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# ANALYSIS OF FERRO-CEMENT IN BENDING

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## ABSTRACT

My purpose in undertaking the work summarized in this paper has been to provide the designer with the basic information and methods with which to design ferro-cement structures. The premise throughout the paper is that the standard methods for analyzing reinforced concrete are applicable.

The paper begins with a description of the physical behavior of ferro-cement experiencing a bending moment. The strength analysis for working loads is covered, and a method is suggested for estimating the ultimate strength. The appendices include test data, strength calculations, and comparisons of calculations with test results.

My particular interest is in marine applications, hence it is my hope that this paper will permit the rational design of ferro-cement boats.

# PREFACE

The questions I hear most often about ferro-cement boats are still: How could it float? Won't it crack? Won't it be too heavy? This appears particularly ironic, for although it is common knowledge that ferro-cement small craft and full sized concrete ships have been constructed for some time, the naval architect who wishes to design a ferro-cement structure to a given loading has very little to go on. Not only is there a shortage of strength data, but also it is difficult to estimate the strength of different types of ferro-cement configurations. The naval architect needs (1) some general qualitative data on behavior and weights, (2) quantitative strength data, and (3) an analysis method for estimating the strength of configuration for which there is no test data.

My principal objective in this paper is to focus on the third category, or specifically, to find an analysis method for ferro-cement in bending.

Rather than assume that ferro-cement is a homogeneous material and then attempt to find an allowable stress, I have reverted to reinforced concrete techniques. I have used working stress analysis to estimate the stress under normal working loads, and ultimate strength analysis for predicting the ultimate strength.

The approach for this work has been to first test the ferro-cement components. That is, to test the mortar, wire mesh, and reinforcing rods separately. Secondly, to fabricate ferro-cement specimens and test them in bending, and thirdly, to compare the test results with the calculated strength. The test results, calculations, and comparisons are given in the appendices whereas the theory is given in the main body of the paper.

In general the predictions are conservative, and the data scatter is within the limits normally encountered in reinforced concrete tests. Hence the methods outlined could be used for analyzing marine structures.

In particular, one could design to the loads, if they are known. In the absence of loading information the structure could be designed to the equivalent strength or stiffness of a known successful structure of a different material. For example, the shell of a ferro-cement boat could be designed to the equivalent strength or stiffness of a similar boat in fiberglass or laminated wood. In any case, the naval architect should be able to estimate the strength and weight of a ferro-cement boat, and to furnish quantitative answers to those three simple questions that he will undoubtedly be asked.

There are a number of people who have given generously of their time in assisting me with this work. The project was originally suggested by Robert Allan and was done under the supervision of Professor Amelio M. D'Arcangelo. I am greatly indebted for the untiring help of Professors Legg and Legatski of the Civil Engineering Department, and for the constant encouragement of Professor Benford, Chairman of the Department of Naval Architecture and Marine Engineering. Thanks also go to Charles Canby for going through my calculations and making helpful suggestions for the format, and to Debbie Moore, for doing a fine job of a very difficult typing task. The photographs 5 and 6 in Appendix E were taken by Terry Little, and the one on page 3 was taken by Dean Runyan. Martin Iorns, of Fibersteel Corporation, deserves credit for his encouragement and support. Finally, a word of thanks goes to M. Rosenblatt and Son, Inc. for assisting with the travel expenses for the presentation of this paper.

CONTENTS

I
FORCE-DEFLECTION CHARACTERISTICS

The typical force-deflection curve of a ferro-cement specimen experiencing a gradually increasing bending moment is shown in Figure 1. The curve has three distinct parts. The first part is in the uncracked range. It is approximately linear, and the bending moments are quite low. The curve then changes slope and becomes linear again in the cracked range. The curve changes slope a second time and becomes horizontal.

In the first part of the curve the specimen behaves elastically, as is suggested by the constant slope. The steel and mortar are stressed below their yield strength on both the tension and the compression sides of the beam. The tensile strength of the mortar, however, is quite low, approximately 400 lb/in.$^2$. Furthermore, since mortar is a brittle material, its tensile strength is not clearly defined. Hence, at a relatively low and somewhat unpredictable bending moment the mortar will crack on the tension side.

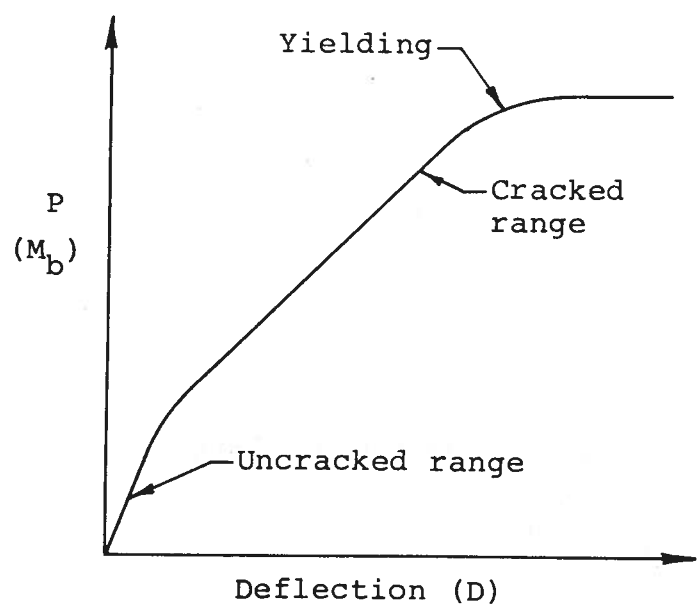



Fig. 1. Load deflection curve.

The cracks will propagate from the tension surface toward the middle of the specimen and will stop at the neutral axis where the stress is zero. Figure 3 illustrates this cracking process. These cracks form and propagate between the uncracked range and the cracked range. As the specimen cracks there is a load transfer. The tensile load carried by the mortar is transferred to the steel in the tension side, and there is a subsequent shift of the neutral axis. With increasing load a stable situation will develop, and the beam will again behave elastically, but in the cracked range. The compressive strength of mortar is high compared to the tensile strength, so the curve follows Hooke's Law (i.e. is linear) in the cracked range. Eventually, as the load is increased, the mortar or the steel, or both, will reach their yield strength. If the steel begins to fail first, the specimen is underreinforced; if the mortar fails first, the specimen is overreinforced; and if both fail simultaneously, the specimen is a balanced beam.

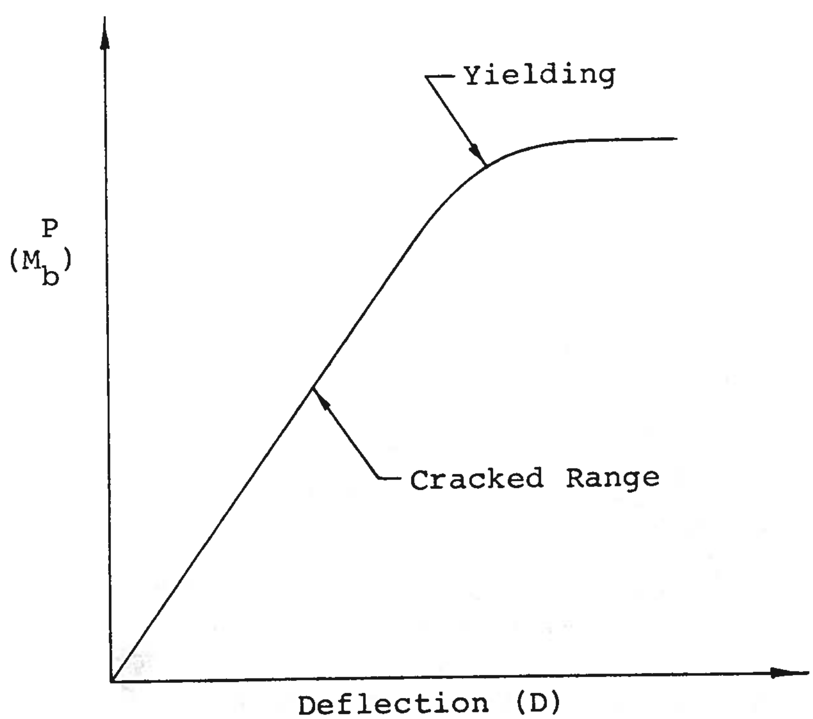



Fig. 2. Load deflection curve for a cracked specimen.

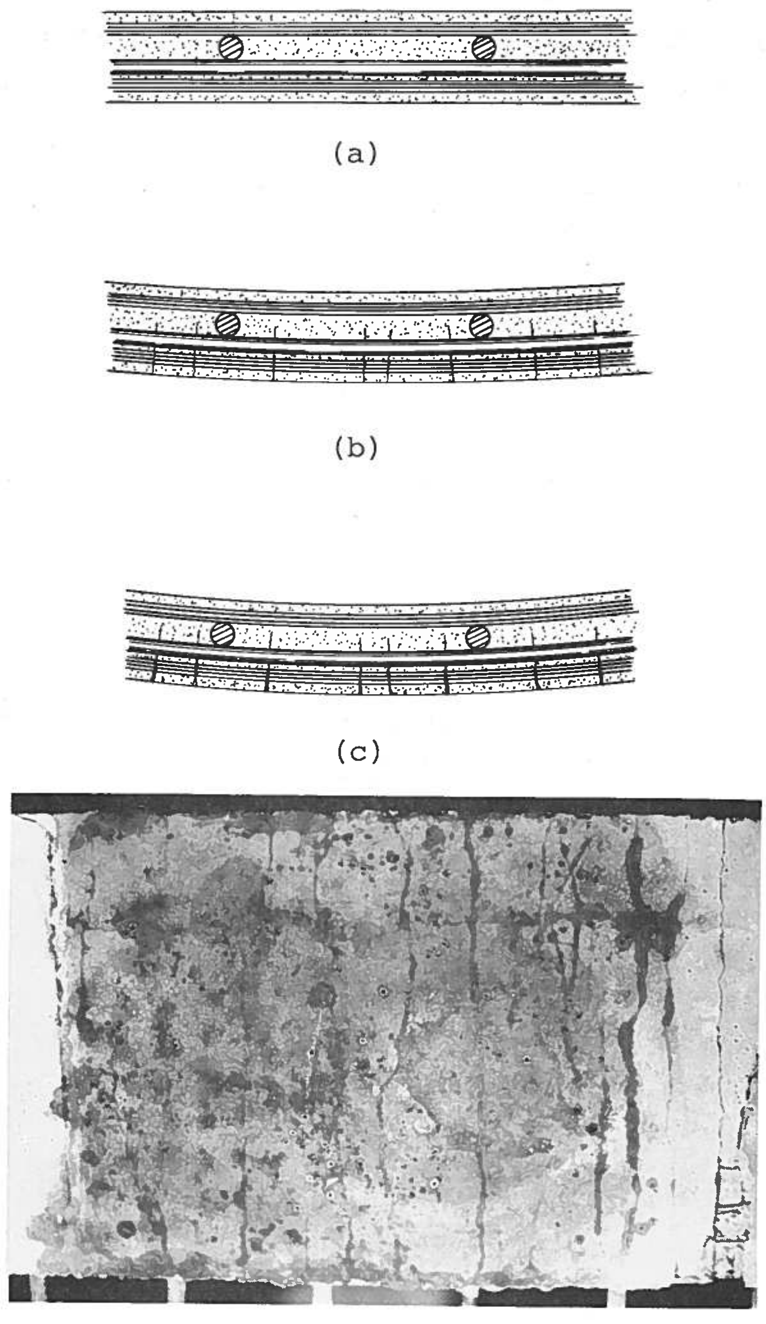


Fig. 3. (a) Specimen is unstressed or lightly stressed and remains uncracked. (b) Specimen is cracked to the neutral axis and is operating in the cracked range. Cracks are microscopic. (c) Specimen is failing. Cracks become visible. (d) Photograph of actual test specimen after failure at point of load application (right side). Cracks were made visible with the aid of a dye.

A beam which is cracked, perhaps because of some previous loading, would not exhibit the initial stiffness of the uncracked beam. The force deflection curve would look like Fig. 2.

## II
## WORKING-STRESS ANALYSIS

Working-stress analysis is valid in the uncracked and in the cracked regions. Since most beams are designed to operate in the cracked region, the analysis method will be described for this range. With one minor modification, it is directly applicable in the uncracked range.

The analysis of composite beams in the elastic range is fairly simple if we assume that the strains vary linearly from the neutral axis and are independent of the material. In other words, at a given distance from the neutral axis, the strains in the different materials, e.g., mortar and steel, are equal. The stresses are then

$$\sigma = \varepsilon E \tag{2.1}$$

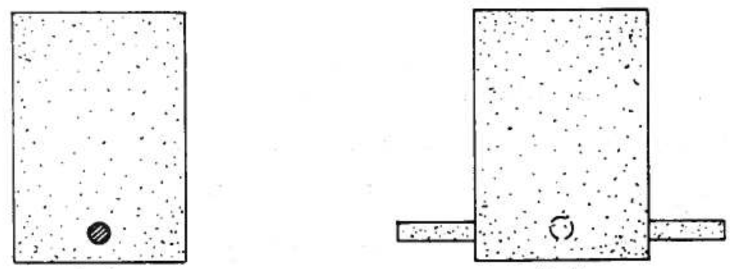
where

$\sigma$ = stress

$\varepsilon$ = strain

$E$ = Young's modulus

If we choose one of the materials as the reference material, for example the mortar, we can transform the other materials into equivalent areas of the reference material. We then analyze the transformed beam as a homogeneous beam by conventional slender-beam theory.



(a) (b)

Fig. 4. (a) Actual reinforced concrete beam. (b) Transformed beam. An equivalent area of concrete has been added to replace the steel reinforcing bar.

A study of the reinforced beam in Figure 4 will illustrate such transformations. We choose the mortar as the reference material and want to replace the steel reinforcing bar with a suitable area of concrete so that the resulting transformed beam behaves like the original beam.

The load carried by the steel bar is

$$F_s = A_s \sigma_s \tag{2.2}$$

where

$F_s$ = total force on the steel bar

$A_s$ = cross-sectional area of the steel bar

$\sigma_s$ = stress in the steel bar

We want the load carried by the equivalent concrete area of the transformed beam to be the same:

$$F_c = A_c\sigma_c \tag{2.3}$$

$$= A_s\sigma_s$$

We assumed that the strain at any distance from the neutral axis would be the same in both materials. Since we are placing the equivalent area of concrete at the location of the steel,

$$\varepsilon_s = \varepsilon_c \tag{2.4}$$

where

$\varepsilon_s$ = strain in the steel

$\varepsilon_c$ = strain in the concrete

Since

$$\sigma_s = \varepsilon_s E_s \tag{2.5}$$

and

$$\sigma_c = \varepsilon_c E_c \tag{2.6}$$

equation (2.3) becomes

$$A_c(\varepsilon_c E_c) = A_s(\varepsilon_s E_s) \tag{2.7}$$

Since

$$\varepsilon_c = \varepsilon_s$$

we get

$$A_c E_c = A_s E_s \tag{2.8}$$

The equivalent concrete area will then be

$$A_c = \frac{A_s E_s}{E_c}$$

The ratio of the Young's moduli is usually referred to as the "modular ratio" (n),

$$\frac{E_s}{E_c} = n \qquad (2.9)$$

Having determined this, we now replace the reinforcement bar with the equivalent concrete area,

$$A_c = nA_s \qquad (2.10)$$

at exactly the same distance from the neutral axis. This is sketched symbolically in Figure 4(b).

With this imaginary transformed beam, we can now determine the stresses as a function of the bending moment,

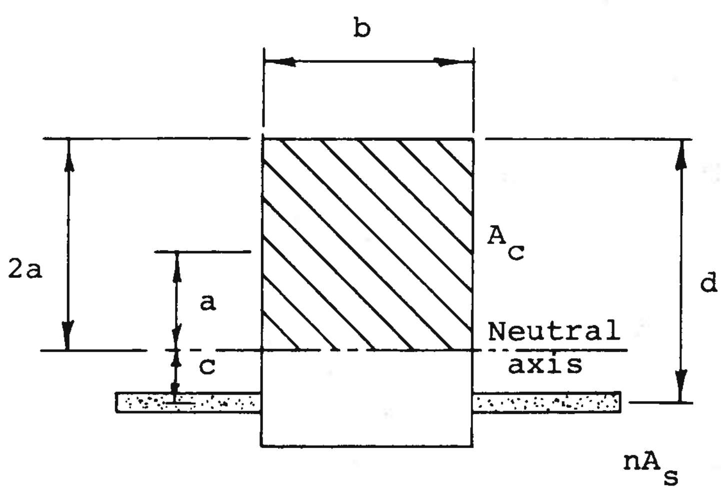



Fig. 5. Dimensions of the cross section of the beam.

or the bending moment as a function of the stresses. In order to do either, we must find the neutral axis of the beam.

The neutral axis will lie at the centroid of the transformed beam. Neglecting the tension concrete, but not the transformed steel area, we calculate the first moment of the area about an assumed neutral axis and set it equal to zero. Using the dimensions of Figure 5,

$$aA_c - c(nA_s) = 0 \tag{2.11}$$

$$a(2a(b)) - c(nA_s) = 0 \tag{2.12}$$

We have the additional information that

$$2a+c = d$$

Hence

$$c = d-2a \tag{2.13}$$

and equation (2.12) becomes

$$a(2a(b)) - (d-2a)(nA_s) = 0 \tag{2.14}$$

The only unknown in this equation is "a." Hence we can solve for it:

$$2a^2b - dnA_s + 2anA_s = 0$$

$$(2b)a^2 + (2nA_s)a - dnA_s = 0$$

$$a = \frac{-(2nA_s \pm \sqrt{(2nA_s)^2 - 4(2b)(-dnA_s)}}{2(2b)} \tag{2.15}$$

i.e., we have a quadratic equation to solve in order to locate the neutral axis.

The bending moment and the stress are related by the equation

$$\sigma_x = \frac{-M_b y}{I_{yy}} \tag{2.16}$$

where

$\sigma_x$ = stress in the x-direction (along the beam) at a location "y" from the neutral axis

$M_b$ = bending moment

$I_{yy}$ = transverse moment of inertia of transformed beam about the neutral axis

For the particular beam we are studying,

$$I = \int zy^2 dy \tag{2.17}$$

$$= nA_s c^2 + \frac{(2a)^3 b}{3} \tag{2.18}$$

If we know the bending moment we can calculate the stress at any location:

$$\sigma_{x_c} = \frac{-M_b y}{I_{yy}} \tag{2.19}$$

$$\sigma_{x_s} = n\sigma_{x_c} \tag{2.20}$$

where

$\sigma_{x_c}$ = concrete stress

$\sigma_{x_s}$ = steel stress

Conversely, for any given stress we can determine the bending moment.

For ferro-cement the procedure is the same, only there are more layers of steel and hence the moment equations have more terms.

Sample Working-Stress Calculation for a Ferro-Cement Specimen

The steps in calculating a ferro-cement specimen are:

(1) determining the dimensions and properties

(2) finding the neutral axis

(3) calculating $I_{yy}$

(4) solving $\sigma_c = \dfrac{-M_b y}{I_{yy}}$

(1) Dimensions and properties

Properties

Wire mesh area ( $A_w$ )[1] .................. .04787 in.$^2$

Rod area ( $A_r$ ) ...................... .0829 in.$^2$

$\sigma_{cu}$ mortar ......................... 5,930 lb/in.$^2$

$\sigma_y$ wire mesh ........................ 91,800 lb/in.$^2$

$\sigma_u$ wire mesh ........................ 107,000 lb/in.$^2$

Specific weight of mortar ( W )........ 145 lb/ft$^3$

[1] A is the total area of the corresponding layer of reinforcement.

