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ON THE DYNAMICS OF CABLES WITH APPLICATION TO MARINE USE

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ON THE DYNAMICS OF CABLES WITH APPLICATION TO MARINE USE

by

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ABSTRACT

Cables play an important role in ship operations, whether in anchoring, in docking, in towing, or in passing through locks. The frequent casualties that result from cable failure attest to the need for a better understanding of cable dynamics. This paper explains analytical techniques that can be used to assess the adequacy of cables of all types. The classical methods for treating static conditions are reviewed. Then dynamic conditions are analyzed in recognition of transient loads applied to cables (e.g. the surging of a ship at anchor). Water resistance and cable elasticity are also recognized. The paper concludes with a numerical example showing how the foregoing principles can be applied to a typical situation of a ship at anchor.

CONTENTS

INTRODUCTION

The term "cables" in shipbuilding and marine fields is understood to imply steel wire ropes, chains, synthetic ropes and cables made from the hemp or other plants which have tough fiber. Their main application is in mooring, towing or anchoring.

Analysis of the forces acting in cables is an important problem of the design of ships' outfitting and mistakes in calculations often have disastrous consequences. There is no need to give many examples because some of them are known to every naval architect. It is enough to mention that according to U. S. Coast Guard annual casualties figures in 1969 nearly 600 ship casualties had the failures of anchor, towing and mooring equipment as primary cause or contributing factors to cause of casualty.

The main properties of the above listed cables are as follows:

<u>flexibility</u> - ability to bend without resistance,

<u>strength</u> - characterized by breaking or test load,

<u>weight</u> - always given per unit length of the cable,

<u>elasticity</u> - defined for given load by the elongation of cable related to its initial length. The nonlinearity of the elasticity with respect to the load makes the problem of its account a very difficult one.

Analysis of the forces or stresses in cables can be divided into two parts: static and dynamic. Static solution involves fewer parameters and less complicated mathematical tools. In many cases it gives satisfactory results and helps on the basis of accumulated experience to make correct decisions.

Unfortunately this approach is not always possible, particularly when solving comparitively new problems, such as deep sea mooring, towing in open sea, etc. The solution of the dynamic problems gives as a by-product a better understanding of known but unexplained facts. At the same time we have to mention that dynamic analysis always follows the static solution as a first initial step.

PART I: SUMMARY OF STATICS

The three main problems that form the static solution of cables are:

1) Equilibrium of heavy flexible cord.
2) Equilibrium of heavy flexible cord in uniform stream.
3) Equilibrium of non-uniform heavy cord.

The solution of the first problem gives the well known catenary equation

$$y = \frac{T_o}{\gamma}\left(\cosh \frac{\gamma x}{T_o} - 1\right) \tag{1}$$

where T_o - horizontal component of tension (Figure 1)
γ - weight of cord of unit length

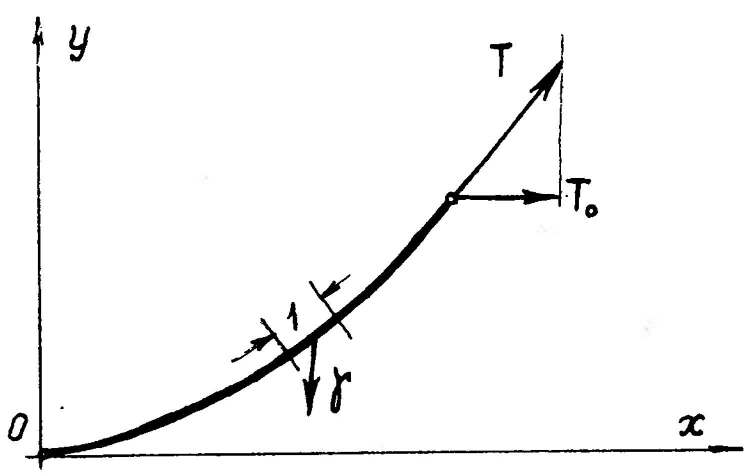

Figure 1

The main features of the catenary form are:

1) The horizontal component of tension is constant.

2) The minimum magnitude of cord tension is equal to the horizontal component of tension T_o.

3) The tension at a given point of the cord is linearly related to the y coordinate at the same point

$$T = T_o + \gamma y \qquad (2)$$

The length of the cord (with respect to the origin O) to the point with coordinate (y) can be given as

$$S = \frac{T_o}{\gamma}\sqrt{\left(\frac{\gamma y}{T_o}\right)^2 - 2\left(\frac{\gamma y}{T_o}\right)} \qquad (3)$$

If we rearrange the coordinate system as indicated in Figure 2 the equation (1) may be transformed as follows

$$y = 2\frac{T_o}{\gamma}\sinh\frac{\gamma x}{2T_o}\cdot\sinh\frac{\gamma(\ell - x)}{2T_o} \qquad (4)$$

where ℓ - distance between two points of support.

