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Fundamentals of Ship Design Economics

Lecture Notes

by

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SYMBOLS AND ABBREVIATIONS

Wherever applicable, the abbreviations used in this book are based on the standards of the Engineering Economy Division of the American Society for Engineering Education.

A	uniform annual return (revenue less operating costs) or annual repayments on a loan (returning capital plus interest)
A'	uniform annual return after tax
AAC	average annual cost (operating cost plus annual cost of capital recovery)
ACCR	annual cost of capital recovery (CR×P)
CA	compound amount factor with single payment (F÷P)
CRF or CR	capital recovery factor (A÷P)
CR'	capital recovery factor after tax (A'÷P)
DCF	discounted cash flow rate of return
F	future sum of money
I	interest payment
i	interest rate (before tax) per annum compounded annually*
i'	interest rate after tax*
L	disposal value
N	number of years, generally the economic life, period of a loan, or depreciation period; also any year in the future
NPV	net present value of all cash flows discounted to the present
NPVI	net present value index (NPV÷P)

(cont.)

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*These may be (a) mutually agreed upon rates, as in a loan, (b) minimum acceptable rates, as used in finding NPV, or (c) internally generated returns from a profitable enterprise expressed as an equivalent rate of interest. On an after-tax basis, this is called "yield" or "interest rate of return."

P — principal (amount of a loan), investment, or present worth of future amount(s)

PV — present value of both investment and operating costs

PW — present worth factor or discount factor (P÷F)

RFR — required freight rate

SCA — series compound amount factor (F÷A)

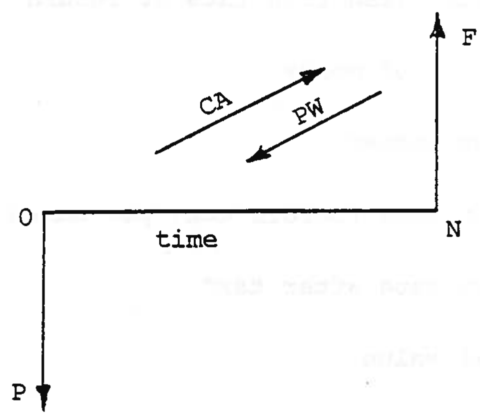
SF — sinking fund factor (A÷F)

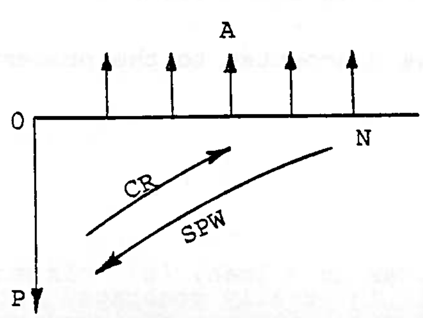
SPW — series present worth factor (cumulative discount factor) (P÷A)

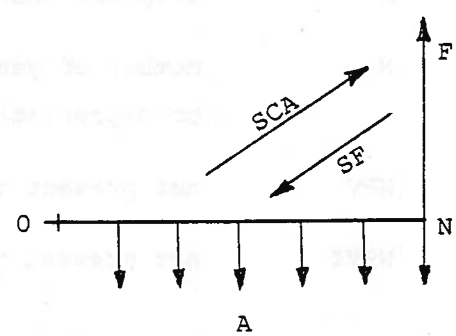
t — tax rate

Y — annual operating costs (wages, repairs, fuel, insurance, overhead, etc.)

Δ — difference or increment







PREFACE

My object in preparing these notes is to introduce students of naval architecture and marine engineering to engineering economy as a tool in ship design. The emphasis is on the principles of engineering economy. Methods of making practical application of these principles to ship design are touched upon, but no effort is made to provide factual cost data. You will find such information available in industry, and years of experience may bring you the facility of making reliable cost projections.

There is no intent here to teach business administration. Overlapping areas exist, however, and I shall try to indicate which decisions are clearly management's prerogative, which may belong either to management or engineering, and which belong to engineering alone.

Problems in communication frequently arise in matters of the sort dealt with here. Businessmen, engineers, and economists all may use different words for the same thought -- or the same word for different thoughts. I make no claim that my particular definitions are better than anyone else's. But I do aim to be clear and consistent, and hope you will concentrate on the thoughts, not the semantics, herein.

CONTENTS

INTRODUCTION

Economics is defined in many ways. Most pertain, not to money *per se*, but to the wise allocation of scarce resources. For our purposes, it is the task of allocating a finite supply of investment funds in the face of infinite possibilities. Most large companies and corporations decide between alternative investment opportunities principally on a basis of predicted profitability.

Engineering also is defined in many ways. Most definitions refer to using scientific knowledge for the benefit of society. In a free economy, society expresses its collective needs through its individual purchases -- which usually places the businessman between engineer and consumer. In short, the engineer must alternately deal, and communicate, with both scientists and business managers.

Engineering economy, then, is an approach to design aimed at meeting society's needs with a maximum effectiveness in the use of scarce resources: man-power, materials (including fuel), machinery and spare money.

In general, if he wants to be convincing to the business manager, the engineer must demonstrate that his proposed design promises to be more profitable than any alternative design. Furthermore, the engineer must realize that the wise business manager will consider several competing investment opportunities. An oil company, for example, may find a pipeline more profitable than a tanker. Or it may find the most profitable alternative is to rely on independent fleets, investing the capital so saved in a new refinery. These concepts are in keeping with both good economics and good engineering.

Economy is also something of a universal solvent, allowing the engineer to weigh the relative merits of design alternatives involving different units. For example: a choice between two engines, one heavier but more compact than the other, involves both weight and volumetric units. Converting both to resulting costs, present and future, allows rational quantitative judgement. In addition, there will always be non-quantifiable considerations that should be weighed. These include quality of life and environmental factors as well as those relating to the owner's personal gratification (e.g., pride in owning a ship that is somehow unique). These intangible factors are normally management's responsibility, although a complete engineering analysis should at least include mention of them.

Many engineers equate sophisticated technology with good design. They slight the economic aspects of their work and the result is over-design. The best wheelbarrows are not made of gold. The same is true of ships.

The topic of inflation is dealt with in more advanced text books. We need say here only that we are using constant-value (rather than face-value) dollars throughout. If the shipowner is free to raise his rates to meet his inflating costs, the impact of inflation on design decisions should be only minimal.

Although the emphasis throughout is on decision making for private industry, the principles are much the same for government-owned service vessels. Even the communist and socialist countries use analytical techniques similar to those espoused here. They, too, look for profits in their merchant ships, but they use another name for it.

In summary of the above, the well-engineered project has a basis of careful economic thought. Such thought is applied not only to the preliminary, major decisions but to many detailed decisions along the way. And, finally, when ready to present to management, the proposal is couched in terms of profitability. A good naval architect, then, must know how to make economic studies and must unendingly develop his ability to estimate future costs of building and operating ships.

THE TIME-VALUE OF MONEY

Misers to the contrary, money does you no good until you spend it on the necessities or pleasures of life. Most of us would rather be given a dollar today then the promise of one in the future, simply because we are humanly impatient to have our necessities and pleasures. In short, we must consider not only how much money changes hands but *when*. This time-value of money means that you should be rewarded if you are willing to lend or invest your spare cash since you are thereby postponing the pleasures it can bring you. The reward, or rent money, we call *interest*, which is generally expressed as an annual charge in percent of the amount owed.

Interest can be thought of in three ways:

1. *Contracted interest* is the type you are most familiar with. Savings deposits, bank loans, mortgages, and bonds all carry mutually agreed-upon interest rates.

2. *Implied interest* is appropriately considered when funds are tied up without any resulting visible reward. For example, if you hide money in your mattress, your decision is, in effect, costing you the interest you would gain from a savings account. This is lost-opportunity interest.

3. *Returned interest* is a measure of the gain (if any) from risk capital invested by an enterprise. This is sometimes called internally generated interest, interest rate of return, or yield. It is a good measure of profitability, expressing the returns from a venture as equivalent to returns from a bank at some rate of interest. Since most nations impose a tax on corporate incomes, we must differentiate between interest rates of return before and after tax (abbreviated i and i').

* * * * *

To repeat, in economic analyses we must study both the amount and the timing of the cash that changes hands. The analysis of timing is handled by the various interest relationships discussed in the next two chapters.

PRESENT WORTH

If you are satisfied with an interest rate of 5 percent, then $1.00 today is exactly equivalent in desirability to the expectation of $1.05 a year hence. Conversely, the expectation of $1.00 a year from now is worth only $0.95 today. That is, the *present worth* of $1.00 one year hence is only $0.95, assuming an interest rate of 5 percent. Thus, interest relationships can be used to transpose dollars to equivalent values in other periods of time. Future cash flows are often analyzed by discounting all future amounts to their individual present worths and finding the sum of those values. This important concept will be illustrated shortly.

Present worth is also called present value. The special variant called net present value (NPV) is explained in the chapter on economic criteria.

INTEREST RELATIONSHIPS

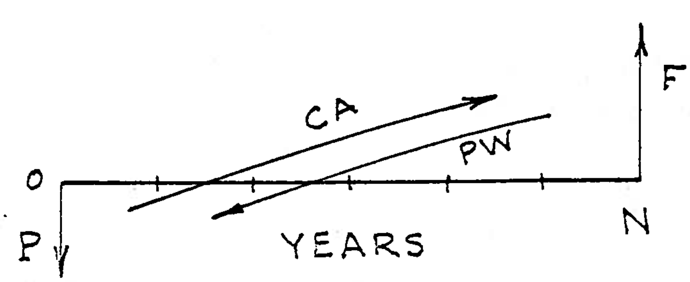
Interest arrangements may be of either the simple or compound variety. With simple interest, the rent money does not fall due until the termination of the debt and applies only to the initial amount. That is a most illogical, hence unusual, arrangement. We normally use compound interest, in which we specify not only the nominal interest rate per year, but also how many times per year the interest payments fall due. If these rents are withheld by the person who owes the money, then the withheld amounts are automatically added to the debt. That is, the debt is allowed to increase exponentially, as the constant interest rate is applied to a periodically increasing debt.

In engineering economy studies we almost always assume annual compounding and the interest rates we talk about are annual rates. As far as decision making in ship design is concerned, the assumption of annual compounding yields accurate conclusions. Money may change hands every day; but that will usually be true for every alternative under consideration. Small, uniform errors such as this will not change the relative rankings of competing proposals.

There are basically six compound interest relationships that we must understand. The first two apply to single-payment arrangements. First, some abbreviations:

- P: present sum, principal (amount of loan), invested cost, or present worth of future money
- F: future sum of money
- A: uniform annual amounts of money
- i: interest rate compounded annually
- N: number of years in the loan period

Plotting time on a horizontal scale and cash flow on a vertical scale:



$$F = (CA)P \quad P = (PW)F \quad \Big\} \quad CA = \frac{1}{PW}$$

where

CA is the compound amount factor, single payment

PW is the present worth factor, single payment; also sometimes called the discount factor.

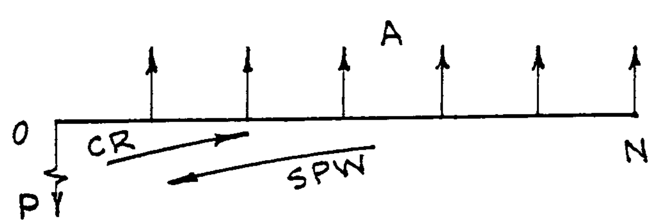
The algebraic values (proof of which you can easily derive yourself) are as follows:

$$CA = (1 + i)^N$$

$$PW = \frac{1}{(1 + i)^N}$$

Numerical values of PW for various combinations of i and N are given in the Appendix. Values of CA can be found by taking the reciprocals.

Where uniform annual amounts (A) are balanced against a present value or investment, we have these two relationships:



$$A = (CR)P \quad P = (SPW)A \quad \Big\} \quad SPW = \frac{1}{CR}$$

Note: Vertical arrows represent either discrete periodic returns, or total net cash flow during the time unit to the left of the arrow.

where

CR is the capital recovery factor

SPW is the series present worth factor.

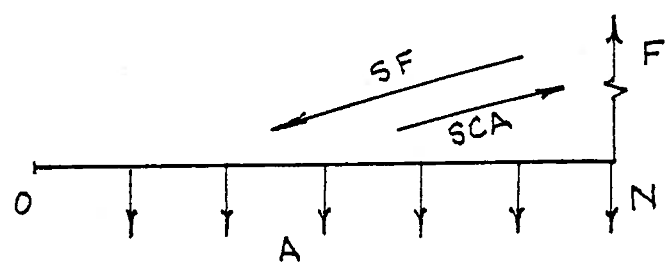
The algebraic value of CR is:

$$\frac{i(1 + i)^N}{(1 + i)^N - 1}$$

Numerical values of CR are found in the Appendix. Reciprocal values give SPW .

A common application of CR is in installment buying. If you wish to find the periodic amounts (A) that will repay a debt (P) in N periods, at an interest rate (i) per compounding period, simply multiply P times CR . Conversely, if you have an opportunity to buy a facility that promises to return A dollars per year for N years, you can find how much you should be willing to pay for it (P) by multiplying A times SPW , the latter based on the years of life of the facility and the interest rate you desire. Note that the greater the interest rate, the less you should be willing to put into the investment.

Where uniform annual amounts (A) are balanced against a single future amount (F) , we have these two relationships:



$$\left.\begin{aligned} A &= (SF)F \\ F &= (SCA)A \end{aligned}\right\} \quad SCA = \frac{1}{SF}$$

where

SF is the sinking fund factor

SCA is the series compound amount factor.

The algebraic value of SF is

$$\frac{i}{(1 + i)^N - 1}$$

Numerical values of SF are found in the Appendix. Reciprocal values give SCA .

