamic heating model into the aerothermoelastic ROM framework.

The progress described has led to further investigation into the aerothermoelastic coupling exhibited in hypersonic flight. A flow-chart of overall framework applied in this work is shown in Fig. 2. The process begins with the calculation of the heat flux on the outer surface of the structure at the initial time. With the boundary conditions and initial conditions of the thermal problem known, the transient temperature distribution is marched forward in time. Solution of the heat transfer problem is carried out in modal space using modes from POD (to be described in a later section) to avoid the computational cost of running full-order finite element analysis. The bypassing of the full-order thermal solution via the reduced-order solution is indicated. This work considers three coupling mechanisms between the thermal and structural solutions. The first involves the thermal stresses that occur in the structure due to differential thermal expansion resulting from the spatially varying temperature distribution. The second is due to the temperature dependence of the Young's modulus resulting from the high temperatures experienced in hypersonic flight. The third involves the thermal loads that are generated on the structure due to thermal expansion. With the stiffness and structural loads known, the structural dynamics equations of motion (EOMs) are transformed to a suitable reduced modal basis to be described in a subsequent section. The reduced modal system is then solved for the modal coordinates to obtain the structural response. The structural deformations couple with the

aerothermal problem due to the effect on aerodynamic flow properties, which change the heat flux. The deformations also result in a change in aerodynamic pressures that modify the structural loads. With the deformations and velocities known at the current time step, the aerodynamic flow parameters are calculated using third-order piston theory [24,25]. Note that the aerodynamic solver makes use of a steady shock/expansion analysis to turn the flow through the angle of attack at the leading edge before the unsteady piston theory calculations. This is performed to decrease the likelihood that the piston velocity will exceed the speed of sound in the undisturbed wall. Finally, the heat flux is updated based on the deformation and wall temperatures and the procedure is repeated at the next time instant.

A time-marching procedure with updates to the thermal and structural boundary conditions at specified intervals is proposed for solution of the coupled aerothermoelastic problem. An outline of the time-stepping schedule is given in Fig. 3. A fundamental assumption to be investigated in the study is that the size of the aeroelastic time step  $\Delta t_{AE}$  will be smaller than the size of the aerothermal time step  $\Delta t_{AT}$ . The procedure begins by calculating the aerodynamic flow properties over the undeformed structure at initial time  $t_0$ . Using the flow properties, the heat flux at the outer surface is found along with the local skin friction coefficients. The aerodynamic pressures and viscous drag components are then integrated to determine the aerodynamic forces and moments at initial time. With the thermal boundary conditions known, a predetermined number of thermal time steps are taken, each of size  $\Delta t_{HT}$ , until the time  $t_0 + \Delta t_{AT}$  is reached. The thermal loads based on the temperature change between  $t_0$  and  $t_0 + \Delta t_{AE}$  are then applied to the structural configuration at  $t_0$ . Additionally, the aerodynamic loads based on the already calculated flow properties are applied to the structure. The structural deformations are then calculated. The displacements are fed back into the aerodynamic solver, and the flow properties are calculated at time  $t_0 + \Delta t_{AE}$  over the updated deformed configuration. Each time the flow properties are recalculated, the aerodynamic pressures are also integrated to allow for characterization of the transient aerodynamic forces and moments on the vehicle. The aeroelastic iterations continue to be carried for a predetermined number of time steps. Once the time instant  $t_0 + \Delta t_{AT}$  has been reached, the time history of the structural dynamic response is used to update the aerodynamic heat flux. The coupling between the aerothermal and aeroelastic solutions is performed in a time-averaged dynamic [18] sense, in which the time average of the structural dynamic response in between aerothermal updates is used in the calculation of the thermal boundary conditions. The details of how this information is used to update the heat flux are given in a later section. With the updated thermal boundary conditions known, the transient thermal solution is

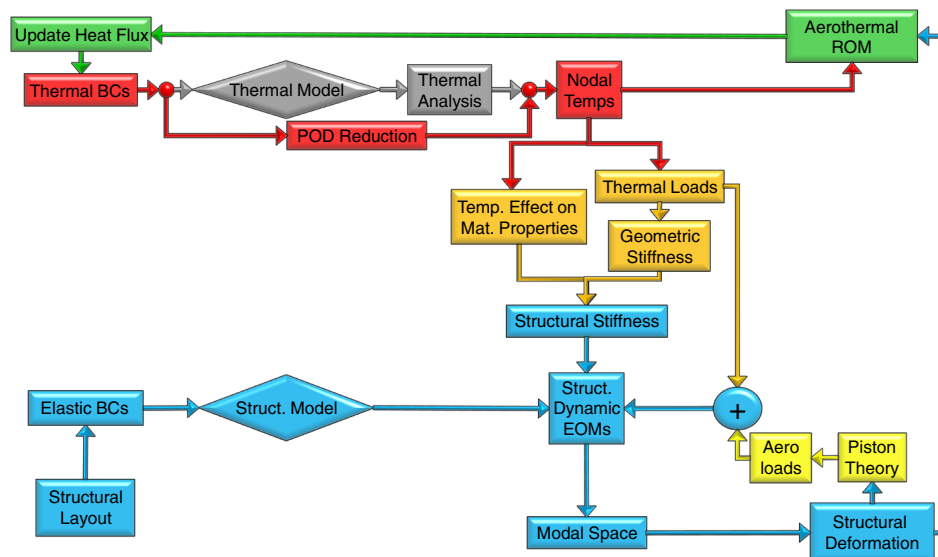


Fig. 2 Reduced-order aerothermoelastic modeling framework implemented in this work.



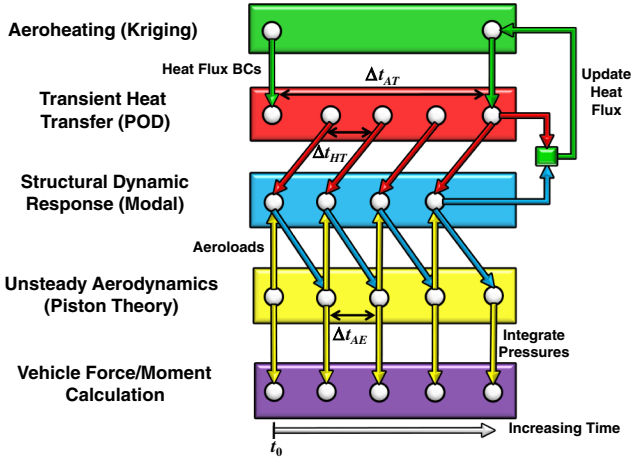


Fig. 3 Overview of aerothermoelastic time-stepping schedule.

marched forward from the time instant  $t_0 + \Delta t_{AT}$  to the time instant  $t_0 + 2\Delta t_{AT}$  and the process is repeated. The following sections describe the formulations for solution of the various aspects of the aerothermoelastic problem.

## II. Reduced-Order Thermal Model

### A. Creation of Proper Orthogonal Decomposition Basis

This work makes use of POD for reduced-order solution of the transient thermal problem. POD (also known as the Karhunen–Loève decomposition, principal components analysis, singular systems analysis, and SVD) is a modal technique in which empirical data are processed using statistical methods to obtain models that capture the dominant physics of a system using a finite number of modes [26]. The fundamental basis for use of POD as a reduced-order modeling method is its ability to represent high-dimensional systems in a low-dimensional approximate manner while still maintaining a high degree of accuracy. The choice of the POD basis as opposed to any other basis, such as an eigenvector basis, is due to its optimality condition of providing the most efficient way of capturing the dominant components of a system with a finite number of modes [27]. In addition to heat transfer problems, the POD has been used in numerous applications, such as turbulence [28], structural dynamics, [29,30], aerodynamics, [31], and control theory [32], among others.

The method of snapshots [33] is used for determination of the POD basis vectors. In this case, the snapshots are defined as vectors of nodal temperatures at various time instants and are computed from high-fidelity finite element analysis. The goal of the POD formulation is to express the vector of nodal temperatures  $T$  at any time instant as a linear combination of the basis  $\varphi(x, y, z)$  with coefficients  $c(t)$ ; that is,

$$\begin{Bmatrix} T_1 \\ \vdots \\ T_s \end{Bmatrix} = c_1(t) \begin{Bmatrix} \varphi_1^{(1)} \\ \vdots \\ \varphi_s^{(1)} \end{Bmatrix} + c_2(t) \begin{Bmatrix} \varphi_1^{(2)} \\ \vdots \\ \varphi_s^{(2)} \end{Bmatrix} + \cdots + c_r(t) \begin{Bmatrix} \varphi_1^{(r)} \\ \vdots \\ \varphi_s^{(r)} \end{Bmatrix} \quad (1)$$

where  $s$  is the total number of degrees of freedom in the finite element model and  $r$  is the total number of POD basis vectors retained after truncation. The basis is computed by first generating the snapshot matrix  $A$  given by

$$A = \begin{bmatrix} T_1^{(1)} & T_1^{(2)} & \cdots & T_1^{(n)} \\ T_2^{(1)} & T_2^{(2)} & \cdots & T_2^{(n)} \\ \vdots & \vdots & \ddots & \vdots \\ T_s^{(1)} & T_s^{(2)} & \cdots & T_s^{(n)} \end{bmatrix} = [a_1, a_2, \dots, a_n] \quad (2)$$

where  $T_i^{(j)}$  indicates the  $i$ th entry of the  $j$ th snapshot,  $n$  is the number of snapshots taken, and  $a_j$  refers to the column vector corresponding to the  $j$ th snapshot. Note that the thermal snapshots are vectors of nodal temperatures at different time instants throughout the transient. The next step is to take the singular value decomposition (SVD) of the snapshot matrix. The full SVD of the  $s \times n$  snapshot matrix is given by

$$A = U\Sigma V^T \quad (3)$$

where  $U$  is a  $s \times s$  orthogonal matrix,  $V$  is an  $n \times n$  orthogonal matrix, and  $\Sigma$  is a  $s \times n$  diagonal matrix with

$$\sigma_{ij} = \begin{cases} 0 & \text{for } i \neq j \\ \sigma_i \geq 0 & \text{for } i = j \end{cases} \quad (4)$$

The diagonal entries of  $\Sigma$  are known as the singular values of  $A$  and are ordered in decreasing order such that  $\sigma_1 \geq \sigma_2 \geq \dots \geq 0$ . The columns of  $U$  and  $V$  are the left and right singular vectors of  $A$ , respectively. The left singular vectors represent the POD basis vectors, i.e.,  $\varphi_k = u_k$ .

The connection between POD and SVD allows for justification of the claim of basis optimality due to the ability of the SVD to provide an optimal low-rank approximation to a matrix. Consider an approximation to  $A$  written as a partial sum of rank-one matrices formed from the outer product of the left and right singular vectors with the singular value as the scalar coefficient given by

$$A = \sum_{j=1}^r \sigma_j u_j v_j^T \quad (5)$$

where  $r$  corresponds to the number of POD basis vectors retained after truncation. It can be shown that the  $r$ th partial sum captures the maximum possible amount of energy of  $A$ , where energy is defined in either the 2-norm or Frobenius norm sense [34–36]. Alternatively stated, no other rank  $r$  matrix can be closer to  $A$  in the 2-norm or in the Frobenius norm. To examine the connection between this optimality property of the SVD and the optimality of the POD basis, consider the normed difference between the snapshot matrix and its projection onto the truncated POD basis. Written in terms of the truncated set of left singular vectors  $\bar{U}$  corresponding to the truncated set of POD basis vectors  $\bar{\Phi}$ , one has

$$\varepsilon = \|A - \bar{U}\bar{U}^T A\|^2 \quad (6)$$

Now, expressing  $A$  in terms of its full SVD leads to

$$\varepsilon = \|U\Sigma V^T - \bar{U}\bar{U}^T U\Sigma V^T\|^2 \quad (7)$$

At this point, the quantity  $\bar{U}\bar{U}^T U$  is recognized as the projection of  $U$  onto the space spanned by  $\bar{U}$  and performs the action of zeroing the columns of  $U$  that correspond to the excluded POD basis vectors. Thus, Eq. (7) becomes

$$\varepsilon = \left\| \sum_{j=1}^n \sigma_j u_j v_j^T - \sum_{j=1}^r \sigma_j u_j v_j^T \right\|^2 \quad (8)$$

and the optimality of the POD basis is demonstrated due to the optimality property of the SVD described. The absolute error associated with the  $r$ -dimensional POD subspace is related to the singular values of the snapshot matrix  $A$  and the eigenvalues of the correlation matrix  $C$ , which is given by

$$C = \frac{1}{n} A^T A \quad (9)$$

The absolute error is expressed as [37]

$$\varepsilon = \sum_{j=r+1}^n \sigma_j^2 = n \sum_{j=r+1}^n \lambda_j \quad (10)$$

where  $\varepsilon$  is defined in the Frobenius norm sense and  $\lambda_j$  is the  $j$ th eigenvalue of  $C$ . A relative error tolerance  $\varepsilon_{\text{rel}}$  can be defined, such that if the relative error is to be less than an error tolerance  $\varepsilon_T$ , i.e.,

$$\frac{\varepsilon}{\|A\|^2} = \varepsilon_{\text{rel}} \leq \varepsilon_T \quad (11)$$

the number of basis vectors retained  $r$  should be the smallest integer that

$$\frac{\sum_{j=1}^r \sigma_j^2}{\sum_{j=1}^n \sigma_j^2} = \frac{\sum_{j=1}^r \lambda_j}{\sum_{j=1}^n \lambda_j} \geq 1 - \varepsilon_T \quad (12)$$

Note that the left-hand side of Eq. (12) can be interpreted as the energy of the included basis vectors relative to the energy of the full set of basis vectors. Furthermore,  $\varepsilon_{\text{rel}}$  can be interpreted as the energy of the excluded basis vectors relative to the energy of the full set of basis vectors given by

$$\varepsilon_{\text{rel}} = \frac{\varepsilon}{\|A\|^2} = \frac{\sum_{j=r+1}^n \sigma_j^2}{\sum_{j=1}^n \sigma_j^2} = \frac{\sum_{j=r+1}^n \lambda_j}{\sum_{j=1}^n \lambda_j} \quad (13)$$

Thus, the magnitude of each of the eigenvalues of the correlation matrix can be used in determining the number of POD basis vectors that can be removed from the set.

