A Naval Architect's Guide to Practical Economics

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(A Teach-Yourself Text)

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October 1991

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Preface

An education enables you to earn more than an educator.
Hans Gaffron

My purpose in writing this little book is to enable students of naval architecture to unravel the mysteries of ship design economics with little or no help from the academic staff. I was asked to provide such a guide, one that could be used by our students in one course at each of three academic levels, namely sophomore, junior and senior years. Each major topic is given its own chapter, and each chapter is accordingly divided into beginning, intermediate, and advanced sections.

I recognize that there may be naval architects outside our own walls who want to educate themselves in this subject. They, too, will find this book suited to their needs. Whereas my intent is that our own students go through the book progressively at each of the three levels, others may choose to master each chapter in its entirety before moving on to the next.

You will find that learning each topic is made easy by the inclusion of numerical examples. In addition, since true mastery requires that you do exercises, you will find plenty of mental calisthenics on which you can strengthen your mind. I urge you to spend time on the examples and also on the answers that I propose. A goodly proportion of the knowledge I wish to share with you comes out in those answers.
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Abbreviations

A: uniform annual amounts, annual returns, annuities
A': annual cash flow after tax
Ap: annual payment on a loan (capital plus interest)
AAB: average annual benefit
AABI: average annual benefit index
AAC: average annual cost
ACR: annual charter rate
a: cost of the first unit of a series
B: ship's beam
CA: compound amount factor
CC: capitalized cost
CN: cubic number
CR: capital recovery factor
CR': capital recovery factor after tax
CV: constant-value dollars
CVA: annual amounts in constant-value dollars
D: annual depreciation allocation, also ship's depth
d: general inflation rate per year, also days in transit
DCF: discounted cash flow rate of return
DW: deadweight
ECT: economic cost of transport
F: a single future amount
FVS: face-value dollars
FVA: fixed annual amounts in face-value dollars
g: a gradient
H: period of a bank loan
IB: uniform annual interest payments on a loan
i: interest rate (usually per year)
i': interest rate after tax
ig: bank interest rate per year
L: resale or disposal value, also ship's length
LCC: life cycle cost
M: million, also compounding periods per year
M&R: maintenance and repair
N: a number of years in the future, life of an investment, also number of identical units
NPV: net present value
NPVI: net present value index
O.H.: overhead cost per year
P: principal, initial investment, present worth, present value
PB: amount of a loan
PR: residual debt
PBP: payback period
PW: present worth, present value, also present worth factor (single payment)
Q: tax life, depreciation period
r: discount rate applied to constant-value dollars
r1: effective annual interest rate
rM: nominal annual interest rate with non-annual compounding
RFR: required freight rate
SCA: series compound amount factor
SF: sinking fund factor
SPW: series present worth factor
SYD: sum of the years digits (depreciation)
t: corporate tax rate
V_{K}: speed in knots
v: unit value of cargo
W_{E}: empty weight, lightship
x: a maverick inflation rate
Y: uniform annual operating costs
Y_{D}: daily operating costs per year
Y_{V}: voyage costs per year
\overline{Y}: cumulative average cost for a series of sister ships
Z: a special non-annual expense, also years remaining in a cash flow
\Delta: difference between two uniform annual amounts

(In the above I have omitted such standard abbreviations as "lb" or "BTU." Several special abbreviations are explained wherever used.)
Chapter 1
Introduction

Economy: Purchasing the barrel of whiskey that you do not need for the price of the cow that you cannot afford.

Ambrose Bierce

Sophomore level

1.0 The engineer's role in society

What exactly is an engineer? There is in truth no fully satisfactory definition. In general, however, we can agree that an engineer is one who takes the facts of nature that scientists have discovered and applies them to the benefit of mankind. He or she is, in short, part of the linkage between the facts of nature and society as a whole.

As illustrated above, engineers seldom have direct contact with the public, but are most likely to deal through some intermediary, usually some person or corporate unit in the business sector. If you want to succeed as an engineer, you must learn to communicate effectively with scientists on the one hand and (usually) business managers on the other. You talk to scientists typically in terms of physics and mathematics. You talk to business people in terms of economics.

Let me put it this way. To succeed in engineering you must be able to sell your technical ideas. To whom? To business managers. And you don't approach them with, "Mr. Paramount, this design I propose represents an ideal combination of section modulus, prismatic coefficient, specific fuel consumption, and seakeeping characteristics." No. Try this: "Mr. Paramount, I understand your commercial needs and constraints, and the design I propose can meet those needs more profitably than any other." That is exactly what the manager wants to hear.

Once you have mastered what I explain in this book you will be in a position to carry out the studies that will enable you to pronounce those potent words to the perspicacious Ms. or Mr. Paramount.

1.1 Engineering economics defined

You saw my attempt to define engineering above. Now what do we mean by economics? The usual understanding is that the subject deals with the wise use of scarce resources: human power (whether mental, physical, or both), materials, physical facilities such as machinery, and spare money (commonly called capital).

Please be clear: Economics is not just about money. We must, nevertheless bring money into the picture. Why? Simply because in deciding how best to use all those scarce resources (in their disparate forms) we must find some common unit of measurement for weighing the importance of each. That common unit is the dollar, or whatever other monetary unit is appropriate in your milieu.
The subject of *finances* deals with raising and repaying capital, a subject we'll touch on later.

Finally, then, we can define *engineering economics* as the art and science of making design decisions that meet society's needs while making best possible use of scarce resources.

### 1.2 Engineering economics as a tool

Practical economics provide a means of making rational decisions. But remember that decisions are between *alternative choices*. If there are no alternatives, no decision is required. If your boss or client says, "This is the way it's going to be..." there's no need to waste your time in weighing the relative merits of other solutions. Conversely, if there is a choice between two or more alternatives, concentrate your attention on their differences. You can more or less ignore those factors that are the same. For example, say you are asked to choose between two different kinds of machinery for a ship. One type may require a larger engine crew than the other. If that is the case you should give careful thought to engine crew wages, but waste little time on those of the deck crew.

### 1.3 Summary

To summarize what we have covered so far in this first chapter, engineering success depends above all on economic success. Every design decision should consider how that decision will affect the overall economics of the unit in question. An engineer who ignores economics will probably never become more than a narrow technician. Moreover, history shows that most successful engineers eventually become at least part-time managers, and some advance to the top of the managerial ladder. As they move up that ladder a knowledge of practical economics becomes ever more important.

### Exercises

1) List the scarce resources that must be considered in economic studies.

2) What do we mean by *capital*?

3) What is the difference between *economics* and *finances*?

4) What is the essential difference between an engineer and a technician?

5) In selling your design proposals to business managers, what sort of factors should you stress?

6) We have been talking about engineering in the private sector. Should a naval architect employed by a government agency follow these same precepts?

7) Can yacht architects ignore economics?

### Answers

1) The scarce resources that must be considered are (a) materials, (b) labor, (c) production facilities, and (d) capital.

2) *Capital* is spare cash that someone is willing to make available for investments in some business venture.
Chapter 1: Introduction, Junior level

3) *Economics* deals with making wise use of scarce resources. *Finances* has to do with raising and subsequently repaying capital.

4) A technician knows all about the technical aspects of some item of hardware or software. An engineer knows those things, too, but also understands how those items may best be used to maximize benefits to the company or government agency. Someone once said, "A technician is a fellow who knows all about his job except why he's doing it."

5) An engineer who wants to sell ideas to business managers should speak in economic terms (no integral signs!).

6) Engineers in the public sector need to consider economics, too. The government agencies in which they are employed must face budget constraints that limit both immediate and future expenses. There will be differences in emphasis, but the same economic principles apply in government and private ventures. This is even true in socialist or communist nations. Communists may never use such words as *profits*, but decisions are usually aimed at gaining benefit from any funds expended.

7) Can yacht architects ignore economics? There is perhaps less stress on the subject than in most other branches of naval architecture. The main emphasis goes to reducing first cost. Most prospective yacht owners pay little heed to operating costs, but there are definite limits on how much they can afford to pay for the boat in the first place.

**Independent thinking**

At the start of the chapter we explained that engineers form a collective link between scientists on the one hand and business managers (or government bureaucrats) on the other. There are situations, however, when an engineer may want to bypass both of those links. Outline some of these situations and suggest how the engineer might go about making the direct contacts implied.

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**Junior level**

*It is vain to do with more what can be done with less.*

"Occam's Razor"

William of Occam

1.4 Systems

First, what is a system? It is a collection of people and objects working as a team toward some common goal.

When we make a decision we want to be sure that in benefiting one thing we do not do so while ignoring its impact on another. (This is known as *sub-optimization*.) Consider the case of the fellow who minimized the cost of his new garage by making it so small that he could hardly open his car doors. We avoid such mistakes by considering the entire system whenever making a decision.

An important step here is to use care in defining the system that we want to optimize. We want to draw the boundaries in such a way that we can make our decisions secure in the knowledge they will not have any untoward impact on anything else within the
Chapter 1: Introduction, Junior level

enterprise (whether private or public) we are trying to benefit. Here are three examples each involving the transport of iron ore:

1) The size of the ship is fixed by the locks of a canal. The aim of the study is to choose the best kind of propulsion plant. No matter what we choose, the cargo handling costs and terminal costs will be the same. In this case we can define the system as the ship itself. We optimize it and ignore the terminal.

2) The ore is to be moved in pellet form. It will be loaded and unloaded at proposed deep offshore terminals. There are no locks to be transited and no real limits on ship size. There are appreciable benefits in making the ship as big as possible. Those benefits, however, will be offset by added costs of providing the correspondingly large terminals. There will be no effect on the inland legs of the movement. The system can now be defined as terminal gate to terminal gate.

3) The question is whether to move the ore in (a) its raw state, (b) in pellet form, or (c) as a slurry. Now we must expand the system to include not only the complete source-to-destination transport, but the processing equipment and operation at each end.

1.5 Systems analysis

Systems analysis is a methodical approach to decision making. In it, we use these distinct steps:

1) A clear definition of the system, and its objective stated in functional terms. (For example: move 500,000 tons of coal each year from Newport News to Yokohama.)

2) A clear understanding of the constraints on the operation. (For example: flag of registry, labor union agreements, loading and unloading facility characteristics, port and canal limits.)

3) A clear definition of the economic measure of merit to be used in choosing among alternative proposals.

4) A menu of all conceivable (but technically feasible) strategies for achieving the objective in the face of the constraints.

5) Estimated quantitative value of the measure of merit likely to be achieved by each of those strategies.

6) A summary of additional, intangible factors that should be considered before making the decision.

Some comments are in order here:

1) The constraints observed in Step 2 should always be considered as subject to relaxation if good enough reason can be found.

2) There is no universally agreed-upon measure of merit. More on this later.

3) Generating the menu of alternative strategies is the truly creative part of this procedure. No two engineers are likely to produce the same selection.
4) Note that we talk about strategies rather than simply designs. This is because we must consider not only the hardware, but also the method of operation.

5) I stress the word estimated in the fifth step because the figures we derive are based on our best guesses about future costs and conditions.

6) If our aim is to maximize human satisfaction, we must recognize that some important considerations cannot readily be expressed in dollar terms. A person's pride in owning a good looking ship surely counts for something. That's just one example.

7) Finally, because of those intangible factors, the alternative that promises the best value of the measure of merit will not necessarily be the best choice. For this reason the big, important decisions should be made by the responsible business manager rather than by an engineer. The engineer's task, then, is to reduce the list of alternatives to a modest menu of choices each of which promises close-to-maximum value of the measure of merit. To this should be appended a few sentences discussing the various intangible considerations that come to mind. The manager is then equipped with a blue ribbon selection from which to choose and a rational basis on which to make the decision.

1.6 Summary
In this section we have learned to avoid the mistake of harming overall economics by concentrating on optimizing just one part of a system. We have learned the logic to be used in defining a system and we have learned about systems analysis: a step-by-step procedure for finding the best possible way to meet some specific objective. We have recognized that there are often important considerations that cannot readily be expressed in dollar terms. Decision makers must give weight as best they can to these intangible factors. In general, because they are best able to weigh the intangibles, the big decisions should be left to owners, business managers or upper level bureaucrats. The engineer's role is to provide the decision maker with a menu of choices that clearly indicates the relative economic merits of each.

Exercises

1) One naval architect has studied a projected transport system and predicts that the cost of service will be $178.23 per ton. Another, working independently, comes up with a figure of $180.77 per ton. Is this a serious discrepancy?

2) What is the difference between designs and strategies?

3) Can you suggest a case in which you might be able to assign a dollar value to what would normally be looked upon as an intangible factor? Explain how you would go about it.

4) Is the most economic choice necessarily the one that promises the best quantitative value of the selected measure of merit?

Answers

1) This is not a serious discrepancy. Indeed, the projections are in excellent agreement, being within two percent of one another. Remember that our cost studies peer into the future and of necessity use guessed-at numbers throughout. You will be doing very
Chapter 1: Introduction, Senior level

well, indeed, if any predicted number turns out to be right in the second significant figure.

2) Designs concentrate on hardware; strategies include operating methods as well as designs.

3) There are many possibilities. Perhaps the simplest is to lay the case before the decision-maker (say a shipowner) and ask how much extra he or she would be willing to pay for that specific intangible benefit. For example, a 12-knot purse seiner might occasionally be able to capture a school of fish that would escape from an 11-knot boat. Being unable to lay a number on "occasionally," the naval architect would do well to turn to the owner/operator of the boat for help.

4) Remember that in economics we attempt to make the wisest use of scarce resources. Since some factors cannot be reduced to dollars, the numbers produced by the analysis, by themselves, do not necessarily point to the wisest choice. The intangible factors must also be given weight. The central aim of engineering is human satisfaction, and some of the most important influences in human satisfaction involve unquantifiable emotions.

Independent thinking

A key element in systems analysis is a clear understanding of the constraints under which the system must operate. Suggest a hypothetical situation in which a major change in some constraint might seriously damage a proposed project. Further, if this change is seen as a distinct possibility within the supposed economic life of the project, suggest one or more means of providing a protective shield against economic disaster.

Senior level

_The moving finger writes; and having writ,_
_Moves on: nor all thy Piety nor Wit _
_Shall lure it back to cancel half a line,_
_Nor all thy tears wash out a Word of it._

Omar Khayyám

1.7 More real-life complications

In Section 1.2 we stressed that economic studies concentrate on the differences between alternatives. A second basic principle is that we must try to project the differences in cash flowing in or out of the enterprise as a result of the decision. One corollary that follows is that we can ignore the past (except for whatever help it may be in predicting the future) because the past is common to all alternatives.

Related to the above is the rule that we engineers give lost opportunity costs just as much emphasis as real costs. Passing up an opportunity to gain a thousand dollars is every bit as bad as making a decision that leads to a thousand-dollar loss. (This is one of the major points of difference between engineers and accountants. Lost opportunity costs never show up in the books, and so are ignored by accountants.)

Keep in mind this important differences between engineers and accountants: accountants are paid to look back, engineers are paid to look ahead. There are other differences, too, and we'll get into them in later chapters. Stay tuned.
Exercises

1) Here is a traditional teaser in engineering economics. Consider the case of the married couple who went to a naval architect and paid $2500 to have drawn a complete set of plans for their dream yacht. Just before contracting with a boatyard for construction, however, the husband happened to find an ad for an inexpensive set of plans for a boat that he and his wife agreed would suit their needs far better than the tailor-made set. The husband said, "Let's use these new plans; they will give us exactly what we want." The wife, however, argued, "Look, Honey, we've got to use the tailor-made set, otherwise we'll have wasted $2500." What would you decide?

2) That same couple has a telephone, which costs $25 a month, fixed rent. They make, on average, only about two calls per day. The average cost per call, then is about 41 cents. They live in an apartment building that has a pay phone in the lobby, where local calls cost only 20 cents. "Heck," says the husband, "calls on our own phone cost twice as much as those down in the lobby. From now on let's economize by using the one down there. Of course we'll still keep this one for the convenience of friends who want to invite us to parties." The wife says she thinks he's crazy but can't quite explain why. What is your opinion?

Answers

1) The husband is right. No matter which decision is made, the $2500 cost will be the same. It was bad enough to have wasted the money. Why keep reminding yourself of it by building the wrong boat? There is nothing to be gained by throwing good money after bad, as the old saying goes. This is an example of how you should not be influenced by matters that are the same for all alternatives. You cannot change history. The moving finger has moved on.

2) The wife should have explained that whether they used the pay phone or not, the monthly rent would be the same, so using that pay phone would be pouring money down a rat hole. This is another example of the wisdom of ignoring costs that are the same for all alternatives and weighing only the differences.

Independent thinking

What is the single most important difference in analyzing the economics of (a) proposed ships, and (b) existing ships?
Chapter 2
The Time-value of Money

*Philosophy triumphs easily over past and future misfortunes, but present misfortunes triumph over philosophy.*
François de la Rochefoucauld

**Sophomore level**

2.0 The human logic

By way of preamble, let me stress that when we talk about the time-value of money we are not discussing just inflation or deflation. Our subject, rather, deals with the plain human instinct for finding more pleasure from money in hand today than the firm expectation of acquiring an exactly equal amount (corrected for inflation) at some time in the future.

Ask yourself, which would you rather hold in your hand: a $1000 bill or a legal document entitling you to withdraw $1000 (plus increment for any inflation) from a bank a year from now? Unless you have something out of alignment in your head, you would surely opt for the first alternative.

Let's carry this psychological self-analysis a bit further. Would you rather have $1000 now or the firm promise of a million dollars a year from now? Unless you are on the verge of starvation you would surely have enough patience to wait for the million dollars. Well, carrying this one more step, what specific amount to be received a year from now would leave you hesitant to decide? If that figure happens to be, let us say, $1200, then your personal time value of money amounts to twenty percent annual interest. *(Interest, in effect, is rent paid for the use of money. It is commonly expressed as a percentage of the initial capital, with rent falling due at the end of every year.) In short, a safe investment opportunity promising repayment equivalent to more than twenty percent interest is one you would find attractive. If it promised anything less you would presumably choose to spend your spare cash on immediate pleasures. (Anyhow, a year from now you may be dead!)*

2.1 The financial logic

So far we have been thinking about the logic of interest in purely human, psychological terms. We can also think about it in cold-blooded financial terms. Let us say your favorite great aunt has gone to her reward and has bequeathed to you a million dollars. With that windfall you have a chance to buy an apartment house from which the steady rent income will leave you with a clear annual profit of $50 000, which amounts to five percent of the investment. Before deciding to buy that apartment house, you would want to weigh that gain against what would be available by some alternative investment of equal reliability. If some well-founded bank offers you six percent interest, you might want to think twice about buying the apartment house.

2.2 Relative merits of the two approaches

If we recognize that the ultimate aim of engineering is to provide human satisfaction, then we can argue that the psychological rationale discussed in Section 2.0 is really more important than the unfeeling tools of financial analysis discussed in Section 2.1. But, we must also admit that the psychological approach is weak in that one's personal time-value of money will tend to change from day to day or even during the day depending on the fluctuating state of one's digestion, love life, or other variable influences. Nevertheless, despite their weaknesses, subjective feelings will dominate in personal decision-making,
whether by ordinary individuals or by wealthy entrepreneurs dealing with their own cash. Corporations, on the other hand, must be more coldly analytical and base most decisions on strictly financial matters. There are, it is true, psychological influences at play because each individual stockholder must be kept happy. The corporate managers, however, can treat those influences only in the crudest manner, asking themselves how little they can pay out in annual dividends before the stockholders revolt. In short, then, both elements have their roles in weighing the time-value of money.

2.3 Three ways of thinking about interest

We can logically ascribe time-value to money in each of these three settings:

*Contracted interest* is the type with which you are most familiar. Savings deposits in banks, loans from banks, mortgages, and bonds all carry mutually agreed-upon interest rates.

*Implied interest* is appropriately considered when funds are tied up to no advantage. If you hide money under your mattress, your action is in effect costing you at least as much as the interest that you could earn by putting the money in a savings account in a bank. (This is an example of a lost opportunity cost.)

*Returned interest* is a measure of gain, if any, from risk capital invested in an enterprise. This is called by various names including *internally generated interest*, *interest rate of return*, or simply yield. It is one good measure of profitability, expressing the benefits of an investment as equivalent to returns from a bank at that derived rate of interest. Most nations impose a tax on business incomes, so we must differentiate between returns before and after tax.

In all of the above, the important thing to remember is that we weigh the time value of money by the exact same means in all three cases. And this brings us to another important rule in engineering economics:

In deciding between alternatives, we must consider for each not only how much money flows in or out, but also when.

Before going further let me state explicitly that throughout this text we shall be talking about *compound*, as distinct from *simple*, interest. In the former, the interest payments are due at the end of each period. If they are left unpaid, they will be added to the debt. Thus the debt would increase exponentially over time. With simple interest, no payments become due until the debt is paid. That is a lot less logical and the plan is seldom used.

2.4 Cash flow diagrams

Before proceeding we must pause and learn about an important convention that engineering economists use in communicating their thoughts. I refer to a simple manner of showing diagramatically how much money is being spent or earned year-by-year. We call such a drawing a *cash flow diagram*. In it, the horizontal scale represents future time, generally divided into years. The vertical scale shows annual amounts of cash flowing in (upward pointing arrows) or out (downward pointing arrows):
Chapter 2: The Time-value of Money, Sophomore level

Annual cash coming in

0

Annual cash going out

Years in future

Part of the convention is that we simplify our work by assuming that all the cash flow occurs on the last day of each year. Whatever inaccuracy this throws into our calculations tends to warp results to pretty much the same degree for all alternatives being considered, and so should have no important effect on the decision.

