REPRESENTATIONS OF MATHEMATICS TEACHING AND THEIR USE IN TRANSFORMING TEACHER EDUCATION: CONTRIBUTIONS TO A PEDAGOGICAL FRAMEWORK

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Discussion document for the Working Group "Representations of Mathematics Teaching"[v]

(a) Brief History of the Working Group

This is the third meeting at PMENA of this Representations of Mathematics Teaching (RMT) working group. The idea of this working group emerged during a series of three-day conferences on representations of mathematics teaching held in Ann Arbor, Michigan in 2009 and 2010 (and earlier workshops in 2007 and 2008) organized by ThEMaT (Thought Experiments in Mathematics Teaching), an NSF-funded research and development project directed by Pat Herbst at the University of Michigan and Daniel Chazan at the University of Maryland. ThEMaT originally created animated representations of teaching using cartoon characters to be used for research, specifically to prompt experienced teachers to share the rationality they draw upon while teaching. The workshops were conceived to begin creating a community of researchers and teacher educators who were interested the use of representations of teaching and the analysis of data collected in response to these representations. The RMT conferences in 2009, 2010, and 2011 gathered developers and users of all kinds of representations of teaching (including video, written cases, dialogues, photographs, comic strips, and animations) to present their work and discuss issues that might be common to using these representations in teacher education and education research. A fourth conference took place on June 6-8, 2012. In proposing a continuation of the working group for PMENA 2012 we'd like to continue the discussion and work we had in recent PMENA working groups at Columbus in 2010, and Reno in 2011 around the elaboration and investigation of a pedagogical framework for teacher development that makes use of representations of teaching, and work toward an edited volume on the subject.

(b) Issues in the Psychology of Mathematics Education that Will Be the Focus of the Work

The use of representations of practice, particularly those that are maintained in a digital form, calls for specialized pedagogical practices from teacher educators. They also open new areas for investigation of how future professionals learn teaching and the role that various technologies play in scaffolding that learning. In the 2010 PMENA discussion paper, Herbst, Bieda, Chazan, and González (2010) briefly reviewed the literature on the use of video records and written cases in teacher education. We noted that classroom scenarios sketched as cartoon animations have begun to be utilized for those purposes and argued that they have affordances that are distinct from those of video and written cases (see also Herbst, Chazan, Chen, Chieu, & Weiss, 2011). We also noted existing literature on the use of written and video cases in teacher education and cited examples that concern mostly face-to-face facilitation. We argued that the increased capabilities of information technologies for creating, manipulating, and collaborating over multimedia point to a promising future for teacher development assisted by representations of practice. A special issue of the ZDM journal dedicated to the theme of representations of teaching added new articles to this literature. In particular Ghousseini and Sleep (2011) and Nachlieli (2011) describe the facilitation of face-to-face discussions around representations of practice and provide two views on what makes these effective for studying practice. Yet the features of novel media and their use with digital technologies, for example in online or blended (face-to-face and online) interactions, may require other pedagogical strategies for teacher education that have not been sufficiently identified and explored.

In the discussion document for the working group meeting in 2011, we complemented the

previous year's review by briefly accounting for three areas of emerging scholarship: (1) information technologies that support teachers' learning from representations of practice; (2) the particular challenge of helping prospective teachers understand students' thinking; and (3) research and theory about what is important or possible to achieve in having prospective teachers look at or work with representations of teaching.

In this document we come back to the central goal of the working group: What are the components of a pedagogy for teacher education assisted by representations of practice? Through an example we demonstrate how that question is anchored in scholarship and activities of interest to the PMENA community. Just as considerations of the subject matter being taught are key in examining pedagogy in K-12 mathematics education, a better grounding of the questions related to the use of representations of teaching in teacher education can benefit from specifications of what is being taught. In the case of mathematics educators, the content being taught to preservice teachers includes mathematics, students' thinking, and instructional practices. To anchor the need for a pedagogy assisted by representations of practice, we examine the use of representations to teach instructional practices. We focus this examination on a generic learning activity called *approximations of practice* and consider how these can be used to teach instructional practices and the kind of questions that arise from that use and that are of interest to mathematics educators gathered at PMENA.