Young's modulus of mortar

$$E_c{}^2 = (W)^{1.5}(33)(\sigma_{cu})^{0.5}$$

$$= (145)^{1.5}(33)(5930)^{0.5}$$

$$= 57{,}400(77.007)$$

$$= 4.42 \times 10^6 \text{ lb/in.}^2$$

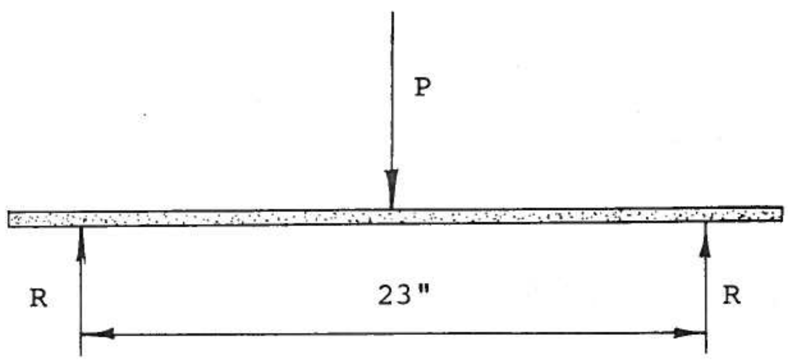



Fig. 6. Beam configuration

2 Ferguson, P. M. Reinforced Concrete Fundamentals. New York: John Wiley and Sons, March 1966. p. 586

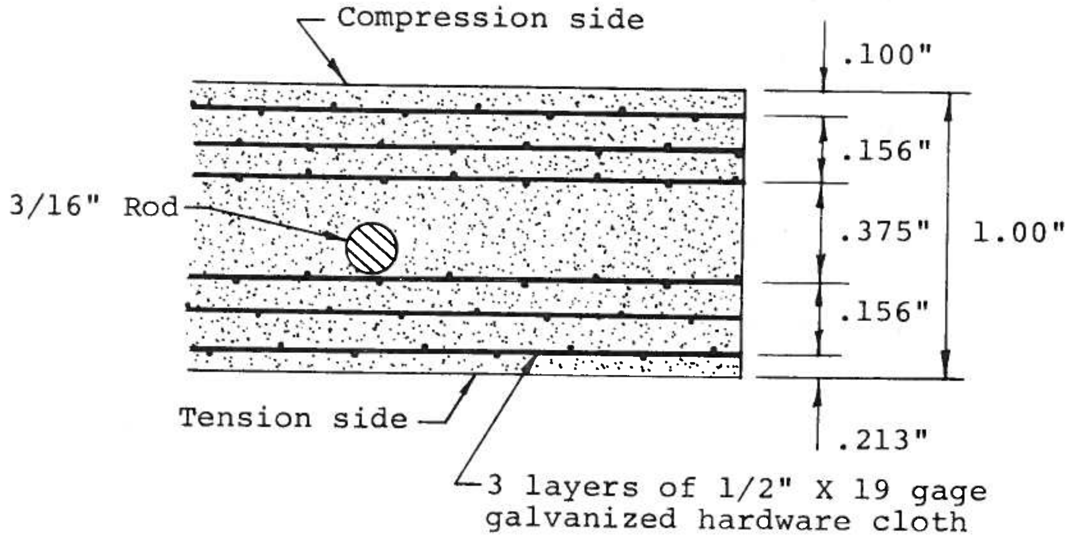



Fig. 7. Section through beam

Modular ratio

$$n = \frac{E_s}{E_c}$$

$$= \frac{29 \times 10^6}{4.42 \times 10^6}$$

$$= 6.561$$

Transformed areas

$$nA_w = 6.561 \times .04787$$

$$= .3140 \text{ in.}^2$$

$$nA_r = 6.561 \times .0829$$

$$= .5439 \text{ in.}^2$$

$$(n-1)\overset{*}{A}_w = 5.561 \times .04787$$

$$= .2662 \text{ in.}^2$$

Mortar Area

Effective area is in the compression side:

$$A_c = 2a(6) \quad \text{(for a 6" wide beam)}$$

$$= 12a$$

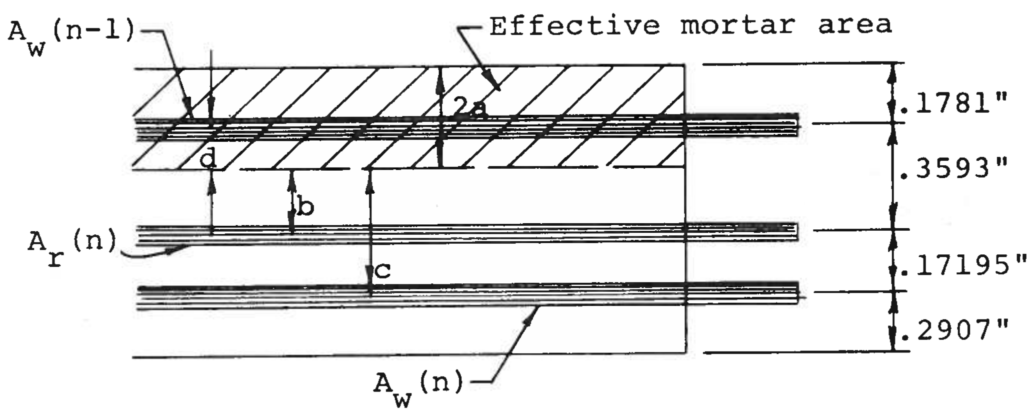



Fig. 8. Section through transformed beam.

Dimensions

From Figure 8,

$$a = \text{unknown}$$
$$b = .5374-2a$$
$$c = .70935-2a$$
$$d = 2a-.1781$$

* For explanation of (n-1) see page 18.

(2) Neutral axis

The neutral axis is at the centroid of the transformed beam. Hence

$$\Sigma M_{n.a.} = 0$$

Substituting from Figure 8,

$$\Sigma M_{n.a.} = 12a(a)+(n-1)A_w(d)-nA_r(b)-nA_w(c)$$

$$= 12(a^2)+(.2662)(2a-.1781)$$

$$-(.5439)(.5374-2a)-(.3140)(.70925-2a)$$

$$= 12a^2+.5324a-.04741-.2923+1.088a$$

$$-.2227+.628a$$

$$= 12a^2+(.5324+1.088+.628)a$$

$$-(.04741+.2923+.2227)$$

$$= 12a^2+2.2484a-.56241$$

$$= 0$$

Hence,

$$a = \frac{-2.2484\pm\sqrt{(2.2484)^2-4(12)(-.56241)}}{24}$$

$$= .1424$$

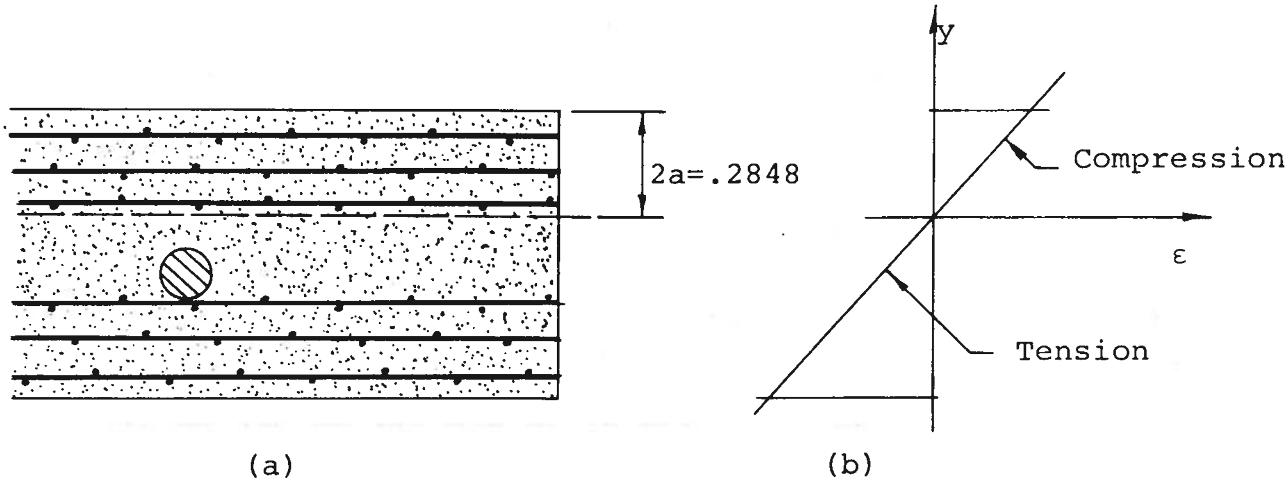



Fig. 9. (a) Neutral axis location. (b) Strain distribution through member.

(3) Moment of inertia

$$I_{yy} = xy^2dy$$

$$= (n-1)A_w(d)^2+nA_r(b)^2+nA_w(c)^2 + \frac{(6)(2a)^3}{3}$$

$$= .2662(.1067)^2+.5439(.2526)^2+ .3140(.42445)^2$$

$$+ \frac{6(.2848)^3}{3}$$

$$= .13787 \text{ in.}^4$$

(4) Stresses and bending moments

Suppose we impose a bending moment of 2,000 in.-lb.

Mortar compressive stress

$$y = .2848$$

$$\sigma_c = \frac{-M_b y}{I_{yy}}$$

$$= \frac{-2{,}000 \text{ in.-lb}(.2848 \text{ in.})}{.13787 \text{ in.}^4}$$

$$= 4{,}131 \frac{\text{lb}}{\text{in.}^2}$$

Steel tensile stress

$$y = -.4026$$

$$\sigma_s = n \left[\frac{-M_b y}{I_{yy}}\right]$$

$$= 6.561 \frac{-2{,}000(-.4026)}{.13787}$$

$$= 38{,}318 \frac{\text{lb}}{\text{in.}^2}$$

Suppose we want to know what the bending moment will be if we load the mortar to its ultimate stress.

$$-M_b = \frac{\sigma_{cu} I_{yy}}{y}$$

$$= \frac{(5930 \text{ lb/in.}^2)(.13787 \text{ in.}^4)}{.2848 \text{ in.}}$$

$$= 2871 \text{ in.-lb}$$

## Compression Steel

As noted in the sample calculation for the 1.00-in. ferro-cement specimen, compression steel is converted to an equivalent mortar area by the ratio (n-1). The explanation for this follows.[3]

Consider a column experiencing a compressive load (P). Let

$A_g$ = gross area of mortar

$A_s$ = steel area

$A_c$ = net mortar area

$$P = A_c\sigma_c + A_s\sigma_s \qquad (2.21)$$

$$\varepsilon = \frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c} \qquad (2.22)$$

Solving equation (2.22) for the steel stress,

$$\sigma_s = \frac{E_s}{E_c}\sigma_c$$

Substituting this into equation (2.21),

$$P = A_c\sigma_c + A_s(n\sigma_c)$$

$$= \sigma_c(A_c + nA_s) \qquad (2.23)$$

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[3] Ferguson, P. M. Reinforced Concrete Fundamentals. New York: John Wiley and Sons, March 1966. p 42

Since

$$A_c = A_g - A_s$$

$$P = \sigma_c(A_g - A_s + nA_s)$$

$$P = \sigma_c(A_g + (n-1)(A_s)) \qquad (2.24)$$

Hence the modular ratio for compression steel is (n-1). Actually it is common practice in concrete design to use a modular ratio of (2n) for long time loads because, after a period of time, the steel is carrying a much greater portion of the load when the structure was new. The increased load is due to concrete creep.

## Analysis in the Uncracked Range

Analysis in the uncracked range would be the same as that in the cracked range, except that a modular ratio of (n-1) is used throughout. This ratio is used for the same reason as that in the case of compression steel.

The end of the uncracked range is difficult to predict because of the brittle nature of the mortar. Concrete has a tensile strength of approximately 10 percent of its compressive strength. Yielding, however, will be highly dependent on stress concentrations which arise from surface roughness and internal imperfections.

## III
## ULTIMATE STRENGTH ANALYSIS

### Yielding of Ferro-Cement

Theoretically, a ferro-cement specimen will begin to yield when

(1) the stress in the mortar reaches the compression strength, or

(2) the stress in the steel reinforcement reaches the yield strength, or

(3) both (1) and (2) occur simultaneously

(a) is termed an overreinforced beam,
(b) an underreinforced beam, and
(c) a balanced beam.

In an underreinforced beam the steel yields first and there is considerable deflection before failure. The failure process is shown in Figure 10. When the steel

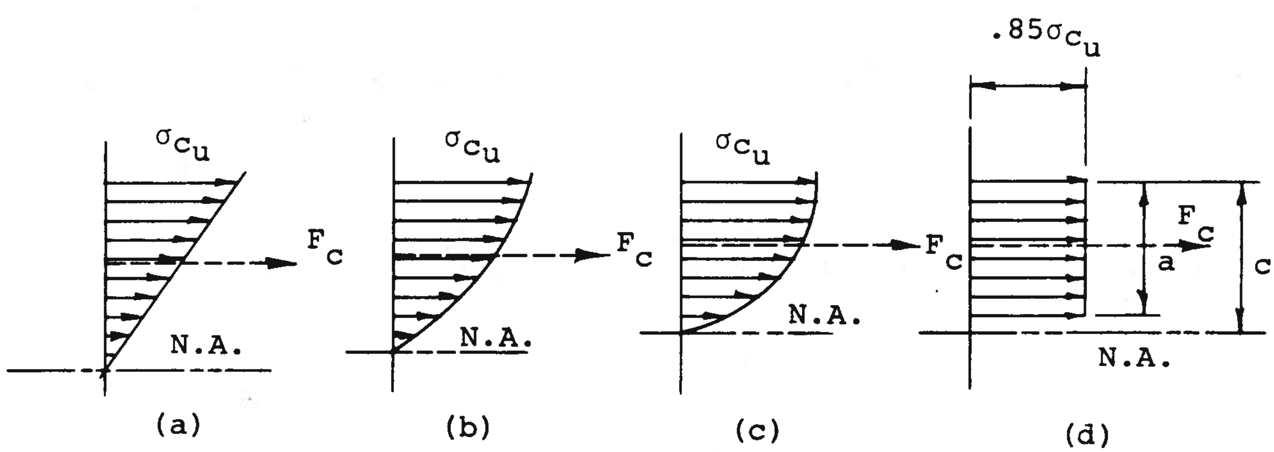


Fig. 10. Failure of an underreinforced beam. (a) to (c) shows the actual failure, and (d) shows the linearized model.

yields, the neutral axis moves up. This increases the moment arm between the resultant of the mortar force ($F_c$) and the steel force ($F_s$), which slightly increases the load-carrying capacity. The stress distribution in the mortar becomes nonlinear, as shown in Figure 10. Eventually the mortar fails in compression and the entire beam fails. The fact that the beam will undergo considerable deflection before failing gives warning of the oncoming failure. For this reason most land structures are underreinforced.

In an overreinforced beam the mortar fails first, as shown in Figure 11. As the outer fibers fail, the neutral axis moves down, hence decreasing the distance between $F_s$ and $F_c$ and diminishing the load-carrying capacity. As a result, the overreinforced beam tends to fail rather suddenly.

The balanced beam is primarily of academic interest and will not be considered here.

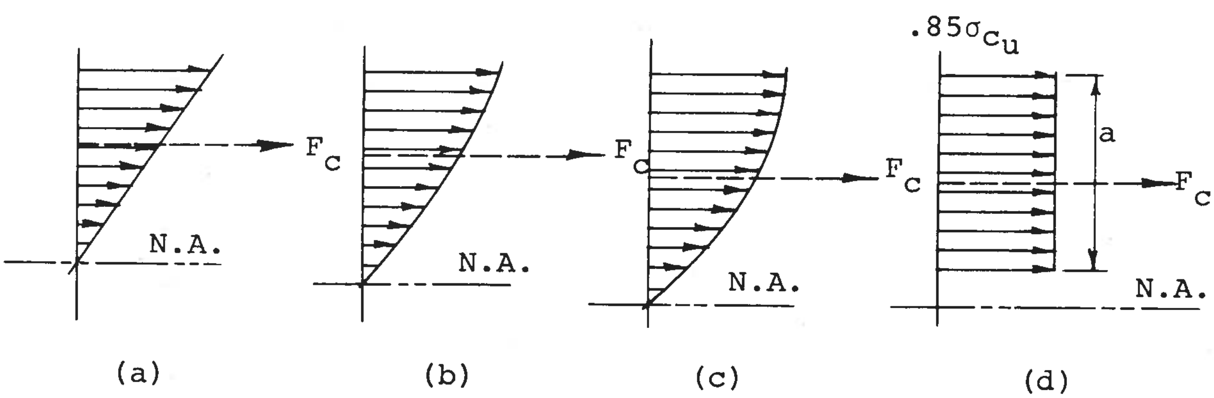


Fig. 11. Failure of an overreinforced beam. (a) to (c) shows the actual failure, and (d) shows the linearized model.

## Estimating Ultimate Strength

The usual procedure for estimating the ultimate strength of a concrete beam is to replace the actual stress distribution in the mortar by an equivalent stress block,

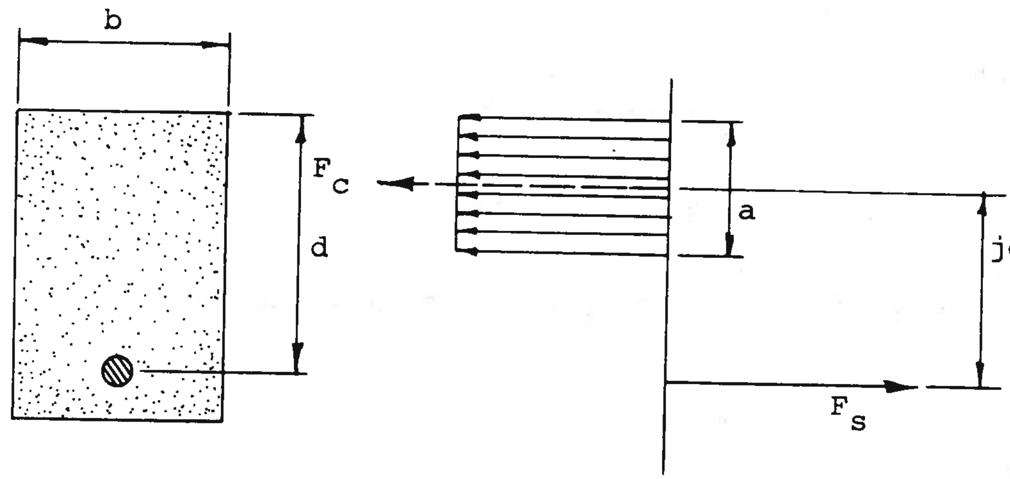


Fig. 12. (a) Dimensions of beam. (b) Stress block and resultant forces.

as shown in Figures 10 and 11. Take, for example, the beam in Figure 12

$$F_c = A_c \sigma_c \qquad (3.1)$$

where

$A_c$ = cross section area of stress block

$\sigma_c$ = concrete stress in stress block

The American Concrete Institute recommends a value of .85 $\sigma_{cu}$ for $\sigma_c$. Hence,

$$F_c = .85\ \sigma_{cu} A_c \qquad (3.2)$$

For the tension steel,

$$F_s = A_s\sigma_s \quad (3.3)$$

where

$A_s$ = steel area

$\sigma_s$ = steel stress

The yield stress is used for $\sigma_s$. Hence,

$$F_s = A_{s_y}\sigma_s \quad (3.4)$$

since

$$\Sigma F_x = 0$$

$$F_s = F_c \quad (3.5)$$

The depth of the stress block (a) remains unknown. However,

$$A_c = ab \quad (3.6)$$

where

b = width of beam

so

$$F_c = .85\ \sigma_{cu}(ab)$$

$$= F_s \quad (3.7)$$

Therefore we can solve for (a):

$$a = \frac{F_s}{.85\sigma_{cu}b} \quad (3.8)$$

The moment arm (jd) is then

$$jd = d - \frac{a}{2} \tag{3.9}$$

and the bending moment is

$$M_b = jdF_s \tag{3.10}$$

For beams with multiple layers of reinforcing, the procedure is similar.

Two simplifying assumptions can be made:

(1) The compression steel is neglected.

(2) All the tension steel is assumed to be yielding.

As in the previous example, we then have

$$F_w = A_w \sigma_{w_y} \tag{3.11}$$

where

$F_w$ = resultant force of wire mesh

$A_w$ = wire mesh area

$\sigma_{w_y}$ = yield strength of wire mesh

and

$$F_r = A_r \sigma_{r_y} \tag{3.12}$$

where

$F_r$ = resultant force of rods

$A_r$ = rod area

$\sigma_{r_y}$ = yield strength of rods

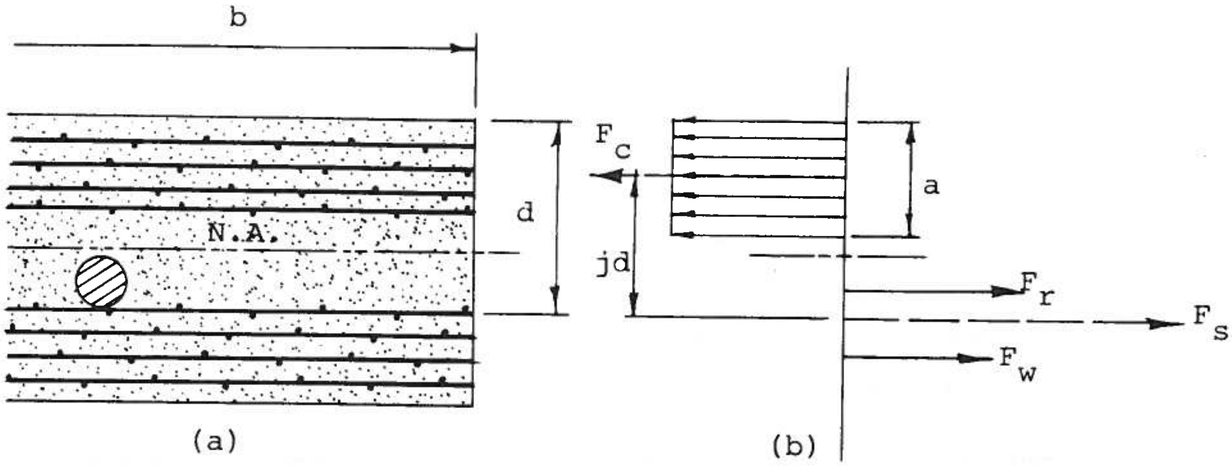


Fig. 13. (a) Dimensions of beam. (b) Stress block and resultant forces.

$$F_s = F_s + F_r \qquad (3.13)$$

$$a = \frac{F_s}{.85\sigma_{c_u} b} \qquad (3.14)$$

$$jd = d - \frac{a}{2} \qquad (3.15)$$

$$M_b = jdF_s = \text{Moment to fail} \qquad (3.16)$$

It should be noted that for the specimens tested, this method predicted ultimate strengths which were less than those actually observed.

# IV
# CONCLUSIONS

(1) The behavior of ferro-cement in bending follows the normal load-deflection curve for reinforced concrete, as shown in Figure 1.

(2) Ferro-cement cracks at very low tensile stresses. These cracks are microscopic and are shown in Figure 3 with the aid of a dye.

(3) Reaching the ultimate strength of either the mortar or the steel does not necessarily mean failure of the member. Loads are transferred from the failed material to the intact material and the behavior may remain linear at slightly increased loads.

(4) The point at which the ultimate strength of the weakest material (usually the mortar) is reached is considerably below the failure point of the member.

(5) Working-stress analysis can safely be used to compute stresses which are due to working loads.

(6) The ultimate-strength analysis outlined can safely be used to estimate the ultimate strength.

(7) Further study of the failure mechanism is clearly needed, not to mention study of corrosion and fatigue. Shear stresses due to bending moments should be investigated.