The diagram shown above is for a typical irregular cash flow pattern. Other common patterns are shown below.

Single investment, single pay back

F

0

P

Single investment, uniform annual returns

0

A

or

A

P

Notice the single-letter abbreviations shown above. These, too, are standard notation used by most engineering economists:

P: principal, (initial investment, treated as a single amount), also present worth or present value
F: future single payment
A: annual amounts (called returns or annuities if positive)

If you are a lender or an investor, your cash flow diagrams would look like those shown just above. If you are a borrower, all those arrows would be reversed, but the method of analysis would be exactly the same.

Ships have long economic lives, usually at least twenty years. We are therefore justified in treating cash flows on an annual basis. For shorter term studies, we may use briefer time periods, perhaps months. The basic principles and mathematics remain the same. You will find more on that at the end of this chapter.

2.5 Six basic interest relationships

Our most basic interest relationships apply to three simple cash flow patterns. First, we have the single-investment, single-payment pattern:
Chapter 2: The Time-value of Money, Sophomore level

If you know the initial amount, P, and want to find the future amount, F, multiple P by what we call the *single payment compound amount factor* (usually shortened to simply *compound amount factor*). If the time period is but a single year, the future amount, F, would equal the initial amount, P, plus the interest due, which would be i×P. In short:

\[ F = P + iP = P(1 + i) \]

If the time period, N, is some integer greater than one, then the debt would have compounded as a function of that number of years, leading to the general expression:

\[ F = P(1 + i)^N \]

The factor \((1 + i)^N\) is the compound amount factor. It is abbreviated CA and, when associated with a given interest rate and number of years, the combination is indicated by this convention: \((CA-i-N)\). For example, the single compound amount factor for 12 percent interest and 15 years would be shown as \((CA-12\%-15)\).

Moving in the other direction, if we know a single future amount and want to find its equivalent initial value, we multiply it by the reciprocal of the compound amount factor. We call this new factor the *single present worth factor*, often shortened to *present worth factor*. This being the case, the abbreviation \(P\) can now be taken to mean *present worth* or *present value*. (The terms are used interchangeably.)

\[ P = F \frac{1}{(1+i)^N} \]

Let me illustrate what we have learned so far with a couple of practical examples. Suppose you have $100 spare cash and decide to put it into a savings deposit with your bank. The bank offers 7% annual interest. If you leave the $100 in the bank for two years, how much should you be able to withdraw at the end of that period?

This calls for the use of compound amount factor:

\[ F = P(CA-7\%-2) \]

\[ F = $100(1 + i)^2 = $100(1.07)^2 \]

Now, an old-fashioned way to continue was to look up the value of \(CA = (1.07)^2\) in an interest table (such as shown here on Page 18). The other way to solve the expression with your pocket calculator. (In days of yore we used slide rules.) The truly up-to-date method is to program your computer so that all you need do is plug in three numerical values: \(P, i, \text{ and } N\), then ask the machine to produce \(F\). You will find that some business-
Chapter 2: The Time-value of Money, Sophomore level

oriented pocket calculators have such programs already built into their circuits. Knowing any three of the four elements (P, F, i, or N), the machine will instantly produce the fourth.

To continue with the problem started on the previous page, I shall use the old-fashioned approach, looking up the compound factor (CA) value in Table 1 (Page 18). The table does not show CA, but it does show its reciprocal, the present worth factor (PW), so we shall use that value (which we see is 0.8734):

\[ F = \$100(\text{CA-7\%}-2) \]

\[ F = \$100 \times \frac{1}{0.8734} = \$114.50 \]

Putting it in words, if we put $100 in the bank today, and allowed it to compound at 7% per year, at the end of two years we could withdraw $114.50. We could say that, given a time-value of money equivalent to 7% interest, $100 today is equal in desirability to $114.50 two years from now. Conversely, then, the firm promise of $114.50 two years from now has a present worth (or present value) of $100. This mental exercise of converting future amounts back into present worths is a valuable tool in economic analysis, and one we shall exploit often.

Here is an example. Assume your personal time value of money amounts to 12% interest. What should you be willing to pay for some sort of financial document that promises to pay $1000 five years from now? Here we should apply the present worth factor (PW):

\[ P = F(\text{PW-12\%}-5) \]

\[ P = \$1000(0.5674) = \$567.40 \]

Now suppose instead of 12% interest, you decided to use 20% because you found an alternative investment opportunity that promised that. Now we have:

\[ P = \$1000(\text{PW-20\%}-5) \]

\[ P = \$1000(0.4019) = \$401.90 \]

Comparing this new present value of $401.90 against the previously found $567.40, we see that the higher interest rate has reduced the present worth of the future $1000. In short, if we ascribe high numbers to the time-value of money, we diminish the importance of future benefits. Something important to keep in mind.

Note: Throughout the rest of this book I shall save space by not showing numerical values of the various interest factors, but shall assume that you have them built into your computer, or at least know how to look them up in the tables.

Our next set of interest relationships apply to a single initial amount, P, balanced against uniform annual amounts, A.
If we can predict the uniform annual amounts, A, and want to find the present worth of them, P, we use this expression (the proof of which can be found in standard texts):

\[ P = \frac{(1+i)^N - 1}{i(1+i)^N} A \]

The component \( \frac{(1+i)^N - 1}{i(1+i)^N} \) is called the series present worth factor. When associated with a given interest rate and number of years (i.e., number of equal payments), it is shown thus: (SPW-1-N). For example, if the interest rate is 12 percent and the number of equal payments is 15: (SPW-12%-15).

This relationship is useful for situations in which you can predict the size of future uniform annual returns from an investment and want to find out how much you could afford to put into that investment.

Please note that, unless otherwise stated, we always assume annual amounts and annual interest rates. Here is an example. A company that commonly earns 10% interest on its investments has a chance to buy an existing ship with a remaining life of 5 years and estimated annual clear profits of $750 000. What is the top price the company should offer for the ship? Here we want to use the series present worth factor (SPW) to convert an expected annual cash flow of $750 000 for 5 years into an equivalent single amount today. So we have:

\[ P = 750000(\text{SPW-10%-5}) = 2 843 000 \]

To be perfectly honest, we should not pretend that our answer is correct to seven significant figures, as implied above. We should more correctly present our outcome as

\[ P = 2.843 \times 10^6 \text{ or, preferably, } \$2.843 \text{ million.} \]

The second way of presenting the outcome is usually better because some business managers lack a technical background, and may not understand the first. Now, moving in the opposite direction, suppose you know the initial amount, P, and want to find the uniform annual amounts of equal present worth. (This is the common situation in which you borrow money from a bank in order to buy an automobile and are required to make uniform periodic repayments that incorporate both return of the initial loan and interest on the residual debt. In that sort of loan the payments usually fall due every month, but the principle is still the same as with annual payments.) We now have:

\[ A = \frac{i(1+i)^N}{(1+i)^N - 1} P \]
The component \( \frac{i(1+i)^N}{(1+i)^N-1} \) is called the *capital recovery factor* and is abbreviated CR. When associated with a given interest rate per compounding period, i, and number of compounding periods, N, we show it as (CR-i-N).

Here is an example. A proposed fishing boat is estimated to cost $2,500,000. The owner weighs his time value of money at 12% annual interest. The boat is expected to last 20 years. In order to justify the investment, what is the minimum annual cash flow the boat should be able to generate? Now we can use the capital recovery factor (CR) to convert the first cost of $2,500,000 to a uniform annual cash flow of equal desirability (A):

\[
A = P(CR-12\%-20) = \$2,500,000 \times (CR-12\%-20) = \$334,700
\]

Here is an example closer to home. Suppose you borrow $8000 from a bank in order to buy a car. The terms of the loan are based on 15% annual interest, payable at monthly intervals over 5 years. How much do you owe the bank each month? Now we shall use a capital recovery factor (CR) to convert the amount of the loan (P = $8000) to a monthly return. In this we must remember that the values for N and i must be on a monthly basis. In short, N (the number of compounding periods) becomes 5\times12, or 60, and i becomes 15%+12, or 1.25%. Thus we have:

\[
A = 8000(CR-1.25\%-60) = 190.32
\]

Our third pair of interest relationships apply to cash flow patterns like this:

![Cash Flow Diagram](image)

A strange quirk about this pattern is that at the end of the final year we have arrows pointing in opposite directions. This is done to simplify the calculations. In truth, of course, in real life the net amount paid would not be F, but F minus A. Another possibility is that within a business setting the annual amounts would actually comprise continual cash deposits during the year. One may nevertheless assume single year-end amounts.

If we know the uniform annual amounts (A) and want to find the equivalent single future amount (F), we multiply A by the series compound amount factor (SCA):

\[
(SCA-i-N) = \frac{(1+i)^N - 1}{i}
\]

For example, if you deposit $100 each year into a bank account paying 7% annual interest, how much should you be able to withdraw at the end 10 years?

\[
F = 100(SCA-7\%-10) = 1381.64.
\]

If you want to reverse the procedure and find out how much you must deposit each year in order to build up some specific future amount (F), you would multiply that future
amount by what we call the sinking fund factor (SF), which would of course be the reciprocal of the series compound amount factor:

\[(SF-i-N) = \frac{i}{(1+i)^N - 1}\]

Let's try an example: You want to build up an amount of $15,000 five years from now so that you can buy a sailboat. You decide to place annual amounts in a bank offering 8% interest compounded annually. How much must you deposit each year?

\[A = 15,000(SF-8\%-5) = 2556.85\]

2.6 Summary and review

We started this chapter by trying to understand the logic of ascribing time-value to money. We saw that its underpinnings are based on financial as well as human psychological considerations. We learned one of the basic rules of engineering economy, namely that in making design decisions we must consider not only how much money flows in or out of an organization, but also when.

We then learned the mechanics of analyzing three kinds of simple cash flows. In the first case, we learned to balance a current amount against a single future amount (such as a single deposit against a single future withdrawal):

Knowing a present worth, to find the equivalent future value, multiple the present worth by the (single payment) compound amount factor.

Knowing a future value, to find the equivalent present value, multiply the future amount by the (single payment) present worth factor.

We then learned how to balance a single current amount against future uniform periodic cash flows:

Knowing a present worth, to find the equivalent uniform periodic returns, multiply present worth by the capital recovery factor.

Knowing the uniform periodic returns, to find the present worth, multiple the periodic returns by the series present worth factor.

Finally, we learned how to balance a single future amount against uniform periodic cash flows:

Knowing the future amount, to find the equivalent periodic amounts, multiply the future amount by the sinking fund factor.

Knowing the uniform periodic amounts, to find the equivalent future amount, multiply the periodic amounts by the series compound amount factor.
Chapter 2: The Time-value of Money, Sophomore level

As you work your way through the following exercises you will learn a few more practical things involving the use of these standard interest relationships. The more advanced sections of this chapter will deal with some real-life complications, but a goodly portion of the problems you will be called upon to solve in your careers can be handled by the concepts explained in this introductory section.

I urge you to go through the following exercises carefully. You need practice to feel at home with these important basic concepts, and I have tossed in a few intellectual bonus points to enhance the intellectual benefits.

Exercises

*One must learn by doing the thing;
though you think you know it,
You have no certainty until you try.*

Sophocles

1) If you borrow $1000 today at 10% interest and repay the debt in a lump sum 5 years from now, how much will you return to the lender?

2) Rework the first problem but use 20% interest.

3) Rework the first problem using 10% interest again, but extending the time to 10 years.

4) Find the present worth of the amount found in Problem #3, using the same values of N and i.

5) Rework #4 using 5% interest.

6) Rework #4 using 20% interest.

7) Comparing the outcomes of the three previous problems, what conclusions do you reach as regards the effect of interest rate on the present worth of future amounts?

8) What is the present worth of $500 two years from now if interest rate is 15%?

9) If you borrow $5000 and repay the debt in five equal annual payments at 12% interest, how large will they be?

10) Find the present worth of the amounts found in the previous problem, again with 12% interest.

11) You think you will need $15 000 in a lump sum 5 years hence. How much must you invest each year at 10% interest in order to meet this objective?
### Table 2.1

#### Interest Factors

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#### Single Present Worth Factors (PW)

Note: Single Compound Amount Factors (CA) are the reciprocals

#### Capital Recovery Factors (CR)

Note: Series Present Worth Factors (SPW) are the reciprocals

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#### Sinking Fund Factors (SF)

Note: Series Compound Amount Factors are the reciprocals

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Chapter 2: The Time-value of Money, Sophomore level

12) Using the same interest rate, find the present worth of that same $15 000.

13) Convert the present worth found in #12 to uniform annual payments over 5 years at 10% interest.

14) If your personal time value of money amounts to 10% interest, which would you rather have: the promise of $15 000 five years hence, the single amount payable today found in #12, or the uniform annual amounts found in #13?

15) Examining Tables 2.1-B and 2.1-C, can you deduce a simple relationship between the numerical values of capital recovery factors (CR) and sinking fund factors (SF)?

16) Which is more desirable, an income of $500 five years hence or one of $2400 twenty years hence, if interest rate is 10%?

**Answers**

1) The amount owed after 5 years will be $1611, found as follows:

\[ F = P(CA) = 1000(CA-10\%-5) = 1000(0.1611) = 1611. \]

*Note: In real life this future amount would be worked out to the nearest penny. Our convention, however, is to round things off after four significant figures. Engineers use economics to choose between alternatives. If we have to develop our final projected outcomes to more than two significant figures, we can only conclude that the alternatives are equal. Moreover, if we are using interest tables that are confined to four significant figures (such as in Table 1), then we should never present outcomes to any greater degree of accuracy.*

2) With 20% interest, the amount owed would be:

\[ F = 1000(CA-20\%-5) = 2488 \]

3) $2594

4) This should be intuitively obvious, but here goes:

\[ P = F(PW) = 2594(PW-10\%-10) = 1000 \]

5) \[ P = F(PW) = 2594(PW-5\%-10) = 1592 \]

6) \[ P = F(PW) = 2594(PW-20\%-10) = 419 \]

7) We can readily see that higher interest rates reduce present worths of future income. By placing greater emphasis on the time-value of money, they diminish the benefits of future incomes -- or lessen the disadvantages of future losses.

8) $378

9) To convert single amounts now to uniform annual amounts in the future, we employ the capital recovery factor (CR):
Chapter 2: The Time-value of Money, Sophomore level

\[ A = P(CR) = 5000(CR-12\%-5) = 1387 \]

10) Again, this should be obvious, but here goes:

\[ P = A(SPW) = 1387 (SPW-12\%-5) = 5000 \]

11) To convert a single future amount ($15,000) to uniform annual amounts (A), we apply the sinking fund factor (SF):

\[ A = F(SF) = 15,000(SF-10\%-5) = 2457 \]

12) \[ P = F(PW) = 15,000(PW-10\%-5) = 9314 \]

13) \[ A = P(CR) = 9314(CR-10\%-5) = 2457 \] (This outcome, coinciding with that of #11, gives us a nice check on our work. Think about it.)

14) All three alternatives have the same present value (at 10% interest) and so there is no apparent reason for you to prefer one over the others. There may be intangible reasons, however.

15) I'll let you figure this out by yourself. It's easy!

16) To weigh the relative advantages of cash flows occurring at different times, the standard approach is to compare their present worths:

For $500 five years hence: \[ P = 500(PW-10\%-5) = 310 \]
For $2400 twenty years hence: \[ P = 2400(PW-10\%-20) = 357 \]
Conclusion: The second alternative is more desirable.

A second way to compare the two is to move either one to the same time as the other. In this case, let's find the present worth of $2400 moved back 15 years to coincide with the timing of the $500 amount:

\[ P \text{ at year } 5 = 2400(PW-10\%-15) = 575 \] (which again indicates that this is the better alternative)

**Independent thinking**

What is the formula for relating debt (P) and uniform annual payments (A) for N years if interest rate is set at zero (i.e., ascribing no time value to money).

What is the relationship between the capital recovery factor (CR) and the interest rate (i) for an investment in real estate with an infinite life (N = \infty)?

The concepts presented in the foregoing section of this chapter are at the heart of engineering economic analysis. You owe it to yourself to work hard to integrate them into your thinking apparatus. Repetition is an important part of the learning process. I urge you to go back and rework all of the exercises using different values of interest rates or time periods.
2.7 Perspective
In the previous section we learned how to handle simple cash flow patterns. Now we
are ready to learn how to handle cash flows of somewhat greater complexity. All the
techniques we shall discuss here are derived from the basic concepts explained in the
preceding section. The details may look complicated, but the individual steps are simple.
If you pay attention both to each individual step and to the overall logic, you should have
no real difficulty.

You will note that I use a lot of rough (i.e., to no exact scale) cash flow diagrams.
These help us clarify the situation and so avoid mistakes in logic. Never be too proud to
use such aids to clear thinking.

2.8 Non-uniform cash flows
Suppose that we have projected cash flows of $100 in year 1, $50 in year 2, nothing
in year 3, a loss of $100 in year 4, and a gain of $200 in year 5 (the end of the project's
economic life). If the interest rate is 10%, what is the present value of this predicted cash
flow? First, here is the cash flow diagram:

Our approach to such a problem is simple. We merely discount each individual amount
back to year zero using 10% interest and appropriate values of N:

For $100 one year hence: $P = 100(PW-10\%-1) = 91$
For $50 two years hence: $P = 50(PW-10\%-2) = 41$
For nothing 3 years hence: $P = 0$
For a loss of $100 4 years hence: $P = (\$100)(PW-10\%-4) = (\$68)$
For $200 five years hence: $P = 200(PW-10\%-5) = 124$

Present worth = net sum of the above = $188$

As you may note in the above, negative cash flows are placed in parentheses.

Many of you will be skillful enough with your computers to plunk in the various
values and let the machine go through all the steps shown above and expectorate the
answer. There will, however, be cases where you will need to convince some business
manager of the validity of your conclusion. Perhaps that manager lacks your mathematical
skills and doesn't trust your magic machine. That being the case, you will want to lay out
your analysis in a neat table exactly as an accountant would do it. (See next page.)
Chapter 2: The Time-value of Money, Junior level

<table>
<thead>
<tr>
<th>N = year</th>
<th>Amount (PW-10%-N)</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$100</td>
<td>0.909</td>
</tr>
<tr>
<td>2</td>
<td>$50</td>
<td>0.826</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.751</td>
</tr>
<tr>
<td>4</td>
<td>($100)</td>
<td>0.683</td>
</tr>
<tr>
<td>5</td>
<td>$200</td>
<td>0.621</td>
</tr>
</tbody>
</table>

Total present worth = $188

2.9 Gradient series

There may be cases where we can predict that future cash flows will be increasing by a fixed amount (which we'll abbreviate "g") each year.

slope = $g$ per year

We could find the present worth of such a cash flow with a year-by-year analysis just as we did above. A more sophisticated (and usually easier) way would be first to find its equivalent uniform annual amount (A) by means of this formula:

\[ A = A_1 + \frac{g}{i} \left(1 - \frac{1}{(1+i)^N}\right) - \frac{Ng}{i} \left(1\right) \]

The present worth could then be found using the appropriate series present worth factor:

\[ P = A(SPW-i-N) \]

If the pattern shows a uniform downward slope, then the equivalent uniform annual amount would be:

\[ A = A_1 - \frac{g}{i} + \frac{Ng}{i} \left(1\right) \]

Here is a numerical example: Find the present worth of a cash flow that starts at $1000 the first year and then increases $200 per year for the next 4 years (i.e., g = $200 and N = 5). Use 15% interest.

To solve this we first find the equivalent uniform annual amount, A:

\[ A = \frac{1000 + \frac{200}{15\%} - \frac{5 \times 200}{15\%} \left(1 - \frac{1}{(1+15\%)^5}\right)}{15\%} = 1344 \]

Then we convert that to its present worth:

\[ P = 1344(SPW-15\%-5) = 4505, \] and that is our answer.
2.10 Stepped patterns

Another common variation involves cash flows that remain uniform for some number of years (or other compounding periods) but then suddenly increase or decrease. In real life this might come about because of the peculiarities of the tax laws, as one example. Let's start with a simple case in which there is no income for the first 5 years and then uniform annual amounts of $175 are expected through the 20th year. We want to find the present worth based on 10% interest. Here's our cash flow diagram:

\[ \text{\$175 per year} \]

One way to solve this problem would be to analyze the cash flow year-by-year in a table, but there are easier ways. One would be to use our standard series present worth factor to find the equivalent value at the year just before the cash flow started, i.e., year 5, and then to discount that to year zero using the single present worth factor. Here goes:

\[
P \text{ at year } 5 = \text{\$175(SPW-10\%.-15)} = \text{\$1331}
\]

\[
P \text{ at year zero (today)} = \text{\$1331 (PW-10\%.-5)} = \text{\$826}
\]

Another logical technique would be to compute the present worth of \$175 per year as though the cash flow occurred throughout the full 20 years, and then subtract the present worth of the first 5 years, in which it did not occur.

\[
P = \text{\$175(SPW-10\%.-20)} - \text{\$175(SPW-10\%.-5)}
\]

\[
P = \text{\$1490 - \$663} = \text{\$827} \quad \text{(in reasonable agreement with the \$826 found above)}
\]

In general, I happen to prefer this second method. As we shall see in later chapters, there are cases where it is the neatest method to use and easiest to understand.