Approximations of Practice

The expression *approximations of practice* was introduced by Grossman and colleagues (2009) to allude to "learning opportunities provided to novices" in which they can get actively involved in "the authentic practices they will be expected to enact." Grossman notes, "Students may be asked to simulate certain aspects of practice through activities such as role-plays. Simulating certain kinds of practice within the professional education classroom can allow students to try piloting the waters under easier conditions. Providing support and feedback while novices learn to paddle may better equip them to navigate the rapids of real practice." We argue that creating and supporting approximations of practice is critical for teaching instructional practices to novices. Two lines of argument contribute to this. On the one hand, Lampert's (2010) argument that teaching needs to be learned in, from, and for practice recommends the creation of opportunities to engage in aspects of the work of teaching that reproduce at least some, if not all, of the complexity of actual teaching practice. The more that the teacher candidate can be engaged in doing the practice, the more this learning will be "in" practice; and while learning "from" practice does not necessarily require representing practice in all its complexity, the more this complexity is represented the better it will ground the teacher education curriculum in actual practice. On the other hand, there is a long tradition of recommending active learning, or learning by doing, across educational levels (Bonweil & Eison, 1991). This approach to teaching seems to concurrently argue for learning activities for teacher candidates where they learn from their own experience engaging with authentic problems of practice and where reference materials such as readings play the role of supporting resources rather than focus. To mathematics teacher educators, approximations of practice offer opportunities and challenges that concern the creation of approximations, activities utilizing these approximations, and the investigation of teacher candidates' learning from these. Belowm we illustrate how representations of practice can feature in these approximations.

What form do approximations of practice take?

One could argue that approximations of practice have always been included in teacher

education programs that contain a practice curriculum—practicum experiences and student teaching are in fact approximations of practice. The literature on student teaching is vast enough to discourage a review, and it has been well represented in handbooks (e.g., McIntyre, Byrd, & Fox, 1996). Beyond practicum experiences in actual classrooms, university classes on teaching methods classes have also contained approximations of practice.

Microteaching (Allen & Eve, 1968; Cruikshank & Metcalf, 1990; McLeod, 1987), or the enactment of short lessons in front of peers, is an example of how approximations of practice have taken shape in the teacher education curriculum. While popular for some time, microteaching has not produced the desired results in supporting teacher candidates in learning to teach (McIntyre, Byrd & Fox, 1996). Over the years teacher educators have worked to improve this technique though the inclusion of targeted guidance and feedback as well as coupling microteaching with the observation of competent performance of the teaching practice being studied (for an example see Chazan, Herbst, Sela, & Hollenbeck, 2011). A related variation has developed to match the medical education practice of the "standardized patient" (Stillman et al., 1991). Dotger, Harris, and Hansel (2008) have proposed a standardized-patient-type approximation for training teachers to talk to parents.

Yet a more recent incarnation of microteaching is that of teaching rehearsals (Lampert et al., 2010). In the process of engaging teacher candidates in performing targeted aspects of the work of teaching, some rehearsal cycles also rely on the use of video recordings of the teaching practice (Kazemi, Franke & Lampert, 2009, Lampert & Graziani, 2009). In these activities the teacher educator guides the teacher candidates in observing live or pre-recorded exemplars of the practice that they will rehearse or watching and debriefing video of the novices' rehearsals.

Another recent approach to engaging teacher candidates with approximations of practice is through the construction and enactment of instructional dialogues, also called "lesson plays" (Crespo, Oslund & Parks, 2011; Ghousseini, 2008; Zazkis, Liliejdahl, & Sinclair, 2009). These activities provide teacher candidates with opportunity to imagine how a classroom scenario might unfold and the specific consequences of the word choices of both the teacher and students.