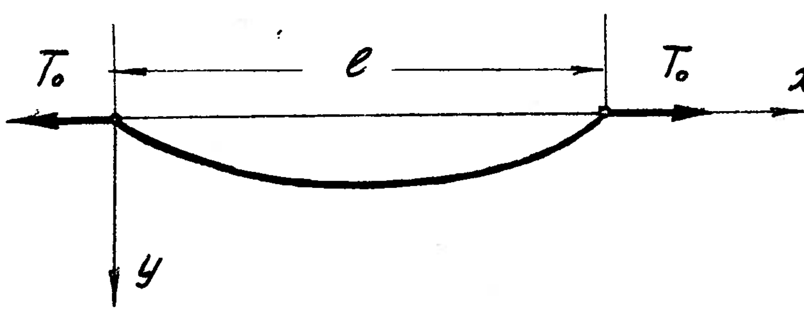

Figure 2

Formula

$$S = \frac{2T_o}{\gamma}\sinh\frac{\gamma\ell}{2T_o} \qquad (5)$$

gives the relationship between S, T, ℓ and γ.

For the part of the catenary curve restricted by two supporting points $(x_1 y_1)$ and $(x_2 y_2)$ [Figure 3] we can write

$$\cosh \frac{\gamma x_2}{T_o} - \cosh \frac{\gamma x_1}{T_o} = H \frac{\gamma}{T_o} \qquad (6)$$

$$\sinh \frac{\gamma x_2}{T_o} - \sinh \frac{\gamma x_1}{T_o} = S \frac{\gamma}{T_o}$$

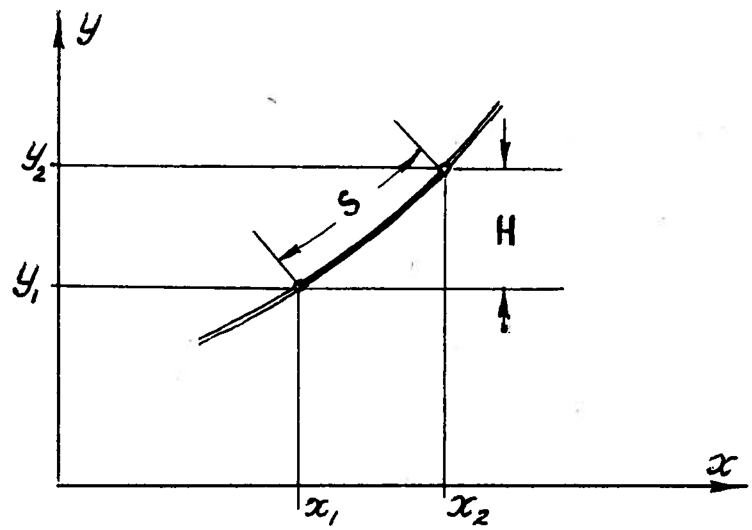

Figure 3

The solution of this system with respect to x_1 and x_2

$$x_1 = \frac{T_o}{\gamma} \ln \left\{ \sqrt{\left[\frac{\gamma}{2T_o}(S+H)\right]^2 + \frac{S+H}{S-H}} - \frac{\gamma}{2T_o}(S+H) \right\}$$

$$x_2 = \frac{T_o}{\gamma} \ln \left\{ \sqrt{\left[\frac{\gamma}{2T_o}(S+H)\right]^2 + \frac{S+H}{S-H}} + \frac{\gamma}{2T_o}(S+H) \right\} \qquad (7)$$

For a given value of $(x_2 - x_1) = \ell$ these equations can help to find one of the three parameters: S, H and T_o when the other two are known.

When the relation $\frac{T_o}{\gamma}$ is large, equation (1) can be replaced by a simpler formula

$$y = \frac{\gamma}{2T_o}(x\ell - x^2) \, . \qquad (8)$$

The horizontal component of the tension then can be found as

$$T_o = \frac{\gamma \ell^{1,5}}{\sqrt{24(S-\ell)}} \, . \qquad (9)$$

The equilibrium of a heavy, flexible cord in a uniform stream can be described by system of differential equations

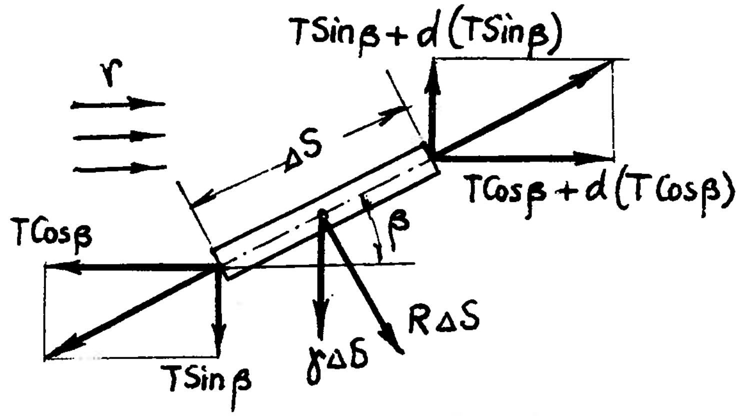

Figure 4

$$dT = \gamma dy$$

$$Td\beta = (c\,\mathrm{Sin}\beta + \gamma\,\mathrm{ctg}\,\beta)\,dy \qquad (10)$$

where $c = c'\frac{\rho v^2}{2}d$ from $R = c'\frac{\rho v^2}{2}d\,\mathrm{Sin}^2\beta$

c' = resistance coefficient

ρ = mass density of water

d = diameter of cord

The solution follows as

$$T = T_o + \gamma y \quad (\text{using } T = T_o \text{ when } y = 0)\ .$$

From the second equation

$$T = C\left(\frac{2\lambda\,\mathrm{Cos}\beta - 1 - \sqrt{4\lambda^2+1}}{2\lambda\,\mathrm{Cos}\beta - 1 + \sqrt{4\lambda^2+1}}\right)^{\frac{1}{\sqrt{4\lambda^2+1}}} \qquad (11)$$

where $\lambda = \frac{C}{\gamma}$.

