Essays on Private Equity Finance and Supply Chain Management

by

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Chapter 1

Introduction

Competition and uncertainty are omnipresent in today's economic environment. This dissertation examines competition and uncertainty in private equity (PE) and supply chains through three papers. Each paper employs Cournot and/or Bertrand models of competition together in conjunction with multiple forms of uncertainty to examine the optimal behavior for decision makers as well as consequences for other involved parties.

Chapter 2 examines the decisions faced by manufacturers dealing with risky suppliers. Through an analysis of four supply chain structures, the paper examines how optimal manufacturers' profits, subsidies provided to suppliers, and quantities released to the retail market vary with manufacturer-level competition and cross-subsidies. It also demonstrates that manufacturers must evaluate a tradeoff between the benefits of cross-subsidies and benefits of reduced competition in making the decision to share a supplier.

Chapter 3 examines the PE fundraising process by analyzing competition, uncertainty, and barriers to entry utilizing a two-stage model. To the authors' knowledge, it is the first paper to examine these three elements collectively, and also the first to apply a theoretical model containing these elements to the PE arena. Among other results, the paper presents evidence demonstrating market size uncertainty increases both expected firm profitability and the expected consumer surplus. This result holds significant implications in an industry where both investors an general partners strive to form long-term relationships: this practice may, in fact, be detrimental to both parties.

Chapter 4 uses a two-stage framework to again examine the effects of uncertainty and competition for investors' capital on PE fundraising. Unlike Chapter 3, however, the market size varies according to geometric Brownian motion, and PE general partners have the option of postponing the fundraising process. A form of uncertainty different from Chapter 3 is employed. Among other results, the work demonstrates that PE firms, especially those facing little competition and perceived by investors to have a "secret sauce," will be most aggressive in the fundraising process when they perceive future degradation in the fundraising climate, perhaps due to decreasing returns. The paper also demonstrates that Accounting Standards Codification (ASC) 820, while well intentioned, may have a detrimental effect on both PE investors and general partners.

This dissertation ends with Chapter 5 which contains concluding remarks regarding results of analyses and provides directions for future research.

Chapter 2

Manufacturer Competition and Subsidies to Suppliers

1 Introduction

Dealing with risky suppliers is a part of everyday business for manufacturers. For example, the domestic automotive supply industry has faced numerous hardships as some of its largest firms have flirted with bankruptcy, or have been subsumed by Chapter 11 over the past few years. Nearly 30% of the pre-existing North American automotive supply base had filed for bankruptcy by the end of 2008. Half are predicted to file for bankruptcy before the end of 2010 (A. T. Kearney, 2009). Despite a $5 billion cash injection from the federal government in early 2009, auto suppliers continue to struggle. With the U.S. economy rebounding more slowly than first expected, the short-term outlook for the entire auto industry is bleak. "Bottomed out" auto sales haven't yet begun to rebound significantly and nearly two-thirds of Tier 1 suppliers remain financially distressed (Harvey, 2009).

If a supplier defaults, its operations may temporarily or permanently cease, crippling downstream manufacturers and starving them of necessary production inputs. In some cases, manufacturers source products from a single supplier. This is especially true of high technology items where suppliers may hold patents on the products they produce. When a supplier defaults, the manufacturer's operations are in jeopardy. Consider the recent hardships faced by American Axle & Manufacturing (AAM). In September 2009, AAM received \$110 million in cash and a \$100 million loan from General Motors to keep the supplier out of bankruptcy (Haywood, September 18, 2009). These payments also prevented shut-downs at numerous GM facilities that were dependent on AAM-sourced parts. Ironically, AAM was once a part of General Motors' Saginaw Steering Division before being sold to a group of investors in 1994. General Motors also experienced a similar situation in December 2008 when Cadence Corporation, a supplier of interior components for GM vehicles, filed for bankruptcy. This bankruptcy caused Cadence to shut down its operations, delaying production of GM's 2010 Chevrolet Camaro.

Financially distressed suppliers pose significant operational risks to manufacturers. Manufacturers may be able to switch suppliers in the event of a default, but in an environment where nearly all suppliers are financially distressed, this is not beneficial. Supplier diversification is expensive for manufacturers when procuring non-commodity products. The only remaining option for manufacturers is subsidies.

Research suggests that publicly-traded firms suffering from supply disruptions experience abnormal stock returns that are roughly 40% lower than their competitors Hendricks and Singhal (1995). Despite the continual emphasis on supply chain robustness both within academia and industry, a majority of supply chain studies and practices have focused on increasing supply chain efficiency rather than mitigating disruptions. Our paper addresses this gap.

In order to mitigate risks arising from supplier financial distress, manufacturers may elect to provide subsidies to their suppliers. These subsidies can take the form of cash, agreements for future contracts, or targeted supplier development. By providing suppliers with subsidies, manufacturers reduce the risk of supplier default, thus increasing the reliability of their supply chain.

In our model, manufacturers can select from two sourcing options. They can procure goods from a dedicated supplier that does not serve other manufacturers (a *dedicated supplier*), or from a supplier that is shared by multiple manufacturers (a *shared supplier*). For example, in 1996, Delphi (then a division of General Motors Corporation) served as a dedicated supplier for GM, and derived 83% of its revenues from its parent company. In 2007, however, 63% of Delphi Corporation's revenues came from non-GM customers.

Manufacturers can also participate in two downstream retail market environments: a *competitive* (or *oligopolistic*) and a *non-competitive* (or *monopolistic*) environment. Over the past 40 years, the North American automotive marketplace has been transformed from an oligopoly dominated by three manufacturers (General Motors Corporation, Ford Motor Company, and Chrysler Corporation) to a fiercely competitive environment with a plethora of players. Increased competition has commensurately decreased profits for automakers, forcing them to pay significant attention to their material procurement decisions in attempting to control costs.

Thus, we consider four supply chain structures (see Figure 2.1): 1) monopolistic manufacturers with dedicated suppliers, 2) oligopolistic manufacturers with dedicated suppliers, 3) oligopolistic manufacturers with a shared supplier, and 4) monopolistic manufacturers with a shared supplier.

Procuring goods from shared suppliers permits manufacturers to share subsidy costs (thus *cross-subsidizing* the supplier), reducing the burden of each manufacturer of ensuring that its supplier is financially viable. On the other hand, by using a dedicated supplier, manufacturers have more direct control over their supplier's reliability and can potentially exploit potential monopoly power should their competitor's supplier falter. In this chapter we study 1) the cross-subsidy benefit to a manufacturer and how this benefit depends on manufacturer competition, 2) the benefit to a supplier from working with several manufacturers and how this benefit depends on manufacturer competition, and 3) the consumer surplus and quantities released to the market and how these quantities depend on the choice of a supply chain structure.

Our analysis shows that when the market size of each monopolistic manufacturer is the same as that of competing manufacturers, suppliers receive less subsidies when manufacturers compete than if they are monopolists. Less competition among manufacturers leads to higher subsidies provided to suppliers, more reliable suppliers, and greater benefits to consumers. If the combined market size of monopolistic manufacturers is equal to that of competing manufacturers, consumers may prefer competing or non-competing manufacturers depending on supplier reliability.

We also find manufacturers' subsidy costs are less when manufacturers share suppliers, irrespective of whether or not manufacturers compete in a retail market. Interestingly, in the scenario where manufacturers do not compete, the total amount of subsidies received by a shared supplier is greater than the payment received by each dedicated supplier. However, in the scenarios where manufacturers compete, manufacturers face a tradeoff between using a shared supplier and dedicated suppliers. By sharing a supplier, manufacturers enjoy decreased subsidy costs because they reap the benefits of competitor-provided subsidies. On the other hand, by using dedicated suppliers, manufacturers may become monopolists when their competitor's supplier defaults. In this scenario, whether a shared or dedicated supplier receives a greater subsidy depends on the difference between monopolistic and oligopolistic manufacturers' profits. When this difference is large, dedicated suppliers receive greater subsidies and are more reliable; if this difference is insignificant, a shared supplier receives greater subsidies and is more reliable.

2 Literature Review

Our paper contributes to two streams of operations management literature: supply risk and manufacturer-level competition. We examine both issues in a novel framework that quantifies the optimal subsidy decisions of manufacturers.

Literature on supply yield uncertainty are surveyed in Yano and Lee (1995). Silver (1976) authored an early paper on this topic using the economic order quantity (EOQ) framework. His paper considered yield uncertainties in which the standard deviation of received goods was proportional to lot size, and also when it was not proportional to lot size. Gerchak and Parlar (1990) use Silver's framework and the EOQ model to jointly determine yield variability and lot sizes when yield variability can be decreased (e.g. improved) through investment. In special cases of the investment function, they derive closed-form solutions for the optimal investment and lot size levels. Gerchak and Parlar's idea of reducing yield variability through investment is very similar to a central theme in our paper: namely, that manufacturers can reduce yield variability by providing financial subsidies to suppliers.

More recent papers by Deo and Corbett (2009) and Tang and Kouvelis (2009) have modeled supply chain disruptions in light of strategic competition among manufacturers, which is represented by Cournot competition in their models.

Deo and Corbett examine the impact of yield uncertainty on manufacturer-level production and entry into a retail marketplace. They use a two-stage model in which, during the first stage, firms decide whether or not they will enter a retail market model, and, during the second stage, each firm selects the target production quantity of goods. They also examine the effects of yield uncertainty on consumers as measured through the consumer surplus. The authors find that yield uncertainty decreases competition at the manufacturer level and also decreases the expected consumer surplus.

Tang and Kouvelis examine the benefits of supplier diversification for competing manufacturers. They consider a two-stage model in which the suppliers' output is affected by proportional random yield similar to Deo and Corbett. In the first stage, manufacturers engage in a sourcing strategy game, while in the second stage, manufacturers compete in the Cournot sense. The authors find that manufacturers should never choose to use the same supplier and that increasing correlation between suppliers' yields decreases manufacturer-level profits.

Our analysis differs from each of the aforementioned papers in several ways: 1) we use a different model of yield uncertainty that is based on the supplier's financial state; 2) we assume manufacturers can directly affect the supplier's financial state through subsidies; and, 3) we focus on the optimal manufacturer subsidy decisions in both competitive and non-competitive manufacturer environments.

Babich (2010) employs financial models of bankruptcy similar to that in our model. He solves an N-period optimization problem, examining both the optimal order quantities and financial subsidies of a manufacturer dealing with a single, risky supplier. He asserts that the supplier's ability to deliver goods is increasing in its state of financial health, defined as the ratio of the supplier's assets to its liabilities. Our analysis differs from Babich's work in that we first seek to quantify the optimal subsidy decisions of manufacturers participating in a competitive retail market. We modify Babich's model of financial health in a multi-manufacturer setting, and analyze the optimal subsidy decisions of such firms when they procure goods from both dedicated and shared suppliers. We also examine the impact of manufacturer-level competition on subsidies and the effect of yield uncertainty on the consumer surplus.

3 Model

We model supply chain interactions as two-stage games of complete information. Each of these stages represents a subgame in our model. In the first stage, manufacturers simultaneously select subsidies, θ. These subsidies represent a promise contingent on the supplier being financially viable by the second stage. Suppliers' capacity is also realized in this first stage. In the second stage, manufacturers release goods, z, to the downstream market if their supplier is financially sound and able to deliver products. Our assumption that the production decision occurs after the uncertainty about the supplier's financial status is resolved is based on current automotive industry practices. Intense competition among Tier 1 manufacturers has afforded much power to manufacturers in sourcing goods. In fact, according to Chrysler Group LLC's Senior Vice President of Purchasing and Supplier Quality, Dan Knott, current contract terms allow manufacturers to "drop a [currently-contracted] supplier because 'I didn't like the way you look'" (Treece, June 11, 2010). In other words, if a manufacturer senses a supplier will be unable to deliver goods, current contract terms generally allow the manufacturer to terminate the relationship without significant repercussions.

We find the subgame perfect equilibrium by backward induction. Recall that our model is used to analyze manufacturers' decisions in each of four supply chain structures that differ along two dimensions: competition among manufacturers and the use of a dedicated or shared supplier (see Figure 2.1).

We assume the total amount of subsidies θ provided by manufacturers is always non-negative and improves supplier reliability by elevating the supplier's financial state. Manufacturers must always reimburse suppliers for their total production costs. Subsidies, then, describe any contributions provided by manufacturers to suppliers in excess of production costs. These subsidies could take the form of promises for future contracts, loans, or cash. Let θ_i denote the subsidies received by the supplier from manufacturer i. In the case where two manufacturers share a common supplier, that supplier receives $\theta_1 + \theta_2$. In the case where two manufacturers use dedicated suppliers, each supplier $i \in \{1, 2\}$ receives θ_i from its dedicated manufacturer.

In each of our four supply chain structures, a supplier is able to provide sufficient capacity to fully satisfy manufacturers' orders or no capacity at all, depending on the supplier's financial state. When a supplier is unable to provide capacity, we assume its manufacturer(s) receive(s) no goods, and cannot sell any products in the retail market. The probability that a supplier who received subsidy θ is able to deliver goods is $p(\theta)$. For each unit of θ that a manufacturer promises its supplier, it must pay $\delta(\theta)$.

We make the following assumptions regarding the functional forms of $p(\theta)$ and $\delta(\theta)$.

Assumption 1. *Function $p(\theta)$ is increasing and log-concave in θ.*

Assumption 2. *Function $\delta(\theta)$ is increasing and convex in θ.*

Assumption 1 holds for many different probability distributions, including the normal and exponential distributions. Assumption 2 is intuitive.

We offer the following lemma that results directly from Assumption 1. All proofs are included in the Appendix.

Lemma 1. *The quantity $\frac{p'(\theta)}{p(\theta)}$ is decreasing in θ.*

Function $p(\cdot)$ can take many forms. For example, a structural model of a supplier's bankruptcy similar to Merton (1974) yields

$$p(\theta) = Pr[E(T) - L + \theta \geq 0], \tag{1}$$

where $E(T)$ is a random variable representing the supplier's earnings by time T and L represents the supplier's financial obligations. Earnings, E, follow a Brownian motion process:

$$dE(t) = \mu dt + \sigma dW(t). \tag{2}$$

In this equation, μ and σ are the drift and diffusion coefficients of the Brownian motion process. Bankruptcy occurs when earnings fall below liabilities by an exogenous level at the end of the period, time T. In our model, we specify this barrier as 0. However, this quantity can be changed without affecting the qualitative results of our analysis. This interpretation of $p(\theta)$ allows $p(\cdot)$ to be increasing and log-concave as required by Assumption 1.

A reduced-form model of default similar to Jarrow and Turnbull (1995) yields

$$p(\theta) = Pr[\tau > T] = e^{-\lambda(\theta)T}, \tag{3}$$

where $\tau(\theta)$ is random variable representing the arrival event of a Poisson process with rate λ. This alternative interpretation of $p(\theta)$ also allows $p(\cdot)$ to be increasing and log-concave as required by Assumption 1 so long as $\lambda(\theta)$ is decreasing and convex in θ.

We define the manufacturer's optimal second stage subgame profit by π using appropriate superscripts and subscripts where necessary in both monopolistic and oligopolistic models. In monopolistic models, π^1 represents a manufacturer's optimal expected profit if its supplier did not declare bankruptcy by stage 2; π^0 represents a manufacturer's profit if the supplier files for bankruptcy before stage 2. We assume $\pi^0 = 0$. In oligopolistic models, we will use two superscripts to indicate the delivery status of suppliers. Subscripts will denote to which manufacturer the equilibrium profit pertains. For instance, π_1^{11} is manufacturer 1's expected equilibrium profit when suppliers of both manufacturers are in sound financial state by stage 2. In this case, manufacturers engage in oligopolistic competition in the second stage subgame and π_i^{11} is the manufacturer i's equilibrium profit. π_1^{10} is the profit of manufacturer 1 when its supplier survived stage 1 and its competitor's (manufacturer 2's) supplier did not survive stage 1. Hence, manufacturer 1 becomes a monopolist in the market in this scenario. π_1^{01} is the profit of manufacturer 1 when its supplier did not survive stage 1, but its competitor's supplier did survive stage 1. In this circumstance, manufacturer 1 has no goods to sell and its competitor is a monopolist. We will assume that $\pi_1^{01} = 0$ and $\pi_2^{10} = 0$. π_1^{00} is manufacturer 1's profit when both suppliers declared bankruptcy. We will assume that $\pi_i^{00} = 0$ for $i \in \{1, 2\}$.

Additionally, we make the following assumption regarding manufacturer profits.

Assumption 3. *Manufacturers' expected profits are positive in equilibrium for all supply chain scenarios.*

We do not need to specify how profits are derived for most of our analysis. However, a convenient illustration is the retail market model where prices are determined by a linear inverse demand function (as in Cournot competition)

$$P(z) = d - z. \tag{4}$$

In (4), d is the market size parameter and z is the quantity of goods released to the retail market. For this illustrative model we will assume that manufacturers have a constant marginal cost of production c. We would like to emphasize that the Cournot competition model is used for illustration and that most of our results hold for more general retail models.

We next analyze the models for the supply chain structures shown in Figure 2.1. We begin with an analysis of a benchmark case consisting of two monopolistic manufacturers and two dedicated suppliers. This scenario is presented in Figure 1(a).

3.1 Benchmark Case: Monopolistic Manufacturers with Dedicated Suppliers

In the benchmark case, there is no strategic interaction among manufacturers. In the second stage subgame, manufacturers are monopolists earning expected profits π^1 or π^0 contingent on their supplier's status. For example, for linear demand model (4), if the supplier is able to deliver goods, the manufacturer chooses an order quantity z to maximize its expected profit $\Pi(z)$

$$\pi^1 = \max_z \{\Pi(z) = zP(z) - cz\}. \tag{5}$$

$P(z)$ is the inverse demand function given by (4).

If the supplier did not default, the equilibrium quantity of goods released to the market by the manufacturer in the second stage game is given by

$$z^* = \arg\max_z \Pi(z) = \frac{(d-c)}{2} \tag{6}$$

for inverse demand (4). When the supplier does default, the manufacturer has no goods to sell in the retail market and $z^* = 0$.

For inverse demand (4)

$$\pi^1 = \frac{(d-c)^2}{4}. \tag{7}$$

Turning now to the first-stage subgame, manufacturers select the amount of subsidies to provide to their dedicated suppliers according to the following optimization problem:

$$\max_{\theta \geq 0} p(\theta)[\pi^1 - \delta(\theta)]. \tag{8}$$

Note that manufacturer's profit and subsidy costs are contingent on the supplier being financially viable by the second stage as we have assumed subsidies represent promises for future contracts. If the supplier is not available by the second stage, these future contracts need not be awarded: the manufacturer will not pay them when the supplier defaults.

The first order condition of (8) is given by

$$p'(\theta)[\pi^1 - \delta(\theta)] - p(\theta)\delta'(\theta) = 0, \tag{9}$$

or, equivalently

$$\frac{p'(\theta)}{p(\theta)} = \frac{\delta'(\theta)}{\pi^1 - \delta(\theta)}. \tag{10}$$

We now offer the following lemma related to this optimization problem.

Lemma 2. *The optimization problem in* (8) *is log-concave.*

As (8) is log-concave, the first order condition represents a sufficient condition for finding optimal subsidy levels.

The following proposition details the manner in which subsidies vary with the manufacturer's optimal profit in our second stage subgame.

Proposition 1. *The optimal amount of manufacturer-provided subsidies* θ^* *is increasing in* π^1*.*

As monopoly profit π^1 increases, so too does the optimal amount of subsidies. Intuitively, as π^1 increases, it becomes increasingly beneficial for manufacturers to ensure their suppliers are reliable. Increased reliability may result from increased subsidies.

For demand model (4), Lemma 1 leads to the following corollary.

Corollary 1. *The optimal amount of manufacturer-provided subsidies* θ^* *is increasing in market size* d *and decreasing in marginal cost of production* c*, assuming* $d > c$*.*

Corollary 1 is analogous to Proposition 1, as an increase in d or a decrease in c will increase π^1 and, hence, the optimal amount of subsidies θ^*.

3.2 Oligopolistic Manufacturers with Dedicated Suppliers

We now analyze the effect of competition on the subsidies by comparing the benchmark case with the competitive scenario shown in Figure 1(b). Manufacturers engage in competition by supplying z_i^{jk} to the retail market, where subscript i is used to distinguish between manufacturers ($i \in \{1,2\}$) and superscripts j and k denote the delivery status of each manufacturer's supplier, similar to the explanation of π_i^{jk} presented earlier in Section 3.

The following proposition describes the second stage equilibrium quantities and profits for demand model (4).

Proposition 2. *For demand model* (4)*, the equilibrium order quantities for manufacturers 1 and 2 in the second-stage game are given by*

$$(z_1^{jk}, z_2^{jk}) = \begin{cases} \left(\frac{d-c}{3}, \frac{d-c}{3}\right); & \textit{Suppliers 1 and 2 did not default (j=1, k=1);} \\ \left(\frac{d-c}{2}, 0\right); & \textit{Supplier 1 did not default while Supplier 2 defaulted (j=1, k=0);} \\ \left(0, \frac{d-c}{2}\right); & \textit{Supplier 1 defaulted while Supplier 2 did not default (j=0, k=1);} \\ (0,0); & \textit{Suppliers 1 and 2 defaulted (j=0, k=0).} \end{cases} \tag{11}$$

The equilibrium profits for manufacturers 1 and 2 are

$$(\pi_1^{jk}, \pi_2^{jk}) = \begin{cases} \left(\frac{(d-c)^2}{9}, \frac{(d-c)^2}{9}\right); & \textit{Suppliers 1 and 2 did not default (j=1, k=1);} \\ \left(\frac{(d-c)^2}{4}, 0\right); & \textit{Supplier 1 did not default while Supplier 2 defaulted (j=1, k=0);} \\ \left(0, \frac{(d-c)^2}{4}\right); & \textit{Supplier 1 defaulted while Supplier 2 did not default (j=0, k=1);} \\ (0,0); & \textit{Suppliers 1 and 2 defaulted (j=0, k=0).} \end{cases} \tag{12}$$

In the first period, manufacturers choose an appropriate amount of non-negative subsidies to provide to their supplier. Manufacturer 1's expected profit is

$$p(\theta_1)\{p(\theta_2)\pi_1^{11} + [1-p(\theta_2)]\pi_1^{10} - \delta(\theta_1)\} = p(\theta_1)[\pi_1^{10} - p(\theta_2)(\pi_1^{10} - \pi_1^{11}) - \delta(\theta_1)]. \tag{13}$$

Similar expressions apply for manufacturer 2's profit.