Here is an example involving two levels of uniform annual cash flow. In this case we can predict an annual cash flow of \$140 for each of the first 5 years of a project and \$100 for years 6 through 10, after which the project will be closed down with no residual value. Apply an interest rate of 10% to find the present value of the predicted cash flow. First, here is our sketch:

\[ \text{\$140 per year} \quad \Delta = \text{\$40 per year} \quad \text{\$100 per year} \]

I propose to start by finding the present worth of the \$100 cash flow over the complete 10 years and then adding the present worth of the difference between \$140 and \$100 (i.e., \( \Delta = \$40 \)) over 5 years:

\[
P = \text{\$100(SPW-10\%.-10)} + \text{\$40(SPW-10\%.-5)}
\]

\[
P = \text{\$614 + \$152} = \text{\$766}
\]
The same logic can be used in situations where the uniform annual amounts increase, rather than decrease at some point. Let us repeat the previous illustration, but reversing the cash flow pattern. Now we shall assume uniform annual amounts of $100 for each of the first 5 years, and $140 for each of the final 5 years.

\[ \Delta = \text{40 per yr} \]
\[ $100 \text{ per yr} \]
\[ \{ \]
\[ $140 \text{ per yr} \]

Again, we shall start by finding the present worth of the second series ($140 per year) but then \textit{subtract} the increment (\( \Delta = \text{40} \)):

\[
P = $140(\text{SPW-10\%-10}) - 40(\text{SPW-10\%-5})
\]
\[
P = 860 - 152 = 708
\]

This second outcome of $708 compares with the $766 found in the previous study. The same total amount of money came in, but the present value in the first case would be greater because more of the money came in during the early years. The quick buck is the good buck.

The analytical technique developed above can be applied to cash flows that involve more than the two levels of income shown. The same technique can also be applied to negative cash flows or combinations of positive and negative flows.

\textbf{Exercises}

1) Find the present worth, using 12% interest, for a cash flow of $100 in year 1, $130 in year 2, $100 in year 3, and $200 in year 4.

2) Repeat the first exercise, but assume the $100 amount in year 3 is negative instead of positive.

3) Convert the present value found in the second exercise to a uniform annual amount.

4) Find the present worth of a cash flow that consists of $1000 during the first year and increases by $200 each year through the fifth year. The interest rate is 12%.

5) Repeat the previous problem using tabular form (i.e., find the present worth of each individual cash flow and then add).

6) Find the present worth of a cash flow consisting of $2000 per year for the first 5 years, $3000 per year for years 6 through 10, and $4200 per year for years 11 through 20. An interest rate of 12% applies.

7) Find the equivalent uniform annual amount for the cash flow described above.

8) Find the equivalent uniform annual cash amount for a cash flow consisting of $1000 immediate expenditure, $1200 expense during the first year, an income of $1000 during the second year, and an income of $5000 during the third year. Use an interest rate of 12%.
Chapter 2: The Time-value of Money, Junior level

**Answers**

1) $391

2) $249

3) To convert a present value to a uniform annual amount, simply multiply it by the capital recovery factor:

\[ A = \frac{249(\text{CR-12\%-4})}{4} = 82 \]

4) To solve this problem we first apply our gradient series equation to find the equivalent uniform annual amount (A), and then use the series present worth factor (SPW) to convert to the present worth.

\[ A = A_1 + \frac{g}{i} \cdot \frac{N_g}{i} (Sf-i-N) \]

\[ A = 1000 + \frac{200}{0.12} \cdot \frac{5 \times 200}{0.12} (SF-12\%-5) = 1355 \]

\[ P = 1355(\text{SPW-12\%-5}) = 4884 \]

5) Your solution should look like this:

<table>
<thead>
<tr>
<th>Yr = N</th>
<th>Amount</th>
<th>(PW-12%-N)</th>
<th>PW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1000</td>
<td>0.8929</td>
<td>$893</td>
</tr>
<tr>
<td>2</td>
<td>$1200</td>
<td>0.7972</td>
<td>$957</td>
</tr>
<tr>
<td>3</td>
<td>$1400</td>
<td>0.7118</td>
<td>$997</td>
</tr>
<tr>
<td>4</td>
<td>$1600</td>
<td>0.6355</td>
<td>$1017</td>
</tr>
<tr>
<td>5</td>
<td>$1800</td>
<td>0.5674</td>
<td>$1021</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total Present Worth</td>
<td>$4885</td>
</tr>
</tbody>
</table>

6) The cash flow diagram looks like this:

And here is the solution:

\[ P = 4200(\text{SPW-12\%-20}) - 1200(\text{SPW-12\%-10}) - 1000(\text{SPW-12\%-5}) \]

\[ P = 31372 - 6780 - 3605 = 20990 \]

7) $2810

8) First find the present value of the cash flow and then multiply by the capital recovery factor (CR) to convert to an equivalent uniform annual amount over the 3 years. Do yourself a favor first and sketch the cash flow diagram. Keep an eye on those treacherous negative numbers!
Chapter 2: The Time-value of Money, Senior level

\[ P = ($1000) + ($1200)(PW-12\%-1) + $1000(PW-12\%-2) + $5000(PW-12\%-3) \]
\[ P = ($1000) + ($1071) + $797 + $3559 = $2285. \]
\[ A = $2285(CR-12\%-3) = $951. \]

**Independent thinking**

Imagine a tent-shaped cash flow, one that features several years with uniformly increasing income and then several years of uniformly decreasing income. Set up a general equation for analyzing such a cash flow pattern. Assign some numbers to each of the variables and solve for the present value. Check your work by slogging through a year-by-year analysis (which suggests using a fairly short overall time span so as to save you labor). Warning: This is tricky!

---

**Senior level**

_Even percent [interest] has no rest, nor no religion; it works nights, and Sundays, and even wet days._

---

Josh Billings

2.11 Periodic discrepancies

Over the life of a ship, typically 20 years, there will usually be periodic special expenses for planned maintenance work. These may occur perhaps every four years. Our projected operating cash flow pattern might then consist of uniform annual expenditures (which we shall abbreviate X) plus special increments, Z, every fourth year:

```
0---------4---------8---------12---------16---------20
\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow
uniform costs of $X per year
\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow
special expense of $Z every fourth year
```

How may we convert the above pattern into an equivalent uniform annual amount? There is the slug-'em-out method of discounting each of those Z values back to year zero, taking the sum, and multiplying it by the capital recovery factor. The more sophisticated method is to convert each Z value to a uniform annual amount by multiplying it by the sinking fund factor appropriate to the number of years between special expenses, in this case 4. The equivalent uniform annual operating cost, Y, would then become:

\[ Y = X + Z(SF-i-4) \]

So far, so good. But, if you are a practical sort of person you may object to our spending money for special maintenance during the final year of the ship's life. That is a valid objection. So, let's remove that final Z amount:

```
0---------4---------8---------12---------16---------20
\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow
X
\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow
Z
```

26
This will require that we reduce the Y value shown above by a uniform annual amount equivalent to Z spread over the entire 20-year life. That uniform decrement will amount to Z times the sinking fund factor based on the full 20 years. Our equation now becomes:

\[
Y = X + Z(SF-i-4) - Z(SF-i-20)
\]

or

\[
Y = X + Z\{(SF-i-4) - (SF-i-20)\}
\]

Let’s illustrate this with a simple example. Suppose the ship’s life is projected to be 25 years. An interest rate of 12% is specified. The uniform annual costs of operation are expected to be $800,000. Every fifth year there will be special survey expenses of $1.5 million. This special cost will be waived during the final year of the ship’s life. Find the equivalent uniform annual expense. Here’s the cash flow pattern:

\[
0 \quad 5 \quad 10 \quad 15 \quad 20 \quad 25
\]

\[
\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow 
\]

\[
X = $800,000 \quad Z = $1.5 \text{ million}
\]

\[
Y = X + Z\{(SF-12\%-5) - (SF-12\%-25)\}
\]

\[
Y = $800,000 + $1.5 \text{ million} \times 0.1574 - 0.0075
\]

\[
Y = $800,000 + $225,000
\]

\[
Y = $1,025,000
\]

2.12 Inflation

*What this country needs is a good five-cent nickel.*

Franklin Pierce Adams

In this section we want to learn how to analyze monetary inflation, particularly as its expectation may influence decision making in ship design. In most cases, as we shall see, the effects will be trivial. There may, however, be special situations in which inflation should not be overlooked. We shall learn what those situations are and how to handle them if they arise. Whether we like it or not, inflation seems to be a fact of life, so we are well advised to learn how to live with it.

If we can assume that our shipowner is free to raise freight rates commensurate with any future inflation in operating costs, then all financial and economic factors will float upward on the same uniform tide. If that occurs, the optimum ship based on no inflation will also be the optimum ship in which inflation is recognized. Inflation need concern us only when some major economic factors are expected to change appreciably faster or slower than the general trend.

A meter is a meter is a meter. The same cannot be said of dollars. Money does us no good until we spend it, and if its purchasing power is rubbery, we should admit as much. If a good meal in a restaurant costs $15 today and is expected to cost $30 in five years, we should be foolish to ignore that threat. To clarify our thinking in all this we must train ourselves to think in terms of constant-value dollars. In short, do not try to analyze long-term cash flows without first adjusting each year's figure according to its purchasing power relative to some convenient base year.
Chapter 2: The Time-value of Money, Senior level

We shall use the following abbreviations to distinguish face-value dollars from constant-value dollars:

FV$ = face-value dollars
CV$ = constant-value dollars

The latter dollars are the ones corrected for inflation and are the ones for which we should develop an affinity.

The question then arises, how may we best convert misleading (even treacherous!) FV$ into virtuous, reliable CV$? There are two alternative methods. Both are based on the same principles and, if correctly carried out, should produce the same final outcome and resulting design decision. One way is to prepare a year-by-year table in which all cash flows are entered in CV$. You are then in a position to apply standard interest relationships to find the present value or equivalent uniform annual cost of this CV$ cash flow in the usual way.

The other approach, as you may have guessed, is to start with FV$ and apply a discount rate that has built into it adjustments for both inflation and time-value of money. This method can be handled by simple algebraic procedures and does not require the boring, error-prone, year-by-year tabular approach described above. It will let you find the present worth (corrected for inflation) of a future cash flow that is subject to predictably changing dollar values.

At this point we had better pause and introduce some abbreviations, as follows:

\[ P = \text{present worth (or present value) in CV$ of future cash flows} \]
\[ r = \text{discount rate (applied to CV$) for finding P. (This rate is the true measure of the time-value of money.)} \]
\[ d = \text{expected rate of general inflation, per year} \]
\[ x = \text{an annual rate of inflation applicable to any given category of income or cost that is changing at a rate other than that of general inflation. (We call this a maverick rate.)} \]
\[ i = \text{discount rate applied to FV$ for finding P (which is in CV$)} \]
\[ A = \text{any initial annual amount} \]

Our task is to derive the value of \( i \) for any given set of assumptions as to the rate of inflation and time-value of money. Remember that \( i \) incorporates both time-value of money and inflation. Let's start with the simple case in which a given category of cost is floating up right along with the general inflation rate, \( d \). That being the case, although it appears to be increasing (in FV$), it is really holding steady in real purchasing power. That is, it is always the same in CV$, so we can ignore inflation and say:

\[ i = r \]

Let's next consider the case in which one category of cash remains fixed in face value dollars during a period of general inflation. A fixed charter fee might lead to such an arrangement. Some tax calculations also involve fixed annual amounts. In any given year:
FVA = A₀

Correcting for inflation:

\[ CVA = \frac{FVA}{(1+d)^N} = \frac{A₀}{(1+d)^N} \]

and

\[ P = \frac{CVA}{(1+r)^N} = \frac{A₀}{(1+r)^N(1+d)^N} \]

That is, we have double discounting, one for the time-value of money, and one for the declining real value of the dollar. In short, where costs remain fixed in FVS$ we use:

\[ i = (1+r)(1+d) - 1 \]

Finally, let's look at the case of a cost factor that changes at an annual rate, \( x \), that differs from general inflation. In face-value terms we have:

\[ FVA = A₀ (1 + x) \]

Correcting for inflation:

\[ CVA = \frac{FVA}{(1+d)^N} = A₀ (1+x)^N \]

and, converting to present worth:

\[ P = CVA \frac{1}{(1+r)^N} = A₀ \frac{(1+x)^N}{(1+d)^N(1+r)^N} \]

so, where maverick costs are concerned:

\[ i = \frac{(1+r)(1+d)}{(1+x)} - 1 \]

This final expression may, in extreme cases, produce a negative interest rate (equivalent to your paying the bank to safeguard your cash, which was originally the custom). This will lead to a present worth exceeding the future amount. This is perfectly reasonable and your calculator will handle it automatically.

Table 2.2 summarizes the interest rates that help us find present values in CV$ in times of inflation.
Table 2.2
Interest Rates Applicable During
Periods of Inflation

<table>
<thead>
<tr>
<th>Cash Flow Characteristic</th>
<th>Interest Rate to Be Applied to Initial Annual Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floats up with general inflation</td>
<td>( i = r )</td>
</tr>
<tr>
<td>Fixed in FV units</td>
<td>( i = (1 + r)(1 + d) - 1 )</td>
</tr>
<tr>
<td>Changes at annual rate, ( x ), other than general inflation rate, ( d )</td>
<td>( i = \frac{(1 + r)(1 + d)}{(1 + x)} - 1 )</td>
</tr>
</tbody>
</table>

Here is a numerical example that will illustrate the concepts explained above. Over a four-year period we have three concurrent cash flows as follows:

- **Wages:** Fixed by contract at $100 000 per year (FV$)
- **Fuel:** Starting at $120 000 per year and increasing at 16% per year
- **All other costs:** Starting at $80 000 per year and rising with inflation

General inflation is expected to amount to 12% per year and the time-value of money is set at 9% per year.

Table 2.3 shows how this problem can be handled in tabular form.

**Important!** A less-than obvious point to notice is that the initial cash amounts are taken *at year zero, not year one.*
### Table 2.3
Handling Inflation with the Tabular Approach

<table>
<thead>
<tr>
<th>Year (N)</th>
<th>Wages</th>
<th>Fuel</th>
<th>Other</th>
<th>Total Costs</th>
<th>(PW-9%-N)</th>
<th>PW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>89</td>
<td>124</td>
<td>80</td>
<td>293</td>
<td>0.9174</td>
<td>269</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>129</td>
<td>80</td>
<td>289</td>
<td>0.8417</td>
<td>243</td>
</tr>
<tr>
<td>3</td>
<td>71</td>
<td>133</td>
<td>80</td>
<td>284</td>
<td>0.7722</td>
<td>219</td>
</tr>
<tr>
<td>4</td>
<td>64</td>
<td>138</td>
<td>80</td>
<td>282</td>
<td>0.7084</td>
<td>200</td>
</tr>
</tbody>
</table>

Total present worth = 931

b. In CV$, wages, \( \frac{A_o}{(1+d)^N} = \frac{100\,000}{(1.12)^N} \)

c. In CV$, fuel cost = $120\,000 \frac{(1.16)^N}{(1.12)^N}

\[ A_o \frac{(1+x)^N}{(1+d)^N} = $120\,000 (1.036)^N \]

d. In CV$; other costs remain fixed at $80\,000

e. Column e = sum of columns b, c and d

f. \( (PW-9%-N) = \frac{1}{(1.09)^N} \)

g. PW = column e \times column f
Chapter 2: The Time-value of Money, Senior level

Turning now to the algebraic approach, we first record these values:

- Time value of money: \( r = 9\% \)
- General rate of inflation: \( d = 12\% \)
- Rate of inflation for fuel: \( x_{\text{fuel}} = 16\% \)

Then, analyzing each cost component in turn, we have:

**Wages:** These are fixed in face value terms, so this equation applies:

\[
i = (1 + r)(1 + d) - 1
\]
\[
i = (1.09)(1.12) - 1 = 22.08\%
\]

To find the present worth in CV$, we need to apply the series present worth factor for 22.08 percent interest and four years:

\[
\text{PW in CV$} = \text{SPW} - 22.08\% - 4 = $249\ 000
\]

**Fuel:** This inflates at its own rate, so this equation applies:

\[
i = \frac{(1+r)(1+d)}{1+x_{\text{fuel}}} - 1
\]
\[
i = \frac{(1.09)(1.12)}{1.16} - 1 = 5.241\%
\]

\[
\text{PW in CV$} = \text{SPW} - 5.241\% - 4 = $423\ 000
\]

**Other:** These costs float up with general inflation, so:

\[
i = r
\]
\[
i = 9\%
\]

\[
\text{PW in CV$} = \text{SPW} - 9\% - 4 = $259\ 000
\]

**Total:** The total present worth in CV$ will equal the sum of the three components derived above:

\[
\text{Total PW} = ($249 + $423 + $259) \times 10^3
\]
\[
= $931\ thousand.
\]

This provides an exact check on the value arrived at in Table 3.

2.13 Non-annual compounding

In most ship design studies we are in the habit of assuming annual compounding when weighing the time-value of money. There may be instances, however, when we should consider other compounding periods. As you may recall, the standard interest formulas introduced at the start of this chapter are applicable to any combination of compounding periods and interest rate *per compounding period*. Take, for example our
standard single payment compound amount factor and apply it to a $10 000 debt with 12% interest compounded annually over a 20-year period. What would be the total debt, F, at the end of that period?

\[ F = P(CA-i-N) \]
\[ F = 10000(CA-12\%-20) = 96460 \text{ (rounded)}. \]

Next, suppose the terms of the loan called for the same 12% interest, but compounded quarterly. Now our number of compounding periods will quadruple to 80, and our interest rate per compounding period will be cut to a quarter, or 3%:

\[ F= 10000(CA-3\%-80) = 106410 \text{ (rounded)}, \text{ which is more than 10\% greater than the figure based on annual compounding.} \]

Clearly, when we changed the frequency of compounding we also changed the weight given to the time-value of money. This is common sense; the more often repayments fall due, the more desirable is the arrangement to the lender, and the less desirable to the debtor. In order to make a valid comparison between debts involving differing compounding periods, we need an algebraic tool that will assign to each repayment plan a measure that is independent of frequency of compounding.

The usual approach to this operation is based on what we call the effective interest rate, abbreviated \( r_1 \). This is an artificial interest rate per annum that ascribes the same time-value to money as some nominal annual rate, \( r_M \), with \( M \) compounding periods per annum. For example, suppose one loan plan is based on quarterly compounding at one interest rate, and another is based on monthly compounding at a somewhat lower rate. We cannot tell by looking at the numbers which is more desirable. If we convert both nominal annual rates to their corresponding effective rates, however, those values will tell us which is the better deal. The question then arises, how do we convert from a nominal annual rate, \( r_M \), to effective rate, \( r_1 \)? Here is the simple key:

\[ r_1 = \left(1 + \frac{r_M}{M} \right)^M - 1 \]

where

- \( r_1 \) = effective interest rate
- \( r_M \) = nominal annual interest rate
- \( M \) = compounding periods per year

For the derivation of this equation, see any standard engineering economy reference.

Let us illustrate this with a numerical example: Suppose banker A offers to lend you money at 12% compounded semi-annually. Banker B offers 11.5% compounded monthly. B's nominal rate is lower, but the compounding is more frequent, so we cannot readily tell which is the better offer. What we need to do is convert each nominal rate to its corresponding effective rate:

Plan A: \( r_M = 12\% \) and \( M = 2 \), so \[ \frac{r_M}{M} = 6\% = 0.06 \]
\[ r_1 = (1.06)^2 - 1 = 12.36\% \]

Plan B: \( r_M = 11.5\% \) and \( M = 12 \), so \( \frac{r_M}{M} = 0.9583\% = 0.009583 \)

\[ r_1 = (1.009583)^{12} - 1 = 12.125\% \]

Comparing the two effective rates, we can conclude that Banker B offers a slightly better deal (i.e., a lower effective interest rate).

The equation for effective rate, \( r_1 \), can be switched to provide this expression for deriving a nominal rate per compounding period from any given effective rate:

\[ \frac{r_M}{M} = \left(1 + \frac{1}{r_1}\right)^M - 1 \]

One important rule to keep in mind: Never use a nominal annual rate all by itself. Always convert it into its corresponding rate per compounding period.

For the sake of brevity I am not going to belabor you with two collateral issues, namely (a) continuous compounding and (b) uniform annual cash flows with non-annual compounding. If you have need of help in those matters, you may refer to standard texts.

2.14 Summary and review

This chapter is dedicated to explaining how to give proper weight to the time-value of money. As such, it deals with the essential core of engineering economic analysis. You are well advised to master all the basic principles explained in the introductory section as well as the important variations explained in the intermediate and advanced sections.

Exercises

1) Find the equivalent uniform annual cost for a cash flow made up of two parts: a uniform expense of $1000 per year for 20 years, and a special expense of $500 that occurs every fourth year starting in year 4. The interest rate is 15%.

2) Find the present worth of the value found above.

3) Please refer back to the numerical example in Section 2.12. Rework the example using an interest rate of 15% instead of 9%. Do this by both methods and make sure your answers agree.

4) Find the effective interest rate for a loan plan offering 10% annual interest compounded weekly.

5) Convert an effective interest rate of 12% to a nominal annual rate based on quarterly compounding.