If you want to accumulate by some future date a certain amount of cash (F) by regularly banking A dollars, find A by multiplying F by SF .

Conversely, if you know you can afford to bank A dollars per year, you can find the compounded future amount (F) owed you by the bank by multiplying A by SCA .

Some miscellaneous notes are appropriate at this time:

a. Although we use annual compounding in most engineering studies, the algebraic relationships and numerical values are valid for other compounding periods. Simply treat i as the interest rate per compounding period and N as the total number of compounding periods. For example, if the interest rate is one percent per month and we compound monthly for two years, the numerical value of any interest factor is the same as for annual compounding at one percent per year for 24 years. Do not be fooled into thinking, however, that one percent monthly interest compounded monthly is equivalent to 12 percent annual interest compounded annually. The sooner the interest is paid, the more desirable it is to the recipient.

b. The vertical arrows in cash flow diagrams (such as those shown in the preceding sketches) represent cash moving in or out of the organization. Imaginary allocations like depreciation are not included, but lost-opportunity costs may be if they differ between alternatives.

c. All vertical arrows in the cash flow diagrams could be reversed without changing the interest relationships. We represent money paid out by a downward pointing arrow, money coming in by an upward pointing arrow. But remember that money out to a lender is money in to a borrower.

d. In choosing between alternatives, relative values are more important than absolute values. Costs that are much the same between alternatives can usually be ignored. It's the differences that count. Study only those cash flows and opportunity costs that result from the decision.

e. Accountants and bankers must make their calculations to the last penny. In engineering economy, however, no such accuracy is required. Engineers try to predict future costs and there is no point in predicting profitability levels to five significant figures when the inputs are seldom accurate in the second figure.

f. An engineer uses engineering economy largely to help in making decisions. Decisions are between alternatives: no alternatives, no decisions.

History offers no alternatives. Therefore an engineer does not analyze past performances except to help predict the future. Accountants look back; engineers look ahead.

g. As a matter of convenience, in many of our studies, we start the measurement of time at the moment of delivery of the ship even though that may be several years in the future. This simplification is perfectly valid unless one alternative promises a significantly different start-up time than the others. Zero on the time scale may also represent the time of making the decision.

h. The recommended standards of the Engineering Economy Division of ASEE are used throughout this book for specifying the annual interest rate and years applied to the various interest relationships. For example, the capital recovery factor for 10 percent interest and 5 years would be represented by (CR - 10% - 5) . In calculations, the numerical value may be appended: (CR - 10% - 5)0.2638 . This may be read, "The capital recovery factor for 10 percent interest and 5 years, whose value is 0.2638..." The general form is (factor - i - N) .

i. As a matter of incidental concern, the sinking fund factor plus the interest rate is numerically equal to the capital recovery factor. Also, the series present worth factor is numerically equal to the sum of all the single present worth factors up to and including the total number of years involved. You can discover other relationships for yourself.

Summary

Good engineering involves economic as well as scientific analysis. Design optimization requires an understanding of the factors influencing the potential success of the ship as an investment. Interest relationships are therefore appropriately applied to ship design economic analyses, which invariably require that we intelligently relate cash flows that occur at different times. The six basic interest factors (CA, PW, CR, SPW, SF, and SCA) can be used in various combinations to analyze cash flow patterns, no matter how complex.

Problems

In the problems for this chapter assume an interest rate of 10 percent unless otherwise noted. Repeat solutions using other arbitrary interest rates if further practice is desired.

1 Calculate numerical values of the following interest factors. Assume a life of five years. Check your answers against the interest tables in the Appendix.

a. CA
b. PW
c. CR
d. SPW
e. SF
f. SCA

Solution to (1a)

The object is to find the numerical value of the single payment compound amount factor, CA , for 10 percent interest (i) and 5 years (N) . Annual compounding is always implied.

$$(CA\text{-}i\text{-}N) = (1 + i)^N$$

$$(CA\text{-}10\%\text{-}5) = (1 + 0.10)^5 = 1.611 \text{ (ans.)}$$

(Value from interest table: $\frac{1}{0.6209} = 1.611$)

Solution to (1b)

$$(PW\text{-}i\text{-}N) = \frac{1}{(1 + i)^N}$$

$$(PW\text{-}10\%\text{-}5) \ \frac{1}{(1 + 0.1)^5} = \frac{1}{1.611} = 0.621 \text{ (ans.)}$$

denominator same as
CA found above

(Value from interest table: 0.6209)

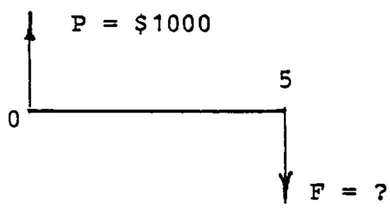
Note: The purpose of the first problem is to convince you of the credibility of your interest tables. Also, there will be occasional combinations of i and N not covered by the tables. In normal cases, however, you should use the tables rather than calculating every factor. Using the tables is both faster and safer.

2 In the problem above, check the relationship:

CR = SF + i

3 If you borrow $1000 today and repay the loan in a lump sum five years from now, how much will you return to the lender?

Solution

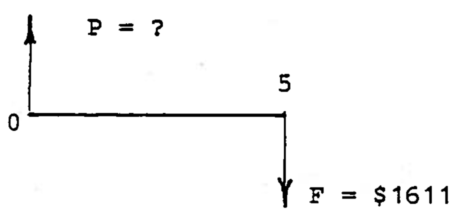


F = (CA-10%-5)1.611 $1000 = $1611 (ans.)

4 Find the present worth of the future amount found in Problem No. 3.

5 Find the present worth of the future amount found in Problem No. 3 using an interest rate of 3 percent.

Solution



P = (PW-3%-5)0.8626 $1611 = $1390 (ans.)

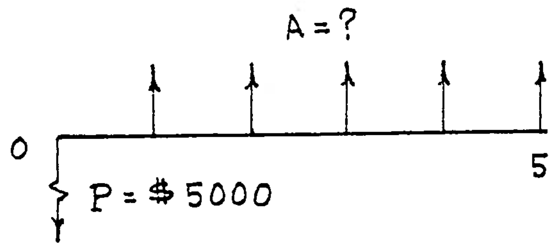
Comparing this answer to the one above, we see that lowering the interest rate increases the present worth of future amounts. Conversely, increasing the rate decreases present worth. With an interest rate of zero, P will equal F.

6 What is the total present worth of $100 five years hence plus $200 three years hence? (answer: $212)

7 If you borrow $5000 and repay the debt in five equal annual payments, how large will these be?

(problem cont.)

Solution

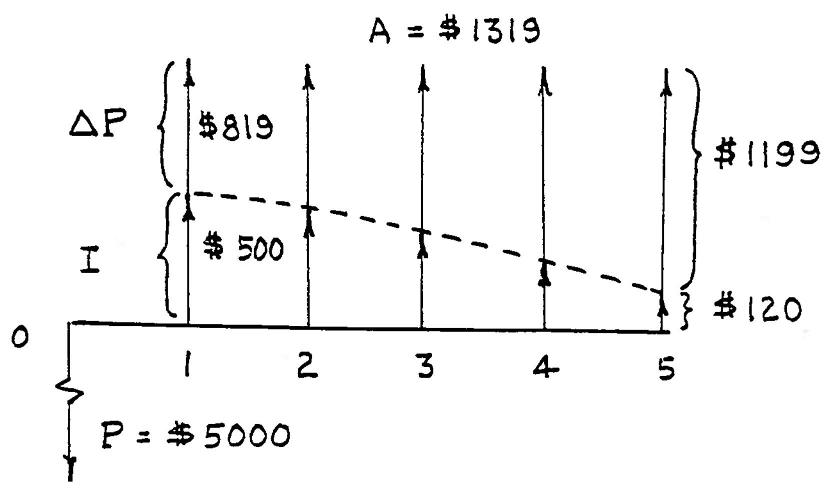


A = (CR-10%-5)0.2638 $5000 = $1319 (ans.)

Let us see how a bookkeeper would keep account of the annual payments of $1319 found above:

End of Year	Residual Debt Before Payment at End of Year	Interest Charge Due at 10%	Total Annual Payment	Reduction In Debt
1	$5000	$500	$1319	$ 819
2	4181	418	1319	901
3	3280	328	1319	991
4	2289	229	1319	1090
5	1199	120	1319	1199
TOTAL	0			5000

Based on the above, we can show on a cash flow diagram how the distribution between interest and debt reduction varies from year to year:



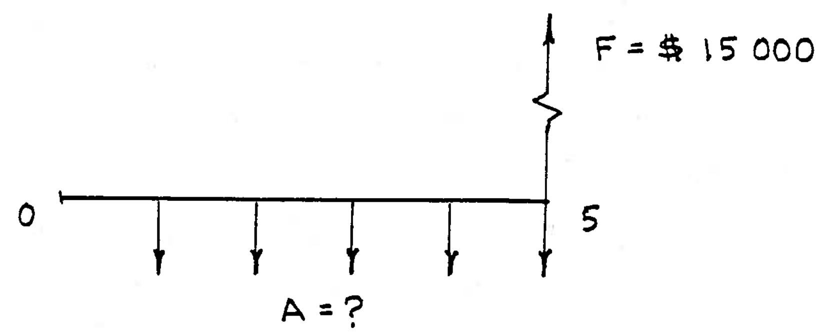
8 Find the present worth of the annual payments found in Problem No. 7.

9 Find the present worth of the annual payments found in Problem No. 7 using

an interest rate of 20 precent. (Answer: $3944)

10 You think you will need $15,000 in a lump sum five years hence. How much must you invest each year at 10 percent interest in order to meet this objective?

Solution



A = (SF-10%-5)0.1638 $15,000 = $2457 (ans.)

11 Find the present worth of $15,000 five years hence.

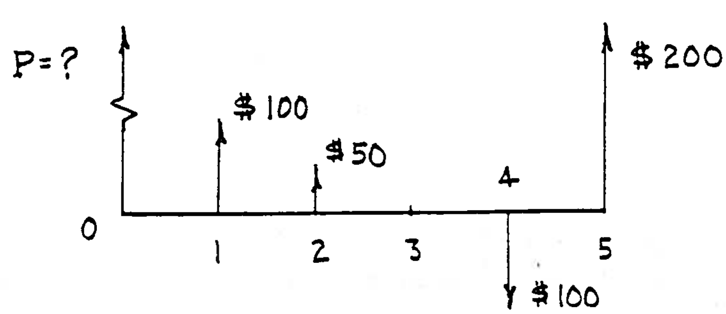
12 Find the total present worth of the annual amounts found in Problem No. 10. Compare this answer with that found in Problem No. 11.

13 What is the present worth of $1000 per year for each of the next five years? Find this in two ways: using SPW , and tabulating single values based on PW .

14 Find the present worth of the following cash flow.

During year 1	$100	
during year 2	50	
during year 3	0	
during year 4	100	loss
during year 5	200	

Solution



(solution cont.)

Year	Amount	(PW-10%-N)	Product
1	$100	0.909	$ 91
2	50	0.826	40
3	0	0.751	0
4	(100)	0.683	(68)
5	200	0.621	124
Total Present Worth			$187 (ans.)

15 Which is more desirable, an income of $500 five years hence or one of $2400 twenty years hence?

16 Find the indifference rate for the alternatives specified above. That is, at what rate of interest do the two become equally desirable?

17 If you borrow $1000 and agree to repay the debt over a two-year period with equal monthly payments at 1% interest per month, how much would you pay each month?

18 What is the formula relating debt (P) and uniform annual payments (A) for zero interest and N years?

19 What is the relationship between CR and i for an investment in real estate having an infinite life?

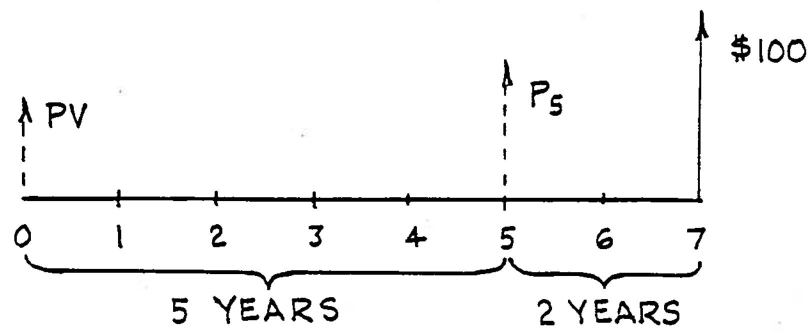
20 Assume your college education costs you $7500 per year for each of four years. If you had not gone to college, by working equally hard, you could have earned $10000 per year during those four years. Recognizing lost opportunity as a cost, how much will your college education cost you? Ignore time-value of money.

21 A loan plan allows you to borrow $10,000; you pay nothing until the end of the sixth year, then you make equal annual payments that retire the debt at the end of the 20th year. How much will you have to pay each year?

22 Use the interest tables in the back of the book to find the present worth of $100 seven years hence.

Solution

Our tables do not happen to give values for N=7 years. They do give values both for 5 years and 2 years, however. We can transpose the $100 from the 7th year to its equivalent value at the end of the 5th year, however, and then find the present value of that amount.