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# APPENDICES

# APPENDIX A

# TEST DATA

TENSILE STRENGTH OF RODS

1/4" Rod

Test 1 - 2,000 lb Yield Force
- 3,120 lb Ultimate Force

Test 2 - 1,900 lb Yield Force
- 3,030 Ultimate Force

Average Yield Force - 1,950 lb

Average Ultimate Force - 3,075 lb

Stresses:

$$\sigma_y = \frac{1,950 \text{ lb}}{.0491 \text{ in.}^2}$$

$$= 39,800 \frac{\text{lb}}{\text{in.}^2}$$

$$\sigma_u = \frac{3,075 \text{ lb}}{.0491 \text{ in.}^2}$$

$$= 62,600 \frac{\text{lb}}{\text{in.}^2}$$

3/16" Rod (Rods showed no yield point)

Test 1 - 2,455 lb Ultimate Force

Test 2 - 2,550 lb Ultimate Force

Average Ultimate Force - 2,503 lb Force

Stresses:

$$\sigma_u = \frac{2,503 \text{ lb}}{.0276 \text{ in.}^2}$$

$$= 90,000 \frac{\text{lb}}{\text{in.}^2}$$

$$\sigma_y \cong 80\% \ \sigma_u = 72{,}500 \ \frac{\text{lb}}{\text{in.}^2}$$

TENSILE STRENGTH OF WIRE MESH

The wire mesh is 19 gage.

Diameter = .0410 in.

Area = .0013 in.$^2$

Assuming that the proportional limit can be taken as $\sigma_y$, and throwing out the inconsistent high value on the plot, forces to yield are

| Test | 1/2 Force (lb) |
|---|---|
| 1 | 61.2 |
| 2 | 61.0 |
| 4 | 62.1 |
| 5 | 60.0 |

Average force = 61.075 × 2
= 122 lb

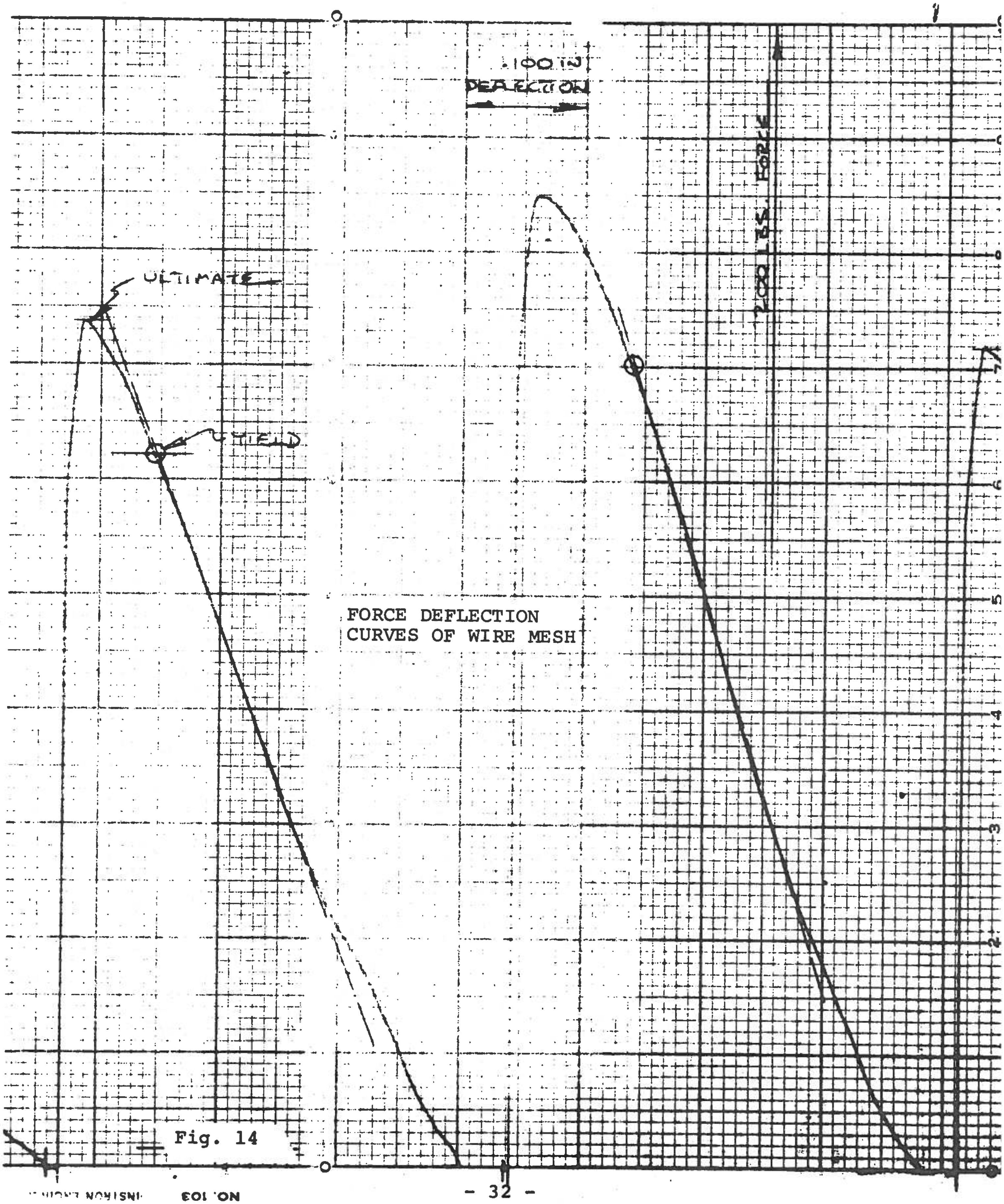
$$\sigma_y = \frac{122 \text{ lb}}{.00133 \text{ in.}^2} = 91{,}800 \ \frac{\text{lb}}{\text{in.}^2}$$

Ultimate forces are

| Test | 1/2 Force (lb) |
|---|---|
| 1 | 70.0 |
| 2 | 71.8 |
| 4 | 72.8 |
| 5 | 70.6 |

Average force = 71.3 × 2
= 142.6 lb

$$\sigma_u = \frac{142.6 \text{ lb}}{.00133 \text{ in.}^2} = 107{,}000 \ \frac{\text{lb}}{\text{in.}^2}$$

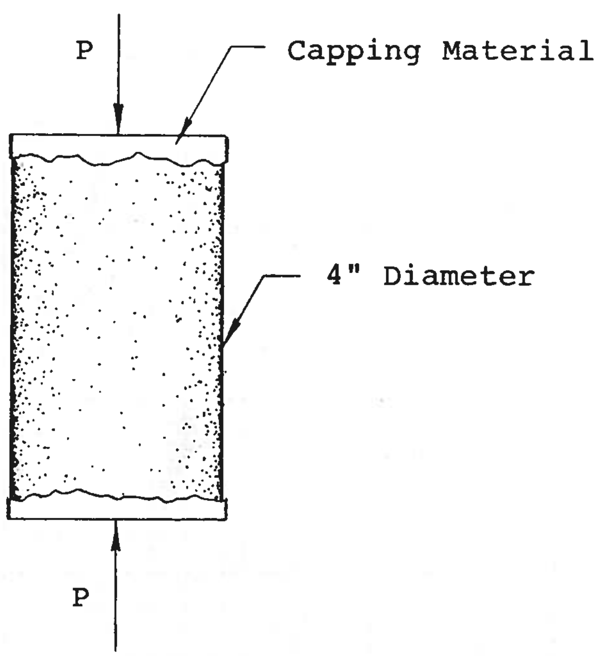



FORCE DEFLECTION
CURVES OF WIRE MESH

Fig. 14

COMPRESSION TESTS

| Series | Specimen | Curing Time | Force (lb) | Average $\sigma_{cu}$ (lb/in.$^2$) |
|---|---|---|---|---|
| 1 | 1 | 7 days | 61,000<br>59,000 | |
| | | | | 4,760 |
| 2 | 1 to 3 | 7 days | 60,500<br>62,500 | |
| | | | | 4,885 |
| 3 | 1 to 3 | 8 days | 76,000<br>73,000 | |
| | | | | 5,930 |
| 3 | 4 | 28 days | 104,000<br>95,500 | |
| | | | | 7,937 |



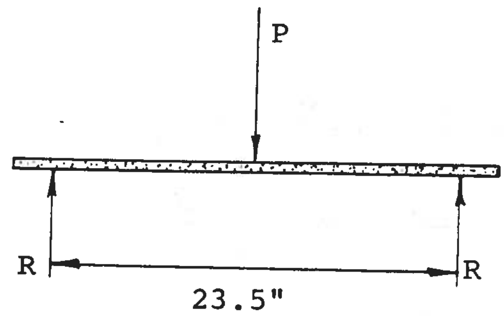

Mortar Mix

Cement ..........16.5 lb
Pozzolan ........ 4.5 lb
Sand.............30.0 lb
Water ...........3,500 cc

Fig. 15. Test cylinder.

## Series 1, Specimen 1

| P (lb) | DEFLECTION (in.) |
|---|---|
| 100 | .0065 |
| 200 | .013 |
| 300 | .020 |
| 400 | .028 |
| 500 | .036 |
| 600 | .045 |
| 700 | .0605 |
| 800 | .082 |
| 900 | .100 |
| 1,000 | .121 |
| 1,100 | .141 |
| 1,200 | .162 |
| 1,300 | .187 |
| 1,400 | .212 |
| 1,500 | .240 |
| 1,600 | .274 |



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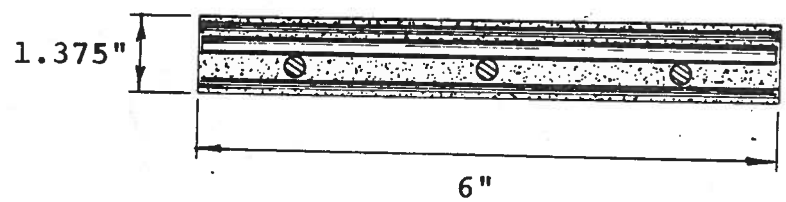


Section through specimen

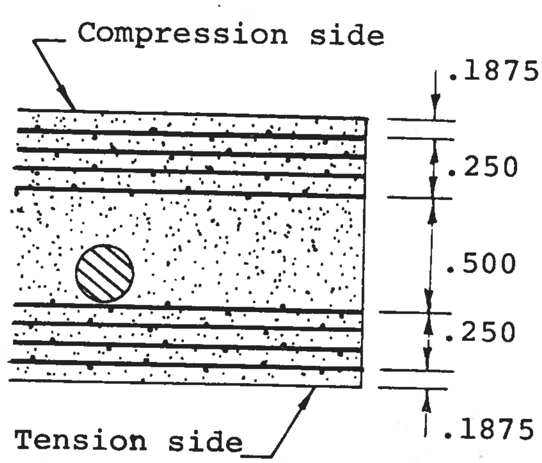
**Mortar** $\sigma_{c_u} = 4760 \frac{\text{lb}}{\text{in.}^2}$

**Wire Mesh**

1/2" x 19 gage galv. hardware cloth. Four layers on each side.

**Rods**

1/4" diameter hot rolled steel. Longitudinal rods spaced 2". Transverse rods spaced 2".



Location of reinforcement

Series 2, Specimen 1

| P (lb) | DEFLECTION (in.) |
|---|---|
| 100 | .0105 |
| 200 | .022 |
| 300 | .031 |
| 400 | .044 |
| 500 | .0705 |
| 600 | .080 |
| 700 | .104 |
| 800 | .127 |
| 900 | .150 |
| 1,000 | .174 |
| 1,100 | .198 |
| 1,200 | .230 |
| 1,300 | .268 |
| 1,400 | .315 |

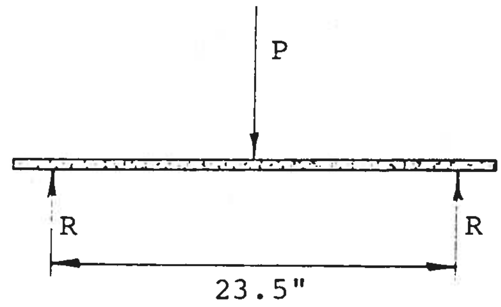
Mortar $\sigma_{c_u} = 4{,}885 \ \frac{\text{lb}}{\text{in.}^2}$

Wire Mesh

1/2" x 19 gage galv. hardware cloth. Four layers each side.

Rods

1/4" diameter hot rolled steel. Longitudinal rods spaced 2". Transverse rods spaced 2".

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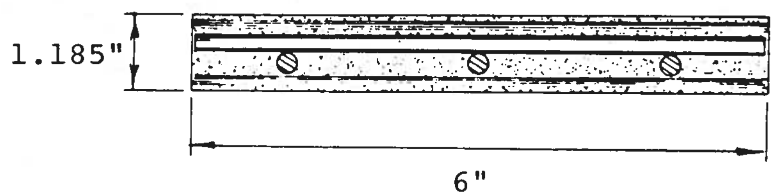
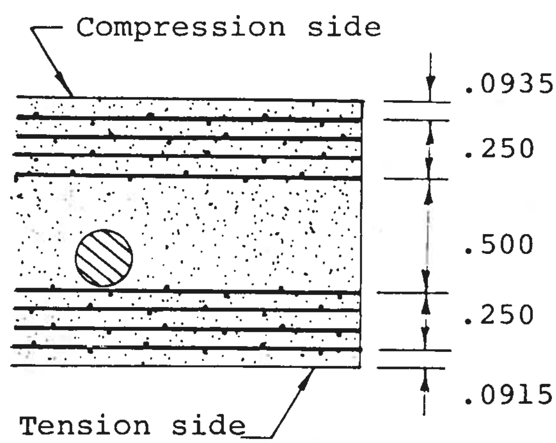



Section through Specimen




Location of reinforcement

Series 2, Specimen 2

| P (lb) | DEFLECTION (in.) |
|---|---|
| 50 | .002 |
| 100 | .009 |
| 150 | .014 |
| 200 | .019 |
| 250 | .0265 |
| 300 | .0375 |
| 350 | .0515 |
| 400 | .068 |
| 450 | .084 |
| 500 | .1025 |
| 550 | .121 |
| 600 | .1405 |
| 650 | .161 |
| 700 | .185 |
| 750 | .209 |
| 800 | .233 |
| 850 | .266 |
| 900 | .292 |
| 950 | .362 |
| 1,000 | .408 |
| 1,050 | .480 |

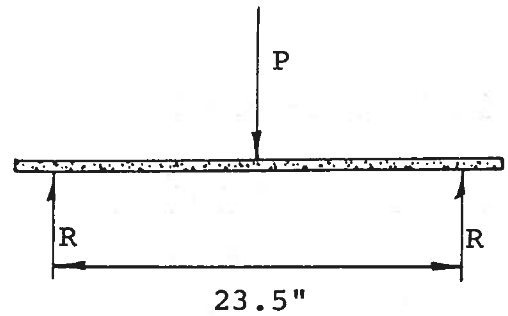
Mortar $\sigma_{c_u} = 4{,}885 \frac{\text{lb}}{\text{in.}^2}$

Wire Mesh

1/2" x 19 gage galv. hardware cloth. Three layers each side.

Rods

3/16" cold rolled steel. Longitudinal rods spaced 2". Transverse rods spaced 2".

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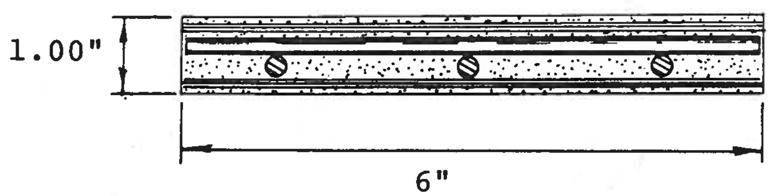
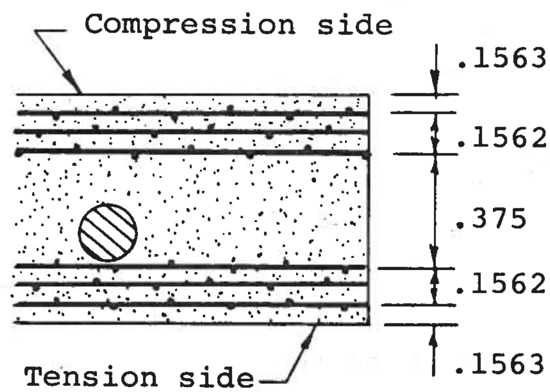


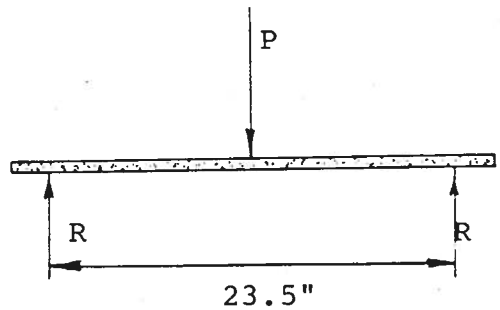

Section through Specimen




Location of reinforcement

Series 2, Specimen 3

| P (lb) | DEFLECTION (in.) |
|---|---|
| 50 | .0075 |
| 100 | .0215 |
| 150 | .0625 |
| 200 | .1275 |
| 250 | .2075 |
| 300 | .2905 |
| 350 | .3755 |
| 400 | .4545 |
| 450 | .5565 |
| 500 | .7075 |

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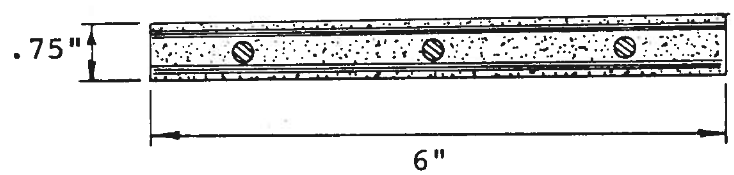
Mortar

$\sigma_{c_u} = 4,885 \ \frac{lb}{in.^2}$



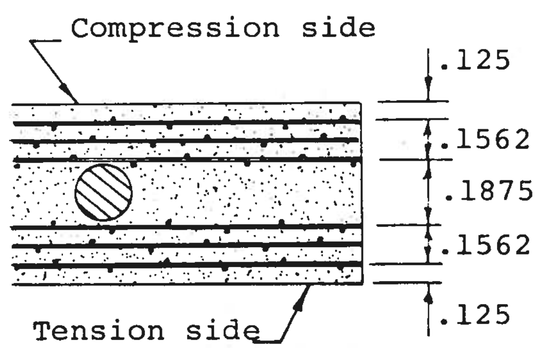

Section through Specimen

Wire Mesh

1/2" x 19 gage galv. hardware cloth. Three layers each side.

Rods

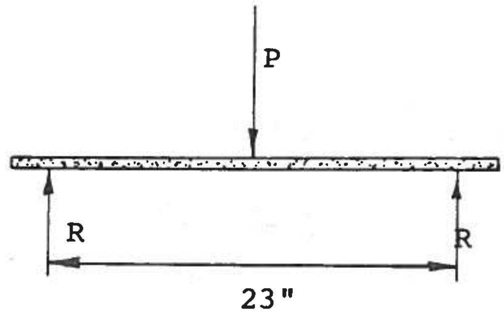
3/16" cold rolled steel. Longitudinal rods spaced 2-1/2". No transverse rods.




Location of reinforcement

Series 3, Specimen 1

| P (lb) | DEFLECTION (in.) |
|---|---|
| 50 | --- |
| 100 | .0735 |
| 150 | .086 |
| 200 | .102 |
| 250 | .124 |
| 300 | .154 |
| 350 | .183 |
| 400 | .212 |
| 450 | .238 |
| 500 | .268 |
| 550 | .298 |
| 600 | .332 |
| 650 | .370 |
| 700 | .410 |
| 750 | .460 |
| 800 | .580 |
| 850 | .700 |

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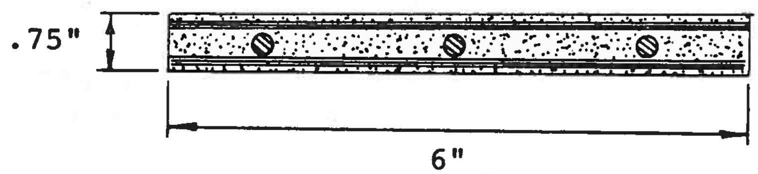



Section through Specimen

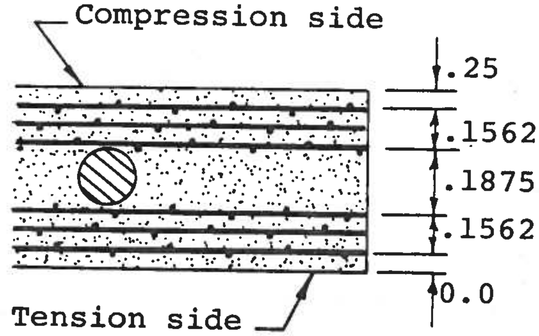
Mortar $\sigma_{c_u} = 5,930 \frac{lb}{in.^2}$

Wire Mesh

1/2" x 19 gage galv. hardware cloth. Three layers each side.

Rods

3/16" cold rolled steel. Longitudinal rods spaced 2-1/2". No transverse rods.




Location of reinforcement

Series 3, Specimen 2

| P (lb) | DEFLECTION (in.) |
|---|---|
| 50 | .094 |
| 100 | .101 |
| 150 | .107 |
| 200 | .113 |
| 250 | .127 |
| 300 | .147 |
| 350 | .168 |
| 400 | .189 |
| 450 | .209 |
| 500 | .234 |
| 550 | .262 |
| 600 | .287 |
| 650 | .310 |
| 700 | .336 |
| 750 | .365 |
| 800 | .400 |
| 850 | .439 |
| 900 | .483 |
| 950 | .575 |

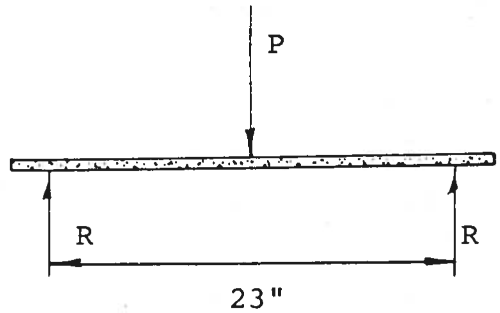
Mortar $\sigma_{c_u} = 5,930 \frac{\text{lb}}{\text{in.}^2}$

Wire Mesh

1/2" x 19 gage galv. hardware cloth. Three layers each side.