6) Take the nominal annual rate found above and convert it back to its effective rate.
Chapter 2: The Time-value of Money, Senior level

7) Suppose you deposit $1000 in a bank that offers 8% interest with daily compounding. If you leave the money in for 20 years, how much will the bank owe you?

**Answers**

1) \[ A = 1000 + 500(SF-15\%-4) = 1000 + 100 = 1100 \]

2) \[ P = 1100(SPW-15\%-20) = 6885 \]

3) (Your own work will provide the correct answer through independent calculations.)

4) 10.51%

5) 11.49%

6) 12% (If you found a different answer, there's something wrong with your approach or your computer. I, for one, would bet on the former.)

7) $4950 (rounded)

**Independent thinking**

There is a simple expression for the series present worth factor for a uniform cash flow of infinite life. You should be able to reason out what it is. Try to do so.

At any given interest rate, how long will it take for a bank deposit to double in value? The rule of thumb, good up to around 15% interest, is to divide the rate (in percent) into 72. Try it. You can use this same rule to estimate how long it will be before inflation cuts the value of your dollar in two.
Chapter 3
Taxes and Depreciation

The art of taxation consists in so plucking the goose
as to obtain the largest amount of feathers
with the least amount of hissing.

Jean Baptiste Colbert

Sophomore level

3.0 Perspective

Most traditional maritime nations impose an annual tax on corporate earnings. If you
are involved in the design of a money-earning ship, the chances are that you should have at
least a rudimentary idea about the applicable tax structure. In many cases a proper
recognition of the tax bite will have a major impact on design decisions. In other cases (as
we'll learn in the next chapter) taxes can be ignored. In any event, you should understand
enough about the subject to discuss it intelligently with business managers.

Tax laws are written by politicians who are swayed by pressures coming from many
directions. As a result the laws are almost always a complex, continually changing jungle.
Thus, most large companies employ full-time experts whose careers are devoted to
understanding the tax laws and finding ways to minimize their impact. I shall make no
attempt here to explain all the complexity of current U.S. tax laws. (I don't pretend to
know them, they are always changing, and many of you will be involved with overseas
operations.) I shall, however, explain the simplest tax concepts and show how they may
affect cash flows coming into a company. Then, in a later chapter I shall show how they
may affect design decisions.

3.1 Tax shields

In most traditional maritime nations (in contradistinction to so-called open-registry
nations) tax rates run around 40 to 50 percent of the before-tax cash flow minus certain tax
shields. Principal among these are an annual allocation for depreciation (which I'll define
in a moment) and any interest paid to a bank or other source of income involving fixed
payments. The impact of bank loans will form the subject of Chapter 4. Here we shall
assume that the owner is able to pay for the ship with his or her own capital. (We call this
an all-equity investment.)

3.2 Depreciation

Depreciation is, in a way, a legal fiction with roots in long-established accounting
practices. When a company makes a major investment it exchanges a large amount of cash
for a physical asset of equal value. In its annual report it takes credit for that asset and
shows no sudden drop in company net worth. Over the years, however, as the asset
becomes less valuable for various reasons, its contribution to the company's worth
declines; that is, it depreciates. You will recognize this as you find the potential resale price
of your automobile dropping off from year to year.

I called depreciation a legal fiction because the tax laws treat it as an expense in a time
period other than when the money was actually spent. (Remember our rule that accurate
economic assessment requires us to consider not only how much cash flows in or out, but
also when.) Another fictitious element is found in the fact that few nations allow owners to
recognize inflation when figuring depreciation.
In summary of what we have covered so far, we have seen that the tax collector's target is not the company's actual annual cash flow (income minus costs), but a distorted version of that cash flow. It recognizes capital investments, not in the year they are made, but rather distributed over a period of years, or what is called depreciation. The principal objective of this chapter, then, is to help you understand some of the major schemes for assigning annual depreciation allocations and their effects on tax liabilities.

3.3 **Straight-line depreciation**

In its simplest form, the ship (or other facility) is assumed to lose the same amount of value every year until the end of its economic life. This is called *straight-line depreciation*. It is found by dividing the depreciable value by the number of years of life:

\[
D = \frac{P - L}{N}
\]

where

- \(P\) = initial investment
- \(L\) = disposal value after \(N\) years
- \(N\) = economic life of the ship, also tax depreciation period

In most cases we are justified in ignoring the disposal value. It is typically less than 5% of the initial investment; it is hard to predict; and, being many years off, has little impact on overall economics. Thus, for design studies, we usually define straight-line depreciation as

\[
D = \frac{P}{N}
\]

3.4 **Cash flows before and after tax**

The bar diagram below shows how annual revenues are treated when figuring corporate income taxes. We assume here that all factors remain constant over the \(N\) years of the project's economic life. (This is what economists call a heroic assumption, but it is frequently good enough for design studies.)

![Diagram of cash flows before and after tax](image)

- **Annual Revenue**
  - \(Y\) = cash flow after tax
  - Tax
    - \(A'\) = cash flow after tax
    - Tax
      - \(A = \text{Cash flow before tax} = \text{Revenue} - Y\)
      - \(A = A - \text{Tax} = A - t(A - P/N)\)
      - \(A = A - tA + tP/N\)
      - \(A = (1 - t) + tP/N\)
      - \(\text{Profit} = A - P/N\)

Annual operating costs
The bar diagram shows that the annual cash flow after tax (A') is related to the cash flow before tax (A) by this simple expression:

\[ A' = A(1 - t) + t \frac{P}{N} \]

where \( t \) = tax rate

or, turning it around:

\[ A = \frac{A' - t \frac{P}{N}}{(1 - t)} \]

An important thing to note in the above is that all of our rational measures of merit (which will be explained in Chapter 5) are based on after-tax cash flows, not profits. **In short, we do not use profits to measure profitability, we use cash flows.** Profits are misleading because they are polluted with depreciation, an expense that is misallocated in time. So much for the infamous bottom line.

**Exercises**

1) A ship that costs $40 million is expected to last 20 years. Assuming zero disposal value and a 20-year life, find its annual depreciation allowance.

2) Returning to the first exercise, if we assume an eventual disposal value of 5% of first cost, what will be the ship's annual depreciation allowance?

3) A ship that costs $55 million has an expected economic life of 15 years. It is expected to earn annual revenues of $4.5 million against operating costs of $750,000. Its projected disposal value after 15 years is $500,000. Assume straight-line depreciation. What would be the annual tax base?

4) A ship that costs $75 million has an expected economic life of 25 years with negligible disposal value. It is expected to earn annual revenues of $5.25 million against operating costs of $1.25 million. Assume a tax rate of 45% and straight-line depreciation. Using the simple algebraic expression relating cash flows before and after tax, find its cash flow after tax.

5) Repeat the fourth problem by setting each factor out in a line in a table, just as an accountant would. Compare answers.

6) Rework the fourth exercise with an assumed eventual disposal value of 4% of the first cost.

**Answers**

1) \( D = \frac{\$40M}{20} = \$2 \text{ million} \)

2) $1.90 million
3) $117,000

4) $3.55 million

5) An accountant would use tabular form, as follows:
   
<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual revenue</td>
<td>$5.25 million</td>
</tr>
<tr>
<td>Annual operating costs</td>
<td>$1.25 million</td>
</tr>
<tr>
<td>Cash flow before tax</td>
<td>$4.0 million</td>
</tr>
<tr>
<td>Depreciation allocation: $75M + 25</td>
<td>$3.0 million</td>
</tr>
<tr>
<td>Tax base</td>
<td>$1.0 million</td>
</tr>
<tr>
<td>Cash flow after tax: $4.0M - $0.45M</td>
<td>$3.55 million</td>
</tr>
</tbody>
</table>

   (Be careful: Do not try to find cash flow by subtracting the tax from the tax base.)

6) $3.50 million (rounded)

   Independent thinking

   Some kinds of investments do not depreciate over time. Can you suggest at least one such?

   Would you expect your new automobile's resale value to decline uniformly from year to year?

   To bring your level of understanding up a notch, rework each of the above problems using arbitrary changes in tax rate, or economic life. Compare results. Repetition is important to your mastering any subject.

---

**Junior level**

*Count that day won when, turning on its axis,*

*this earth imposes no additional taxes.*

Franklin Pierce Adams

3.5 Fast writeoff

In the first section we made the standard assumption that the ship's tax life coincided with its economic life. This is not always the case because owners are sometimes permitted to base depreciation on a shorter period. This is one way the politicians can help businesses without offending their soak-the-rich constituency by lowering the corporate tax rate. It is called fast writeoff. It is advantageous to the investor. This is so because it provides a more favorable after-tax cash flow pattern. Over the life of the ship (or whatever) the same total taxes must be paid, but their worst impact is delayed. Remember always, the quick buck is the best buck.

Some nations allow ship owners freedom to depreciate their ships as fast as they like. In that setting the owner can make the depreciation allocation equal to the cash flow before tax. That will reduce the tax basis to zero, and no taxes need be paid during the early years of the ship's life. After that, of course, the depreciation tax shield will be gone, and higher
Chapter 3: Taxes and Depreciation, Junior level

taxes will ensue. Again, however, the total tax bill over the ship's life will remain unchanged. The advantage is simply in the pattern of payments.

More typically, the owner will not be given a free hand in depreciating the ship. Rather, the tax life (i.e., depreciation period) will be set at some period appreciably shorter than the expected economic life. This will result in cash flow projections that feature uniform annual amounts with a step down after the depreciable life is reached. Let me explain how we can handle such a situation.

First, we give separate attention to two distinct time periods. The first of these comprises the years during which depreciation allowances are in effect. We shall identify the final year of this period by the letter Q. The second time period follows Q and extends to the final year of the ship's economic life, designated with the letter N. If we have straight-line depreciation, our cash flows before (A) and after (A') tax will be related as shown below:

\[ A' = A(1 - t) + \frac{tP}{Q} \]

\[ (\text{Annual depreciation} = \frac{P}{Q}) \]

\[ \Delta = \frac{tP}{Q} \]

\[ A' = A(1 - t) \]

\[ (\text{No depreciation}) \]

Now, recalling what we learned about handling stepped cash flows in Section 2.10, we can find the present worth of the above as follows:

\[ PW = A(1 - t) (SPW\cdot i'\cdot N) + t \frac{P}{Q} (SPW\cdot i'\cdot Q) \]

where \( i' \) is the owner's specified after-tax interest rate.

Let me illustrate this concept with a numerical example. We shall assume that an owner expects a ship to have an economic life of 20 years, with negligible disposal value. The tax depreciation period is 12 years. The tax rate is 40%. The initial cost is $24 million. The annual revenues are $3.2 million and operating costs are $800 000. Find the after-tax cash flows during years 1 - 12 and 13 - 20, then find the present worth of the cash flows using 12% interest. Here is a schematic of the cash distributions:

\[ \text{Economic life} \]

\[ \text{Depreciable life} \]

\[ 0 \quad Q = 12 \quad N = 20 \]

\[ A = \text{Rev} - Y = 3.2M - 0.8M = 2.4M \]

During the initial 12 years:
Chapter 3: Taxes and Depreciation, Junior level

\[ A' = A(1 - t) + \frac{P}{Q} = \$2.40M(1 - 0.40) + 0.40 \frac{\$24M}{12} = \$2.24M \]

After that, with no tax shield:

\[ A' = A(1 - t) = \$2.40M(1 - 0.40) = \$1.44M \]

Our cash flow pattern now looks like this:

![Cash flow diagram](image)

We can now find the present worth of the cash flow:

\[ PW = \$1.44M(\text{SPW-12\%-20}) + \$0.80M(\text{SPW-12\%-12}) \]
\[ PW = \$10.76M + \$4.96M = \$15.72M \]

Let us next consider the case where fast write-off is not allowed, i.e., where the tax life and economic life are the same: N years. Given that assumption, the after-tax cash flow, \( A' \), would be equal to:

\[ A' = A(1 - t) + \frac{P}{N} \]

\[ A' = \$2.4M(1 - 0.40) + 0.40 \frac{\$24M}{20} = \$1.44M + \$0.48M = \$1.92M \]

so

\[ PW = \$1.92M(\text{SPW-12\%-20}) = \$14.34M \]

which compares with \$15.72M attained with fast writeoff, as shown above, nearly a 10% advantage in favor of fast writeoff.

Next, let us compare the total tax amounts paid over the life of the ship in each case. With fast writeoff the annual tax during the first 12 years would amount to

\[ \text{Tax} = t(A - \frac{P}{Q}) = 0.40(\$2.40M - \frac{\$24M}{12}) = \$0.16M \text{ per year} \]

The annual tax during the final 8 years would amount to:

\[ \text{Tax} = t(A) = 0.40(\$2.40M) = \$0.96M \text{ per year} \]
Chapter 3: Taxes and Depreciation, Junior level

The total tax over the 20-year life would be:

\[ \text{Total tax} = 12 \times ($0.16M) + 8 \times ($0.96M) = $1.92M + $7.68M = $9.60M \]

Without fast writeoff, the annual tax during each of the 20 years would be:

\[ \text{Tax} = t\left( A - \frac{P}{N} \right) = 0.40 \times ($2.40M - \frac{24M}{20} \right) = 0.40\times ($2.40M - $1.20M) = $0.48M \]

The total tax over 20 years would be:

\[ \text{Total tax} = 20 \times ($0.48M) = $9.60M \text{ (which is the same as before)} \]

The two outcomes, being the same, show that fast writeoff does not reduce the tax burden; it merely gives it a less onerous distribution.

**Exercises**

1) Rework the numerical example shown above but change the depreciation period to 8 years.

2) Rework problem number 1, but cut the interest rate to 6%.

3) Convert the present worth found in the second exercise to an equivalent series of uniform annual payments over a 20-year period.

**Answers**

1) Skeleton solution:
   
   During first 8 years: \( A' = $2.64M \)
   During final 12 years: \( A' = $1.44M \)
   \( \Delta = $2.64M - $1.44M = $1.20M \)
   \( \text{PW} = $1.44M \times \text{SPW-12%-20} + $1.20 \times \text{SPW-12%-8} = $16.72 \text{ million}. \)

2) $23.97 million

3) Do you remember how to convert a present value to a uniform annual series? If not, go back to Chapter 2, Section 2.5. There you'll find the magic number is the capital recovery factor. So \( A' = $23.97M \times \text{CR-6%-20} = $2.090M \)

**Independent thinking**

Once the ship is fully depreciated, or "written off the books," accountants will show that profits suddenly jump up. (Review the cash distribution pattern shown on Page 38 to convince yourself of this.) They may then say, "These are the best years of the investment. Look how profits have increased!" Can you find any flaw in their logic? Hint: What happens to taxes?
Senior level

What is the difference between a taxidermist and a tax collector?
The taxidermist takes only your skin.
Mark Twain

3.6 Variable tax rates
Current tax laws in the United States give a break to small businesses by using a lower tax rate on the first $25,000 of taxable income. We generally overlook this in ship economic studies because the overall effect is so trivial.

3.7 Dual tax rates
In some other nations the tax laws assign one tax rate against taxable income that is turned over to the stockholders in the form of dividends, and a much higher rate against income that the corporation retains (probably in order to expand operations or simply overcome inflation). There is logic in assigning a lower rate against dividends. The government will get its due from the individual income taxes paid by the stockholders (something our own government has never been willing to admit). If faced with a dual tax rate setting, you may ask the shipowner's advice. Alternatively, assume a 50/50 distribution, leading to an average tax rate applied to the entire taxable amount.

3.8 Accelerated depreciation
Some tax laws recognize that straight-line depreciation is based on an unrealistic assessment of actual resale values of physical assets. This leads to various depreciation schemes that feature a large allocation during the first year of the asset's life and diminishing allocations thereafter. These declining amounts may continue over the entire economic life, or they may lead to complete writeoff in some shorter period. We may thus find accelerated depreciation combined with fast writeoff. In any event, the total taxes over the asset's life will once more be the same. The primary advantage of such schemes is to offer the corporation a more favorable distribution of after-tax cash flows.

Among the accelerated depreciation schemes, perhaps the most popular is called sum of the years' digits. In any given year (designated M) the depreciation allocation, D, is found using this expression:

\[ D = (P - L) \frac{N - M + 1}{N(N + 1)} \]

The denominator in that fraction produces the sum of the years digits for the number of years over which the depreciation plan runs. That is where the plan gets its name.

The cumulative present worth of those allocations over the depreciable life is found by this expression:

\[ PW = \frac{2(P - L)}{N(N + 1)i'} [N - (SPW - i' - N)] \]
You may wish to note that some of the standard references contain tables showing the present worth per depreciable dollar for various combinations of interest rate and years of life.

Knowing the present worth of depreciation allocations (PW of D), we can find the present worth of the after-tax cash flow with this relationship:

\[
PW \text{ of } A' = A(1 - t)(SPW-i'\text{-N}) + t(PW \text{ of } D)
\]

Please note how this resembles the expression for present worth of an after-tax cash flow when straight-line depreciation applies:

\[
PW \text{ of } A' = A(1 - t)(SPW-i'\text{-N}) + \frac{P}{N} (SPW-i'\text{-N})
\]

To clarify this exposition, let us consider a simple example. Assume a facility has an economic life of 5 years. The invested cost is $6 million and the ultimate disposal value is nil. The before-tax cash flow (A) is expected to be $3 million. The tax rate is 40% and sum of the years digits will be used for depreciation. The objective is to find the present value of the after-tax cash flow assuming an interest rate of 12%.

We shall first solve this problem by setting up a table showing what is happening year-by-year.

<table>
<thead>
<tr>
<th>Year</th>
<th>M</th>
<th>N − M + 1</th>
<th>D</th>
<th>A − D</th>
<th>Tax</th>
<th>A'</th>
<th>PW of A'</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.333</td>
<td>$1.998M</td>
<td>$1.022M</td>
<td>$0.401</td>
<td>$2.599M</td>
<td>$2.321M</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.267</td>
<td>$1.602M</td>
<td>$1.398M</td>
<td>$0.558</td>
<td>$2.442M</td>
<td>$1.947M</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.200</td>
<td>$1.200M</td>
<td>$1.800M</td>
<td>$0.720</td>
<td>$2.280M</td>
<td>$1.623M</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.133</td>
<td>$0.798M</td>
<td>$2.202M</td>
<td>$0.881</td>
<td>$2.119M</td>
<td>$1.347M</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.067</td>
<td>$0.402M</td>
<td>$2.598M</td>
<td>$1.039</td>
<td>$1.961M</td>
<td>$1.113M</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1.000</td>
<td>$6.000M</td>
<td></td>
<td></td>
<td></td>
<td>$8.351M</td>
<td></td>
</tr>
</tbody>
</table>

Note that I have added up the depreciation factors in the second column. They should total one. Likewise, the numbers in the third column must add up to the depreciable amount.

The final outcome of the table indicates that the project's after-tax cash flows will have a present value of $8.351 million. Let us check this figure using the algebraic approach. Our first step will be to solve for the present worth of the depreciation allocations, D:

\[
PW \text{ of } D = \frac{2(P - L)}{N(N+1)i'} [N - (SPW-i'\text{-N})]
\]

\[
PW \text{ of } D = \frac{2\times6M}{5(6)0.12} [5 - (SPW-12\% - 5)]
\]

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Chapter 3: Taxes and Depreciation, Senior level

\[ \text{PW of D} = \$3.333M[5 - 3.605] = \$4.650M \]

Now we are ready for the final step:

\[
\begin{align*}
\text{PW} &= A(1 - t)(\text{SPW-i'-N}) + t(\text{PW of D}) \\
\text{PW} &= \$3M(1 - 0.40)(\text{SPW-12%-5}) + 0.40(\$4.650M) \\
\text{PW} &= \$6.489M + \$1.860M = \$8.349 \text{ million} \quad \text{(which is in almost perfect agreement with the value found in the table).}
\end{align*}
\]

If you will go over this section on accelerated depreciation carefully, you should develop confidence that the seeming complexity of such plans should cause no fears. And always remember, even the most complicated cash flow patterns can be analyzed on a year-by-year basis.

3.9 Some other complications

Among other entangling vines in the jungle of taxation is something called the investment tax credit. When a government finds the economy slowing, it will want to encourage business managers to spur the economy through new capital investments. The obvious way to do this would be to lower the corporate tax rate. Political leaders may lack the courage to do that, so they look for less visible ways. One such way is the investment tax credit. This allows the organization to reduce its first year’s tax on a new project by some modest fraction of the initial investment. This tax reduction in no way reduces the depreciation allocations and gives business managers added confidence that they will be able to get their money back in a hurry.

How do you handle depreciation calculations when the system under analysis includes components with differing depreciable lives? An example would be a new containerized cargo transport system. There you might find investments in real estate (infinite life), ships (20-year life), cranes (15-year life), and buildings (50-year life). The answer is clear: You must treat each such component separately. The principle is simple and so are the calculations; they just look complicated when taken in total.

3.10 Closure

By this time you are probably somewhat discouraged by all the complexities sketched above — to say nothing of the ones lying hidden in wait. I can assure you, when dealing with ship owners you will probably find yourself talking with earnest accountants who know all the tax rules and want to apply them to your design analysis. You must of course listen to what these people have to say. But you must also realize that you are usually safe in applying massive amounts of simplifying assumptions, at least in the preliminary design stages.