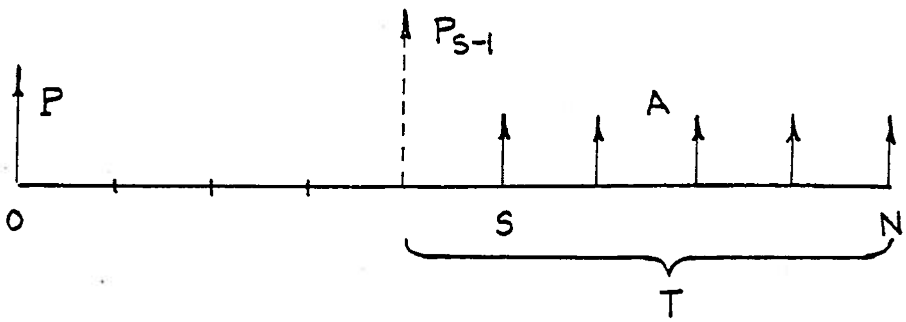
P_5 = (PW-10%-2)0.826 $100 = $83

PV = (PW-10%-5)0.621 $83 = $52 (ans.)

From this we can derive the general relationship that the present value of an amount F that changes hands X + Y years hence equals the product of the present worth factors for X and Y multiplied by F :

PV = (PW-i-X) (PW-i-Y)F

Problem 14, in the previous section, shows the universal solution to finding the present worth of complex cash flows: simply tabulate the individual annual amounts, multiply each by the appropriate present worth factor, and add the products. Frequently, however, alternative solutions are more convenient. Consider the problem of finding the present worth of A dollars per year for each of T years, starting in year S :



This can be done in several ways. One is to find the present worth at year (S-1) , then transfer that present worth back to year zero:

$$P_{S-1} = (\text{SPW-i-T})A$$

$$P = (\text{PW-i-S-1})P_{S-1}$$

combining

$$P = (\text{PW-i-S-1})(\text{SPW-i-T})A$$

Another method is to find the series present worth of A for N years, then subtract the series present worth of A for (S-1) years:

$$P = (\text{SPW-i-N})A - (\text{SPW-i-}[S-1])A$$

or

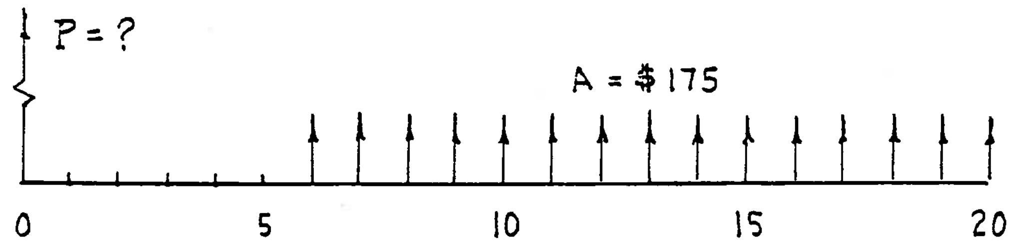
$$P = \{(\text{SPW-i-N}) - (\text{SPW-i-}[S-1])\}A$$

Example 23 illustrates the foregoing approach.

23 Find the present worth of $175 per year for 15 years starting in the sixth year. Assume an interest rate of 10 percent.

(solution on next page)

Solution



Using the first method:

$$P \text{ at year } 5 = (\text{SPW-10\%-15})\frac{1}{0.1315}\ \$175 = \$1331$$

$$P = (\text{PW-10\%-5})0.6209\ \$1331 = \$828 \quad (\text{ans.})$$

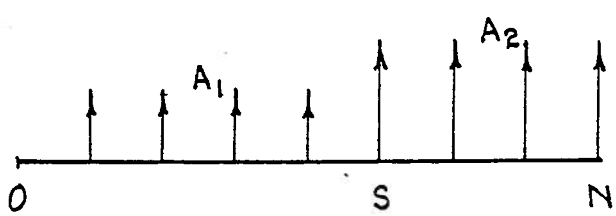
Using the second method:

$$P = \{(\text{SPW-10\%-20})\frac{1}{0.1175} - (\text{SPW-10\%-5})\frac{1}{0.2638}\}\$175$$

$$P = (8.51-3.78)\$175 = (4.73)\$175 = \$828 \quad (\text{ans.})$$

Devise and solve by a third method. (There are several alternatives.)

A slightly more complex problem involves a step in annual amounts:

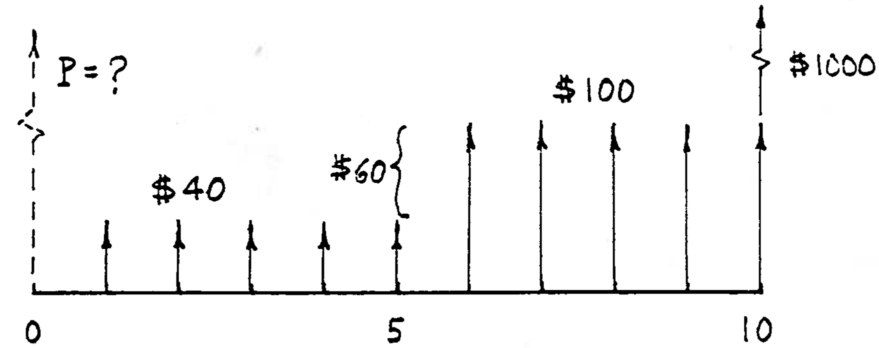


The present worth could, as before, be found in any of several ways. One solution would be to find the present worth of A_2 dollars per year for N years, then subtract the present worth of (A_2-A_1) dollars per year for $(S-1)$ years:

$$P = (\text{SPW-i-N})A_2 - (\text{SPW-i-S-1})(A_2-A_1)$$

24 Find the present worth of \$40 per year for the first five years, \$100 per year for years 6 through 10, and \$1000 at the end of the tenth year. Interest rate is 10 percent.

(cont.)



Solution

let

P_1 = present worth of \$100/yr for 10 years

P_2 = present worth of \$60/yr for 5 years

P_3 = present worth of \$100 at end of tenth year.

$$P = P_1 - P_2 + P_3$$

$$P = (\text{SPW-10\%-10})\frac{1}{0.1627}\ \$100 - (\text{SPW-10\%-5})\frac{1}{0.2638}\ \$60$$

$$+ (\text{PW-10\%-10})0.3855\ \$1000$$

$$P = \$614 - \$227 + \$385 = \$772 \quad \text{(ans.)}$$

Repeat this problem using 20% interest rate. Check your answer using tabular form.

<u>25</u> Find the amount of a single payment during the third year that is equivalent in present value to the following cash flow, using an interest rate of 10 percent.

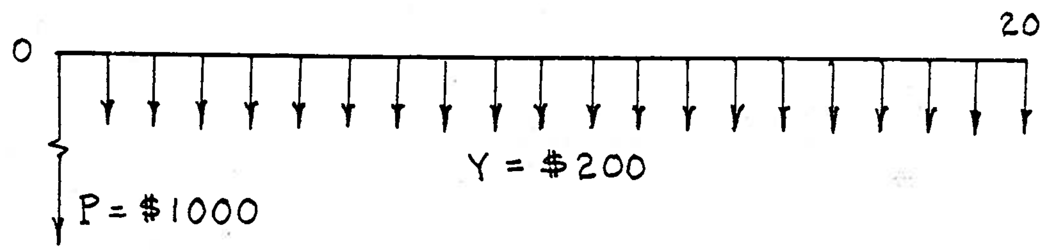
Year	Thousands of Dollars
1	2000 outflow
2	2000 outflow
3	2000 outflow
4 through 13, each	1000 return
14 through 28, each	800 return

(The answer is \$1,870,000)

26 Find the present worth of the following cash flow. Do this year-by-year (tabular form) and by at least one other method. Use an interest rate of 17 percent.

Year	Amount
1 through 4	$100
5 through 10	$300
11	($100)
12 through 15	$200

27 What is the present worth of a facility that will cost $1000 to start with and cost another $200 per year for 20 years? There is no income. Interest rate is 6 percent.



There are two things to note here. One is that we are really talking about costs, not worths. The same principles nevertheless apply. Secondly, the initial investment is already designated "P," we shall therefore call present worth of both present and future amounts "PV."

$$PV = P + (SPW\text{-}i\text{-}N)Y$$

$$PV = (\$1000) + (SPW\text{-}6\text{-}20)\frac{1}{0.0872}(\$200)$$

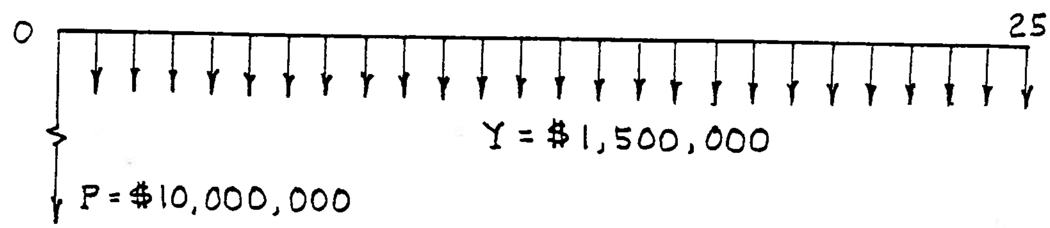
$$PV = (\$1000) + (\$2295) = (\$3295) \quad (ans.)$$

Repeat the above using an interest rate of 3 percent; also add an income from resale of $1000 at the end of the 20th year.

AVERAGE ANNUAL COST

The previous section has used present worth to measure the cost or value of various cash flows. An alternative is to find the *average annual cost*, (AAC), which converts all costs to an equivalent uniform annual amount, taking due recognition of the time-value of money. The following examples illustrate the principle of average annual cost.

28 Find the average annual cost for a ship that requires an initial investment of $10,000,000 and annual operating costs of $1,500,000. The ship is expected to last for 25 years. An interest rate of 18 percent is specified.



The average annual cost of operation is, of course, $1,500,000. To this must be added the annual cost of capital recovery (ACCR). This is found by converting the invested cost (P) to an equivalent annual cost by multiplying by the CR corresponding to the specified years and interest rate:

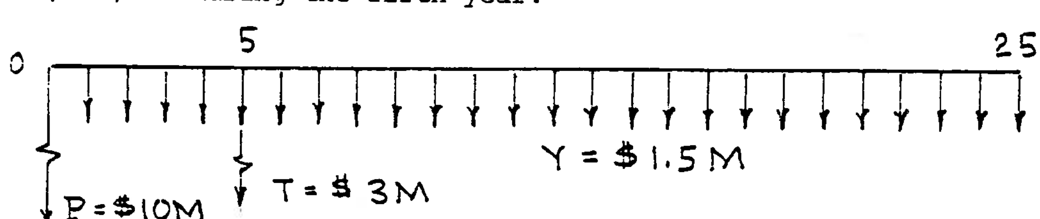
AAC for operations (Y)	$1,500,000	
ACCR = (CR-18%-25)0.1829 × $10M =	$1,829,000	
Total AAC	$3,329,000	(ans.)

In algebraic form:

AAC = Y + (CR-i-N)P

Note: M = million

29 Recalculate Problem 28 with the added assumption of a special cash outlay (T) of $3,000,000 during the fifth year:



(solution on next page)

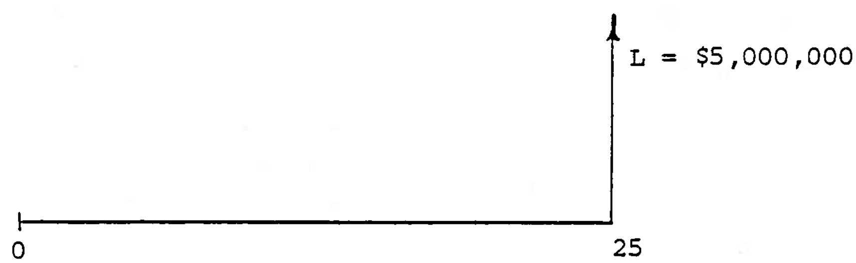
Solution

The average annual cost of a single expenditure such as T is found by first calculating its present worth and then multiplying by CR. The complete average annual cost would then be:

AAC = Y +(CR-18%-25)P + (PW-18%-5)(CR-18%-25)T

(To be completed by the student)

<u>30</u> Find the average annual cost (better phrased: equivalent annual return) of a predicted $5,000,000 resale 25 years hence. Interest rate is 18 percent.



Solution

There are two ways to do this. One is to treat L just as T was treated in the previous problem.

AAC = -(PW-28%-25)(CR-18%-25)L

An easier method is to multiply L by SF :

AAC = -(SF-18%-25)L

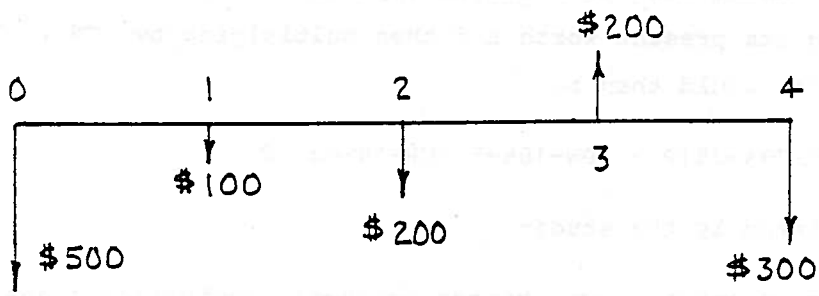
Note that incomes are treated as negative costs.

(You should complete this problem by both methods.)

Complex cash flows can be converted to average annual costs in two steps: (1) Find the total present worth as illustrated in Problem 14; (2) Multiply the total present worth by the capital recovery factor:

$$AAC = (CR)\{\sum_{0}^{N} F(PW) + P\}$$

31 Find the average annual cost of the following, if interest rate is 15 percent:



Solution

Use the tabular form to find the present worth of the future costs; add the initial cost of $500; then multiply by CR :

Year	Amount	(PW-15%-N)	Product
0	$500	1.000	$500
1	100	0.8696	87
2	200	0.7561	151
3	(200)	0.6575	(132)
4	300	0.5718	172
PW			$778

AAC = (CR-15%-4)0.3505 $778 = $273 (ans.)