Historically, teacher preparation programs have also engaged teacher candidates with approximations of teaching through lesson planning and lesson anticipation. Lesson planning, or asking teacher candidates to create a timeline for a lesson, has long been used as a tool for preparing teacher candidates for the work of teaching and scholars have begun to examine the techniques commonly used with teacher candidates and their effectiveness (Harris & Hofer, 2009; John, 1991, 2006; Mutton, Hagger & Burn, 2011; Rusznyak & Walton, 2011). As Americans became more knowledgeable about Japanese lesson study (Hiebert & Stigler, 2000), some educators have begun to use lesson study in preservice education (Fernandez, 2002; Hiebert, Morris & Glass, 2003; Parks, 2008.). Pre-service lesson study teams may be comprised of a group of teacher candidates, although some work has involved teams of mentor teachers and teacher candidates collaborating together (Burroughs & Lubeck, 2010). These teams may work in microteaching or lab-type settings (Fernandez, 2005), as well as in the context of actual classrooms. Distinctions in who contributes to planning, observing, and debriefing the lesson, as well the context of the lesson, determine how closely lesson study approximates actual teaching practice. For instance, when lesson study teams work on developing a lesson to be taught in an actual classroom, the school curriculum, norms, and student characteristics must be taken into consideration and heighten the authenticity of the lesson planning when compared to a lesson study for fellow teacher candidates in a microteaching setting. But approximations of teaching may also be deployed in virtual settings.

Approximations of teaching in virtual settings: A use of Depict

Herbst and Chieu (2011) introduced the *Depict* tool (a component of the *LessonSketch* environment in www.lessonsketch.org). *Depict* enables users to create a classroom scenario using text, inscriptions, and graphics (see also Herbst, et al., 2011). Chen (2012) has shown that when preservice teachers were asked to anticipate a lesson using Depict they were able to think through the tasks they would propose in more detail than when they merely talked through a lesson plan they had written before. As a result it is possible to envision a new kind of approximation of practice that moves above and beyond activities in which novices construct dialogues to show how they would handle problems of practice. The teacher educator can depict the beginning of a classroom scenario, using text and graphics, and leave it to the novice to complete the scenario and submit it to the teacher educator, who in turn might insert comments or alternatives and return that to the novice. Thus approximations of practice can be deployed and transacted through the use of multimedia representations. We exemplify this use below.

Teaching the instructional practice of 'explaining a concept' using approximations of practice. One thing the first author does as part of his methods course is teach novices how to explain concepts. Explaining concepts can be a teacher-centered activity in that the teacher may "provide" all the explanation, but it may also be a blend, using discussion or brief explorations as parts of the explanation. In teaching novices how to explain concepts, however, the goal is not so much to identify the best activity type for them to use, but to teach novices about what things need to be included in an explanation of a concept. As Leinhardt and Steele (2005) have shown, dialogue-based lessons can be constructed to share features of instructional explanations found in the instructional explanations documented of expert teachers (Leinhardt, 1989, 2001). To support the teacher candidates in learning how to explain concepts, Herbst uses a decomposition of practice for the practice of 'explaining concepts and propositions' (Herbst, 2011). This decomposition of practice builds on Leinhardt's work on instructional explanations by describing and exemplifying the components of an explanation in text form. Components such as problematizing the concept, exemplifying the concept, and so on are described and illustrated in documents such as Herbst (2011). Until 2011 students in Herbst's methods course would practice what they learn in the context of approximations of practice like the problem shown on Figure 1.

4. Abilene Clark has been teaching her Algebra II class the exponential function and properties about the multiplication and division of exponents. She wants to make sure students use those properties well so she points out possible errors related to operations with exponents. Write a dialogue in which you show how Abilene could a. demonstrate one of those errors, and b. explain why it is an error.