Manufacturer 1's best response function r_1 for the first stage subgame is

$$r_1(\theta_2) = \arg\max_{\theta_1 \geq 0} p(\theta_1)[\pi_1^{10} - p(\theta_2)(\pi_1^{10} - \pi_1^{11}) - \delta(\theta_1)]. \tag{14}$$

The first order condition of (13) for manufacturer 1 is

$$p'(\theta_1)[\pi_1^{10} - p(\theta_2)(\pi_1^{10} - \pi_1^{11}) - \delta(\theta_1)] - p(\theta_1)\delta'(\theta_1) = 0, \tag{15}$$

or equivalently,

$$\frac{p'(\theta_1)}{p(\theta_1)} = \frac{\delta'(\theta_1)}{\pi_1^{10} - p(\theta_2)(\pi_1^{10} - \pi_1^{11}) - \delta(\theta_1)}. \tag{16}$$

Assuming $\pi_1^{10} = \pi_2^{01}, \pi_1^{00} = \pi_2^{00}$, and $\pi_1^{11} = \pi_2^{11}$ (a "symmetric equilibrium"), $\theta = \theta_1 = \theta_2$ and

$$\frac{p'(\theta)}{p(\theta)} = \frac{\delta'(\theta)}{\pi_1^{10} - p(\theta)(\pi_1^{10} - \pi_1^{11}) - \delta(\theta)}. \tag{17}$$

As the following lemma shows, (13) is log-concave in θ_1.

Lemma 3. *Manufacturer 1's expected profit* (13) *is log-concave in* θ_1*.*

Similarly, manufacturer 2's profit is log-concave in θ_2. Lemma 3 yields the existence of a Nash Equilibrium in the first stage game between manufacturers.

Lemma 4. *There exists a Nash Equilibrium in the oligopolistic manufacturers with dedicated suppliers scenario.*

3.2.1 Insights on Manufacturer Competition

Having presented an analysis of supply chain structures with and without competition, we can examine the effect of competition on expected quantities of goods released to the retail market, subsidies, manufacturer profits, and the consumer surplus. We introduce subscripts "m" and "o" to denote the monopolistic and oligopolistic manufacturers, respectively, and the superscript "d" to denote a dedicated supplier scenario.

Proposition 3 examines the effect of competition on subsidies.

Proposition 3. *If $\pi^1 = \pi_1^{10}$, then, as long as $\pi_1^{10} > \pi_1^{11}$, the level of subsidies provided by manufacturers to suppliers will be higher when manufacturers are monopolists than when they are oligopolists ($\theta_m^d > \theta_o^d$) in dedicated supplier scenarios.*

We call $\pi_1^{10} - \pi_1^{11}$ the *competition intensity* as it represents the loss of profits experienced by manufacturers in competitive settings over non-competitive settings. The condition $\pi^1 = \pi_1^{10}$ (i.e., the profit of the manufacturer in the monopolistic model when its supplier delivers goods equals the profit of the manufacturer in the oligopolistic model when its supplier delivers goods while its competitor's supplier does not) in model (4) with linear inverse demand is the same as assuming that the market size d is the same in the monopolistic and oligopolistic models.

We see from (48) that subsidies decrease with higher competition intensity (lower π_1^{11}, holding π_1^{10} constant) in the oligopoly model. Intuitively, these subsidies decrease as competition intensity increases because the expected profits of manufacturers decrease. Hence, manufacturers will reduce the amount of subsidies provided to manufacturers to curb subsidy costs.

Let us now assume that the market size increases in the oligopoly model, that is $\pi_1^{10} > \pi^1$. This market size increase could be due to synergies (e.g. combined advertising) realized from multiple manufacturers producing and selling goods. This effect is fairly common within the automotive industry when new products are introduced. For instance, as more and more vehicle manufacturers added hybrid vehicles to their lineup, demand increased considerably. Hybrid vehicle sales rose from 210 thousand units in 2005 to 324 thousand in 2008. Over the same time period, the number of vehicle nameplates offering hybrid vehicles expanded from eight to 15 (Electric Drive Transportation Association, 2009).

If $\pi_1^{10} > \pi^1$, the size of the manufacturer-provided subsidies in the oligopoly model may be higher than the monopoly model. Referring to (47) and (48), we see that when $\pi_1^{10} > \pi^1$, the left hand side of (48) is greater than the left hand side of (47), as the right hand side of both equations is an increasing function of θ.

Turning now to analyze manufacturers' optimal profits, we offer the following proposition.

Proposition 4. *If $\pi^1 = \pi_1^{10}$, expected profits are lower for manufacturers in the competitive setting as compared to the non-competitive setting for symmetric manufacturers.*

The previous proposition demonstrates that, not surprisingly, manufacturers prefer operating in a monopolistic environment, all other things equal.

Total quantities of goods released to *all* retail markets in the monopolistic manufacturers with dedicated suppliers scenario (Q_m^d) are

$$Q_m^d = \begin{cases} 0, & \text{w.p. } [(1 - p(\theta_m^d)]^2, \\ z^1, & \text{w.p. } 2p(\theta_m^d)[1 - p(\theta_m^d)], \\ 2z^1, & \text{w.p. } p(\theta_m^d)^2. \end{cases} \tag{18}$$

In the oligopolistic manufacturers with dedicated suppliers scenario, the quantities released to retail markets, Q_o^d, are

$$
Q_o^d = \begin{cases} 0, & \text{w.p. } [1-p(\theta_o^d)]^2, \\ z^{10}, & \text{w.p. } p(\theta_o^d)[1-p(\theta_o^d)], \\ z^{01}, & \text{w.p. } p(\theta_o^d)[1-p(\theta_o^d)], \\ 2z^{11}, & \text{w.p. } p(\theta_o^d)^2. \end{cases} \tag{19}
$$

The expected total quantity of goods released to retail markets is then

$$
EQ_m^d = 2p(\theta_m^d)z^1, \tag{20}
$$

$$
EQ_o^d = p(\theta_o^d)[2z^{11}p(\theta_o^d) - (z^{01}+z^{10})(p(\theta_o^d)-1)]. \tag{21}
$$

If $z^{10} = z^{01} = z^1$, (21) becomes

$$
EQ_o^d = 2p(\theta_o^d)[z^1 - p(\theta_o^d)(z^1 - z^{11})]. \tag{22}
$$

As discussed in Proposition 3, $p(\theta_m^d) > p(\theta_o^d)$ under reasonable assumptions. Under these assumptions, $EQ_m^d > EQ_o^d$ as long as $z^1 > z^{11}$, which is the case in the Cournot model.

While our discussion up to this point has focused on the manufacturer and its decisions, it is also important to consider the benefits consumers can reap as a consequence of each supply chain structure. We measure benefits to consumers using the consumer surplus. Lemma 5 discusses this quantity in the benchmark scenario.

Lemma 5. *For demand model* (4)*, the expected consumer surplus (CS) in the monopolistic manufacturers with dedicated suppliers scenario is*

$$
E(CS_m^d) = p(\theta_m^d)\cdot\frac{(d-c)^2}{4}. \tag{23}
$$

Lemma 6 examines the consumer surplus in the oligopolistic manufacturers with dedicated suppliers scenario.

Lemma 6. *The expected consumer surplus in the oligopolistic manufacturers with dedicated suppliers scenario, $E(CS_o^d)$, assuming a symmetric equilibrium and perfectly correlated supplier asset shocks, is given by*

$$E(CS_o^d) = p(\theta_o^d)^2 \cdot \frac{2(d-c)^2}{9} + p(\theta_o^d)[1-p(\theta_o^d)] \cdot \frac{(d-c)^2}{4}. \tag{24}$$

It is important to note which scenario consumers prefer. We now compare the results shown in Lemmas 5 and 6.

Proposition 5. *The consumer surplus is larger with manufacturers who are monopolists in the retail marketplace.*

The result shown in Proposition 5 arises from the fact that $p(\theta_m^d) > p(\theta_o^d)$. Because of this relationship, suppliers will be more reliable when manufacturers are monopolists, and, in expectation, consumers will reap greater benefits because manufacturers will be more likely to release goods to the retail market in this scenario.

We wish to point out that our previous discussion of expected quantities released to retail markets and the consumer surplus examined these quantities across *all* retail markets. In other words, when manufacturers operate as monopolists, each addressed its own market, whereas when manufacturers operate as oligopolists, they addressed one market in total. If, instead, we normalize the combined size of the retail markets in the monopolistic manufacturers with dedicated suppliers scenario to equal that of the single market in the oligopolistic manufacturers with dedicated suppliers scenario, we have a different expression for Q_m^d:

$$Q_m^d = \begin{cases} 0, & \text{w.p. } [(1-p(\theta_m^d)]^2, \\ \frac{z^1}{2}, & \text{w.p. } 2p(\theta_m^d)[1-p(\theta_m^d)], \\ z^1, & \text{w.p. } p(\theta_m^d)^2. \end{cases} \tag{25}$$

The expected consumer surplus $E(CS_m^d)$ would also differ from (23):

$$E(CS_m^d) = p(\theta_m^d) \cdot \frac{(d-c)^2}{8}. \tag{26}$$

Conventional wisdom would suggest that manufacturer-level competition would benefit consumers and increase the consumer surplus. However, we find through Proposition 5 that manufacturer-level competition leads to decreased subsidies, causing suppliers to become less reliable. Therefore, the net effect of competition on the consumer surplus with normalized market sizes is dependent on the reliability of suppliers, inclusive of subsidies. For instance, when suppliers for competing manufacturers are highly reliable, $E(CS_o^d)$ will be greater than $E(CS_m^d)$, but when suppliers for competing manufacturers are not reliable, it is the case that $E(CS_m^d)$ will exceed $E(CS_o^d)$.

3.3 Monopolistic Manufacturers with a Shared Supplier

Having analyzed the effects of manufacturer-level competition, we now examine the effect of using a shared supplier on the manufacturers' decisions when manufacturers do not compete. Specifically, we will analyze the supply chain structure in Figure 1(d) against the benchmark case in Figure 1(a).

Because manufacturers do not compete, the equilibrium quantity of goods released to the market is the same as in the benchmark model. If its supplier does not default, a manufacturer releases z^* to the market. For linear demand model (4), $z^* = \frac{(d-c)}{2}$ when the supplier does not default, and $z^* = 0$ when the shared supplier defaults. Definitions of π^1 and π^0 are given in our explanation of the benchmark model.

In the first-stage subgame, manufacturer 1 solves

$$\max_{\theta_1 \geq 0} p(\theta_1 + \theta_2)[\pi^1 - \delta(\theta_1)], \tag{27}$$

where $p(\cdot)$ is now a function of θ_1 and θ_2 because the supplier receives subsidies from both manufacturers. The first order condition of (27) is given by

$$p'(\theta_1 + \theta_2)[\pi^1 - \delta(\theta_1)] - p(\theta_1 + \theta_2)\delta'(\theta_1) = 0, \tag{28}$$

or equivalently,

$$\frac{p'(\theta_1 + \theta_2)}{p(\theta_1 + \theta_2)} = \frac{\delta'(\theta_1)}{\pi^1 - \delta(\theta_1)}. \tag{29}$$

In a symmetric equilibrium, (29) simplifies to

$$\frac{p'(2\theta_m^s)}{p(2\theta_m^s)} = \frac{\delta'(\theta_m^s)}{\pi^1 - \delta(\theta_m^s)}, \tag{30}$$

where the subscript "m" denotes monopolistic manufacturers and the superscript "s" denotes a shared supplier scenario. Similar to Lemma 2, (27) is log-concave. Therefore, the first order condition represents a sufficient condition for finding optimal subsidy levels. The results from Proposition 1 and Corollary 1 also hold in this supply chain scenario.

3.3.1 Insights on Cross Subsidies

We now analyze the impact of cross subsidies on the expected quantities of goods released to the retail market, subsidies, manufacturer profits, and the consumer surplus.

The following proposition compares the amount of subsidies provided by each manufacturer in the monopolistic manufacturers with a shared supplier scenario to those in the monopolistic manufacturers with dedicated suppliers scenario.

Proposition 6. *The optimal amount of subsidies provided by each manufacturer in the monopolistic manufacturers with a shared supplier scenario, θ_m^s is less than the optimal amount of subsidies in the monopolistic manufacturers with dedicated suppliers scenario, θ_m^d.*

If, instead of examining the subsidies *provided* by each manufacturer, we examine the subsidies *received* by each supplier, we must take into account the fact that a shared supplier receives subsidies from two manufacturers. In other words, if each manufacturer that uses a shared supplier provides θ_m^s, the supplier receives $2\theta_m^s$. Proposition 7 compares the amount of subsidies received by shared and dedicated suppliers when manufacturers are monopolists.

Proposition 7. *The amount of subsidies received by a shared supplier, $2\theta_m^s$, is greater than the amount of subsidies received by a dedicated supplier θ_m^d.*

One can see that 1) manufacturers each provide less subsidies to a shared supplier than they do to a dedicated one, and 2) the total subsidies received by a shared supplier is greater than the total subsidies received by a dedicated supplier.

Proposition 8 discusses the implications of cross subsidies on manufacturer-level profits.

Proposition 8. *Manufacturer profits in monopolistic settings are higher when manufacturers share a supplier than if they used dedicated suppliers.*

In the monopolistic manufacturers with a shared supplier scenario, the quantities released to *all* retail markets, Q_m^s, are

$$Q_m^s = \begin{cases} 0, & \text{w.p. } 1 - p(2\theta_m^s), \\ 2z^1, & \text{w.p. } p(2\theta_m^s), \end{cases} \tag{31}$$

while the quantities released to *all* retail markets in the monopolistic manufacturers with dedicated suppliers scenario, Q_m^d, is specified by (18).

The expected quantities released to the market are then:

$$EQ_m^s = 2z^1 p(2\theta_m^s), \tag{32}$$

$$EQ_m^d = 2z^1 p(\theta_m^d). \tag{33}$$

As $2\theta_m^s > \theta_m^d$, $EQ_m^s > EQ_m^d$.

With respect to the preferences of consumers, we examine the consumer surplus in the monopolistic manufacturers with a shared supplier scenario and compare it to the consumer surplus in the monopolistic manufacturers with dedicated suppliers scenario.

Lemma 7. *The expected consumer surplus in the monopolistic manufacturers with a shared supplier scenario is given by*

$$E(CS_m^s) = p(2\theta_m^s) \cdot \frac{(d-c)^2}{4}. \tag{34}$$

The expected consumer surplus in the monopolistic manufacturers with dedicated suppliers scenario is identical to that stated in Lemma 5. Comparing these two expressions, we have the following proposition.

Proposition 9. *Consumers are always better off when non-competing manufacturers use a shared supplier.*

3.4 Combined Effects of Competition and Cross Subsidies

In the previous sections, we saw that manufacturer-level competition decreases the amount of subsidies to suppliers while simultaneously decreasing manufacturers' profits. We have also seen that cross subsidies make the total subsidy received by a supplier larger while increasing manufacturers' profits. In this section we answer the question of which of these effects will dominate the other when they are combined. We do so by incorporating an analysis of the supply chain structure shown in Figure 1(c).

The results in this section provide an interesting extension to an observation in Tang and Kouvelis (2009) that competing manufacturers should never choose to share a supplier. We show that when cross-subsidy benefits are significant, a shared supplier is the preferred choice for manufacturers.

Within the oligopolistic manufacturers with dedicated suppliers scenario, recall that manufacturer 1 solved best response function (14) which yielded first order condition (15).

The first-stage subgame optimization problem for manufacturers in the oligopolistic manufacturers with a shared supplier scenario is

$$\max_{\theta_1 \geq 0} p(\theta_1 + \theta_2)[\pi_1^{11} - \delta(\theta_1)]. \tag{35}$$

It is easy to see that this problem is again log-concave, and that the symmetric equilibrium in this scenario is given by

$$\frac{p'(2\theta)}{p(2\theta)} = \frac{\delta'(\theta)}{\pi_1^{11} - \delta(\theta)}. \tag{36}$$

We offer the following proposition which compares subsidies provided by competitive manufacturers when they use dedicated suppliers or a shared supplier.

Proposition 10. *The amount of subsidies provided by each manufacturer to its supplier in the oligopolistic manufacturers with a shared supplier scenario is less than the amount of subsidies provided by each manufacturer in the oligopolistic manufacturers with dedicated suppliers scenario when* $\pi_1^{10} > \pi_1^{11}$.

In analyzing the amount of subsidies received by suppliers we must again compare the first order conditions associated with the oligopolistic manufacturers with dedicated suppliers ("*od*") and oligopolistic manufacturers with a shared supplier ("*os*") scenarios. For the oligopolistic manufacturer with dedicated suppliers scenario ("*od*")

$$\frac{p'(\theta_o^d)}{p(\theta_o^d)} = \frac{\delta'(\theta_o^d)}{\pi_1^{10} - p(\theta_o^d)(\pi_1^{10} - \pi_1^{11}) - \delta(\theta_o^d)}, \tag{37}$$

and, for the shared supplier scenario ("*os*")

$$\frac{p'(\theta_o^s)}{p(\theta_o^s)} = \frac{\delta'\left(\frac{\theta_o^s}{2}\right)}{\pi^{11} - \delta\left(\frac{\theta_o^s}{2}\right)}. \tag{38}$$

We define

$$G(\theta_o^d) = \frac{\delta'(\theta_o^d)}{\pi_1^{10} - p(\theta_o^d)(\pi_1^{10} - \pi_1^{11}) - \delta(\theta_o^d)}, \tag{39}$$

$$H(\theta_o^s) = \frac{\delta'\left(\frac{\theta_o^s}{2}\right)}{\pi^{11} - \delta\left(\frac{\theta_o^s}{2}\right)}. \tag{40}$$

Examining (39) and (40), we can see that if $\pi_1^{10} \approx \pi_1^{11}$, total subsidies are lower in the oligopolistic manufacturers with dedicated suppliers scenario than the shared supplier scenario because the $\frac{\theta}{2}$ effect will dominate. However, when competition intensity $(\pi_1^{10} - \pi^{11})$ increases, subsidies to dedicated suppliers increase. In the extreme case, when $\pi_1^{10} >> \pi_1^{11}$, the amount of total subsidies in the oligopolistic manufacturers with dedicated suppliers scenario will exceed the amount of total subsidies in the shared supplier scenario.

We now compare the quantity of goods released to retail markets in the oligopolistic manufacturers with dedicated suppliers (Q_o^d) and oligopolistic manufacturers with a shared supplier (Q_o^s) scenarios.

Q_o^d is specified by (19), while Q_o^s is specified by

$$Q_o^s = \begin{cases} 0, & \text{w.p. } 1 - p(2\theta_m^s), \\ 2z^{11}, & \text{w.p. } p(2\theta_m^s). \end{cases} \tag{41}$$

If $z^{10} = z^{01} = z^1$, expected quantities released to retail markets in both scenarios are

$$EQ_o^d = 2p(\theta_o^d)[z^1 - p(\theta_o^d)(z^1 - z^{11})], \tag{42}$$

$$EQ_o^s = 2z^{11}p(2\theta_o^s). \tag{43}$$

We know $z^{11} < z^1 - p(\theta_o^d)(z^1 - z^{11})$. Therefore, if $2\theta_o^s < \theta_o^d$, $EQ_o^d > EQ_o^s$.

However, we have previously seen that $2\theta_o^s > \theta_o^d$ when $\pi^{10} - \pi^{11} \approx 0$. Therefore, if competition is weak, the manufacturer using a shared supplier will, in expectation, release more goods to the retail market.

4 Conclusions

Intense competition in the automotive retail marketplace has forced manufacturers to carefully examine their cost structures. A major component of these costs is purchases from suppliers. Manufacturers strive to procure the highest quality goods at the lowest prices from reliable suppliers. They can elect to purchase goods from a dedicated supplier, or share a supplier with other manufacturers. Manufacturers' profitability is directly related to supplier reliability, as well as subsidies provided to suppliers. These subsidies can directly influence supplier reliability.

We have examined how optimal manufacturers' profits, subsidies provided to suppliers, and quantities released to the retail market vary with manufacturer-level competition and cross-subsidies by analyzing the four supply chain structures shown in Figure 2.1.

We show that there exists a tradeoff between the benefits of cross-subsidies and benefits of reduced competition in making the decision to share a supplier. The use of shared or dedicated suppliers will largely depend on whether the effects of competition or cross-subsidies dominates.

On one hand, cross-subsidies achieved through the use of a common supplier allow manufacturers to share subsidy costs. We show that the amount of subsidies provided by each manufacturer is less when manufacturers share a supplier. However, because suppliers receive subsidies from two manufacturers, the total subsidy received can be larger for a shared supplier. Thus, cross-subsidies can improve supplier reliability.

But, on the other hand, the use of shared suppliers poses additional problems for competing manufacturers. When suppliers are shared, all manufacturers are affected by their supplier's status. Manufacturers cannot differentiate themselves from their peers by the availability of supplies, so retail competition intensifies. If, instead, suppliers are dedicated, manufacturers possess an option to capture a larger portion of the market if their competitor's supplier fails.

Our model is applicable to the behavior of firms within the US automotive industry. For example, General Motors (GM) procured many goods from internal, dedicated suppliers for a large majority of its corporate life. These internal suppliers were organized as divisions–largely run autonomously–and provided GM with everything from radios and engine control modules (Delco Division), air conditioning compressors (Harrison Division), engine electrical systems (Packard Division), fuel delivery systems (Rochester Products Division), headlamps (Guide Division), and car bodies (Fisher Body Division). While some components produced by these divisions were sold to other automakers, their primary responsibility was to fulfill GM's needs. However, in the late 1990s, GM elected to combine many of these organizations into a single division, which it renamed Delphi. GM spun Delphi off in 1997 through an initial public offering, thus creating an independent parts supplier. In divorcing itself from Delphi, GM was hoping to reap the benefits of cross-subsidies from other manufacturers.

After its initial public offering, Delphi Corporation moved to diversify its customer base. In 2007, the year before it filed for bankruptcy, 63% of Delphi's revenues came from non-GM customers, up from 17% in fiscal year 1996. Delphi increasingly became a "shared supplier." However, as Delphi continued to diversify its customer base, competition in the automotive industry increased. According to Ward's Automotive, sales of cars produced by U.S. automakers in 2009 represented 45% of total U.S. vehicle sales, compared with over 70% of vehicle sales in 1996. This decreased market share is largely due to increased competition within the American marketplace.

As competition in the U.S. market intensified, subsidies to Delphi from manufacturers began to decrease. One could argue that Delphi's new customers elected to use the firm for the same reason as GM: namely, to reap the benefit of cross-subsidies. However, as our paper has demonstrated, under certain conditions, the amount of subsidies received by a shared supplier decreases. This decreased level of subsidies makes shared suppliers less reliable than dedicated ones, and is exemplified by the continuing financial struggles facing Delphi: from 1996 through 2007, Delphi's pre-tax operating income decreased from $1.3 billion to -$2.0 billion in spite of the company's diversification efforts. (Here, we cite pre-tax income to negate the impact of tax-loss carry forwards and non-recurring/extraordinary items.)

Delphi is not alone in its financial struggles. Its fate largely mirrors that of Visteon Corporation, a parts maker that was formed in 1997 and spun off from the Ford Motor Company in 2000. Visteon filed for Chapter 11 bankruptcy in May 2009. Sales to Ford accounted for 28% of Visteon's revenues in 2009 compared with 84% of its revenues in 2000, the year it went public. In spite of its attempts to become a profitable "shared supplier," Visteon's strategy did not allow it to avoid a bankruptcy that was arguably caused, in part, by the decreased level of subsidies it received from its customers.