(Repeat, using an interest rate of 10 percent.)

32 Find the average annual cost of a ship that has an invested cost of $6,000,000, a predicted resale value of $3,000,000 after five years, and operating costs as listed in the table below. The interest rate is 20 percent.

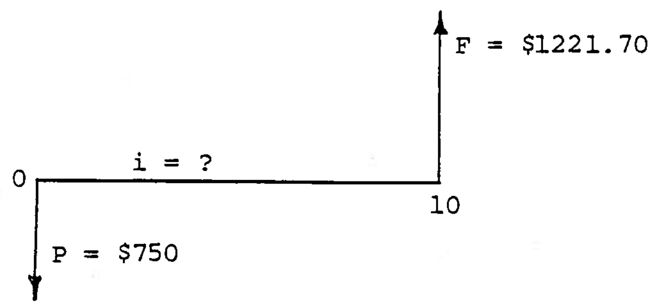
Year:	1	2	3	4	5
Operating Costs:	$1M	$1.2M	$3M	$3M	$3.2M

(M: million)

FINDING AN UNSPECIFIED INTEREST RATE

The principle of this section is best presented in an example such as the following:

<u>33</u> Suppose a banker offers to give you a sum of \$1221.70 in ten years if you will deposit \$750 in his bank today. What interest rate does he have in mind?



Solution

We know P , F , and N . Dividing P by F gives us the single payment present worth factor the banker used:

$$P = (PW)F$$

$$\frac{P}{F} = \frac{\$750}{\$1221.70} = (PW\text{-}i\text{-}10)$$

$$(PW\text{-}i\text{-}10) = 0.6139$$

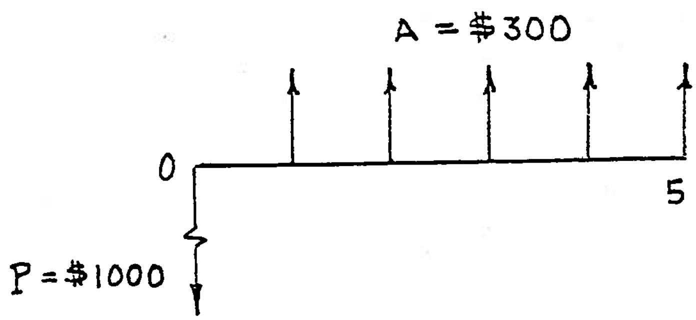
If we examine our single payment present worth table, we will find that the calculated value of 0.6139 is appropriate to 10 years and 5 percent interest. We can then infer that 5 percent interest was what the banker had in mind.

<u>34</u> A banker offers to give you a sum of \$121.60 five years from now if you will give him \$100 today. What interest rate is implied?

<u>35</u> A retirement plan involves your paying annual premiums of \$445 for the next 50 years. At the end of that period you will be given \$50,000. What interest rate is implied?

<u>36</u> A banker says he will give you \$300 for each of five years if you will lend him \$1000 at the beginning of the first year. What interest rate is implied?

(cont.)



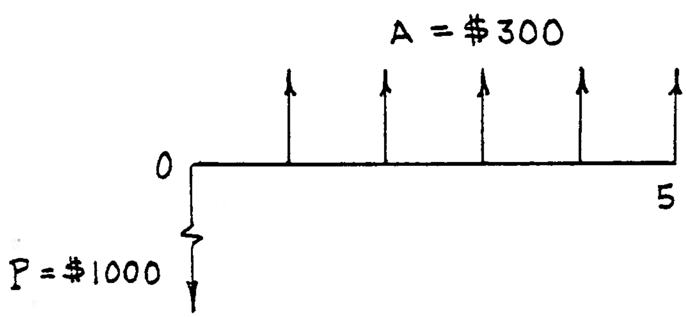
Solution

$$(CR\text{-}i\text{-}5) = \frac{\$300}{\$1000} = 0.30$$

$i = 15.3\%$ (ans.)

Note: Figure 1 may be used in place of interest tables when CR is known.

37 Suppose you can predict that a $1000 investment in a business venture will return $300 to you each year for five years. How advantageous is this as measured by an equivalent rate of interest?



As you may see, this cash distribution is exactly the same as that shown in the previous problem. The principle is the same and the method of solving is the same. The only difference is that the returns are not guaranteed.

Solution

$$(CR\text{-}i\text{-}5) = \frac{A}{F} = \frac{\$300}{\$1000} = 0.30$$

$i = 15.3\%$ (ans.)

In short, the predicted returns from this investment are equivalent to repayment of the principal at 15.3 percent interest.

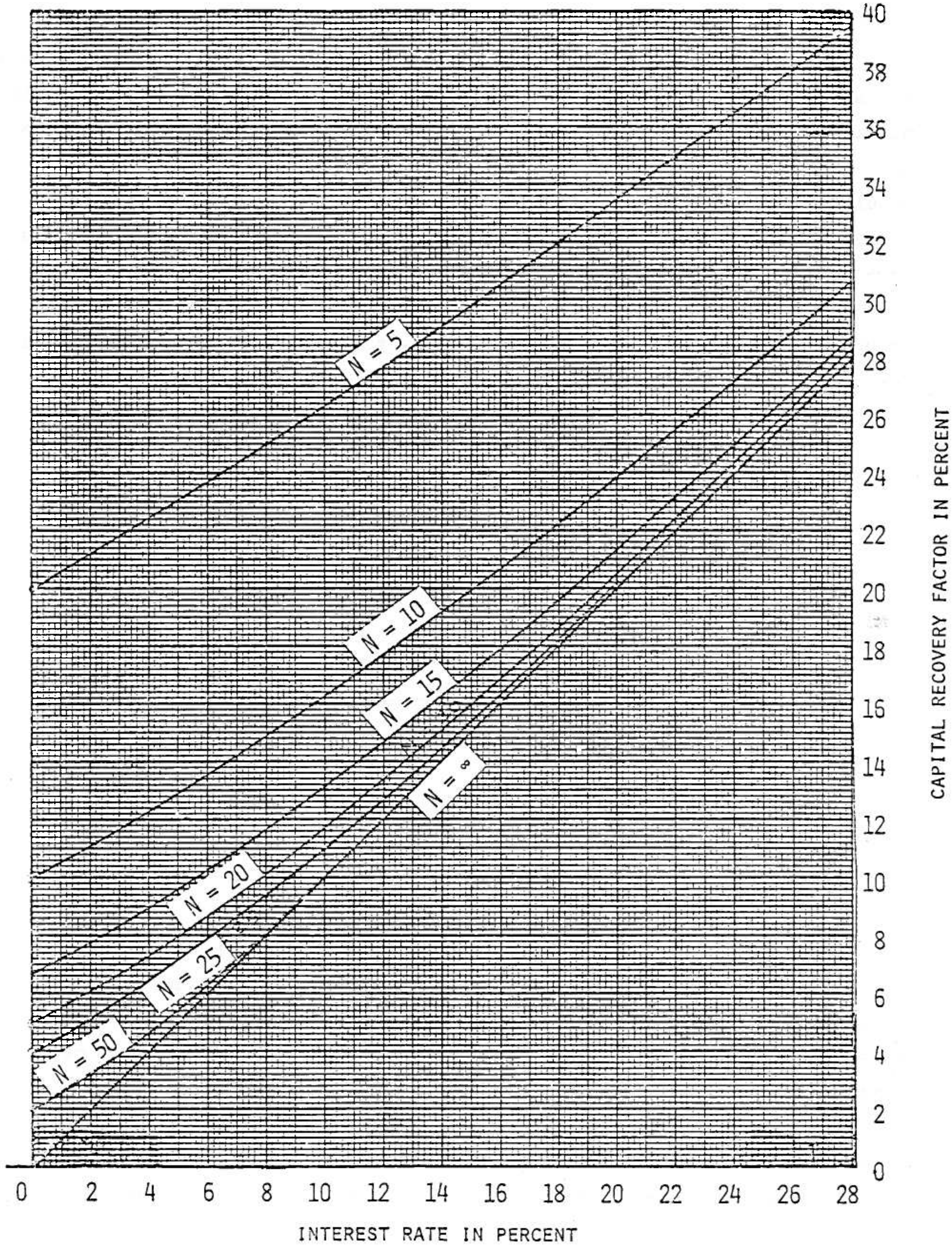
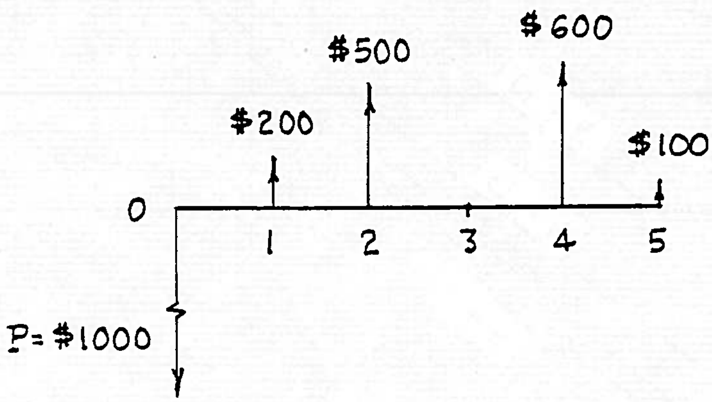


Figure 1. Capital Recovery Factor Versus Interest Rate.

The term *rate of return* is used by some to mean the ratio of the uniform annual returns to the initial investment, i.e. the ratio of A to P . Others use that term to mean the equivalent interest rate. The two coincide in value only when the investment has an infinite life. To make our meaning clear, we should use the term *capital recovery factor* when referring to the ratio of A to P . The interest rate can be called just *interest rate*, although terms such as *interest rate of return*, *internally generated interest* or *yield* help distinguish this type of interest from the contracted type that is fixed in amount and guaranteed. These terms, incidentally, are usually reserved for measures of profitability *after* tax. See next chapter.

38 What is the equivalent rate of interest for an investment of $1000 that promises returns of $200 the first year, $500 the second, zero the third, $600 the fourth, and $100 the fifth?



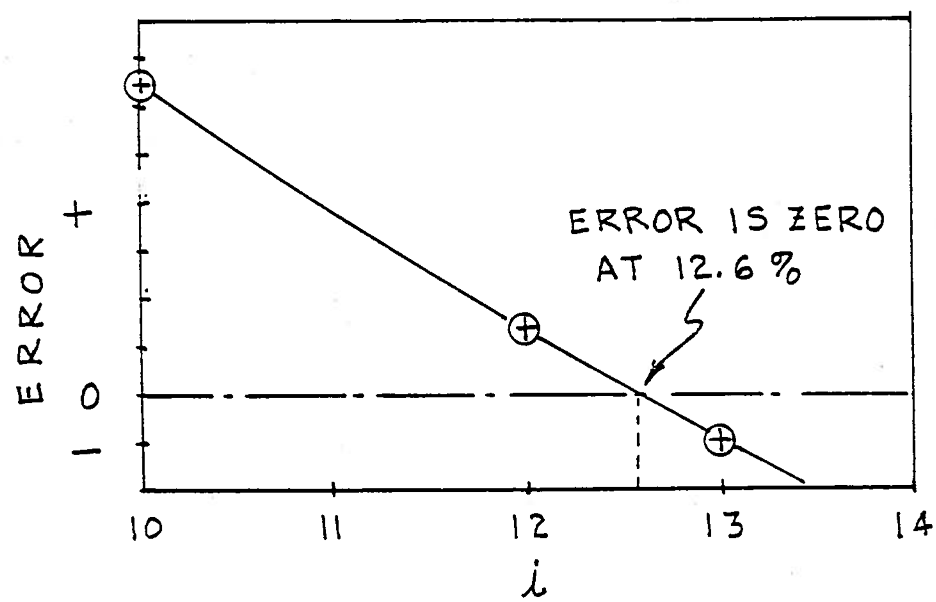
Solution

The key idea for a problem of this type is as follows: There is one and only one interest rate that will make the present worth of the future returns exactly equal to the investment. If we find that interest rate, we have the answer. This is essentially what we did by a shortcut method (using CR) in the previous problems. For complex patterns we must resort to a trial-and-error approach. That is, we guess at an interest rate, find present worth, and compare with the investment. We repeat this process with other rates until we get agreement between invested cost and present worth of future returns.

(cont.)

Year	Return	i = 10% (PW-10%-N)	P	i = 12% (PW-12%-N)	P	i = 13% (PW-13%-N)	P
1	$200	.9091	180	.8929	178	.8850	177
2	500	.8264	413	.7972	398	.7831	392
3	0	.7513	-	.7118	-	.6931	-
4	600	.6830	410	.6355	381	.6133	368
5	100	.6209	62	.5674	57	.5428	54
Total			1065		1014		991
error			+65		+14		-9

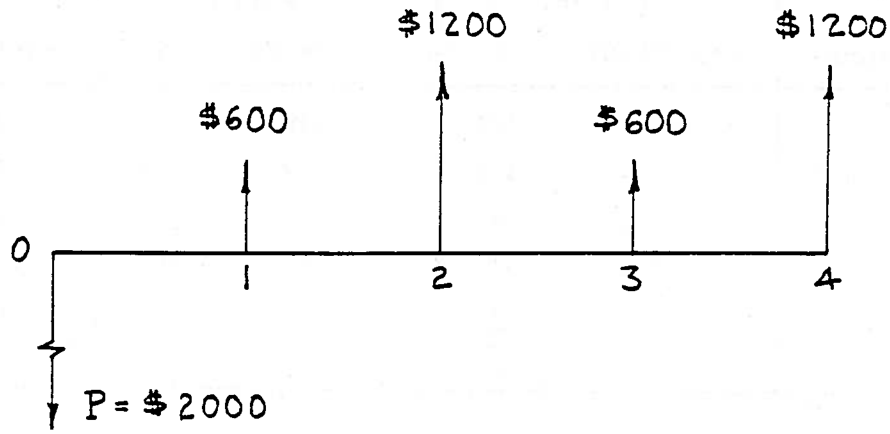
Plotting error vs interest rate:



This method has a potential enigma. When one or more of the years produce a loss, more than one interest rate may bring the present worth of all returns into balance with the investment. Such a situation would be infrequent in ship design studies, however.