### B. Solution for Proper Orthogonal Decomposition Modal Coordinates

In this work, the POD modal coordinates are computed numerically as opposed to decoupling the equations and solving analytically. Numerical solutions of the coupled thermal problem are expected to be more useful than decoupled analytical solutions within a HSV aerothermoelastic simulation framework. This is due to the fact that the heat flux depends on various factors, such as deformation and aerodynamic flow properties, that are not known ahead of time. The thermal load vector must therefore be sampled at specific time instants. Note that one could decouple the equations and integrate the thermal loads numerically; however, at that point, the ability to obtain a purely analytical solution is lost. Additionally, solving the coupled system directly avoids the need to solve an eigenvalue problem to diagonalize the equations.

For the numerical solution, the transient equations are solved using a numerical time-marching algorithm. To begin the formulation, consider the full-order system of first-order ordinary differential equations governing transient heat transfer

$$M\dot{T} + KT = F(t) \quad (14)$$

where  $M$  is the thermal capacitance matrix,  $K$  is the thermal conductivity matrix, and  $F$  is the thermal load vector due to aerodynamic heating. In this work,  $F$  includes the aerodynamic heat flux and external radiation at the outer surface. Internal radiation is not included in the present formulation. The first step is to project the full-order system onto the truncated POD basis and transform from physical space to modal space using

$$\bar{\Phi}^T M \bar{\Phi} \dot{c} + \bar{\Phi}^T K \bar{\Phi} c = \bar{\Phi}^T F \quad (15)$$

The generalized thermal capacitance matrix  $m$ , generalized thermal conductivity matrix  $k$ , and generalized load vector  $f$  are then identified such that

$$m = \bar{\Phi}^T M \bar{\Phi} \quad (16a)$$

$$k = \bar{\Phi}^T K \bar{\Phi} \quad (16b)$$

$$f = \bar{\Phi}^T F \quad (16c)$$

Note that, because the POD basis vectors are not eigenvectors of the generalized eigenvalue problem,  $m$  and  $k$  will still not be diagonal at this stage. For the numerical solution of the equations, the system is

not diagonalized by solving an eigenvalue problem of the reduced system. Rather, the coupled system is integrated numerically at this stage using a second-order-accurate Crank–Nicolson scheme. Note that, since the equations are solved in modal space and the initial temperatures are known in physical space, the initial values of the POD modal coordinates must first be calculated before time marching can proceed. This is accomplished via the relation

$$c_0 = \bar{\Phi}^T T_0 \quad (17)$$

where  $c_0$  is the vector of initial values of the POD modal coordinates.

### III. Reduced-Order Structural Dynamic Model

While the full-order system of structural dynamic equations of motion is used for comparison purposes, its solution within the aerothermoelastic framework presented here is not suitable for control system design and simulation purposes. The problem of solving for the structural dynamic response of HSV structures within a control design and evaluation framework is complicated by various factors. Because of the large number of degrees of freedom involved in a traditional finite element solution, steps must be taken to reduce the order of the structural dynamics system of equations. A common approach is to employ a normal modal transformation in which the structural displacements are written as a linear combination of a small number of basis vectors, which are the free vibration mode shapes of the structure. However, this approach cannot be applied directly for HSV applications, as the mode shapes change over time due to modification of the stiffness from geometric stiffness and material degradation effects. The approach taken in this work is to first perform an offline calculation and select a reduced number of Ritz modes based on the free vibration modes and static modes of the structure at a reference temperature distribution. These Ritz modes are then used as the modal basis for solution of the structural response throughout the simulation. This procedure is applicable, as the Ritz modes need only to satisfy the geometric boundary conditions [38], which will always be the case, regardless of the stiffness distribution. The modal matrix containing the structural reference modes  $\Phi_{\text{ref}}$  will not be updated throughout the simulation, thus preventing the need to solve an eigenvalue problem of the full system during the course of the simulation. Although the reference modes will not be updated throughout the simulation, the modified stiffness matrix will be updated each time the structural dynamic response is calculated to account for the dependence on temperature. Updating of the conventional stiffness matrix is performed using the temperature dependence of the material properties of the various materials. The geometric stiffness matrix is updated by solving a static finite element problem based on the thermal loads from temperatures at the current time step and the material coefficients of thermal expansion to calculate the internal loads.

The full-order system of structural dynamic equations of motion in physical space is given by

$$M_s \ddot{x} + K_s^*(T)x = F_s(t) \quad (18)$$

where  $M_s$  is the mass matrix,  $F_s$  is the load vector, and  $x$  are the physical degrees of freedom. The modified stiffness matrix  $K_s^*$  is given by

$$K_s^*(T) \equiv K_s(T) + K_G(T) \quad (19)$$

where  $K_s(T)$  is the conventional stiffness matrix that varies due to the temperature dependence of the material properties and  $K_G(T)$  is the geometric stiffness matrix resulting from thermal stresses. The reduced-order system is obtained by first expressing the physical degrees of freedom as a linear combination of the reference free vibration modes such that

$$x = \Phi_{\text{ref}} d \quad (20)$$

where  $d$  represents the modal coordinates of the reference modes that are stored as columns of the modal matrix  $\Phi_{\text{ref}}$ . Note that, since the number of reference modes used in the modal expansion is much less

than the number of physical degrees of freedom in the model, the computational cost of the numerical solution of the system is relatively inexpensive. Once the modified stiffness matrix is known at the current time instant, the system is reduced by substituting Eq. (20) into Eq. (18) and premultiplying the system by  $\Phi_{\text{ref}}^T$ ; that is,

$$\Phi_{\text{ref}}^T M_s \Phi_{\text{ref}} \ddot{d} + \Phi_{\text{ref}}^T K_s^*(T) \Phi_{\text{ref}} d = \Phi_{\text{ref}}^T F_s(t) \quad (21)$$

The generalized mass matrix  $m_s$ , generalized stiffness matrix  $k_s^*$ , and generalized load vector  $f_s$  are then given by

$$m_s = \Phi_{\text{ref}}^T M_s \Phi_{\text{ref}} \quad (22a)$$

$$k_s^* = \Phi_{\text{ref}}^T K_s^* \Phi_{\text{ref}} \quad (22b)$$

$$f_s = \Phi_{\text{ref}}^T F_s \quad (22c)$$

As the mass of the structure is taken to be constant in this work, the reference modes will be orthogonal with respect to the mass matrix, and the generalized mass matrix  $m_s$  will reduce to the identity matrix. Since the modified stiffness matrix will be continuously changing, we have no guarantee of orthogonality of the reference modes with respect to stiffness, and the equations will be coupled. As such, the reduced-order system of equations in modal space is integrated numerically to calculate the vector of modal coordinates at each time instant. As the high-fidelity structural dynamic response solution is treated as the truth model, the numerical integration scheme used for the high-fidelity model (based on Nastran Sol 109) is implemented for integration of the reduced-order system to eliminate any discrepancies in the response due to differences in numerical integration schemes. The numerical integration method is similar to the Newmark- $\beta$  method, except that the load vector is averaged over three time instants and the stiffness matrix is modified such that the dynamic equation of motion reduces to a static solution if no inertial effects or damping exists [39]. The scheme uses a central finite difference representation for the velocity and acceleration at discrete times given by [39]

$$\dot{d}_n = \frac{1}{2\Delta t_{\text{AE}}} (d_{n+1} - d_{n-1}) \quad (23a)$$

$$\ddot{d}_n = \frac{1}{\Delta t_{\text{AE}}^2} (d_{n+1} - 2d_n + d_{n-1}) \quad (23b)$$

where  $n$  refers to the time level. The initial conditions  $d_0$  and  $\dot{d}_0$  are used to generate the vectors  $d_{n-1}$ ,  $f_{s,n-1}$ , and  $f_{s,n}$  for the initial time step,  $n = 0$ , using

$$\dot{d}_{-1} = d_0 - \dot{d}_0 \Delta t_{\text{AE}} \quad (24a)$$

$$f_{s,-1} = k_s^* d_{-1} \quad (24b)$$

$$f_{s,0} = k_s^* x_0 \quad (24c)$$

Note that this formulation assumes that the initial acceleration for all points is zero (initial velocity is constant). Substituting the finite difference approximations of the velocity and accelerations, Eqs. (23), into the equations of motion (18), and averaging the applied loads over three adjacent time instants, the equations of motion are rewritten as

$$H_1 d_{n+1} = H_2 + H_3 d_n + H_4 d_{n-1} \quad (25)$$

where

$$H_1 = \frac{1}{\Delta t_{\text{AE}}^2} m_s + \frac{1}{3} k_s^* \quad (26a)$$

$$H_2 = \frac{1}{3} (f_{s,n+1} + f_{s,n} + f_{s,n-1}) \quad (26b)$$

$$H_3 = \frac{2}{\Delta t_{\text{AE}}^2} m_s - \frac{1}{3} k_s^* \quad (26c)$$

$$H_4 = \frac{-1}{\Delta t_{\text{AE}}^2} m_s - \frac{1}{3} k_s^* \quad (26d)$$

The vector of modal coordinates at the next time step  $d_{n+1}$  is obtained by decomposing  $H_1$  and applying it to the right-hand side of Eq. (25).

#### IV. Reduced-Order Aerothermal Model

To compute the aerodynamic heat flux, a reduced-order aerothermodynamic model (ROAM) is generated from CFD solutions to the Navier–Stokes equations using kriging [23,40–45]. Kriging is a useful method for replacing expensive computer models (i.e., CFD) with computationally efficient approximations, or surrogates, of nonlinear functions [44,46,47]. Typical surrogate prediction times are on the order of a fraction of a second, whereas a single CFD computation may take on the order of minutes to hours [23].

A kriging interpolation is generated from training data, or snapshots, of the full-order simulation of the computer model. The kriging approximation of the function of interest is characterized by local deviations  $C(x)$  from a global approximation  $R(x)$ , as defined by [41]

$$y(d) = R(d, X) + C(d, X) \quad (27)$$

where  $y(d)$  is the kriging prediction at a desired point  $d$ , and  $X$  are the sample points of the training data. For this study, the kriging surrogates are computed using the Design and Analysis of Computer Experiments (DACE) [40] toolbox in MATLAB®. Typically,  $R(d, X)$  is a polynomial regression function, which is assumed to be either constant, linear, or quadratic, and the constants of the polynomials are generally determined in a least-squares sense. The quantity  $C(d, X)$  provides the local deviations by means of a correlation function, defined by the user. In this study, the Gaussian correlation function [40] and quadratic polynomial regression function are used for the kriging surrogate. Note that, in order to minimize the computational burden of generating the CFD sample data, it is assumed in this work that the primary features of the aerodynamic heating are captured using a steady-state analysis. Culler and McNamara [18], found that aerothermal–aeroelastic coupling for a skin panel in hypersonic flow is primarily quasi-static due to disparate timescales between the heat transfer and aeroelastic subsystems. Thus, for the present study, the full-order CFD sample points for the aerodynamic heating are computed using a steady-state flow analysis, enabling a significant reduction in the expense of generating the sample points.

For the present study, the ROAM is constructed to compute the aerodynamic heat flux on the control surface as a function of 1) freestream Mach number, 2) angle of attack, 3) altitude, 4) surface deformation, and 5) surface temperature. Both the structural deformation and surface temperature are represented as consistent with the modal bases determined for the reduced-order thermal and structural models; that is,

$$w(x, y) = d_1 \Phi_{\text{ref},1}(x, y) + d_2 \Phi_{\text{ref},2}(x, y) + \dots + d_{n_s} \Phi_{\text{ref},n_s}(x, y) \quad (28)$$

$$T_w(x, y) = c_1 \begin{Bmatrix} \varphi_1^{(1)} \\ \vdots \\ \varphi_k^{(1)} \end{Bmatrix} + c_2 \begin{Bmatrix} \varphi_1^{(2)} \\ \vdots \\ \varphi_k^{(2)} \end{Bmatrix} + \dots + c_{n_t} \begin{Bmatrix} \varphi_1^{(n_t)} \\ \vdots \\ \varphi_k^{(n_t)} \end{Bmatrix} \quad (29)$$

where  $\{\varphi_1^{(i)} \dots \varphi_k^{(i)}\}^T$  represent the contribution of only the surface nodal temperatures of the  $i$ th POD mode of the thermal problem. Therefore, the ROAM will include two trajectory parameters (Mach number and altitude), one control input parameter (angle of attack),  $n_s$  structural parameters ( $d_i$ ), and  $n_t$  thermal parameters ( $c_i$ ). The

**Table 1** Bounds of parameters used to construct the aerodynamic heating ROAM

Parameter	Values
$M_\infty$	5.0–10.0
$\alpha$	–5.0–5.0°
Alt.	25.0–45.0 km
$w/L$	–10–10%
$T_w$	$T_\infty$ –1500 K

bounds for these parameters are shown in Table 1, where  $w$  is the nodal deflection of the wing in the  $z$  direction,  $L$  is the distance from the attachment point (located at the root midchord) to each node, and  $T_\infty$  is the freestream temperature at the altitude of the sample point. These bounds are selected based on expected operating conditions, linearity of deformation, and maximum temperature limits of the heat shield. A flowchart of the ROAM construction is provided in Fig. 4.