Rods

3/16" cold rolled steel. Longitudinal rods spaced 2". Transverse rods spaced 2".

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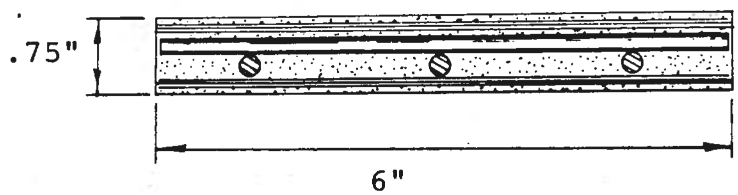
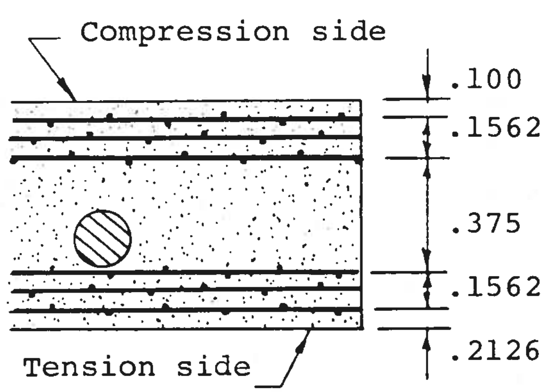


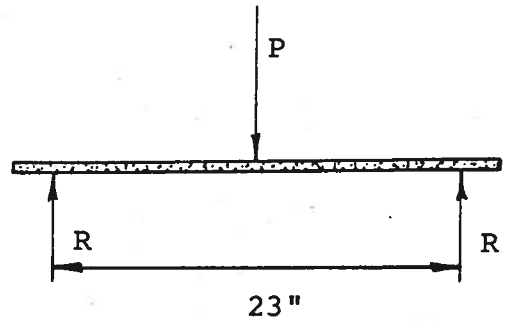

Section through Specimen




Location of reinforcement

Series 3, Specimen 3

| P (lb) | DEFLECTION (in.) |
|---|---|
| 100 | .061 |
| 200 | .069 |
| 300 | .077 |
| 400 | .085 |
| 500 | .0925 |
| 600 | .101 |
| 700 | .109 |
| 800 | .121 |
| 900 | .136 |
| 1,000 | .156 |
| 1,100 | .179 |
| 1,200 | .203 |
| 1,300 | .249 |
| 1,400 | .309 |
| 1,500 | .383 |

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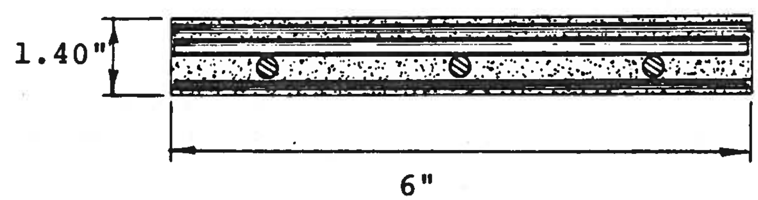



Section through Specimen

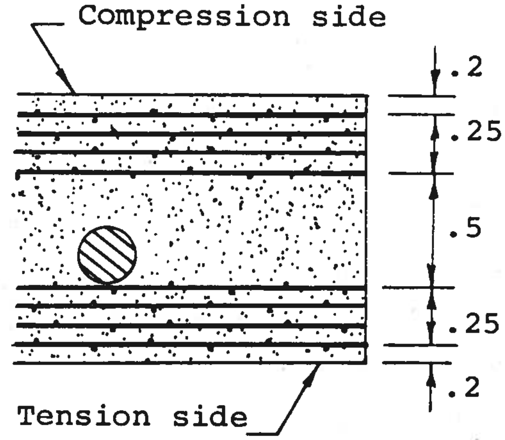
Mortar $\sigma_{c_u} = 5,930 \frac{\text{lb}}{\text{in.}^2}$

Wire Mesh

1/2" x 19 gage galv. hardware cloth. Three layers each side.

Rods

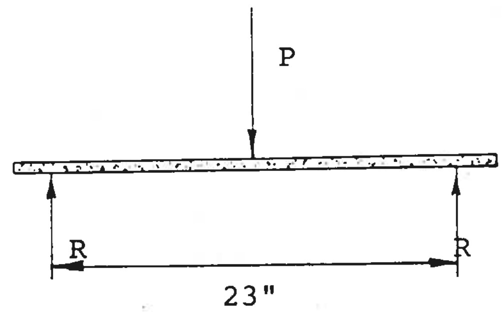
1/4" diameter hot rolled steel. Longitudinal rods spaced 2". Transverse rods spaced 2".




Location of reinforcement

Series 3, Specimen 4

| P (lb) | DEFLECTION (in.) |
|---|---|
| 50 | .032 |
| 100 | .047 |
| 150 | .063 |
| 200 | .084 |
| 250 | .112 |
| 300 | .144 |
| 350 | .177 |
| 400 | .210 |
| 450 | .243 |
| 500 | .279 |
| 550 | .314 |
| 600 | .351 |
| 650 | .394 |
| 700 | .444 |
| 750 | .496 |
| 800 | .562 |
| 850 | .756 |

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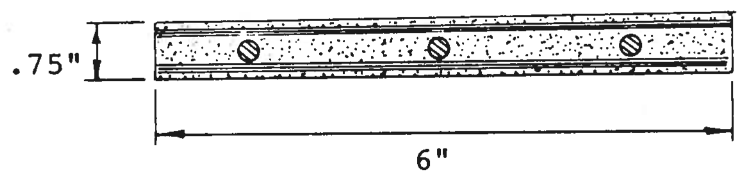



Section through Specimen

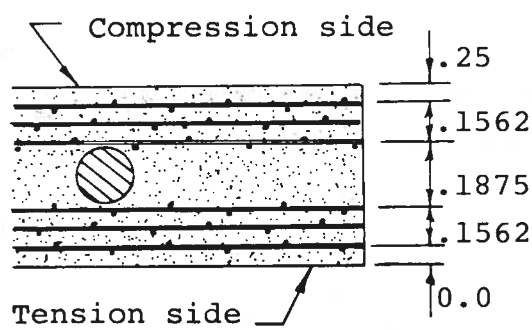
Mortar $\sigma_{c_u} = 7{,}937 \frac{\text{lb}}{\text{in.}^2}$

Wire Mesh

1/2" x 19 gage galv. hardware cloth. Three layers each side.

Rods

3/16" cold rolled steel. Longitudinal rods spaced 2-1/2". No transverse rods.




Location of reinforcement

# APPENDIX B

## COMPUTER PROGRAM FOR WORKING-STRESS ANALYSIS

## COMPUTER PROGRAM FOR WORKING-STRESS ANALYSIS

The computer program shown on pages 47 and 48 was used to do the stress analysis of the ferro-cement specimens by working stress theory. The computer is not necessary for this analysis. If available, however, it saves time, minimizes the chance of error, and makes sensitivity studies easy to perform. In this case the input consisted of the material properties and the beam geometry, while the output consisted of the stress-bending moment relationship as well as some intermediate data. The program could probably be streamlined to use computer time more efficiently. As the program stands, it uses about 10 seconds of CPU time on the IBM 360 computer.

### Language

The computer language is FORTRAN IV.

### Theory

The program uses the stress analysis outlined in Chapter II. Some changes in notation were made. For example, the layers of reinforcement have been numbered starting from the bottom and working up. Hence we have steel areas $A_1$, $A_2$, $A_3$, ... at distances $y_1$, $y_2$, $y_3$, ... from the bottom surface.

Another superficial change is the generalization of the neutral axis calculation. The explanation follows.

Taking moments about the neutral axis, (see Figure 16 )

$$\Sigma M_{NA} = (T - NA)^2 \frac{W}{2} + (y_3 - NA)A_3 + (y_2 - NA)A_2 + (y_1 - NA)A_1 \qquad W = \text{width of beam}$$

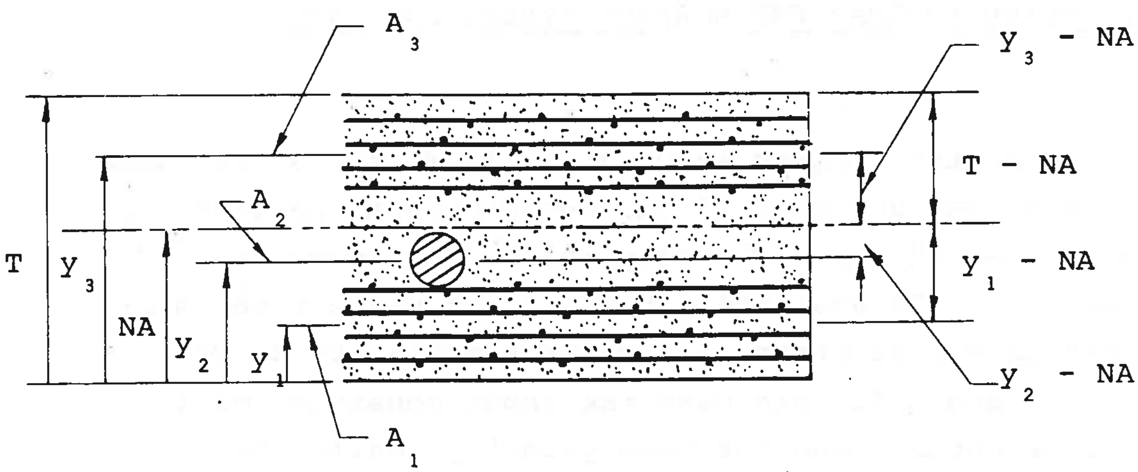



Fig. 16. Transverse section through a ferro-cement specimen, showing nomenclature.

$$\Sigma M_{NA} = [T^2 - 2T\,(NA) + (NA)^2]\,\frac{W}{2}$$

$$+ A_3 y_3 - A_3 NA$$

$$+ A_2 y_2 - A_2 NA$$

$$+ A_1 y_1 - A_1 NA$$

$$= T^2\frac{W}{2} - 2T\frac{W}{2}\,(NA) + \frac{W}{2}\,(NA)^2$$

$$+ A_3 y_3 - A_3 NA$$

$$+ A_2 y_2 - A_2 NA$$

$$+ A_1 y_1 - A_1 NA$$

$$= \frac{W}{2}\,(NA)^2 + (NA)\,\left(-2T\frac{W}{2} - A_3 - A_2 - A_1\right)$$

$$+ \left(T^2\frac{W}{2} + A_3 y_3 + A_2 y_2 + A_1 y_1\right)$$

Let $b = (-TW - A_3 - A_2 - A_1)$

and $c = (T^2 \frac{W}{2} + A_3 y_3 + A_2 y_2 + A_1 y_1)$

$$NA = \frac{-b \pm \sqrt{b^2 - 4 \frac{W}{2} c}}{2 \frac{W}{2}}$$

$$= \frac{-b \pm \sqrt{b^2 - 2WC}}{W}$$

Take the plus sign.

It should also be noted that in using this program we assume an initial neutral axis location. When calculating the transformed areas, the steel area below the assumed neutral axis will be multiplied by the factor (n), while the steel above will be multiplied by (n-1). If the calculated neutral axis is found to be between layers of reinforcement different from the assumed one, then we rerun the program with the calculated neutral axis location as the new assumed location.

Data Input

Card 1

Punch
- length (in.)
- width (in.)
- thickness (in.)
- mortar compressive strength (lb/in.$^2$)
- density of mortar (lb/ft$^3$)
- E of steel (lb/in.$^2$)
- assumed N.A. location from bottom (in.)
- yield strength of steel (lb/in.$^2$)

Format (8F10.4)

Card 2

Punch number of layers of steel
kode - 0 for cracked analysis
- 1 for uncracked analysis

Format (2I2)

Card 3

Punch area of each steel layer (in.$^2$)

Format (8F10.4)

Card 4

Punch distance of each layer of steel from bottom of beam, in same order as on Card 3 (in.)

Format (8F10.4)

Results

The object of the stress analysis was to determine the stresses as a function of the material, the location (y), and the bending moment. The stresses in any of the component materials will vary linearly with the bending moment as well as with (y). Let us study the stress as a functional bending moment. We know that b = 0 at $M_b$ = 0. Hence, if we know one more point on the curve, the curve is defined. The computer program provides one more point for both the steel and the mortar stress curves. We can therefore plot stress as a function of bending moment. Note that the curves for the steel stresses shown on the following pages apply at the extreme fibers on the tension side, and those for the mortar stress apply at the extreme fibers on the compression side. Stresses at other (y) distances can be found by changing the values in statements 25 and 26 in the program.

```
      $COMPILE
            DIMENSION A(10),ATR(10),Y(10)
            REAL L,NAA,NA,I,N
          1 READ(5,200)L,W,T,SCU,WC,ES,NAA,SSY
      C
      C     KODE = 0 , CRACKED ANALYSIS
      C     KODE = 1 , UNCRACKED ANALYSIS
      C
            READ(5,210)NC,KODE
            READ(5,200)(A(J),J=1,NC)
            READ(5,200)(Y(J),J=1,NC)
      C
      C     E OF MORTAR
            EC=(WC**1.5)*33*SQRT(SCU)
      C
      C     MODULAR RATIO
            N=ES/EC
      C
      C     TRANSFORMED AREAS
            DO 11 J=1,NC
            IF(KODE.EQ.1)GO TO 10
            IF(Y(J).GT.NAA)GO TO 10
            ATR(J)=N*A(J)
            GO TO 11
         10 ATR(J)=(N-1)*A(J)
         11 CONTINUE
      C
      C     COEFFICIENTS FOR QUADRATIC FORMULA
      C
            B=-T*W
            C=T*T*W/2
            DO 13 J=1,NC
            B=B-ATR(J)
         13 C=C+ATR(J)*Y(J)
      C
      C     SOLVING QUADRATIC FORMULA TO FIND NEUTRAL AXIS
            NA=(-B-SQRT(B*B-2*W*C))/W
      C
      C     MOMENT OF INERTIA
            I=((T-NA)**3)*W/3
            DO 14 J=1,NC
         14 I=I+(Y(J)-NA)*(Y(J)-NA)*ATR(J)
      C
      C     CALCULATING BENDING MOMENTS
            BMSCUC=SCU*I/(T-NA)
            BMSSY=SSY*I/((Y(1)-NA)*N)
            IF(KODE.EQ.0)GO TO 15
            BMSCUT=0.1*SCU*I/(-NA)
      C
      C     PRINTING OUT INPUT DATA
         15 WRITE(6,206)T
            WRITE(6,201)
            WRITE(6,202)L,W,T,WC,SCU,SSY,ES,NAA,NC,KODE
            WRITE(6,229)
            DO 16 J=1,NC
         16 WRITE(6,230)J,Y(J),A(J),ATR(J)
      C
      C     PRINTING OUT RESULTS
            WRITE(6,203)EC,N,NA,I,BMSSY,BMSCUC
            IF(KODE.EQ.0)GO TO 1
```

```
37          WRITE(6,205)BMSCUT
38          GO TO 1
      C
      C     FORMATS
39     200  FORMAT(8F10.4)
40     210  FORMAT(2I2)
41     206  FORMAT( '1RESULTS FOR ',F8.4,' INCH SPECIMEN')
42     201  FORMAT( /,'INPUT DATA',//)
43     202  FORMAT( 'LENGTH=',F10.3,/,
           1'WIDTH = ',F10.3,/,
           1' DEPTH = 'F10.5,/,
           1'DENSITY OF MORTAR = ',F10.2,/,
           1'ULTIMATE STRENGTH OF MORTAR = ',F10.0,/,
           1'YIELD STRENGTH OF STEEL = ',F10.0,/,
           1'YOUNGS MODULUS OF STEEL = ',F10.0,/,
           1'ASSUMED NEUTRAL AXIS = ',F10.4,/,
           1'NUMBER OF STEEL LAYERS = ',I5,/,
           1'KODE = ',I5)
44     229  FORMAT( /,'STEEL REINFORCEMENT DATA ',//,
           1'LAYER NUMBER      DISTANCE(Y)                AREA              TRANSFORMED
           1 AREA')
45     230  FORMAT( 1I7,3F20.5)
46     203  FORMAT( //,'RESULTS',//,
           1'YOUNGS MODULUS OF MORTAR = ', F10.0,/,
           1'MODULAR RATIO (N) = ', F10.5,/,
           1'NEUTRAL AXIS = ', F10.5,/,
           1'MOMENT OF INERTIA = ',F10.5,/,
           1'BENDING MOMENT AT YIELD STRESS OF STEEL = ',F10.0,/,
           1'BENDING MOMENT AT ULTIMATE MORTAR COMPRESSIVE STRENGTH = ',
           1F10.0)
47     205  FORMAT( 'BENDING MOMENT AT TENSILE STRESS OF MORTAR = ',F10.2)
48          END
```

RESULTS FOR 1.3750 INCH SPECIMEN Series 1

INPUT DATA

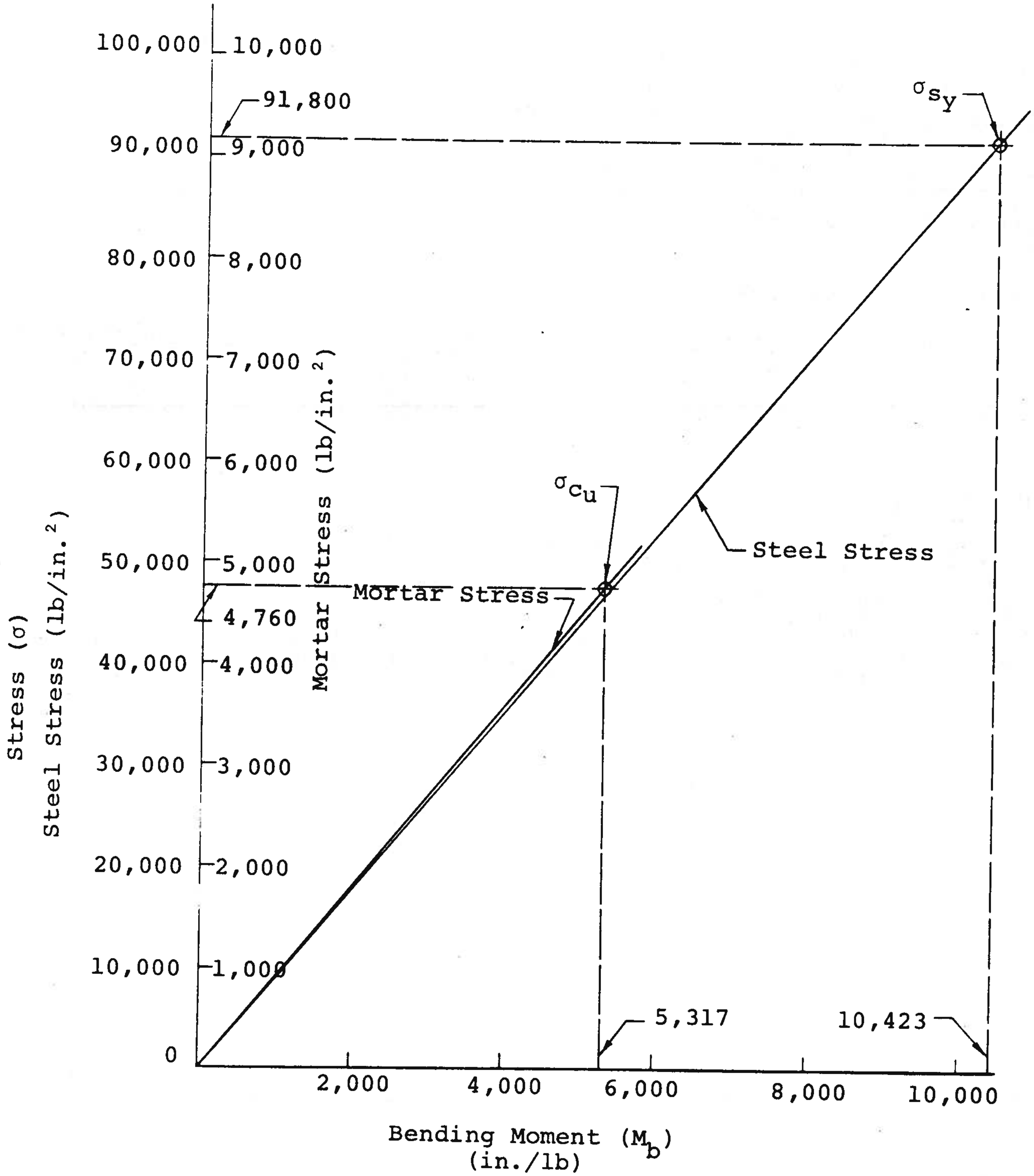
LENGTH= 23.500
WIDTH = 6.000
DEPTH = 1.37500
DENSITY OF MORTAR = 145.00
ULTIMATE STRENGTH OF MORTAR = 4760.
YIELD STRENGTH OF STEEL = 91800.
YOUNGS MODULUS OF STEEL = 29000000.
ASSUMED NEUTRAL AXIS = 0.7000
NUMBER OF STEEL LAYERS = 3
KODE = 0

STEEL REINFORCEMENT DATA

| LAYER NUMBER | DISTANCE(Y) | AREA | TRANSFORMED AREA |
|---|---|---|---|
| 1 | 0.31250 | 0.06380 | 0.46542 |
| 2 | 0.56250 | 0.14700 | 1.07237 |
| 3 | 1.06250 | 0.06380 | 0.40162 |

RESULTS

YOUNGS MODULUS OF MORTAR = 3975294.
MODULAR RATIO (N) = 7.29506
NEUTRAL AXIS = 0.92261
MOMENT OF INERTIA = 0.50534
BENDING MOMENT AT YIELD STRESS OF STEEL = -10423.
BENDING MOMENT AT ULTIMATE MORTAR COMPRESSIVE STRENGTH = 5317.