The trial-and-error procedure described above is commonly called the discounted cash flow (DCF) rate of return method. Another term for this procedure and its result is "profitability index."

39 Find the interest rate of return for the investment and return shown at the top of the next page:



40 Find the yield for a facility that cost $5,525,000 during the first year and $7,870,000 during the second year. It was placed in service during the second year and returned $2,570,000 before the end of that year. It then returned $6,540,000 during the third year, $7,210,000 during the fourth year, and $5,120,000 during the fifth year, plus a net disposal value of $1,630,000 at the end of the fifth year.

41 Find the estimated yield for a ship that is expected to cost $15,000,000, paid in equal monthly amounts over a three-year period, and then returns $3,275,000 per year after tax for each of 30 years. The disposal value is negligible.

Note: Treat those monthly payments as though they all occurred at end of year.

CORPORATE INCOME TAX

Most of the traditional maritime nations tax the incomes of corporations. The tax laws not only vary between countries but are also likely to change over the years within each country. We cannot, of course, deal with all possible tax situations here. We can however present the principles involved in the simplest sort of tax. You can then develop analytical solutions to suit more complex situations.

Figure 2 shows the distribution of annual revenue from a capital investment such as a ship. Taxes are actually based on the total returns of the corporation. But, for our purposes, we compute the tax as though it applied to a single ship. Figure 2 is for the simple case of an all-equity (no loan) investment, and straight-line depreciation with the depreciable life equal to the anticipated useful life of the ship and zero net disposal value.

To find the return after tax (A'), we must subtract from revenue the operating costs (Y) and the tax itself. The tax is usually taken as a percentage of what the government calls profit: return (A) minus depreciation allocation $(P \div N)$. (When bank loans are involved, fixed interest payments to the bank are also excluded from taxable income.)

The relationship between returns before and after tax, as shown by Figure 2, is:

$$A' = A - t(A - \frac{P}{N})$$

or

$$A' = A - tA + \frac{tP}{N}$$

so

$$A' = A(1 - t) + \frac{tP}{N}$$

Dividing both sides by P :

$$\frac{A'}{P} = \frac{A}{P}(1-t) + \frac{t}{N}$$

But, since (assuming returns to be uniform)

$$\frac{A'}{P} = CR'$$

and

$$\frac{A}{P} = CR$$

we have

$$CR' = CR\,(1-t) + \frac{t}{N} \qquad (1)$$

or, inverting:

$$CR = \frac{CR' - \frac{t}{N}}{1 - t} \qquad (2)$$

In a similar manner, the net gain in after-tax returns $(\Delta A')$ from an incremental investment (ΔP) that promises uniform annual savings (ΔY) can be shown to be:

$$\Delta A' = \Delta Y\,(1 - t) + \frac{t\Delta P}{N} \qquad (3)$$

Equation (3) is based on the simplifying assumptions applied to Figure 2.

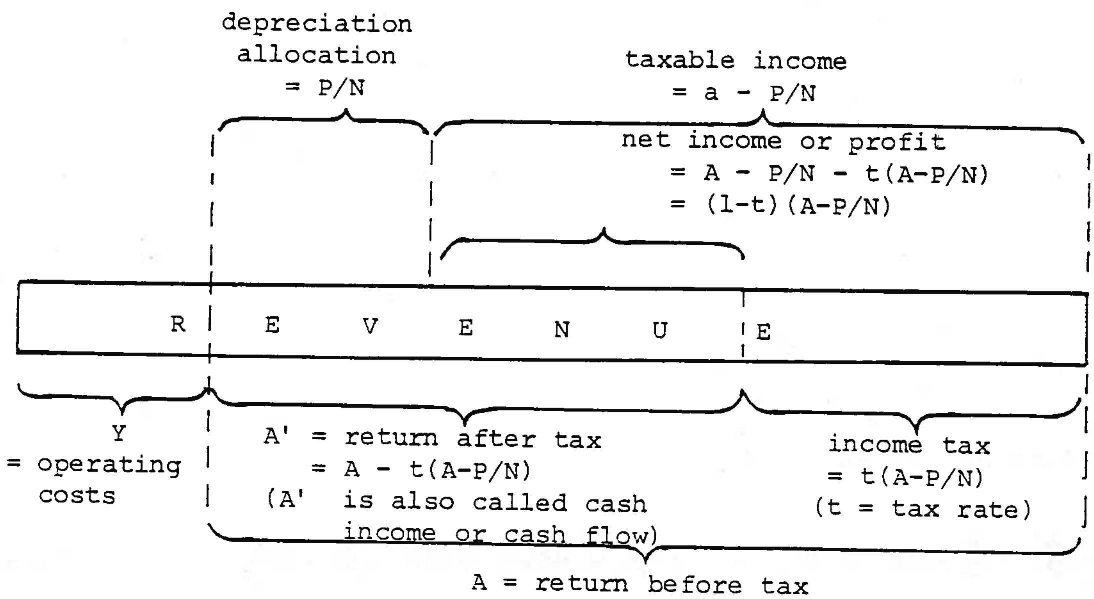


Figure 2. Division of Revenue. (all amounts are on an annual basis)

Note: The words profit and profitability are not synonymous. A $1,000,000 investment that generates profits of $2 per year is not as profitable as a $10 investment that yields $1 profit per year.

The following examples illustrate the use of Equations (1) and (2).

<u>42</u> An owner specifies a yield of 15 percent and a life of 25 years for his proposed ship. The tax rate is 48 percent and a straight-line depreciation is used. What interest rate should you use in relating present and future amounts before tax?

Solution

$i' = 15\%$ $\qquad$ $t = 0.48$

$N = 25$ $\qquad$ $i = ?$

The solution requires three steps. First: find CR' from interest tables. Second: convert to CR by means of Equation (2). Third: find i from Figure 1 or from interest tables.

$$(CR'-15\%-25) = 0.1547$$

$$CR = \frac{CR' - \frac{t}{N}}{1 - t}$$

$$CR = \frac{0.1547 - \frac{0.48}{25}}{1 - 0.48} = 0.261$$

$i = 25.9\%$ (ans.)

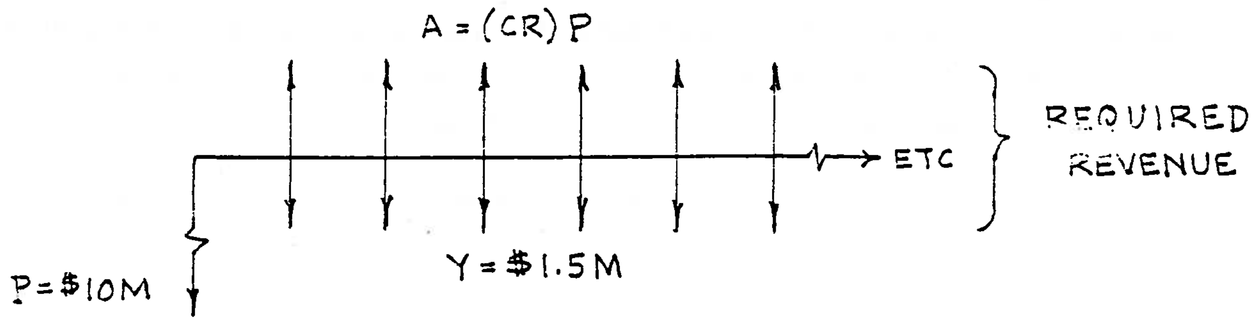
<u>43</u> Continuing the above, how much annual revenue would be required if the first cost of the ship is $10,000,000 and the annual operating costs are $1,500,000?

Solution

P = $10,000,000

Y = $ 1,500,000

(cont.)



Required revenue = Y + (CR)P

= $1,500,000 + 0.261 × $10,000,000

= $1,500,000 + $2,610,000

= $4,110,000   (ans.)

Note: the required revenue is simply the average annual cost: direct operating costs (Y) plus annual cost of capital recovery, or (CR) P .

44 Starting with the $4,110,000 annual revenue calculated above, set up a table and derive the yield. In short, reverse the procedure of Problems 42 and 43. Keep Figure 2 in view.

Solution

Item	$ per year	Notes
Revenue	4,110,000	
Operating costs	1,500,000	
Return before tax	2,610,000	
Depreciation allocation	400,000	1
Taxable income	2,210,000	
Tax at 48%	1,060,000	
Net income	1,150,000	
Return after tax (A')	1,550,000	2
CR'	0.155	3
i' (yield)	15% (ans.)	4

(cont.)

Notes

1. Depreciation allocation = P ÷ N = \$10,000,000 ÷ 25
2. Return after tax = return before tax minus tax
   also = net income plus depreciation
3. CR' = A' ÷ P
4. Yield is found from Figure 1.

The validity of Equations (1) and (2) is shown by the two examples above.

<u>45</u> A shipowner stipulates a yield of 13 percent. If ship's life is 20 years and tax rate is 48 percent, find interest rate before taxes.
(answer: 22.4%)

<u>46</u> Based on above, find top price owner should be willing to pay for a gadget that would last ten years and save a net amount of \$1000 per year before tax. Gadget has zero scrap value.
(answer: \$3860)

<u>47</u> Repeat Problem 45 using several different tax rates each time, including zero tax. Plot your results.

<u>48</u> A ship costs \$15,000,000 and is expected to last 30 years. Its annual gross income is \$4,000,000 and its annual operating costs are \$750,000. There are no fixed interest charges. Tax rate is 42 percent. Find capital recovery factors before and after tax, using tabular form.

<u>49</u> Check CR and CR' relationship in Problem 48 by means of Equations (1) and (2).

CHOOSING AN INTEREST RATE

There are often circumstances when cost studies require that a minimum acceptable yield (or target yield) be specified. Specifying yield is primarily a responsibility of management. Nevertheless, engineers should develop some insight along these lines.

In stipulating yield, a business manager will weigh alternative investment opportunities and relative risks. Furthermore, if free-market economic conditions exist, he knows that they will set reasonable upper and lower limits on profitability over the life of the ship.

Most business managers think about after-tax levels of profitability, so stipulated yields should be adjusted upwards (as in Problem 42) before applying to before-tax returns.

What constitutes a reasonable interest rate of return for a commercial ship is a matter of opinion that varies in different economic environments. In normal times, manufacturing enterprises in the United States earn average after-tax interest rates (yields) of about 10 percent. Owners of captive fleets in this country frequently aim for rates of about 12 percent. Independent operators, with their greater risks, are more likely to aim for about 15 percent. These target rates should not be confused with the cut-off, or minimum acceptable, rates used in net present value calculations. That topic is discussed in the next chapter. The aforementioned levels are based on idealized cost studies; actual returns are usually somewhat lower. In all too many cases they are considerably lower. You should give weight to your realistic pessimism by raising either projected costs *or* stipulated rates, but not both.

Some engineers advocate lower rates than those mentioned above. They argue that half or more of the investment comes from a bank loan with interest rates of only about six percent. Thus, if half the investment requires six percent interest and half requires twelve percent, the overall interest would be nine percent. There is less merit in this argument than you might at first suppose. Since bank repayment takes priority, the return of profits on equity capital is less certain than if no bank loan were involved. Which means that expected returns on equity capital must thereby be higher than normal. The Civil Aeronautics Board considers an overall yield of 10.5 percent on total invested capital to be "just and reasonable." This represents a 16 percent return on equity capital and five percent on debt, with an investment made up of half debt, half equity, capital. Remember, too, that a banker is unlikely to lend money at six percent unless he is convinced that the entire project is likely to generate a good deal more than that. In short, use "leverage" to increase the owner's yield on his equity capital (consonant with his greater risk) rather than to whittle the acceptable rate of return. In most cases, then, you can almost ignore the influence of bank loans in engineering economy studies.

Service vessels owned by governments seldom earn income. The time-value of money invested in them is real, nevertheless. Political economists are at odds as to the selection of a reasonable rate. Some argue for the cost of the national debt. Others argue for the lost-opportunity cost of leaving the money in private hands, which might justify 11 percent. Recent Department of Defense studies have used rates ranging from six to ten percent. The whole issue is still a long way from being fully understood, much less universally agreed upon.

ECONOMIC CRITERIA

The table on the next page outlines the most common combinations of circumstances under which we may have to analyze a ship design or operation. It also shows, by abbreviation, one or more economic criteria suited to each set of circumstances. The criteria are more fully defined and discussed in the text of this chapter.

All of the recommended criteria adhere to the following principles:

I. A commercial ship is an investment that earns its returns as a socially useful instrument of transport. In such a ship the best measure of engineering success is economic success.

II. Even in ships (or components, or systems) that earn no income, engineering success hinges on minimizing costs, with due recognition given to the time-value of money.

III. Once the ship is built the investment is fixed and need not be considered in deciding on how the ship should be operated. The simple objective of maximizing annual net income (revenue minus expenses) will automatically lead to maximum yield, NPV, etc. If revenue is fixed, try to minimize operating costs. In either case, of course, we should recognize the need to keep the ship in good repair in recognition of a reasonable economic life.

In keeping with the above principles, my recommended criteria either predict the returned interest rate, or make use of a specified rate.