To obtain the constant C we can apply the surface boundary condition

$$T \cos\beta = P_o$$

where P_o - horizontal force acting upon the ship.

For the case of non-uniform stream with a given stepwise velocity distribution the configuration and tension of the cord can be found as a combination of solutions for the depth intervals with constant speed. (Fig. 5)

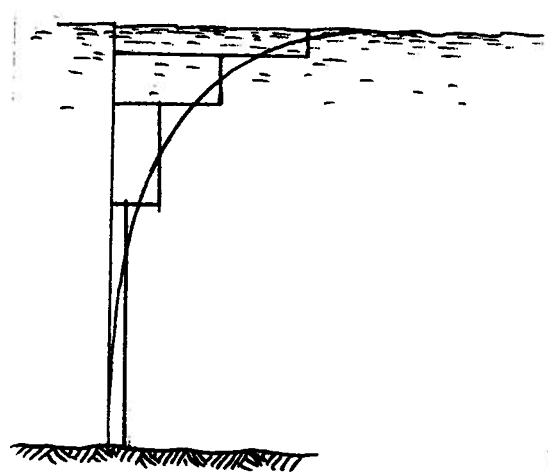

Figure 5

The analysis of formulas (2), and (11) shows that to a very large extent the tension depends upon the cable weight and the vertical distance between the supporting points. The most rational type of cable from this point of view is one with the constant stress

$$\sigma = \frac{T}{F} = Const. \tag{12}$$

Using (12) we can establish a relation between the tension (T) and γ (weight of the unit length of cable)

$$T = \sigma F = \frac{\gamma}{z} \tag{13}$$

where z - is a coefficient, which can be easily calculated for each type of cable.

The differential equations of equilibrium can be written as

$$\frac{dT}{dy} = \gamma \; ; \; \frac{d\beta}{dy} = \frac{\gamma}{T} ctg\beta \tag{14}$$

or, taking into account (13)

$$\frac{dT}{dy} = zT \; ; \; \frac{d\beta}{dy} = z\, ctg\, \beta \tag{15}$$

The final results of solution are given as:

$$\begin{aligned} T &= C_1 e^{zy} \\ \cos\beta &= C_2 e^{-zy} \\ y &= \frac{1}{z} \ln \frac{C_2}{\cos(zx + C_3)} \\ S &= \frac{1}{z} \ln \left(C_2 e^{zy} \sqrt{C_2^2 e^{2zy} - 1}\right) + C_4 \end{aligned} \tag{16}$$

where C_1, C_2, C_3 and C_4 are constants, which can be found accordingly to the boundary conditions of the particular problem to be solved.

PART II: THE DYNAMICS OF CABLES

In actual conditions of marine application cables for the most part are subjected to dynamic loading due to oscillations and jerks that are unavoidable when mooring, towing, etc. Using the simplified mathematical model we can replace the motion of real ships by the fluctuation of one of the cord supporting points. Taking into account the high order of forces causing ships to heave and pitch we can accept the assumption that the parameters of the motion of supporting point are independent of the cord motion parameters. Very frequently the displacements of the supporting point can be thought of as small in comparison to the length of cord. This enables us to make several additional useful assumptions. One of them is that the displacements of the cord from position of its static equilibrium are small and we can use the static configuration as a form of cables involved in dynamic motion.

We must mention another important assumption. For a cord hanging freely between two supports positioned on the same level we assume that the "suspension" (distance from the lowest point of cord to the level of supports) is small. This assumption is justified because we are interested in the load combination which produces the maximum stress. The corresponding maximum static tension invariably leads to the small suspension. This assumption leads to considerable mathematical simplifications such as

$$\cos\beta \rightarrow 0, \quad \Delta S \sim \Delta x, \quad T \sim T_0. \qquad (17)$$

Let us evaluate the range of suspension values where we can accept this assumption without losing much accuracy. Using the simplified equation of cable form

$$y = \frac{\gamma}{2T_o}\left(\ell x - x^2\right) \tag{18}$$

we can see that the tangent of β

$$tg\beta = \frac{dy}{dx} = \frac{\gamma}{2T_o}\left(\ell - 2x\right)$$

with maximum value

$$tg\beta_{max} = \frac{\gamma\ell}{2T_o} \tag{19}$$

If we denote the suspension as y_{max}, then from (18)

$$T_o = \frac{\gamma\ell}{8\, y_{max}} \tag{20}$$

Substituting T_o in (19) we get

$$tg\beta_{max} = \frac{4y_{max}}{\ell} ,$$

or

$$\sqrt{\frac{1}{Cos^2\beta_{max}} - 1} = \frac{4y_{max}}{\ell}$$

Taking into account comparitively small influence of the parts of cable near its supports and accepting 10% inaccuracy the above mentioned assumption is valid for $\frac{Y_{max}}{\ell}$ in the range 0 to 0.167, $(\frac{1}{6})$. This is quite sufficient for all practical purposes. Let us now prove that the case of the cord with small suspension or "flat cord" is most interesting from the point of dynamic also.