Given the control surface configuration and the bounds on the parameter space defined in Table 1, ROAM construction begins with generation of training data. First, Latin hypercube sampling (LHS) is used to generate a random, yet uniformly distributed, sampling of the selected parameter space. The sample points are then passed to the CFD solver and the aerodynamic heating solutions for the flow field are collected;  $n_e$  snapshot solutions are saved for evaluation of the ROAM accuracy, while  $n_k$  snapshot solutions are used in conjunction with the DACE toolbox [40] in MATLAB. The DACE toolbox is used to construct the kriging surrogate models. This toolbox allows for the rapid interchange of different polynomial regression functions  $R(x)$  and correlation functions  $C(x)$  in order to identify the most accurate kriging surrogate method. If the desired accuracy of the surrogate is not met, more sample points can be added to the ROAM process. For the current study, the LHS is generated from 2000 sample points, optimized to maximize the minimum distance between sample points over 500 iterations.

## V. Influence of Flexibility on Aerodynamic Performance and Control Input

A goal of this work is to assess the aerothermoelastic effects on the vehicle control effectors. These effects ultimately result in changing aerodynamic forces and moments on the vehicle over time, which will in turn affect its flight dynamics. To maintain the desired trajectory, it is essential that the vehicle possess adequate control authority to account for these effects. As the HSV control surface is

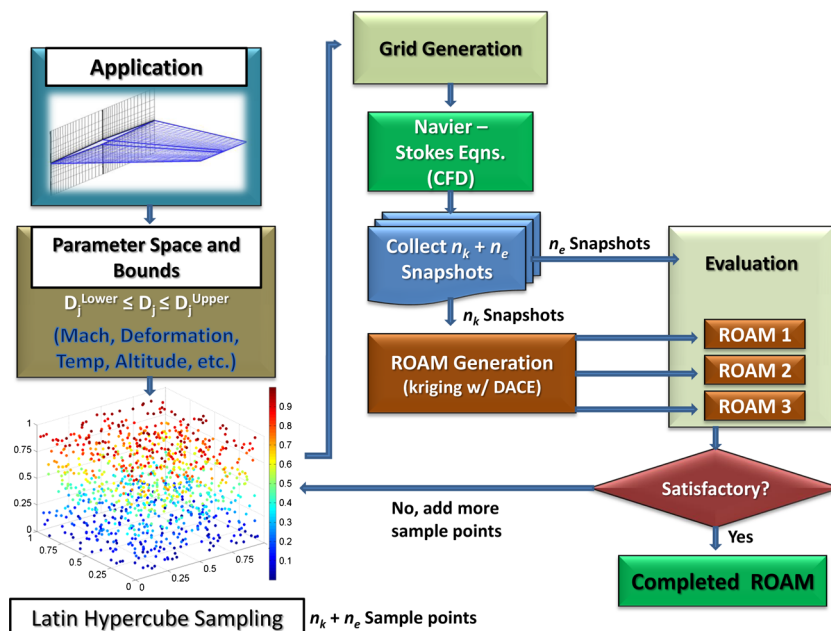
expected to have a strong influence on the dynamics of the vehicle, it is considered as a case study in this work. At each time step, the aerodynamic pressure loads and thermal loads cause the control surface to deform, leading to a change in the forces and moments acting on the vehicle and altering the trim state of the vehicle. As such, an iterative routine is employed for which the objective is to quantify the control input necessary to account for control surface flexibility. The routine uses the control surface angle of attack  $\alpha$  as the control input and the lift produced by a rigid control surface  $L_R$  as the objective. The lift produced by a rigid control surface at particular flight conditions and a specific rigid angle of attack  $\alpha_0$  is first calculated. Aerothermoelastic simulation of the elastic control surface at these flight conditions is then carried out at  $\alpha_0$ . At predetermined time intervals within the aerothermoelastic simulation, the analysis is paused and the deformed configuration at that time instant is stored. The deformed configuration is then used within the aerodynamic solver, and the angle of attack is iterated on to minimize the residue  $\mathcal{R}$  given by

$$\mathcal{R} = |L_R - L_E| \quad (30)$$

where  $L_E$  is the lift produced by the flexible (elastic) control surface. The residue is minimized by iterating on  $\alpha$  using `fminbnd`, an internal MATLAB minimization routine. This routine is based on a golden section search algorithm combined with successive parabolic interpolation. The angle of attack that minimizes the residue  $\alpha_{\min}$  is then stored at the current time instant, and the simulation resumes at  $\alpha_0$  to the next time instant. A flowchart of the iterative routine is given in Fig. 5. The introduction of such a routine provides insight as to the necessary robustness of the vehicle control effector under aerothermoelastic effects.

## VI. Control Surface Model

The HSV configuration considered in this study, as developed in a previous work [48], is shown in Fig. 6. A finite element model of a representative HSV elevator has been created for use in this study. The thickness from the top skin layer to the bottom skin layer is 4% chord length [49–51]. The top and bottom skin layers are each equipped with two 3.8-mm-thick thermal protection system (TPS) layers; thus, the thickness of the outer mold line is 4% chord length plus the 15.2 mm of TPS material. The chord length at the root is 5.2 m [48] and the leading edge makes an angle of 34° with the  $y$  axis, while the trailing edge makes an angle of 18° with the  $y$  axis [52].



**Fig. 4** Schematic of ROAM framework.



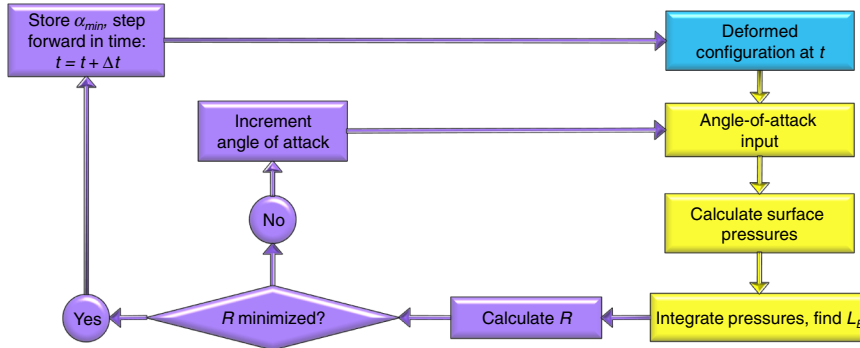


Fig. 5 Flowchart illustrating influence of aerothermoelastic effects on vehicle dynamics and control.

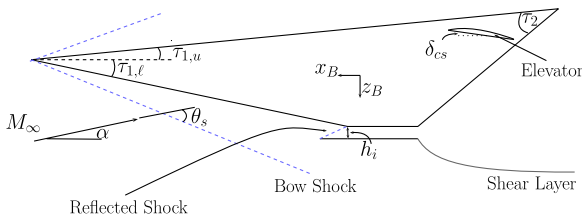


Fig. 6 Overall HSV geometry illustrating position of control surface.

Planform and cross-sectional views of the airfoil are given in Figs. 7 and 8, respectively.

A survey of the literature revealed a wide range of design strategies for mitigating the high temperatures experienced in hypersonic flight [53–60]. This study considers a TPS consisting of an outer heat shield and middle insulation layer on top of the structure, as shown in Fig. 9. The material for the heat shield is chosen to be René 41, as it was found to be efficient in terms of mechanical properties at elevated temperatures [59]. For the insulation layer, three different materials were considered in the preliminary materials evaluation: internal multiscreen insulation (IMI) [54], high-temperature flexible Min-K [59], and Q-Fiber felt [54]. Of these, the Min-K insulation, which is a proprietary silica-based material faced with astroquartz cloth [59], is selected due to its relatively low thermal diffusivity. For the structure, the titanium-alloy TIMETAL 834 (formerly known as IMI 834) is chosen. The advantage of using this alloy is its relatively high maximum application temperature (600°) when compared with that of Ti-6242S (520°), Ti-6242 (450°C), Ti-811 (400°C), and Ti-6-4 (300°C) [61]. The thermal and mechanical properties of the three materials employed in the model are given in Table 2, where T-dep. indicates that the property is temperature dependent [52,59,62,63]. Note that the temperature-dependent material properties for René 41

are extrapolated beyond the available data, and the maximum application temperature  $T_{max}$  is set to 1500 K. The emissivity of the heat shield is taken to be 0.85 [52,64]. The thermal strain is calculated based on the temperature change with respect to a reference stress-free temperature. The reference temperature used for calculation of thermal strain is taken to be 293 K for all materials.

The finite element model used for the thermal and structural modeling aspects of the study is shown in Fig. 10, with the top surface removed for visualization purposes. The model consists of the thermal protection layers system described, along with chordwise and spanwise stiffeners. The material used for the stiffeners is TIMETAL 834, and the thickness of all stiffeners is 25.4 mm (1 in.). The model contains 2812 thermal degrees of freedom and 8074 structural degrees of freedom. The heat shield and insulation are each modeled using one layer of six-node solid wedge elements, resulting in two elements in the thickness direction of the TPS. Recent work [65] suggests that refinement of the mesh in the thickness direction of the TPS may improve the thermal solution; however, this is not explored in the current work. The top and bottom skins and stiffeners are modeled using three-node 2-D triangular elements. Of the 6886 elements in the model, 3456 are solid elements and 3430 are triangular elements. The control surface is taken to be all moveable about a hinge line located at the midchord [48] and will thus be connected to the vehicle main body through a torque tube. This attachment is modeled by constraining the region indicated by the circle in Fig. 10 in all degrees of freedom. In addition, the nodes at the root are constrained against translation in the y direction. Because the stiffness of the insulation layer is neglected, rigid (RBE2) elements are used between each skin node and the corresponding node at the outer surface of the insulation layer to prevent singularities in the solution.

The NASA Langley Research Center CFL3D code [66,67] is used in this study for full-order CFD solutions for the aerodynamic heating. The CFL3D code uses an implicit, finite volume algorithm based on upwind-biased spatial differencing to solve the Reynolds-averaged Navier–Stokes (RANS) equations. Multigrid and mesh sequencing are available for convergence acceleration. The algorithm, which is based on a cell-centered scheme, uses upwind-differencing based on either flux-vector splitting or flux-difference splitting, and it can sharply capture shock waves. The Menter  $k-\omega$  shear stress transport [68] turbulence model is used in this study for closure of the RANS equations. Note that the applicability of this turbulence model in the hypersonic regime was verified recently in

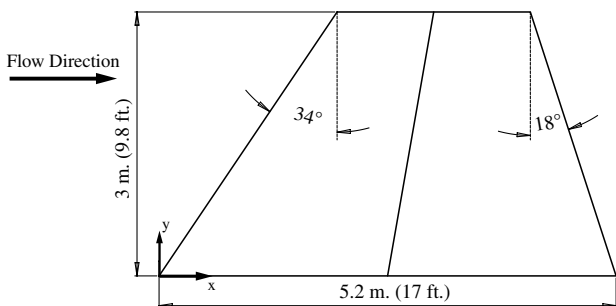


Fig. 7 Planform geometry of control surface model.

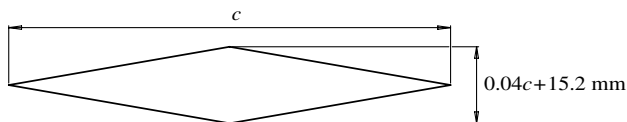


Fig. 8 Cross-sectional geometry of control surface model.

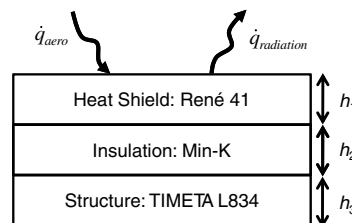


Fig. 9 Schematic of material stacking scheme at outer mold line of structure.

**Table 2 Structural and thermal material properties used in study**

	$\rho$ , kg/m <sup>3</sup>	$E$ , Pa	$\nu$	$\alpha_T$ , $\mu\text{m}/\text{m}/\text{K}$	$k_T$ , W/m/K	$c_p$ , J/kg/K	$T_{\text{max}}$ , K	$h$ , mm
René 41	8240	T-dep.	0.31	T-dep.	18	541	1500	3.8
Min-K	256	Neglect	Neglect	Neglect	0.052	858	1250	3.8
TIMETAL 834 <sup>a</sup>	4550	T-dep.	0.31	11	7	525	873	3.175

<sup>a</sup>Properties  $\nu$ ,  $k_T$ , and  $c_p$  for TIMETAL 834 were obtained from <http://www.matweb.com/search/DataSheet.aspx?MatGUID=a74096c99aa6486382a9c9e1be0883c4> [retrieved 24 January 2011].

two separate studies [69,70]. Also, note that CFL3D is an ideal gas code. It is well known that an ideal gas assumption represents an overprediction of the heat load in hypersonic flow. For instance, a recent study [18] has shown that at moderate hypersonic Mach numbers, the ideal gas assumption results in up to a 14% overprediction of the heat load. However, in this same study, this variation was less than the variation between different turbulence models. Thus, while using an ideal gas assumption introduces error into the present analysis, it is expected to be within the standard bounds of uncertainty for computational aerothermodynamic predictions of RANS CFD codes.

Note that the heat flux in CFL3D is computed in the form of the Stanton number, as given by [66]

$$\dot{q}_{\text{aero}} = c_p V_{\infty} \rho_{\infty} St (T_w - T_i) \quad (31)$$

where  $St$  is the CFL3D Stanton number and  $T_i$  is the total or stagnation temperature.

The fluid mesh for CFD computations, shown in Fig. 11, is an H–H grid with 43 points spanwise, 109 points chordwise, and 49 points extending vertically from the surface (435,000 cells). Note that cells are clustered near the surface, leading edge, and midchord, since these locations correspond to maximum flow gradients [17]. This grid does not include any flow sections upstream or downstream of the wing surface, since the considered flow is hypersonic and disturbances cannot propagate upstream. Coefficients of lift and drag for this grid are within 0.05 and 0.25%, respectively, of a similar 3.5 million cell grid.