Graph 1. Working stress as a function of bending moment for specimen 1, series 1 (1.375 in.)

RESULTS FOR 1.1850 INCH SPECIMEN Series 2

INPUT DATA

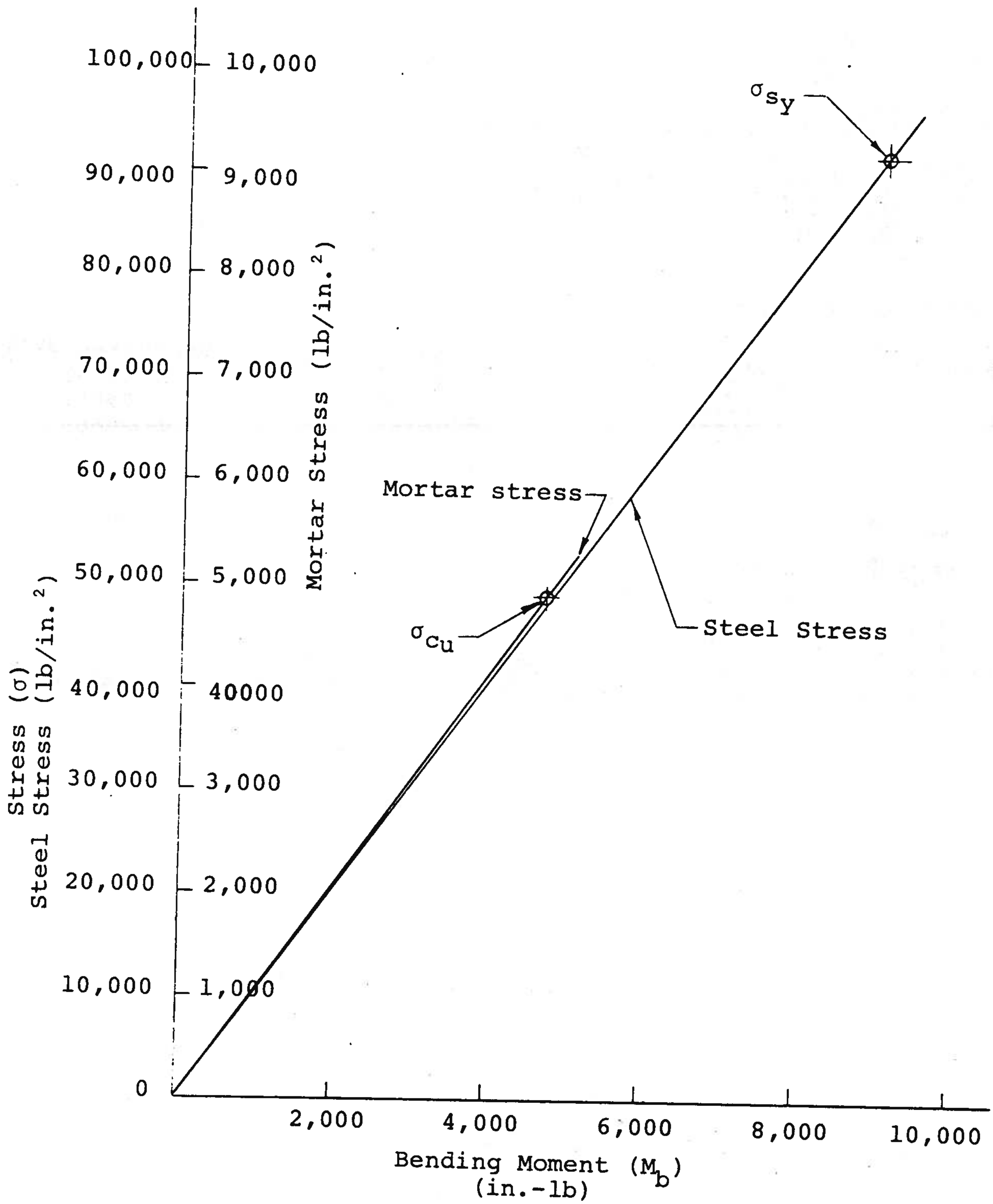
```
LENGTH=     23.500
WIDTH =      6.000
DEPTH =     1.18500
DENSITY OF MORTAR =      145.00
ULTIMATE STRENGTH OF MORTAR =      4885.
YIELD STRENGTH OF STEEL =     91800.
YOUNGS MODULUS OF STEEL =  29000000.
ASSUMED NEUTRAL AXIS =     0.7000
NUMBER OF STEEL LAYERS =      3
KODE =      0
```

STEEL REINFORCEMENT DATA

| LAYER NUMBER | DISTANCE(Y) | AREA | TRANSFORMED AREA |
|---|---|---|---|
| 1 | 0.21650 | 0.06380 | 0.45943 |
| 2 | 0.46650 | 0.14700 | 1.05856 |
| 3 | 0.96650 | 0.06380 | 0.39563 |

RESULTS

```
YOUNGS MODULUS OF MORTAR =   4027152.
MODULAR RATIO (N) =    7.20112
NEUTRAL AXIS =    0.77429
MOMENT OF INERTIA =    0.39640
BENDING MOMENT AT YIELD STRESS OF STEEL =      -9060.
BENDING MOMENT AT ULTIMATE MORTAR COMPRESSIVE STRENGTH =      4715.
```




Graph 2. Working stress as a function of bending moment for specimen 1, series 2 (1.185 in.)

RESULTS FOR 1.CCCC INCH SPECIMEN Series 2

INFUT CATA

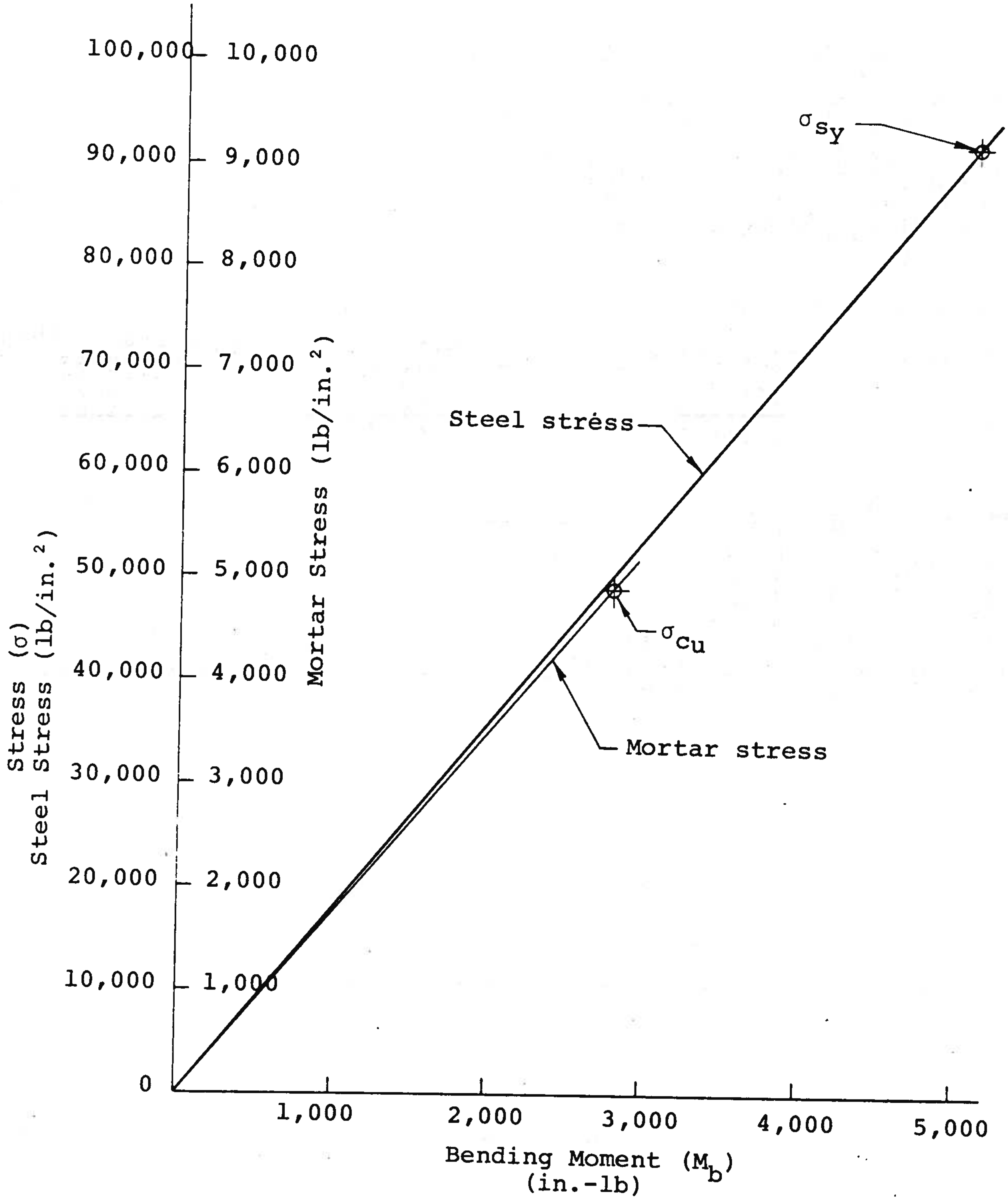
LENGTH= 23.500
WIDTH = 6.000
DEPTH = 1.CCCCC
CENSITY CF MCRTAR = 145.00
ULTIMATE STRENGTH CF MCRTAR = 4885.
YIELD STRENGTH OF STEEL = 91800.
YCUNGS MODULUS CF STEEL = 2900000C.
ASSUMED NEUTRAL AXIS = 0.6000
NUMBER OF STEEL LAYERS = 3
KODE = 0

STEEL REINFORCEMENT CATA

| LAYER NUMBER | CISTANCE(Y) | AREA | TRANSFORMEC AREA |
|---|---|---|---|
| 1 | 0.23440 | 0.04787 | 0.34472 |
| 2 | 0.40625 | C.C829C | C.59697 |
| 3 | 0.76555 | 0.04787 | 0.29685 |

RESULTS

YOUNGS MCDULUS CF MCRTAR = 4027152.
MODULAR RATIC (N) = 7.20112
NEUTRAL AXIS = C.68495
MOMENT OF INERTIA = 0.18081
BENDING MOMENT AT YIELC STRESS OF STEEL = -5116.
BENDING MOMENT AT ULTIMATE MORTAR CCMPRESSIVE STRENGTH = 2804.




Graph 3. Working stress as a function of bending moment for specimen 2, series 2 (1.00 in.)

RESULTS FOR  C.7500 INCH SPECIMEN  Series 2

INPUT CATA

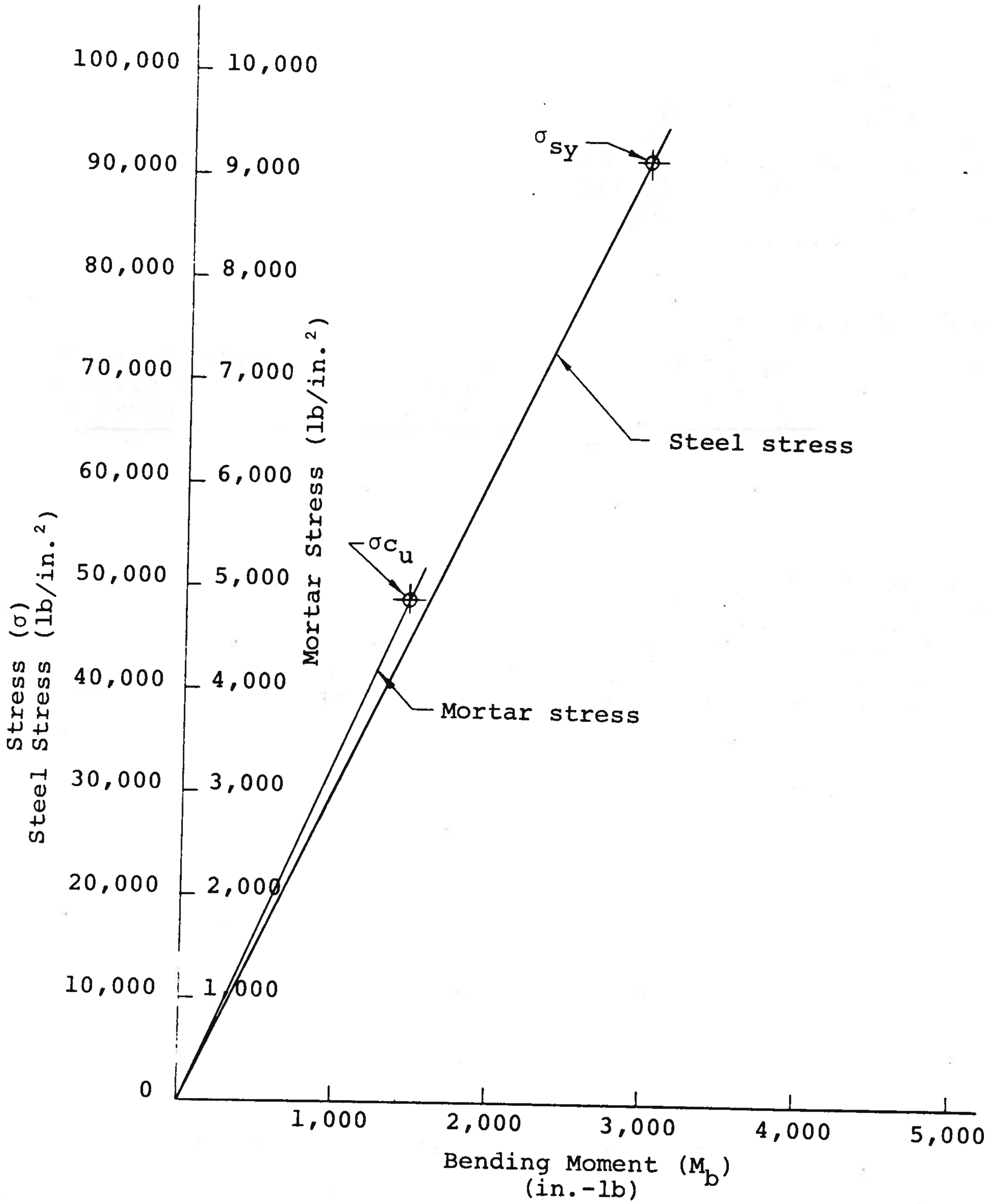
LENGTH=    23.500
WIDTH =     6.000
DEPTH =    C.75CCC
DENSITY CF MORTAR =      145.00
ULTIMATE STRENGTH CF MORTAR =      4885.
YIELD STRENGTH OF STEEL =      91800.
YOUNGS MODULUS CF STEEL =   290000CC.
ASSUMED NEUTRAL AXIS =     0.4000
NUMBER OF STEEL LAYERS =      3
KODE =     0

STEEL REINFORCEMENT CATA

| LAYER NUMBER | CISTANCE(Y) | AREA | TRANSFORMEC AREA |
|---|---|---|---|
| 1 | 0.20310 | C.04787 | C.34472 |
| 2 | 0.37500 | C.C829C | C.59697 |
| 3 | 0.5469C | 0.04787 | C.29685 |

RESULTS

YOUNGS MODULUS CF MORTAR =   4C27152.
MODULAR RATIO (N) =    7.20112
NEUTRAL AXIS =    C.509C2
MOMENT OF INERTIA =    0.07140
BENDING MOMENT AT YIELC STRESS OF STEEL =      -2975.
BENDING MOMENT AT ULTIMATE MORTAR COMPRESSIVE STRENGTH =      1447.




Graph 4. Working stress as a function of bending moment for specimen 3, series 2 (.75 in.)

RESULTS FOR 0.7500 INCH SPECIMEN Series 3

INPUT DATA

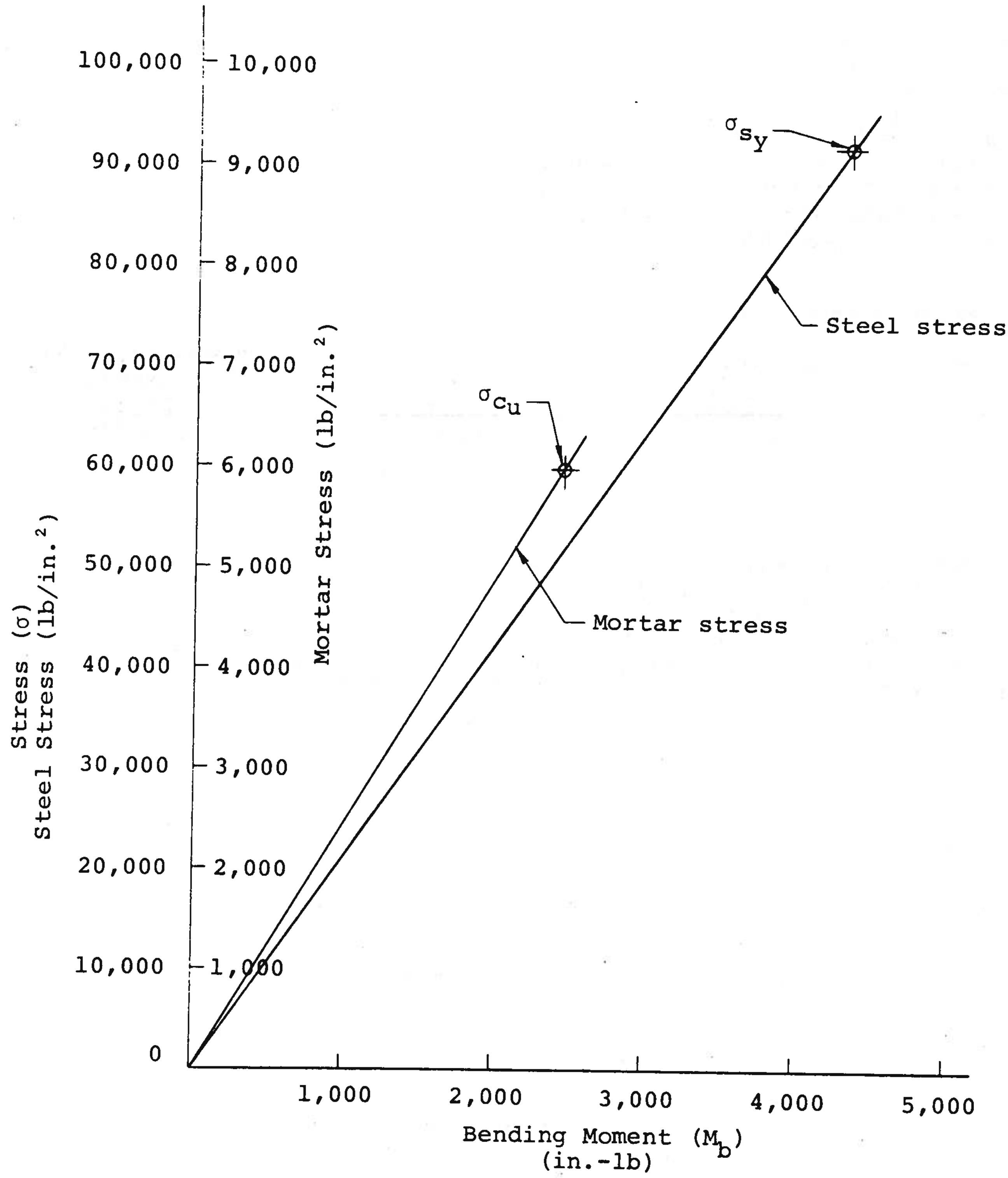
LENGTH= 23.000
WIDTH = 6.000
DEPTH = 0.75000
DENSITY OF MORTAR = 145.00
ULTIMATE STRENGTH OF MORTAR = 5930.
YIELD STRENGTH OF STEEL = 91800.
YOUNGS MODULUS OF STEEL = 29000000.
ASSUMED NEUTRAL AXIS = 0.4800
NUMBER OF STEEL LAYERS = 3
KODE = 0

STEEL REINFORCEMENT DATA

| LAYER NUMBER | DISTANCE(Y) | AREA | TRANSFORMED AREA |
|---|---|---|---|
| 1 | 0.07800 | 0.04787 | 0.31287 |
| 2 | 0.25000 | 0.08290 | 0.54183 |
| 3 | 0.42200 | 0.04787 | 0.31287 |

RESULTS

YOUNGS MODULUS OF MORTAR = 4437038.
MODULAR RATIO (N) = 6.53589
NEUTRAL AXIS = 0.46245
MOMENT OF INERTIA = 0.11876
BENDING MOMENT AT YIELD STRESS OF STEEL = -4339.
BENDING MOMENT AT ULTIMATE MORTAR COMPRESSIVE STRENGTH = 2449.




Graph 5. Working stress as a function of bending moment for specimen 1, series 3 (.75 in).