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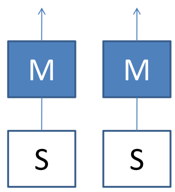
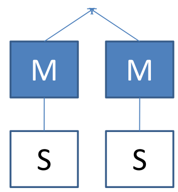
Figures



(a)



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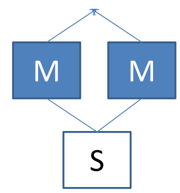

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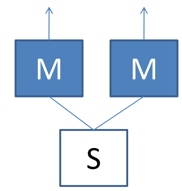

(d)

Figure 2.1: Supply Chain Structures. *Shaded boxes labeled with an "M" represent manufacturers. Unshaded boxes labeled with an "S" represent suppliers. Retail competition is denoted by two intersecting arrows. Figures a) and d) represent non-competitive retail market scenarios, while Figures b) and c) represent competitive scenarios.*

Appendices

Appendix 2.1: Proofs of Figures, Corollaries, and Lemmas

Proof of Lemma 1. Since $p(\theta)$ is log-concave by Assumption 1, the first-order derivative of $\log[p(\theta)]$ is decreasing. That is, $\frac{d}{d\theta}$ $[\log\, p(\theta)] = \frac{p'(\theta)}{p(\theta)}$ is decreasing.

Proof of Lemma 2. Since $p(\theta)$ is log-concave by Assumption 1, we must demonstrate $\pi^1 - \delta(\theta)$ is log-concave. Define

$$f(\theta) \stackrel{\text{def}}{=} \log[\pi^1 - \delta(\theta)]. \tag{44}$$

The function $f(\theta)$ is a composition of an increasing concave and concave function. Therefore, $f(\theta)$ is itself concave and $\pi^1 - \delta(\theta)$ is a log-concave function.

Proof of Proposition 1. Taking the cross partial derivative of (8), with respect to π^1 and θ yields

$$\frac{\partial^2}{\partial\pi^1\partial\theta}\{p(\theta)[\pi^1 - \delta(\theta)]\} = p'(\theta) > 0. \tag{45}$$

Therefore, (8) is supermodular in (θ, π^1) as we have assumed $p(\cdot)$ is an increasing function. Because (8) is supermodular in (θ, π^1), the optimal amount of manufacturer-provided subsidies θ^* is increasing in π^1 as shown in (Heyman and Sobel, 2004, Chapter 8).

Proof of Corollary 1. The assumption of linear demand (4), and a monopoly environment for the manufacturer means that manufacturers will each sell the monopoly quantity, $\frac{d-c}{2}$, of goods in the downstream retail market. This means that the optimal manufacturer's profit when the supplier is able to deliver goods is

$$\pi^1 = \frac{(d-c)^2}{4}. \tag{46}$$

We have already shown θ^* is increasing in π^1. Therefore, with equation (46), θ^* is increasing in d and decreasing in c.

Proof of Proposition 2. When manufacturer 1 is unable to supply goods to the retail market, its equilibrium order quantity is necessarily 0. When manufacturer 1 can supply goods to the retail market while manufacturer 2 cannot, manufacturer 1 acts as a monopolist and releases the monopoly quantity to the downstream market. This result is similar to that discussed in the previous section. When both manufacturers can supply goods to the retail market, manufacturer 1 releases the standard oligopoly quantity. These equilibrium quantities yield the optimal profits shown above through substitution into (4).

Proof of Lemma 3. Assumption 1 states $p(\cdot)$ is log-concave in its argument. Cost function $\delta(\cdot)$ is convex by Assumption 2. Hence, (13) is log-concave as it is a product of a log-concave function, $p(\theta_1)$, with a positive concave function: $\pi_1^{10} - p(\theta_2)(\pi_1^{10} - \pi_1^{11}) - \delta(\theta_1)$ is concave in θ_1. The result is log-concave function (13).

Proof of Lemma 4. Manufacturers' action spaces are compact and convex when their suppliers are available. Additionally, all components of (13) are continuous, therefore, their payoff functions are continuous. We have also shown in Lemma 3 that payoff functions are log-concave. As log-concavity implies quasi-concavity, there exists at least one pure strategy Nash Equilibrium in the oligopolistic manufacturers with dedicated suppliers scenario.

Proof of Proposition 3. Rewriting (10) and (17), we compare solutions of the following two equations:

$$\text{Monopolistic Model:} \qquad \pi^1 = \frac{\delta'(\theta_m^d)p(\theta_m^d)}{p'(\theta_m^d)} + \delta(\theta_m^d) \tag{47}$$

$$\text{Oligopolistic Model:} \qquad \pi_1^{10} = \frac{\delta'(\theta_o^d)p(\theta_o^d)}{p'(\theta_o^d)} + \delta(\theta_o^d) + p(\theta_o^d)(\pi_1^{10} - \pi_1^{11}) \tag{48}$$

Lemma 1 states that the quantity $\frac{p'(\cdot)}{p(\cdot)}$ is decreasing when it is governed by Assumption 1. Therefore, the first expression in the right-hand side of both equations is increasing in θ. Also, using Assumption 2, $\delta(\cdot)$ is increasing in its argument. Given $\pi^1 = \pi_1^{10}$, θ_o^d must be lower than θ_m^d as long as $\pi_1^{10} - \pi_1^{11} > 0$ in order for the right hand side of (47) and (48) to remain the same.

Proof of Proposition 4. Expected profits in the monopoly setting are given by

$$p(\theta_m^d)[\pi^1 - \delta(\theta_m^d)], \tag{49}$$

and by the following expression in the oligopoly setting

$$p(\theta_o^d)[\pi_1^{10} - p(\theta_o^d)(\pi_1^{10} - \pi_1^{11}) - \delta(\theta_o^d)] = p(\theta_o^d)[\pi_1^{10} - \delta(\theta_o^d)] - p(\theta_o^d)^2(\pi_1^{10} - \pi_1^{11}). \tag{50}$$

When $\pi^1 = \pi_1^{10}$, because θ_m^d maximizes (8), we have

$$p(\theta_m^d)[\pi^1 - \delta(\theta_m^d)] \geq p(\theta_o^d)[\pi_1^{10} - \delta(\theta_o^d)]. \tag{51}$$

Hence, $p(\theta_m^d)[\pi^1 - \delta(\theta_m^d)] > p(\theta_o^d)[\pi_1^{10} - \delta(\theta_o^d)] - p(\theta_o^d)^2(\pi_1^{10} - \pi_1^{11})$, in other words, expected profits are lower in the oligopoly setting as compared to the monopoly setting.

Proof of Lemma 5. Using the inverse demand function specified in (4), the equilibrium price of goods in the retail market is

$$P(z^*) = d - \frac{d-c}{2} = \frac{d+c}{2}, \tag{52}$$

when the supplier is not in default. Using the standard formula for consumer surplus with linear demand and taking into account the fact that manufacturers serve two retail markets yields

$$CS_m^d(z^*) = 2 \cdot \frac{1}{2} \cdot \frac{d-c}{2} \cdot \left(d - \frac{d+c}{2}\right) \tag{53}$$

$$= 2\frac{(d-c)^2}{8}. \tag{54}$$

When the supplier is bankrupt, the manufacturer cannot release goods to the market, hence the consumer surplus is 0 as no goods are available for consumers to buy. Therefore, the expected consumer surplus, $E(CS_m^d)$ is given by (23).

Proof of Lemma 6. Using the inverse demand function specified in (4), we find that the equilibrium price of goods in the retail market for the oligopolistic manufacturers with dedicated suppliers scenario is

$$P(z^*) = d - \frac{2(d-c)}{3} = \frac{d+2c}{3}, \tag{55}$$

when both suppliers are not bankrupt. Using the standard formula for consumer surplus with linear demand yields

$$CS_o^d = \frac{1}{2} \cdot \frac{2(d-c)}{3} \cdot \left(d - \frac{d+2c}{3}\right) \tag{56}$$

$$=\frac{2(d-c)^2}{9}. \tag{57}$$

When one supplier is bankrupt while the other is not, the consumer surplus is specified by an expression similar to CS_m^d (however, p_o^d is substituted for p_m^d) because only one manufacturer can sell goods in the retail market. The consumer surplus when both suppliers are bankrupt is zero due to the fact that no manufacturer can sell goods in the downstream retail market. The expected consumer surplus follows directly from these values and the probability they occur.

Proof of Proposition 5. Rewriting expression (24) for $E(CS_o^d)$ yields

$$p(\theta_o^d)^2\left[\frac{2(d-c)^2}{9}-\frac{(d-c)^2}{4}\right]+p(\theta_o^d)\frac{(d-c)^2}{4} \tag{58}$$

$$=-\,p(\theta_o^d)^2\cdot\frac{(d-c)^2}{36}+p(\theta_o^d)\frac{(d-c)^2}{4}. \tag{59}$$

As we have shown previously, for any given level of expected quantities released to the market, $p(\theta_m^d)>p(\theta_o^d)$. Therefore, comparing (59) with the expression for $E(CS_o^d)$ given in (23), $E(CS_m^d)>E(CS_o^d)$ and consumers are better off if manufacturers are monopolists in retail markets.

Proof of Proposition 6. Comparing (30) and (10), one can see that the right hand sides of both expressions are identical. The only difference between these expressions is that θ has been replaced by 2θ in the left-hand side of (30). We also know $\frac{p'(\cdot)}{p(\cdot)}$ is decreasing by Lemma 1, and $\frac{\delta'(\theta)}{\pi^1-\delta(\theta)}$ is increasing as

$$\frac{\partial}{\partial\theta}\left[\frac{\delta'(\theta)}{\pi^1-\delta(\theta)}\right]=\frac{\delta''(\theta)[\pi^1-\delta(\theta)]+[\delta(\theta)^2]}{[\pi^1-\delta(\theta)]^2}>0. \tag{60}$$

Therefore, θ_m^s must be less than θ_m^d in order for first order condition (30) to hold.

Proof of Proposition 7. The first order condition in the shared supplier scenario is

$$\frac{p'(2\theta_m^s)}{p(2\theta_m^s)} = \frac{\delta'(\theta_m^s)}{\pi^1 - \delta(\theta_m^s)}, \tag{61}$$

and, for the dedicated suppliers scenario

$$\frac{p'(\theta_m^d)}{p(\theta_m^d)} = \frac{\delta'(\theta_m^d)}{\pi^1 - \delta(\theta_m^d)}. \tag{62}$$

Because $\frac{p'(\cdot)}{p(\cdot)}$ is decreasing by Lemma 1 and $\frac{\delta' \cdot}{\pi^1 - \delta(\cdot)} > 0$, it follows that $\theta_m^d < 2\theta_m^s$.

Proof of Proposition 8. Manufacturer-level profits in the monopolistic manufacturers with dedicated suppliers and monopolistic manufacturers with a shared supplier models are, respectively

$$p(\theta_m^d)[\pi^1 - \delta(\theta_m^d)], \tag{63}$$

$$p(2\theta_m^s)[\pi^1 - \delta(\theta_m^s)]. \tag{64}$$

From Propositions 6 and 7,

$$2\theta_m^s > \theta_m^d, \tag{65}$$

$$\theta_m^s < \theta_m^d. \tag{66}$$

Using this information together with Assumptions 1 and 2 yields

$$p(2\theta_m^s) > p(\theta_m^d), \tag{67}$$

$$\delta(\theta_m^s) < \delta(\theta_m^d). \tag{68}$$

In other words, supplier reliability increases when manufacturers use a shared supplier in non-competitive settings while decreasing their costs of subsidies. Therefore, manufacturer-level profits in monopolistic settings are higher when manufacturers use a shared supplier.

Proof of Lemma 7. The expected consumer surplus in this scenario is identical to the expected consumer surplus in the monopolistic manufacturers with dedicated suppliers scenario except that the shared supplier receives subsidies from both manufacturers. These subsidies total $2\theta_m^s$.

Proof of Proposition 9. The proof results directly from (65) and Assumption 1.

Proof of Proposition 10. Equations (17) and (36) determine the equilibrium amount of subsidies for dedicated suppliers and shared supplier settings, respectively, when manufacturers compete (reproduced here with distinguishing notation):

$$\frac{p'(\theta_o^d)}{p(\theta_o^d)} = \frac{\delta'(\theta_o^d)}{[1 - p(\theta_o^d)]\pi_1^{10} + p(\theta_o^d)\pi_1^{11} - \delta(\theta_o^d)}, \tag{69}$$

and

$$\frac{p'(2\theta_o^s)}{p(2\theta_o^s)} = \frac{\delta'(\theta_o^s)}{\pi_1^{11} - \delta(\theta_o^s)}. \tag{70}$$

Let $\bar{\theta}$ satisfy

$$\frac{p'(\bar{\theta})}{p(\bar{\theta})} = \frac{\delta'(\bar{\theta})}{\pi_1^{11} - \delta(\bar{\theta})}. \tag{71}$$

Comparing (69) and (71) yields $\theta_o^d > \bar{\theta}$, and (70) and (71) yield $\bar{\theta} > \theta_o^s$. Therefore, $\theta_o^d > \theta_o^s$.

Chapter 3

Determinants of Private Equity Fundraising: An Analysis of Competition, Uncertainty, and Barriers to Entry

1 Introduction

Private equity (PE)[1], including leveraged buyout (aka "buyout") and venture capital (VC) finance, is a sizeable asset class whose importance in the US economy is well established. PE fundraising levels have ebbed and flowed in recent times, but these levels represent a nontrivial fraction of the US's gross domestic product (see Figure 3.1).

In the 55 years proceeding the establishment of American Research and Development (ARD), the first VC firm, VC has evolved into, according to Gompers and Lerner (2001), an important financial markets intermediary. Buyout firms and executed transactions first became an increasingly important staple of the US economy in the 1980s (see Kaplan and Stromberg (2008)) and late 2000s. Some, including Jensen (1989), believe buyout firms may ultimately surpass the public corporation as the dominant organizational form.

As PE has become an increasingly popular asset class, competition for investors' capital among PE firms ("competition") has increased (see Figure 3.2). Similar to fundraising levels, competition varies significantly year-to-year.

[1] Here we use PE as an umbrella term for alternative assets, focusing on leveraged buyout and venture capital finance. Buyout firms generally inject capital into existing companies in hopes of increasing efficiencies. Venture capital firms generally focus on providing capital to nascent companies. For an introduction to PE finance, see J. Lerner (1997), Fraser-Sampson (2010), or Cendrowski et al. (2008).

This paper examines the PE fundraising process by analyzing competition, uncertainty, and barriers to entry through a two-stage model. While numerous papers in the economics literature have examined each of the previously-mentioned elements (see the subsequent Literature Review), this paper is the first to examine them collectively and comparatively in n-firm, differentiated Cournot (1838) (quantity) and Bertrand (1883) (price) competition models. Our paper thus utilizes a novel model and innovatively applies it to analyze PE fundraising environments.

The inclusion of both Cournot and Bertrand models is motivated by a primary contention. We assert, and present empirical evidence demonstrating, brand name PE funds generally compete in quantities, while non-brand name firms compete in price. This empirical evidence has numerous implications for our theory-based results. Firms prefer Cournot competition, but are unable to select this method of competition when devoid of requisite brand equity.

We assume PE firms are risk neutral and compete to raise capital for their underlying funds. For expositional simplicity, we assume firms are contractually bound to investors (aka "limited partners") by the limited partnership agreement; our results also apply to other organizational forms.

Firms sell partnership units in their funds to investors. The "sale price" of firm partnership units is the net present value (NPV) of various financial and non-financial terms present in a PE firm's limited partnership agreement. Among other items, financial terms include an annual fund-level management fee (generally assessed based on an investor's committed capital) and carried interest (the portion of capital gains on investments retained by the PE firm). In addition to these revenue streams, PE funds receive additional fees from investors, including transaction fees incurred when the fund purchases or sells a company; fees associated with scouting deals; and advisory and monitoring fees (see Philippou (2009) for additional information). Each of these items is included in our definition of the sale price.

The market level of fees in the PE industry generally includes a 2% annual management fee and 20% carried interest: Gompers and Lerner (1999b) found roughly 80% of PE funds charged this level of fees to their investors. Their results are further supported by Metrick and Yasuda (2010). Non-financial terms include key man provisions, redemption rights, and items that are not necessarily expressed in financial terms, but which have an economic value associated with them (see Sahlman (1990) and Gompers and Lerner (1996)). We do not explicitly model the economics of fee receipts as they are not the focus of this paper.

In the first stage of our model, a large, finite number of firms simultaneously decide whether they will participate in the fundraising environment. If they elect to participate, firms pay a fixed setup cost. This setup cost represents barriers to entry and includes the monetary cost of starting and marketing a PE fund, the opportunity cost of the firm's managers, and the reputational capital required to establish a fund. In the second stage, firms sell partnership units in their funds and compete in an environment with market size uncertainty.

Our model yields numerous insights through an analysis of expected equilibrium prices and quantities, firm profits, and the consumer surplus. Among other results, we find

- brand name firms achieve greater expected profits and are more incentivized to enter the fundraising marketplace than non-brand name firms;
- market size uncertainty increases expected firm profits, the expected consumer sur-

plus, and incentives for firms to enter the marketplace;

- firm profits are more sensitive to market size uncertainty when investors purchase partnership units from brand name firms than when they purchase partnership units from non-brand name firms;
- the expected consumer surplus is more sensitive to market size uncertainty when investors purchase partnership units from non-brand name firms than when they purchase partnership units from brand name firms; and
- Bertrand competition decreases the ultimate level of competition in the marketplace, but remains advantageous for investors.

These results are used to deduce insights into the PE fundraising process as the paper progresses. Results and insights are summarized in the Conclusion.

2 Literature review

This paper develops a new theoretical model and novelly employs it to analyze competition, uncertainty, and barriers to entry in the PE fundraising process. It contributes to and extends different streams of research, including literature that–

1. Compares Cournot (quantity) and Bertrand (price) competition models;
2. Analyzes competition in uncertain environments; and
3. Examines the effect of competition and uncertainty in the PE fundraising process.

Sonnenschein (1968) first examined and demonstrated the duality of Cournot and Bertrand competition. Other seminal comparisons of Cournot and Bertrand competition include Kreps and Scheinkman (1983), in which the authors demonstrate that if capacities are selected by firms before Bertrand competition occurs, firms will select a capacity equal to the Cournot output level, and the resulting market price will be the Cournot price. Singh and Vives (1984) utilize the linear differentiated duopoly model proposed by Dixit (1979) to examine the duality of Cournot and Bertrand competition models. Through the use of a two-stage framework, Singh and Vives (1984) show that the choice of a quantity (price) contract is optimal when goods are substitutes (complements). Vives (1984) examines a duopoly model where firms have private information about uncertain demand. He shows that it is a dominant strategy to share information in Bertrand (Cournot) competition if goods are substitutes (complements).

Cournot and Bertrand models have previously been employed in the literature to examine the effects of uncertainty on competitive environments. Many papers in the oligopoly literature, including Novshek and Sonnenschein (1982), Basar and Ho (1976), and Clarke (1983) focus on uncertain demand and incomplete information in Cournot competition with homogeneous products. Other papers, including Klemperer and Meyer (1986) and Hackner (2000), contrast Cournot and Bertrand models. In the former paper, Klemperer and Meyer (1986) examine the effects of uncertainty, modeled in several different ways, on both Cournot and Bertrand Competition in a duopoly setting. They examine the sensitivity of their results to assumptions regarding uncertainty. Hackner (2000) extends the work of Singh and Vives (1984) to an n-firm model under certainty, and shows the latter authors' results are sensitive to their duopoly assumption. Hackner (2000) concludes that it is not evident whether Cournot or Bertrand competition is more efficient.

We extend the abovementioned theory-based work by examining market size uncertainty in differentiated, n-firm Cournot and Bertrand competition models. Furthermore, we extend the work of Hackner (2000) to examine the welfare implications of both competition mechanisms and uncertainty, and contrast these implications with the optimal decisions of PE firms.

Our analysis presents a novel contribution to the abovementioned literature as well as PE research centered on fundraising. Theoretical research on PE fundraising has generally focused on the effects of changes in capital gains tax rates (see Poterba (1987) and Poterba (1989)) and optimal contracting between VC firms and entrepreneurs (see Schmidt (2003), Repullo (1999), and Admati and Pfleiderer (1994)). The authors are unaware of papers utilizing theoretical models to explain the effects of competition and uncertainty on the PE fundraising process.

Numerous papers utilize an empirical approach to analyze PE fundraising; many center on venture capital finance. Gompers and Lerner (1999b) empirically determined macroeconomic factors, including rates of return on US Treasuries, capital gains tax rates, and previous equity market returns, that influenced the level of venture capital fundraising in any given year. Through an analysis of venture partnership agreements, Gompers and Lerner (1999a) found compensation in older and larger venture capital funds was closely related to historical performance. They also determined the oldest and largest VC funds received roughly 1% higher carried interest than less established funds. Black and Gilson (1998) demonstrated the strength of venture capital markets is dependent on the strength of the public markets. Kaplan and Schoar (2005) examined the performance of PE funds, the persistence in PE fund performance, and the relation of fund performance to capital flows, fund size, and fund survival. Through an empirical analysis of a novel dataset, Kaplan and Schoar (2005) found better performing funds were likely to raise larger follow-on funds and survive longer. Kaplan and Stromberg (2008) presented evidence illustrating how PE fundraising, activity, and transaction characteristics have varied over time, demonstrating a high level of cyclicality in each of these items. Metrick and Yasuda (2007) examined and provided evidence regarding deal and monitoring fees charged to investors by fund managers. Our paper extends the abovementioned literature by using theoretical models to examine the impact of competition, uncertainty, and barriers to entry on the optimal fundraising decisions of PE firms, and the effect of these decisions on PE investors.

3 Empirical evidence of fundraising competition methods

The PE industry contains elements of both Cournot and Bertrand competition. We assert that competition among brand name PE firms most closely resembles Cournot competition, while competition among non-brand name firms most closely resembles Bertrand competition. Firms prefer Cournot competition while investors prefer firm-level Bertrand competition; firms can only select Cournot competition if they have sufficient brand equity. Our aim within this section is not to provide exhaustive evidence to prove this assertion or to quantify brand equity, but rather to provide the reader with anecdotal evidence that validates the assertion.

Quantitatively defining a brand name firm is difficult, but, at a minimum, a brand name firm is one that has successfully raised multiple funds of above average size[2]. Brand name firms

[2]For purposes of comparison, from 2005-2010, the average venture capital fund raised $17.1 million in capital while the average buyout fund raised $1.0 billion in capital. *Source: Authors' analysis of data from Thomson ONE Banker.*

include buyout firms Blackstone Capital Partners; Goldman Sachs & Co.; TPG Capital; and Kohlberg, Kravis Robers & Co, as well as venture capital firms Sequoia Capital, Benchmark Capital, and Kleiner Perkins Caufield & Byers. Research, including Kaplan and Schoar (2005), has demonstrated that brand name firms are generally larger than their peers. Other papers in the literature, including Gompers and Lerner (1999b), use fund size as a proxy for the experience and success of a PE firm, items we deem components of brand equity.