You should know that some managers use the simplest sort of analysis in choosing projects and in deciding whether or not to go ahead with them. This is so even though they intend to use every possible tax-reducing trick if the project does indeed come to fruition. This suggests the wisdom of using simple methods (e.g., straight-line depreciation) in the early design stages when dozens or hundreds of alternatives are under consideration. But then, having narrowed the choice down to half a dozen alternatives, letting the accountants add all the baroque decorations they like to the chosen few.

If you do start out with gross simplifications, such as straight-line depreciation, you can look ahead to the effect of the more elaborate tax schemes by recognizing that their net effect is to produce some modest increase in present values of future incomes. You may
account for this by assuming a slightly lower tax rate. Alternatively, you may discount future cash flows with a slightly lower interest rate.

**Exercises**

1) Rework the example shown above but assume a 60% tax rate. Do this by both tabular and algebraic methods to check your work. Make sure they agree.

2) Extend the first problem by assuming a six-year life and a disposal value of $0.50 million. Again, do this by both methods.

3) If you borrow $5000 and repay the debt in five equal annual payments at 10% interest, how large will they be?

4) Prepare a table showing how a bookkeeper would keep track of the debt in the third exercise. This should show year-by-year four figures: (a) Residual debt before payment at end of year, (b) Interest charge due at end of year, (c) Total annual payment, and (d) Reduction in debt. Work only to the nearest dollar, to save labor.

**Answers**

1)-4) Your own work should prove itself.

**Independent thinking**

There is a certain kind of ship propulsion plant that might lead to a negative disposal value. Can you think what that plant might be?
Chapter 4
Leverage

Alas! how deeply painful is all payment!
They hate a murderer much less than a claimant.

Lord Byron

Sophomore level

4.0 Perspective

Many, if not most, business managers have ambitions beyond the reach of their equity capital. This leads them to "leverage up" their operation by obtaining a loan from a bank. The same is true of individuals who want to own a yacht. It is also often true of governments who sell bonds so as to finance a share of current expenditures. In nearly every case the lender requires repayment of the loan within a given time and at a given interest rate. Typically, the repayments are made in periodic bits and pieces comprising both interest and some reduction in the debt itself. In short, the periodic payments are determined by multiplying the amount of the loan (which we shall abbreviate \( P_B \)) by the capital recovery factor appropriate to the loan period (which we shall abbreviate \( H \)) and the agreed-upon interest rate (\( i_B \)). The typical repayment period is monthly, but for ship design studies we generally assume annual payments (which we shall abbreviate \( A_B \)). In short:

\[
A_B = P_B(CR-i_B-H)
\]

As an alternative to applying to a bank, managers may choose to raise capital by selling bonds. As far as we are concerned here, the effect is the same: the debt must be repaid at some agreed-upon rate of interest.

In the third chapter you learned how depreciation plans affect the corporate income tax. In the United States, as well as in many other maritime nations, the interest paid to the bank or bond holder is treated as an operating expense and so it, too, reduces the tax.

Bank loans are popular with managers because that source of capital usually implies a lower interest rate than would be demanded by owners of common stock. But, as explained in the appendix (A-5), there are also added risks to be considered.

4.1 Cash flows before and after tax

The bar diagram shown in the middle of the next page is like the one you studied in Section 3.4 except that here we add the complication of a bank loan. We assume here that the bank loan period is the same as the ship's economic life (\( H = N \)). We also assume straight-line depreciation with depreciation period equal to economic life (\( Q = N \)). A final assumption is that the before-tax cash flow (\( A \)) remains constant. For many design studies these assumptions are reasonable. At the junior level we shall consider cases where \( N, Q \), and \( H \) all differ.

Before analyzing the cash flow distribution shown on the next page, I want to make things easier for you by introducing one more simplifying assumption, which involves substituting a uniform annual value of the interest payments (abbreviated \( I_B \)) for the actual, ever-diminishing values.
Chapter 4: Leverage, Sophomore level

The left hand sketch below shows how the uniform periodic bank payments, $A_B$, are divided between interest payments and reduction of the amount owed (abbreviated $\Delta P_B$). The right hand sketch shows our simplified version.

\[
A_B = P_B(CR - i_B - N)
\]
\[
\Delta P_B = A_B - I_B
\]

\[
A_B = P_B(CR - i_B - N)
\]
\[
\Delta P_B = \frac{P_B}{N}
\]
\[
I_B = A_B - \frac{P_B}{N}
\]

Shown next is the distribution of the annual revenue when both bank loans and straight-line depreciation are involved.

\[
I_B \quad \rightarrow \quad \frac{P}{N} \quad \rightarrow \quad \text{Tax base} = A - I_B - \frac{P}{N}
\]

\[
\text{Annual Revenue}
\]

\[
\begin{align*}
Y & \quad \rightarrow \quad A_B \quad \rightarrow \quad A_0 \quad \rightarrow \quad \text{Tax} \\
\text{Annual operating costs} & \quad \rightarrow \quad \text{Return to bank} \quad \rightarrow \quad \text{Owner's cash flow} \quad \rightarrow \quad t(A - I_B - \frac{P}{N}) \\
& \quad \rightarrow \quad A' = \text{cash flow after tax} \\
& \quad \rightarrow \quad A = \text{cash flow before tax}
\end{align*}
\]

An examination of the diagram above leads to this expression relating cash flows before and after tax:

\[
A' = A - \text{Tax} = A - t(A - I_B - \frac{P}{N})
\]

\[
A' = A - tA + tI_B + t\frac{P}{N}
\]

\[
A' = A(1 - t) + tI_B + t\frac{P}{N}
\]
Further, the residual annual return to the owner, $A_o$, will be:

$$A_o = A' - A_B$$

where $A_B = P_B \times (CR-i_B-N)$

4.2 Numerical example

Assume that a ship that cost $75 million has an expected economic life of 25 years. The owner uses $25 million equity capital and the rest comes from a bank loan payable over the 25-year life at 9% annual interest. The ship is expected to earn annual revenues of $8.25 million against operating costs of $1.25 million. Assume a tax rate of 45% and straight-line depreciation. Find these four annual cash flows: (a) before tax, (b) after tax, (c) to bank and (d) to owner.

Our first step in solving this problem will be to subtract the owner's equity from the total cost of the ship; that will tell us how much must be borrowed from the bank:

$$P_B = P - P_o = \$75M - \$25M = \$50M$$

where $M$ = millions

Now we can find the annual payment owed the bank:

$$A_B = P_B \times (CR-i_B-N) = \$50M \times (CR-9\%-25) = \$5.09M \text{ (rounded)}$$

Then, our approximate value of the annual interest payment will be:

$$I_B = A_B \times \frac{P_B}{N} = \$5.09M \times \frac{\$50M}{25} = \$5.09M - \$2.00M = \$3.09M$$

Next, we need to find the annual cash flow before tax:

$$A = \text{revenue} - \text{operating costs} = \$8.25M - \$1.25M = \$7.00M \text{ [which is the answer to part (a)].}$$

Now we are ready to convert to cash flow after tax:

$$A' = A(1 - t) + tI_B + t\frac{P}{N}$$

$$A' = \$7M(1 - 0.45) + 0.45 \times \$3.09M + 0.45 \times \frac{\$75M}{25}$$

$$A' = \$3.85M + \$1.39M + \$1.35M = \$6.59M \text{ [the answer to part (b)]}$$

We have already found the annual payment to the bank as the first step in finding $I_B$: $\$5.09M \text{ [the answer to part (c)]}$

Finally, we can find the owner's after-tax cash flow:

$$A_o = A' - A_B = \$6.59M - \$5.09M = \$1.50M \text{ [the answer to part (d)]}$$
Chapter 4: Leverage, Sophomore level

4.3 Summary

In this introductory section we have learned how bank loans (or other kinds of bonded indebtedness) may reduce the corporate income tax. Specifically, we have learned how to derive the after-tax cash flow under the simplifying assumption that tax life, and economic life are both the same as the loan period. We also assumed straight-line depreciation. At the next level we shall learn how to handle cases where those simplifying assumptions do not apply. First, however, you should enhance your understanding of these matters by working out the following exercises.

Exercises

1) A $5000 loan from a bank is to be repaid in five equal annual payments with 10% interest. How much must be paid each year?

2) Prepare a table showing year-by-year how an accountant might keep track of the above loan and its repayments.

3) Find the present value of the interest payments found in the second example (based on 10% interest), then convert to an equivalent uniform annual amount.

4) Use our simplified approximation to find the annual interest payments due in the loan outlined in the second exercise. Compare outcome with amount found in the third exercise.

5) Rework the numerical example given in Section 4.2, but change the owner's equity to $35 million and the bank interest rate to 8%.

Answers

1) This calls for use of the capital recovery factor: $A_B = P(CR-i_B-N)$

$A_B = $5000(CR-10%-5) = $1319$ (rounded)

2) $5000 Account:

<table>
<thead>
<tr>
<th>End of year</th>
<th>Residual debt before payment at end of year</th>
<th>Interest due @ 10%</th>
<th>Total annual</th>
<th>Reduction in debt payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5000</td>
<td>$500</td>
<td>$1319</td>
<td>$819</td>
</tr>
<tr>
<td>2</td>
<td>$4181</td>
<td>$418</td>
<td>$1319</td>
<td>$901</td>
</tr>
<tr>
<td>3</td>
<td>$3280</td>
<td>$328</td>
<td>$1319</td>
<td>$991</td>
</tr>
<tr>
<td>4</td>
<td>$2289</td>
<td>$229</td>
<td>$1319</td>
<td>$1090</td>
</tr>
<tr>
<td>5</td>
<td>$1199</td>
<td>$120</td>
<td>$1319</td>
<td>$1199</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td>$5000</td>
</tr>
</tbody>
</table>

(Note that the final-year figures in the second and fifth columns must be the same and that the total of the fifth column must equal the initial debt.)

3) To find the present worth of the interest payments shown above:
Chapter 4: Leverage, Junior level

<table>
<thead>
<tr>
<th>Year</th>
<th>Interest IB</th>
<th>Present worth of IB @ 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$500</td>
<td>$455</td>
</tr>
<tr>
<td>2</td>
<td>$418</td>
<td>$345</td>
</tr>
<tr>
<td>3</td>
<td>$328</td>
<td>$246</td>
</tr>
<tr>
<td>4</td>
<td>$229</td>
<td>$156</td>
</tr>
<tr>
<td>5</td>
<td>$120</td>
<td>$75</td>
</tr>
</tbody>
</table>

Total: $1277

Then, to convert to a uniform annual amount, apply the appropriate capital recovery factor:

\[ A = 1277(CR-10\%-5) = 337 \]

4) Here is our approximate value of IB:

\[ IB = PB \left\{ \left( CR_{IB} - N \right) \right\} \frac{1}{N} \]

\[ IB = 5000\left( CR-10\%-5 \right)^{\frac{1}{5}} = 319 \] (which is about 5% lower than the exact value found above)


Independent thinking

Would you think it reasonable for a prospective ship owner to approach a banker for a loan with a repayment period greater than the expected economic life of the proposed ship? Justify your answer.

---

*Unto a stranger thou mayest lend upon usury; but upon thy brother thou shalt not.*

Deuteronomy 23:20

Junior level

4.4 Differing time periods

To this point we have assumed that the period of the bank loan and the tax depreciation period both coincided with the economic life of the ship. Now we shall find how to analyze cash flows before and after tax when those periods are all different. Initially we shall assume that the loan period, H, is shorter than the depreciation period, Q, which in turn is shorter than the economic life, N. Our cash flow diagram would then contain three segments:
During Period (A) the cash flows before and after tax would be as developed in the earlier part of this chapter except that we must now be careful to identify the differing time periods: H, Q, and N:

\[ A' = A(1 - t) + \frac{tP}{Q} + tI_B \]

During Period (B) the interest payments would no longer be a factor, so the only tax shield would be the depreciation allocation:

\[ A' = A(1 - t) + \frac{tP}{Q} \]

During Period (C) there would be no tax shields at all, so:

\[ A' = A(1 - t) \]

Putting these individual cash flow patterns back together will yield this after-tax cash flow diagram:

\[ A' = A(1 - t) + t \frac{P}{Q} \]

Applying the techniques we learned in Chapter 2, we can find the present worth of this cash flow as follows:

\[ PW = A(1 - t)(SPW - i' - N) + \frac{P}{Q} (SPW - i' - Q) + tI_B (SPW - i' - H) \]

Thus, if we have a uniform cash flow before tax and a stepped-pattern of cash flows after tax, we can find the present worth of the after-tax cash flows by means of this relatively
simple equation. In Chapter 5 we shall learn how to apply this concept to decision making in design.

Exercises

1) Find the present worth (based on 11% interest) of the after-tax cash flows projected for a ship that is expected to cost $125 million, of which $100 million will be obtained from a bank loan at 9% interest to be repaid in uniform annual amounts (capital plus interest) over 12 years. The economic life is set at 20 years; the tax rate is 40% based on straight-line depreciation over 15 years. The before-tax cash flow is estimated at $21 million per year.

2) Rework the first exercise using 15% interest instead of 11% in discounting future amounts.

Answers

1) To solve this problem we need do little more than plug into the equation derived in Section 4.4. First, however, we must derive an approximate value of the annual interest paid to the bank:

\[ I_B = P_B \left( (CR - i_B - H) - \frac{1}{H} \right) \]

\[ I_B = 100M \left( (CR - 9\% - 12) - \frac{1}{12} \right) \]

\[ I_B = 100M(0.1397 - 0.0833) = 5.64M \]

Now we are ready to apply our equation for present worth of the stepped cash flow:

\[ PW = A(1 - t)(SPW - i' - N) + t \frac{P}{Q}(SPW - i' - Q) + tI_B(SPW - i' - H) \]

\[ PW = 21M(1 - 0.40)(SPW - 11\% - 20) + 0.40 \frac{125M}{15}(SPW - 11\% - 15) + 0.40 \times 5.64M(SPW - 11\% - 12) \]

\[ PW = 100.34M + 23.97M + 14.65M \]

\[ PW = 139 \text{ million (rounded)} \]

2) $111 \text{ million (rounded)}$

Independent thinking

In Section 4.4 we assumed that the loan period would be less than the tax depreciation period. Rework the equation for present worth under the reverse assumption. Compare results.
If you lend money you make a secret enemy; 
if you refuse it, an open one.

Voltaire

Senior level

4.5 Accelerated depreciation

In Section 4.4 we learned how to find the present worth of stepped cash flow patterns resulting from a bank loan period (H), depreciation period (Q) and economic life (N) all being different. A basic assumption was that straight-line depreciation would be applied. Suppose instead of straight-line we have sum of the years digits depreciation. What effect will that have on the present worth of the after-tax cash flow? The approach is simple; replace the straight-line depreciation component:

\[ t \frac{P}{Q} (SPW-i'-Q) \] with the SYD expression explained in Section 3.8:

\[ t \text{(PW of } D) = \frac{2(P - L)}{Q(Q + 1)i'} [Q - (SPW-i'-Q)] \]

This technique is demonstrated in the final exercise (and its solution) below.

4.6 Residual debt

Imagine yourself in this happy situation: Five years ago you took out a $150 000 mortgage on your new house, agreeing to repay the bank in 15 equal annual payments with interest set at 12%. The annual payments are found as follows:

\[ A_B = 150 000(\text{CR-12\%-15}) = 22 023.64. \]

Now you find that your Uncle Vladimir has died and left you some half a million dollars. You are now in a position to pay off the mortgage and enjoy a debt-free home. The question then arises, how much do you still owe? The banker could tell you readily enough, but suppose you want an independent check. The approach is direct and easy; at any point during an ongoing series of payments the residual debt is simply the present value of the remaining payments. In this case, 10 payments are still due, so the residual debt, \( P_R \), will be:

\[ P_R = 22 023.64(\text{SPW-12\%-10}) = 124 438.48 \]

To generalize the logic developed above, let \( X \) = the number of years since the start of a loan period of \( H \) years at an interest rate \( i_B \). The remaining years, which we'll call \( Z \), will then be \( H - X \). The residual debt will then be found this way:

\[ P_R = P_B (\text{CR-}i_B-H)(\text{SPW-}i_B-Z) \]

4.7 Balloon mortgages

A ship owner faced with a heavy mortgage on a new ship may have great difficulty in meeting the periodic payments, particularly where the loan is a major part of the total
investment (i.e., heavily leveraged), the repayment period is relatively brief, and the transport business is still newly developing. Under those circumstances the owner and bank may agree on a mortgage scheme that will require the owner to pay an appreciable portion of the debt by perhaps the ship's half life, leaving the owner responsible for paying the rest in a lump sum at that time. If at that time the owner cannot produce that amount of capital, there are two major options: (a) sell the ship, or (b) obtain a new loan from the same, or other, bank. This kind of an arrangement is known as a balloon mortgage.

One logical way to set the amount of the residual debt (i.e., the balloon payment) is to apply the technique explained in the previous section.

Here is a numerical example. A ship owner wants to borrow $35 million to help pay for his proposed ship. His banker offers the loan at 10% annual interest payable over 6 years. This leads to an annual payment of $35M(CR-10%-6), or $8.04 million. The owner is worried that he might not be able to generate enough cash to pay at that rate. The banker then offers to base the payments on a 10.25% interest rate and a 10-year schedule, but with a balloon payment due at the end of 6 years. The annual payments, \( A_B \), will be:

\[
A_B = 35M(CR-10.25%-10) = 5.757 \text{ million}
\]

At the end of 6 years the residual debt will equal the present worth of the remaining 4 years of payments, each of $5.757 million:

\[
P_R = 5.757M(SPW-10.25%-4) = 18.15 \text{ million}
\]

As an alternative to balloon payments, some lending plans allow a period of years before the first payment falls due. This leads to some extra risk to the lender, which will have to be balanced by an increase in the interest rate, or an addition to the total debt.

4.8 Summary and review

In this chapter we have examined various ways in which a ship owner (or yacht owner) may go into debt in order to expand the scope of his or her operations. We have seen that the interest payments incurred may reduce the tax base and so must be recognized in assessing after-tax cash flows.

We have looked at increasingly complicated loan arrangements. There are times when you, as a naval architect, will want to apply simple schemes. There will be times when you will want to apply complex schemes. In general, in the preliminary design stages (when dozens or hundreds of alternatives are under consideration), you should be satisfied to use the simplest schemes. At the other end of the scale (when the choice has been narrowed down to half a dozen), your client, the business manager, may want to apply some more realistic assumptions.

In general, the more realistic (complex) assumptions will slightly reduce the impact of the income tax. In the early design stages, when assuming simple loan plans, you may recognize this effect by adding a small increment to the actual tax rate or to the interest rate. The same thought applies to assumptions regarding tax depreciation plans. By so doing, the optimum design as indicated by the simple assumptions will also be the optimum as indicated by the more realistic, elaborate assumptions.
Exercises

1) Assume that two years ago you borrowed $15,000 to buy a new car. The loan was to be repaid in equal monthly installments at 18% annual interest over 5 years. Now, having just won a lottery, you want to pay off the debt. How much will you owe?

2) Find the effective interest rate for the installment plan outlined in the first problem.

3) A ship owner wants to borrow $80 million to finance a new ship. The bank offers to lend that amount at 10.5% interest payable over 8 years. This leads to annual payments of $80M(CR-10.5%-8) = $15.27 million. The owner doubts that the ship will be able to generate that level of cash flow during its early years. What about postponing the first payment until the end of the second year, and then paying off during the next 6? The banker would then either add a bit to the interest rate, or simply let the debt build up at 10.5% interest for 2 years and then collect larger repayments during the final 6 years. If the second option is selected, how big would the annual payments be?

4) Repeat the first exercise called for under the junior level above, but assume sum of the years digits depreciation. To refresh your memory, you were asked to find the present worth (based on 11% interest) of the after-tax cash flows projected for a ship that was expected to cost $125 million, of which $100 million was to be be obtained from a bank loan at 9% interest to be repaid in uniform annual amounts (capital plus interest) over 12 years. The economic life was set at 20 years; the tax rate was 40% and was based on straight-line depreciation over 15 years. The before-tax cash flow was estimated at $21 million per year.

Answers

1) Now we come to the more realistic picture of monthly installments, so you may want to review the subject of non-annual compounding in Section 2.13. In this case we have 12 compounding periods per year, so \( M = 12 \) and the total number of compounding periods, \( MN \) will equal \( 5 \times 12 = 60 \). The interest rate per compounding period will be \( 18 \div 12 = 1.5\% \). The monthly payments will be:

\[
15,000(CR-1.5\%-60) = 380.90
\]

After two years, the remaining payment periods will be \( (5 - 2) \times 12 = 36 \), so the residual debt will be the present value of a stream of 36 payments of $380.90 each:

\[
P_R = 380.90(SPW-1.5\%-36) = 10,535.96
\]

2) \( r_1 = \left(1 + \frac{rM}{M}\right)^M - 1 \)

\[
r_1 = \left(1 + \frac{0.18}{12}\right)^{12} - 1 = 19.56\%
\]

3) $22.76 million
4) The present worth of the SYD depreciation can be found with the equation presented in Section 3.8, being careful to substitute Q for N:

\[ \text{PW of } D = \frac{2(P - L)}{Q(Q + 1)i} [Q - (SPW - i' - Q)] \]

\[ \text{PW of } D = \frac{2(125M - 0)}{15(15 + 1)11\%} [15 - (SPW - 11\% - 15)] \]

\[ \text{PW of } D = 9.470M[15 - 7.191] = 73.95M \]

\[ t(\text{PW of } D) = 0.40(73.95M) = 29.58 \text{ million} \] (which compares with the $23.97 figure with straight-line depreciation found before).