## VII. Results and Discussion

### A. Selection of Thermal and Structural Modes

To use the various ROMs in the aerothermoelastic solution, thermal and structural mode shapes must first be selected. The thermal POD modes are extracted from temperature snapshots, which are representative of the expected dynamics in order for the basis to most closely span the space of the transient thermal solution throughout the simulation. The temperature snapshots are taken by sampling the transient temperature distribution every second from 0–5000 s for the case of  $\alpha = 6^\circ$ , Mach 8, at an altitude of 26 km (85,000 ft). For this simulation,  $\Delta t_{\text{AE}}$  and  $\Delta t_{\text{HT}}$  are each 1 s and  $\Delta t_{\text{AT}}$  is 4 s. To calculate the aerodynamic heating in this simulation, the Eckert reference temperature method [71,72] is used. The details of

the implementation of the Eckert reference temperature method are given in a previous work [21]. Using the 5001 snapshots, the snapshot matrix is assembled, its SVD is taken, and the POD modes are extracted. The first 24 eigenvalues of the correlation matrix are given in the semilog plot shown in Fig. 12. Based on the magnitudes of the eigenvalues, the first 12 POD modes are used in the reduced-order thermal solution.

The strategy for selecting the structural reference modes is to solve the free vibration problem at a reference thermal state that approximates the average temperature distribution (and thus average stiffness distribution) over the length of a mission. These modes contain both the effect of material degradation with temperature as well as geometric stiffness effects due to thermal stresses. The thermal state at which to evaluate the structural reference modes is calculated by averaging the temperatures for each node of the model over the 5001 snapshots used in generating the thermal modes. Contour plots showing the temperature distribution of the reference thermal state are given in Fig. 13. Although the maximum temperature on the bottom surface for the reference thermal state exceeds the maximum application temperature of the heat shield, temperature-

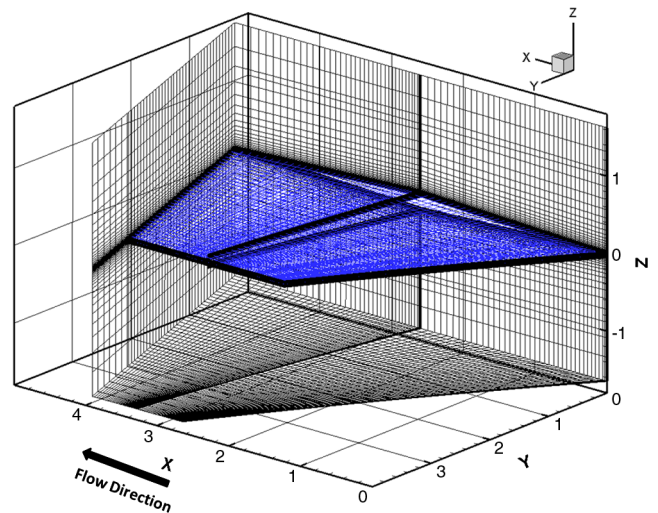


Fig. 11 CFD model of control surface used in this study.

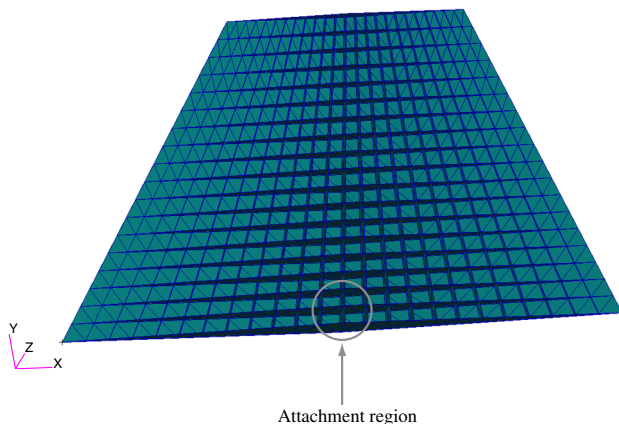


Fig. 10 Finite element model of control surface used in study.

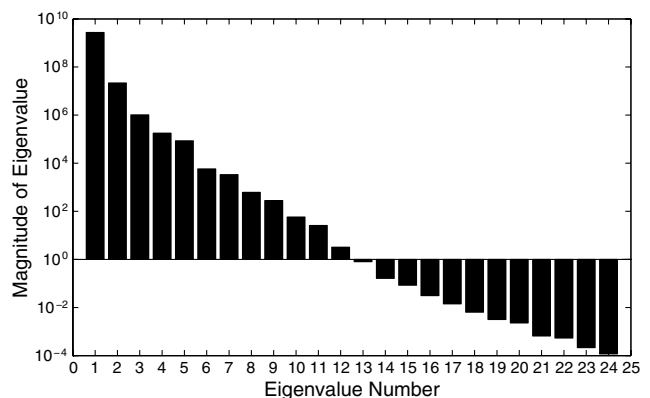


Fig. 12 First 24 eigenvalues of correlation matrix.

dependent material property data have been extrapolated beyond this point and subsequent simulations will bound the flight parameters to ensure that the maximum temperature stays within feasible range.

The structural ROM uses the first six structural dynamic reference modes at the reference thermal state, which are given in Fig. 14. The structural displacements are expected to be dominated by thermal loads early in the transient as the structure approaches thermal equilibrium and the bottom surfaces heats faster than the top surface for positive angles of attack. The thermal loads change on a slow timescale, and it is expected that the structural response to the thermal loads can be approximated in a quasi-static manner. Therefore, accuracy of the structural ROM is expected to increase by augmenting the original six-mode basis with a seventh mode that is computed based on the static deformed configuration due to the

applied thermal loads in the reference thermal state. The inclusion of this mode is likely to capture the quasi-static response due to slowly changing thermal loads that would otherwise not be captured with the original six structural dynamic eigenmodes. This additional seventh mode is shown in Fig. 14g. To assess the effect of including this mode, structural ROM validation cases are run with both the six-mode and seven-mode bases.

**B. Summary of Reduced-Order Model Error Characterization**

To validate the various ROMs, sample aerothermoelastic cases are run to compare each ROM against the corresponding high-fidelity solution. The error incurred due to model reduction is quantified using two different error metrics. The first is the normalized

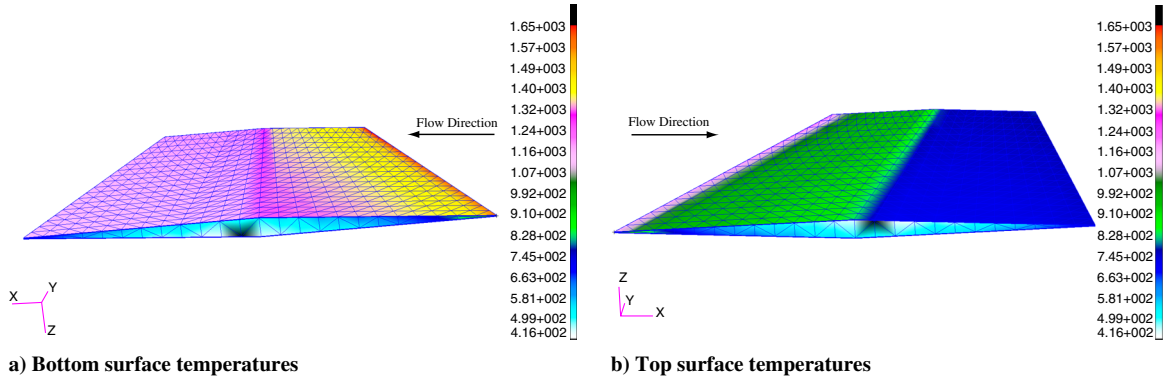


Fig. 13 Temperature distribution (K) at reference thermal state over outer surfaces of structure.

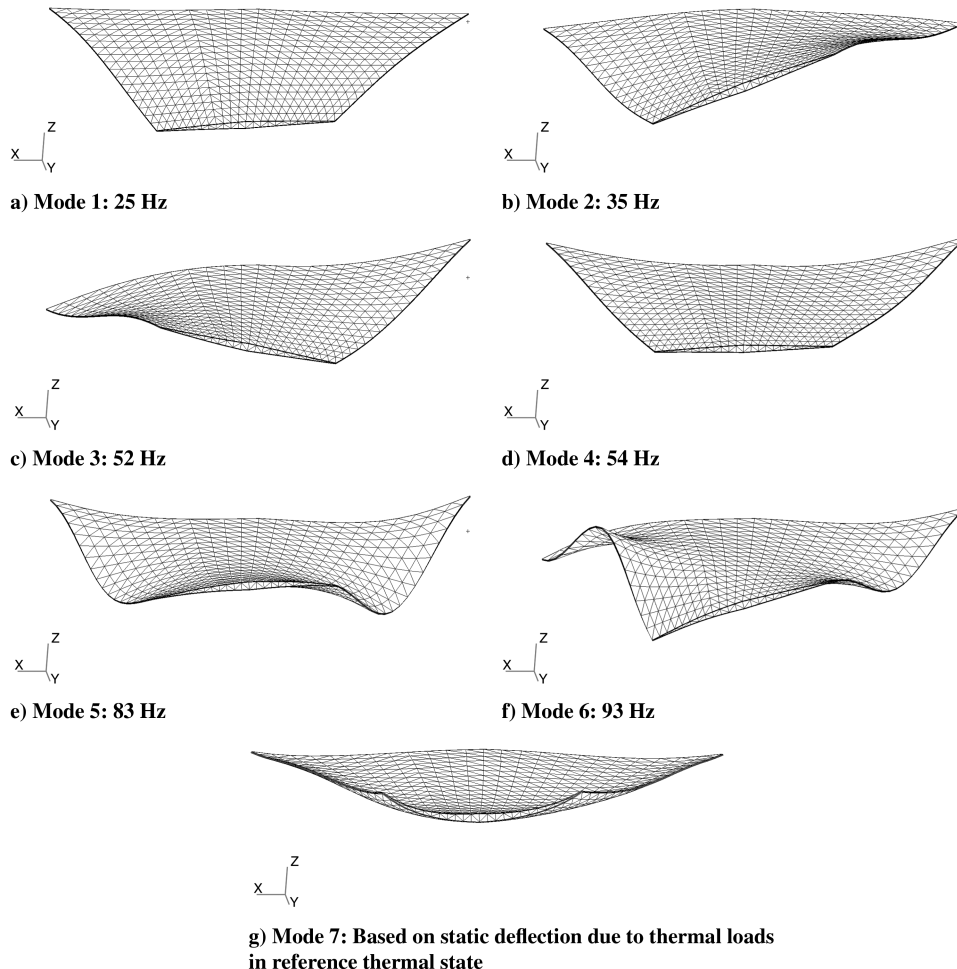


Fig. 14 Structural mode shapes based on reference thermal state.



**Table 3** Summary of average error of various ROMs for selected cases

ROM	Case	Average NRMSE, %	Average $L_\infty$ , %
Thermal	1RF	2.19	16.4
	2RF	4.52	28.8
Structural	4FR (7 mode)	8.52	28.7
Aerothermal	From LHS	1.46	7.74

root-mean-squared error (NRMSE). Expressed as a percentage, the NRMSE is given by

$$\text{NRMSE} [\%] = \frac{\sqrt{\frac{1}{s} \sum_{i=1}^s (\text{ROM}_i - \text{Full}_i)^2}}{\text{Max}(\text{Full}) - \text{Min}(\text{Full})} \times 100 \quad (32)$$

where  $i$  is the  $i$ th nodal value of interest, ROM represents a solution vector of the ROM, Full represents a solution vector of the full-order model, and  $s$  is the total number of data points in the solution vector. The second error metric to be used in validation of the ROMs is the  $L_\infty$  error. Expressed as a percentage, the  $L_\infty$  error is given by

$$L_\infty [\%] = \frac{\text{Max}(|\text{ROM} - \text{Full}|)}{\text{Max}(\text{Full}) - \text{Min}(\text{Full})} \times 100 \quad (33)$$

A summary of the average NRMSE and  $L_\infty$  for each ROM is given in Table 3. Note that the average error is calculated differently for the thermal and structural ROMs than for the aerothermal ROM due to differences in the way the validation of each are carried out. For validation of the thermal and structural ROMs, aerothermoelastic simulations are run, and the error of the ROM with respect to the corresponding full-order model is computed over the duration of the simulation as a function of time. Thus, the average errors given in Table 3 are calculated by averaging the corresponding error metric over the duration of the simulation. The case identifications (IDs) over which the simulation errors are time-averaged are given in the table. The flight conditions corresponding to each case ID are detailed in the next section. Validation of the aerothermal ROM is carried out by generating 500 evaluation cases with the parameters chosen based on Latin Hypercube parameter space sampling. The average error of the aerothermal ROM is calculated by averaging each error metric over the 500 evaluation cases. The use of the 500 evaluation cases from LHS is denoted in table as ‘‘From LHS’’. As this is the first attempt to integrate the ROMs into a unified aerothermoelastic framework, specific areas for ROM improvement have been brought to light by this study. However, the primary focus of this paper is to implement the ROM framework and use it to assess the effect of aerothermoelasticity on HSV performance. Detailed analysis of the tradeoff between the number of aerothermoelastic states and ROM accuracy, as well as development of techniques for improving the error of the ROMs, is reserved for future work.

### 1. Validation of Reduced-Order Thermal and Structural Models

Two different flight conditions are used for validation of the thermal and structural ROMs. The altitude, Mach number, angle of attack, time-step sizes, and initial temperatures for each case are summarized in Table 4. The first letter in the case ID specifies the thermal model used, and the second letter specifies the structural model used, where R denotes the ROM and F denotes the full-order model. Note that the full-order structural model is used in the thermal

ROM validation cases (cases 1RF and 2RF), and the full-order thermal model is used in the structural ROM validation cases (cases 1FR and 2FR). The Eckert reference temperature method is used to calculate aerodynamic heat flux for all thermal and structural ROM validation cases. The initial conditions for the thermal problem are a uniform temperature distribution of 293 K for all cases. Larger time steps are taken for the thermal ROM validation cases, as the timescale of the heat transfer process is slower than that of the structural dynamics, and resolution of high-frequency structural dynamic oscillations is not of interest for thermal ROM validation.