To better understand the nature of Cournot and Bertrand competition in the PE fundraising environment, we turn to the words of Kreps and Scheinkman (1983),

> *"The Cournot story concerns producers who simultaneously and independently make production quantity decisions, and who then bring what they have produced to the market, with the market price being the price that equates the total supply with demand. The Bertrand story, on the other hand, concerns producers who simultaneously and independently name prices. Demand is allocated to the low-price producer(s), who then produce (up to) the demand they encounter [sic]. Any unsatisfied demand goes to the second lowest price producer(s), and so on."*

Similar to Cournot competitors, we assert brand name PE firms generally set a target size for their funds and adjust their fee structure to equate the supply of fund partnership units with investor demand. Many brand name firms strictly adhere to target sizes set by their fund managers. For example, during the buyout boom of the late 2000s, numerous brand name funds were "oversubscribed" by investors: some investors were not permitted to invest in the funds at all, while others were not provided the number of partnership units they desired. Though it seems counterintuitive that PE firms would turn away investors' capital, brand name firm managers generally feel that they cannot efficiently deploy large amounts of capital in excess of target levels. In light of the previous statements, a comparison of target and actual fund sizes for brand name funds should demonstrate marked similarities between these two quantities.

In contrast, we assert non-brand name firms generally release their fee structure to investors, and raise the maximum amount of capital they can obtain. They thus compete in price. If this statement is correct, a comparison of target and actual fund sizes for non-brand name funds should demonstrate large discrepancies between these two quantities.

In support of the first assertion, Table 3.1 presents recent fundraising information pertaining to some of the largest funds ever raised, those over $10 billion in size. Employing the implications of Kaplan and Schoar (2005) and Gompers and Lerner (1999b), these funds should have a high degree of brand equity: a significant amount of brand equity is required for a firm to raise such a large fund and our assertion regarding brand name firms should be most evident among this peer group.

The fifth and sixth columns of the table present the variance between the actual and target fund sizes ("variance") for PE funds in excess of $10 billion in size as well as the average variance of all buyout and venture capital funds, respectively, in the year of the fund's vintage. The variance between the actual and target fund size for the funds in Table 3.1 is smaller than the average variance for all PE funds.

In contrast to the data on large funds in Table 3.1, Table 3.2 presents similar data for PE funds under $100 million in size. If our second assertion is correct, and such funds compete in price, one would expect to see not only larger than average variances, but also fund target sizes in excess of actual fund sizes: target sizes for funds competing in price should represent an upper bound on the fund's capacity as firm managers would not want to cap the fund's size below its residual demand level. These expectations are borne by the data in Table 3.2. The magnitude of the variance for funds in Table 3.2 is significantly less than the average variance.

4 Differentiated Cournot and Bertrand Competition under uncertainty

We examine the decisions of risk neutral private equity (PE) firms competing in an n-firm differentiated-products oligopoly using a two-stage model.

1. In the first stage, firms simultaneously decide if they will enter the competitive market for raising capital. If a firm enters, it pays a fixed entry cost.
2. In the second stage, all firms in the competitive market participate in either Cournot or Bertrand competition and offer partnership units in their funds for sale.

We find the subgame perfect Nash equilibrium (SPNE) of each stage by backward induction; we first describe the solution to the second-stage game before proceeding to analyze the first-stage game.

In our initial solution to the second-stage game, the number of firms competing in the fundraising environment is exogenously specified. We subsequently use results from our first-stage model to later endogenize this parameter and explore additional results.

4.1 Second Stage Model

We have an n-firm economy where firms produce differentiated goods. Similar to the n-firm model of Hackner (2000), and to the duopoly models in Singh and Vives (1984), Vives (1985), and Dixit (1979), prices are set according to inverse demand function

$$p_i = \theta a - q_i - \gamma \sum_{i \neq j} q_j. \tag{1}$$

In equation (1), p_i is the market price of partnership units for firm i, a is the strictly positive market size parameter, q_i is the positive quantity of partnership units released by firm i to the market, and θ is the realization of strictly positive random variable Θ with $E(\Theta) = 1$ and $\text{Var}(\Theta) = s^2$. Klemperer and Meyer (1986) employ a similar multiplicative form of uncertainty in their model.

We restrict substitutability $\gamma \in (0, 1]$. Our restriction on γ ensures quantities remain positive is intuitive. Limited partners often set a ceiling for the maximum amount of capital they will invest in a particular asset class, for instance, venture capital or buyouts. Therefore, an increase in the consumption of partnership units from one fund necessarily means a decrease in the consumption of similar partnership units from another firm: partnership units are generally substitutes. However, if PE firms are viewed to serve differing niches in the limited partner's portfolio, purchases of partnership units from differing firms may be nearly independent of one another.

4.1.1 Results

In Cournot competition, risk neutral firm i seeks to maximize profit Π_C by selecting the optimal quantity q_i of partnership units to release

$$\max_{q_i \geq 0} \Pi_C = \left(\theta a - q_i - \gamma \sum_{i \neq j} q_j \right) q_i. \tag{2}$$

Similar to Hackner, and employing our own assumptions, the equilibrium quantity of fund partnership units ("quantity") in Cournot competition, q_C^*, is

$$q_C^* = \frac{\theta a}{\gamma(n-1)+2}, \tag{3}$$

where n is the number of firms participating in the fundraising environment. The equilibrium price of partnership units offered for sale ("price") in Cournot competition p_C^* is identical to q_C^*.

Hackner demonstrates one can invert (1) and solve for prices as a function of quantities to derive firm i's optimization problem in Bertrand competition. A similar approach is employed in Singh and Vives (1984), Vives (1985), and Dixit (1979).

To illustrate Hackner's approach in Bertrand competition, we begin with (1), and rearrange:

$$\sum_{i \neq j} q_j = \frac{\theta a - q_i - p_i}{\gamma}. \tag{4}$$

Summing (1) over all n firms, and noting $\sum_{i=1}^{n} \eta_i = \eta_k + \sum_{i \neq j} \eta_i$, we have

$$\left(p_i + \sum_{i \neq j} p_j\right) = n\theta a - \left(q_i + \sum_{i \neq j} q_j\right) - \gamma(n-1)\left(q_i + \sum_{i \neq j} q_j\right) \tag{5}$$

Manipulating (5) through substitution with (1) and (4), we find

$$q_i(p_i, p_{-i}) = \frac{(\theta a - p_i)[\gamma(n-2)+1] - \gamma \sum_{j \neq k}(\theta a - p_j)}{(1-\gamma)[\gamma(n-1)+1]}. \tag{6}$$

Profit maximization by firm i yields

$$p_i(p_{-i}) = \frac{\theta a}{2} - \frac{\gamma \sum_{i \neq j}(\theta a - p_j)}{2[\gamma(n-2)+1]}. \tag{7}$$

Summing (7) over all firms, and noting $p_i = p \; \forall i$ in our model (as compared to Hackner's where they may differ), we have quantity q_B^* is

$$q_B^* = \frac{\theta a[\gamma(n-2)+1]}{[\gamma(n-1)+1][\gamma(n-3)+2]}, \tag{8}$$

while the price p_B^* is

$$p_B^* = \frac{a(1-\gamma)}{\gamma(n-3)+2}. \tag{9}$$

Note that when $\gamma = 1$, $q_B^* = \frac{a}{n}$ and $p_B^* = 0$, both familiar symmetric Bertrand competition results.

Propositions 1 and 2 summarize the relationship between quantities and prices and underlying variables in Cournot and Bertrand competition, respectively. All proofs are contained in the Appendix. Quantities and prices are unaffected by s.

Proposition 1. *In Cournot competition, quantities and prices are decreasing in n and are decreasing in γ when $n > 1$.*

Proposition 2. *In Bertrand competition, quantities are decreasing in n and γ when $n > 2$. Prices are decreasing in n and decreasing in γ when $n > 1$.*

Propositions 1 and 2 demonstrate quantities and prices are sensitive to the number of competitive firms n and partnership unit substitutability γ. Prices and quantities are decreasing in the number of firms participating in a fundraising environment. Investors generally benefit by purchasing partnership units from firms participating in highly competitive environments. Highly competitive environments are disadvantageous for firms. An increase in the substitutability of partnership units generally increases quantities and decreases prices by effectively increasing competition.

Quantities are lower and prices are higher in Cournot competition versus Bertrand competition when $n > 1$. The difference in Cournot and Bertrand competition, $q_C^* - q_B^*$, is

$$q_C^* - q_B^* = \frac{\theta a \gamma^2 (n-1)}{[\gamma(n-3)+2][\gamma(n-1)+1][\gamma(n-1)+2]}. \tag{10}$$

Equation (10) is positive for $n > 1$ and is equal to zero for $n = 1$.

The difference in prices between Cournot and Bertrand competition, $p_C^* - p_B^*$, is

$$p_C^* - p_B^* = \frac{\theta a \gamma^2 (n-1)}{[\gamma(n-3)+2][\gamma(n-1)+2]}. \tag{11}$$

When $n = 1$, a price differential does not exist between the Cournot and Bertrand competitive scenarios.

When firms compete in Bertrand competition, as is the case with non-brand name firms, investors receive more partnership units at lower prices. All other things equal, an investor will find it advantageous to purchase partnership units from a firm that competes in prices. For example, assuming constant gross returns, an investor will achieve greater returns net of fees when firms compete in price (non-brand name firms) than when they compete in quantities (brand name firms).

Returning to our analysis, in Cournot competition, equilibrium profit ("profit") in Cournot competition Π_C^* is

$$\Pi_C^* = \frac{(\theta a)^2}{[\gamma(n-1)+2]^2} \tag{12}$$

and, in Bertrand competition,

$$\Pi_B^* = \frac{(\theta a)^2 (1-\gamma)[\gamma(n-2)+1]}{[\gamma(n-1)+1][\gamma(n-3)+2]^2}. \tag{13}$$

It is also beneficial to examine expected equilibrium profits ("expected profits"). The expressions for expected profits are

$$E(\Pi_C^*) = \frac{a^2(1+s^2)}{[\gamma(n-1)+2]^2} \tag{14}$$

and

$$E(\Pi_B^*) = \frac{a^2(1+s^2)(1-\gamma)[\gamma(n-2)+1]}{[\gamma(n-1)+1][\gamma(n-3)+2]^2}. \tag{15}$$

Propositions 3 and 4 summarize the relationship between expected profits in Cournot and Bertrand competition, respectively, and underlying variables.

Proposition 3. *Expected profits in Cournot competition are increasing in s. They are decreasing in n and γ when $n > 1$.*

Proposition 4. *Expected profits in Bertrand competition are increasing in s. They are decreasing in n and γ when $n > 2$.*

Propositions 3 and 4 demonstrate expected profits are increasing in market size uncertainty s. Holding constant the expected market size a, our analysis suggests firms participating in more volatile markets should achieve higher profitability. Though many firms view uncertainty negatively, our analysis shows it serves to increase profitability irrespective of the whether firms compete in Cournot or Bertrand competition.

Expected firm profits are decreasing in both n and γ, indicating that a high degree of competition and high substitutability detrimentally affect profits. As demonstrated previously, firms participating in highly competitive environments receive less revenue from selling partnership units and also are not able to sell as many partnership units as firms that participate in less competitive markets.

Corollary 1 examines the sensitivity of expected profits to market size uncertainty when firms compete in Cournot and Bertrand competition.

Corollary 1. *Expected profits are more sensitive to market size uncertainty in Cournot competition than in Bertrand competition when $n > 1$.*

Firms competing in quantities (brand name firms) see expected profits increase more greatly with market size uncertainty than firms competing in price (non-brand name firms). The intensity of competition in Bertrand environments[3] mutes the effect of market size uncertainty as compared to Cournot competition.

Expected profits in Cournot competition are higher than in Bertrand competition. Subtracting (14) from (15) yields

$$E(\Pi_C^* - \Pi_B^*) = \frac{a^2(1+s^2)\gamma^3(n-1)^2[\gamma(n-2)+2]}{[\gamma(n-1)+1][\gamma(n-3)+2]^2[\gamma(n-1)+2]^2}. \tag{16}$$

This quantity is positive when $\gamma > 0$ and $n > 1$. However, it is unclear analytically how $E(\Pi_C^* - \Pi_B^*)$ varies with respect to γ and n. We examine this relationship numerically in Figure 3.3.

The expected difference in equilibrium profits is increasing in γ until a threshold is reached. It is then decreasing in γ. Figure 3.3 similarly demonstrates the expected equilibrium profit difference is decreasing in n after a threshold is reached. In both instances, large values of n decrease the expected difference in equilibrium profits: as $n \to \infty$, profits in both Cournot and Bertrand competition tend to zero.

Turning now to examine the welfare implications of our analysis, we find the consumer surplus when funds compete in Cournot competition is

$$\begin{aligned} CS_C &= \frac{n}{2} \cdot \frac{\theta a}{\gamma(n-1)+2}\left\{\theta a - \left[\theta a - \frac{\theta a}{\gamma(n-1)+2} - \gamma(n-1)\frac{\theta a}{\gamma(n-1)+2}\right]\right\} && (17) \\ &= \frac{n(\theta a)^2[\gamma(n-1)+1]}{2[\gamma(n-1)+2]^2}. && (18) \end{aligned}$$

Similar calculations yield the consumer surplus when funds compete in Bertrand competition:

$$CS_B = \frac{n(\theta a)^2[\gamma(n-2)+1]^2}{2[\gamma(n-1)+1][\gamma(n-3)+2]^2}. \tag{19}$$

[3]Some authors, including Singh and Vives (1984) and Darrough (1993), argue Bertrand competition is more intense than Cournot competition.

The expected consumer surplus for both Cournot and Bertrand competition are presented below.

$$E(CS_C) = \frac{na^2(1+s^2)[\gamma(n-1)+1]}{2[\gamma(n-1)+2]^2}. \tag{20}$$

$$E(CS_B) = \frac{na^2(1+s^2)[\gamma(n-2)+1]^2}{2[\gamma(n-1)+1][\gamma(n-3)+2]^2}. \tag{21}$$

The relationship of the expected consumer surplus in Cournot and Bertrand competition to underlying variables is presented in Propositions 5 and 6.

Proposition 5. *The expected consumer surplus in Cournot competition is increasing in s and n. It is also decreasing in γ when $n > 1$.*

Proposition 6. *The expected consumer surplus in Bertrand competition is increasing in s. It is also increasing in n when $n > 2$ and decreasing in γ when $n > 5$.*

Propositions 5 and 6 illustrate that the expected consumer surplus varies in a similar manner as expected firm profitability for a number of variables. Market size uncertainty increases the expected consumer surplus. Implications of this result generally run counter to current industry norms. Investors and PE firms often strive to establish and maintain long-term relationships. These relationships serve to decrease uncertainty as firms have more stable access to capital when such relationships are established. Our results suggest, holding constant gross returns, long-term relationships detrimentally affect investors by decreasing the expected consumer surplus.

Both expected profits and the expected consumer surplus are decreasing in substitutability γ. While increased substitutability benefits investors by decreasing prices, increased levels of substitutability cause firms to decrease the amount of partnership units for sale, and overall, decreases the consumer surplus. However, while firm profitability is decreasing in

n, the consumer surplus is increasing in n. Competitive environments benefit investors and harm firm profitability.

Corollary 2 examines the sensitivity of the expected consumer surplus to market size uncertainty in Cournot and Bertrand competition.

Corollary 2. *The expected consumer surplus is more sensitive to market size uncertainty in Bertrand competition than in Cournot competition when $n > 1$ and $\gamma > 0$.*

Corollary 2 demonstrates that, contrary to expected profits, the expected consumer surplus is more sensitive to market size uncertainty when firms compete in Bertrand competition than in Cournot competition. This result suggests investors receive the most benefit from market size uncertainty when they invests with non-brand name firms. While investors generally flock to brand-name funds in uncertain times, our analysis demonstrates there are significant benefits to investing with non-brand name firms.

The expected consumer surplus in Cournot competition is lower than in Bertrand competition when $n > 2$. The expected difference in the consumer surplus between Cournot and Bertrand competition, $E(CS_C - CS_B)$, is

$$-\frac{n(n-1)\gamma^2[a^2(1+s^2)\gamma^2(2n^2-7n+5)+2\gamma(3n-5)+4]}{2[\gamma(n-1)+1][\gamma(n-1)+2]^2[\gamma(n-3)+2]^2}. \tag{22}$$

This quantity is positive for $n > 2$.

Corollary 3 summarizes the relationship between the expected difference in consumer surplus between Cournot and Bertrand competition and underlying variables.

Corollary 3. *The expected difference between the consumer surplus in Cournot and Bertrand competition scenarios, $E(CS_C - CS_B)$, is decreasing in s and increasing in n when $n > 2$.*

Corollary 3 demonstrates that $E(CS_C - CS_B)$ decreases as market size uncertainty grows. An increase in n also increases this quantity.

We cannot analytically verify how $E(CS_C - CS_B)$ varies with γ. However, Figure 3.4 demonstrates how $E(CS_C - CS_B)$ varies with γ through numerical analysis, and generally demonstrates that this quantity is increasing for small values of γ and decreasing for large values of γ.

4.2 First-stage model

In our second-stage model, we analyzed oligopolistic market outcomes in both Cournot and Bertrand competition pertaining to PE fundraising environments. The number of active firms was fixed exogenously. We now endogenize this parameter in our first stage model to better understand the cyclicality with which PE firms enter and exit the fundraising market. (See Figure 3.2 for additional information.)

We presuppose a finite, large number of PE firms may enter the competitive market and raise funds. In order to enter the market, a firm must pay setup cost $K > 0$. Once this cost has been paid, firms compete in Cournot or Bertrand competition according to the models formulated in the previous section.

In any subgame perfect Nash equilibrium (SPNE) of our first stage game, no firm would want to deviate from its decision to enter the competitive fundraising market. If we assume that a firm will enter the market if it is beneficial to do so or indifferent, there exists an equilibrium with n^* firms in the market if and only if

$$E(\Pi_{n^*}) \geq K \tag{23}$$

and

$$E(\Pi_{n^*+1}) < K, \tag{24}$$

where $E(\Pi_i)$ denotes the expected profit of a firm entering the fundraising market with i competitive participants. Equations (23) and (24) apply in both Cournot and Bertrand competition. The condition specified in (23) states that a firm that enters the market and anticipates competition with n^* firms expects to be weakly better off than if it opted not to do so. The condition specified in (24) states that a firm that forgoes entering the market given anticipated competition with $n^* + 1$ firms must we weakly better off than if it had opted to compete in the market.

Given that $E(\Pi)$ is decreasing in n and that $E(\Pi_n) \to 0$ as $n \to \infty$, there is a unique integer $\tilde{n}$ such that $E(\Pi_{\tilde{n}}) \geq K$ for all $n \leq \tilde{n}$ and $E(\Pi_n) < K$ for all $n > \tilde{n}$. As such, $n^* = \tilde{n}$ is the unique equilibrium number of PE firms participating in the market.

4.2.1 Results

We first examine the expected equilibrium number of firms competing in the fundraising market given that firms compete in Cournot competition. As in our previous section, we assume firms are risk neutral. When the second-stage game corresponds to our Cournot competition model, we are able to analytically solve for the equilibrium number of firms n^*. This result is summarized in Proposition 7.

Proposition 7. *The equilibrium number of firms in Cournot competition is*

$$n^* = \frac{1}{\gamma}\left(\sqrt{\frac{a^2(1+s^2)}{K}} - 2\right) + 1. \tag{25}$$

Proposition 8 summarizes the manner in which the equilibrium number of firms in Cournot competition varies with selected variables in (25).

Proposition 8. *The equilibrium number of firms n^* in Cournot competition is increasing in s, and is decreasing in K and γ.*

Proposition 8 illustrates that market size uncertainty helps incentivize firms to enter a fundraising environment. As such, the pursuit of long-term relationships with PE firms, a general goal for investors, may ultimately decreases marketplace competition.

Setup costs decrease the amount of competition in the marketplace. Investors placing increased burdens on fund managers attempting to raise funds (e.g. by making lofty reputational or performance demands), are behaving in a manner that will ultimately lead to decreased competition. Additionally, substitutability decreases fundraising competition.

We cannot analytically solve for the equilibrium number of firms in Bertrand competition in the general case. We can however, analytically solve for the equilibrium number of firms when $\gamma = 1$. As $\gamma \to 1$, $E(\Pi_B) \to 0$ when $n \geq 2$. When $n = 1$, $E(\Pi_B) \to E(\Pi_B^m) > 0$, the monopoly profit level. Assuming $E(\Pi_B^m) > K$, the SPNE when firms compete in Bertrand competition must have $n^* = 1$. Comparing this result with (25) demonstrates n^* in Bertrand competition is always less than n^* in Cournot competition when $\gamma = 1$.

When $\gamma \in (0, 1]$, we must investigate the equilibrium number of firms in Bertrand competition, and the relationship of this quantity to the variables in Proposition 8 numerically. Figure 3.5 presents graphs illustrating the relationship between n^* and the variables specified in Proposition 8 for both Cournot and Bertrand competition. As shown in the figures, similar relationships apply in both forms of competition. However, our numerical analysis reveals the equilibrium number of firms is higher in Cournot competition than Bertrand competition. While Bertrand competition is generally viewed as a more intense form of competition by economists, our results indicate Bertrand competition actually lowers the number of firms participating in the fundraising environment.

4.2.2 An endogenized number of firms

It remains unclear how ultimate, decreased levels of competition affect the preferences of investors toward Cournot or Bertrand competition. To answer this question, we return to our second-stage model and numerically analyze equilibrium prices, quantities, and the expected consumer surplus with the number of firms endogenized for varying levels of setup costs. Figure 3.6 presents a summary of this analysis.

As shown in the figure, equilibrium prices of partnership units are higher in Cournot competition than Bertrand competition, and equilibrium quantities of partnership units are lower in Cournot competition than Bertrand competition when n is endogenized. Though less firms participate in the fundraising environment when firms compete in price, each price-competing firm releases a level of partnership units that is high enough to ensure the total quantity of partnership units remains above Cournot competition levels. When combined, these factors ensure the consumer surplus in Bertrand competition exceeds that in Cournot competition in spite of decreased competition.

The degree to which the expected consumer surplus in Bertrand competition exceeds the expected consumer surplus in Cournot competition is dependent on setup costs. As setup costs increase, so too does the difference between the expected consumer surplus in Cournot and Bertrand competition. Setup costs also serve to decrease the expected consumer surplus, as well as decrease the equilibrium number of firms, increase prices, and increase the quantity of partnership units release by each firm in both forms of competition. However, our analysis indicates the expected consumer surplus is less sensitive to setup costs in Bertrand competition (non-brand name firms) than in Cournot competition (brand name firms).

This analysis also reveals a key result: by lowering setup costs, investors can achieve higher expected consumer surplus values when firms compete in both Cournot and Bertrand competition. Our result suggests the very existence of brand name firms, in which a firm must incur significant setup costs to achieve this status, is detrimental to investors.

Our analysis also reveals the difference between the equilibrium number of firms, prices, quantities, and expected consumer surplus in Cournot and Bertrand competition is decreasing in γ. Intuitively, as $\gamma \to 0$, these items will take on the same value, irrespective of the prevailing form of competition.

5 Conclusions

We assert and present evidence demonstrating brand name PE firms compete in quantities while non-brand name firms compete in price. Firms prefer Cournot competition while investors prefer firm-level Bertrand competition. By competing in quantities, firms are able to better control the market for their partnership units, charge higher prices, and attain greater profits. When firms compete in prices, investors achieve higher consumer surplus levels, prices are lower, and quantities greater than in Cournot competition. While Bertrand competition effectively increases barriers to entry for PE firms and decreases the ultimate level of competition in the marketplace, it remains the favorable form of competition for investors.