The total present worth of the after-tax cash flow now becomes:

\[ \text{PW} = 21M(1 - 0.40)(\text{SPW} - 11\% - 20) + 29.58M + 0.4 \times 5.64M (\text{SPW} - 11\% - 12) \]

\[ \text{PW} = 100.34M + 29.58M + 14.64M = 144.6 \text{ million} \] (rounded).

This new result is some $5.6 million better than was the case with straight-line depreciation. Again, the total tax paid will be the same, but SYD offers a better pattern of cash flows.

**Independent thinking**

In most instances a banker will want the period of the loan to be considerably shorter than the projected economic life of a ship or yacht. What are some of the factors that may lead to such a view?
Chapter 5
Measures of Merit

A student who can weave his technology into the fabric of society can claim to have a liberal education;
a student who cannot weave his technology into the fabric of society cannot claim even to be a good technologist.
Lord Eric Ashby

Sophomore level

5.0 Perspective
Up to this point you have learned (and I hope mastered) the basic principles of engineering economics. You have learned how to assess the relative values of cash exchanges that occur at different times. You have also learned how to analyze the impact of taxes and interest payments on cash flows. Now we come to the critical question of how to apply all of the foregoing to decision making in ship, or boat, design.

The first thing I have to tell you is that there is no universally agreed upon technique for weighing the relative merits of alternative designs or strategies. Business managers, for example, agree that our aim in designing a merchant ship should be to maximize its profitability as an investment. But then they fail to agree on how to measure profitability. Likewise, government officials who are responsible for designing non-commercial vessels (such as for military or service functions) have a hard time agreeing on how to go about deciding between alternatives. The truth of the matter is that there are good arguments in favor of each of several economic measures of merit and you should understand how to handle each of them. That is what this chapter is all about.

5.1 Menu of measures of merit
Table 5.1 identifies thirteen measures of merit, each based on sound economic principles. Each is of potential value in marine design, and several have strong adherents among people in authority. They are placed in three categories depending on whether the analyst wants to assign (versus derive) a level of income and assign (versus derive) an interest rate.

As you will note in the table, there are but three primary measures of merit; the other ten are each closely related to one of those three. Most of the rest of this chapter is devoted to explaining the mechanics of each measure and when it is most suitably applied. The abbreviations used in the table are explained on the next page.

Table 5.1

<table>
<thead>
<tr>
<th>Required Revenue</th>
<th>Assumptions Interest Rate</th>
<th>Primary Measure of Merit</th>
<th>Surrogates or Derivatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>yes</td>
<td>NPV</td>
<td>NPVI, AAB, AABI</td>
</tr>
<tr>
<td>yes</td>
<td>no</td>
<td>Yield</td>
<td>CR, CR', PBP</td>
</tr>
<tr>
<td>no</td>
<td>yes</td>
<td>AAC</td>
<td>LCC, CC, RFR, BCT</td>
</tr>
</tbody>
</table>
Chapter 5: Measures of Merit, Sophomore level

Table 5.2

Abbreviations Used in Table 5.1

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>Net present value</td>
</tr>
<tr>
<td>NPVI</td>
<td>Net present value index</td>
</tr>
<tr>
<td>AAB</td>
<td>Average annual benefit</td>
</tr>
<tr>
<td>AABI</td>
<td>Average annual benefit index</td>
</tr>
<tr>
<td>CR</td>
<td>Capital recovery factor before tax</td>
</tr>
<tr>
<td>CR'</td>
<td>Capital recovery factor after tax</td>
</tr>
<tr>
<td>PBP</td>
<td>Pay-back period</td>
</tr>
<tr>
<td>AAC</td>
<td>Average annual cost</td>
</tr>
<tr>
<td>LCC</td>
<td>Life cycle cost</td>
</tr>
<tr>
<td>CC</td>
<td>Capitalized cost</td>
</tr>
<tr>
<td>RFR</td>
<td>Required freight rate</td>
</tr>
<tr>
<td>ECT</td>
<td>Economic cost of transport</td>
</tr>
</tbody>
</table>

In this introductory part of the chapter you will learn about the four most important measures of merit. These are the three primary measures shown in the middle column of the table on the previous page (net present value, yield, and average annual cost) plus required freight rate. At the more advanced levels you will learn about the other measures and when they should be applied.

5.2 Net present value (NPV)

The net present value, commonly abbreviated NPV, is a good place to start. It is by far the most popular of all these economic measures of merit among American business managers. It is also one of the easiest to understand and use. As indicated in the table, it requires an estimate of future revenues and it assigns an interest rate for discounting future (usually after-tax) cash flows. The discount rate is usually taken as the minimum rate of return acceptable to management. As implied by its name, NPV is simply the present value of the projected cash flow including the investments.

\[
\text{NPV} = A'(\text{SPW-i'-N}) - P
\]

In the simple cash flow pattern shown above, A' represents a uniform annual level of cash flows after tax and P represents a single lump investment. Given that pattern, the net present value is found by subtracting the investment from the present worth of the future cash flows. In short:

With more complex cash flows, perhaps involving multi-year investments, the NPV can be found using a year-by-year table. Consider, for example, a project that is expected to involve the investments and after-tax returns shown in this diagram:

\[
\text{NPV} = A'(\text{SPW-i'-N}) - P
\]
Chapter 5: Measures of Merit, Sophomore level

Table 5.3

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flow</th>
<th>PW @ 9%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>($50)</td>
<td>($45.87)</td>
</tr>
<tr>
<td>2</td>
<td>($60)</td>
<td>($50.50)</td>
</tr>
<tr>
<td>3</td>
<td>$30</td>
<td>$23.17</td>
</tr>
<tr>
<td>4</td>
<td>$60</td>
<td>$42.51</td>
</tr>
<tr>
<td>5</td>
<td>$80</td>
<td>$51.99</td>
</tr>
</tbody>
</table>

Net present value = $21.30

The NPV of $21.30, being positive, would cause the proposed project to be looked upon with favor. Of course it might not be accepted if some alternative project promised an even higher value. Had the NPV turned out to be negative, the project would not be given further thought.

What would happen to the NPV above if the minimum acceptable interest rate were to be raised? Suppose we double, it to 18%:

<table>
<thead>
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</tr>
<tr>
<td>3</td>
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<td>$18.26</td>
</tr>
<tr>
<td>4</td>
<td>$60</td>
<td>$30.95</td>
</tr>
<tr>
<td>5</td>
<td>$80</td>
<td>$34.97</td>
</tr>
</tbody>
</table>

Net present value = ($1.28)

Now, the NPV being negative, the project would be rejected. What has caused the change? The answer is that the higher interest rate has strengthened the time-value of money, thus reducing the apparent benefits of future incomes.

5.3 Yield

An important fact to understand about NPV is that you find it by discounting future cash flows at the manager's minimum acceptable interest rate. Because the predicted value of an acceptable project must always be positive, the actual expected interest rate will be something higher than the minimum rate used in the calculation. Instead of applying that minimum acceptable rate, we could look at the expected cash flow pattern and infer the interest rate implied.

Take, for example, the projected cash flow analyzed just above. There is some interest rate that will make the NPV equal to zero. When we find it, that will be the yield. The mechanics of the process are to start by guessing at an interest rate and using it to find the corresponding NPV. If the number comes out positive, we know our rate was too low, so we try again, this time with a higher interest rate. After about three repetitions we may plot our results (NPV vs interest rate), draw a curve through the points and find where the NPV is zero. That will be the derived yield -- and an excellent measure of merit.

In this particular case, starting with the NPV values derived above for interest rates of 9% and 18%, I interpolated a third value (15%). This led to an NPV of $4.95. I then plotted those results, as shown next, leading to a derived yield of about 17.25%.
When I made a table and derived the NPV with 17.25% interest, the NPV came out to be $0.18 - just a little too big, so I tried again with 17.35%. That led to an NPV of minus three cents, which was close enough to zero for our purposes. Conclusion: the cash flow in question leads to a yield of about 17.35%. Put another way, if a banker offers to pay you 17.35% per year for the use of your money, you could deposit $50 now and $60 a year from now. Then in two years you could withdraw $30, in three years $60, and in four years $80.

If you are clever with your computer you can probably develop a program that will quickly go through the mechanics of what I have outlined above and zero in on that unique interest rate that will bring the NPV down to zero, thus telling you the yield for the cash flow pattern in question. Alternatively, you may be able to find software packages with such tools already built in.

In most preliminary ship design studies you will probably not be afflicted with complex cash flow patterns, but will rather be looking at a single investment (at year zero) and uniform annual after-tax returns. Take, for example, a ship with an initial cost of $30 million and uniform annual after-tax returns of $4.50 million. The economic life is expected to be 20 years and the disposal value can be ignored. What is the projected yield? Here is the pattern:

\[ A' = 4.5 \text{ million} \]

\[ P = 30 \text{ million} \]

20 years

Your computer may be able to use the three values of initial investment, uniform returns, and period of years, to derive the interest rate (which turns out to be about 13.9%). If your machine can't do that for you, you can turn to Figure 5.1 for help. To use the figure, first take the ratio of annual returns (A') to investment (P). That will give you the after-tax capital recovery factor:
Figure 5.1
CR' = \frac{A'}{P} = \frac{\$4.5M}{\$30M} = 0.15

Then turn to Figure 5.1 and locate 0.15 on the vertical scale (capital recovery factors), go over to the sloping contour for 20-year life and then down to the horizontal scale, where you will read the corresponding interest rate of 13.9%. Having derived an interest rate (or yield) of 13.9%, you should check to see if that will lead to the projected annual returns. Like this:

A' = P(CR'-i'-N) = \$30M(CR'-13.9\%-20) = \$4.50 million (okay!)

Yield is a logical measure of merit. The popularity of the concept is reflected in the many things it is called. Among these are Discounted cash flow rate of return, Internally generated interest, Rate of return, Profitability index, Percentage return, Investor's method, and Equivalent return on investment.

Some advocates of NPV point to situations where yield may be misleading. You will learn about those arguments in the appendix.

5.4 Average annual cost (AAC)

We come now to a measure of merit that is useful in designing ships that are not expected to generate income: naval vessels, Coast Guard vessels, and yachts immediately come to mind. Now our cash flow pattern will feature only money flowing out. When that is the case, a logical and popular measure of merit is called average annual cost (AAC). In the simplest case we would have a single initial investment (P) at time zero, and uniform annual operating expenses (Y) for N years thereafter:

In the above, the average annual cost would be found by converting the initial investment, P, to an equivalent uniform annual amount, which would be added to the annual operating costs, Y:

AAC = P(CR-i-N) + Y

The interest rate here should be some logical measure of the decision maker's time-value of money. In the case of a government-owned ship it might reflect the current rate of interest paid on government bonds.

Whereas in using NPV or yield, we seek the alternative promising highest values, in using AAC we seek lowest values.

Average annual cost may also be applied to commercial ship designs where all alternatives would happen to have equal incomes.

Here is a simple numerical example. Find the average annual cost for a proposed oceanographic research vessel that is projected to cost $12 million to buy and $3 million per year to operate. The expected life is 25 years and an interest rate of 12% will apply. Using the equation developed above, we have:

For more complex cash flows, simply discount everything back to year zero, (including the initial investment), then multiply the total figure by the capital recovery factor. That will give you the average annual cost.

Here is an example. A survey ship is expected to cost $17 million. Its operating costs will come to $2 million in the first year, $3 million in the second and third years, and $4 million in the fourth year. After that it is to be sold at an expected net resale value of $9 million (leading to a net inflow of $5 million in year four). An interest rate of 15% is stipulated. The cash flow pattern is sketched below, together with a table showing year-by-year present values.

```
0  1  2  3  4
\  \  \  \  \  ~
\ $2m $3m $3m ~
\   \   \   ~
\  \  \  \  ~
\ $17m
```

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<td>$2M</td>
<td>$1.74M</td>
</tr>
<tr>
<td>2</td>
<td>$3M</td>
<td>$2.27M</td>
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<tr>
<td>3</td>
<td>$3M</td>
<td>$1.97M</td>
</tr>
<tr>
<td>4</td>
<td>($5M)</td>
<td>($2.86M)</td>
</tr>
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</table>

Total present worth $20.12M

AAC = $20.12M(CR-15\%-4) = $7.05 million (rounded)

Notice in the above that any positive cash flow, such as that resulting from the resale, is treated as a negative cost.

There you have the principles of finding average annual cost. What you have learned about gradient series (in Section 2.9), stepped cash flows (in Section 2.10), and so forth, can all be applied in finding AAC. The main difference is that now we recognize initial costs as well as operating costs.

5.5 Required freight rate (RFR)

Suppose two competitive designs promise the same average annual cost, but vessel B promises to be more productive than vessel A. Clearly that should tip the scales in B's favor. We can quantify this difference by relating the AAC to productivity. In the case of cargo ships we could divide the average annual cost by the tons of cargo that could be carried each year on some particular trade route. This would give us what we call the required freight rate (RFR). The same concept could be applied to other measures of productivity such as automobiles per year for a ferry, tons of fish per year for a trawler, passengers per year for a passenger ship, and so forth.
Chapter 5: Measures of Merit, Sophomore level

Where we can assume a single invested amount (P) at year zero, uniform annual operating costs (Y), and annual tons of cargo (C), our equation for required freight rate becomes:

\[ RFR = \frac{AAC}{C} = \frac{P(CR-i-N)+Y}{C} \]

Choosing an interest rate here is ticklish. Assuming free market forces at play and all competitors facing equal costs (another brave assumption!), the interest rate should be just high enough to bring a balance between demand for transport service on the trade route in question and the supply of ships capable of providing that service. Higher rates would attract too many ships; lower rates would drive ships to other services. Adam Smith called this the natural rate. It is closely akin to what economists call the shadow rate.

What is the significance of RFR? It is the rate the ship owner must charge the customer if the ship owner is to earn a "reasonable" return on the investment. The theory is that the owner who can enter a given trade route with a ship offering the lowest RFR will best be able to compete.

Required freight rate is especially appropriate for student design projects because it does not require any prediction of actual revenues (confidential data seldom available in academia).

A key step in finding RFR is to convert the initial investment to an equivalent uniform annual cash flow before tax. These annual amounts must be large enough to pay the income tax, and return the original investment to the owner at the specified level of interest. We must, in short, find a suitable value for the capital recovery factor before tax. To do this, let me first remind you of the basic relationship between cash flows before and after tax that you learned about in Section 3.4:

\[ A' = A(1 - t) + \frac{P}{N} t \]

To make this non-dimensional, we can divide through by the initial investment, P:

\[ \frac{A'}{P} = \frac{A}{P}(1 - t) + \frac{t}{N} \]

But, since \( \frac{A'}{P} = CR' \), and \( \frac{A}{P} = CR \), we have:

\[ CR' = CR(1 - t) + \frac{t}{N} \]

Then, solving for CR, we have:

\[ CR = \frac{CR' - \frac{t}{N}}{1 - t} \]
Chapter 5: Measures of Merit, Sophomore level

This, then, is a simple way of converting an after-tax interest rate to a before-tax capital recovery factor. It assumes an all-equity investment and a tax depreciation period equal to the ship's economic life. We shall deal with more complex relationships later on.

Let us clarify the above with a numerical example. Here is a proposed ship that can move 3.5 million tons of cargo over a given trade route each year. Its estimated first cost is $40 million. Its economic life is set at 20 years. The tax rate is 45%. The annual operating costs are estimated at $2.5 million. The owner stipulates a yield of 12%. What is this ship's required freight rate?

We start by finding the after-tax capital recovery factor based on 12% interest and 20-year life:

\[
CR' = (CR - 12\% - 20) = 0.1339
\]

This leads us to:

\[
CR = \frac{CR' - t}{N} = \frac{0.1339 - 0.45}{20} = 0.2025
\]

and, finally:

\[
RFR = \frac{P(CR) + Y}{C} = \frac{40M(0.2025) + 2.5M}{3.5M} = \$3.03 \text{ per ton}
\]

Having found our required freight rate, let us turn the problem around by starting with that figure and deriving the attainable yield. Here is how an accountant would handle the job:

| Annual revenue = $3.03 \times 3.5M tons | $10.605M |
| Annual operating costs | $2.500M |
| Annual cash flow before tax | $8.105M |
| Depreciation: $40M/20 | $2.000M |
| Annual tax base | $6.105M |
| Tax @ 45% | $2.747M |
| Annual cash flow after tax | $5.358M |
| After-tax capital recovery factor | 0.1339 |
| Corresponding after-tax yield | 12% (which agrees with the initial specification) |

This bears out the soundness of the way we handled the task of converting from an after-tax yield to a before-tax level of income.

A reminder: notice in the table above that the after-tax cash flow is found by subtracting the tax from the before-tax cash flow. Most of us, unless wide awake, are apt to subtract it from the tax base instead, and that gives us profit instead of cash flow.
Chapter 5: Measures of Merit, Sophomore level

Exercises

1) A $5000 investment promises annual returns of $700 over 20 years. Find the net present value based on 9% interest.

2) Find the NPV at 8% for this cash flow: $1000 expense now, $2000 expense a year from now, $2000 income two years from now, and $2000 income three years from now.

3) Find the yield for a $10,000 investment that promises returns of $1000 per year for 20 years.

4) Find the yield to the nearest one percent for an investment of $1000 that promises a return of $500 in the first year and $800 the second.

5) Find the required freight rate for a dry bulk carrier that has an initial cost of $80 million, annual operating costs of $1.85 million, and an annual transport capacity of 4.25 million tons. The owner's target interest rate is 14% after tax. The tax rate is 25% and the ship's life is 20 years.

6) Rework the fifth problem but assume a doubling of the tax rate.

Answers

1) 

$A' = $700$

$P = $5000$

$NPV = $700(SPW-9%-20) - $5000 = $139$

2) 

<table>
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<td>$1715</td>
</tr>
<tr>
<td>3</td>
<td>$2000</td>
<td>$1588</td>
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</table>

Total = NPV = $451

3) 7.75%

4) My approach was to start with a guessed-at interest rate of 15%. That led to an NPV of $39.90. Being greater than zero, I realized that I needed a higher interest rate, so I tried 20%. That led to an NPV of ($27). That led me to conclude that the yield had to fall between 15% and 20% and somewhat closer to the higher figure. So I tried 18% and found an NPV of ($1.45), which was close enough. Conclusion: The yield would be 18%
Chapter 5: Measures of Merit, Junior level

5) $3.91 per ton.
6) $5.18 per ton.

**Independent thinking**

Would you consider NPV to be a reliable criterion in cases where competing alternatives have different economic lives? How about yield? How about AAC? How about RFR?

In yacht design, what would you consider to be the single most important measure of merit?

Consider the outcome of exercises 5 and 6 above. What can you infer about the influence of corporate income taxes on your own cost of living?

---

**Junior level**

*We keep an extremely small prophet, a prophet
Who brings us unbounded returns.*

W. S. Gilbert

5.6 Net present value index (NPVI)

Despite its popularity, net present value (NPV) can lead to faulty decisions unless used with a certain amount of care. One weakness arises from its being dimensionally-dependent. As a result, it will always tend to favor large proposals even though smaller, more numerous proposals might well lead to greater cumulative NPVs. (We always have to assume that the supply of investment dollars is limited.) To correct that weakness, we can simply divide each proposal's NPV by the investment: NPV + P. I call this *net present value index*, and abbreviate it NPVI.

5.7 Average annual benefit (AAB)

A second weakness of NPV is that it makes unfair comparisons between long and short term investments. Consider a new ship with a projected life of 20 years that is in competition with a secondhand ship with a projected life of, say, 10 years. If the new ship’s NPV is estimated to be $20 million, and the other’s $15 million, what does that prove? The comparison is obviously unfair because the secondhand ship, after 10 years could presumably be replaced with another old ship and that would add to the NPV of the secondhand ship option. Our standard approach to such comparisons is to develop the NPV for a succession of identical units. In this case we should add to the first ship’s $15 million NPV the present worth of a like amount 10 years in the future.

The approach outlined above is easy enough where the competing lives have some neat common multiple. But suppose the secondhand ship has a projected life of, say, 8 years? That being the case we can make a valid comparison by converting each projected NPV to a uniform annual income stream of equivalent value. How do we convert a single present amount to a uniform annual amount? In case you've forgotten: simply multiply the present amount by the capital recovery factor (CR) appropriate to the unit’s expected life and the interest rate used in finding NPV. I call this uniform amount the *average annual benefit* (AAB). Note its exact parallel to average annual cost, AAC. Moreover, like AAC, it automatically corrects for differing life expectancies, because each succeeding unit must be assumed to have the same average annual cost on into infinity.
5.8 Average annual benefit index (AABI)

Now we can overcome NPV's two weaknesses simultaneously, if we divide the average annual benefit by the investment to give us average annual benefit per dollar investment. I call this criterion average annual benefit index (AABI).

These three variations on NPV are such obvious common sense corrections that they are commonly used without attaching names to them. Don't try to look them up in the literature.

When comparing two alternatives where initial investments are unequal, some analysts consider what use would be made of the savings if the less expensive option were chosen. Similarly, if lives differ, they would project the cash flow arising from the replacement of the shorter-lived option. This kind of approach allows reliance on NPV without the corrections involved in NPVI, AAB, or AABI. Although reasonable when comparing limited numbers of alternatives, such approaches would be ill-fitted in preliminary design studies involving large numbers of choices.