Validation of the thermal ROM is performed by running aerothermoelastic simulations on the control surface using the Eckert reference temperature formulation for the thermal boundary conditions with both the full-order and reduced-order thermal models. Plots of the time history of the POD error for cases 1RF and 2RF are given in Figs. 15 and 16, respectively. The NRMSE percentage error shows good agreement with the full-order model in both cases. Based on the time history of the  $L_\infty$  error, it is evident that the thermal ROM has some difficulty capturing the steep temperature gradients within the initial portion of the transient. However, as the structure approaches thermal steady state and the temperature gradients and loads decrease, the accuracy of the thermal ROM improves.

The structural ROM is compared against the high-fidelity structural model using the same two validation cases used for validation of the thermal ROM. The aeroelastic time-step size  $\Delta t_{\text{AE}}$  is chosen based on the desire to resolve the structural reference modes with a minimum of 10 temporal sampling points for the highest frequency mode. The highest frequency of the chosen reference modes is 93 Hz; however, recall that this mode was calculated at the reference thermal state with elevated temperatures. Thus, to account for the fact that the frequency of this mode may be slightly higher when the structure is at lower temperatures,  $\Delta t_{\text{AE}}$  is chosen to be 1 ms based on a maximum frequency of 100 Hz with 10 temporal sampling points within one cycle. The heat transfer time step is chosen to be of the same size as the aeroelastic time step ( $\Delta t_{\text{HT}} = \Delta t_{\text{AE}}$ ), and the time between updates of the thermal boundary conditions  $\Delta t_{\text{AT}}$  is chosen to be 10 ms. The solution vectors used to compute the difference between the high-fidelity and ROM solutions are vectors of the displacement in the  $z$  direction for the nodes at the outer surface of the finite element model at each time instant. Because of the small size of the time step and relatively long run times, the error is computed for the time range 0–6 s. Plots of the time history of the error for case 1FR using both the six-mode and seven-mode bases are given in Fig. 17. The error for case 2FR is shown in Fig. 18. Both error metrics for both cases show an oscillating error in the initial part of the transient. This is due to initial high-frequency structural oscillations that are not captured by the structural ROM. As these oscillations are damped out, a decrease in the error of the structural ROM is observed. Note that these cases do not show a large difference in accuracy between the six-mode solution and seven-mode solution. This is likely due to the fact that the temperatures do not get high enough in the time range considered for the thermal loads to make a significant enough impact on the response. The structural dynamic response for longer time histories is studied next and will allow for further assessment of the difference between the six-mode and seven-mode structural ROMs at longer flight times.

While a small aeroelastic time-step size allows for the resolution of structural dynamic oscillations, in situations in which the thermal loads are dominant over the aerodynamic loads, it may be possible to capture the structural response with a larger aeroelastic time step because the thermal loads change on a much slower timescale than the aerodynamic loads. The largest heat flux in the preceding

**Table 4** Parameters for aerothermoelastic simulations used to validate thermal and structural ROMs

Case ID	Thermal model	Structural model	Alt, km	$M_\infty$	$\alpha$ , °	$\Delta t_{\text{AE}}$	$\Delta t_{\text{HT}}$	$\Delta t_{\text{AT}}$	$T_0$ , K
1RF	ROM	Full order	26	8	3	1 s	1 s	4 s	293
2RF	ROM	Full order	36	6	1.5	1 s	1 s	4 s	293
1FR	Full order	ROM	26	8	3	1 ms	1 ms	10 ms	293
2FR	Full order	ROM	36	6	1.5	1 ms	1 ms	10 ms	293



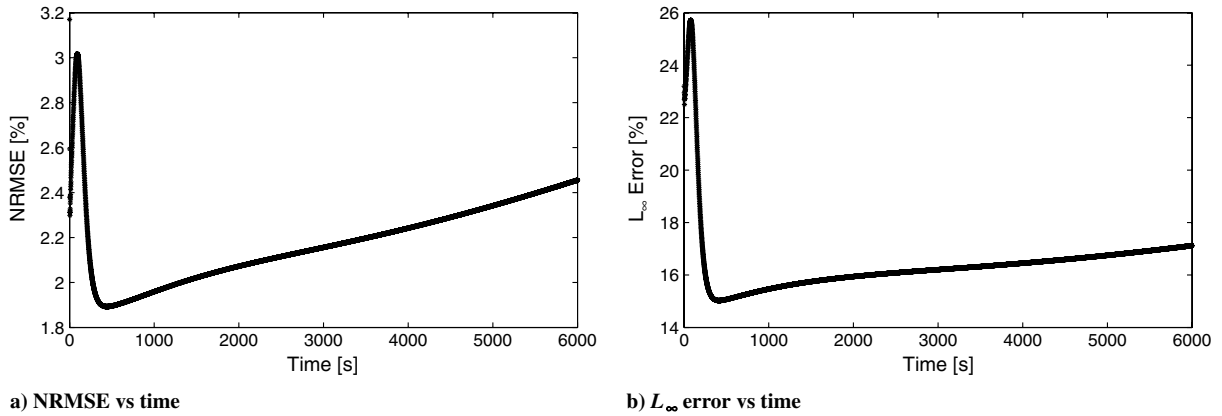


Fig. 15 Error of POD thermal model for case 1RF based on nodal temperatures.

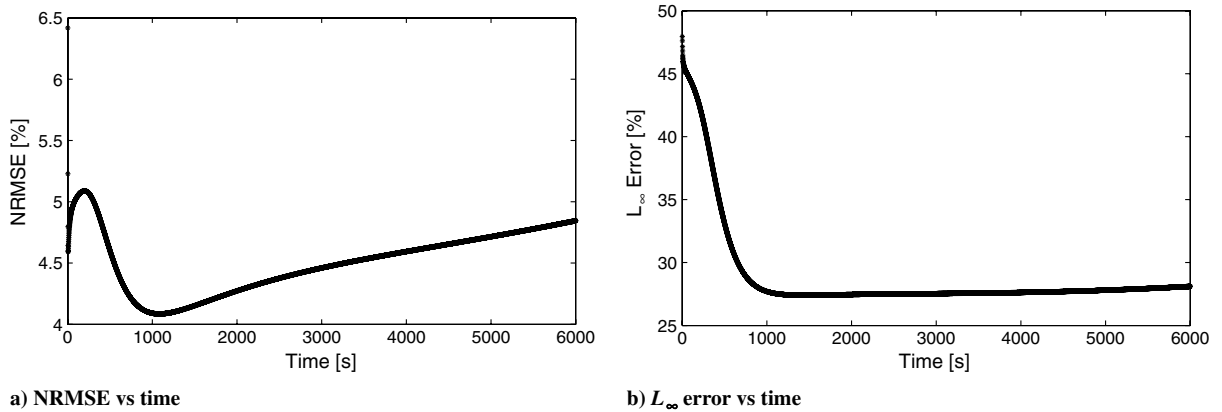


Fig. 16 Error of POD thermal model for case 2RF based on nodal temperatures.

validation cases is most likely to occur in the initial transient of the simulation. This is due to the fact that the difference between the initial wall temperature of the structure and the initial recovery temperature is large, which leads to high aerodynamic heating. Furthermore, the difference between the initial wall temperature and the environment temperature is small, which means that the heat flux radiated from the structure to the environment is small. Therefore, one might expect the highest thermal loads to occur in the initial part of the transient in cases where the structure has a low initial temperature. To assess the effect larger time-step sizes on the accuracy of the structural response, simulations are run with the full-order thermal and structural models using different time-step sizes. These cases are denoted 3FF and 4FF and are summarized in Table 5. Note that the time-step sizes for case 4FF are each 100 times larger than those for case 3FF. Additionally, the difference in structural response between

the full-order structural model and structural ROM for equal-sized larger size time steps is assessed using case 4FR. This case is also summarized in Table 5.

A comparison of the  $z$  displacements of node 247, which is located at the midchord of the tip on the bottom surface, is given for both cases in Fig. 19. Although case 4FF fails to capture the high-frequency oscillations due to the initial excitation because of its larger time steps, these oscillations are subsequently damped out and the nodal responses show good agreement following the initial oscillations. To assess the difference in  $z$  displacements between cases 3FF and 4FF for all nodes at the surface, the NRMSE and  $L_\infty$  error are calculated for the time range 0.1–6 s in intervals of 0.1 s. Case 3FF is treated as the reference, and the errors are calculated based on the difference in the solution vectors from cases 3FF and 4FF at each time instant. The plots of the error given in Fig. 20 show

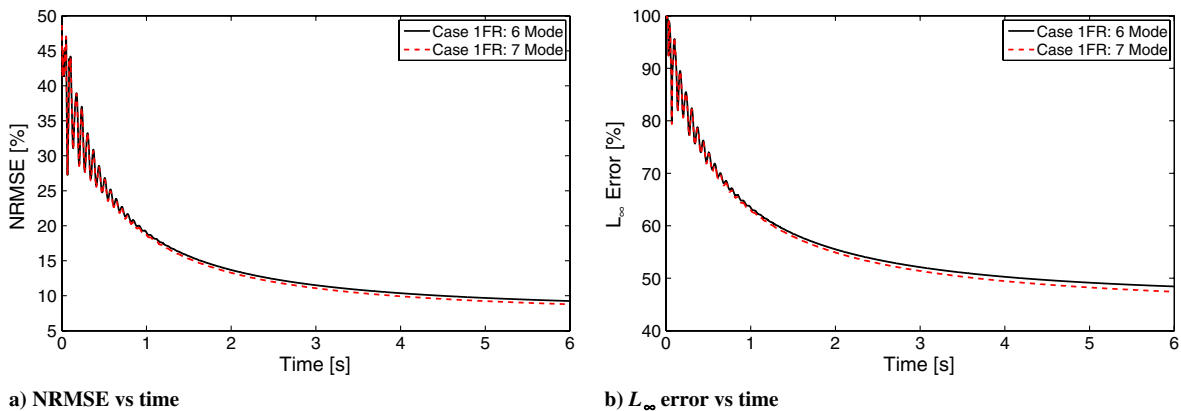


Fig. 17 Error of structural ROM for case 1FR based on  $z$  displacements of surface nodes.

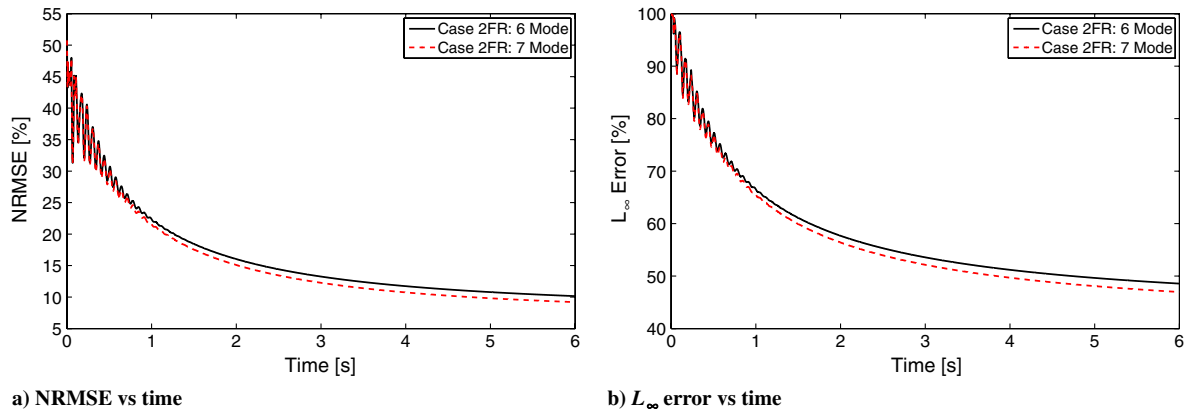


Fig. 18 Error of structural ROM for case 2FR based on  $z$  displacements of surface nodes.

good agreement between the two solutions beyond approximately 1 s. The error of the structural ROM for larger time steps is assessed by computing the NRMSE and  $L_\infty$  error between cases 4FF and 4FR. The use of larger time steps allows for error quantification over a longer time record for the same amount of computational time. Plots of the error as a function of time are given in Fig. 21. Improvement of the structural ROM by addition of the seventh mode is observed in Fig. 21 after approximately 50 s. This is likely due to the fact that, by this time, the temperatures have increased to the point where the thermal loads have a significant impact on the structural response. Although the initial NRMSE of case 4FR is approximately 39%, it improves to below 10% within 5 s. The displacement histories of node 247 for cases 4FF and 4FR with both the six-mode and seven-mode models are given in Fig. 22 to illustrate the level of structural displacements for these flight conditions.

## 2. Validation of Reduced-Order Aerothermal Model

Since this work represents an initial step in modeling the coupled aerodynamic heating/heat transfer and aerodynamic heating/structural displacement response of a hypersonic control surface, the number of modes required from the heat transfer and structural ROMs in order to accurately model the aerodynamic heating is uncertain. Thus, as a first step in accessing the aerothermoelastic response of the control surface, the first five thermal POD modes and the first five free vibration structural modes are included in the ROAM.

Previous work [23] has demonstrated the need for a large number of snapshots to accurately capture the aerodynamic heat flux on a hypersonic control surface. Thus, 2500 snapshots are selected through LHS for the parameter space defined in Table 1. Two-thousand snapshots are used to construct the kriging ROAM, and 500 are used to evaluate the ROAM accuracy. The NRMSE and  $L_\infty$  error metrics are used to illustrate the accuracy of the ROAM. These error metrics are computed for each evaluation case, resulting in a vector of 500 NRMSE and  $L_\infty$  values for the ROAM. To provide a scalar quantification of the model quality, the average NRMSE and average  $L_\infty$  over the 500 evaluation cases are computed, and the overall max  $L_\infty$  of the 500 is identified. The average NRMSE is 1.46%, the average  $L_\infty$  is 7.74%, and the overall maximum  $L_\infty$  at any point on the control surface for all 500 evaluations is 27.54%. It is important to note that a concern for this approach in high-Mach-number flows is accurate shock capturing [73]. However, the average NRMSE and

average  $L_\infty$  demonstrate that this does not significantly degrade the model.