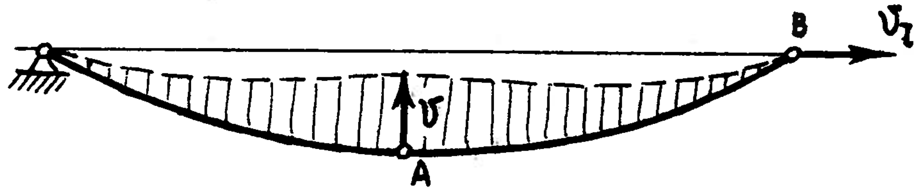

Figure 6

When the point of support moves horizontally with certain speed (in Figure 6 the right point B moves the right with speed V_z) all points of cord acquire the speed with distribution of its vertical components as shown in Figure 6. The relation between V_z and amplitude meaning of cord speed (V point A) can be established on energy basis

$$T_o V_l = \gamma \int_0^l v \sin\frac{\pi x}{e}\, dx, \qquad (21)$$

where $v \sin\frac{\pi x}{e}$ approximately described the distribution of the vertical components of cord speed. After integration

$$T_o V_l = 2\gamma v \frac{l}{\pi} \qquad (22)$$

with the help of (20) we get

$$v = V_l \frac{\pi l}{16 y_{max}} = \frac{1}{5} V_l \frac{l}{y_{max}} \qquad (23)$$

Note that when $\frac{y_{max}}{e}$ is decreasing, the vertical speed of the cord increases with a corresponding increase in resistance and inertia forces.

Summarizing,

1. The change in cord tension has neglected effect upon forces which generate cord oscillation.

2. The static form of cord may be used to describe position of the cord in dynamic motion.

3. We shall consider the flat cord, whose suspension is small in comparison with length.

A. Motion of flexible, non-elastic cord generated by fluctuation of supporting point

The given displacement of supporting point can be resolved in vertical and horizontal parts. Let us investigate the motion caused by each component seperately. We might as well start from the vertical motion of support.

Al. Considering the balance of forces acting on an infinitesimal cord element with displacement y(x,t) from its static position we have

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \tag{24}$$

where $a = \sqrt{T\frac{q}{\gamma}}$ and q - acceleration of freely rolling body. When t = 0 we may assume linear distribution of vertical speed therefore the initial conditions are

$$y\Big|_{t=0} = 0\ ; \quad y'\Big|_{t=0} = \frac{2h\pi}{\tau}\,\frac{x}{l} \ . \tag{25}$$

The boundary conditions

$$y\Big|_{x=0} = 0\ ; \quad y\Big|_{x=l} = h\sin\frac{2\pi t}{\tau} \tag{26}$$

where τ and h - period and amplitude of support motion. Solution of (24) can be found as a sum

$$y = U - U_o$$

where U_o - arbitrary function satisfying given boundary condition.

For example $U_o = \varphi(x) \sin \frac{2\pi t}{\tau}$ to obtain $\varphi(x)$ may be used the form of cord in limit condition, when $T \to \infty$

$$U_o = \frac{hx}{l} \sin \frac{2\pi t}{\tau},$$

and

$$y = U - \frac{hx}{l} \sin \frac{2\pi t}{T} \tag{27}$$

After substitution in (24) we get

$$\frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 U}{\partial x^2} - \chi \sin \frac{2\pi t}{\tau}; \quad \chi = \frac{h}{l}\left(\frac{2\pi}{\tau}\right)^2 \tag{28}$$

The function U can be introduced as a sum of free and forced oscillations. The final result of solution is

$$y = \sum_n \frac{2h}{n\omega_n}\left[\frac{4}{\tau} \sin \omega_n t + \frac{2\frac{\pi}{\tau} \sin \omega_n t - \omega_n \sin \frac{2\pi t}{\tau}}{1 - \left(\frac{\omega_n \tau}{2\pi}\right)^2}\right] \sin \frac{n\pi x}{l} - \frac{hx}{l} \sin \frac{2\pi t}{\tau} \tag{29}$$

where $\omega_n = \frac{na\pi}{l}$.