5.9 Capital recovery factor after tax

As I pointed out in Section 5.3, in most preliminary design studies we assume the simplest possible cash flow pattern: a single investment (made on the day of delivery) and uniform annual after-tax returns. Such a pattern hinges on several other assumptions:

a) The tax depreciation period equals the economic life of the ship.

b) Taxes are based on straight-line depreciation.

c) The ship's net disposal value will be zero.

d) There are no bank loans or bonded debt (i.e., we have an all-equity investment).

e) No working capital is required. (That is temporary cash paid out but to be recovered later -- like a key deposit.)

f) No fancy tax-softening schemes (e.g., tax credit or tax deferral) are used.

g) Revenues and operating costs will both remain uniform throughout the economic life (after adjustment for inflation).

h) There are no major components (e.g., cargo containers) with an economic life that differs from that of the ship.

I am quick to admit that these are exceedingly bold assumptions. Yet, in the majority of ship economic studies they are reasonably safe because the errors induced tend to be the same for all alternatives. (Remember, in choosing between alternatives, it's the differences that count.) As I have mentioned before, some owners, especially former accountants, will want to embellish your estimates with all manner of elaborate complications. For your own sanity, and in consideration of your computer's less-than-infinite circuits, you should try to make a compromise. Apply the baroque gingerbread to the final few alternatives, but start your work with the simplifying assumptions that lead to the neat cash flow pattern shown in Section 5.3.

Given that simple pattern, you could find the yield (as you may remember) by first finding the capital recovery factor after tax:
Chapter 5: Measures of Merit, Junior level

\[ CR' = \frac{A'}{P} \]

and then using the chart on Page 65 to find the corresponding interest rate \((i')\), which would then be the investment's yield.

Now, if you will look again at the chart you will note that the contours aim ever-upward, so you can easily infer that the alternative design promising the highest capital recovery factor after tax will automatically promise the highest yield. In short, \(CR'\) is a valid surrogate for yield (if all those bold assumptions are accepted) and is just a little easier to find.

5.10 Pay-back period (PBP)

Another related measure of merit is the pay-back period (PBP), which answers the entrepreneur's invariable question: how soon do I get my money back? Assuming uniform annual returns, the answer is easily supplied:

\[ PBP = \frac{P}{A'} \]

This is the reciprocal of \(CR'\) and so incorporates all that criterion's strengths and weaknesses. Its main problem is that it has often been misused (ignoring comparative cash flows that may occur after the pay-back period) and has acquired an unsavory reputation. It cannot do anything for us that \(CR'\) or yield cannot do, so we need concern ourselves with it no further.

5.11 Life cycle cost (LCC)

In non-income producing projects, some analysts use a criterion consisting of the initial cost plus the cumulative value of the discounted future costs. This is usually called life cycle costs (LCC). With uniform operating costs, we would have:

\[ LCC = P + Y(\text{SPW}-i-N) \]

Whereas average annual cost (AAC) totals all present and discounted future costs and then spreads them out into a uniform annual stream of equivalent value, LCC simply brings everything back to the present. If all the alternatives have equal lives, then LCC and AAC will lead to the same conclusion as to which alternative is best. If lives differ, however, LCC will be unreliable. I consider life cycle cost to be inferior to average annual cost in range of applicability and so suggest that we give it short shrift.

Some students have trouble telling the difference between NPV and LCC. There are two important differences. NPV applies to cases where incomes can be predicted. LCC applies to cases where either there is no income, or all alternatives have equal incomes. NPV discounts future amounts based on a minimum acceptable interest rate. LCC use a somewhat higher, target rate.
Exercises

1) Here are projected figures for four proposed ships:
   Ship B: $50M, Annual returns: $8M, Life: 20 years.
   Ship C: $100M, Annual returns: $20M, Life: 10 years.
   Ship D: $50M, Annual returns: $11M, Life: 10 years.

The owner is a believer in NPV and specifies discounting future amounts at 9%. Make a table showing for each alternative your calculated values for NPV, NPVI, AAB, and AABI. Underscore the best outcome for each measure of merit.

2) A proposed investment promises annual returns over 15 years of $250,000. The estimated first cost is $1 million and the expected resale value is $500,000. How would you go about finding the projected yield?

3) Find the pay-back period for the cash flow projected in the second exercise.

4) A proposed Coast Guard ship has these projected figures: First cost $85 million, annual costs of operation for first ten years: $7 million, for second ten years: $8 million. Resale value after 20 years: $12 million. Target interest rate: 13%. Find (a) LCC, and (b) AAC.

5) Before going on, let’s take time for a little refresher. Name the standard compound interest factors relating:
   a) The present value of a single future amount
   b) A series of uniform annual amounts derived from a single present amount
   c) A single future amount equivalent to a single present amount
   d) A single future amount equivalent in value to a uniform series
   e) A uniform series equivalent in value to a single future amount, and
   f) A single present amount equivalent in value to a series of uniform annual amounts.

Answers

1) Alternative   NPV       NPVI    AAB     AABI
   A             $36.93M  0.369   $4.05M  0.0405
   B             $23.03M  0.461   $2.52M  0.0504
   C             $28.35M  0.283   $4.42M  0.0442
   D             $20.59M  0.412   $3.21M  0.0642

2) With uniform annual returns you might think that you could easily find CR' (= A' + P), and use that to derive yield (the corresponding i'). That would be a mistake, however, because that appreciable resale value would make the CR' value meaningless. The only way to find yield would be through trial-and-error: DCF, in short.

3) PBP = P + A' = $1M + $0.25M per year = 4 years. (As you will note, the PBP criterion is utterly blind to that hefty resale value.)

4) (a): LCC = $85M + (SPW-13%-20)$8M - (SPW-13%-10)$1M - (PW-13%-20)$12M = $134.7M
    (b): AAC = (CR-13%-20)$134.7M = $19.18M
Chapter 5: Measures of Merit, Senior level


**Independent thinking**

In applying NPV to a particular pair of competing proposals in which one bears considerably greater risk than the other, can you suggest a logical way to give proper weight to the relative risks?

Two competing proposals promise equal values of the average annual benefit index. One projection is based on a 10-year life, the other on a 20-year life. Which would you select? Why?

---

**Senior level**

*If Karl, instead of writing a lot about Capital, made a lot of Capital, it would have been much better.*

Karl Marx's mother

5.12 Capital recovery factor before tax as a measure of merit

In Section 5.9 you learned that under a set of commonly assumed circumstances the capital recovery factor after tax (CR') could serve as a reliable surrogate for yield. If you will recall, our list of assumptions included the specification that the tax would be based on straight-line depreciation with tax life equal to the economic life. Given that, the capital recovery factors before and after tax would be related as follows:

\[ CR' = CR(1 - t) + \frac{t}{N} \]

If all alternatives have equal lives (N), and since the tax rate (t) would be the same for all, it becomes clear that the alternative promising highest capital recovery before tax would also promise the highest capital recovery factor after tax. Further, then, we can conclude that capital recovery factor before tax is a valid surrogate for yield (as long as all those standard simplifying assumptions hold true). In short, the simple ratio of before-tax returns to first cost can serve as a reliable measure of merit:

\[ CR = A + P \]

5.13 Economic cost of transport

If the ships under study are to carry a high-value cargo, then the required freight rate (RFR) should be adjusted in recognition of the inventory value of the goods in transit. If this is done, the faster ships will receive deserved credit for reducing the time the merchant's investment is tied up. We can call this adjusted freight rate the *economic cost of transport*, abbreviated ECT. Its value can be derived from this expression:

\[ ECT = RFR + \frac{ivd}{(1 - t)365} \]
where

\begin{align*}
    i &= \text{applicable interest rate per annum} \\
    v &= \text{value per ton of cargo as loaded aboard} \\
    d &= \text{days in transit} \\
    t &= \text{tax rate}
\end{align*}

5.14 Ships in service

Once a ship is built and paid for, the first cost (P) is no longer a variable and should therefore be ignored in making decisions about its operation. Maximizing profitability now hinges simply on maximizing the annual difference between income and operating costs. In doing this, you should take a long-term view and not try to save money by neglecting maintenance and repairs. You must also be a good citizen and give due regard to protecting the environment.

Exercises

1) Find the economic cost of transport (ECT) for a ship on a given voyage in which the required freight rate (RFR) is predicted to be $3.27, the value of the cargo as loaded aboard is $25 per ton, the tax rate is 55%, the owner uses an interest rate of 17.5%, and the days in transit are 36.

2) In the first problem, what would be the unit value of the cargo at the end of the voyage?

3) Re-work the first problem but change the tax rate to 35%, the interest rate to 11%, and the days in transit to 40.

Answers

1) \[ ECT = RFR + \frac{ivd}{(1-t)365} = \$3.27 + \frac{0.175\times25\times36}{(1-0.55)365} = \$3.27 + \$0.96 = \$4.23 \text{ per ton}. \]

2) The value at the end of the voyage would be the sum of the initial value and the economic cost of transport, or \$25.00 + \$4.23 = \$29.23 \text{ per ton}. 

3) \$3.73 \text{ per ton.}

Independent thinking

In the second part of the expression for ECT we find the factor \((1-t)\). Why is that necessary?

Reminders

1) Most of these measures of merit are valid only as long as their built-in assumptions hold true. **Please be careful!**

2) Chapter 6 and the appendix both include additional information on measures of merit.
Chapter 6
Constructing Your Analysis

Man is an animal that makes bargains; no other animal
does this – no dog exchanges bones with another.
Adam Smith

Sophomore level

6.0 Perspective

Now that you have assimilated the principles of economic analysis, you must develop
rational methods for applying them to real-life. There are few immutable, all-purpose rules
I can serve up to you on a silver platter. The best I can hope for is to help you develop a
feeling for how to construct economic comparisons that will lead to wise decisions in
choosing between design alternatives. Stimulating you to learn to think for yourself is
what this chapter is all about.

My first bit of advice is to stress that, in all this, common sense is king and simplicity
is queen. In slogging through the innumerable steps involved in economic analysis you
will find it all too easy to be so overwhelmed by details that you forget where it is you hope
to travel and why. You may find yourself dealing with accountants whose aim in life
seems to be to turn simple problems into baroque snarls. They have good reasons for
doing this, and you should try to understand them. They, as accountants, want to predict
to the nearest penny the profitability of each alternative. You, as an engineer, want only to
make sure you have a way to rank the alternatives, i.e., to show which ones promise to be
most profitable. In most cases relatively simple approaches will suit your needs.
Accounting elaborations will tend to confuse the situation and needlessly burden your
computer. The logical compromise is to use simple, qualitative methods to narrow the field
of contenders, and then satisfy the accountants by applying their quantitative methods only
to the more promising candidates.

In most of what follows I shall be stressing the design of merchant ships. Much of
what I say, however, can be modified to apply to all manner of engineering concepts.

6.1 Know your goal

Your basic aim in all this is to sell to some prospective ship owner some strategy (say
a ship design) for maximizing the profitability of his or her investment. To be a convincing
salesman you must avoid technical jargon and talk about economics. Right from the start
learn the person's preferred measure of merit and be ready to deal in those terms.

Along with learning the preferred measure of merit, you must determine the owner's
functional needs and under what constraints the project must operate. There are other
details to be learned from the owner: tax rate, depreciation plan, interest rates, perhaps
charter rates, and so forth. If some of those figures are confidential, the owner should still
be willing to bracket them in upper and lower values.

The owner should be explicit as to the form in which the cargo is to be moved (i.e.,
bulk, break-bulk, on pallets, in containers, etc.) You must also learn the details of the
pertinent port facilities. If these do not yet exist, you should expand the definition of your
system to include the design and operation of the terminals as well as the ships. This leads
to the next section.
6.2 Define the system

To reach proper decisions you must establish the boundaries of the system under study. They should be so chosen that your decision will have little if any effect on the rest of the enterprise (or the outside world, for that matter). Let me repeat the example given in the first chapter, junior level. There we looked at three examples, all involving water transport of iron ore.

Example 1: The size of the ship is fixed by the locks of a canal. The aim of the study is to select the type of propulsion machinery. No matter what decision is reached, the cargo handling costs and terminal costs will remain the same. In this case you can define the system as the ship itself. Optimize it and ignore the terminal.

Example 2: The ore is to be moved in pellet form. It will be loaded and off-loaded at deep offshore terminals. There are no canals or locks to be transited and no physical limitations on ship size. The economic benefits of large ship size must, however, be balanced against the economic disadvantages to the terminals. There will be no effect on the inland parts of the transport. The system, then, should be defined as terminal gate to terminal gate.

Example 3: The question is whether to ship the ore in its raw state, in pellets, or in a slurry. Here we must expand the system to include not only the complete source-to-destination journey, but the processing equipment and operation at each end.

6.3 Be prepared

To equip yourself for a career in design you must continually strive to collect data on weights, building costs, operating costs, and income potential. You must also learn how to use such data to predict the profitability potential of competing design alternatives. (Again, how to use such data is what this chapter is all about.) Keeping your data bank up to date is no easy task, but doing so is an important professional responsibility.

6.4 Selecting the structure

By way of preface to this topic, let me point out a bit of irony, which is that in general the more important the decision, the less applicable are sophisticated analyses. You can find equations for the diameter of a destroyer’s propeller shaft, but what equation will tell the government whether to declare war? What equation tells you which job offer to accept or whom to choose as your spouse? This does not mean that you should ignore rational decision-making methods; it only points to the logic of selecting an appropriate degree of sophistication.

There are situations in which you need consider only two alternatives. I refer to technical feasibility studies such as coal versus oil for ship propulsion. Here you may establish feasibility by comparing one well thought-out challenger (coal) against one equally well thought-out defender (oil). In doing this, select an operating environment that favors the challenger. Then, if the challenger fails to measure up to the defender you are probably safe in deciding against the challenger. If it looks good under those favorable circumstances, then you can seek to expand the operating environment in which it offers promise.

In more thorough feasibility studies you should seek to optimize both challenger and defender (by considering many alternatives in each) and then let each camp be championed by its own best contender. Does this strike you as being too obvious to be worth saying? Perhaps, but you can find cases in the marine literature where this common sense rule is blatantly ignored.
In optimizing the design of a merchant ship, the logical procedure will hinge first of all on whether the size is to be limited by external constraints (allowable draft or limits on overall dimensions) or by the availability of cargo, passengers, or whatever the ship is to transport.

Let us consider first the case where cargo comes in virtually unlimited supply. Examples include most bulk commodities such as crude oil, iron ore, and grain. In ships for such cargoes the cardinal rule is the bigger the better. There are all manner of economic benefits in making them as big as external constraints will reasonably allow. Do not make the mistake of starting with an arbitrarily established deadweight or cargo capacity. Those characteristics should drop out at the end and not affect your thinking along the line. In most bulk trades the same is true of sea speed.

Frequently the only important external constraint may be the allowable draft. That being the case, maximum values of length, beam and depth will be determined by reasonableness of proportions. Setting maximum values is not altogether easy, but the following suggestions may be taken as a start for seagoing ships:

- Beam/draft: 3.00
- Depth/draft: 1.43
- Length/depth: 14.0

Figure 6.1 shows a typical (now somewhat dated) attempt to use data on existing ships of a given type to establish reasonable upper bounds on proportions. The question of how far you should go in trying to maximize proportions is not easy to settle, and you should not expect to find any sharply defined optimum value. Like everything else in design economics, trade-offs are involved, and the true optimal point is on a curve exhibiting strong attributes of flat laxity. (By this we mean that you could select a design characteristic that was well off the best value with only negligible impact on the overall economics.)

One important exception to the above rule comes from discontinuities such as you might encounter if, as you moved along some design parameter such as installed power, you suddenly had to switch from single screw to twin screw propulsion. Other exceptions may arise from arbitrary man-made rules, such as abrupt changes in crewing requirements above certain values of gross tonnage, or IMO rules for pollution abatement.

Where draft is the only external constraint you may find economic benefit in choosing a design draft (and corresponding dimensions) that is about ten percent greater than the allowable operating draft. The economic benefits of the greater cargo capacity thus allowed more than offset the disadvantages of operating at partial draft.

Having selected the overall dimensions, you next predict the economic outcome promised by various combinations of power and block coefficient. In many cases you will find that the best economic value of the block coefficient falls close to, or on, the upper boundary of its technically feasible values. This implies a forebody as full as seagoing factors will allow and an afterbody as full as propeller flow factors will allow. In any event, with only a couple of dozen arbitrary combinations of power and block coefficient, you should be able to select the best. The work is relatively straightforward.
Where cargo is limited in availability, optimal ship size is not so easily found. You will usually be forced into some trial-and-error procedure, perhaps logically organized along the lines of Harvey Evans's well known design spiral. Aiming to propose a ship
Chapter 6: Constructing Your Analysis, Sophomore level

with some owner's specified deadweight and speed, your path should follow something like this sequence:

1) Based on data from previous similar ships, use the aimed-for deadweight to make a first-stab guess at the required displacement.
2) Based on that displacement and the specified speed, select appropriate values of length, beam and depth. (A maximum draft will normally be specified.)
3) Estimate the power required to move a ship of that displacement, length, and proportions at the specified speed.
4) Estimate hull weights based on dimensions, machinery weights on installed horsepower.
5) Add those weights to get light ship weight.
6) Subtract light ship weight from displacement to get deadweight.
7) Compare (contrast?) that derived deadweight with the specified value. If the derived value is too high or too low, make an adjustment to the displacement, which means going back to step 1.
8) Keep this up until the derived deadweight comes to within a percent or two of the specified value. It's usually best to err on the high side, of course. Life is full of surprises and very few of them are helpful.

This spiraling procedure is a lot of work, but well-conceived computer programs can overcome that problem.

The question of sea speed must be addressed next. If the ship is to enter the liner trades, then speed will be neither a continuous variable nor one that drops out at the end (such as it does in bulk carrier design). If the ship is to be integrated into an existing fleet, then speed will usually be fixed. If a new fleet is contemplated, then different combinations of speed and numbers of ships can provide the desired frequency of service. The economics of each combination will need to predicted. You should know that most liner operators like to offer easily-remembered sailings, such as every Friday or every other Friday, from a given port.

This brings up the matter of the economics of speed in the liner trades. Most liner operators belong to ocean conferences (cartels) that set freight rates, and these are fixed regardless of quality of service. Competition comes, then, in trying to offer the best service, including speed of delivery. Thus, high speed, although fundamentally uneconomic, may be highly profitable. As a ship designer, there is little that you can do to make a science of optimum speed under such conditions. You simply have to accept what the owner says.

6.5 Exploiting lighter weight components

In selecting components for a ship the question of weight saving advantage may arise. How this is handled will depend on whether the ship falls into the first or second categories discussed above. If cargo is unlimited, any saving in weight can be directly converted into an increase in annual transport capability. If cargo is limited, however, any saving in weight should be exploited by changing the design to produce a smaller ship that can carry the same amount of cargo. This will usually cause you to start into the design spiral again, with changes in power as well as hull dimensions. In either case, decisions can be made on the basis of selecting the component that does the most to enhance the ship's overall measure of merit.
6.6 Recapitulation

Endless paragraphs could be written about how to structure your analysis not only for merchant ships (as above) but for all the other kinds of vessels, large or small. You can arrive at logical approaches every time if you will keep the following thoughts always in mind:

a) Be sure you give each alternative a fair chance to succeed. Let each seek out its own best strategy.

b) Keep reminding yourself what the owner's real needs are and the circumstances under which the enterprise must operate.

c) In choosing between alternatives it is the differences that count. Concentrate on those differences in cash flow that will result from each decision.

d) Keep an open mind and do not impose unnecessary constraints on your design; yet do not feel obligated to investigate every conceivable variation, combination, permutation, and color scheme that comes along. Use common sense to lop some of them off.

e) Except in the final stages steer clear of complex financial and tax-minimizing schemes. They only tend to magnify the work by an order of magnitude without appreciable benefit. Why try to attain super-accuracy when you must of necessity guess at most of the numbers you put in? The proper place for such analytical curlicues is in the final stages, where you want to sell some business manager on the virtues of your proposal.

f) Computers and their programs are your servants and not your masters. They should be used with intelligent understanding of their built-in assumptions and shortcomings.

g) The final product of your study should not be a single best-of-all-possible solutions, but a limited menu of close-to-optimum choices. That will allow the decision-maker (usually a business manager) to temper the objective results of your economic analysis with his or her own subjective, intuitive judgement about intangible considerations. The wisest decision will always be a mixture of art and science.

h) If intangible considerations occur to you along the way, first make sure they really are intangible, i.e., irreducible to monetary terms. If that is the case you should at least mention them in your report. You might also add a few words about which way each factor will tend to push the decision.

i) And, finally, keep reminding yourself that, in all this, common sense is king and simplicity queen.

Exercises

1) List some attributes of a trade route that would favor the introduction of commercial sailing ships.

2) During the design stage of an iron ore ship a salesman shows up with a proposal to install aluminum alloy, instead of steel, hatch covers. His chief selling point is the saving in weight. How would you go about giving proper credit to that advantage?

3) How would you go about giving proper credit to that weight saving if the hatch covers were to go on a container ship in the liner trades?