To gain insight into the sources of error for the present problem, two additional error metrics are computed as a function of surface location. As before, both average and maximum error metrics are considered. In Eq. (34), the absolute value of the percent error at each surface location is averaged over the total number of evaluation cases  $\bar{E}$ :

$$\bar{E}[\%] = \left( \frac{1}{500} \sum_{j=1}^{500} \frac{[|\text{ROAM} - \text{Full}|]_j}{[\text{Max}(\text{Full}) - \text{Min}(\text{Full})]_j} \right) \times 100 \quad (34)$$

where  $j$  refers to the  $j$ th evaluation case. To identify the locations where maximum errors most often occur, the surface location of the  $L_\infty$  error for each evaluation case is recorded. The total number of times an  $L_\infty$  error occurs at a given location is then tabulated.

These error metrics as a function of surface location are shown in Figs. 23a–23d for the ROAM. Several general observations can be made from these results. First, it is clear that the average error as a function of surface location, shown in Figs. 23a and 23c, is generally small. The average error over the entire control surface is generally less than 2%. Furthermore, the average error is generally highest near the leading edge of the control surface. Note that these surface regions are also most likely to correspond with the maximum errors over the control surface, as shown in Figs. 23b and 23d.

## C. Influence of Aerothermoelastic Effects on Aerodynamic Forces

The ROM error analysis presented has revealed that enrichment of the basis representations for the transient thermal and structural dynamic ROMs may improve the level of errors in the ROMs, particularly early in the transient when the dynamics are changing on a relatively fast timescale. This is the subject of ongoing investigation by Falkiewicz and Cesnik [74] and is not discussed further in the present work. The ROMs are now used within the aerothermoelastic framework to assess the effect of aerothermoelasticity on the total lift and drag forces acting on the control surface. All simulations from this point forward use the corresponding ROM for the aeroheating, heat transfer, and structural dynamic response components. For these simulations, aeroelastic iterations are carried out to bring the structure to aeroelastic equilibrium before beginning the aerothermoelastic simulation. To find the total aerodynamic forces, the pressures

Table 5 Parameters for aerothermoelastic simulations used to assess effect of time-step size on structural response

Case ID	Thermal model	Structural model	Alt, km	$M_\infty$	$\alpha, ^\circ$	$\Delta t_{AE}$	$\Delta t_{HT}$	$\Delta t_{AT}$	$T_0, K$
3FF	Full order	Full order	26	8	3	1 ms	1 ms	10 ms	293
4FF	Full order	Full order	26	8	3	0.1 s	0.1 s	1 s	293
4FR	Full order	ROM	26	8	3	0.1 s	0.1 s	1 s	293

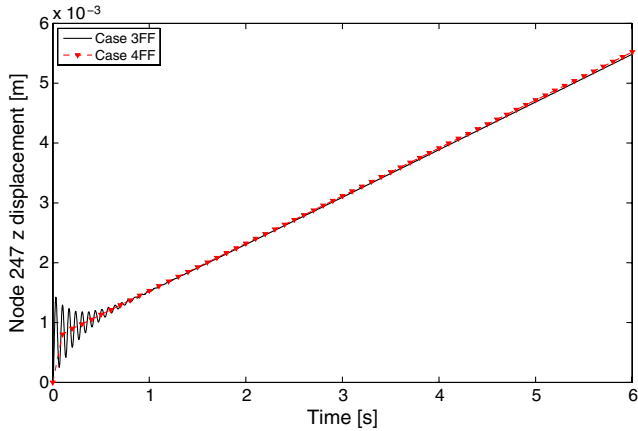


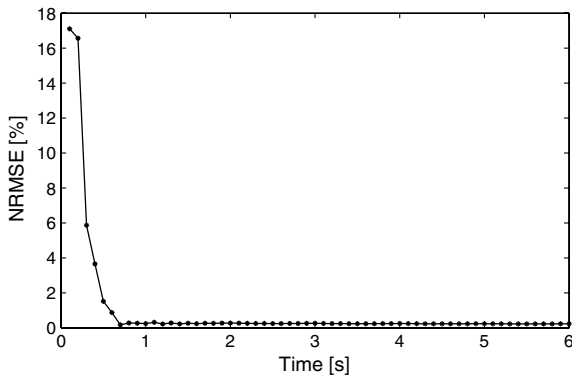
Fig. 19 Node 247 z displacements: case 3FF vs case 4FF.

are integrated over the outer surfaces of the structure. The pressures consist of a component in the direction of the local normal to the deformed configuration due to aerodynamic pressures and a component in the direction of the local tangent to the deformed configuration due to viscous stress. The viscous stress is computed using local skin friction coefficients for each finite element using an Eckert reference temperature formulation [21]. Note that local normal and tangent vectors for each finite element are updated at each aeroelastic time step based on the deformed configuration at the end of the time step.

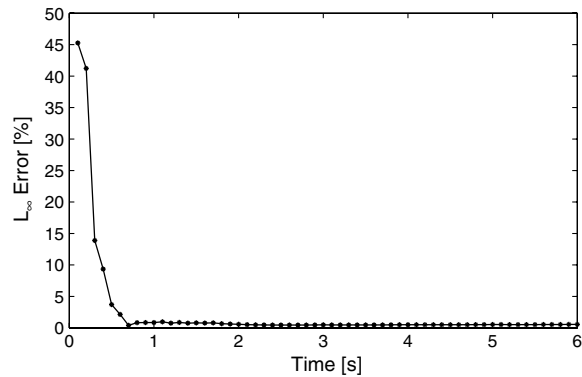
The time-marching aerothermoelastic cases used for this part of the study are summarized in Table 6, where LD denotes that these cases are used for lift and drag assessment. Cases 1LD and 2LD represent constant-Mach constant- $\alpha$  cruise at an initial uniform

stress-free temperature of 293 K. These cases are run for longer time histories (0–1200 s) to allow the temperature distribution to more fully evolve and to assess the response over a wide range of thermal conditions. Cases 3LD and 4LD are based on the ascent phase of a proposed trajectory for a single-stage-to-orbit mission of a blended wing body configuration typical of a transatmospheric HSV [58]. Curve fits to the data points given in [58] are used to obtain the time dependence of the altitude and Mach number, and two different angles of attack are selected. As the ROMs in this work are valid for a specific range of flight parameters, only the Mach 5–Mach 10 portion of the ascent trajectory is considered. Difficulty arises in determining the initial temperatures for the Mach 5–Mach 10 ascent, because the ROMs do not permit simulation from takeoff to Mach 5. Thus, to obtain the initial temperatures, a cruise condition at the initial altitude (25 km) and initial Mach number (Mach 5) is simulated for 163 s, which is the amount of time the vehicle takes to reach Mach 5 for the given trajectory. This initial temperature distribution is denoted as 163 s cruise in the table. After the initial cruise segment, the temperature distribution is stored for use as the initial temperature distribution for the ascent segment, time is reset to zero, and the ascent trajectory is commenced. As the simulation time for these cases is shorter (0–172 s), the corresponding time-step sizes for these cases are chosen to be smaller.

First, the total lift force  $L$  and drag force  $D$  are calculated over the undeformed (rigid) control surface at initial flight conditions for each case. Aerothermoelastic simulation for the elastic control surface is then carried out for each case, and total lift and drag are calculated at each aeroelastic time step. Plots of the transient lift and drag force are given in Figs. 24 and 25, respectively, for both the rigid and elastic control surfaces. Note that the rigid case contains no aerothermoelastic effects, and the skin friction is computed at initial flight conditions at a uniform temperature of 293 K. To assess the relative impact of flexibility on the total forces, the percentage differences

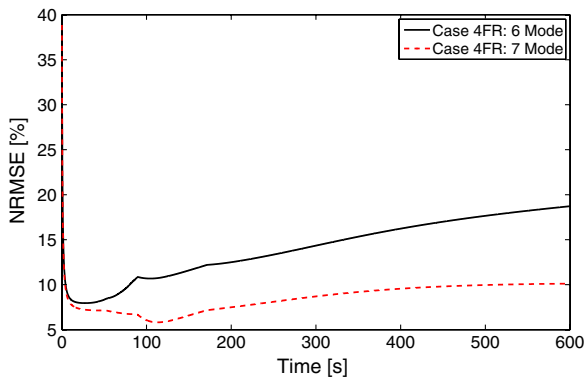


a) NRMSE vs time

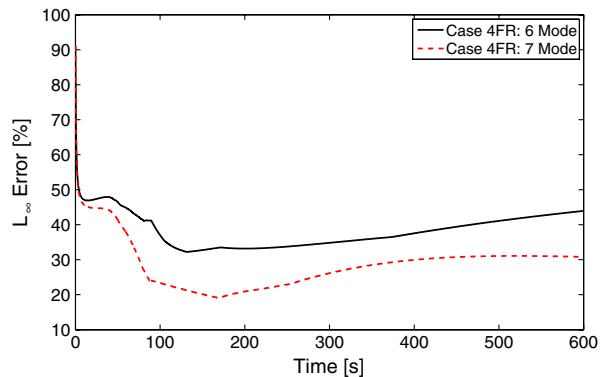


b)  $L_\infty$  error vs time

Fig. 20 Error of case 3FF with respect to case 4FF based on  $z$  displacements of surface nodes.



a) NRMSE vs time



b)  $L_\infty$  error vs time

Fig. 21 Error of case 4FR with respect to case 4FF based on  $z$  displacements of surface nodes.

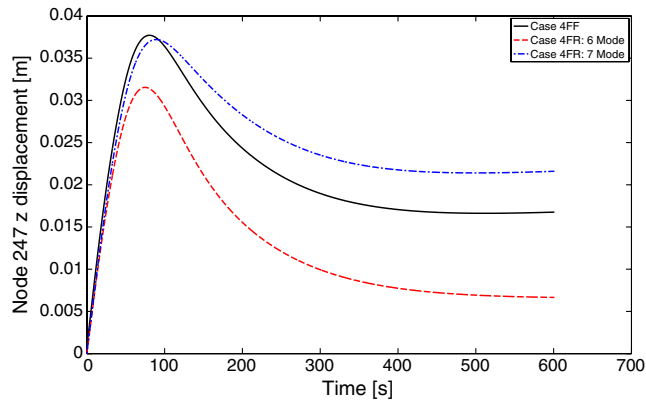


Fig. 22 Node 247 z displacements: case 4FF vs case 4FR.

between the lift and drag of the elastic structure are calculated with respect to the lift and drag produced by the rigid structure. Results for the relative change lift and drag are given in Figs. 26 and 27, respectively. For cases 1LD and 2LD, there is an initial abrupt change in the lift and drag forces due to the high-temperature gradients as the structure begins to heat from room temperature and approach equilibrium temperature. At the end of the time history shown for these cases, the total lift and drag begin to level off as the temperature gradients in the structure decrease. For ascent trajectory represented by cases 3LD and 4LD, both the lift and drag are monotonically decreasing for the complete duration of the simulation. The initial abrupt change in the aerodynamic forces is not present in these cases, because the initial temperature distribution is much higher than that

used for cases 1LD and 2LD. Although the trend of the elastic and rigid control surfaces is similar for both lift and drag in cases 3LD and 4LD, the percentage difference between rigid and elastic drag is more pronounced than that for the lift. Note that the initial percentage change in lift and drag for cases 3LD and 4LD is nonzero due to the fact that the aerothermoelastic solution is converged at cruise before beginning the simulation such that the structure is in a thermoelastically deformed state at initial time (see Figs. 25c and 25d).

To assess the level of heating for each of the cases, the maximum nodal temperature is plotted as a function of time for each of the cases. Results are given in Fig. 28. For cases 1LD and 2LD, there is a steep increase in the maximum temperature in the initial portion of the transient as the structure heats up from room temperature. As the structure approaches equilibrium temperature, the gradient in the maximum temperature begins to decrease. For the ascent trajectory of cases 3LD and 4LD, we see a monotonic increase in maximum temperature for the time range considered. For all cases, the maximum temperature remains below 1500 K.