The average meaning of inertia forces is

$$q_{av} = \frac{\int_0^S \frac{\gamma}{q} \ddot{y}_t \, dx}{S} \tag{30}$$

where $\ddot{y}_t$ - the second derivative with respect to time.
The additional tension due to inertia forces is

$$\Delta T_o \approx \Delta T = \frac{q_{av} S}{2} \left(\frac{S}{y_{max}}\right)\left(1 - \frac{y_{max}^2}{S^2}\right) \tag{31}$$

A2. Let us now consider the effect of horizontal displacement of supporting point. The solution of this problem in full details practically is outside the reach of contemporary mathematical methods. Without losing much generality we may accept several new assumptions:

1. The relation between vertical speed of cord mid-point and horizontal speed of support is limited by

$$V = \frac{1}{5} V_l \frac{l}{y_{max}}$$

2. Only vertical component of cord motion must be taken into account.

With the account of all mentioned, the differential equation of cord motion can be written as

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} + f(x,t), \tag{32}$$

where $f(x,t) = -\frac{\partial^2 y_o}{\partial t^2}$ and y_o - function of forced motion of cord.

The initial and boundary conditions are

$$y\Big/_{x=0} = y\Big/_{x=e} = y\Big/_{t=0} = y'\Big/_{t=0} = 0$$

The solution may be found in the form

$$y(x,t) = \sum_{n=1}^{\infty} T_n(t) \sin \frac{n\pi x}{l}. \tag{33}$$

After substituting (33) in (32) we obtain

$$\sum \left[T_n''(t) + \omega_n^2 T_n(t) \right] \sin \frac{n\pi x}{l} = f(x,t) \tag{34}$$

if for the expression inside brackets we have

$$T_n''(t) + \omega_n^2 T_n(t) = f_n(t)$$

where

$$f_n(t) = \frac{2}{l} \int_0^l f(x,t) \sin \frac{n\pi x}{l} dx$$

then

$$T_n(t) = \frac{1}{\omega_n}\int_0^t f_n(t)\sin\omega_n(t-t_1)\,dt_1 \, .$$

Assuming that $y_0(x,t) = h\sin\frac{2\pi t}{\tau}\varphi(x)$ we get

$$f_n(t) = \frac{2h}{l}\left(\frac{2\pi}{\tau}\right)\sin\frac{2\pi t}{\tau}\int_0^l \varphi(x)\sin\frac{n\pi x}{l}dx \quad (35)$$

If, for instance $\varphi(x) = \sin\frac{\pi x}{l}$ then for all $n \neq 1$ the integral in the last expression is equal to zero, and

$$y(x,t) = \frac{lh}{a\pi}\,\frac{\frac{2\pi}{\tau}\sin wt - w\sin\frac{2\pi t}{\tau}}{1-\left(\frac{w\tau}{2\pi}\right)^2}\sin\frac{\pi x}{l} \quad (36)$$

where $w = \frac{a\pi}{l}$

B. Motion of suspended cord with account of water resistance

In the above solutions it has been taken for granted that air resistance has negligible effect on the parameters of cord motion. More frequently we have to steady the behavior of submerged cable when water resistance substantially affects its form and tension. Here again the cases of horizontal and vertical motion of the cord supporting point are treated separately.

B1. With the account of all previous assumptions and neglecting the inertia forces, the equation of cord motion generated by vertical fluctuation of its support can be written as

$$\frac{\partial^2 y}{\partial x^2} = a^2\left(\frac{\partial y}{\partial t}\right)^2 , \quad a^2 = \frac{c}{T_0} = c'\frac{\rho d}{2}\frac{1}{T_0} \, . \quad (37)$$

This equation describes the motion of cord in close proximity to the position of its static equilibrium, where inertia forces are small. Comparably small change of the speed in mentioned period justifies the linearization of the resistance

$$\frac{\partial^2 y}{\partial x^2} = a^2 \frac{\partial y}{\partial t} \tag{37a}$$

with corresponding adjustment of coefficient a^2.

For the initial and boundary conditions

$$y\Big/_{t=0} = 0\,; \quad y\Big/_{x=0} = y\Big/_{x=l} = h \sin \frac{2\pi t}{\tau} \tag{38}$$

the final solution of (37a) can be written as

$$y = \frac{4h}{\pi} \sum_{1,3,5\ldots}^{\infty} \frac{\omega_n \sin \frac{n\pi x}{l}}{n\left[\omega_n^2 + \left(\frac{2\pi}{\tau}\right)^2\right]} \left[\omega_n \sin \frac{2\pi t}{\tau} - \frac{2\pi}{\tau} \cos \frac{2\pi t}{\tau} + \frac{2\pi}{\tau} e^{-\omega_n t}\right] \tag{39}$$

where

$$\omega_n = \frac{n\pi}{la}$$

The numerical analysis of the solution (39) indicated that for the practical range of parameters T_o, ℓ, d only very small part of the cord (approximately 1/20 of length) near the point of support is involved in motion.

Consequently the cord motion generated by vertical fluctuation of the supporting point doesn't cause the substantial change of tension and may be neglected.