RESULTS FOR 1.0000 INCH SPECIMEN Series 3

INPUT DATA

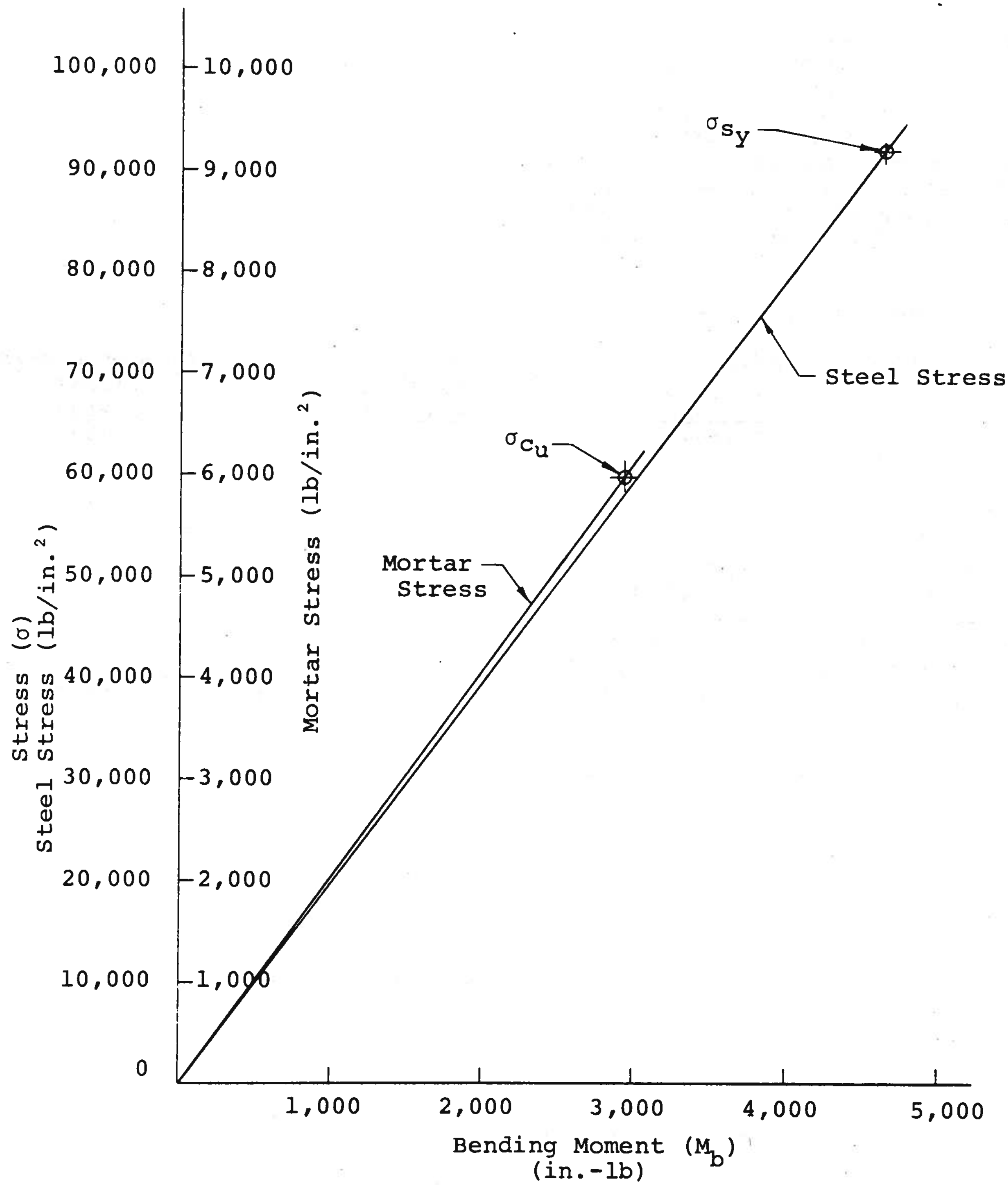
LENGTH= 23.000
WIDTH = 6.000
DEPTH = 1.00000
DENSITY OF MORTAR = 145.00
ULTIMATE STRENGTH OF MORTAR = 5930.
YIELD STRENGTH OF STEEL = 91800.
YOUNGS MODULUS OF STEEL = 29000000.
ASSUMED NEUTRAL AXIS = 0.7000
NUMBER OF STEEL LAYERS = 3
KODE = 0

STEEL REINFORCEMENT DATA

| LAYER NUMBER | DISTANCE(Y) | AREA | TRANSFORMED AREA |
|---|---|---|---|
| 1 | 0.29070 | 0.04787 | 0.31287 |
| 2 | 0.46255 | 0.08290 | 0.54183 |
| 3 | 0.82185 | 0.04787 | 0.26500 |

RESULTS

YOUNGS MODULUS OF MORTAR = 4437038.
MODULAR RATIO (N) = 6.53589
NEUTRAL AXIS = 0.71587
MOMENT OF INERTIA = 0.14018
BENDING MOMENT AT YIELD STRESS OF STEEL = -4631.
BENDING MOMENT AT ULTIMATE MORTAR COMPRESSIVE STRENGTH = 2926.




Graph 6. Working stress as a function of bending moment for specimen 2, series 3 (1.00 in.)

RESULTS FOR 1.4000 INCH SPECIMEN Series 3

INPUT DATA

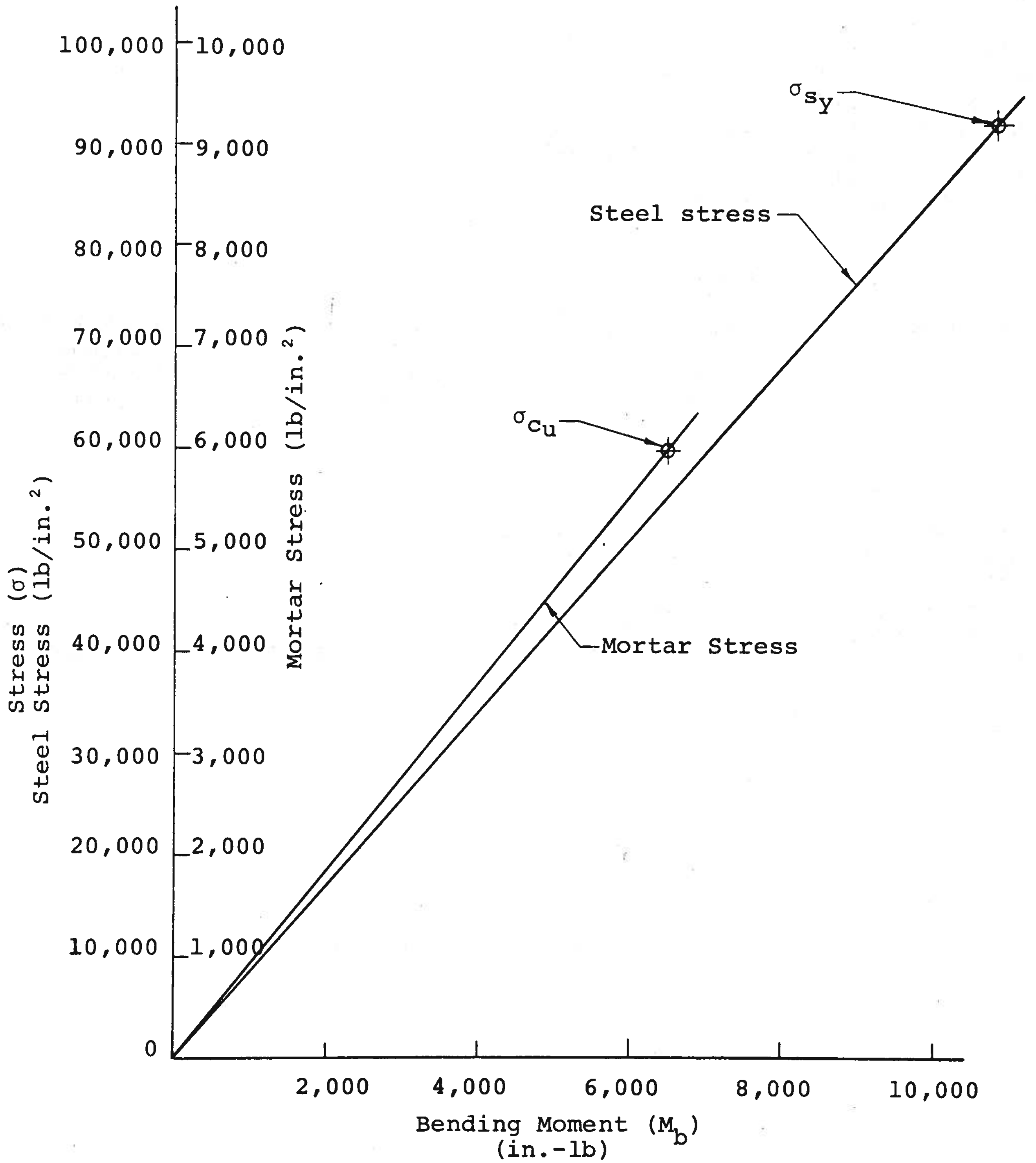
LENGTH= 23.000
WIDTH = 6.000
DEPTH = 1.40000
DENSITY OF MORTAR = 145.00
ULTIMATE STRENGTH OF MORTAR = 5930.
YIELD STRENGTH OF STEEL = 91800.
YOUNGS MODULUS OF STEEL = 29000000.
ASSUMED NEUTRAL AXIS = 0.9000
NUMBER OF STEEL LAYERS = 3
KODE = 0

STEEL REINFORCEMENT DATA

| LAYER NUMBER | DISTANCE(Y) | AREA | TRANSFORMED AREA |
|---|---|---|---|
| 1 | 0.32500 | 0.06380 | 0.41699 |
| 2 | 0.57500 | 0.14700 | 0.96078 |
| 3 | 1.07500 | 0.06380 | 0.35319 |

RESULTS

YOUNGS MODULUS OF MORTAR = 4437038.
MODULAR RATIO (N) = 6.53589
NEUTRAL AXIS = 0.95697
MOMENT OF INERTIA = 0.48555
BENDING MOMENT AT YIELD STRESS OF STEEL = -10791.
BENDING MOMENT AT ULTIMATE MORTAR COMPRESSIVE STRENGTH = 6499.




Graph 7. Working stress as a function of bending moment for specimen 3, series 3 (1.4 in.).

RESULTS FOR C.75CO INCH SPECIMEN Series 3

INPUT CATA

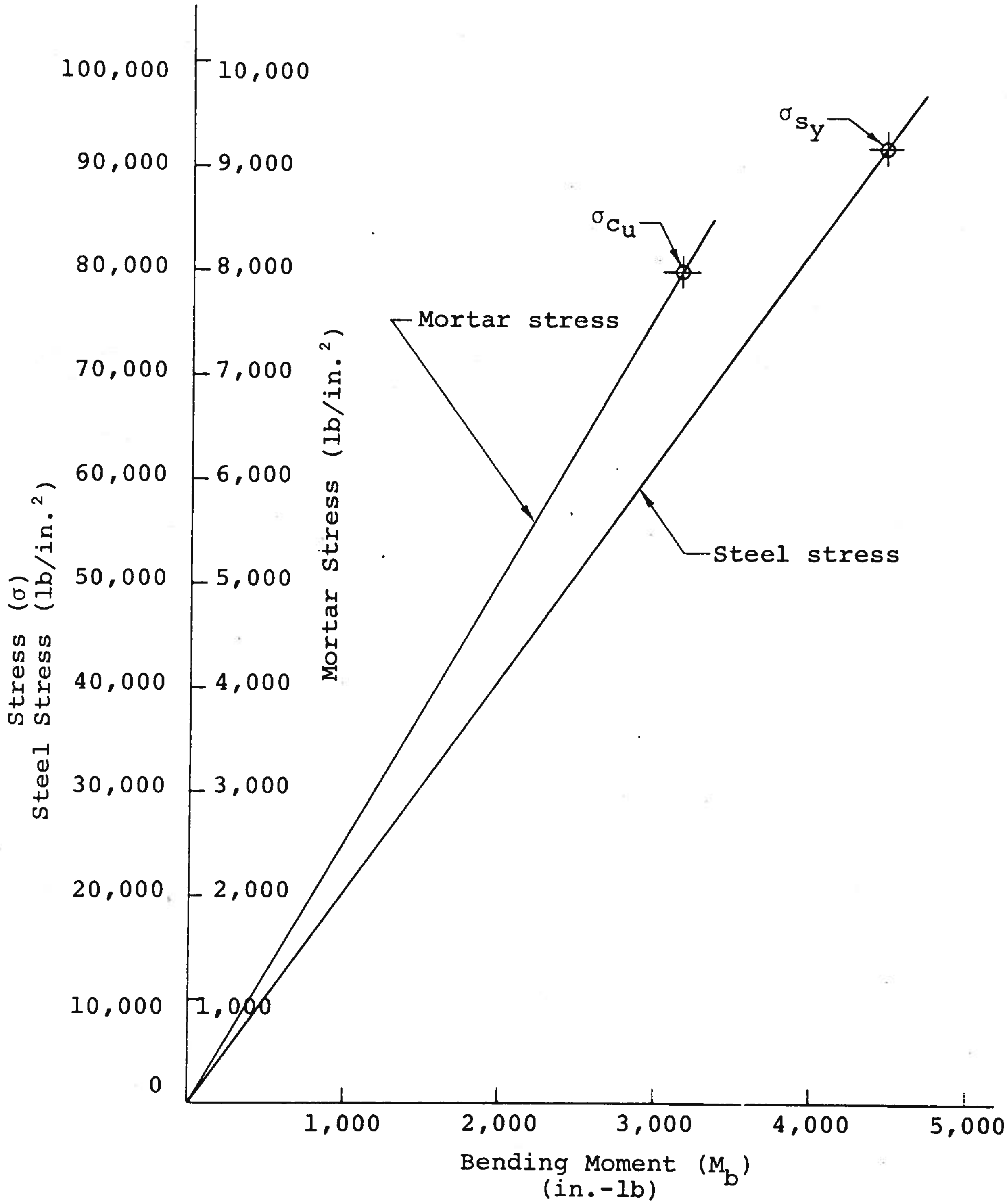
LENGTH= 23.000
WIDTH = 6.000
DEPTH = C.75COC
DENSITY OF MORTAR = 145.00
ULTIMATE STRENGTH OF MORTAR = 7937.
YIELD STRENGTH OF STEEL = 9180C.
YOUNGS MODULUS OF STEEL = 29000000.
ASSUMED NEUTRAL AXIS = 0.4800
NUMBER OF STEEL LAYERS = 3
KODE = 0

STEEL REINFORCEMENT CATA

| LAYER NUMBER | CISTANCE(Y) | AREA | TRANSFORMED AREA |
|---|---|---|---|
| 1 | 0.07800 | 0.04787 | 0.27044 |
| 2 | 0.25000 | C.C829C | C.46834 |
| 3 | 0.42200 | C.C4787 | C.27044 |

RESULTS

YOUNGS MODULUS OF MORTAR = 5133268.
MODULAR RATIO (N) = 5.64942
NEUTRAL AXIS = 0.47493
MOMENT OF INERTIA = 0.10869
BENDING MOMENT AT YIELD STRESS OF STEEL = -4449.
BENDING MOMENT AT ULTIMATE MORTAR COMPRESSIVE STRENGTH = 3136.




Graph 8. Working stress as a function of bending moment for specimen 4, series 3 (.75 in.).

# APPENDIX C

# ULTIMATE STRENGTH CALCULATIONS

Prediction for Specimen 1, Series 1 (1.375")

$$F_w = \sigma_{wy} A_w \qquad A_w = .0638 \text{ in.}^2$$

$$= 91{,}800 \times .0638 \qquad A_r = .147 \text{ in.}^2$$

$$= 5{,}857 \text{ lb} \qquad \sigma_{cu} = 4{,}760 \frac{\text{lb}}{\text{in.}^2}$$

$$F_r = \sigma_{ry} A_r$$

$$= 39{,}800 \times .147 \qquad y_1 = .3125 \text{ in.}$$

$$= 5{,}851 \text{ lb} \qquad y_2 = .5625 \text{ in.}$$

$$F_s = F_w + F_r$$

$$= 5{,}857 + 5{,}851$$

$$= 11{,}708 \text{ lb}$$

$$a = \frac{F_s}{.85\sigma_{cu} b}$$

$$= \frac{11{,}708}{.85 \times 4760 \times 6} = .4823 \text{ in.}$$

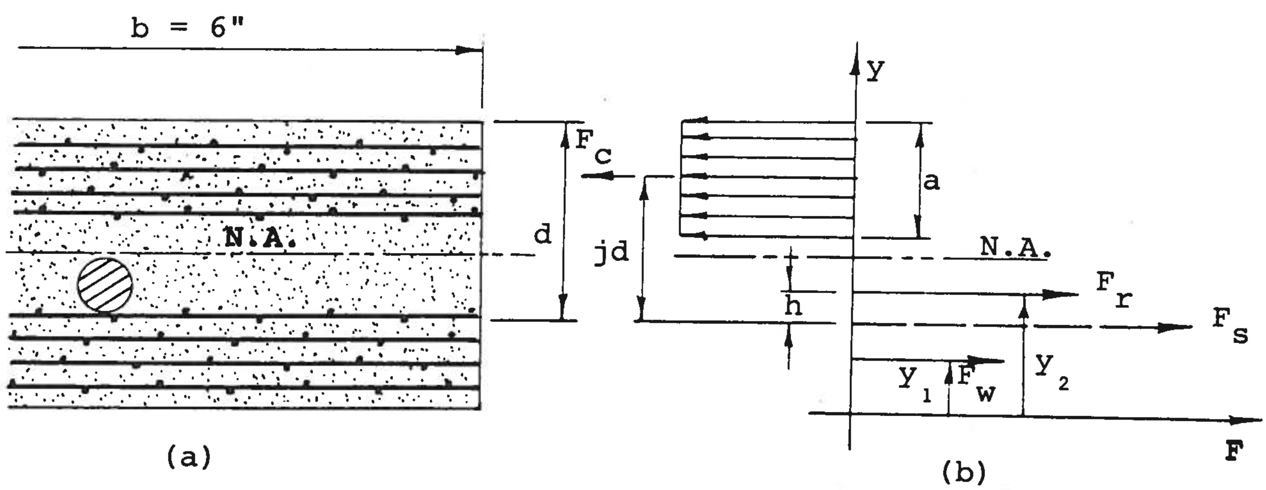



Fig. 17.(a) Dimensions of beam. (b) Stress block and resultant forces.

Taking moments about $F_r$,

$$\Sigma M_{F_r} = F_w(y_2 - y_1) + F_r(0)$$

$$h = \frac{\Sigma M_{F_r}}{F_s}$$

Evaluating:

$$\Sigma M_{F_r} = 5,857 \times (.5625 - .3125)$$

$$= 1,464 \text{ in.-lb}$$

$$\frac{\Sigma M_{F_r}}{F_s} = \frac{1,464}{11,708}$$

$$= .125 \text{ in.} = h$$

$$jd = (1.375 - y_2) + h - \frac{a}{2}$$

$$= (1.375 - .5625) + .125 - \frac{.4823}{2}$$

$$= .9375 - .2412$$

$$= .6963 \text{ in.}$$

$$M_b = jdF_s$$

$$= .6963 \times 11,708$$

$$= 8,152 \text{ in.-lb}$$

$$P = \frac{4M_b}{L}$$

$$P = \frac{4 \times 8,152}{23.5}$$

$$\underline{P = 1,388 \text{ lb}}$$

Prediction for Specimen 1, Series 2 (1.185")

$F_s = 11{,}708$ lb

(from page 66)

$$a = \frac{F_s}{.85\sigma_{cu} b}$$

$$= \frac{11{,}708}{.85 \times 4{,}885 \times 6}$$

$$= .4699 \text{ in}$$

$h = .125$ (same as page 67)

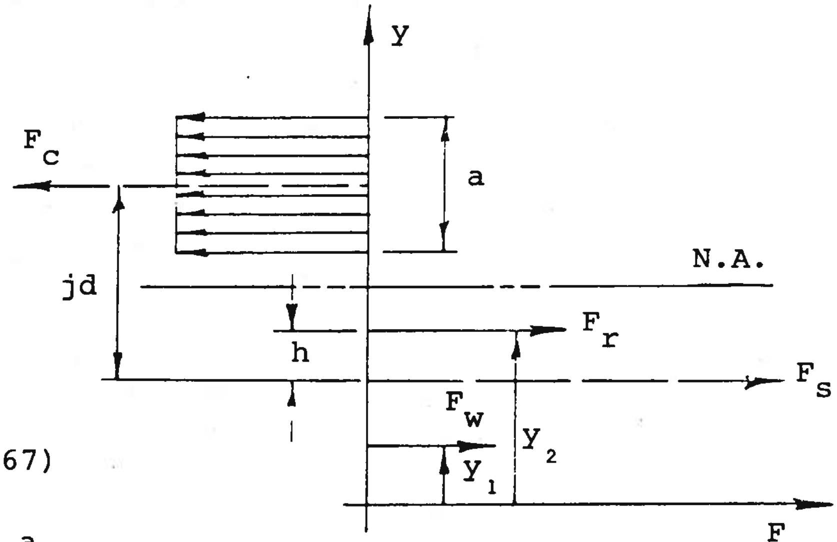



Fig. 18. Stress block and resultant forces.

$$jd = (1.185 - y_2) + h - \frac{a}{2}$$

$$= (1.185 - .4665) + .125 - \frac{.4699}{2}$$

$$= .8435 - \frac{.4699}{2}$$

$$= .8435 - .2350$$

$$= .6085$$

$$M_b = jdF_s$$

$$= .6085 \times 11{,}708$$

$$= 7{,}124 \text{ in.-lb}$$

$$P = \frac{4 \times 7{,}124}{23.5}$$

$P = 1{,}213$ lb

Prediction for Specimen 3, Series 3 (1.4")

$F_s$ = 11,708 lb (from page 66)

$$a = \frac{F_s}{.85\sigma_{cu}b}$$

$$= \frac{11,708}{.85 \times 5,930 \times 6}$$

$$= .3871 \text{ in}$$

h = .125 in. (same as page 67)

$$= (1.4 - y_2) + h - \frac{a}{2}$$

$$= (1.4 - .575) + .125 - \frac{.3871}{2}$$

$$= .950 - \frac{.3871}{2}$$

$$= .950 - .1936$$

$$= .7564$$

$$M_b = jdF_s$$

$$= .7564 \times 11,708$$

$$= 8,856 \text{ in.-lb}$$

$$P = \frac{4 \times 8,856}{23.0}$$

P = 1,540

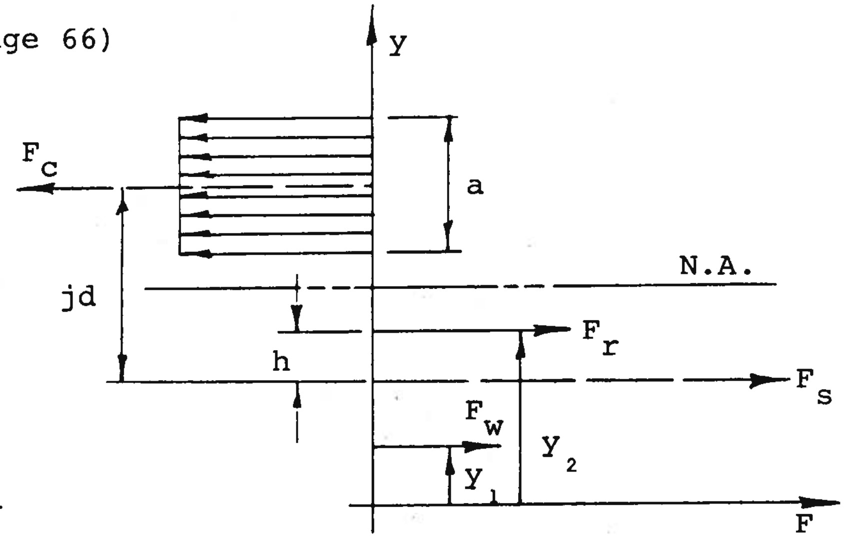



Fig. 19. Stress block and resultant forces.