Usually we assume that the investment is made as a single payment upon delivery of the ship. This is not normally true, but making that assumption eases our work and will seldom invalidate our selection of the optimal design. If the investment time-patterns differ appreciably between alternatives, then we must use more laborious techniques such as the DCF approach (see Problem 38). When alternatives have non-uniform returns that are generally similar in time pattern, we are usually safe in simplifying our work by finding each one's measure of merit based on average returns.

When analyzing a complex system that includes components of widely different life expectancy, the usual practice is to assume a series of identical replacements for each unit. There may be an indefinitely long life for the entire system (e.g., ships, containers, terminals); in that case we arbitrarily select a planning horizon (perhaps 20 years) and ignore all cash flows beyond that point.

A GUIDE FOR SELECTING MEASURES OF MERIT

General Notes: 1) The system to be optimized can be an entire transport system, a single ship, or a component of a ship

2) In every case we assume a knowledge of the invested cost and the annual costs of operation

Task:	Design a System or Select & Buy a Second-Hand System					Operate an Existing System (See Note m in Text)				
Circumstances	System Produces Income			System Produces No Income		System Produces Income			No Income	
	Revenue Is Predictable		Revenue Is Not Predictable			Modify & Operate		Operate As Is	Modify and Operate	Operate As Is
	Equal Lives	Unequal Lives		Equal Lives	Unequal Lives	Revenue Predictable	Revenue Not Predictable			
Suitable Measure(s) of Merit	Yield or NPV or NPVI	Yield or AAB	RFR or AAC	AAC or PW (LCC) or CC	AAC or CC	Yield or NPV or NPVI	AAC	A	AAC	Y
See Notes In Text	a,b,c d	e	f,g,h	h,i,j	h,j	a,b,c d,k	h,k	l	h,k	

Abbreviations

A	:	Uniform annual returns before tax
A'	:	Uniform annual returns after tax
AAB	:	Average annual benefit
AAC	:	Average annual cost
CC	:	Capitalized cost
CRF	:	Capital recovery factor
LCC	:	Life cycle cost
NPV	:	Net present value
NPVI	:	Net present value index
PW	:	Present worth
RFR	:	Required freight rate
Y	:	Uniform annual operating cost

Notes on Applicability of Measures of Merit

The following notes apply to the table shown on the previous page:

a) When we can predict both revenues and operating costs generated by an investment, we can estimate the after-tax returns (A') and from them derive the yield (i') by relating those returns to the initial investment. This is the measure of merit most likely to be favored by entrepreneurs. Also, an investment in a component that may save money in the future can be treated as though the money so saved (after tax) were cash income.

b) In place of yield, conservative corporate administrators are more likely to favor net present value (NPV) as their measure of merit. To find NPV , use a minimum acceptable interest rate (also abbreviated i') to find the present value of the entire cash flow pattern, including investments.

c) NPV has a bias in favor of larger, more ambitious projects. To temper that influence, take the ratio of NPV to the investment: the net present value index:

$$NPVI = \frac{NPV}{P}$$

d) When alternatives have equal lives and annual returns are uniform (i.e., assumed unchanging from year to year, but not necessarily the same for various alternatives) CRF is a simple surrogate for yield; so is its reciprocal, the pay-back period (PBP) . All three will indicate the same point of optimality, which is also true of NPVI .

e) The average annual benefit (AAB) may be used to neutralize the bias in NPV that results when alternatives have different lives. It is the product of NPV times a capital recovery factor based on the same interest rate used in finding NPV and the years of life appropriate to each alternative.

f) The required freight rate (RFR) is usually defined as the average annual cost per ton of cargo carried:

$$RFR = \frac{AAC}{C} = \frac{Y + ACCR}{C} = \frac{Y + (CR)P}{C}$$

where

C = tons of cargo carried per year

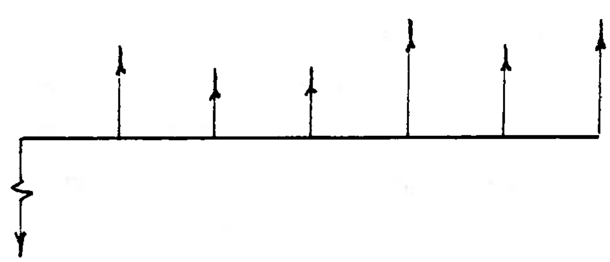
ACCR = annual cost of capital recovery, using a CR factor derived from the owner's stipulated target yield (often 10%) with correction for corporate income tax.

For complex trade patterns (perhaps both multi-port and multi-cargo), RFR may be defined as AAC per ton-mile, or container-mile, or passenger-mile, etc.

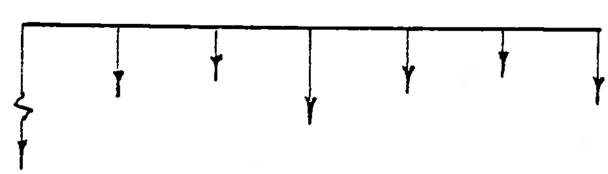
g) RFR is akin to what economists call a shadow rate, or what Adam Smith called a natural rate. It assumes that under free market conditions the best ship for any given trade is the one that can offer the service to the customer at the lowest price, while returning to the owner a reasonable profit. Although good for selecting the best ship, it cannot of itself prove that even the best will be profitable. Yes or no decisions on an investment demand an estimate of future income, leading to a derivation of yield, NPV , or one of their surrogates.

h) In the case of service craft, such as tugboats, average annual cost (AAC) is appropriate. If all alternatives have equal functional capabilities, then the one with minimum AAC is presumably the best. If capabilities differ, the choice becomes a matter of subjective judgment.

i) The present worth (PW) is as already defined. It is also called life cycle cost (LCC) . When alternatives have equal lives PW and AAC will both point to the same optimum. There are two important differences between PW and NPV . The latter is used where we expect positive net incomes



whereas PW is used when we have primarily cash out-flows:



Moreover, in NPV we use minimum acceptable discount rates; in PW we are more likely to use expected, or target, rates.

j) The capitalized cost (CC) is the present worth of providing a perpetual service. It assumes a never-ending stream of identical replacements. Numerically, it equals AAC divided by the interest rate. It is seldom used in the marine industry, nor can it do anything that AAC can't do just as well; therefore we shall say no more about it here.

k) In analyzing proposed alterations to a ship, the investment is composed of the cost of the alteration plus any potential returns that may be lost while the ship is out of service. The original cost of the ship is ignored. The future returns are taken as the added cash that will result from the alteration.

l) Except under unusual circumstances, the decision leading to highest before-tax return (A) will also lead to the highest after-tax return (A') -- which is the real goal.

m) Another alternative, not shown, is that of selling the existing system.

* * * * *

One final word of caution: Yield, whether used as input or output, is an accurate criterion only when all alternatives carry equal risks. If some are riskier than others, apply different rates.

Problems

50 A proposed ship has a life expectancy of twenty years. The predicted uniform annual before-tax returns are \$2,000,000. The estimated first cost is \$12,500,000. Would it be economic to add a heat exchanger that would have an installed cost of \$130,000? The unit would save \$25,000 per year in fuel; it would not affect crew costs or cargo capacity. Annual insurance costs would be one percent of investment; repairs would cost \$2,000 per year for the unit. Salvage value would be nil.

Solution

For the ship: $CR = \frac{A}{P} = \frac{\$2M}{\$12.5M} = 0.16$

(cont.)

Incremental annual returns from the heat exchanger:

Fuel saving	+ $25,000
Insurance at 1%	- 1,300
Repairs	- 2,000
Net incremental return	+ $21,700

For the heat exchanger:

$$CR = \frac{\$21,700}{\$130,000} = 0.167 \text{ (versus 0.16 for ship)}$$

Conclusion: The advantage of the heat exchanger would be barely worth the added cost. (ans.)

51 Repeat Problem 50 but assume the ship's first cost is dropped to $10,000,000.

Comparing the results of the last two problems leads to a curious point: High technical efficiency is most appropriate in ships of low profitability. The owner of the highly profitable ship might better use his spare cash to build another ship, not add refinements. Of course it may be that he can't afford another ship but can afford a heat exchanger.

52 What economic criterion or criteria would be suitable for finding the superior alternative below. *Explain*

Alternative	A	B
Investment	$8,000	$12,000
Life, years	12	20
Annual return after tax	$2,000	$ 2,400
Resale value	$ 200	$ 6,200

53 A company, which generally earns before-tax returns equivalent to 18 percent interest, has the following alternatives. Which would you recommend?

Alternative	A	B	C
Annual gross revenue	$ 4,000	$4,500	Alternative
Invested cost	$10,000	$9,000	C is to turn
Life, years	5	10	down both A and
Annual operating costs	$ 2,000	$2,700	B and seek other
Resale value	0	0	investments.

54 A ship needs a new compressed air tank. Tank A will cost $4,000 (installed) and last ten years. Tank B will cost $6,000 (installed) and last 20 years. The ship itself is expected to last another 20 years, also. Desired rate of interest is 20 percent before tax. Which tank would you recommend?

Solution

Revenues are unknown but equal. Lives are unequal, therefore use the AAC criterion.

tank A: AAC = (CR-20%-10)0.2385 $4,000 = $995

tank B: AAC = (CR-20%-20)0.2054 $6,000 = $1,230

Choose tank A (ans.)

55 Rework Problem 54 using an interest rate of five percent.

56 Find the indifference rate for tanks A and B (Problem 54). That is, find the interest rate at which the two tanks would be equally desirable.

57 A Baratarian shipowner expects his new ore carrier to move 200,000 tons of cargo per year on a given trade route. The ship will cost $6,000,000 to build and will last 25 years. Its average annual operating costs will be $1,000,000. The Baratarian corporate profits tax rate is 33 percent. What revenue per ton must he charge if he is to earn 12 percent interest on his investment, after taxes?

Solution

The method is to find required freight rate, which equals the average annual cost per ton of cargo. In finding average annual cost, the invested cost must be converted to an annual cost by a capital recovery factor gaged to produce the stipulated after-tax level of profitability. The first step is to find CR :

(CR'-12%-25) = 0.1275

$$CR = \frac{CR' - \frac{t}{N}}{1 - t} = \frac{0.1275 - \frac{0.33}{25}}{1 - 0.33} = 0.1705$$

(cont.)

The second step is to find AAC :

AAC for operations.................	$1,000,000
ACCR=(CR) 0.1705 $6,000,000..........	$1,023,000
Total AAC	$2,023,000

Dividing AAC by annual cargo capacity gives us the required freight rate:

$$RFR = \frac{\$2,023,000}{200,000} = \$10.11 \text{ per ton} \quad \text{(ans.)}$$

58 If the above owner can reduce average building costs by 15 percent through buying several identical ships, what will the new RFR become?

59 Rework Problem 57 assuming an increase in the Baratarian tax rate to 50 percent.

60 Reverting to the original conditions of Problem 57, how profitable would the investment be if the attainable freight rate were only $9.00 per ton?

61 Find the superior alternative if desired interest rate is 18 percent and there are no taxes.

Alternative	A	B
Life, years	25	30
Invested cost	$10,000,000	$11,000,000
Annual operating costs	$ 1,500,000	$ 1,250,000
Salvage value	$ 1,000,000	$ 1,000,000

62 In Problem 61 would the present value criterion be as suitable as the one you chose? Explain.

63 Which of these proposed ships would you recommend? All have an expected life of 25 years. (Costs shown are in $1000, weights in 1000's of tons.)

Ship	A	B	C	D	E	F
Invested cost	4000	4200	4500	4900	5400	6000
Annual oper. costs	600	610	620	630	640	650
Cargo per year	50	55.5	62	64.5	67	70
Annual revenue	1000	1100	1240	1290	1340	1400

64 Explain why you did or did not include depreciation charges in the above analysis.

UNSOUND CRITERIA

The marine industry harbors several persistent economic criteria that do not give proper weight to the time-value of money. For example, some studies compare alternatives by adding the cumulative total operating costs to the first cost. That is present value with zero interest.

Another common misconception is to design for maximum annual profit. Remember, however, that a small investment yielding a modest profit is more profitable than a large investment yielding only a slightly greater profit. In the design stage, profit and profitability are not the same. (Once the ship is built, however, the investment is no longer a variable and maximum profit means maximum CR hence maximum profitability.)

One of the most common criteria is the minimum cost method or its cousin: minimum cost per ton. These methods are akin to the average annual cost and required freight rate with one important exception: they use unrealistically low interest rates (or none at all) and overlook the corporate profits tax. They measure the cost to the owner for providing the service, whereas the average annual cost and required freight rate measure the cost to the customer. The cost to the customer is, of course, what is germane in optimizing a design. This slighting of the time-value of money means that high capital costs are (in appearance) easily offset by future savings. In fixed-income studies, this leads to over-design. On the other hand, cost-per-ton may lead to under-design when used as a variation of RFR .

Those who use the minimum-cost approach usually arrive at what they call the cost of capital as follows:

Item	Cost per year as a percent of initial investment
Depreciation (N = 20)	5
Interest	3
Insurance	1
Maintenance and repair	2
Total	11

Since the insurance and maintenance and repair costs are really part of the annual operating costs, the true annual cost of capital recovery implied above is $0.05P + 0.03P = 0.08P$, i.e. 8 percent of the investment. In short, the capital recovery factor is 8 percent, which allows us to infer (by means of Figure 1) an interest rate of 5 percent -- which is the minimum borrowing rate from a bank during times when little inflation is anticipated. The net result is that the minimum cost method uses a capital recovery factor that is less than half of what is necessary to pay the tax and leave enough profit to attract equity capital.

Here are three related problems that illustrate the danger of using the cost of service criterion. All are based on a proposal that a fuel-saving device be included in the specifications for a new cargo ship. The device would cost $200,000 installed and save $25,000 each year over the 20-year life of the ship. Insurance costs per year for the device would be one percent of its first cost, maintenance and repair: two percent.