B2. Let us now consider the role of horizontal fluctuation of the support. Similarly to the previous case for all parts of cord we will take into account only vertical components of motion, assuming for the vertical speed the following distribution

$$V_x = V \sin \frac{\pi x}{l} \tag{40}$$

The differential equation of the cord configuration for the moment when V reaches its maximum value is

$$\frac{d^2y}{dx^2} = -\frac{cV^2}{T_o}\sin^2\frac{\pi x}{l} - \frac{\gamma}{T} \quad . \tag{41}$$

After integrating this equation twice, we have

$$y = \frac{cV^2}{T_o}\left[\frac{lx}{4} - \frac{x^2}{4} + \frac{l^2}{8\pi^2}\left(1-\cos\frac{2\pi x}{l}\right)\right] + \frac{\gamma}{T_o}\left(\frac{xl}{2} - \frac{x^2}{2}\right) \tag{42}$$

where the following boundary conditions were used

$$y = 0, \text{ when } x = 0 = \ell$$

$$\frac{dy}{dx} = 0, \text{ when } x = \frac{\ell}{2} \quad .$$

Introducing $\varkappa = \frac{y_{max}}{e}$ as a "form parameter", where

$$y = y_{max} \text{ when } x = \frac{\ell}{2} \quad ,$$

instead of (42) we have

$$\varkappa = \frac{1}{T_o}\left(\frac{cV^2l}{11,4} + \frac{\gamma l}{8}\right)$$

$$T_o = \frac{1}{\varkappa}\left(\frac{cV^2l}{11,4} + \frac{\gamma l}{8}\right) = \frac{1}{\varkappa}\frac{cV^2l}{11,4} + \frac{\gamma l}{8\varkappa}$$

The second form of the sum represents the static tension of the suspended cord ($T_{o\ st}$) therefore

$$T_o = T_{o\,st} + \frac{cV^2l}{11,4\varkappa} \tag{44}$$

After replacing v by (23) we have

$$T_o = T_{o\,st} + \frac{cl}{270\varkappa^3}V_l^{\ 2} \tag{45}$$

Several numerical examples of calculation were made for the practical problems of towing and mooring. They indicated that figures given by formula (45) are far outside the breaking load limits for all existing wire ropes and chains. The careful analysis suggested that only one assumption - non-elasticity of the cord - may be taken as a reason for the above controversy. In fact, all types of existing cables are stretchable enough to cover considerable part of displacement of the supporting point.

B3. Effect of elasticity on the cord tension

In view of the fact that the assumption of rigidity leads to an unrealistic result we can suggest that the displacement of cord supporting point may be represented as a sum of two components: stretch of the cord and change of its configuration. These two parts are tightly interconnected, because the stretch of cord is always the result of the increase of tension caused by change of form and corresponding water resistance.

Let us denote the speed component related to the change of form as V, and the second component due to the cord stretch as V_s. The additional tension (ΔT) can be expressed as follows:

$$\Delta T = \frac{c\,l}{270\,\text{æ}^3}\left(\Delta V\right)^2 \qquad (46)$$

Cord elongation

$$\Delta S = \Delta T \frac{S}{\mathcal{E} d^2} \qquad (47)$$

where S - length of cable

$\mathcal{E}$ - modulus of elasticity

Assuming that S ≈ l

$$V_s = \frac{c}{\text{æ}^3}\,\frac{l^2}{270\,\mathcal{E} d^2}\,\frac{dV^2}{dt} \qquad (48)$$

and finally the sum of V_s and V must be equal to V_l

$$V + V_s = V_l$$

$$V + \mu \frac{dV^2}{dt} = V_l(t) \tag{49}$$

where

$$\mu = \frac{cl}{270\,\varkappa^3 \varepsilon d^2} \tag{50}$$

For the most part, the function at the right hand side of (49) is periodic with τ as time of full cycle. Because the maximum value of ΔT is the center of our interest, solution may be limited to the interval of time when the distance between supporting points increases. Within this interval the speed V_z is positive and it changes from 0 to its maximum value and back. Assuming the linear dependence of the speed on time we can write two separate equations

$$V + \mu \frac{dV^2}{dt} = V_l \frac{4t}{\tau} \quad ; \; 0 < t < \frac{4t}{\tau} \tag{51}$$

$$V + \mu \frac{dV^2}{dt} = V_l\left(1 - \frac{4t}{\tau}\right); \; \frac{4t}{\tau} < t < \frac{2t}{\tau} \tag{52}$$

The first equation can be solved using the substitution

$$V = bt$$

The final result is

$$\frac{V}{V_l} = \frac{\tau}{16\mu V_l}\left[\left(\frac{32\mu V_l}{\tau} + 1\right)^{0.5} - 1\right] t \tag{53}$$

The maximum ratio $\frac{V}{V_l}$ according to (53) is when $t = \frac{\tau}{4}$

$$\frac{V}{V_l} = \frac{1}{2\varphi}\left(\sqrt{4\varphi + 1} - 1\right), \tag{54}$$

where

$$\varphi = \frac{8\mu V_l}{\tau}.$$

The second equation (52) after substitution $V^2 = y$ can be presented in general form

$$y' = f(x)y^n + q(x)y + h(x)$$

with solution

$$y = \left[\frac{h(x)}{f(x)}\right]^{\frac{1}{n}} u_1 \quad .$$

Omitting all intermediate operations the final result may be presented as

$$\left(\frac{4t}{\tau} - 1\right) = C \frac{\exp\left[\frac{1}{\sqrt{4\varphi-1}} \operatorname{arctg} \frac{2\varphi z + 1}{\sqrt{4\varphi - 1}}\right]}{(\varphi z^2 + z + 1)^{0,5}} ; \quad (\varphi > 0{,}25) \qquad (55)$$