To determine the effect of aerothermoelasticity on aerodynamic performance and necessary control input, the iterative routine described in Sec. V is incorporated into the aerothermoelastic framework. Cases 1LD and 2LD are used as example cases, and  $\alpha_{min}$  is updated and stored every time the thermal boundary conditions are updated. Thus,  $\alpha_{min}$  is updated every 10 aeroelastic iterations (once per second) based on the relative time-step sizes used for these cases. Recall that the initial (baseline) angle of attack is  $3^\circ$  for case 1LD and  $1.5^\circ$  for case 2LD. Plots of the time history of  $\alpha_{min}$  for the two cases are given in Fig. 29. Note that the maximum residue  $\mathcal{R}$  is 0.3 N for case 1LD and 0.05 N for case 2LD. As seen in the figures, the maximum absolute departure of  $\alpha_{min}$  from the initial angle of attack is 7.6% for case 1LD and 2.8% for case 2LD. However, note that the

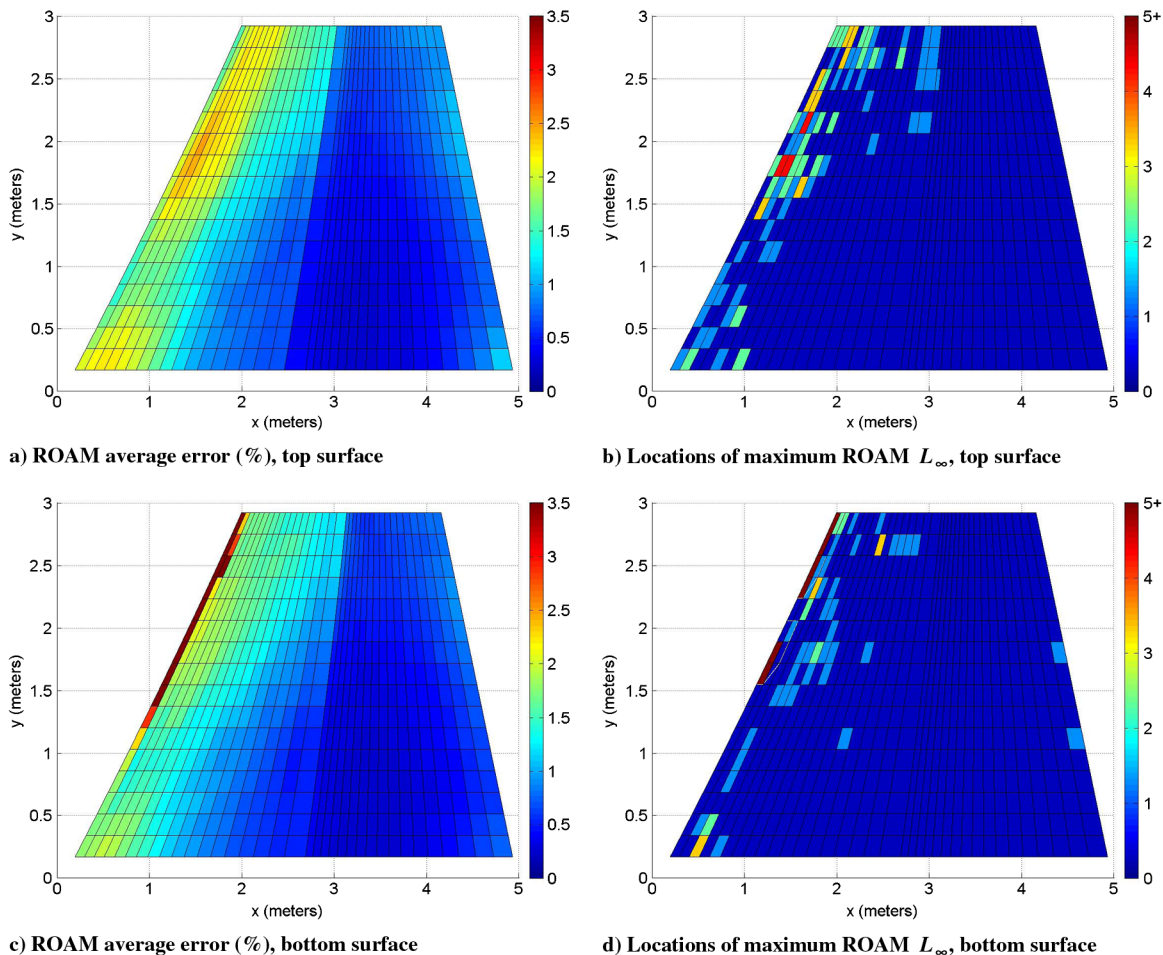


Fig. 23 Spatial distribution of ROAM errors for the 500 test cases.



**Table 6 Cases for assessment of transient lift and drag**

Case	Alt, km	$M_\infty$	$\alpha, ^\circ$	$\Delta t_{AE}, s$	$\Delta t_{HT}, s$	$\Delta t_{AT}, s$	$T_0$
1LD	26	8	3	0.1	0.1	1	293 K unif.
2LD	36	6	1.5	0.1	0.1	1	293 K unif.
3LD	$-7 \times 10^{-5}t^2 + 0.06t + 24.97$	$0.03t + 5$	3	0.01	0.01	0.1	163 s cruise
4LD	$-7 \times 10^{-5}t^2 + 0.06t + 24.97$	$0.03t + 5$	1.5	0.01	0.01	0.1	163 s cruise

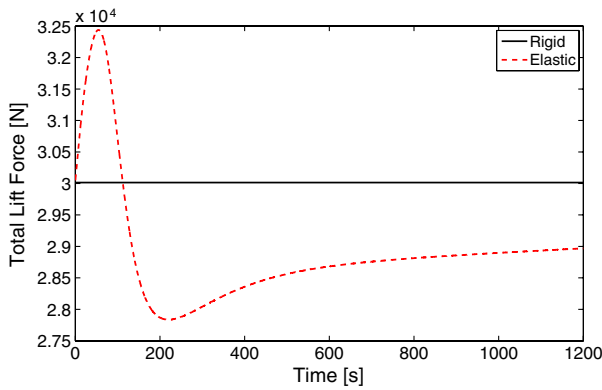
largest departures from initial angle of attack occur early in the transient, which is likely an effect of the low initial temperature and resultant large temperature gradients. For cases in which the initial temperature of the structure is higher, the deformation due to thermal loads will be lower and the impact on overall lift and drag will be less.

**D. Assessment of Computational Cost**

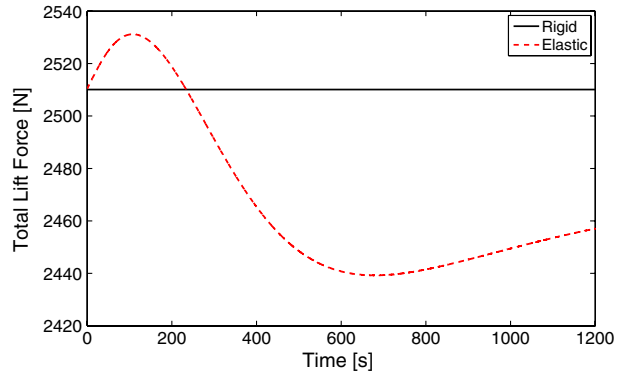
As this research is aimed at making aerothermoelastic analysis of HSVs feasible in a control design and simulation setting, one important metric is the computational cost associated with the reduced-order framework. While reducing the number of aerothermoelastic states for the purpose of control studies is the prime motivation for this work, simulation time is a practical consideration that must be taken into account. The average computational time for one iteration of each of the various components of the solution is given in Table 7 for both the full-order model and ROM. Note that the heat flux iteration for the ROM also consists of calculation of the local skin friction coefficients for each element at the outer surface using the Eckert reference temperature method. The data assumes that the size of the aeroelastic time step is equal to the size of the heat transfer time step, and that one complete aerothermoelastic iteration contains 10 aeroelastic subiterations ( $\Delta t_{AE} = \Delta t_{HT}, \Delta t_{AT} = 10\Delta t_{AE}$ ). Therefore, one heat transfer iteration contains 10 time steps of the thermal solution. The computational times for the unsteady aerodynamic iteration and aerodynamic force calculation are the same in both

cases, because the same model is used in both the full-order and reduced-order solutions. Table 7 also gives the average computational time to complete one full aerothermoelastic iteration for both the full-order model and ROM. Based on the relative time-step sizes assumed, one aerothermoelastic iteration includes one heat flux and heat transfer iteration and 10 structural iterations, unsteady aerodynamic iterations, and aerodynamic force calculations. The last column in Table 7 gives the ratio of the computational time for the full-order solution to that for the reduced-order solution.

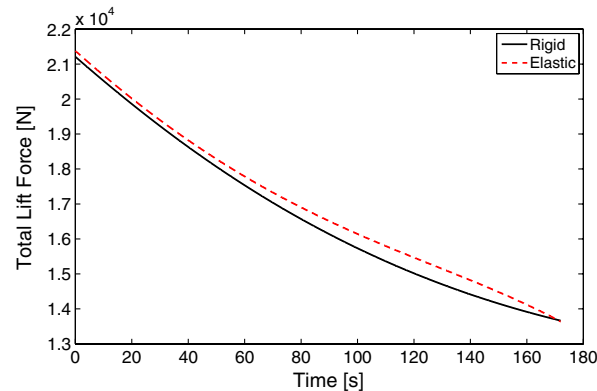
The component of the reduced-order aerothermoelastic solution with the greatest computational cost is the structural iteration. Although a fixed basis is used for the structural dynamic response, the current framework still requires that finite element solver be called at every structural iteration to update the  $8074 \times 8074$  physical stiffness matrix and premultiply and postmultiply by the structural reference modes to obtain the generalized stiffness matrix. Recall that the physical stiffness update consists of updating the conventional stiffness matrix based on the temperature dependence of material properties, as well as solving a linear static problem to generate the geometric stiffness matrix. There exists potential for significant improvement in overall computational cost by using reduced-order techniques to update the generalized stiffness matrix directly based on the temperature distribution. A methodology for accomplishing this is the subject of a recent work by Falkiewicz and Cesnik [74].



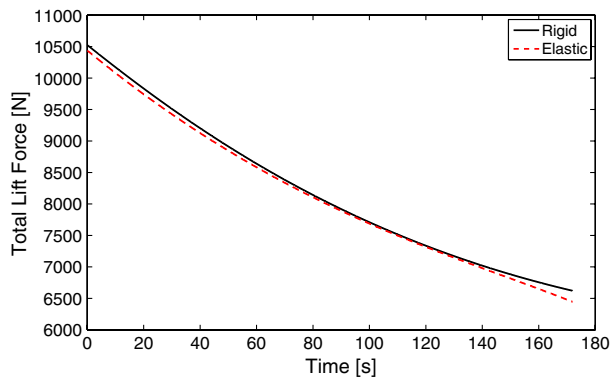
a) Case 1LD



b) Case 2LD



c) Case 3LD



d) Case 4LD

**Fig. 24 Time history of total lift force for various flight conditions.**

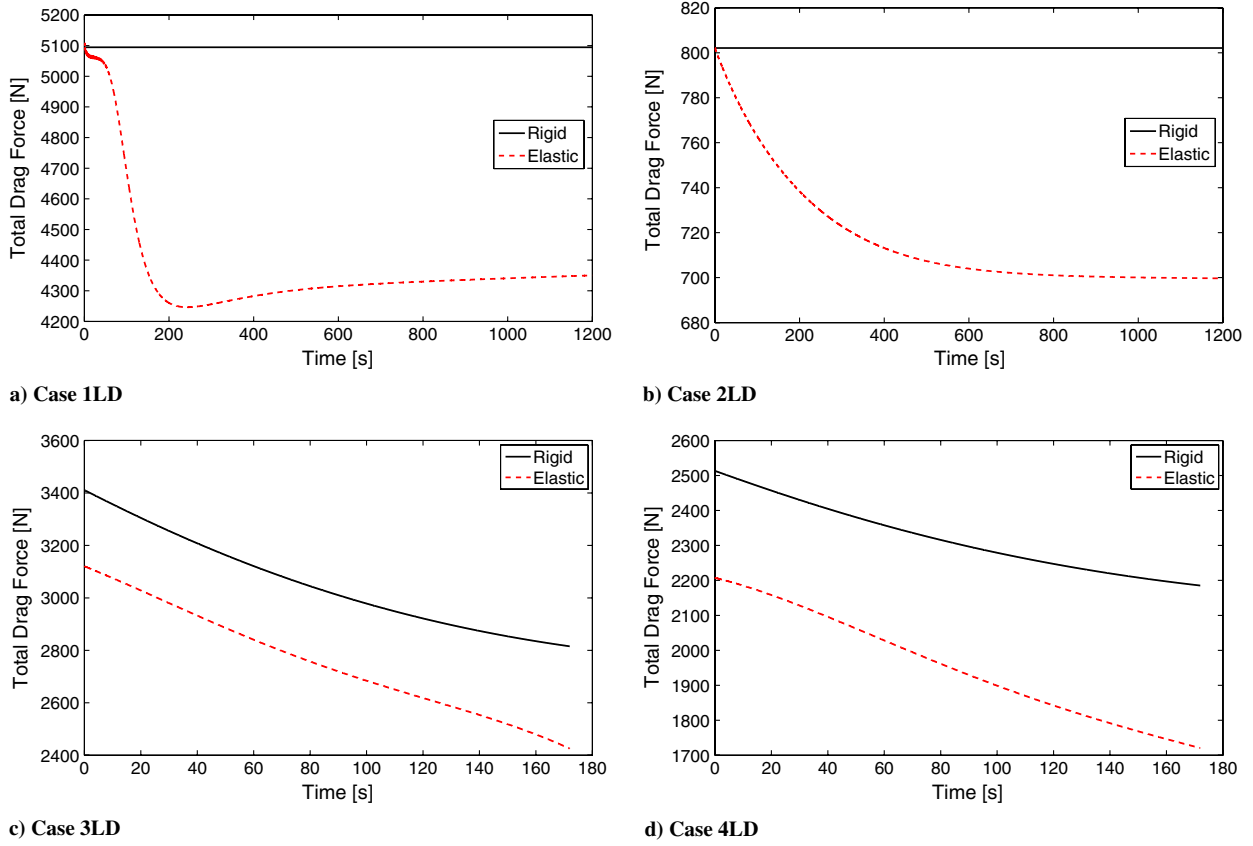


Fig. 25 Time history of total drag force for various flight conditions.