65 Analyze the above proposal using the minimum cost approach. Assume an interest rate of 6 percent.

Solution

The advocates of the minimum cost approach would first find the annual cost for interest as a fraction of the investment:

Item		Fraction of Investment
Capital recovery factor (CR-6%-20)	=	0.087
Depreciation $\left(\frac{1}{N}\right)$	=	0.050
Annual interest (difference of the two)	=	0.037

They would next tabulate all the costs that were estimated as a fraction of the investment:

	Costs per Year
Depreciation	0.050P
Interest	0.037P
Insurance	0.010P
Maintenance and repair	0.020P
Total capital costs	0.117P

Then they would summarize the calculation:

Total capital related costs per year		
= 0.117 $200,000	=	($23,400)
Fuel cost saving per year	=	$25,000
Net saving per year		$1,600 (ans.)

Based on costs alone, the proposal appears beneficial. But let us analyze its true level of profitability in the following problem.

66 What is the after-tax interest rate (i.e., yield) that would result from the proposed investment in the fuel-saving device? Assume an all-equity investment, straight-line depreciation, and a tax rate of 50 percent.

Solution

The investment is $200,000; the before-tax savings (i.e. returns) would be:

Item	Annual Saving
Fuel Cost	$25,000
Insurance = 0.01 $200,000 =	(2,000)
Maintenance and repair = 0.02 $200,000 =	(4,000)
Net saving (A) =	$19,000

$$CR = \frac{A}{P} = \frac{\$19,000}{\$200,000} = 0.095$$

$$CR' = CR\ (1 - t) + \frac{t}{N}$$

$$CR' = 0.095\ (1 - 0.50) + \frac{0.50}{20}$$

$$= 0.0475 + 0.025 = 0.0725$$

from Figure 1:

$$i' = 3.8\% \quad \text{(ans.)}$$

Does this still look like a good investment?

67 Given all of the other conditions mentioned in Problems 65 and 66, what annual fuel cost savings would be needed to justify the invested cost of

the heat exchanger if the shipowner stipulates a yield of 11 percent?

Solution

$$i' = 11\%$$

$$(CR'\text{-}11\%\text{-}20) = 0.1256$$

$$CR = \frac{CR' - \frac{t}{N}}{1 - t}$$

$$= \frac{0.1256 - \frac{0.50}{20}}{1 - 0.50} = 0.2012$$

A = Annual return from the device

A = (CR)0.2012 $200,000 = $40,240

also:

$$A = F - (N + M)$$

$$\therefore F = A + N + M$$

where

F = Annual saving in cost of fuel

N = Annual cost of insurance

M = Annual cost of maintenance and repair

$$N = 0.10P$$

$$M = 0.02P$$

N+M = 0.03P = 0.03 $200,000 = $6,000

F = $40,240 + $6,000 = $46,240 (ans.)

(This compares with the predicted annual fuel saving of $25,000.)

The three foregoing examples illustrate how the cost of service criterion can lead to over-design in the case of fixed-income studies. When applied to variable income studies, however, the criterion leads to under-design. The next three examples illustrate this point.

68 Choose the optimal SHP for a proposed tanker, given the costs and revenues tabulated below. The owner wants to earn a yield of 12 percent. The ships are expected to last for 20 years and the tax rate is 48 percent.

(Costs and weights are in 1000's)

Tanker	A	B	C	D
SHP	10,000	15,000	20,000	25,000
Speed in knots	12.83	14.59	15.90	16.93
Cargo: tons/year	315.0	351.3	376.9	395.2
Investment: $	13,425	14,010	14,511	14,977
Annual oper. costs: $	1,174	1,347	1,509	1,668

Solution

Revenues, which are unknown, would vary between alternatives because of differences in annual transport capability. Required freight rate (RFR) would be the appropriate criterion. The first step is to find CR :

$$i' = 12\%$$

$$(CR'-12\%-20) = 0.1339$$

$$CR = \frac{CR' - \frac{t}{N}}{1 - t} = \frac{0.1339 - \frac{0.48}{20}}{1 - 0.48} = 0.2115$$

Applying this capital recovery factor to each invested cost gives us the annual cost of capital recovery (ACCR) for each design: Continuing the table we have:

Tanker	A	B	C	D
ACCR $	2840	2962	3070	3167
Annual oper. costs: $	1174	1347	1509	1668
AAC	4014	4309	4579	4835
Cargo: tons per year	315.0	351.3	376.6	395.2
RFR	$12.75	$12.26	$12.16 (min.)	$12.23

Conclusion: Tanker C (20,000 SHP) is optimal by virtue of promising the lowest required freight rate.

69 Rework Problem 68 using the minimum cost (cost of service) criterion. Assume six percent interest (before tax).

Solution

The annual capital costs are now:

In the usual manner of presentation:

Depreciation	0.050P
Interest	0.037P
Total annual capital cost	0.087P

(Annual cost of insurance and maintenance and repair are already included in the operating costs in this instance.)

Using this CR, we find the cost per ton of cargo by modifying the table as follows:

Tanker		A	B	C	D
Annual capital costs:	$	1168	1219	1262	1303
Annual oper. costs:	$	1174	1347	1509	1668
Fully distributed costs:	$	2342	2566	2771	2971
Cargo: tons per year		315.0	351.3	376.6	395.2
Costs per ton cargo		$7.43	$7.30 (min.)	$7.36	$7.52

On this basis, we would choose tanker B (15,000 SHP). This would be a mistake, however, because under normal market conditions tanker C would be more profitable.

70 Assume owner X buys tanker B and owner Y buys tanker C above. Assume the freight rate is an average of RFR found for tankers B and C in Problem 68. Find the after-tax interest rate of return in each case.

71 Assume a freight rate of $12.16 per long ton. Analyze proposed tankers A, B, C, and D on the maximum annual profit basis, pre-tax, (profit = revenue - operating costs - depreciation allocation).

(cont.)

Solution

Tanker		A	B	C	D
Annual revenue:	$	3830	4270	4579	4803
Annual oper. costs:	$	1174	1347	1509	1668
Depreciation (0.05P):	$	671	701	726	748
Annual Profit:	$	1985	2222	2344	2387 (max.)

72 Under the conditions of Problem 71, which tanker would be the most profitable?

These examples have illustrated the shortcomings of the minimum cost and maximum profit criteria, both of which fail to relate correctly present and future cash flows. There are other criteria that have similar weaknesses, and some do not even use monetary units. The concept of deadweight times speed divided by horsepower is an example.

There are certain other devious arts engaged in by overeager exponents of one proposal or another. Here are some actual examples:

1. One study of air cushion vehicles concluded that they were economically feasible because their estimated passenger-mile costs were slightly lower than those currently charged by aircraft. Its author privately admitted to me that he had used zero interest rate (i.e., no profit) in figuring the annual costs of capital recovery for his air cushion vehicles. A poor start for any enterprise.

2. One aircraft salesman, in comparing his company's proposal with a rival's, was discovered to be applying a higher CR to his rival's plane. He argued that his own plane had a higher first cost and therefore deserved a longer life.

3. Many cost studies present hundreds of pages of parametric cost and weight curves in meticulous detail. These are followed by a cost study that invariably proves the proposed innovation to be economically superior to established methods. These studies seldom state the assumptions, which (if truth were known) would all be found biased in favor of the innovation. The outcome of a long series of barely credible assumptions is not itself at all credible.

4. Some years ago a heavily-financed study reported that "Nuclear ships can economically compete now with conventional ships on long trade routes at

high speed." Upon studying the details, it became apparent that a more reliable statement would have been: "*If* you can find us an owner who is so ill-advised as to operate a 100,000-ton tanker at 22 knots on a 20,000 mile trade route, then nuclear power is economically competitive today -- provided also that the owner is satisfied with a 10 percent return before tax." This is part of the sales syndrome: the new device is designed and operated by individuals who are exceedingly smart, whereas those associated with the old device are exceedingly stupid (although neither is interested in making a profit or paying a tax).

5. A more recent study essayed to prove the economic feasibility of nuclear power by showing that a proposed 30-knot nuclear cargo liner would have lower transport costs than a 20-knot conventional liner. Aside from the usual implication that shipowners should be happy with a yield of 3 or 4 percent, the study gave the 30-knot nuclear ship the advantage of containerized cargo -- leaving the 20-knot ship with break-bulk cargo. $(A + B) > (C + D)$; therefore $A > C$.

6. Another way to hide a high first cost is to engage in some jiggery pokery by which capital recovery costs are supposedly lowered through shrewd reinvestment of the returns. Don't fall for that one. Reinvestment is an option that is open to all alternatives -- including that of building no ship at all, but banking the money instead. Remember two things: (a) in engineering economy, it is the differences that count, and (b) the money does the investor no good until he gets it back and spends it for his own personal wants and needs. Reinvesting is hardly fun.

* * * * *

Here are a few more problems:

<u>73</u> Suppose you are asked to find the optimum ship design in a situation where the owner has given you a predicted freight rate. You analyze several possible designs and find the best using CRF . Its value is 0.70. Would you be satisfied with CRF as a criterion? Discuss.

<u>74</u> Suppose you are asking to decide whether to add to a new ship a device costing $1000 and saving $700 per year. The device will last as long as the ship. Would you be satisfied with CRF as a criterion? Discuss.

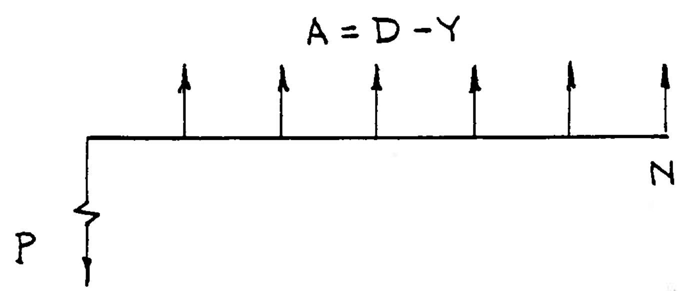
75 How would you compare two proposals both of which would reduce both the building cost and the operating cost of the ship?

76 We have noted that corporate income taxes increase RFR . The U.S. Navy pays no taxes. Does this mean that it can provide its own transport needs at less cost than if it paid a private fleet?

CAPTIVE FLEETS

Managers of subsidiary fleets, wholly owned by some industrial corporation, are apt to aim for minimum cost rather than maximum profitability in their ships. They reason that the fleet is not really producing any income and is simply a service facility to the corporation. In truth, however, each ship is producing a hidden income: annual cost of chartering the services of an independent operator minus the annual operating costs for the captive vessel.

Put another way, the corporation has a choice. It can invest in a ship and pay relatively small annual operating costs, or it can invest in another mine, oil refinery, or whatever, and pay large annual amounts for chartering. We can infer a cash flow resulting from the decision to invest P dollars in a ship:



where

D = annual chartering costs including internal administration

Y = annual operating cost for captive ship and, of course

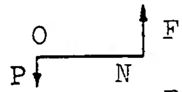
$$CR = \frac{A}{P}$$

That is on a before-tax basis. A more reliable comparison would consider after-tax conditions:

$$CR' = \frac{(D-Y)(1-t) + t\frac{P}{N}}{P}$$

This can be used to derive the yield or net present value, if preferred.

There is one serious omission in the approach outlined above. It does not recognize a disadvantage to the parent company in granting a long-term charter to an independent operator. Such a charter involves the parent company in long-term responsibilities that must be met regardless of future business conditions. The situation is much like taking out a bank loan and does, indeed, affect one's credit rating -- without giving the visible benefit of tax-free interest payments. Walsh's paper in the October, 1966 issue of *Marine Technology* explains in detail how to handle this.