$$\left(\frac{4t}{\tau} - 1\right) = C \frac{\left[\frac{2\varphi z + 1 - \sqrt{1-4\varphi}}{2\varphi z + 1 + \sqrt{1 - 4\varphi}}\right]^{\frac{1}{\sqrt{1-4\varphi}}}}{(\varphi z^2 + z + 1)^{0,5}} ; \quad (\varphi < 0{,}25) \qquad (56)$$

where

$$z = \frac{V}{V_l} \cdot \frac{1}{\frac{4t}{\tau} - 1} \quad .$$

The constant C may be obtained using the initial condition for the second interval:

$$z = -\frac{V}{V_l} = -\frac{1}{2\varphi}\left(\sqrt{4\varphi + 1} - 1\right) \; ; \quad t = 0.$$

Unfortunately the solution doesn't establish the direct relationship between the speed ratio $\frac{V}{V_l}$ and time. This ratio depends upon parameter φ and time, its maximum value is dependent only on parameter φ. The corresponding inter-relations are presented in Figures 7 and 8 in graphical form.

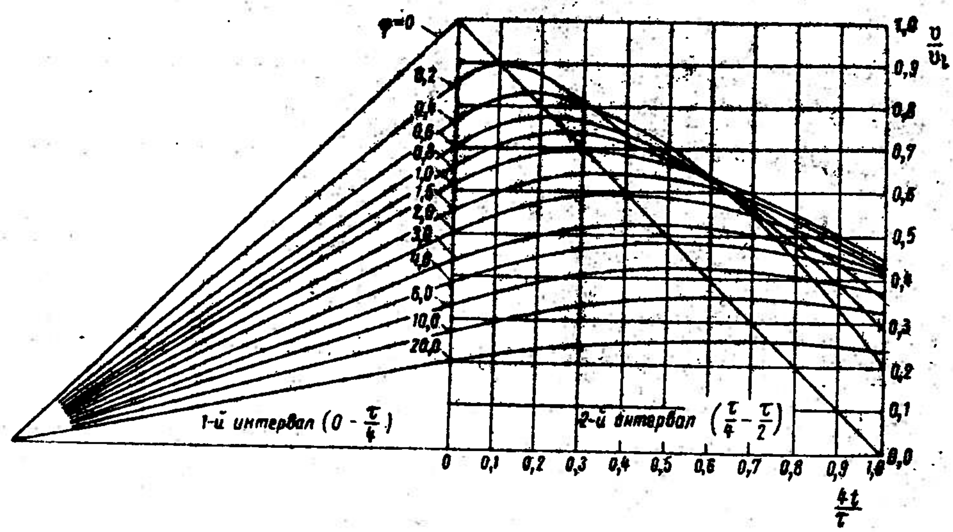

Figure 7

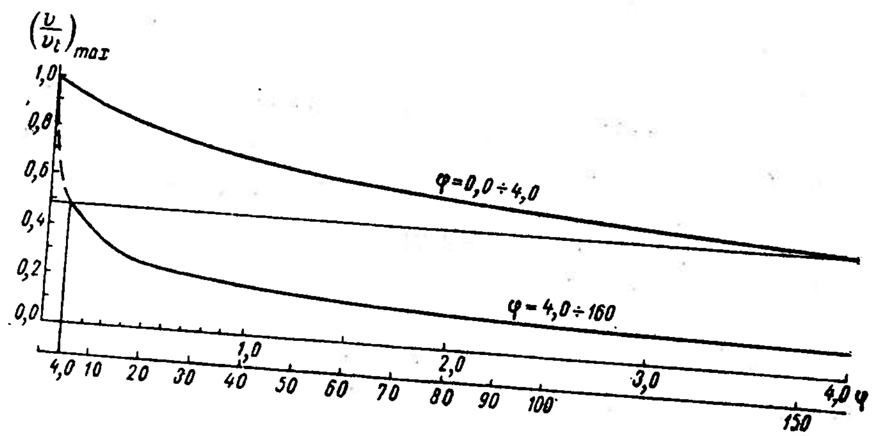

Figure 8

After obtaining the maximum magnitude of speed related to change of form V_{max}, we can find the maximum additional dynamic tension

PART III: SOME PRACTICAL CONSIDERATIONS

Any calculating method would be of no use without two elements -- the forces acting on the system and corresponding safety factor. This study was initiated after an unrealistically large safety factor was obtained when the conventional anchorage problem was analyzed from the static point of view. When the new technique described here was applied to this problem the safety factor was within practical limits and what was even more important, it was almost constant for the whole range of ship size covered by the Rules for construction of Register of Shipping of U.S.S.R.:

$$\frac{T_B}{T_{max}} = 2{,}0 \div 2{,}3 \qquad (57)$$

where T_B - breaking or test load,

$T_{max} = T_{st.} + \Delta T_{max}$.

The length of towing cable may be taken as distance between ships. The dynamic component of towing cable tension can be obtained as

$$\Delta T = \frac{T_B}{2{,}2} - T_s$$

where 2.2 - safety factor.