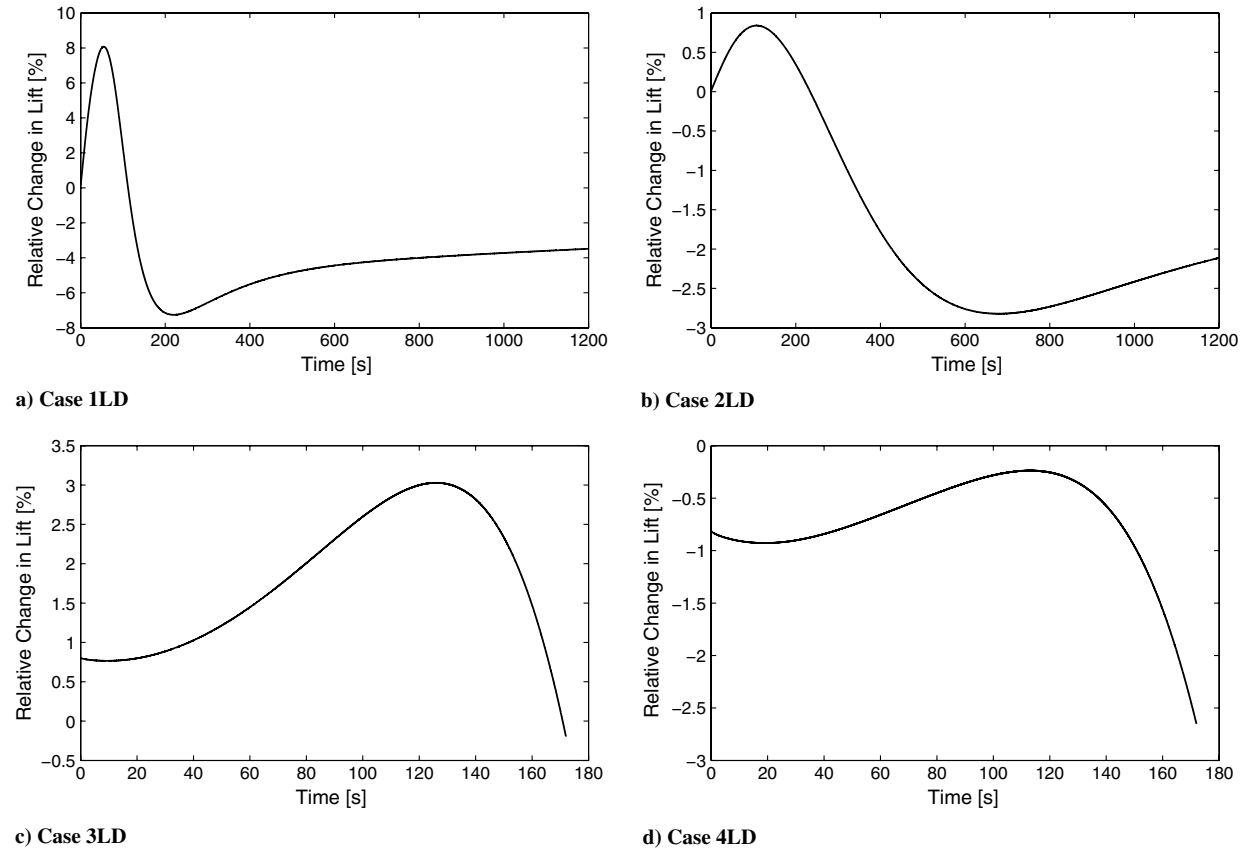
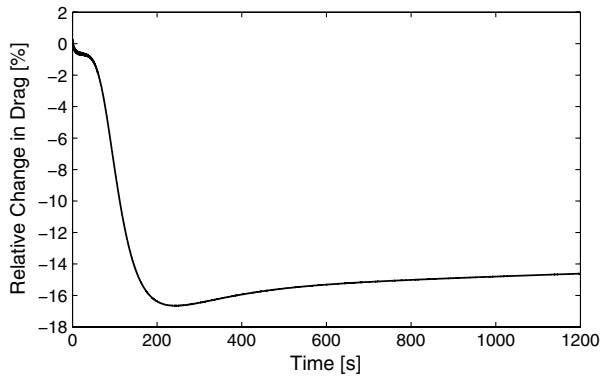
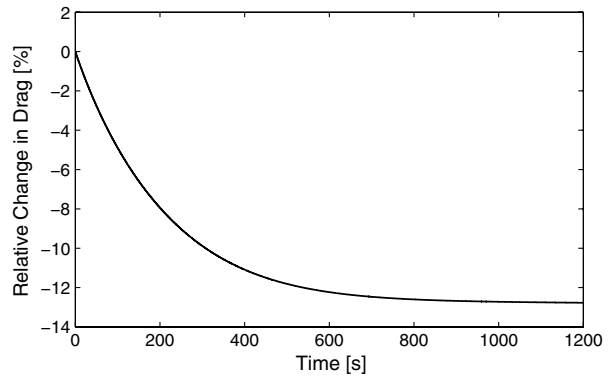


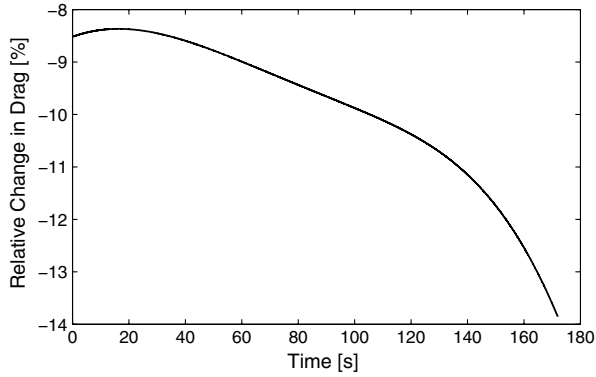
Fig. 26 Time history of percentage change in lift force for various flight conditions.



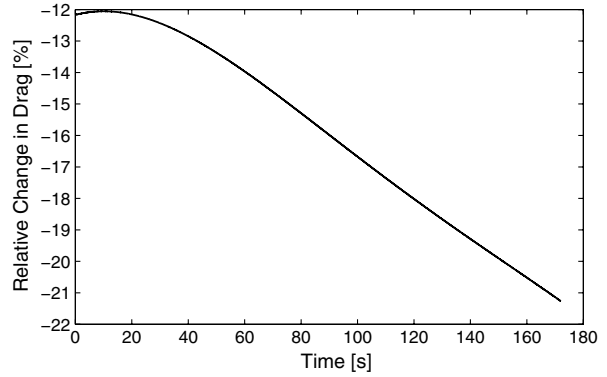
a) Case 1LD



b) Case 2LD

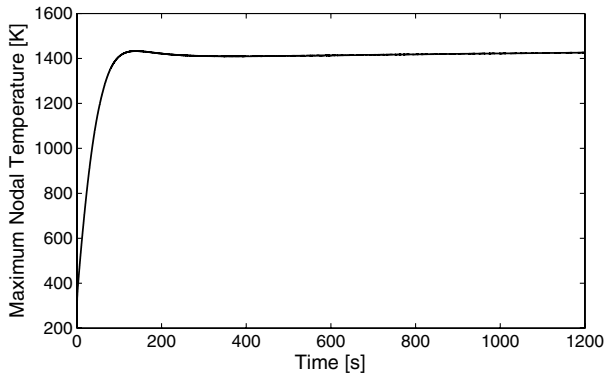


c) Case 3LD

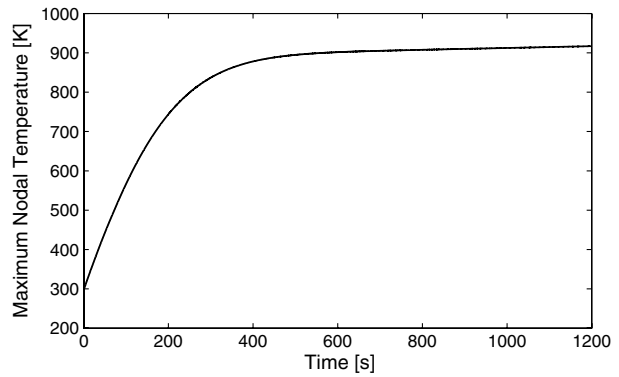


d) Case 4LD

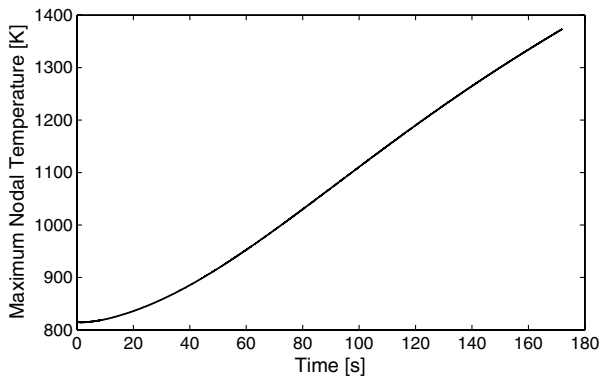
Fig. 27 Time history of percentage change in drag force for various flight conditions.



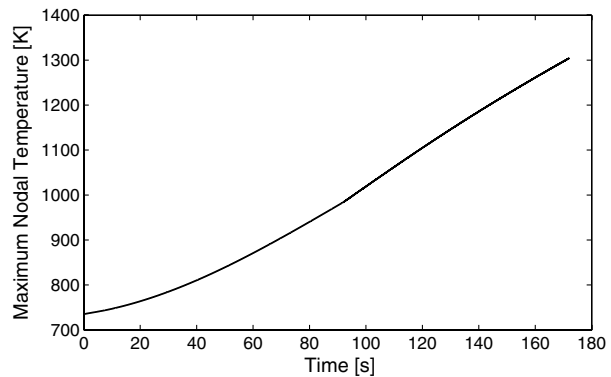
a) Case 1LD



b) Case 2LD



c) Case 3LD



d) Case 4LD

Fig. 28 Time history of maximum nodal temperature for various flight conditions.

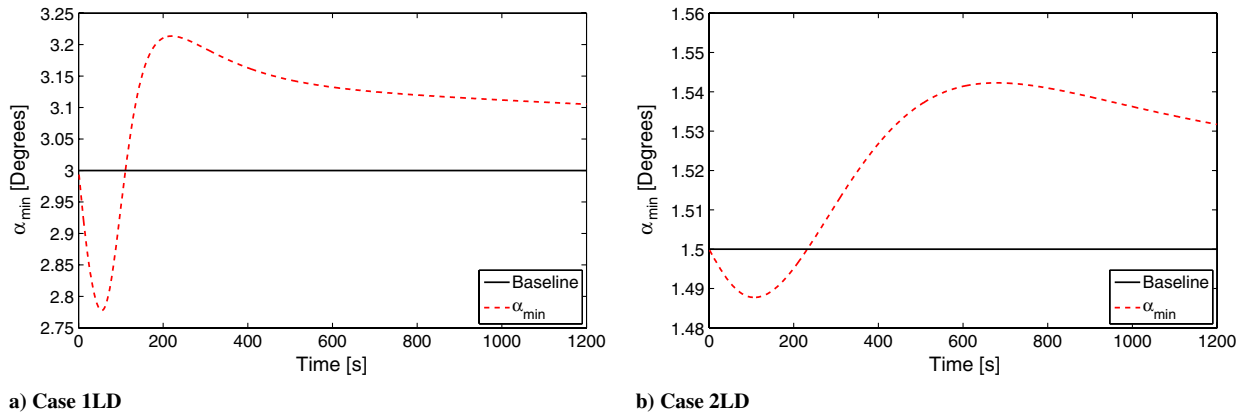


Fig. 29 Time history of  $\alpha_{\min}$  for cases 1LD and 2LD.

Table 7 Comparison of computational cost between full-order model and ROM

	Iteration type		Ratio
	Full order	Reduced order	
	CPU time/iteration, s	CPU time/iteration, s	
Heat flux iteration	375 <sup>a</sup>	0.0924 <sup>b</sup>	4058
Heat transfer iteration (10 time steps)	0.215 <sup>b</sup>	0.00246 <sup>b</sup>	87.4
Structural iteration (1 time step)	6.23 <sup>b</sup>	4.78 <sup>b</sup>	1.30
Unsteady aerodynamic iteration	0.162 <sup>b</sup>	0.162 <sup>b</sup>	N/A
Aerodynamic force calculation	0.392 <sup>b</sup>	0.392 <sup>b</sup>	N/A
Aerothermoelastic iteration ( $\Delta t_{AE} = \Delta t_{HT}$ , $\Delta t_{AT} = 10\Delta t_{AE}$ )	443 <sup>c</sup>	53.4 <sup>b</sup>	8.30

<sup>a</sup>17 2.60 GHz Opteron processors, 2.0 GB RAM.

<sup>b</sup>1 2.53 GHz Intel Xeon E5540 processor, 3.0 GB RAM.

<sup>c</sup>Predicted time.

## VIII. Conclusions

A time-marching aerothermoelastic framework is presented that makes use of reduced-order aerothermal, heat transfer, and structural dynamic models for computationally efficient simulation of HSVs. The various components of the framework are fully coupled to capture the interactions among the various disciplines in a dynamic sense. The major components of the framework used for the various disciplines consist of the following:

1) The aerothermal component was a CFD-based kriging surrogate with LHS to calculate the aerodynamic heat flux at the outer surface.

2) The heat transfer component was a reduced-order POD modal formulation to obtain the transient temperature distribution.

3) The structural dynamics component was a Ritz-based modal formulation to calculate the transient structural dynamic response.

4) The unsteady aerodynamics component was a piston theory unsteady flow solver with shock-expansion analysis to obtain steady flow conditions by turning flow through angle of attack.

The time-step scheduling of the aerothermal, heat transfer, and structural dynamics solutions are intentionally partitioned to allow for different sizes of time steps so that the different timescales governing the various processes can be resolved efficiently. The aerothermoelastic framework is applied to a representative HSV control surface, as such a component is expected to have a strong impact on the overall vehicle flight dynamics.

Assessment of the transient lift and drag for four different sets of flight conditions provides insight into the impact of aerothermoelastic effects on the total aerodynamic forces. Of the two cruise trajectories considered, the maximum absolute relative changes in total lift and drag are 8 and 15%, respectively. Of the two ascent trajectories, the maximum absolute relative changes in total lift and drag are 3 and 21%, respectively. In general, it appears that aerothermoelasticity has a larger effect on total drag than total lift. An iterative routine based on the angle of attack necessary to match the lift of the elastic control surface to that of the rigid control surface for

the two cruise trajectories is a step toward analysis of HSV controllability under fully coupled aerothermoelastic effects. Of the two cruise trajectories considered here, the maximum departure from the initial angle of attack is found to be 7.6%. However, this value is likely to be highly dependent on flight conditions and further study can provide insight into necessary variation in angle of attack for various maneuvers and trajectories. Furthermore, the degree to which aerothermoelasticity affects the forces generated by the control surfaces can be influenced by configuration parameters, such as TPS thickness, skin thickness, overall size, etc. Use of the reduced-order framework provides insight into the impact of aerothermoelasticity on HSVs in a computationally efficient manner.

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E. Livne  
Associate Editor