Market size uncertainty is beneficial for both PE firms and investors: both expected firm profits and the expected consumer surplus are increasing in market size uncertainty. Market size uncertainty also incentivizes a greater number of firms to enter the PE fundraising market. While market size uncertainty is generally viewed as a negative attribute when present in a fundraising environment, it is important that firms and investors not confuse the expected market size with market size uncertainty. In difficult fundraising environments, the expected market size shrinks markedly in comparison to ebullient fundraising environments. However, our analysis suggests uncertainty helps temper uncertain fundraising environments.

It is currently the norm for investors and firms to form long-term relationships; our analysis indicates this is not always beneficial for investors or firms. Long-standing relationships deter new market entrants by decreasing expected profits: firms that might otherwise enter the market choose to hold off on fundraising for fear that they will not receive their portion of commitments. Our analysis indicates investors concede more financial and non-financial terms to firms by forming long-term relationships.

Setup costs decrease levels of competition in the market place. They also increase prices, decrease quantities, and decrease the expected consumer surplus. The expected consumer surplus is especially sensitive to setup costs when firms compete in Cournot competition (brand name funds), or in Bertrand competition (non-brand name funds) when the substitutability of funds is low. Setup costs are often highest in difficult fundraising environments when investors generally exhibit a “flight to quality” (i.e. a flight to established, brand name firms) among their investment selections.[4] Our analysis suggests investors would benefit from lowering setup costs in such scenarios to entice new entrants into the marketplace.

[4]This flight to quality is illustrated in Figure 3.2 by the marked decrease in the number of new funds participating in fundraising environments during periods of below-average fundraising.

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Figures

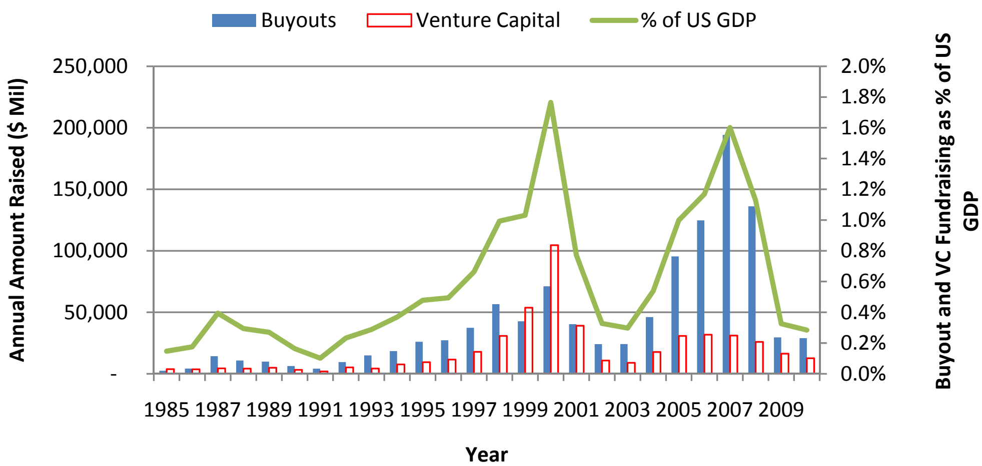


Figure 3.1: Historical buyout and VC fundraising levels and total buyout and VC fundraising as a percent of US GDP, 1985-2010. Data from Thomson ONE Banker and US Department of Commerce.

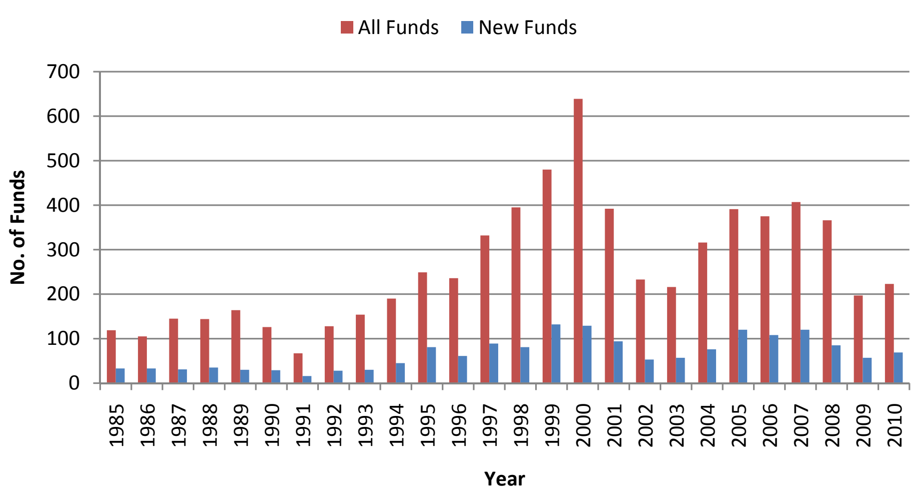
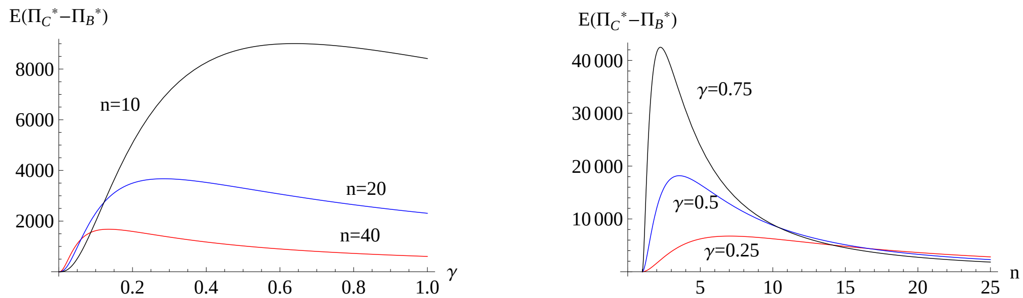
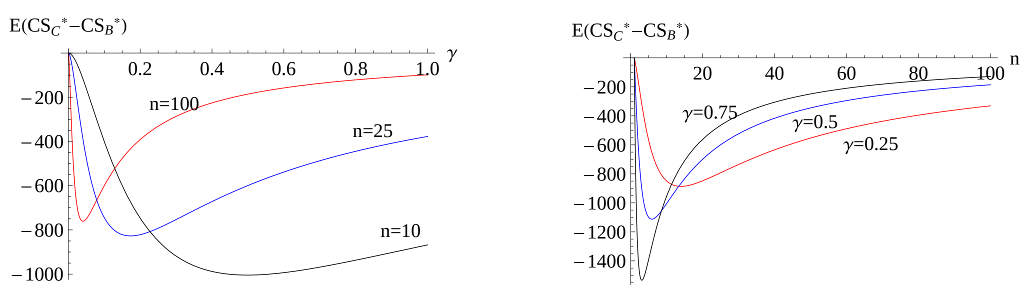


Figure 3.2: Total number of funds and number of new funds seeking fundraising, 1985-2010. Data from Thomson ONE Banker.



(a) (b)

Figure 3.3: Expected difference in profits versus substitutability for varying number of firms and expected difference in profits versus number of firms for varying substitutability. Where not stated otherwise $a = 90, s = 10$, and $\gamma = 0.5$.



(a) (b)

Figure 3.4: Expected difference in consumer surplus between Cournot and Bertrand models versus substitutability for varying number of firms and expected difference in consumer surplus versus number of firms for varying levels of substitutability. Where not stated otherwise $a = 90, s = 10, \gamma = 1$, and $n = 25$.

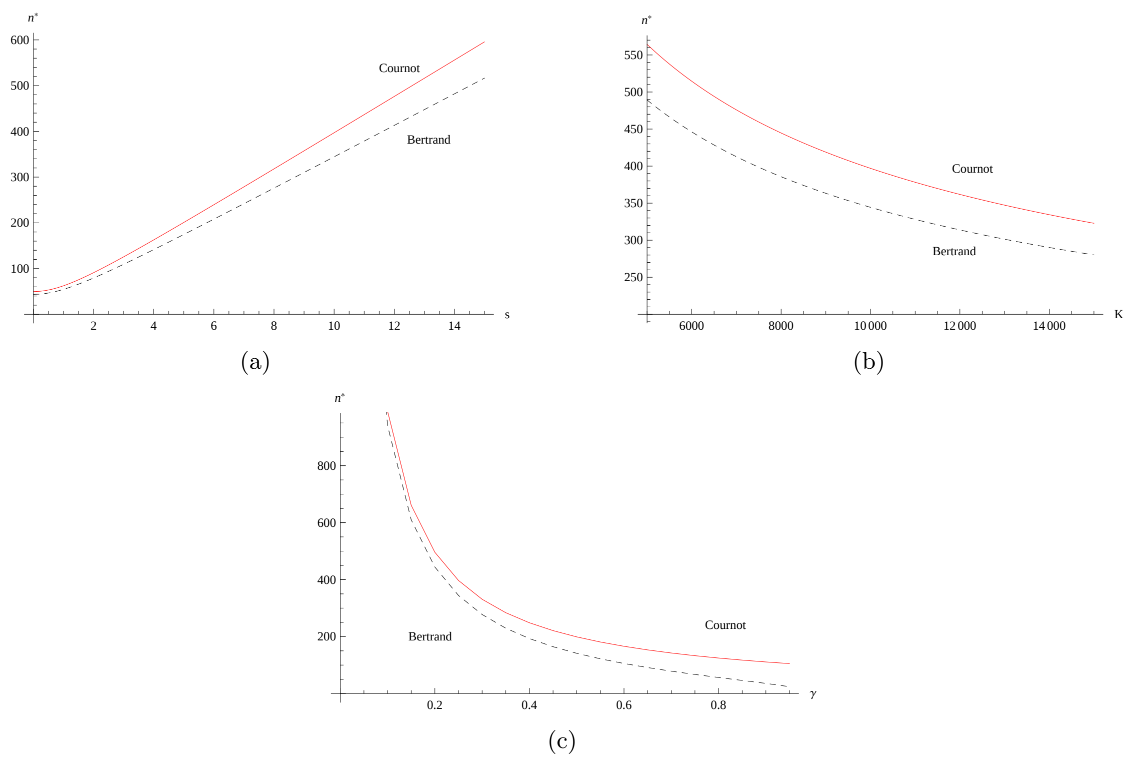


Figure 3.5: Comparison of equilibrium number of firms in Cournot and Bertrand competition. Where not stated otherwise $a = 900, s = 10, \gamma = 0.5$, and $K = 10,000$.

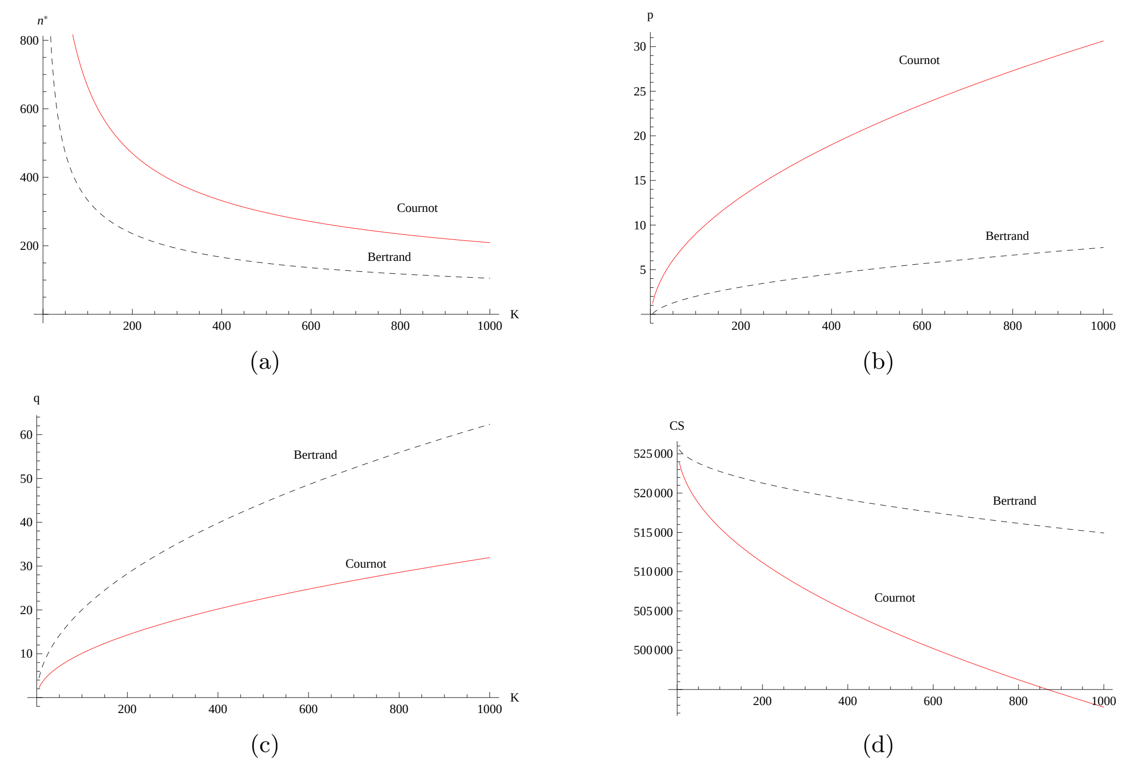


Figure 3.6: Comparison of equilibrium number of firms, prices, quantities, and consumer surplus in Cournot and Bertrand competition versus setup costs. Where not stated otherwise $a = 900, s = 10$, and $\gamma = 0.75$.

Tables

Table 3.1: Historical fundraising information for PE funds in excess of $10 billion, 2005-2010. Data from Thomson ONE Banker.

Year	No. of Funds	Actual Size ($ Mil)	Target Size ($ Mil)	Variance	Avg. Variance
2005	3	41,825	40,000	4.6%	23.5%
2006	5	75,500	73,000	3.4%	10.8%
2007	8	118,512	117,486	0.9%	8.4%
2008	8	116,812	117,886	-0.9%	-9.7%
2009	2	29,600	30,800	-3.9%	-15.8%
2010	1	14,700	15,000	-2.0%	-20.3%

Table 3.2: Historical fundraising information for PE funds under $100 million, 2005-2010. Data from Thomson ONE Banker.

Year	No. of Funds	Actual Size ($ Mil)	Target Size ($ Mil)	Variance	Avg. Variance
2005	138	5,069	7,420	-31.7%	23.5%
2006	126	5,248	6,979	-24.8%	10.8%
2007	143	5,804	10,808	-46.3%	8.4%
2008	133	5,271	9,442	-44.2%	-9.7%
2009	95	3,699	7,577	-51.2%	-15.8%
2010	120	5,020	12,775	-60.7%	-20.3%

Appendices

Appendix 3.1: Proofs of Propositions, Corollaries, and Lemmas

Proof of Proposition 1. Equation (3) is clearly decreasing n. To demonstrate equation (3) is decreasing in γ when $n > 1$, we examine

$$\frac{\partial q_C^*}{\partial \gamma} = \frac{\partial p_C^*}{\partial \gamma} = -\frac{a(n-1)}{[\gamma(n-1)+2]^2}. \tag{26}$$

The sign of this expression is negative $\forall n > 1$.

Proof of Proposition 2. With respect to quantities in Bertrand competition, equation (8) is decreasing in n when $n > 2$:

$$\frac{\partial q_B^*}{\partial n} = -\frac{a\gamma[\gamma^2(n^2-4n+5)+\gamma(2n-5)+1]}{[\gamma(n-1)+1][\gamma(n-3)+2]} < 0 \quad \forall n > 2 \tag{27}$$

as $n^2 - 4n + 5 > 0$ and $\gamma(2n-5)+1 > 0$ for all $\{n \in \mathbb{Z} | n > 2\}$.

Additionally,

$$\frac{\partial q_B^*}{\partial \gamma} = -\frac{a(n-1)[\gamma^2(n^2-5n+6)+2\gamma(n-3)+1]}{[\gamma(n-1)+1]^2[\gamma(n-3)+2]^2} < 0 \quad \forall n > 2 \tag{28}$$

as $n^2 - 5n + 6 \geq 0$ and $2\gamma(n-3)+1 \geq 0$ for all $\{n \in \mathbb{Z} | n > 2\}$.

Equation (9) is clearly decreasing in n. It is also decreasing in γ for $n > 1$:

$$\frac{\partial p_B^*}{\partial \gamma} = -\frac{a(n-1)}{[\gamma(n-3)+2]^2} < 0 \quad \forall n > 1. \tag{29}$$

Proof of Proposition 3. Proposition 3's first assertion is intuitive. The quantity $E(\Pi_C^*)$

is also clearly decreasing in n and γ.

Proof of Proposition 4. Similar to Proposition 3, Proposition 4's first assertion is intuitive. However, proving $E(\Pi_B^*)$ is decreasing in n and γ over the specified parameter range is not intuitive.

For profits in Bertrand competition,

$$\frac{\partial E(\Pi_B^*)}{\partial n} = -\frac{a^2(1+s^2)(1-\gamma)\gamma[\gamma^2(2n^2-7n+7)+4\gamma(n-2)+2]}{[\gamma(n-1)+1]^2[\gamma(n-3)+2]^3}. \tag{30}$$

As (30) is always negative when $n > 2$, $E(\Pi_B^*)$ is decreasing in n. Checking the sign of $\frac{\partial E(\Pi_B^*)}{\partial \gamma}$ amounts to checking the sign of the following expression in the numerator of the partial derivative:

$$\text{sign}\left(\frac{\partial E(\Pi_B^*)}{\partial \gamma}\right) = \text{sign}[\gamma^2(n^2-5n+6)-2\gamma(n^2-5n+7)-4n+10]. \tag{31}$$

In order for (31) to be negative, we require

$$\gamma^2(n^2-5n+6)-2\gamma(n^2-5n+7) < 4n-10. \tag{32}$$

As $n > 2$, we can bound the inequality's left expression by 0

$$\gamma^2(n^2-5n+6)-2\gamma(n^2-5n+7) < 0, \tag{33}$$

and, since $\gamma > 0$, we can divide the inequality by γ:

$$\gamma(n^2-5n+6)-2(n^2-5n+7) < 0. \tag{34}$$

As γ is at most 1, we have proven $\text{sign}\left(\frac{\partial E(\Pi_B^*)}{\partial \gamma}\right) < 0$ and $E(\Pi_B^*)$ is decreasing in γ when $n > 2$.

Proof of Corollary 1. To prove the corollary, we examine

$$\frac{\partial E(\Pi_C)}{\partial s} - \frac{\partial E(\Pi_B)}{\partial s} = \frac{2s\gamma^3 a^2(\gamma(n-2)+2)(n-1)^2}{(\gamma(n-3)+2)^2(\gamma(n-1)+1)(\gamma(n-1)+2)^2}. \tag{35}$$

This quantity is always positive $\forall n > 1$.

Proof of Proposition 5. The proposition's first assertion is intuitive.

To verify how the expected consumer surplus changes with n and γ in Cournot competition, we examine the following expressions:

$$\frac{\partial E(CS_C)}{\partial n} = \frac{a^2(1+s^2)[\gamma(3-\gamma)(n-1)+2]}{2[\gamma(n-1)+2]^3}, \tag{36}$$

and

$$\frac{\partial E(CS_C)}{\partial \gamma} = -\frac{a^2(1+s^2)n\gamma(n-1)^2}{2[\gamma(n-1)+2]^3}. \tag{37}$$

The first expression is always positive; the second expression is always negative when $n > 1$.

Proof of Proposition 6. The proposition's first assertion is intuitive.

To verify how the expected consumer surplus changes with n in Bertrand competition, we check the partial derivative

$$\frac{\partial E(CS_B)}{\partial n} = \frac{a^2(1+s^2)(1-\gamma)[\gamma(n-2)+1]\{2+\gamma^2[n(3n-7)+6]+5\gamma n-7\gamma\}}{2[\gamma(n-3)+2]^3[\gamma(n-1)+1]^2}. \tag{38}$$

This quantity is always positive for $n > 2$. We also verify how the expected consumer surplus changes with γ in Bertrand competition:

$$\frac{\partial E(CS_B)}{\partial \gamma} = -\frac{a^2(1+s^2)\gamma n(n-1)[\gamma(n-2)+1][\gamma(n-2)(n-3)+n-5]}{2[\gamma(n-1)+1]^2[\gamma(n-3)+2]^3}. \tag{39}$$

This quantity is always positive for $n > 5$.

Proof of Corollary 2. To prove the proposition, we examine

$$\frac{\partial E(CS_C)}{\partial s} - \frac{\partial E(CS_B)}{\partial s} = \frac{n\eta s a^2}{\gamma(n-1)+1} \tag{40}$$

where

$$\eta = \frac{(\gamma(n-1)+1)^2}{(\gamma(n-1)+2)^2} - \frac{(\gamma(n-2)+1)^2}{(\gamma(n-3)+2)^2}. \tag{41}$$

Checking the sign of $\frac{\partial E(CS_C)}{\partial s} - \frac{\partial E(CS_B)}{\partial s}$ amounts to checking the sign of η. This expression is always negative when $n > 1$ and $\gamma > 0$.

Proof of Proposition 7. The equilibrium number of firms is found by solving for the number of firms n at which $E(\Pi_C^*)$ from (14) equals setup cost K.

Proof of Proposition 8. The proposition is a direct consequence of a comparative statics analysis on (25).

Chapter 4

Uncertainty and Competition for Investors' Capital in Private Equity Fundraising

1 Introduction

Private equity (PE)[1] is a central component of modern finance and of numerous investors' portfolios. Commitments to US PE funds have grown from roughly \$2 billion in 1980 to nearly \$42 billion in 2010. In 2007, over \$225 billion of capital was placed with US PE funds. (See Figure 4.1 for more information.) Additionally, as Phalippou and Gottschalg (2011) have mentioned, the economic impact from PE funds is much greater than these figures would suggest, given that many PE investments are highly levered.

The maximization of net returns, or returns after all fees have been paid to PE firms for managing money, is a central goal in PE investing. Investors in PE funds expect to achieve net returns that outperform the market, and numerous empirical studies, including Kaplan and Schoar (2005), have verified that has indeed been the case. However, fees, as noted by Ljungqvist and Richardson (2003), can significantly debase net returns. Ljungqvist and Richardson (2003) believe this is one reason why their PE return calculations are much lower than those of empirical studies such as Cochrane (2005) that examine gross returns.

This paper examines how uncertainty and competition affect both PE investors and firms.

[1]Though other definitions exist, we use private equity as an umbrella term encompassing venture capital and buyout finance.

It examines how these items influence the quantity of limited partnership units released by PE firms to investors, the price of these partnership units, the consumer surplus, and PE firm profits. This analysis is derived through a theoretical, two-stage model.

In the first stage, PE firms independently select the optimal time to begin the fundraising process in an environment with market size uncertainty. Market size uncertainty is modeled with geometric Brownian motion. In the second stage, PE firms compete in differentiated, n-firm Cournot competition where the quantity of partnership units sold to investors is uncertain. The two-stage model is solved by backward induction. Results from analyses of both stages are compared with empirical data to supplement theory-based conclusions.