Prediction for Specimen 2, Series 2 (1")

$F_w = \sigma_{w_y} A_w$

$= 91{,}800 \times .04787$

$= 4{,}395$ lb

$F_r = \sigma_{r_y} A_r$

$= 72{,}500 \times .0829$

$= 6{,}010$ lb

$F_s = F_r + F_w$

$= 6{,}010 + 4{,}395$

$= 10{,}405$ lb

$a = \frac{F_s}{.85\sigma_{c_u} b}$

$= \frac{10{,}405}{.85 \times 4{,}885 \times 6}$

$= .4176$ in.

$\Sigma M_{F_r} = F_w \times .17185$

$= 4{,}395 \times .17185$

$= 755.3$ in.-lb

$h = \frac{\Sigma M_{F_r}}{F_s}$

$= \frac{755.3}{10405}$

$= .0726$

$jd = (1.000 - y_2) + h - \frac{a}{2}$

$= (1.000 - .40625) + .0726 - \frac{.4176}{2}$

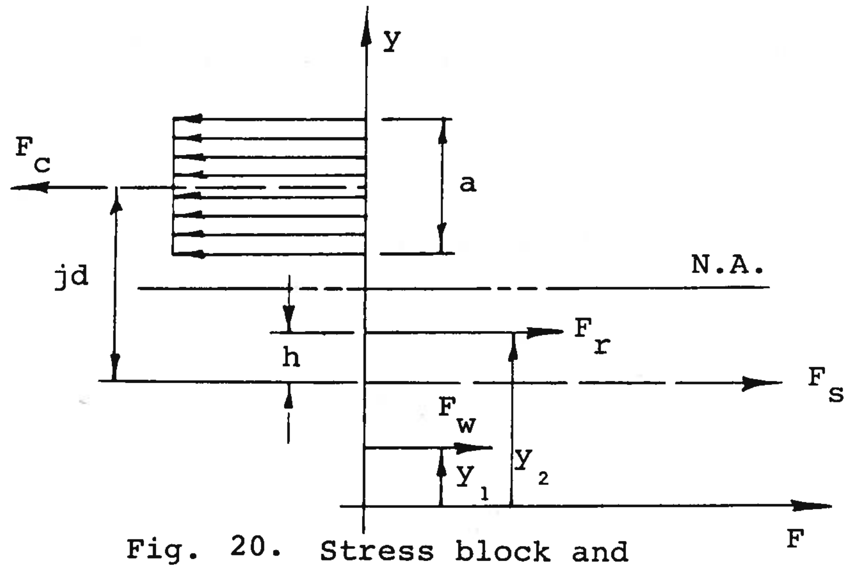



Fig. 20. Stress block and resultant forces.

Prediction for Specimen 2, Series 2 (1") (Cont'd.)

$$= .6663 - \frac{.4176}{2}$$

$$= .6663 - .2088$$

$$= .4575 \text{ in.}$$

$$M_b = jdF_s$$

$$= .4575 \times 10{,}405$$

$$= 4{,}760 \text{ in.-lb}$$

$$P = \frac{4 \times 4{,}760}{23.5}$$

$$\underline{P = 810 \text{ lb}}$$

Prediction for Specimen 2, Series 3 (1")

$F_s = 10{,}405$ (from page 70)

$$a = \frac{F_s}{.85\sigma_{cu} b}$$

$$= \frac{10{,}405}{.85 \times 5{,}930 \times 6}$$

$$= .34404 \text{ in.}$$

$h = .0726$ (from page 70)

$$jd = (1.000 - y_2) + h - \frac{a}{2}$$

$$= (1.000 - .46255)$$

$$+ .0726 - \frac{.34404}{2}$$

$$= .61005 - \frac{.34404}{2}$$

$$= .61005 - .17202$$

$$= .43803$$

$$M_b = jdF_s$$

$$= .4385 \times 10{,}405$$

$$= 4{,}563 \text{ in.-lb}$$

$$P = \frac{4 \times 4{,}563}{23.0}$$

$P = 794$ lb

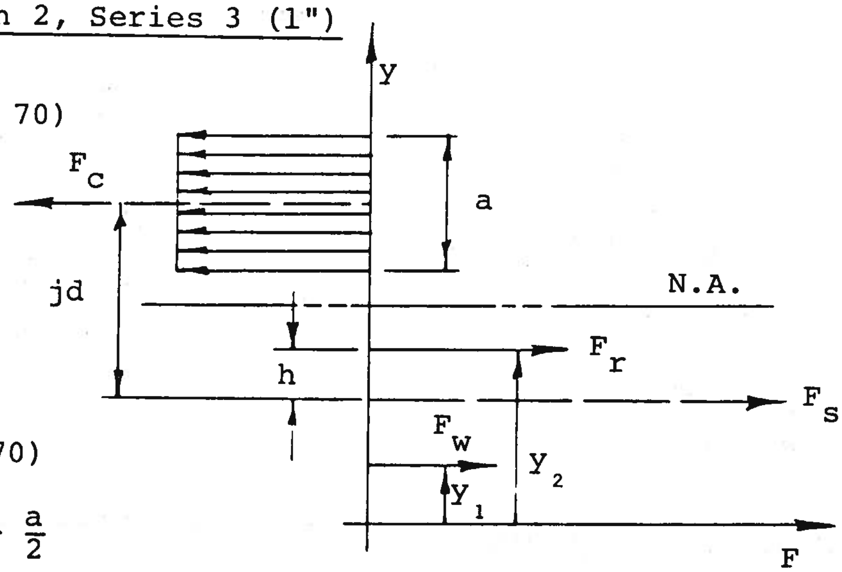



Fig. 21. Stress block and resultant forces.

Prediction for Specimen 3, Series 2 (3/4")

$F_s$ = 10,405 (from page 70)

$$a = \frac{F_s}{.85\sigma_{cu}b}$$

$$= \frac{10,405}{.85 \times 4,885 \times 6}$$

$$= .41764$$

. Impossible situation (see Fig. 22) i.e. overreinforced. Suppose the assumption is made that the beam will fail when the stress block reaches the metal rods (see Fig. 23).

$$a = .75 - y_2$$

$$= .75 - .375$$

$$= .375$$

$$F_c = .85\sigma_{cu}ba$$

$$= .85 \times 4,885 \times 6 \times .375$$

$$= 9,343 \text{ lb}$$

$$F_s = F_c$$

$$F_w = 4,395 \text{ lb}$$

$$F_r = F_s - 4,396$$

$$= 4,948 \text{ lb}$$

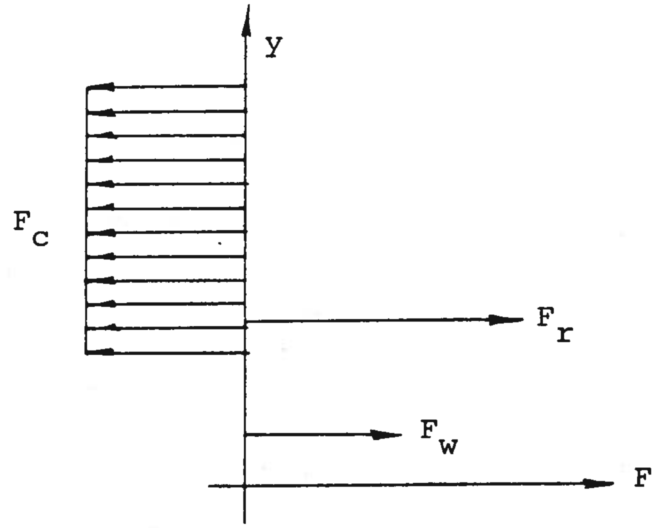



Fig. 22. Stress block and resultant forces. Note that the stress block required to balance the steel forces encroaches on the tension steel. Since there cannot be both tension and compression at the same (y) location, this is an impossible situation.

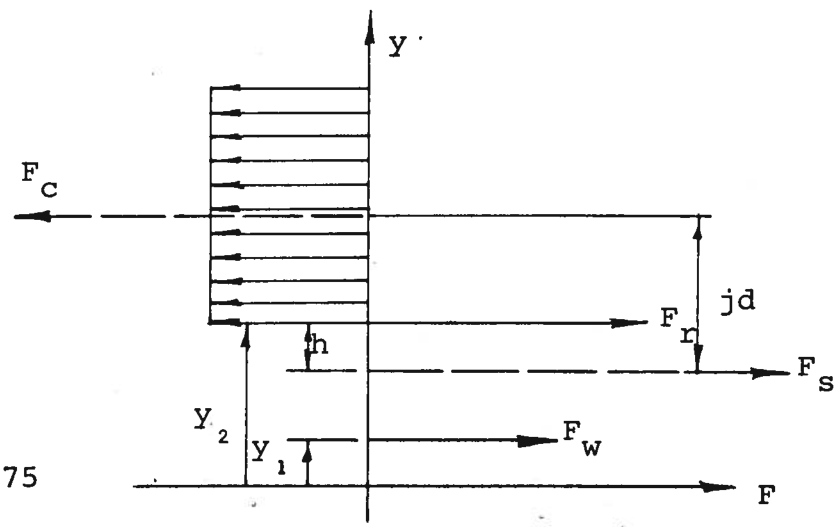



Fig. 23. The assumption is made that the beam will fail when the stress block reaches the tension steel.

Prediction for Specimen 3, Series 2 (3/4") (Cont'd.)

$$\Sigma M_{F_r} = F_w\ (y_2 - y_1) + F_r\ (0)$$

$$= 4{,}395\ (.375 - .2031)$$

$$= 755.5 \text{ in.-lb}$$

$$h = \frac{\Sigma M_{F_r}}{F_s}$$

$$= \frac{755.5}{9343}$$

$$= .08086$$

$$jd = \frac{a}{2} + h$$

$$= \frac{.375}{2} + .08086$$

$$= .26836$$

$$M_b = jdF_s$$

$$= .26836 \times 9{,}343$$

$$= 2{,}507 \text{ in.-lb}$$

$$P = \frac{2{,}507 \times 4}{23.5}$$

$$\underline{P = 427 \text{ lb}}$$

Prediction for Specimen 1, Series 3 (3/4")

$F_s$ = 10,405 (from page 70)

$$a = \frac{F_s}{.85\sigma_{cu} b}$$

$$= \frac{10,405}{.85 \times 5,930 \times 6}$$

$$= .34405$$

$$\Sigma M_{F_r} = F_w\ (y_2 - y_1) + F_r\ (0)$$

$$= 4,395\ (.1720)$$

$$= 755.9$$

$$h = \frac{\Sigma M_{F_r}}{F_s}$$

$$= \frac{755.9}{10,405}$$

$$= .07265$$

$$jd = (.75 - y_2) + h - \frac{a}{2}$$

$$= .50 + .07265 - \frac{.34405}{2}$$

$$= .4006$$

$$M_b = jdF_s$$

$$= .4006 \times 10,405$$

$$- 4,169$$

$$P = \frac{4 \times 4,169}{23.0}$$

**P = 725 lb**

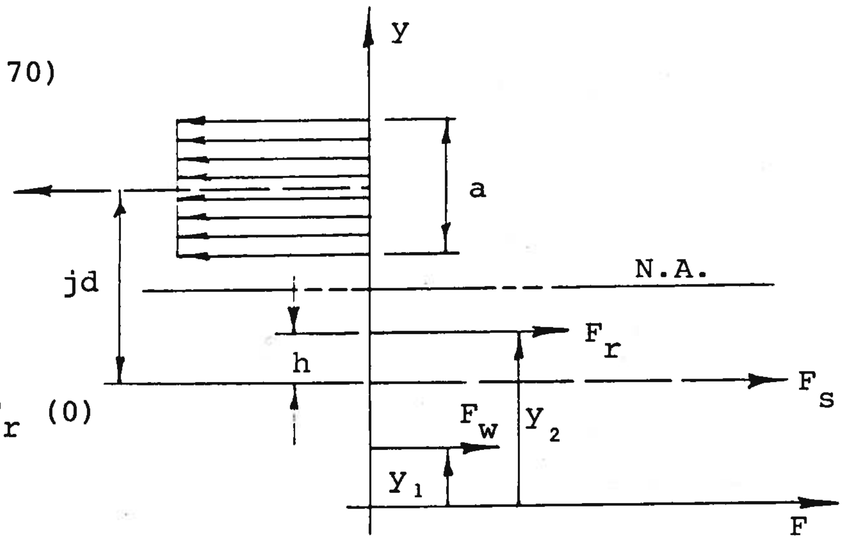



Fig. 24. Stress block and resultant forces.

$y_1$ = .0780

$y_2$ = .7500

Prediction for Specimen 4, Series 3

$F_s = 10,405$ (from page 70)

$$a = \frac{F_s}{.85\sigma_{c_u} b}$$

$$= \frac{10,405}{.85 \times 7,937 \times 6}$$

$$= .2571$$

$h = .07265$ (from page 75)

$$jd = (.75 - y_2) + h - \frac{a}{2}$$

$$= .5 + .07265 - \frac{.2571}{2}$$

$$= .4441$$

$$M_b = jdF_s$$

$$= .4441 \times 10,405$$

$$= 4,621$$

$$P = \frac{4 \times 4,621}{23.0}$$

$P = 804$ lb

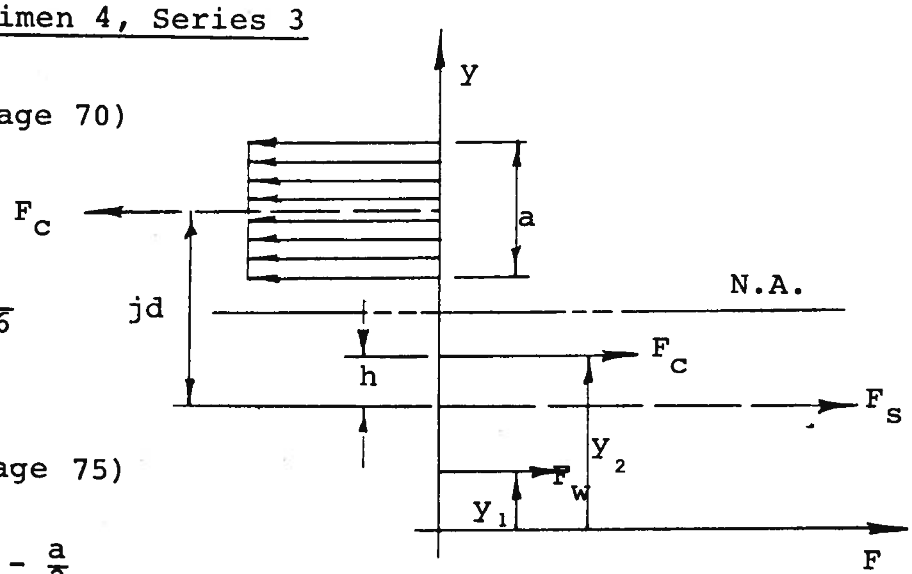


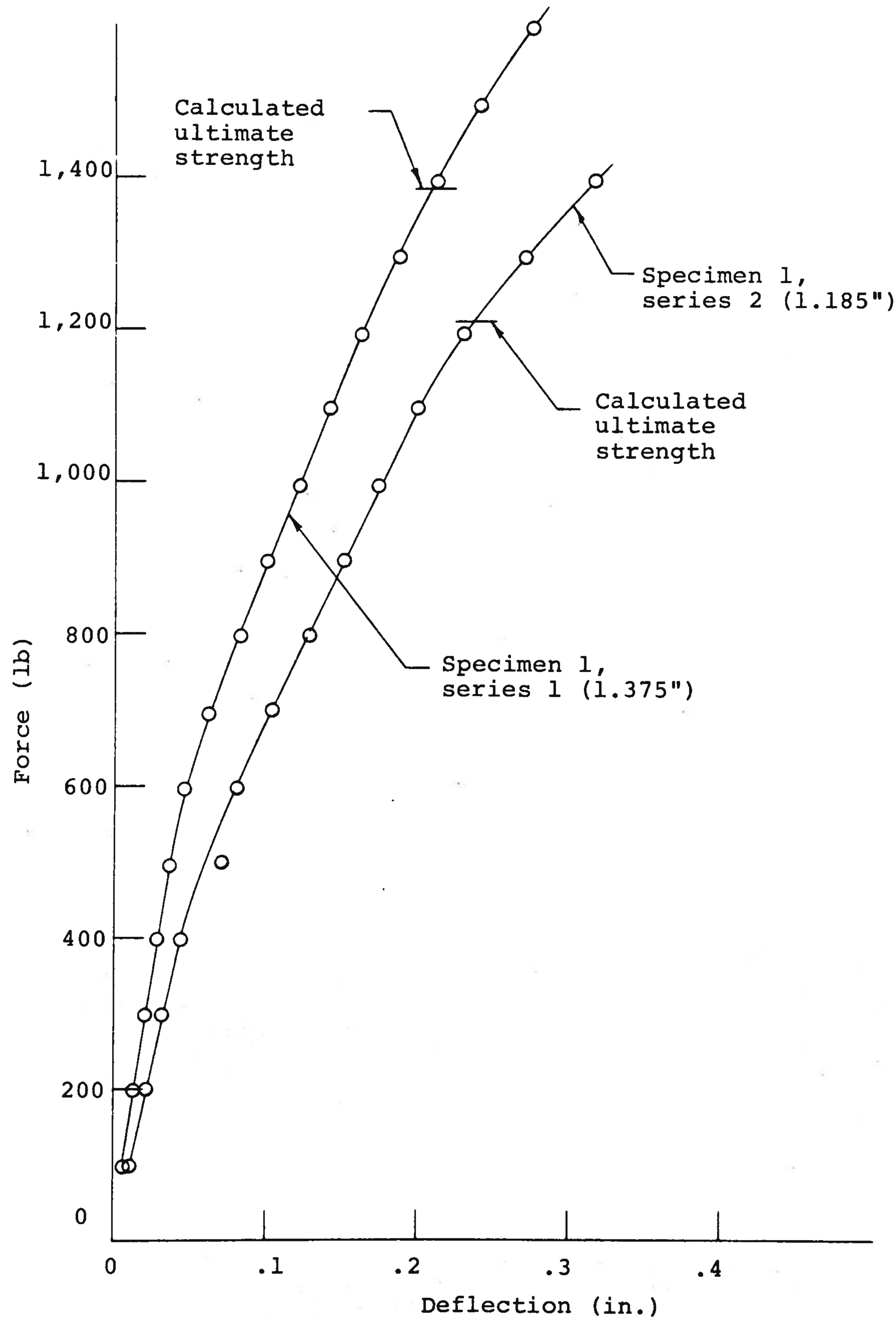

Fig. 25. Stress block and resultant forces.