Figure 1. A dialogue based approximation of the practice of explaining a concept (co-designed by Gloriana González, Pat Herbst, and Adam Poetzel)

In fall 2011, Herbst and his team started using an illustrated version of the decomposition for explaining concepts and created graphic approximations of practice using *Depict*. The problem in Figure 1, which gives novices the opportunity to practice examining common errors, which is one aspect of explaining concepts, was then posed using the depiction shown in Figure 2 and in the context of an online homework assignment including three problems like it. Novices were told that the depiction shows how what Ms. Clark has done so far and asks, "What are some of the conceptual errors that are at the root of common mistakes students could make when working with the logarithmic function? Write your comments in the box below." Then it prompts them, "Now, please press View and then Edit to edit the slideshow, so that you can complete what Ms.

Clark should say to the class to point out common errors that students make when using the logarithmic function. Your edited slides should demonstrate one common error and Ms. Clark's explanation to the class of why it is an error (and how they might avoid it). [The red text on the whiteboard and the text between brackets in the speech bubbles] shows where you can fill in what the teacher should say and write on the board. Feel free to add more slides…."

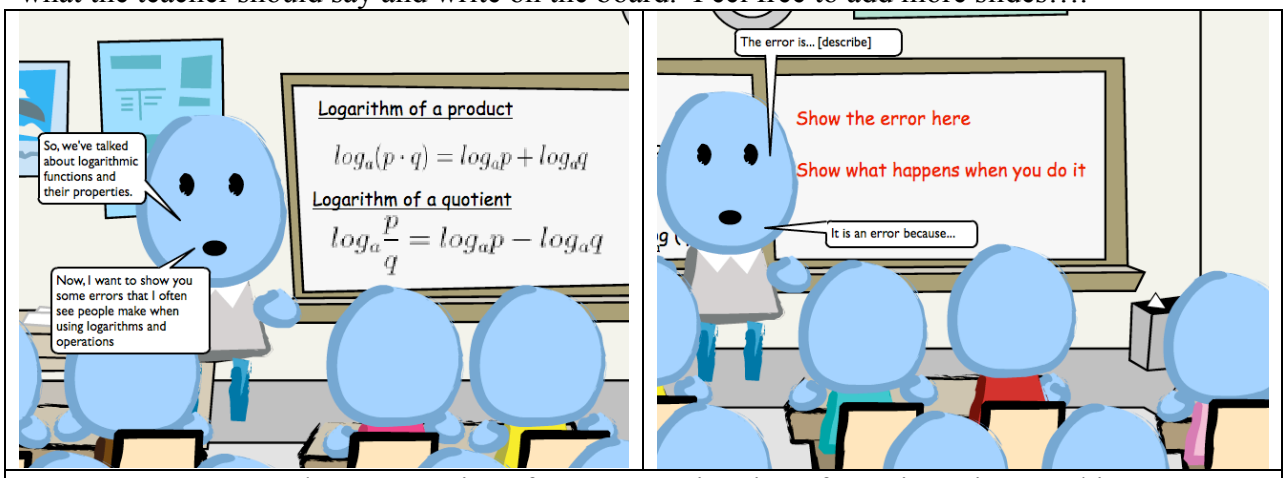

Figure 2. The context given for an approximation of practice using graphics

Figure 3 shows parts of the depiction that one novice (D) made continuing the provided slides. D's depiction contained five original frames that followed the provided ones. D's work shows not only how the approximation got novices involved in practicing but also, as Chen (2012) had found, it made them think of the details of tasks and multimodal student involvement.

A teacher educator could provide feedback to that depiction, or perhaps sketch an alternative scenario. In Herbst's class, teacher candidates not only created the depictions that showed what a teacher could say and do when providing an explanation, but they also rehearsed them in front of their peers on the following class. Peers and instructors could then provide constructive criticism about choices made in planning and about the qualities of their performance.

Through depicting the continuation of scenarios like those, teacher candidates can develop and display their knowledge of instructional practices. The decomposition of practice provides teacher candidates with a framework for thinking about the essential aspects of the practice while the work of depicting the scenario provides teacher candidates with the opportunity to explore the possibilities for what the practice will look like when it is enacted in the classroom.