We find that uncertainty decreases the quantity of limited partnership units available for sale and increases the price of partnership units. The consumer surplus, consequently, decreases. However, while uncertainty is always detrimental to investors, small amounts of uncertainty may benefit PE firms. Recent empirical fundraising data is examined in light of these implications.

Our model also examines the effect of random shock correlation ("correlation"), and demonstrates correlation negatively affects investors, but small amounts of correlation may positively affect firms. Empirical data pertaining to the implementation of Accounting Standards Codification (ASC) 820 is examined.

Lastly, our model demonstrates PE firms will be most aggressive in fundraising when they perceive a forthcoming drop in investors' appetites for PE investment. As empirical data, including Kaplan and Schoar (2005), demonstrates a positively-correlated relationship between fundraising and returns, it is possible that PE firms are most aggressive in their fundraising efforts when they recognize poor returns are on the horizon.

2 Literature review

This paper uses novel theoretical models to extend literature centered on the effect of competition and uncertainty in the PE fundraising process.

Numerous empirical papers examine PE fundraising. Many of these papers focus on VC finance. Most recently, Kaplan and Stromberg (2008) presented evidence demonstrating a high level of cyclicality in PE fundraising over time. Previous work by Kaplan includes Kaplan and Schoar (2005), which analyzed PE fund returns and the relationship between fund returns, capital flows, fund size, and fund survival. Kaplan and Schoar (2005) found better performing funds were likely to raise larger follow-on funds and survive longer than lesser performing peers. We later exploit this finding of Kaplan and Schoar (2005) to arrive at numerous conclusions.

Other empirical papers in the fundraising literature include analyses of VC partnership agreements (Gompers and Lerner (1999a)), macroeconomic factors of VC fundraising (Gompers and Lerner (1999b)), deal and monitoring fees charged to investors (Metrick and Yasuda (2007)), and the relationship between venture capital and public markets (Black and Gilson (1998)). This paper extends the abovementioned literature by using theoretical models to examine the impact of competition and uncertainty on the fundraising decisions of PE firms.

Theoretical papers centered on competition and uncertainty include Novshek and Sonnenschein (1982), Basar and Ho (1976), and Clarke (1983). These works examine uncertain demand and incomplete information in homogeneous-product Cournot competition. This paper relaxes the homogeneous products assumption. Other papers, including Klemperer and Meyer (1986), Hackner (2000), and Singh and Vives (1984) compare the effects of uncertainty in Cournot and Bertrand models. We do not compare Cournot and Bertrand competition in this paper, but instead utilize a two-stage model to analyze PE fundraising. To our knowledge, none of the abovementioned papers discuss applications to PE finance.

Real options models have been used to explain delays in investment; however, many papers employing real options in this context assume a monopolistic or perfectly competitive setting. McDonald and Siegel (1986), Pindyck (1986), and Dixit (1989) examine monopolistic firms' investment opportunities, while Dixit (1991) examines market entry in a perfectly competitive setting. Other papers, including Paxson and Pinto (2005) and Lambrecht (2005) have employed real options in oligopolistic settings, but typically in a two-firm economy. This paper builds upon the oligopolistic framework of Grenadier (2002) and Baldursson (1998) and uses real options to examine the investment decisions of firms participating in an n-firm environment under Cournot competition.

This paper most closely resembles the work of Wadecki and Brophy (2011), where the authors examine the determinants of PE fundraising in an environment of uncertainty, competition, and barriers to entry. Uncertainty occurs in Wadecki and Brophy (2011) with respect to market size and subsidies and PE firms must make a now-or-never decision with respect to the fundraising process. Wadecki and Brophy (2011) do not explicitly examine uncertainty occurring in the partnership sale process itself. In contrast, this paper directly examines this latter form of uncertainty by employing a modified version of the model in Deo and Corbett (2009). The modified model relaxes Deo and Corbett (2009)'s assumption of random shock independence and introduces product differentiation into their model. This paper also examines the timing of firms' fundraising decisions when the market size is uncertain by allowing market size to follow geometric Brownian motion. In brief, the analysis herein directly complements Wadecki and Brophy (2011) and provides a complimentary look at competition and uncertainty in the PE fundraising process.

3 Effect of quantity uncertainty

We model the second-stage decisions of PE firms competing for investors' capital in an n-firm economy through a game of complete information with uncertainty. Firms sell limited partnership interests, otherwise known as units, in their underlying funds. We assume limited partnerships units are differentiated goods and that firms face Cournot competition in the abovementioned fundraising process.

Firm i targets the sale of $q_i\ \forall i \in \{1, 2, ..., n\}$ partnership units, but only $\beta_i q_i\ \forall i \in \{1, 2, ..., n\}$ are purchased by investors, where β_i is a random variable associated with market uncertainty. A firm's target quantity q_i represents the amount of partnership units it wishes to sell, but there exists randomness in the fundraising process. For example, Kaplan and Schoar (2005) demonstrate a firm's fundraising ability is generally a function of a its previous net returns to investors. A firm may establish target quantity q_i at a time when its historical track record of net returns to investors is beyond reproach. By the time a firm engages in the fundraising process, its historical track record may have changed, favorably or unfavorably, and this change may alter its ability to sell the targeted quantity of fund partnership units. Another factor that contributes to uncertainty in the fundraising process is the loss of a senior partner at a PE firm. Senior partners often have extensive professional networks, affording them high fundraising ability. If a senior partner leaves a PE firm in between the time it sets target quantity q_i and the fundraising process begins, it may affect the firm's fundraising ability Both of the previous examples demonstrate fundraising shocks that are independent of market-level uncertainty and the price of partnership units.

We assume $E(\beta_i) = 1$ and $\mathrm{Var}(\beta_i) = \sigma^2$ and that the distribution of β is not dependent on p or market size. Variable $\beta_i > 0$, and when β_i exceeds (is less than) 1, a firm has sold more (fewer) partnership units than it expected. We omit an explicit marginal cost of production similar to Singh and Vives (1984) or Dixit (1979).

The assumption of independence between β and other terms is motivated by the many forms underlying causes of uncertainty may take. For instance, loss of a "key man" (for instance, a named principal or general partner of a PE firm), through departure, retirement, or other means, at a PE firm will cause uncertainty in the partnership unit sale process independent of other factors. Additionally, returns to PE funds are continually evolving. If returns to PE firms' predecessor funds change significantly between the time firms announce the fundraising process and the time they begin the fundraising process, uncertainty will arise in the partnership unit sale process.

We define the correlation coefficient between random variables ("correlation") β_i and β_j as $\rho_{ij} = \rho \; \forall i \neq j$. This assumption is made for mathematical convenience and to preserve intuition. One might view ρ as the average fundraising shock correlation. We assume $\rho > 0$, an assumption supported by data subsequently presented in this paper.

Prices for firm i are set according to a linear inverse demand function

$$p_i = a - \beta_i q_i - \gamma \sum_{i \neq j} \beta_j q_j, \tag{1}$$

where a is the market size parameter and γ is the substitutability of partnership units. Similar to Wadecki and Brophy (2011), price p denotes the net present value of any management fees, carried interest, and other fees received by a PE firm for its management of investors' capital.

We assume $\gamma \in [0,1]$. This assumption is justified by investors' strict allocation limits of capital to PE funds: an increase of investor commitments to PE firm i necessarily requires a decrease in commitments to PE firm j $(i \neq j)$, all other this equal. The linear nature of (1) allows us to derive closed-form expressions for our results.

Expected profits in the Cournot model are given by

$$E[\Pi_i(q_i, q_j)] = E\left[\left(a - \beta_i q_i - \gamma \sum_{i \neq j} \beta_j q_j\right) \beta_i q_i\right], \tag{2}$$

and firm i solves

$$\Pi_i^* = \max_{q_i \geq 0} E[\Pi_i(q_i, q_j)]. \tag{3}$$

Equilibrium target quantities are found by solving:

$$\left.\frac{\partial \Pi_i^*}{\partial q_i}\right|_{q_i = q_i^*} = a\ E(\beta_i) - 2q_i\ E(\beta_i^2) - \gamma\ E\left[\beta_i \sum_{i \neq j} \beta_j q_j\right] = 0, \tag{4}$$

As $\frac{\partial^2 \Pi_i^*}{\partial q_i^2} < 0$, (4) represents a sufficient condition for a maximum.

Lemma 1 presents preliminary results associated with the abovementioned model. All proofs are contained in the Appendix.

Lemma 1. *The Cournot game has a unique equilibrium in which:*

1. *The target quantity of partnership units for firm i is $q_i^* = \frac{a}{2(1+\sigma^2)+\gamma(n-1)(1+\rho\sigma^2)}$;*
2. *The expected quantity ("quantity") of partnership units released for sale by firm i is $E(q_i^*) = E(\beta_i)\ q_i^* = \frac{a}{2(1+\sigma^2)+\gamma(n-1)(1+\rho\sigma^2)}$;*
3. *The expected profit ("profit") of firm i is $\Pi_i^* = \frac{a^2[1+2\sigma^2+\gamma(n-1)\rho\sigma^2]}{[2(1+\sigma^2)+\gamma(n-1)(1+\rho\sigma^2)]^2}$*

4. *The expected consumer surplus ("consumer surplus") when n firms sell partnership units is* $CS = \frac{1}{2} \cdot \frac{a^2 n[\gamma(n-1)+1]}{[2(1+\sigma^2)+\gamma(n-1)(1+\rho\sigma^2)]^2}$.

When $\sigma = 0$, and $\gamma = 1$, $q_i^* = \frac{a}{n+1}$ and $\Pi_i^* = \frac{a^2}{(n+1)^2}$; both are familiar Cournot results.

The socially optimal (competitive) level of the quantities–the level of quantities where firm achieve no profits–specified in Lemma 1 is calculated in Lemma 2. Lemma 2 also summarizes the dependence of these quantities on uncertainty σ and correlation ρ.

Lemma 2. *In the Cournot game, the following socially-optimal quantities are not dependent on uncertainty* σ *or the correlation between random shocks* ρ *in the partnership unit sale process:*

1. *The target quantity of partnership units for sale* $q_i^o = \frac{a}{\gamma(n-1)+1}$*;*

2. *The quantity of partnership units for sale* $E(q_i^o) = \frac{a}{\gamma(n-1)+1}$*;*

3. *The profit of firm* i, $\Pi_i^o = 0$*; and*

4. *The consumer surplus,* $CS^o = \frac{a^2 n}{2\gamma(n-1)+2}$.

Lemmas 1 and 2 are subsequently employed in analyzing PE firms' fundraising decisions. Results focus on an analysis of quantities and omit an analysis of target quantities as both are identical.

3.1 Effects of uncertainty on investors

We examine the effects of uncertainty, modeled through σ, on the quantity and price of partnership units released by firm i as well as the consumer surplus and firm i's profit.

As firms solve symmetric profit functions, $q_i^* = q_j^*$, $E(q_i^*) = E(q_j^*)$, and $\Pi_i^* = \Pi_j^*$ $\forall i \neq j$. Subscripts are thus suspended when discussing these quantities. Proposition 1 presents results pertaining to quantities, prices, and the consumer surplus.

Proposition 1. *The quantity of partnership units released to the market and the consumer surplus are decreasing in uncertainty. Partnership unit prices are increasing in uncertainty.*

Uncertainty negatively affects investors. When PE firms are unsure of their ability to sell partnership units, they elect to release fewer quantities to the market. With fewer quantities available for purchase, prices for partnership units increase. Hence, in an environment of uncertainty, investors are allowed access to fewer partnership units at higher prices than in an environment of certainty, decreasing the consumer surplus.

Though the consumer surplus is always negatively affected by uncertainty, the surplus' sensitivity to uncertainty is dependent on model parameter values. Figure 4.2 illustrates how the consumer surplus changes with uncertainty for varying n, ρ and γ.

Figure 2(a) demonstrates that the consumer surplus decreases most significantly with uncertainty for high values of n. One explanation is that the consumer surplus itself is increasing in n: when more PE firms participate in the market, competition is intensified, driving prices down and the total amount of partnership units available to investors up. Therefore, it is in the best interest of investors to decrease uncertainty when a large number of firms compete for capital. When a small number of firms compete for capital, the consumer surplus is less sensitive to uncertainty.

Figure 2(b) demonstrates the consumer surplus is more sensitive to uncertainty as ρ increases. When the random shock correlation between firms' fundraising processes increases, firms are more likely to curb the amount of partnership units for sale in uncertain environments. A decrease in partnership units available also translates into increased prices.

Figure 2(c) illustrates that the consumer surplus is highly sensitive to uncertainty for low values of γ, but less sensitive for high values of γ. The consumer surplus may be so sensitive to uncertainty for low values of γ that a scenario in which a high degree of substitutability between partnership units exists may be preferable to one in which a low degree of substitutability exists for $\sigma > 0$. Furthermore, as γ increases, the consumer surplus is less sensitive to changes in γ.

Decreased uncertainty is always beneficial for investors. Our results, however, indicate investors should most strive to decrease uncertainty in environments with–

- a large number of competitive firms;
- a high degree of random shock correlation between firms' fundraising abilities; and
- low levels of substitutability between funds.

Corollary 1 examines how uncertainty affects the relationship between the expected quantity of partnership units, the consumer surplus, and the socially optimal values of these quantities.

Corollary 1. *The spread between the expected quantity and price of partnership units and the socially optimal levels of these quantities $E(q_i^*) - E(q^o)$ and $E(p_i^*) - E(p^o)$, respectively, is increasing in uncertainty. The spread between the expected consumer surplus and the socially optimal consumer surplus $CS - CS^o$ is also increasing in uncertainty.*

Corollary 1 demonstrates uncertainty drives a wedge between the socially optimal and expected quantity and price of partnership units available for sale as well as the socially optimal and expected consumer surpluses. Mitigating uncertainty allows investors to achieve results more closely resembling socially optimal levels. When combined with previous results, Corollary 1 suggests the formation of long-term relationships with PE firms and consistent participation in follow-on funds are beneficial for investors. Both elements provide PE firms with a level of comfort regarding their fundraising efforts. When uncertainty regarding fundraising efforts is reduced, PE firms will release additional partnership units into the market and prices will fall.

3.2 Effects of uncertainty on PE firms

Uncertainty also affects PE firms' expected profits. Notably, while uncertainty is always detrimental to investors, Proposition 2 shows it may benefit PE firms.

Proposition 2. *Profit is decreasing in uncertainty when* $\sigma > \sqrt{\frac{\gamma(n-1)}{\gamma\rho(n-1)+2}} = \underline{\sigma}$. *It is otherwise increasing in uncertainty.*

Proposition 2 demonstrates that the effect of uncertainty on PE firms is dependent on the level of uncertainty and threshold $\underline{\sigma}$. When PE firms sell independent goods ($\gamma = 0$), they each act as a monopolist, releasing a level of partnership units that allows them to achieve optimum profits. In this scenario, the presence of uncertainty always decreases firms' profits as $\underline{\sigma} = 0$. However, when fund partnership units are substitutes ($\gamma > 0$), uncertainty may increase firm profitability. For example, an uncertain fundraising environment may benefit a PE firm while detrimentally affecting its competitors. This realization allows the firm to increase its market share *and* charge a higher price for partnership units than it would in a fundraising environment devoid of uncertainty.

A crucial determinant of the effect of uncertainty on PE firms is threshold $\underline{\sigma}$. Figure 4.3 explores this threshold by presenting graphs that plot the expected profit for firm i, Π_i^*, versus uncertainty σ for varying levels of n, ρ, and γ. Figure 3(a) shows that while Π_i is decreasing in n, $\underline{\sigma}$ (represented by the inflection point in the figure) is increasing in n. An increased number of market participants divides the market and decreases profits. However, as the number of firms in a competitive space grows, there is a greater chance uncertainty may detrimentally affect some market participants while benefitting others when $\rho < 1$. This increases expected profits, and pushes threshold $\underline{\sigma}$ further to the right.

Figure 3(b) demonstrates the region over which uncertainty proves beneficial to firms, $\sigma \in [0, \underline{\sigma})$, increases as the correlation value between random shocks decreases. This is because $\underline{\sigma} \propto \frac{1}{\sqrt{\rho}}$. Intuitively, as $\rho \to 0$, it is more likely that random shocks will have opposite effects on competitive PE firms, allowing firms benefitting from uncertainty to profit off of those who were harmed by it. As $\rho \to 1$, random shocks are more likely to affect firms similarly, decreasing the chance that firms will be able to profit off of less fortunate competitors.

Figure 3(c) illustrates that as the substitutability of partnership units increases, the region over which uncertainty is beneficial to firms and expected profits decrease. As stated earlier, when firm partnership units are independent ($\gamma = 0$), $\underline{\sigma} = 0$ and any amount of uncertainty is detrimental to firm profit. When $\gamma \to 1$, uncertainty provides less benefits to firms and firms compete more directly for investors' capital.

In summary, the previous results suggest the most profitable PE firms–

- have a small amount of uncertainty in their fundraising process;
- participate in environments with few competitors;

- have low fundraising correlation with their peers; and
- are viewed by investors as having a "secret sauce."

The last result is synonymous with a lack of substitutability between fund partnership units.

While we previously stated the formation of long-term PE-investor relationships is beneficial for investors, these relationships may cause firms to achieve suboptimal profit levels if they decrease uncertainty to levels below threshold $\underline{\sigma}$. Our analysis indicates the formation of long-term relationships is most beneficial to PE firms when–

- few PE firm competitors exist;
- fundraising correlation is high; and
- there exists a high degree of substitutability between firms' partnership units.

3.3 Implications of empirical data for fundraising uncertainty

Reliable data on target and actual fund sizes is not available for a lengthy historical period. However, we can observe historical fundraising volatility in venture capital and private equity funds, an appropriate proxy for fundraising process randomness.

Table 4.1 demonstrates fundraising volatility has been most prevalent among firms raising the largest venture capital and buyout funds. Fundraising levels associated with these firms have displayed the greatest standard deviation over the past 15 years. Additionally, Table 4.1 demonstrates fundraising volatility for venture capital funds has exceeded that in the buyout space for all tabulated fund sizes. Holding constant all other factors, this empirical data, coupled with our theoretical results, suggests it would behoove investors to place capital in smaller buyout funds. Lower levels of fundraising volatility will incentivize parent PE firms to release a greater amount of fund partnership units, leading to decreased prices and increased net returns.

Table 4.2 presents historical information regarding the number of firms actively fundraising in a specified year. The tables indicate the highest level of competition exists among small venture capital funds. However, among funds in excess of $500 million in size, competition is more fierce in the buyout arena. In conjunction with our theoretical results, this empirical data regarding competition presents implications counter to those discerned from fundraising volatility data. While data presented in Table 4.1 demonstrated investors should place capital in small buyout funds due to low fundraising volatility, information contained in Table 4.2 suggests investors should place capital in small venture capital funds due to high levels of competition. The results of Tables 4.1 and 4.2 indicate investors may face a tradeoff between fundraising volatility and fund-level competition when placing capital. In an ideal world, our theoretical results demonstrate investors would place capital in highly-competitive environments with low levels of fundraising volatility.

Fundraising correlation and fund substitutability are difficult to empirically quantify and test given the private nature of PE data. Investors, however, may be able to observe manifestations of these items in placing capital given their access to such data. In evaluating fundraising correlation, investors should look to place capital with firms whose fundraising levels are largely insensitive to overall fundraising trends. For example, our results suggest it would have benefitted investors to place capital in funds that maintained historically-high fundraising levels in the midst of the recent economic downturn. This example also suggests fundraising correlation and fund substitutability may be interrelated.

Some brand name PE funds were able to successfully weather the recent economic recession and demonstrated strong fundraising ability in spite of market headwinds. It is possible that these funds were able to achieve notably high fundraising levels because investors believed these funds possessed a "secret sauce"; in brief, investors thought these funds offered non-substitutable partnership units for sale. Ideally, investors would place capital with such funds as they offer increased consumer surplus to investors, so long as fundraising uncertainty is low (see Figure 2(c)).

3.4 Effects of random shock correlation on investors

The following sections examine the effects of random shock correlation. It begins with an analysis of the implications of correlation on investors. These results are summarized in Proposition 3.

Proposition 3. *The expected quantity of partnership units released to the market and the expected consumer surplus are decreasing in ρ when $\gamma > 0$. Expected partnership unit prices are increasing in ρ when $\gamma > 0$.*

Proposition 3 demonstrates increases in ρ detrimentally affect investors when $\gamma > 0$. As ρ increases, firms become more reluctant to release partnership units knowing that random shocks will similarly affect all competitors. Decreased quantities of partnership units lead to increased prices. Though the consumer surplus is always detrimentally affected by correlation, its sensitivity to ρ is dependent on other parameters. Figure 4.5 illustrates how the consumer surplus changes with correlation for varying n, σ, and γ.

As n increases, the consumer surplus becomes more sensitive to ρ (see Figure 4(a)). Consumers reap the benefits of high quantities and low prices when n is large and ρ is small.

When ρ increases, PE firms begin to restrict partnership units in an environment with uncertainty.

Figure 4(b) illustrates the level of uncertainty significantly affects the impact of ρ on the consumer surplus. When no uncertainty exists in the fundraising environment, the consumer surplus is insensitive to ρ. As uncertainty grows in environments with $\rho > 0$, firms begin to restrict partnership unit sales.

Figure 4(c) shows that the consumer surplus' is generally more sensitive to correlation when γ is small. When γ is small, the quantity of goods released to the market decreases rapidly as ρ increases, leading to a decrease in the consumer surplus.

Corollary 2 examines how random shock correlation affects the relationship between the expected quantity and price of partnership units, the consumer surplus, and the socially optimal values of these quantities.

Corollary 2. *The spread between the expected quantity and price of partnership units and socially optimal levels of these quantities, $E(q_i^*) - E(q^o)$ and $E(p_i^*) - E(p_o)$, respectively, is increasing in random shock correlation. The spread between the expected consumer surplus and the socially optimal consumer surplus $E(CS) - E(CS^o)$ is also increasing in random shock correlation.*

Similar to Corollary 1, Corollary 2 shows random shock correlation drives a wedge between the socially optimal and expected quantity and price of partnership units as well as the socially optimal and expected consumer surplus. Investors are better off when random shock correlation is low as quantities, prices, and the consumer surplus approach socially optimal levels.

3.5 Effects of random shock correlation on PE firms

This section examines how random shock correlation affects PE firms. Results are summarized in Proposition 4.

Proposition 4. *Expected profit is decreasing in correlation when* $\rho > \frac{\gamma(1-n)+2\sigma}{\gamma\sigma(1-n)} = \underline{\rho}$ *and* $\gamma > 0$. *It is otherwise increasing in correlation. When* $\gamma = 0$, *random shock correlation does not affect any of the aforementioned quantities.*

Proposition 4 specifies that while investors are always harmed by random shock correlation, firms may benefit from it. More specifically, correlation increases firm profitability when $\rho < \underline{\rho}$ and $\gamma > 0$. This result is counterintuitive. It would appear that firm profits would be maximized when $\rho = 0$ as this would afford firms the chance to profit off of less fortunate competitors. However, firms play simultaneous, complete information games and understand this possibility. They, consequently, release too many partnership units to the market when $\rho < \underline{\rho}$ to maximize profits. When $\rho = \underline{\rho}$, firms anticipate the effects of random shocks to be less favorable and release fewer partnership units, increasing prices and maximizing profits.