$y_1 = .0780$

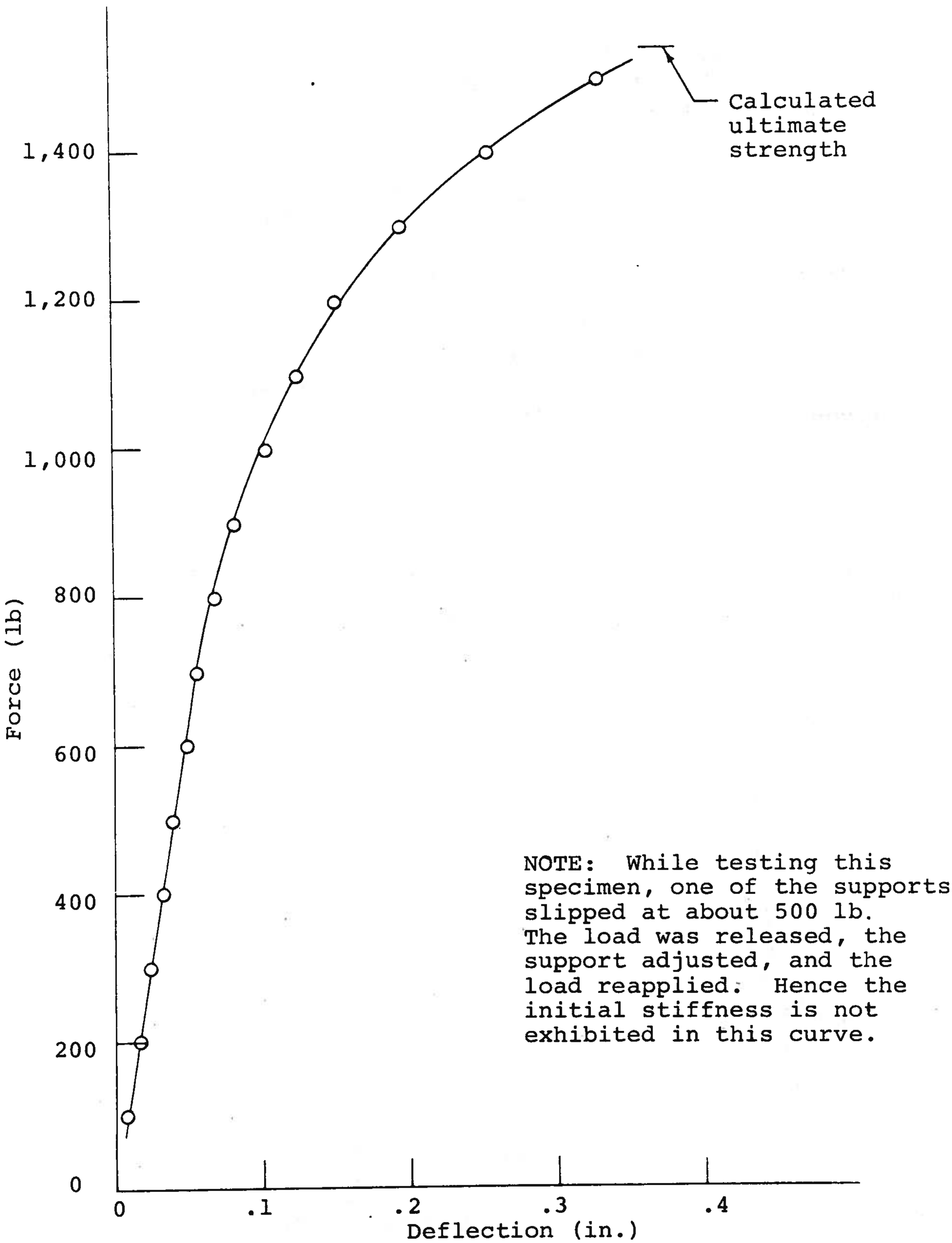
$y_2 = .250$

# APPENDIX D

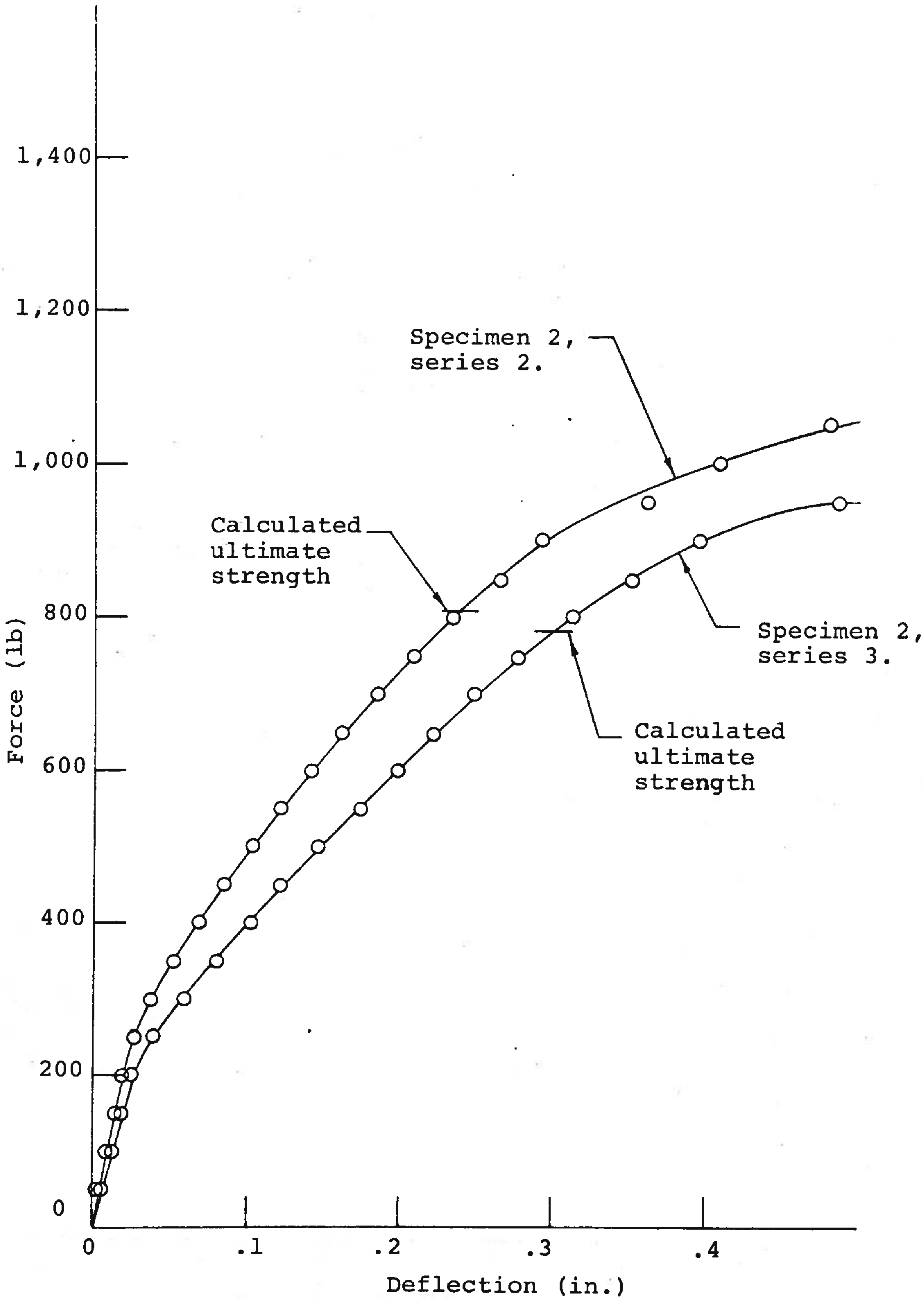
# GRAPHS



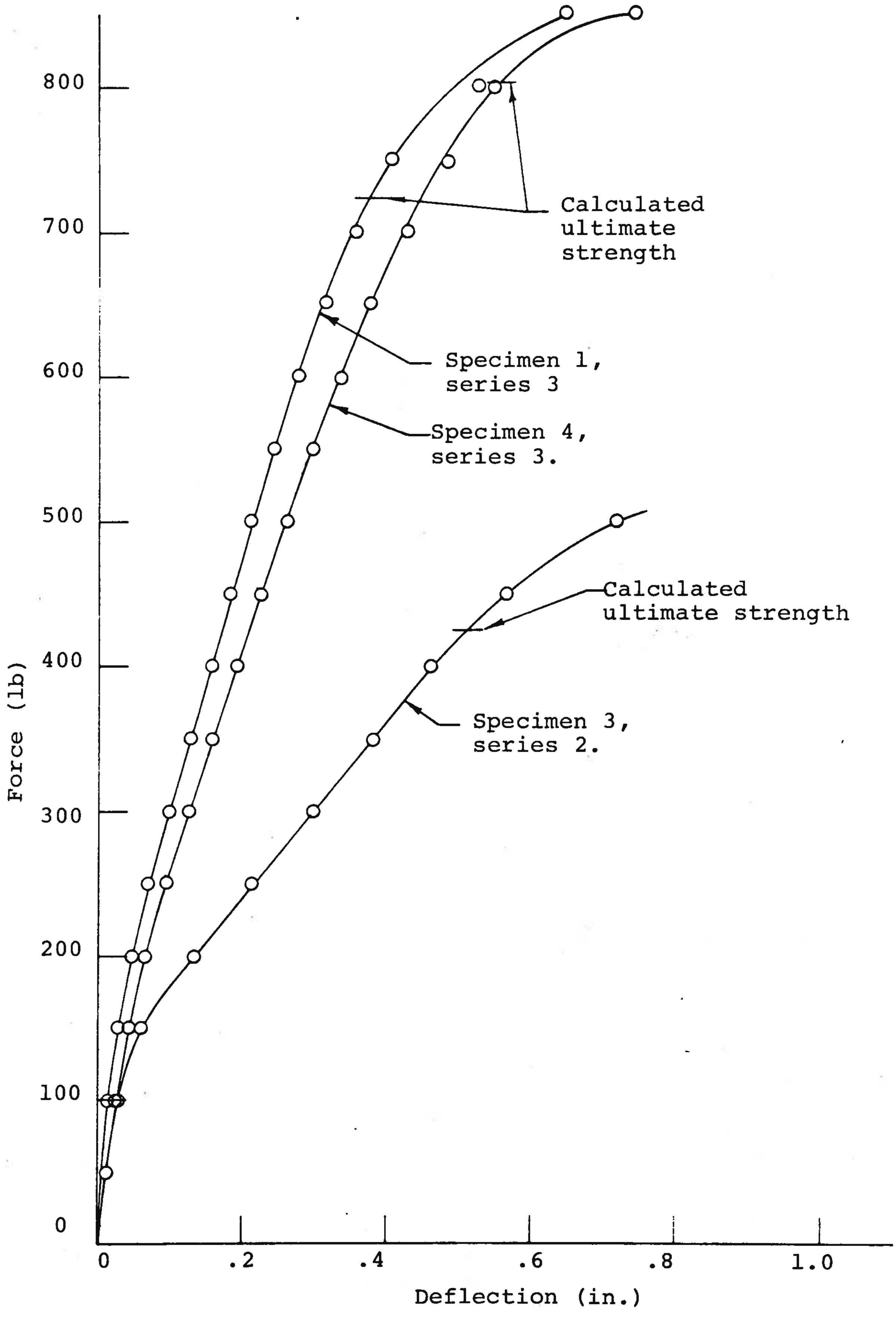

Graph 1. Force plotted as a function of deflection for large specimens.




Graph 2. Force plotted as a function of deflection for specimen 3, series 3 (1.4").




Graph 3. Force plotted as a function of deflection for 1.0" specimens.




Graph 4. Force plotted as a function of deflection for 3/4" specimens.

# APPENDIX E

# PHOTOGRAPHS

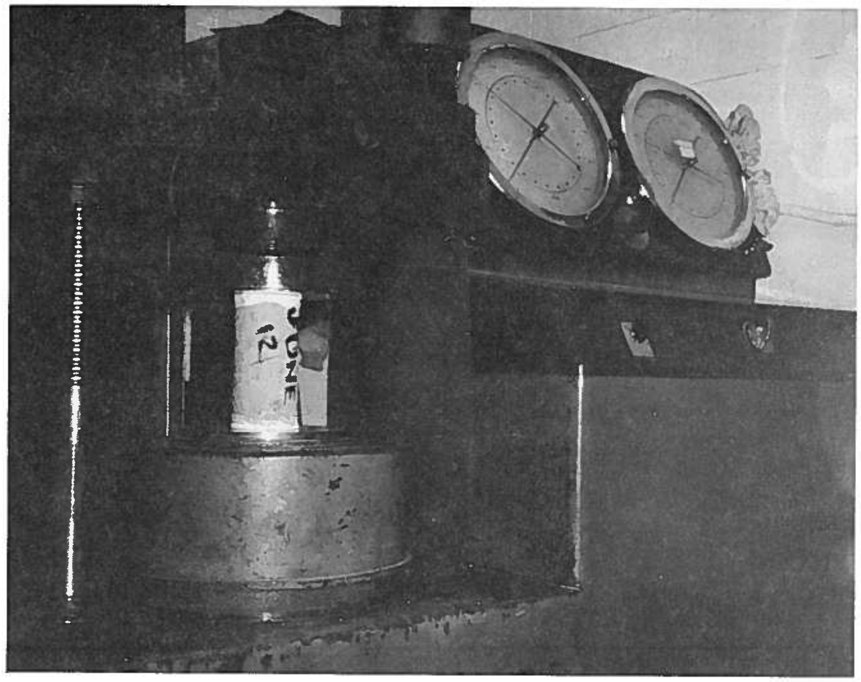



Photo. 1. Compression test of mortar cylinder.

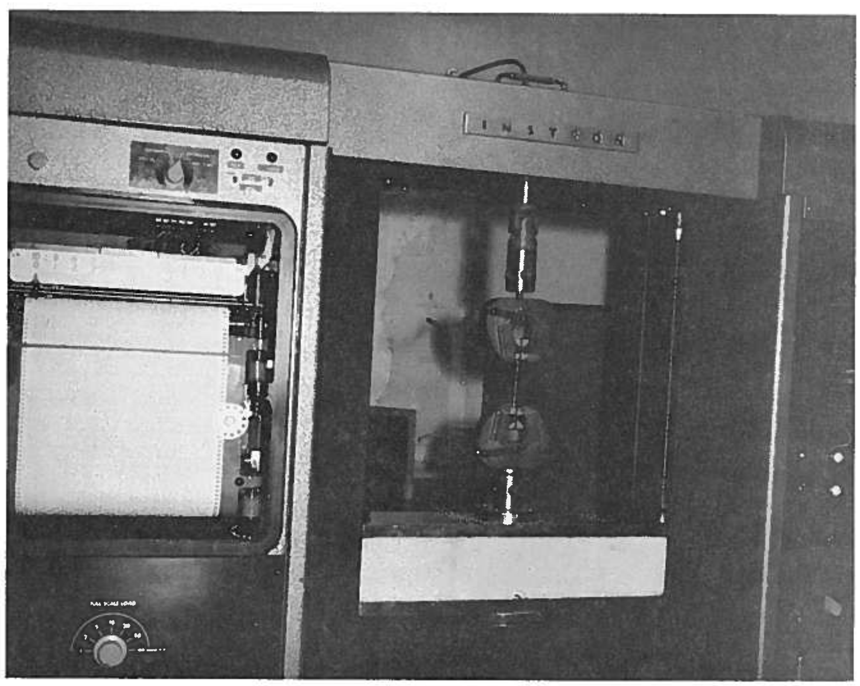



Photo. 2. Tensile test of wire mesh segments.

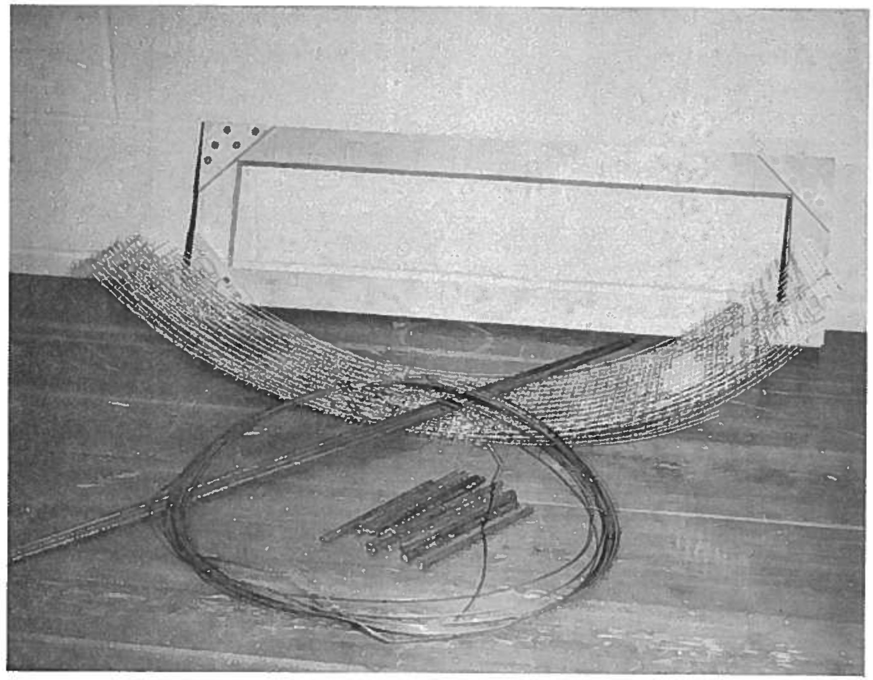


Photo. 3. The components of a ferro-cement specimen.

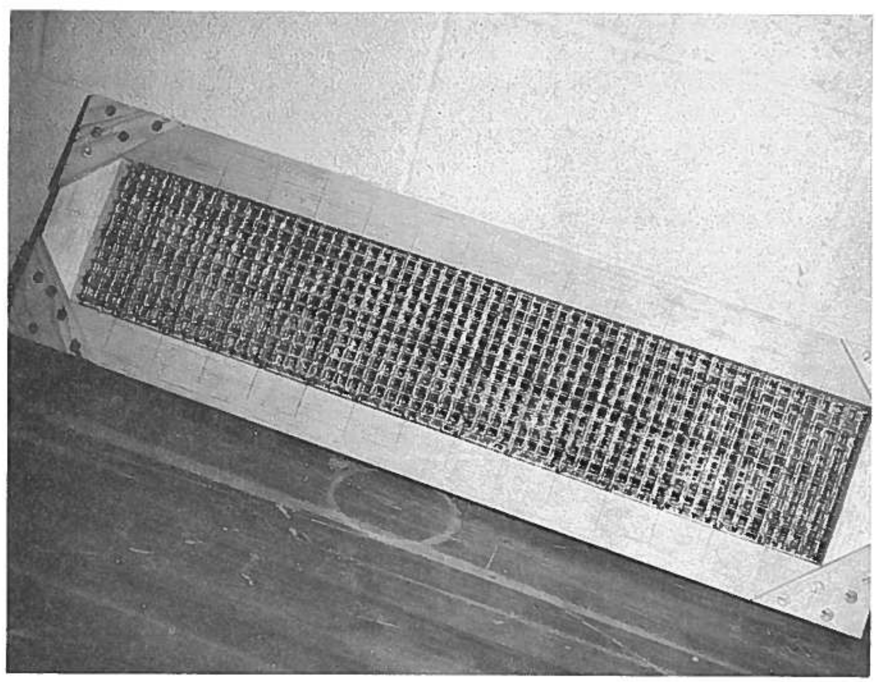


Photo. 4. A ferro-cement specimen ready for mortar.

Photo. 5. Working cement mortar into the reinforcing.

Photo. 6. A ferro-cement specimen ready for curing.

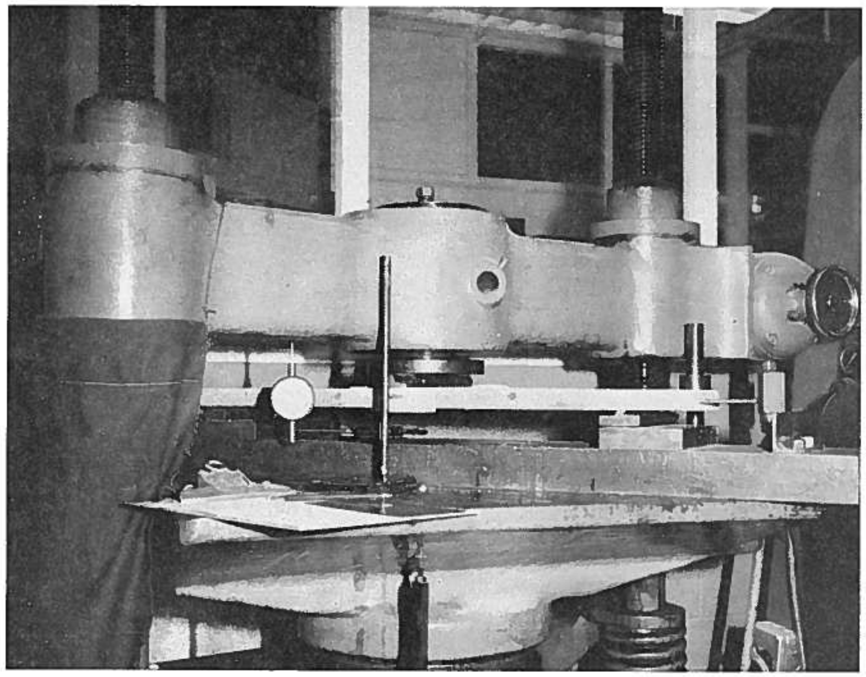


Photo. 8. A ferro-cement specimen set up for testing.

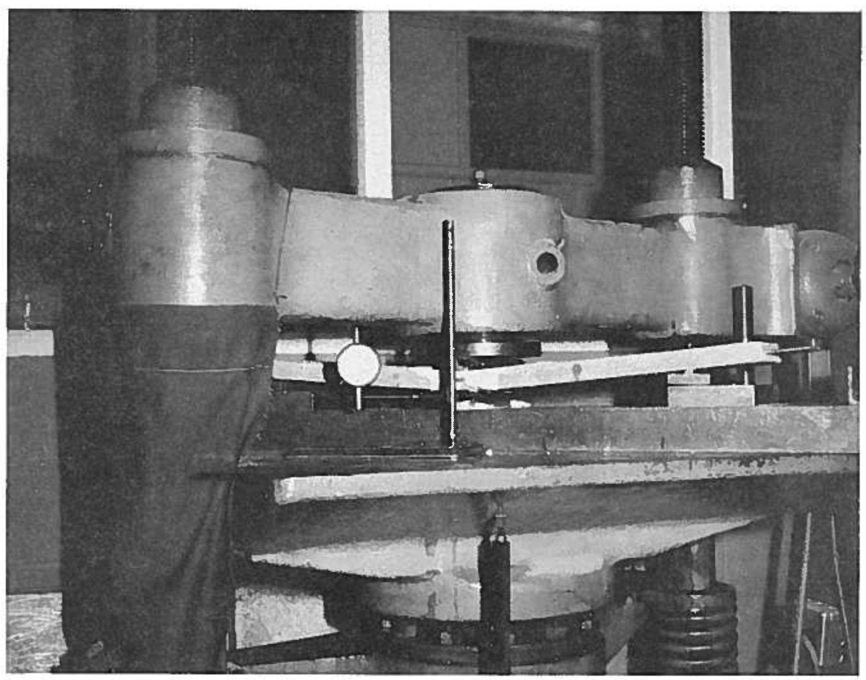


Photo. 9. A ferro-cement specimen tested to failure.