As this example shows, approximations of practice can be used to create opportunities to learn a practice (explaining concepts, and its component of examining common errors) *in* practice (by providing an explanation in a depicted classroom) and *from* practice (where the feedback addresses alternative choices and ways of enacting them). A more sophisticated case of using approximations of practice has been proposed by Chieu & Herbst (2011) who describe the features of a teaching simulator, where teacher candidates practice teaching by choosing what the cartoon teacher would do and the simulator provides the ensuing events in a simulated class.

Clearly, *Depict* is only one example of how technology can support learning from approximations of practice (in the case shown, homework problems based on the work of teaching are used to support teacher candidates in learning how to preform an instructional practice). The same approximation could be realized using video; the teacher educator could record the initial scenes of the scenario, while playing the role of teacher, and could ask the teacher candidates to record themselves doing the ensuing actions. Feedback could come in the

form of an annotation of the video or a video response, where the teacher educator demonstrates how he would modify what the teacher candidate did. In both cases one can see how technology-supported representations of practice can be used to create approximations of practice than blend active learning with learning in and from practice. This activity can involve novices in figuring out what to do in particular circumstances, while blended with microteaching it can also address the development of skills in performing the work of teaching; that is, learning *for* practice.

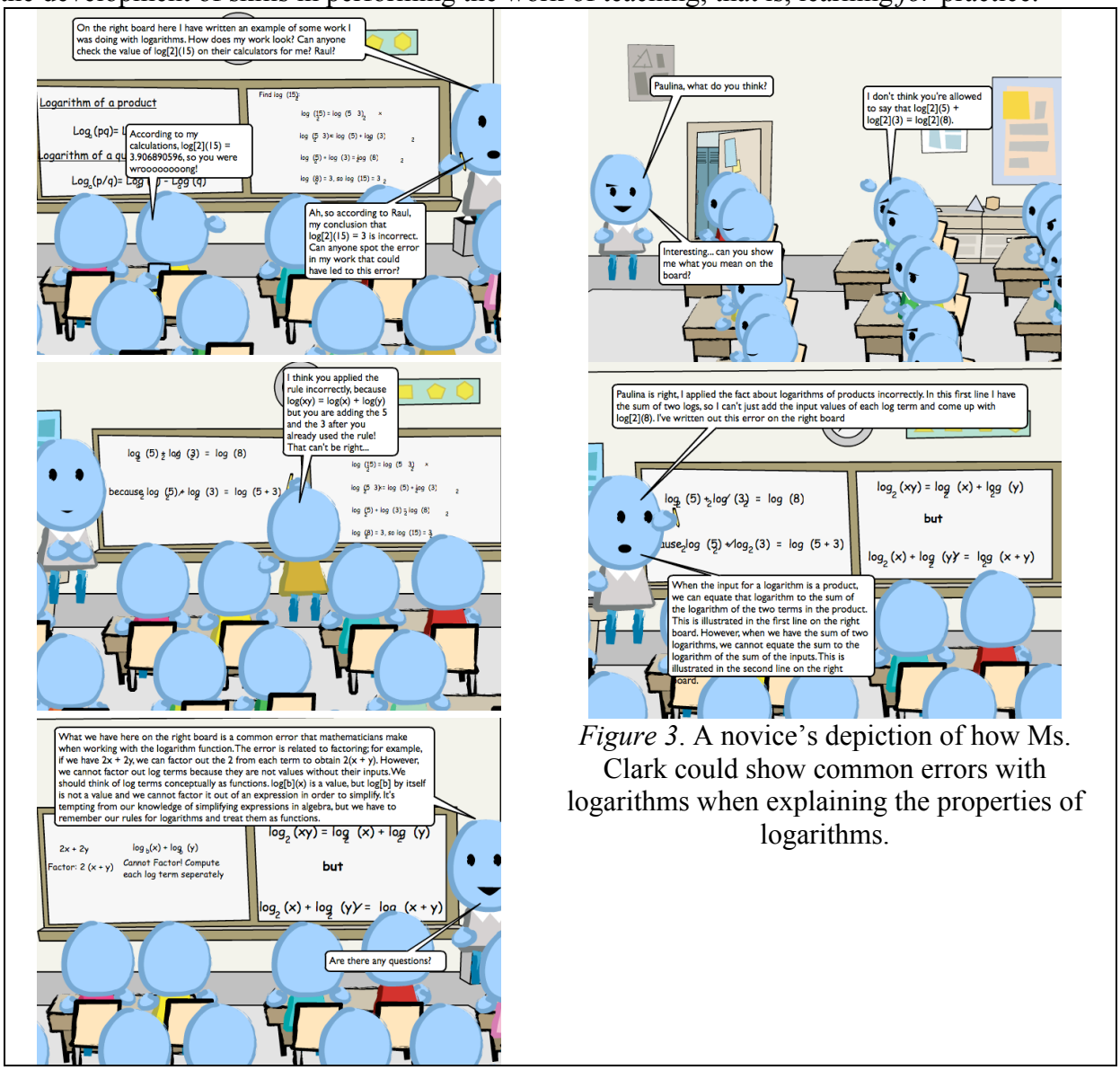

Figure 3. A novice's depiction of how Ms. Clark could show common errors with logarithms when explaining the properties of logarithms.

The prior discussion of approximations of practice suggests that a generic learning activity in teacher education could involve teacher candidates in studying a practice (e.g., viewing illustrations that decompose a practice) then doing problems of practice where they engage in virtually enacting those practices. The learning materials and the problems could be posed in some information technology based environment such as LessonSketch, and the teacher candidates would produce their responses using tools integrated in that environment (in this case *Depict*). This generic learning activity helps raise questions that are of interest to the PMENA community and that illustrate why the working group fills a gap in the community. Some