The effect of correlation on firm profitability is dependent on various model parameters. Figure 4.5 plots firm profitability versus ρ for varying n, σ, and γ.

Firm profits are most sensitive to correlation when n is small (see Figure 5(a)). Firms are able to reap greater profits when they have fewer competitors. The effects of ρ are thus magnified in such scenarios when a larger profit might be negatively affected by correlation. The quantity $\underline{\rho}$ is increasing in n as $\frac{\partial \underline{\rho}}{\partial n} = \frac{2}{\gamma(n-1)^2} > 0$. Therefore, as n grows, it is less likely that correlation will detrimentally impact firm profits. Our results indicate firms participating in fundraising environments with few competitors and a minor degree of correlation between firms' fundraising abilities would enjoy the highest levels of profitability. However, firms participating in environments with numerous competitors would be less sensitive to the effects of random shock correlation.

Figure 5(b) shows uncertainty is a significant determinant in the effects of ρ on firm profitability. As σ increases, it becomes more likely that $\rho > \underline{\rho}$ as $\frac{\partial \underline{\rho}}{\partial \sigma} = -\frac{1}{\sigma^2}$. Therefore, for large (small) σ, it is likely that ρ negatively (positively) affects firm profits.

Figure 5(c) illustrates a similar pattern with respect to substitutability. The quantity $\underline{\rho}$ is increasing in γ as $\frac{\partial \underline{\rho}}{\partial \gamma} = \frac{2}{\gamma^2(n-1)} > 0$. Therefore, as γ increases (decreases), the interval over which correlation benefits firms, $(0, \underline{\rho})$, increases (decreases).

3.6 Implications of empirical data for random shock correlation

Beginning December 31, 2008, private equity funds were required by the Financial Accounting Standards Board (FASB) to comply with a new rule, Accounting Standards Codification (ASC) 820 (formerly known as FAS 157). Previous rules under Generally Accepted Accounting Principles (GAAP) "required general partners to determine the 'fair value' of their portfolio companies. But [ASC 820]...altered how that figure must be determined, requiring general partners to value companies based on the exit price those companies would fetch at the time of the valuation rather than what it cost to buy the asset." More specifically, prior to ASC 820, general partners "typically could hold their investments at cost until an actual realization, or until the company saw a material decline in its fortunes"[2].

Requiring PE professionals to perform mark-to-market portfolio company valuations based

[2] Kreutzer, Laura. "New Fair Value Rules Add Extra Complexity To Credit Crunch." *Private Equity Analyst.* May 1, 2008.

on public market performance should hypothetically increase the correlation between PE fund returns and those of public markets.

Figure 4.6 presents rolling five-year correlation values for returns to all US private equity, buyout, and venture capital funds with those of the public markets. The latter figure was estimated using capitalization-weighted average of NYSE/NASDAQ/AMEX returns from the Center in Research for Security Prices (CRSP). Immediately prior to the implementation of ASC 820, the correlation between returns to PE funds and public markets increased significantly. Though correlation values have declined from the highest values seen at the end of 2008, they remain above their 1995-2008 historical average values.

Correlations between returns to various forms of VC and buyout funds have also increased subsequent to the implementation of ASC 820. Table 4.3 presents correlation data prior to and after ASC 820 implementation (Tables 4.3(a) and 4.3(b), respectively). Correlations between returns to each form of PE funds increased after ASC 820 implementation.

Table 4.4 further reinforces the findings of Table 4.3 by demonstrating increased correlation between new and follow-on funds after ASC 820 implementation.

The implications of data in Tables 4.3 and 4.4 on fundraising are substantial. Empirical research, including Kaplan and Schoar (2005), has demonstrated a strong link between PE fund returns and the ability of PE firms to raise capital. If fund returns are converging, empirical research suggests that so, too, will the correlation between the fundraising abilities of various PE firms. Our theoretical model has demonstrated increased random shock correlation across PE firms is always detrimental to investors. Increased random shock correlation is also detrimental for PE firms when ρ exceeds threshold $\underline{\rho}$. As Tables 4.3 and 4.4 demonstrate return correlations approaching 1 across PE funds, our theoretical research, combined with empirical data and the results of previous empirical research, suggests the affects of ASC 820 may be detrimental to both investors and PE firms. PE firms, knowing their fundraising abilities will be highly correlated with those of other firms, may curb partnership unit sales. With fewer partnership unit sales available, prices will increase. Thus, while ASC 820 was implemented with favorable intentions, our results suggest ASC 820 may be detrimental for investors and firms in the PE arena.

4 Dynamic offering of PE partnerships

Through this point in our analysis, we have tacitly assumed PE firms have offered partnership units for sale as of a certain date; they were not able to delay the sale of partnership units, or optimally select a time for offering the units for sale.

This section addresses this issue by examining the optimal decisions of PE firms in an oligopolistic environment where the market size parameter a is uncertain. Firms in the n-firm economy separately decide to enter the fundraising process knowing they will face Cournot competition. We revise (1) to include random process X with respect to market size uncertainty:

$$p_i(X, Q) = aX - \beta_i q_i - \gamma \sum_{i \neq j} \beta_j q_j, \tag{5}$$

where $Q \equiv \sum_i q_i$, and $p_i(X, Q) \equiv p(X, Q)\ \forall i$. Random variable X follows geometric Brownian motion

$$dX = \mu X dt + \zeta X dW, \tag{6}$$

where dW is the increment of the Wiener process. We also assume the firm has an opportunity cost of capital r exists and that $\mu < r$ to guarantee a solution. This condition is required for a solution and is employed in the literature (for example, see Dixit and Pindyck (1994)).

Market size uncertainty affects the expected profit a firm will receive from partnership unit sales. Firm i controls quantity q_i and recognizes that its competitors control $Q_{-i} = \sum_{j \neq i} q_j$. For each level of output q_i, firm i possesses a perpetual, American option that allows it to increase its marginal profit flow $q_i + dq$ by paying an exercise price $K \cdot dq$. The choice made by firm i in our model is a specific example of the general case exposited by Grenadier (2002).

Grenadier (2002) extends the work of Baldursson (1998) to a general framework and demonstrates that, for various forms of uncertainty and inverse demand functions, the optimal exercise policy of the abovementioned options follows a trigger policy: options will be exercised when the value of the shock process $X(t)$ exceeds a pre-specified threshold. The general case of Grenadier (2002) applies in oligopolistic settings whenever the inverse demand function is 1) twice continuously differentiable; 2) strictly increasing in random shock process X; and 3) strictly decreasing in Q. One must also ensure $\frac{\partial \Pi_i}{\partial X \partial q_i} > 0$ and $\frac{\partial^2 \Pi_i}{\partial q_i^2} < 0$. All conditions are applicable in our model, and our model is, thus, a special case of Grenadier (2002).

The optimal trigger point at which an option is exercised in a competitive framework can be reduced to a myopic option problem where competitors' actions are ignored. We denote the value of myopic firm i by $M^i(X, q_i, Q_{-i})$, and the value of the myopic firm i's marginal output by $m^i(X, q_i, Q_{-i}) = \frac{\partial M^i}{\partial q_i}(X, q_i, Q_{-i})$. In a symmetric Cournot equilibrium, $q_i = \frac{1}{n}Q$ and $Q_{-i} = \frac{n-1}{n}Q$.

Employing Proposition 3 of Grenadier (2002) demonstrates the value of myopic PE firm i's marginal fundraising process $m(X,Q) \equiv m^i(X, \frac{1}{n}Q, \frac{n-1}{n}Q)$ is determined by solving

$$0 = \frac{1}{2}\zeta^2 m_{XX} + \mu m_X - rm + E[p(X,Q)] - E[p_Q(X,Q)] \tag{7}$$

subject to

$$m[X^*, Q] = K, \tag{8}$$

$$\frac{\partial m}{\partial X}[X^*, Q] = 0. \tag{9}$$

where subscripts denote partial derivatives. Expression $E[p(X,Q)]$ distributes the expected value operator throughout (5) in a symmetric equilibrium and $E[p_Q(X,Q)]$ denotes the expected value of the partial derivative of $p(X,Q)$ with respect to Q:

$$E[p(X,Q)] = aX - \frac{1}{n}[\gamma(n-1)+1]Q, \tag{10}$$

$$E[p_Q(X,Q)] = -\frac{1}{n}[\gamma(n-1)+1]. \tag{11}$$

Under these conditions, the expected equilibrium investment trigger is

$$X^* = \left(\frac{\omega}{\omega-1}\right)\left(\frac{r-\mu}{a}\right)\left[K + Q\left(\frac{n+1}{n}\right)\cdot\frac{\gamma(n-1)+1}{nr}\right], \tag{12}$$

where

$$\omega = \frac{-(\mu - \frac{1}{2}\zeta^2) + \sqrt{(\mu - \frac{1}{2}\zeta^2)^2 + 2r\zeta^2}}{\zeta^2} > 1. \tag{13}$$

Proposition 5 summarizes how the equilibrium investment trigger varies with numerous parameters.

Proposition 5. *Equilibrium investment trigger X^* is decreasing in n, and is increasing in K, Q, ζ, γ, and μ.*

As the number of competitive PE firms increases, expected firm profitability decreases, lowering the investment trigger. Conversely, when little competition exists, the importance of the $\frac{n+1}{n}$ term increases, and firms will delay the fundraising process until a higher investment trigger is met.

Additionally, the equilibrium investment trigger is increasing in the cost of exercising the option, market size drift, market size uncertainty, the quantity of goods released to the market, and partnership unit substitutability. The first two results are intuitive; the third is a standard option pricing result whereby the value of the option increases as volatility increases; the fourth occurs due to increased profits when more goods are sold, fixing other parameters; the fifth result occurs due to increased profitability as substitutability decreases.

4.1 Implications of empirical data on the dynamic offering model

Results from the first-stage, dynamic offering model indicated that, holding constant other variables, PE firms participating in the most volatile environments with little competition would be most likely to delay their fundraising process, as these elements increase investment trigger X^*. High substitutability and low market size drift would also raise investment trigger X^*. Juxtaposing these results with the data presented in Tables 4.1 and 4.2 leads to novel conclusions.

Table 4.1 demonstrates the largest funds, those over \$1 billion in size, exhibited the greatest amount of fundraising volatility from 1995-2010. If one views the exhibited fundraising volatility as a proxy for market size volatility ζ, investment trigger X^* would be highest for these firms. Moreover, Table 4.2 demonstrates the largest funds face the lowest levels of competition. Low levels of competition increase investment trigger X^*. In addition to high volatility and relatively low competition, one might argue that partnership units in funds over \$1 billion in size also have low levels of substitutability: many of these PE firms are viewed by investors as having a "secret sauce" that cannot be replicated. Low levels of substitutability increase investment trigger X^*.

In examining the data of Tables 4.1 and 4.2, one finds these large funds exhibited extreme growth in fundraising immediately prior to subsequent asset crashes. For example, Table 4.1(a) illustrates VC funds in excess of \$1 billion in size experienced a near six-fold increase in fundraising levels between 1999 and 2000 (the year of the dot-com bust), as well as a near 25-fold increase from 1997 to 2000. Table 4.1(b) illustrates buyout funds in excess of \$1 billion experienced a near seven-fold increase from 2004 to 2007 (the beginning of the current economic crisis). If firms participating in highly volatile fundraising environments, with little competition, and low partnership unit substitutability are most likely to delay fundraising, as a high investment trigger X^* would suggest, the previous marked increases in fundraising immediately preceding crises must, in part, be attributable to funds' perceptions that future fundraising environments would be considerably harsher than the environment at the time such vast sums of money were raised. Our model captures this element through market size drift μ. In the vernacular of our model, firms must have perceived a strong decrease in μ immediately prior to economic crises. This decrease in μ eroded the value of investment trigger X^*, greatly incentivizing firms to raise all the money they could before the environment soured. This conclusion has significant implications for investors.

Investors are continually looking ways to anticipate future returns in opaque, illiquid asset classes such as PE. Given the link between fundraising and returns demonstrated in numerous empirical papers such as Kaplan and Schoar (2005), it is plausible that PE firms anticipated poor returns in times subsequent to high fundraising levels, and that these returns would decrease the market's appetite for PE capital. Thus, investors should be most wary of investing in PE funds when fundraising levels among the largest PE funds are especially high compared to historical norms. High fundraising levels at large firms signify an erosion of the investment trigger value; our model demonstrates an erosion of this option value may be caused by PE firms' perceiving a forthcoming shift in the market environment.

5 Conclusions

This paper employs a two-stage, theory-based model to examine the effects of competition and uncertainty on PE investors and firms. While uncertainty always negatively affects investors, we demonstrate that small amounts of uncertainty may be beneficial for PE firms. This is especially true in environments with a low number of competitors, low correlation, and high product differentiation. When coupled with empirical data, we find investors are most likely better off when they invest in small venture capital or buyout funds rather than large funds.

Similar to uncertainty, our model indicates correlation negatively affects investors, but small amounts of correlation may positively affect PE firms. Through an analysis of empirical data, coupled with our theoretical results, we demonstrate the implementation of ASC 820 may negatively affect both investors and PE firms, in spite of the regulation's laudable intentions.

Our model also demonstrates PE firms, especially those participating in an environment with little competition and low substitutability of partnership units, will be most aggressive in the fundraising process when they perceive a forthcoming drop in investors' appetites for

PE investment. Given the established empircial link between fundraising and returns, it is possible that PE firms will be most aggressive in pursuing fundraising when they foresee a drop in returns. Though complete transparency does not exist in the PE environment, our results imply investors can potentially gauge PE firms' market perceptions by examining their fundraising behavior, specifically the fundraising behavior of large firms.

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Figures

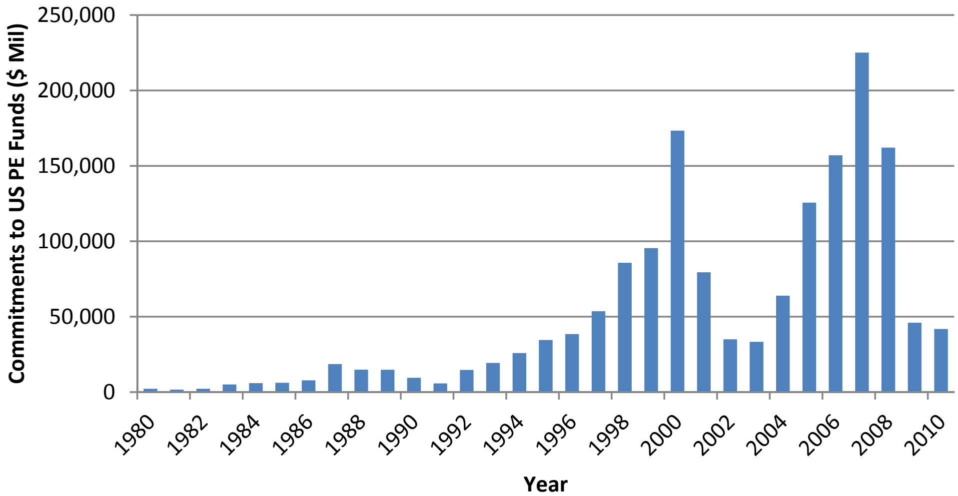


Figure 4.1: Historical commitments to US PE funds, 1980-2010. Data from Thomson ONE Banker.

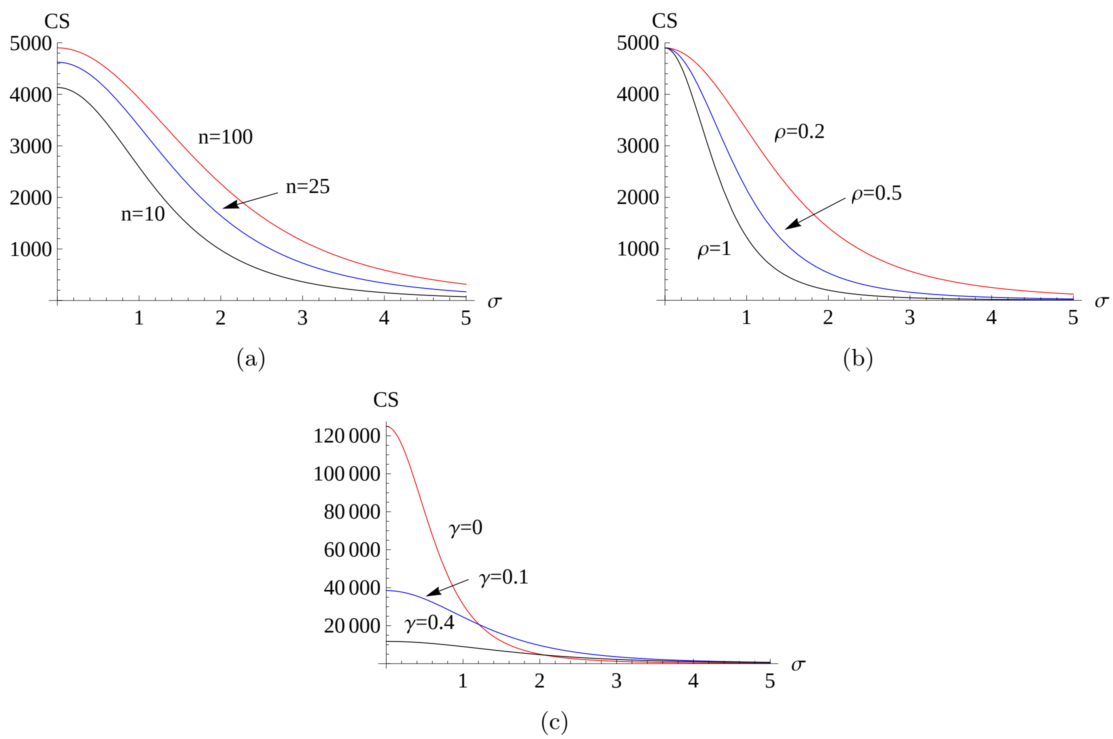


Figure 4.2: Consumer surplus CS versus uncertainty σ for varying n, ρ and γ. The consumer surplus is always decreasing in uncertainty. It is most sensitive to uncertainty when n is large, ρ is small, and γ is small. Where not stated otherwise $a = 100, \gamma = 1, n = 100, \rho = 0.1$.

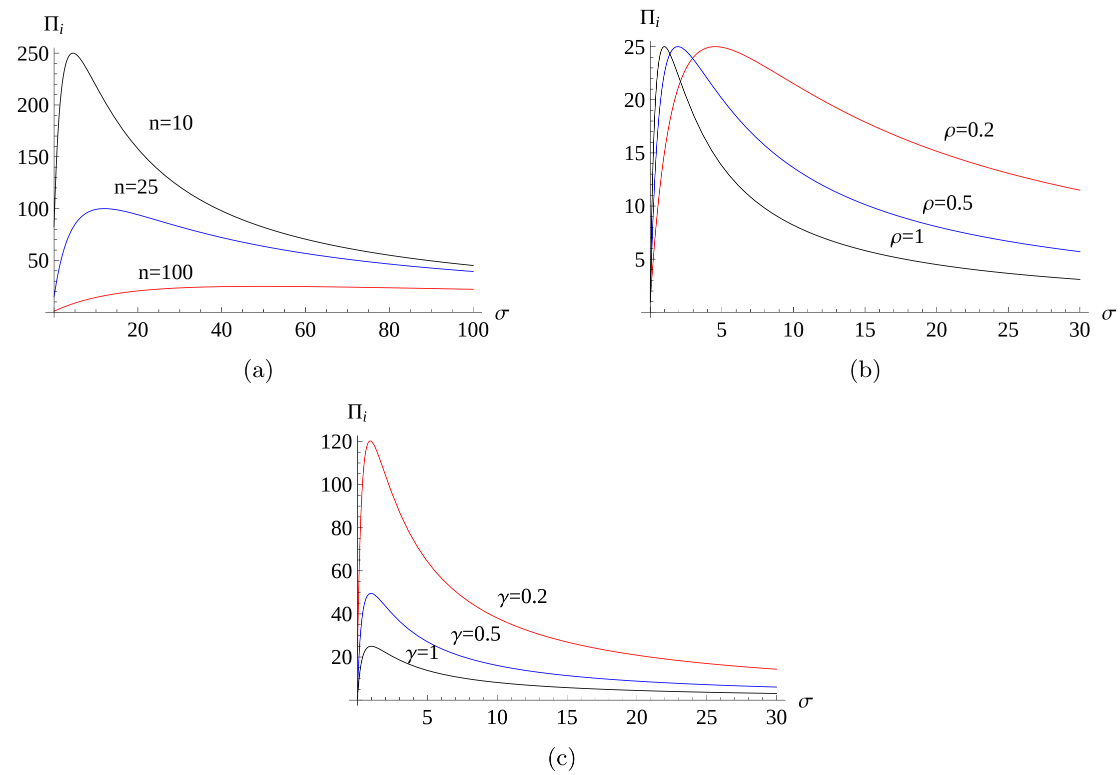
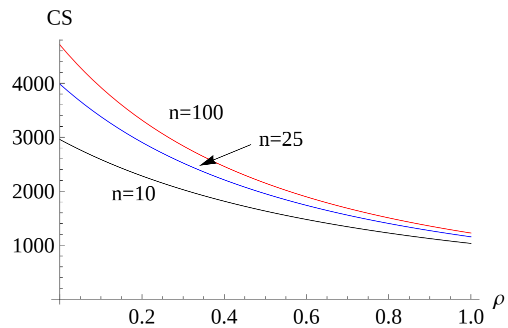
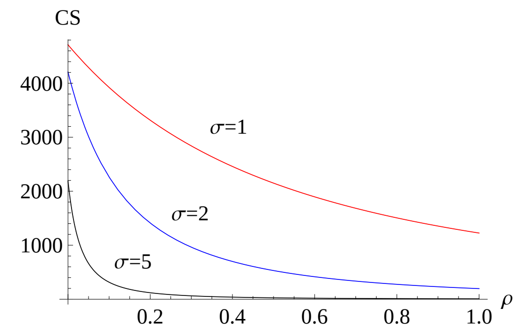


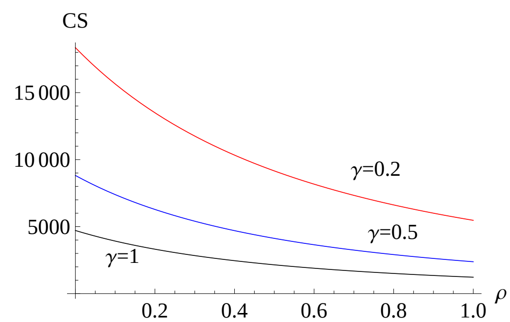
Figure 4.3: Expected profit for firm i, Π_i, versus uncertainty σ for varying n, ρ, and γ. Expected profit increases until threshold $\underline{\sigma}$ is reached. After this threshold is exceeded, expected profit decreases. Where not stated otherwise $a = 100, \gamma = 1, n = 100, \rho = 0.5$.



(a)



(b)



(c)

Figure 4.4: Expected consumer surplus versus correlation ρ for varying n, σ, and γ. Correlation negatively affects the consumer surplus. Where not stated otherwise $a = 100, \gamma = 1, n = 100, \sigma = 5$.