APPENDIX

Single Payment Present Worth Factors

$$PW = \frac{1}{(1+i)^N}$$

$$CA = \frac{1}{PW}$$

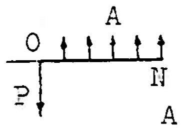
P = (PW)F   F = (CA)P

N	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	.9901	.9804	.9709	.9615	.9524	.9434	.9346	.9259	.9174	.9091
2	.9803	.9612	.9426	.9246	.9070	.8900	.8734	.8573	.8417	.8264
3	.9706	.9423	.9151	.8890	.8638	.8396	.8163	.7938	.7722	.7513
4	.9610	.9238	.8885	.8548	.8227	.7921	.7629	.7350	.7084	.6830
5	.9515	.9057	.8626	.8219	.7835	.7473	.7130	.6806	.6499	.6209
10	.9053	.8203	.7441	.6756	.6139	.5584	.5083	.4632	.4224	.3855
15	.8613	.7430	.6419	.5553	.4810	.4173	.3624	.3152	.2745	.2394
20	.8195	.6730	.5537	.4564	.3769	.3118	.2584	.2145	.1784	.1486
25	.7798	.6095	.4776	.3751	.2953	.2330	.1842	.1460	.1160	.0923
30	.7419	.5521	.4120	.3083	.2314	.1741	.1314	.0994	.0754	.0573
50	.6080	.3715	.2281	.1407	.0872	.0543	.0339	.0213	.0134	.0085

N	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	.9009	.8929	.8850	.8772	.8696	.8621	.8547	.8475	.8403	.8333
2	.8116	.7972	.7831	.7695	.7561	.7432	.7305	.7182	.7062	.6944
3	.7312	.7118	.6931	.6750	.6575	.6407	.6244	.6086	.5934	.5787
4	.6587	.6355	.6133	.5921	.5718	.5523	.5337	.5158	.4987	.4823
5	.5935	.5674	.5428	.5194	.4972	.4761	.4561	.4371	.4190	.4019
10	.3522	.3220	.2946	.2697	.2472	.2267	.2080	.1911	.1756	.1615
15	.2090	.1827	.1599	.1401	.1229	.1079	.0949	.0835	.0736	.0649
20	.1240	.1037	.0868	.0728	.0611	.0514	.0433	.0365	.0303	.0261
25	.0736	.0588	.0471	.0378	.0304	.0245	.0197	.0160	.0129	.0105
30	.0437	.0334	.0256	.0196	.0151	.0116	.0090	.0070	.0054	.0042
50	.0054	.0035	.0022	.0014	.0009	.0006	.0004	.0003	.0002	.0001

N	21%	22%	23%	24%	25%	26%	27%	28%	29%	30%
1	.8264	.8197	.8130	.8065	.8000	.7937	.7874	.7813	.7752	.7692
2	.6830	.6719	.6610	.6504	.6400	.6299	.6200	.6104	.6009	.5917
3	.5645	.5507	.5374	.5245	.5120	.4999	.4882	.4768	.4658	.4552
4	.4665	.4514	.4369	.4230	.4096	.3968	.3844	.3725	.3611	.3501
5	.3855	.3700	.3552	.3411	.3277	.3149	.3027	.2910	.2799	.2693
10	.1486	.1369	.1262	.1164	.1074	.0992	.0916	.0847	.0784	.0725
15	.0573	.0507	.0448	.0397	.0352	.0312	.0277	.0247	.0219	.0195
20	.0221	.0187	.0159	.0135	.0115	.0098	.0084	.0072	.0061	.0053
25	.0085	.0069	.0057	.0046	.0038	.0031	.0025	.0021	.0017	.0014
30	.0033	.0026	.0020	.0016	.0012	.0010	.0008	.0006	.0005	.0004
50	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

N	31%	32%	33%	34%	35%	36%	37%	38%	39%	40%
1	.7634	.7576	.7519	.7463	.7407	.7353	.7299	.7246	.7194	.7143
2	.5827	.5739	.5653	.5569	.5487	.5407	.5328	.5251	.5176	.5102
3	.4448	.4348	.4251	.4156	.4064	.3975	.3889	.3805	.3724	.3644
4	.3396	.3294	.3196	.3102	.3011	.2923	.2839	.2757	.2679	.2603
5	.2592	.2495	.2403	.2315	.2230	.2149	.2072	.1998	.1927	.1859
10	.0672	.0623	.0577	.0536	.0497	.0462	.0429	.0399	.0371	.0346
15	.0174	.0155	.0139	.0124	.0111	.0099	.0089	.0080	.0072	.0064
20	.0045	.0039	.0033	.0029	.0025	.0021	.0018	.0015	.0014	.0012
25	.0012	.0010	.0008	.0007	.0006	.0005	.0004	.0003	.0003	.0002
30	.0003	.0002	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0000



APPENDIX **Capital Recovery Factors**

$$CR = \frac{i(1+i)^N}{(1+i)^N - 1}$$

$$SPW = \frac{1}{CR}$$

A = (CR)P P = (SPW)A

N	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000
2	.5076	.5155	.5226	.5303	.5376	.5455	.5529	.5606	.5685	.5760
3	.3401	.3466	.3534	.3604	.3671	.3741	.3811	.3880	.3951	.4021
4	.2564	.2625	.2691	.2735	.2820	.2886	.2952	.3019	.3086	.3155
5	.2062	.2121	.2183	.2246	.2309	.2374	.2439	.2505	.2371	.2638
10	.1056	.1113	.1172	.1233	.1295	.1359	.1424	.1490	.1558	.1627
15	.0721	.0778	.0838	.0899	.0963	.1030	.1098	.1168	.1241	.1315
20	.0554	.0612	.0672	.0736	.0802	.0872	.0944	.1018	.1095	.1175
25	.0454	.0512	.0574	.0640	.0710	.0782	.0858	.0937	.1018	.1102
30	.0387	.0447	.0510	.0578	.0651	.0726	.0806	.0888	.0973	.1061
50	.0255	.0318	.0389	.0465	.0548	.0634	.0725	.0817	.0912	.1009

N	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	1.1100	1.1200	1.1300	1.1400	1.1500	1.1600	1.1700	1.1800	1.1900	1.2000
2	.5839	.5917	.5994	.6074	.6150	.6231	.6308	.6388	.6467	.6545
3	.4092	.4164	.4236	.4308	.4380	.4453	.4526	.4599	.4673	.4747
4	.3223	.3292	.3362	.3432	.3503	.3574	.3646	.3717	.3790	.3863
5	.2706	.2774	.2843	.2913	.2983	.3054	.3126	.3198	.3270	.3344
10	.1698	.1770	.1843	.1917	.1993	.2069	.2146	.2225	.2305	.2385
15	.1391	.1468	.1547	.1628	.1710	.1794	.1878	.1964	.2051	.2139
20	.1256	.1339	.1424	.1510	.1598	.1687	.1777	.1868	.1960	.2054
25	.1187	.1275	.1364	.1455	.1547	.1640	.1734	.1829	.1925	.2021
30	.1150	.1241	.1334	.1428	.1523	.1619	.1715	.1813	.1910	.2008
50	.1106	.1204	.1303	.1402	.1501	.1601	.1701	.1801	.1900	.2000

N	21%	22%	23%	24%	25%	26%	27%	28%	29%	30%
1	1.2100	1.2200	1.2300	1.2400	1.2500	1.2600	1.2700	1.2800	1.2900	1.3000
2	.6625	.6705	.6785	.6865	.6944	.7025	.7105	.7187	.7266	.7348
3	.4822	.4897	.4972	.5047	.5123	.5199	.5275	.5352	.5429	.5507
4	.3936	.4010	.4083	.4159	.4234	.4310	.4386	.4462	.4539	.4616
5	.3417	.3492	.3567	.3642	.3719	.3795	.3872	.3949	.4027	.4106
10	.2467	.2549	.2632	.2716	.2801	.2886	.2972	.3059	.3147	.3235
15	.2228	.2317	.2408	.2499	.2591	.2684	.2777	.2871	.2965	.3060
20	.2147	.2242	.2337	.2433	.2529	.2626	.2723	.2820	.2918	.3016
25	.2118	.2215	.2313	.2411	.2510	.2608	.2707	.2806	.2905	.3004
30	.2107	.2206	.2305	.2404	.2503	.2603	.2702	.2802	.2901	.3001
50	.2100	.2200	.2300	.2400	.2500	.2600	.2700	.2800	.2900	.3000

N	31%	32%	33%	34%	35%	36%	37%	38%	39%	40%
1	1.3100	1.3200	1.3300	1.3400	1.3500	1.3600	1.3700	1.3800	1.3900	1.4000
2	.7429	.7510	.7591	.7673	.7755	.7838	.7920	.8002	.8085	.8167
3	.5584	.5662	.5740	.5818	.5896	.5975	.6055	.6134	.6214	.6293
4	.4594	.4772	.4850	.4929	.5008	.5087	.5167	.5246	.5327	.5408
5	.4183	.4264	.4344	.4424	.4505	.4585	.4667	.4749	.4831	.4913
10	.3323	.3413	.3502	.3593	.3683	.3774	.3866	.3958	.4050	.4143
15	.3153	.3250	.3347	.3443	.3539	.3636	.3733	.3831	.3928	.4026
20	.3114	.3213	.3311	.3410	.3509	.3608	.3707	.3806	.3905	.4005
25	.3104	.3203	.3303	.3402	.3502	.3602	.3701	.3801	.3901	.4001
30	.3101	.3201	.3301	.3401	.3500	.3600	.3700	.3800	.3900	.4000

APPENDIX — Sinking Fund Factors

$$SF = \frac{i}{(1+i)^N - 1}$$

$A = (SF)F$ $\qquad$ $F = (SCA)A$ $\qquad$ $SCA = \frac{1}{SF}$

N	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.4975	.4950	.4926	.4902	.4878	.4854	.4831	.4808	.4785	.4762
3	.3300	.3268	.3235	.3203	.3172	.3141	.3111	.3080	.3051	.3021
4	.2463	.2426	.2390	.2355	.2320	.2286	.2252	.2219	.2187	.2155
5	.1960	.1922	.1884	.1846	.1810	.1774	.1739	.1705	.1671	.1638
10	.0956	.0913	.0872	.0833	.0795	.0759	.0724	.0690	.0658	.0627
15	.0621	.0578	.8538	.0499	.0463	.0430	.0398	.0368	.0341	.0315
20	.0454	.0412	.0372	.0336	.0302	.0272	.0244	.0219	.0195	.0175
25	.0354	.0312	.0274	.0240	.0210	.0182	.0158	.0137	.0118	.0102
30	.0287	.0246	.0210	.0178	.0151	.0126	.0106	.0088	.0073	.0061
50	.0155	.0118	.0089	.0066	.0048	.0034	.0025	.0017	.0012	.0009

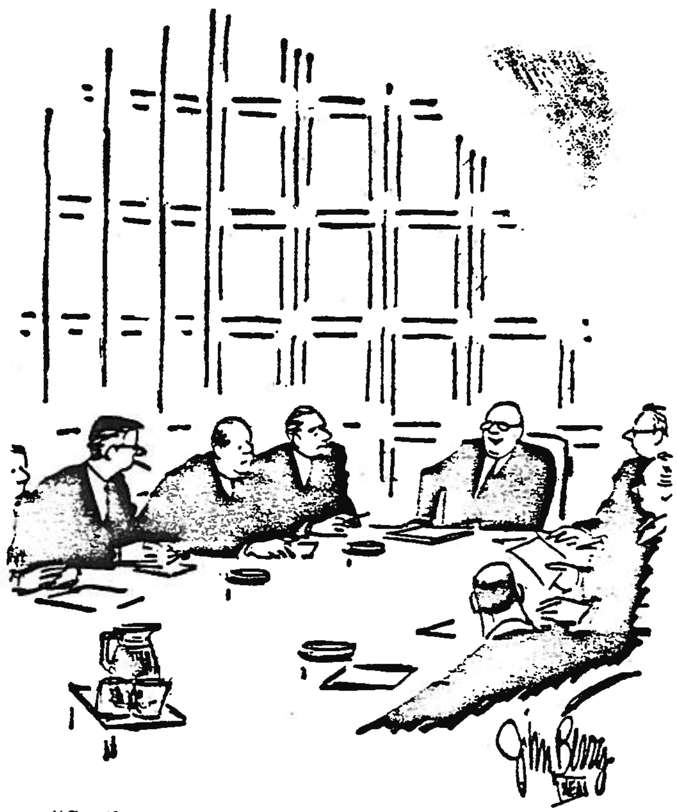
N	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.4739	.4717	.4695	.4673	.4651	.4630	.4608	.4587	.4566	.4545
3	.2992	.2963	.2935	.2907	.2880	.2853	.2826	.2799	.2773	.2747
4	.2123	.2092	.2062	.2032	.2003	.1974	.1945	.1917	.1890	.1863
5	.1606	.1574	.1543	.1513	.1483	.1454	.1426	.1398	.1371	.1344
10	.0598	.0570	.0543	.0517	.0493	.0469	.0447	.0425	.0405	.0385
15	.0291	.0268	.0247	.0228	.0210	.0194	.0178	.0164	.0151	.0139
20	.0156	.0139	.0124	.0110	.0098	.0087	.0077	.0068	.0060	.0054
25	.0087	.0075	.0064	.0055	.0047	.0040	.0034	.0029	.0025	.0021
30	.0050	.0041	.0034	.0028	.0023	.0019	.0015	.0013	.0010	.0008
50	.0006	.0004	.0003	.0002	.0001	.0001	.0001	.0000	.0000	.0000

N	21%	22%	23%	24%	25%	26%	27%	28%	29%	30%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	..4525	.4505	.4484	.4464	.4444	.4425	.4405	.4386	.4367	.4348
3	.2722	.2697	.2672	.2647	.2623	.2599	.2575	.2552	.2529	.2506
4	.1836	.1810	.1785	.1759	.1734	.1710	.1686	.1662	.1639	.1616
5	.1318	.1292	.1267	.1242	.1218	.1195	.1172	.1149	.1127	.1106
10	.0367	.0349	.0332	.0316	.0301	.0286	.0272	.0239	.0247	.0235
15	.0128	.0117	.0108	.0099	.0091	.0084	.0077	.0071	.0065	.0060
20	.0047	.0042	.0037	.0033	.0029	.0026	.0023	.0020	.0018	.0016
25	.0018	.0015	.0013	.0011	.0009	.0008	.0007	.0006	.0005	.0004
30	.0007	.0006	.0005	.0004	.0003	.0003	.0002	.0002	.0001	.0001
50	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

N	31%	32%	33%	34%	35%	36%	37%	38%	39%	40%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.4329	.4310	.4292	.4274	.4235	.4237	.4219	.4202	.4184	.4167
3	.2484	.2462	.2440	.2418	.2397	.2376	.2355	.2334	.2314	.2294
4	.1594	.1572	.1550	.1529	.1508	.1487	.1467	.1447	.1427	.1408
5	.1085	.1064	.1044	.1024	.1005	.0985	.0967	.0949	.0931	.0914
10	.0223	.0212	.0202	.0192	.0183	.0174	.0166	.0158	.0150	.0143
15	.0055	.0051	.0046	.0043	.0039	.0035	.0033	.0031	.0028	.0026
20	.0014	.0012	.0011	.0010	.0009	.0008	.0007	.0006	.0005	.0005
25	.0004	.0003	.0003	.0002	.0002	.0002	.0001	.0001	.0001	.0001
30	.0001	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000

EPILOGUE

A Closing Reflection on the Minimum Cost Criterion



"Gentlemen—we've achieved the ultimate in cost reduction . . . we've gone out of business!"