questions are about the novices' learning: What do novices learn by engaging with representations of teaching in the context of activities that approximate practice? Do different kinds of representations afford different learning opportunities when similar approximations of practice are used (e.g., text only vs. *Depict* vs. video)? In other words, are there cognitive or performance changes in novice teachers that go along (conceptually or statistically) with different kinds of representation-based activities or different kinds of representations? On the other hand the extent to which this kind of approximation of practice involves media and communication technologies provides a snippet of how much pedagogical innovation is possible and needed in order to handle representations of practice with novices. Teacher educators need to choose what their novices will learn indeed. But they also need to design how they are going to create opportunities for them to learn. Approximations of practice illustrate the complexities involved in this work. Teacher educators are not limited to selecting media artifacts; they can also produce them. This requires making choices of symbol systems, content, and form. Beyond producing or selecting representations to use, teacher educators need to design or choose activities in which novices will engage with those representations, and they need to design how to propose those activities to novices. Teacher educators need to identify the medium within which to share those activities and the representations associated with them. Face-to-face group encounters with a projector screen are only one of the many choices available, which include notably, online environments like *LessonSketch* that can be used at distance (e.g., when students are doing homework at home) or co-located (e.g., when students browse through a representation in class, using their own laptops). Teacher educators also need to design the tasks they pose to the novices—these tasks may be "what do you notice?" but they may also be "what would you do or say next?" In sum, teacher educators need to design learning environments for the learning of teaching—recent improvements in internet broadband speed, web-based software, user experience standards, and graphics technologies have made the choices available for that design much greater, and more diverse, than ever before at least in terms of their possibilities. We argue that this presents the challenge and the opportunity of developing a pedagogical framework with which to conceive these learning environments. The case of approximations of practice and its application in the teaching of how to explain a concept in secondary mathematics methods exemplifies how the pedagogy of teacher education may be expanding in response to existing technologies. A framework can help us direct technology development as well.