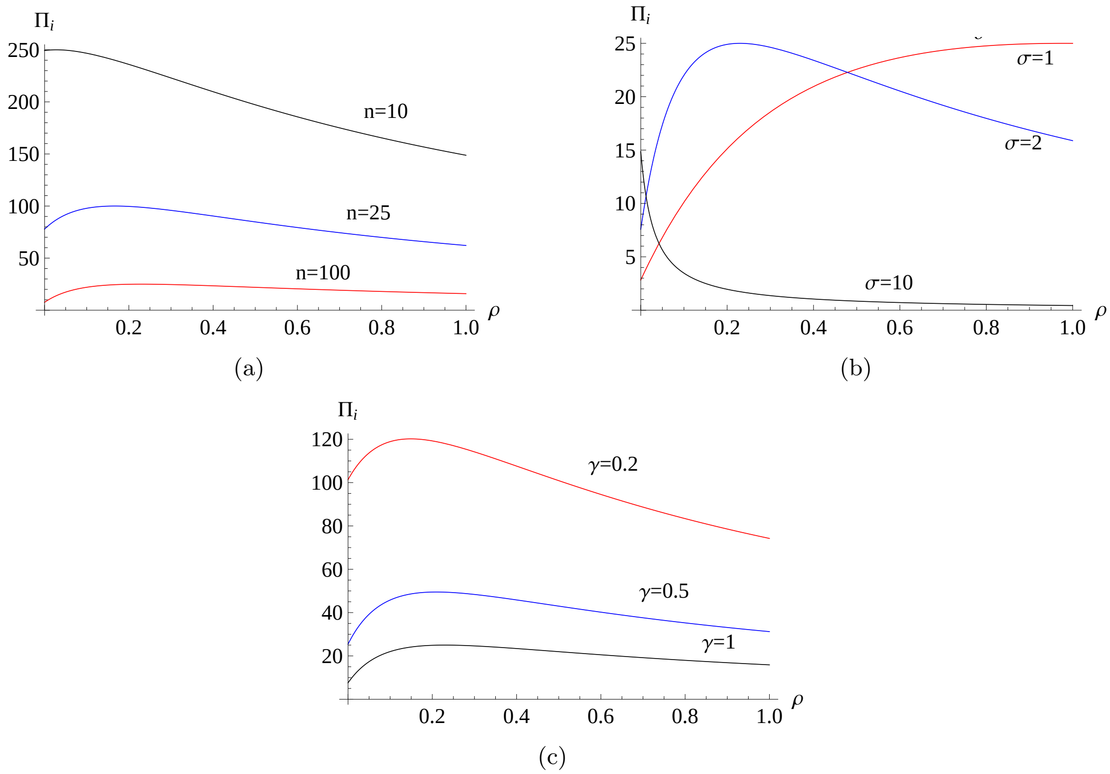


Figure 4.5: Expected profit for firm i, Π_i, versus random shock correlation ρ for varying n, σ, and γ. Expected profit increases until threshold $\underline{\rho}$ is reached. After this threshold is exceeded, expected profit decreases. Where not stated otherwise $a = 100, \gamma = 1, n = 100, \rho = 1, \sigma = 5$.

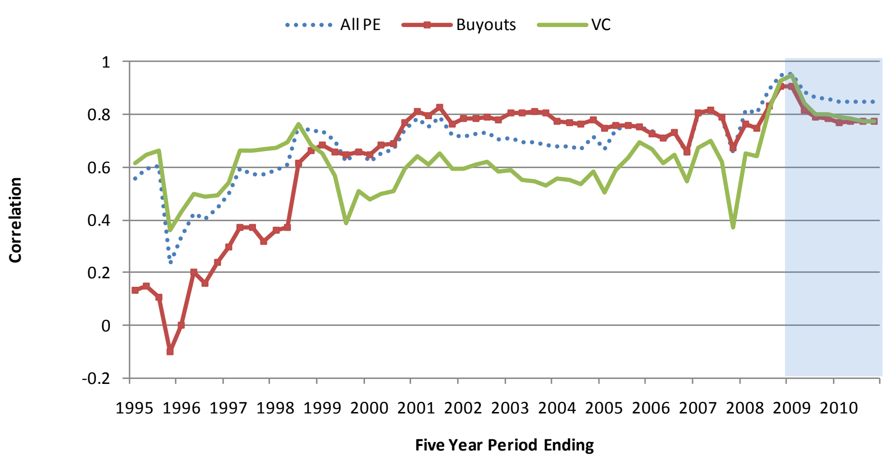


Figure 4.6: Rolling five-year quarterly correlation values for returns to all US private equity, buyout, and venture capital funds, respectively, with public market returns. Capitalization-weighted average of NYSE/NASDAQ/AMEX returns from Center in Research for Security Prices (CRSP) used as a proxy for public market returns. All other data from Thomson ONE Banker. Shaded area denotes period in which ASC 820 is effective.

Tables

Table 4.1: Historical fundraising levels and percent change in fundraising levels for venture capital and buyout funds by size of fund, 1995-2010. All fundraising information in millions of dollars. Negative fundraising figures denote a net return of pre-committed capital from PE firms to their investors. Data from Thomson ONE Banker.

(a)

Year	$1-100M	% Chg.	$100-500M	% Chg.	$500M-1B	% Chg.	$1B+	% Chg.
1995	$3,606		$4,408		$610		$522	
1996	$3,515	-2.5%	$6,513	47.7%	$800	31.1%	$660	26.4%
1997	$5,588	59.0%	$10,040	54.2%	$800	0.0%	$700	6.1%
1998	$5,782	3.5%	$14,657	46.0%	$2,917	264.6%	$6,853	879.0%
1999	$8,472	46.5%	$27,461	87.4%	$14,516	397.6%	$3,286	-52.1%
2000	$13,087	54.5%	$44,825	63.2%	$26,684	83.8%	$18,075	450.1%
2001	$6,341	-51.6%	$14,418	-67.8%	$12,951	-51.5%	$5,349	-70.4%
2002	$2,627	-58.6%	$3,149	-78.2%	-$1,901	-114.7%	$6,625	23.8%
2003	$2,057	-21.7%	$6,466	105.3%	$964	-150.7%	-$405	-106.1%
2004	$2,365	15.0%	$13,129	103.1%	$695	-27.9%	$1,662	-510.0%
2005	$2,525	6.8%	$15,920	21.3%	$6,540	841.0%	$5,700	243.0%
2006	$2,568	1.7%	$13,616	-14.5%	$10,292	57.4%	$5,085	-10.8%
2007	$2,994	16.9%	$17,045	37.1%	$6,406	79.0%	$4,300	55.3%
2008	$2,408	-19.6%	$12,803	-24.9%	$9,768	52.5%	$836	-80.6%
2009	$1,581	-34.4%	$6,454	-49.6%	$3,933	-59.7%	$4,212	403.8%
2010	$2,696	70.5%	$7,281	12.8%	$3,330	-15.3%	$40	-99.1%
St. Dev.	$3,019	38.1%	$10,281	64.0%	$7,308	285.5%	$4,505	238.2%

(b)

Year	$1-100M	% Chg.	$100-500M	% Chg.	$500M-1B	% Chg.	$1B+	% Chg.
1995	$1,282		$6,306		$4,405		$13,315	
1996	$1,264	-1.3%	$6,168	-3.6%	$7,887	79.0%	$11,633	-12.6%
1997	$1,457	15.2%	$10,301	67.0%	$7,996	1.4%	$16,745	43.9%
1998	$1,396	-4.2%	$12,016	16.7%	$13,316	66.5%	$28,783	71.9%
1999	$1,250	-10.5%	$11,231	-6.5%	$6,128	-54.0%	$23,108	-19.7%
2000	$1,457	16.6%	$9,131	-18.7%	$9,073	48.1%	$51,074	121.0%
2001	$971	-33.4%	$5,914	-35.2%	$9,533	5.1%	$23,832	-53.3%
2002	$887	-8.7%	$4,877	-17.5%	$2,412	-74.7%	$16,351	-31.4%
2003	$895	1.0%	$5,174	6.1%	$6,291	160.8%	$11,901	-27.2%
2004	$923	3.0%	$7,714	49.1%	$10,836	72.2%	$26,600	123.5%
2005	$1,128	22.2%	$12,213	58.3%	$10,511	-3.0%	$70,930	166.7%
2006	$939	-16.7%	$11,181	-8.5%	$9,443	-10.2%	$103,712	46.2%
2007	$1,098	16.9%	$15,330	37.1%	$16,903	79.0%	$161,033	55.3%
2008	$1,095	-0.3%	$11,762	-23.3%	$11,513	-31.9%	$111,874	-30.5%
2009	$540	-50.7%	$4,589	-61.0%	$3,112	-73.0%	$21,513	-80.8%
2010	$812	50.6%	$6,155	34.1%	$4,306	38.4%	$17,218	-20.0%
St. Dev.	$256	23.9%	$3,294	36.7%	$3,881	65.8%	$44,549	73.0%

Table 4.2: Historical number of venture capital and buyout funds actively engaged in the fundraising process, 1995-2010. Data from Thomson ONE Banker.

(a)

Year	$1-100M	$100-500M	$500M-1B	$1B+
1995	132	27	1	1
1996	124	40	1	1
1997	174	65	1	1
1998	189	90	5	4
1999	264	144	24	4
2000	378	207	42	12
2001	202	95	21	8
2002	126	56	17	2
2003	99	53	8	2
2004	127	74	7	2
2005	133	85	12	3
2006	117	92	21	2
2007	119	98	15	2
2008	119	75	17	1
2009	88	48	12	4
2010	109	44	7	2
Avg.	156	81	13	3

(b)

$1-100M	$100-500M	$500M-1B	$1B+
35	39	8	12
28	33	13	7
33	49	15	10
33	58	24	18
37	63	13	17
33	51	17	29
26	40	19	19
28	27	7	12
24	32	11	8
32	45	20	18
27	69	23	27
28	70	20	35
42	80	28	45
37	66	23	45
21	39	11	23
34	37	10	16
31	50	16	21

Table 4.3: Correlation between various quarterly returns to VC and buyout funds from 1995-2008 (Table 3(a)) and from 2009-2011 YTD (Table 3(b)). All correlations significant at a 99% level of confidence. Small buyouts are funds between $1M and $100M in size; medium buyouts are funds between $100M and $1B in size. Large buyouts are funds over $1B in size. Data from Thomson ONE Banker.

(a)

	Seed Stage VC	Early Stage VC	Later Stage VC	Small Buyout	Medium Buyout
Early Stage VC	0.55				
Later Stage VC	0.52	0.89			
Small Buyout	0.34	0.52	0.57		
Medium Buyout	0.31	0.59	0.66	0.57	
Large Buyout	0.26	0.36	0.50	0.58	0.52

(b)

	Seed Stage VC	Early Stage VC	Later Stage VC	Small Buyout	Medium Buyout
Early Stage VC	0.59				
Later Stage VC	0.60	0.94			
Small Buyout	0.43	0.74	0.73		
Medium Buyout	0.59	0.91	0.91	0.92	
Large Buyout	0.52	0.98	0.95	0.79	0.92

Table 4.4: Correlation between new and follow-on VC and buyout funds from 1995-2008 (Table 4(a)) and from 2009-2011 YTD (Table 4(b)). All correlations significant at a 99% level of confidence. Data from Thomson ONE Banker.

(a)

	New VC Funds	Follow-on VC Funds	New Buyout Funds
Follow-on VC Funds	0.90		
New Buyout Funds	0.32	0.35	
Follow-on Buyout Funds	0.41	0.52	0.38

(b)

	New VC Funds	Follow-on VC Funds	New Buyout Funds
Follow-on VC Funds	0.93		
New Buyout Funds	0.91	0.97	
Follow-on Buyout Funds	0.90	0.99	0.95

Appendices

Appendix 4.1: Proofs of Propositions, Corollaries, and Lemmas

Proof of Lemma 1. Firm i solves the concave maximization problem specified in (4). As such, the first order condition is necessary and sufficient to solve for equilibrium target quantities.

Denoting q_{-i}^* as the optimal actions of all firms except firm i and using (4) yields

$$a - 2q_i^*(1+\sigma^2) - \gamma(1+\rho\sigma^2)q_{-i}^* = 0, \tag{14}$$

which can be solved for firm i's reaction function

$$q_i^* = \frac{a - \gamma(1+\rho\sigma^2)q_{-i}^*}{2(1+\sigma^2)}. \tag{15}$$

Since firms solve symmetric profit functions, we can sum the reaction functions of all firms

$$\sum_i q_i^* = \sum_i \frac{a - \gamma(1+\rho\sigma^2)q_{-i}^*}{2(1+\sigma^2)} \tag{16}$$

$$= n \cdot \frac{a}{2(1+\sigma^2)} - (n-1) \cdot \frac{\gamma(1+\rho\sigma^2)}{2(1+\sigma^2)} \sum_i q_i^* \tag{17}$$

Solving for $\sum_i q_i^*$ yields

$$\sum_i q_i^* = n \cdot \frac{a}{2(1+\sigma^2) + \gamma(n-1)(1+\rho\sigma^2)}, \tag{18}$$

or, equivalently,

$$q_i^* = \frac{a}{2(1+\sigma^2) + \gamma(n-1)(1+\rho\sigma^2)}, \tag{19}$$

The expected quantity produced by firm i is $E(\beta_i)q_i^*$, where $E(\beta_i) = \mu = 1$:

$$E(q_i^*) = \frac{a}{2(1+\sigma^2) + \gamma(n-1)(1+\rho\sigma^2)}. \tag{20}$$

Expected profit is found by substituting (19) into (2).

The expected utility experienced by investors is $E(\int_0^{nq}(a-x)dx)$, and the expected payment made by investors to each PE firm is $E\{[a - q_i^* - \gamma q_{-i}^*(n-1)]q_i^*\}$. Therefore, the expected consumer surplus when PE funds experience uncertainty is

$$CS = \frac{1}{2} \cdot \frac{a^2 n[\gamma(n-1)+1]}{[2(1+\sigma^2)+\gamma(n-1)(1+\rho\sigma^2)]^2}. \tag{21}$$

Proof of Lemma 2. The socially optimal target quantity is derived by setting expected price equal to marginal cost:

$$E(a - \beta_i q_i^o - \gamma \sum_{i \neq j} \beta_j q_j^o) = 0 \tag{22}$$

$$a - q_i^o - \gamma q_{-i}^o = 0. \tag{23}$$

Solving for $q_i^o =$ yields

$$q_i^o = a - \gamma q_{-i}^o. \tag{24}$$

Summing over all q_i^o, we obtain

$$\sum_i q_i^o = na - \gamma(n-1)\sum_i \tilde{q}_i{}^o, \tag{25}$$

or, equivalently,

$$q_i^o = \frac{a}{\gamma(n-1)+1} \qquad \forall i \tag{26}$$

as firms solve symmetric profit functions. This quantity is independent of σ and ρ. Similar calculations show the expected value of q_i^o is also independent of σ and ρ.

The expected profit when firms release the socially optimal level of partnership units is

$$\Pi_i^o = 0, \tag{27}$$

and the expected consumer surplus when firms release the socially optimal level of partnership units is

$$CS^o = \frac{a^2 n}{2\gamma(n-1)+2}. \tag{28}$$

Both quantities are independent of σ and ρ.

Proof of Proposition 1. As derivatives with respect to σ^2 and $\sigma \geq 0$ have the same sign, we focus on the former due to ease of exposition. With respect to the expected quantity of partnership units released to the market, we have

$$\text{sign}\left(\frac{\partial E(q_i^*)}{\partial \sigma^2}\right) = \text{sign}\left(-\frac{a[2+(n-1)\gamma\rho]}{[2(1+\sigma)+(n-1)\gamma(1+\rho\sigma)]^2}\right) = -. \tag{29}$$

As the sign of (29) is always negative, the expected quantity of partnership units released by firm i is decreasing in uncertainty. Prices are increasing in uncertainty due to inverse demand (1).

Similarly, we examine how the consumer surplus changes with uncertainty:

$$\text{sign}\left(\frac{\partial CS}{\partial \sigma^2}\right) = \text{sign}\left(-\frac{a^2 n(\gamma(n-1)+1)(\gamma\rho(n-1)+2)}{[2(1+\sigma^2)+\gamma(n-1)(1+\rho\sigma^2)]^3}\right) = -. \tag{30}$$

This quantity is always negative.

Proof of Corollary 1. A straightforward comparison of Lemma 1 and 2, along with an application of (1), yields the desired result.

Proof of Proposition 2. For expected profit, we have

$$\text{sign}\left(\frac{\partial \Pi_i^*}{\partial \sigma^2}\right) = \text{sign}\left(-\frac{a^2[\gamma\rho(n-1)+1][2\sigma^2+\gamma(n-1)(\rho\sigma^2-1)]}{[2(1+\sigma^2)+\gamma(n-1)(1+\rho\sigma^2)]^3}\right), \tag{31}$$

which is negative for $\sigma > \underline{\sigma}$ and positive otherwise.

Proof of Proposition 3. The sign of the partial derivative of the expected quantity of partnership units with respect to the random shock correlation ρ is always negative when $\gamma > 0$:

$$\text{sign}\left(\left.\frac{\partial \tilde{q}^*}{\partial \rho}\right|_{\gamma>0}\right) = \text{sign}\left(-\left.\frac{a\gamma(n-1)\sigma^2}{[2(1+\sigma^2)+\gamma(n-1)(1+\rho\sigma^2)]^2}\right|_{\gamma>0}\right) = -. \tag{32}$$

Combining (32) with (1), demonstrates expected partnership unit prices are increasing in ρ when $\gamma > 0$.

The sign of the partial derivative of the expected consumer surplus with respect to ρ is always negative for $\gamma > 0$:

$$\text{sign}\left(\left.\frac{\partial CS}{\partial \rho}\right|_{\gamma>0}\right) = \text{sign}\left(-\left.\frac{a^2n^2\gamma(n-1)\sigma^2}{[2(1+\sigma^2)+\gamma(n-1)(1+\rho\sigma^2)]^2}\right|_{\gamma>0}\right) = -. \tag{33}$$

Proof of Proposition 4. The sign of the partial derivative of firm i's profit with respect to ρ is negative for $\rho > \underline{\rho}$ and $\gamma > 0$:

$$\text{sign}\left(\left.\frac{\partial \Pi_i^*}{\partial \rho}\right|_{\gamma>0,\rho>\underline{\rho}}\right) = \text{sign}\left(-\left.\frac{a^2\gamma(n-1)\sigma[2\sigma+\gamma(n-1)(\rho\sigma-1)}{[2(1+\sigma)+\gamma(n-1)(1+\rho\sigma)]^3}\right|_{\gamma>0,\rho>\underline{\rho}}\right) = -. \tag{34}$$

It is positive when $\gamma > 0$ and $\rho < \underline{\rho}$:

$$\text{sign}\left(\left.\frac{\partial \Pi_i^*}{\partial \rho}\right|_{\gamma>0,\rho<\underline{\rho}}\right) = \text{sign}\left(-\left.\frac{a^2\gamma(n-1)\sigma[2\sigma+\gamma(n-1)(\rho\sigma-1)}{[2(1+\sigma)+\gamma(n-1)(1+\rho\sigma)]^3}\right|_{\gamma>0,\rho<\underline{\rho}}\right) = +. \tag{35}$$

Proof of Corollary 2. A straightforward comparison of Lemma 1 and 2, along with an application of (1), yields the desired result.

Proof of Proposition 5. To prove the proposition, we examine the following quantities (signs of each expression are shown below):

$$\frac{\partial X^*}{\partial n} = -\frac{\omega Q(n+2-2\gamma)(r-\mu)}{n^3 ar(\omega-1)} < 0 \tag{36}$$

$$\frac{\partial X^*}{\partial \gamma} = \frac{Q\omega(r-\mu)(n^2-1)}{an^2 r(\omega-1)} > 0 \tag{37}$$

$$\frac{\partial X^*}{\partial K} = -\frac{\omega(r-\mu)}{a(\omega-1)} < 0 \tag{38}$$

$$\frac{\partial X^*}{\partial Q} = -\frac{\omega(n-1)[\gamma(n-1)+1](r-\mu)}{an^2 r(\omega-1)} < 0 \tag{39}$$

$$\frac{\partial X^*}{\partial \zeta} = \frac{\{Kn^2 r + Q[\gamma(n^2-1)+n+1]\}(r-\mu)}{an^2 r(\omega-1)} \tag{40}$$

$$\cdot \frac{2[2r\zeta^2 - \mu(\zeta^2 - 2\mu + \sqrt{(2\mu-\zeta^2)^2 + 8r\zeta^2})]}{\zeta^3\sqrt{(\zeta^2-2\mu)^2 + 8r\zeta^2}} < 0 \tag{41}$$

$$\frac{\partial X^*}{\partial \mu} = -\frac{\{Kn^2 r + Q[\gamma(n^2-1)+n+1]\}(r-\mu)}{an^2 r(\omega-1)} \cdot \frac{\frac{2\mu-\zeta^2}{\sqrt{(\zeta^2-2\mu)^2+8\zeta^2 r}} - 1}{\zeta^2} > 0 \tag{42}$$

Chapter 5

Conclusion

This dissertation has examined competition and uncertainty in private equity (PE) and supply chains. Each of the three papers included in this dissertation utilized Cournot and/or Bertrand models of competition and multiple forms of uncertainty to analyze the optimal behavior of decision makers, and the implications of their decisions on other parties. Empirical data has also been employed throughout the document to corroborate theory-based findings and to provide additional, practical insights.

A central finding of this dissertation is the positive benefits associated with subsidies. While paying above market-level prices for goods or services is generally taboo in today's economy, this practice may, in fact, prove beneficial to firms providing the subsidies as well as subsidy recipients. Paying above market-level prices increases the stability of goods or service providers. Subsidies, of course, are not a panacea for industry participants, but they should not be regarded with complete disfavor. High levels of instability within many industries might be partially mitigated by subsidies.

Another central finding of this dissertation is that market size uncertainty can benefit those who receive goods or services ("recipients") from a provider ("providers"). If shocks associated with uncertainty are not perfectly correlated, a possibility exists that uncertainty may benefit one firm while harming another. The previously-described scenario allows the firm benefitting from uncertainty to capitalize on its competitors' weaknesses and increase its profitability. The smaller the correlation between uncertainty shocks across firms, the greater the probability the previously-described scenario is realized.

Market size uncertainty also provides a mechanism for investors to better understand general partner behavior. Uncertainty increases the option value associated with waiting to begin the fundraising process, especially in environments with little competition and low substitutability of partnership units. Thus, investors can accurately gauge general partners' perception of the marketplace by examining their fundraising behavior. If general partners believe the market size is expanding, they will be more likely to delay their fundraising process; if they perceive market size erosion, they will be more likely to fundraise immediately.

Overall, the results included in this dissertation suggest that while numerous industry practices, and even regulations, are used in an effort to maximize benefits to recipients and participants, some counterintuitive practices, such as the granting of subsidies, or seemingly-detrimental market conditions, such as high uncertainty, may in fact help recipients and/or participants better gauge the market and achieve optimum levels of consumer surplus and profitability.