Using expression (55) and the magnitude of dynamic tension ΔT the speed ratio $\frac{V}{Ve}$ may be found and consequently can be calculated for all possible combinations of τ and h for the previously given wind speed.

In 1949 A. N. Krilov introduced the assumption that the parameters of ship motion in vertical longitudinal plane are equal to the period and amplitude of water particles involved in wave motion. Acceptance of this assumption here leads to a conservative mistake, which is bigger for bigger ships.

For the calculations of T_{max} were taken the following conditions:

wind speed - 50 knots,
deepness - 80-100 m.,
head sea with the wave length equal to the length of ship.

When applying given above safety factor (sf), we guarantee the same reliability standard as accepted by contemporary construction rules for anchor cables.

Towing cables - Application of the method described here may help to solve two main problems of towing: determination of possible wind and sea conditions when using towing cable with given diameter; and determination of the size of towing cable for given sea and wind conditions.

In the first case the following calculation procedure may be given:

1) Definition of the towed ship resistance as a function of its speed and speed of wind (T_{st}).

2) Towing cable form parameter can be found using expression (8).

APPENDIX

Calculation of anchor chain tension
(numerical example)

Ship particulars: Length $L = 120$ m.

Breadth $B = 14$ m.

Draft $D = 7.5$ m.

Block coefficient $\delta = 0.73$ (C_B)

$\beta = 0.96$ (C_M)

Sea deepness $H_s = 80$ m.

According to the Rules for ship construction of Register of Shipping of the U.S.S.R., a ship must be provided with an anchor chain:

length = 2 × 250 m.

diameter $d = 53$ mm $(\gamma = 52$ kq/m)

1) To obtain the static tension and form of anchor cable we assume that

V_o - wind speed = 25 m/sec (approx. 50 knots),
speed of current = 3 km/h (∿ 2kn)

Wind pressure $R_N = k_w S_f V_w^{\,2}$

where $k_w = 0.0001$
S_f = modified ship front area

$$S_f = S_s + 0.3S_h$$

S_s - front area of superstructures (95 m^2)

S_h - front area of ship's hull above waterline (49 m^2)

$$S_f = 110 \ m^2$$

V_w - effective speed of the wind.

Taking into account 50% increase of speed in the gust (V_q)

$$V_w = V_o \sqrt{1 + \frac{2}{\pi}\left(\frac{V_g}{V_o}\right) + \frac{1}{2}\left(\frac{V_g}{V_o}\right)^2} = 30 \ m/sec \ .$$

Finally

$$R_w = 9{,}9 \ t.$$

Hull resistance can be calculated using simplified formula

$$R_h = \beta \frac{BDV^2}{A}$$

where A - numerical coefficient ($A \sim 400$)

$$R_h = 2{,}3 \ t.$$

With the account of propeller resistance $R_h = 2.7$ t.

Force acting on one anchor chain

$$T_o = \frac{R_w + R_h}{2} = 6{,}3 \ t.$$

The suspended length (length of chain without its part on the bottom) is

$$S = H_s \sqrt{\frac{2T_o}{\gamma H_s} + 1} = 160 \ m.$$

The configuration of suspended part of chain is described by the equation

$$y = \frac{T_o}{\gamma}\left(ch \frac{\gamma x}{T_o} - 1\right) = 120\left(ch \frac{x}{120} - 1\right)$$

The calculations are given in the table

x	$\frac{\gamma x}{To}$	ch $\frac{\gamma x}{To}$	ch $\frac{\gamma x}{To}$ - 1	y
30	0.25	1.0314	0.0314	3.8
60	0.50	1.1276	0.1276	15.3
90	0.75	1.2947	0.2947	35.4
120	1.00	1.5431	0.5431	65.2
150	1.25	1.8884	0.8884	107.5

The form parameter $\varkappa = \frac{1}{7.85}$

The maximum static tension

$$T_{st} = T_o + \gamma H = 10,5t.$$

2) The additional dynamic tension

$$\Delta T = c \frac{v^2 S}{270 \varkappa^3}$$

where $C = \frac{c'pd}{2}$ for the chain the resistance coefficient $c'd = 1.B_c$ where B_c - the size of link (0.053 m · 36)

$$C = 9,7$$

The coefficient of elasticity Ed^2 can be obtained on the basis of the chain test results. The relative elongation for the test load (78t. for the chain under consideration) is equal to 5 - 5.5%.

So the elasticity factor $\mathcal{E}d^2$ = 1.560,000 kg. And finally

The parameters of the motion of supporting point (ship bow) are τ = 8.75 sec, V_l = 49 m/sec.

Speed ratio (Figure 8) $\frac{V}{V_l} = 0.64$

$$\Delta T = \frac{9,7(0,64 \cdot 4,9)^2 \cdot 160 \cdot 7,85}{270} = 27200 \text{ kg}$$

$$T_{max} = T_{st} + \Delta T = 37,7\,t.$$

The safety factor

$$n = \frac{T_B}{T_{max}} = \frac{78.0}{37.7} = 2,07$$

is in acceptable region.