Toward a Pedagogy for Teacher Development Assisted by Representations of Practice

Building on the proposals from previous years, the working group's purpose is to design a pedagogical framework for teacher development. The framework is aimed at assisting teacher educators who want to help teacher candidates actively learn teaching *in*, *from*, and *for* practice by taking advantage of representations of practice and new technologies. This enterprise may require conceptual developments, for example in articulating connections between theories of teaching and the design of a curriculum for teacher education. The enterprise also requires the creation of pedagogical templates or generic learning activities/environments that can be particularized for the specific goals of individual teacher educators, the needs of their students, and the media artifacts or software tools that are available. Thus far, the working group has proposed a framework articulated by a number of categories of things that are involved in different ways in the process of teaching with the assistance of representations of teaching. These categories include *boundary objects*, *activity types*, *technology tools*, *problem types*, and *teacher education goals*. Each of those categories contains elements from which choices can be made to design learning activities for novices. The working group has been operationalizing those

categories by using *Plan*, a software tool included in *LessonSketch*. *Plan* allows teacher educators to design a learning module for their clients, putting together media artifacts, tools, and tasks and to sequence them in a desired order that may include individual or group work. In the discussion document for last year (Herbst et al, 2011) we described the framework in considerably more detail. Our work this year will include asking questions like: Given that a teacher educator has a specific learning goal in mind for her students, such as learning how to probe student thinking or learning how to demonstrate the subtraction algorithm, what are appropriate representations of teaching, activity structures, problems types, and technology tools to use to reach that goal? The group will work on articulating goals of teacher education and a pedagogy of teacher education.

The convenors of this working group are particularly interested in exploring how cartoon-based representations of practice facilitate teacher learning. Over the past few years teacher educators have begun to use LessonSketch in content and methods courses for teachers. This working group is an opportunity for users to share their experiences and insights. These contributions are invaluable to the life of the working group and, more generally, to the development of the knowledge base for use of these resources. We expect the continued use and development of *LessonSketch* will help improve the framework of the pedagogy and further develop specifications for yet other technologies that respond to the needs of the field.

We have proposed that a pedagogy of teacher preparation assisted by representations of practice needed at least four categories of elements: boundary objects (or open ended expressions), activity types, problem types, and technology tools or screens. This year we add the category of *teacher education goals* that one needs to consider when planning educative experiences for teacher candidates around representations of practice—since different resources and tasks may be needed depending on those goals. These teacher education goals can include having novices learn instructional practices such as "explaining concepts" or other, even "high-leverage practices" (Hatch & Grossman, 2009) such as "facilitating classroom discussions." The goals can be student-centered too (such as having novices develop capacity to notice, describe, and explain students' errors). Also, they can be mathematical, such as when one wants novices to map the terrain of a given problem (Lampert, 2001). We refer the reader to the discussion document of last year's working group for a detailed description of the other categories.

(c) Plan for active engagement of participants and (d) anticipated follow-up activities

The plan includes starting with a brief exposition by the authors of the structure and contents of the present framework for which we will illustrate how the use of approximations of practice narrated here maps onto elements of the framework. We will engage the audience in creating learning activities they would like to use to engage their clients. The idea is to use the collective planning of these sessions to probe the framework and possibly enrich it by adding more items to the lists considered, possibly also adding new categories of elements. Participants will then form groups and spend the second half of the first session and the first half of the second session creating exemplars. Then the second half of the second session and the closing session will be dedicated to sharing these exemplars and improving the framework, including discussions about the questions raised earlier in this document, paving the way for an edited publication.

By the time this working group meets we will have had the fourth conference on Representations of Mathematics Teaching in Ann Arbor (June 6-8, 2012). We will be proposing a session slot at the AMTE Annual Meeting in 2013 to continue this work. We plan to use that slot to mirror the work done at the PMENA meeting and to engage in further work on (1)

improving the exemplars and (2) using the exemplars to improve the taxonomies. We hope we will be able to use those products to continue this working group at next year's PMENA.

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