Answers

1) a) It should not be in the liner trades because sailing ships cannot promise fixed schedules.
Chapter 6: Constructing Your Analysis, Junior level

b) It should involve long voyages so as to exploit the weight saving inherent in carrying little fuel.

c) Its trade route should fall entirely within the northern or southern hemisphere so as to avoid having to cross the calms found near the equator.

d) It should avoid harbors with relatively low bridges across their entrances.

e) It should serve ports with cargo handling gear that will not be unduly inhibited by the ship's rigging.

f) It should be in a trade operating in a north-south direction so that prevailing winds (usually from east or west) will seldom require beating upwind.

g) (You can probably think of more.)

2) In an iron ore carrier weight saving is important and the cargo is usually found in unlimited supply. The weight saved can be converted directly into increased cargo deadweight, hence increased annual transport capability and revenue.

3) Saving weight in hatch covers will not produce any more cargo to carry. The benefit can be exploited by holding the cargo deadweight while reducing the displacement. This means reducing the underwater dimensions or the block coefficient, or both. If those reductions are appreciable, a smaller engine may be appropriate.

Independent thinking

List the trade route characteristics that would favor the introduction of nuclear power in merchant ships.

Some years ago a MarAd-sponsored study showed that in a given passenger trade an advanced concept water vehicle could offer fares competitive with current air fares. When I asked the consultants what level of profitability they had assumed for their proposed vehicle, they replied, "We thought it would be presumptuous of us to dictate to an owner his level of profit, so we set it at zero." Discuss.

Junior level

"Let us work without theorizing," said Martin;
"it is the only way to make life endurable."
Voltaire (in Candide)

6.7 Charter parties

What do we mean by charter parties? The term derives from the Latin charta partita, literally, split document. This dates back to the old days when contracts between two people were written in duplicate and then torn or cut in two, with one half going to each person involved. The commonality of the parts could later be proven by matching the indentures, and that is the source of such terms as "indentured servant." In maritime matters charter parties apply to various formal arrangements (generally confined to complete ship loads of some commodity) by which a ship owner can provide such service to someone who is willing to pay. The level of service can take several different forms as outlined below.

In a bareboat charter the owner rents the ship out to an operator who runs the ship pretty much as though he or she owned it. The operator pays all expenses except the
original capital cost. The terms usually cover several years, with payment on an annual basis.

In a voyage charter (sometimes called a spot charter) the owner runs the ship and pays all the expenses, but places it at the service of the cargo owner, usually for only a single voyage. The cargo owner pays so much per ton delivered.

In a contract of affreightment the owner agrees to carry a given quantity of cargo from one port to another within a given period of time. Normally several voyages would be required. No specific ship need be named.

In a time charter the owner runs the ship but the cargo owner dictates the voyages. The owner pays those parts of the expenses that are more or less the same regardless of the voyage: crew costs, insurance, stores and supplies, and maintenance and repair. (The charter rate, of course, is high enough to cover all those plus the capital costs.) The cargo owner pays those costs that vary with the chosen voyage, notably fuel, port fees, and canal fees. The contract normally covers at least a full year, often far longer. The terms of payment are usually so many dollars per deadweight ton per month.

In most of these charter parties the economic incentives are pretty much the same as those where no charters are involved, which means that we can ignore them in this course of study. The single exception is the time charter, so we had better give it some special thought.

6.8 Time charters
First we have to clarify a couple of terms. In earlier chapters we talked about capital costs and operating costs. Now, in dealing with time charters, we have to divide operating costs into two components:

a) Those for which the user pays:
   * Fuel
   * Port Fees
   * Tug service
   * Canal fees

b) Those for which the owner pays:
   * Crew wages and benefits
   * Victuals
   * Hull & machinery insurance
   * Protection & indemnity insurance
   * Maintenance & repair
   * Stores & supplies

The first collection is called voyage costs. We shall abbreviate it here as $Y_v$. The second is sometimes called the operating costs. To avoid confusing the second term with what we have used before (comprising both categories), we shall call it here daily costs, and abbreviate it $Y_D$. Note that, despite the terminology, both are annual costs.

Now our equation for required freight rate becomes:

$$RFR = \frac{(CR)P + Y_v + Y_D}{C}$$

If the ship is being designed for long-term time charter hire, RFR has no real meaning, so we can use as our measure of merit a variant on RFR, namely the required charter rate, RCR:
RCR = \frac{(CR)P + Y_D}{11.5 \times DW}

where DW is the deadweight in tons, and the 11.5 figure is based on the number of months per year in which the ship will be available (leaving half a month for dry docking and repairs). RCR is given, then, in its usual terms of dollars per month per ton of deadweight.

If you want to find the net present value (NPV), the cash flow before tax (A) will be:

A = 11.5 \times r \times DW - Y_D

in which r is the charter rate in dollars per deadweight ton per month.

6.9 Charter versus buy

Integrated corporations, such as we find in the petroleum industry, frequently move their commodities under a variety of contractual arrangements. They may own some tankers, they may charter some on long-term bases, and when times are good they may use spot charters to round out their needs. Let us consider the relative economic advantages of owning versus long-term contracts of affreightment. If the oil company owns a ship it will require a first cost, P, and annual operating costs, Y:

If, on the other hand, the company chooses to retain the services of an independent tanker company (and let's say for an entire ship life), then there will be no initial investment required, but the annual charter costs will far exceed Y. Why? Because that independent operator will have not only the usual operating costs, but also annual costs of capital recovery. Let's abbreviate the annual charter rate as ACR:

The easiest way to compare these two alternatives is to examine the differences in cash flow. If we (representing the owner) choose to buy our own ship, we'll have the disadvantage of the invested cost, P, but that will be offset by the saving in annual costs, namely ACR - Y. Our cash flow pattern, then will look like this:
Given that pattern (and assuming for the moment that no taxes are to be paid), we can easily find NPV or yield and decide which alternative we prefer on that basis. This illustrates another important way to structure your analysis: examine the differences in cash flow resulting from the decision. Let's go on with that logic in the next section.

6.10 Analyzing differences

Suppose you have to choose between two alternatives both of which have equal annual incomes, (including the possibility that both are zero). The alternative with the higher first cost will have lower operating costs. The question then arises: if we choose the alternative with higher first cost, will that higher cost ($\Delta P$) be more than offset by the future savings in operating costs ($\Delta Y$)? If no taxes are involved, the cash flow pattern, and methods of analysis, will be exactly like that shown in the previous section. If taxes are involved, then the annual saving in operating cost will be reduced by the amount of the tax, but tempered by the increased depreciation allowance. The net gain in annual cash flow after tax ($\Delta A'$) will then be:

$$\Delta A' = \Delta Y(1 - t) + t \frac{\Delta P}{N}$$

Some economists apply the same concept to multiple-choice situations, such as optimization studies. To do this, they rank the alternatives in order of ascending first costs. Then they use NPV or yield (or whatever) to analyze the benefit of going from first to second alternative. If that meets their standard of profitability, they go on and look at the benefit of going from the second step to the third, and so forth until the incremental cash flow is no longer great enough to justify the incremental investment. There are settings where this approach is satisfactory; there are others where it is not. Its fundamental weakness shows up perhaps most clearly when NPV is the criterion. If you keep increasing the first cost, the NPV of the differences will be large at first and will diminish as you advance up the scale. When at last it shrinks to zero, and you select that point on the design scale, you will in effect settle on a design that promises minimum acceptable profitability. But that is not what NPV is all about. NPV aims to find the alternative that will exceed the minimum acceptable level of profitability by the greatest margin. What this means is that the incremental approach will almost always lead to overdesign. This may be acceptable in that rare circumstance where you have an unlimited supply of capital and no alternative investment opportunities. That might occur, for example with your electric company, which has government-imposed limits on profitability.

**Exercises**

1) Of the various kinds of charter parties, which is most closely akin to (a) riding in a taxi, (b) retaining the services of a chauffeured limousine for protracted vacation travel through Barataria (you pay for gas and turnpike fees), (c) leasing a car for a year.
2) An oil company can meet a projected long-term increase in transport needs by building a ship with a projected 20-year life or by signing a 20-year contract of affreightment with an independent operator. The construction cost of the ship, including owner's expenses would be $75 million. Its estimated operating costs would total $3.25 million per year. The same service could be provided by the independent operator with charter fees of $11 million per year. The oil company's tax rate is 35%. Find the net present value for the build alternative using a discount rate of 9.5%

**Answers**

1) (a): voyage charter.  
(b): time charter.  
(c): bareboat charter

2) The before-tax annual saving for the build alternative would be $11 million minus $3.25 million, or $7.75 million. The resulting difference in after-tax cash flow would be:

\[
A' = A(1 - t) + \frac{P}{N} = 7.75M(1 - 0.35) + 0.35 \frac{75M}{20} = 5.04M + 1.31M
\]

\[
A' = 6.35M
\]

NPV = A'(SPW-i'-N) - P

NPV = 6.35M (SPW-9.5%-20) - 75M = ($19.0M).

Conclusion: Opt for the charter party.

**Independent thinking**

Can you think why signing a 20-year charter may lower a tanker company's credit rating?

---

**Senior level**

*Proof of Trotsky's long-range foresight is seen in the fact that none of his predictions has yet come to pass.*  
(attributed to one of his admirers)

**6.11 Planning horizons**

You may find yourself asked to analyze the economics of complex systems that incorporate a variety of facilities each with a different economic life. An example is a container transport system involving terminals and their cranes as well as ships and containers. Life expectancies may vary from 10 years for the containers to 20 for the ships to 25 for the cranes to 50 for the buildings to infinity for the real estate. To find the complete system's NPV, for example, how far into the future should you look? Most economists will simply select an arbitrary cut-off date at some moderately far-future time -- perhaps 20 or 25 years. Remember that those future cash flows are going to be severely discounted, so gross oversights need be of little concern. Are you troubled by the thought
of dropping the curtain 25 years from now on a replacement ship that will then be only 5 years old? If so, introduce that ship's potential resale value at that time, as explained in the next section.

6.12 Residual values
At any point during the life of a ship it has a residual or disposal value. For example, how much insurance coverage do you need on an old ship? Enough to buy an equivalent new ship? The original cost of the old ship? Neither of those. What value have you lost? You have lost the present worth of the projected after-tax cash flows over the presumed remaining years of life. To protect your investment, that is the logical amount of coverage.

The same kind of thinking applies to what you should ask if someone offers to buy your ship, or if you want to buy someone else's.

6.13 Uncertainty
Economic studies are built on a foundation of guesses about future costs, incomes, and operating conditions. Nearly every element of the analysis may prove wrong in actual fact. This does not mean that we should throw up our hands in resignation. There are rational ways to minimize the dangers inherent in the unknown future, and we shall mention two of the more commonly-used ones here.

By way of preface I may add that successful business managers cannot always explain how they go about making decisions. There is clearly room for intuition in this, but the logical methods explained below should also have a place.

Perhaps the most justifiable approach to an uncertain future is called expectation. In this method you consider the economic results expected from each alternative under each of several possible future conditions. Then you take as your measure of merit the weighted average of each outcome. Here is an example. Suppose you are asked to select between two kinds of power plants. One will be more profitable than the other only if fuel prices continue to rise. Your estimates of the relative probability of future fuel prices and corresponding yields are shown in the table below.

### Predicted yields

<table>
<thead>
<tr>
<th></th>
<th>Future fuel prices (in CV$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Remain fixed</td>
</tr>
<tr>
<td>Probability</td>
<td>0.25</td>
</tr>
<tr>
<td>Power plant A</td>
<td>19%</td>
</tr>
<tr>
<td>Power plant B</td>
<td>24%</td>
</tr>
</tbody>
</table>

The expectation for A is found as follows:

\[
E \text{ for } A = 0.25 \times 0.19 + 0.60 \times 0.15 + 0.15 \times 0.13 = 0.157
\]

The expectation for B is:

\[
E \text{ for } B = 0.25 \times 0.24 + 0.60 \times 0.14 + 0.15 \times 0.12 = 0.162
\]

Based on expectation, we should favor power plant B.
Chapter 6: Constructing Your Analysis, Senior level

Another logical approach is called \textit{maxi-min} (short for the maximum of the minimum outcomes). This is the criterion favored by the pessimist (an individual who usually turns out to be right, but who gets no pleasure out of it). He knows that whatever the decision, it will anger the gods and the worst will always happen. He therefore looks at all the worst outcomes and selects the alternative whose worst outcome is the least bad. Based on that logic, the pessimist would select power plant A. Risk aversion is an important factor in managerial decisions.

There are other logical approaches to handling uncertainty, which you can look up in standard books on management. The two explained above should be good enough for most situations, however.

If your economic study considers large numbers of alternatives, you would normally start out using only single most likely values of each parameter (this is called the \textit{deterministic approach}). The more elaborate procedures mentioned above would be applied only to the final few contenders. This is simply a matter of keeping the computational load within reason.

6.14 Closure

Here is one final philosophical thought to keep in mind when structuring your analysis: always remember that your chief aim should be to design a ship that will satisfy some customer. To do this, put yourself in the customer's shoes. Take, for example, a customer who ships goods in containers. Don't start by visualizing a ship that may happen to have room for containers. Start by visualizing a flow of $X$ containers per week from Port A to Port B, and then start thinking about the ship you might wrap around them. Function before hardware.

Exercises

1) Rework the numbers in the table of Section 6.13 under the assumption that all future fuel cost alternatives are equally likely. Which plant would be favored by expectation and which by maxi-min?

Answers

1) The expectation of power plant A is 15.7\%, while that of B is 16.7\%, so B would still be favored. The maxi-min remains in favor of A. Although the change in probability of the outcomes might change the relative expectations, it will not affect the maxi-min.

\textbf{Independent thinking}

In our discussion of uncertainty we used changing fuel costs as an example of a factor that is hard to predict and whose actual value would appreciably influence the outcome of any economic projection. What are some other factors that are subject to major changes and also highly influential on outcomes?
Chapter 7
Building Costs

For which of you, intending to build a tower, sitteth not down first, and counteth the cost, whether he have sufficient to finish it?
Luke 14: 28

Sophomore level

7.0 Perspective

Engineering economic studies almost always involve an estimate of invested costs. Indeed, the first cost of a project is usually the single largest (hence most important) factor entering into the study. Although shipbuilding costs may be estimated for several different reasons, we shall concentrate in this first section on only one: to help make rational decisions in preliminary design.

First, an important disclaimer: this is not a cook book that you can use to predict costs. It is, rather, an explanation of how you can structure a procedure for estimating the costs of alternative design concepts. You will need to complement what is explained here with appropriate real-life data dredged up from various sources. I shall cite a few useful publications at the end of the chapter, but you must realize that even the best of them go quickly out of date. Discouragingly little shipbuilding cost information is available in the literature, so naval architects (and especially students) must develop ingenious ways to discover the necessary factual information. (I know that's not much help, but it's the best I can do.)

7.1 What's important

In preliminary ship design we normally want to predict the economics of large numbers of alternative designs. This means that our estimating methods should be relatively simple. The alternatives under consideration usually exist only as imaginary concepts about which few details have been established. This, too, suggests that our techniques must be relatively simple. Moreover, except in rare cases, we need not worry about exact costs; relative costs are what matter. This suggests that our estimating methods should strive to emphasize differences in costs between the various alternatives. Absolutely accurate costs are not necessary.

7.2 Two common bases

Most cost estimating techniques boil down to questions of costs related to some understandable characteristic of the subject under study, whether it be a ship or not. These characteristics fall into two major categories: functional capability such as deadweight and speed, or technical characteristics such as major dimensions and power. The second family of techniques is usually better suited to our purposes and it is on them that we shall concentrate most of our attention. But, to start, we shall take a brief look at the first group.

7.3 Functional capability as a costing basis

Among ship owners, a popular estimating rule of thumb is to talk about shipbuilding costs in terms of so many dollars per ton of deadweight. This answers two questions of paramount importance to the prospective owner of a merchant ship: how much can it carry and how much must I pay? It may take forms such as these:

\[ P = C_1 \text{ (DW)} \]

or

\[ P = C_2 \text{ (DW)}^B \]
where

\[ P = \text{shipbuilding cost} \]
\[ DW = \text{deadweight tons} \]
\[ B \text{ is an exponent somewhat less than unity} \]
\[ C_1 \text{ and } C_2 \text{ are coefficients derived from known data on similar ships.} \]

Needless to say, such methods will be highly unreliable unless confined to ships closely akin to those that served as sources of data. They lack the versatility that we need for most preliminary design studies, so we shall move on.

7.4 Technical characteristics as a costing basis

Perhaps the simplest technical characteristic that we might use as a basis for estimating cost is the light ship weight \((W_E)\). That, after all, is the single most basic measure of what the owner buys. Some years ago, when working with some aeronautical engineers, we concluded that the cost of almost any kind of vehicle could be approximated by means of the simple expression:

\[ P = C(W_E)^{0.87} \]

In the above, the value of \(C\) was some 24 times higher for a 400-knot aircraft than for a simple cargo ship.

Again, such a simple approach has its limitations, but can be useful in situations where returned costs are rare, such as in newly developing kinds of vehicles.

When shipyard cost estimators prepare a bid for a proposed ship, they, too, look at unit costs based on technical characteristics. But now, rather than basing their work on a single characteristic, they look at one part of the ship at a time and try to predict both material and labor costs for building each part. Typically, they may make individual estimates for about 200 physical components of the finished ship: shell plating, framing, decks, bulkheads, and so forth, through gauges and sight glasses in the engine room. Most of their unit costs are based on weights, which can be fairly accurately predicted during the bidding phase. In preliminary design work, however, not enough is known about the ship to go into such detail. Some simplification is needed. Some examples are given below.

In the early design stages, before any drawings have been prepared, the alternative designs are in the form of concepts about which nothing is known beyond perhaps the principal dimensions and power. The ship can be broken down into two parts: hull and machinery. Hull costs can be based on the cubic number \((CN)\) and machinery costs on power \((BHP)\). This might lead to this expression for first cost \((P)\):

\[ P = C_1 (CN)^E + C_2 (BHP)^F \]

where

\[ CN = \frac{L \times B \times D}{100} \]
\[ C_1 \text{ and } C_2 \text{ are derived from previous similar ships} \]
\[ E \text{ is an exponent that may vary from 0.8 to 0.9} \]
\[ F \text{ is an exponent that varies from 0.5 to 0.8} \]
Again, such simple methods become wildly inaccurate unless narrowly confined. We can increase our confidence in our predictions if we employ techniques that are considerably more accurate and yet require no more knowledge about the alternative ships than what is implied above: main dimensions, power, and perhaps block coefficient. Now we want to break the ship down into three major parts; hull, outfitting plus hull engineering, and machinery. We can also divide expenses between material, labor, and overhead (to be explained shortly). Normally, we try to establish material and labor costs for each of the three major components, but take overhead as a single, overall cost.

Our first step is to estimate the hull component weights based on the cubic number. Those are used to predict material and labor costs for hull and outfit plus hull engineering. Machinery material and labor costs may be based directly on BHP.

Table 7.1, taken from a paper I published in 1985, is a typical example of a cost estimate based on the sort of technique described just above. Its degree of elaboration is sufficient to give reasonably accurate estimates, and yet simple enough to allow you to analyze hundreds of alternative designs (assuming access to computer).

<table>
<thead>
<tr>
<th>Table 7.1: Simple Cost Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>COST ESTIMATES</strong></td>
</tr>
<tr>
<td>Ship component</td>
</tr>
<tr>
<td>Structural hull</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Outfitting and hull engineering</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Machinery</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
</tr>
</tbody>
</table>

Notes:

- $W_s$ = weight of structural steel (net) = 3900 tonnes;
- $W_0$ = weight of outfitting and hull engineering = 1800 tonnes;
- BHP = maximum continuous rating brake horsepower = 12 800;
- Hourly labor rate = $10; and Overhead cost = 85% of labor cost.
- M = million
Table 7.2: Summary of Costs

<table>
<thead>
<tr>
<th></th>
<th>millions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>$15.26</td>
</tr>
<tr>
<td>Labor at $10 per hour</td>
<td>$4.57</td>
</tr>
<tr>
<td>Overhead at 85%</td>
<td>$3.88</td>
</tr>
<tr>
<td>SUB TOTAL</td>
<td>$23.71</td>
</tr>
<tr>
<td>Profit at 10% (arbitrary)</td>
<td>$2.37</td>
</tr>
<tr>
<td>Appended costs†</td>
<td>$0.50</td>
</tr>
<tr>
<td>Shipyard bill</td>
<td>$26.58 (say $26.6 M)</td>
</tr>
</tbody>
</table>

† Appended costs include classification society fees and similar costs that the shipyard normally passes on to the owner without mark-up for profit. They also include tug and drydock charges based upon standard rates that already include profit. The figure used here is arbitrary and might well be omitted in preliminary design studies.

What is meant by overhead? This division comprises all costs necessary to running the shipyard, but which cannot be associated with any particular ship under construction. Examples include salaries for executive officers, cost estimators, and watchmen. Bills for electricity, real estate taxes, income taxes, and depreciation are also included. There are lots more, but that should give you the idea.

Something else to note is that what we usually call material costs should more accurately be called costs for outside goods and services. Many shipyards, for example, use subcontractors to do the joiner work or, perhaps, the deck covering. Consulting service bills would come in this category, too.

7.5 Owner's costs

The total invested cost of a ship is more than the shipyard bill. The ship owner has some appreciable costs of his own that would never arise had the ship not been built. Peter Swift cited these figures for a large merchant ship built in 1978:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Spare parts</td>
<td>$600 000</td>
</tr>
<tr>
<td>Owner-furnished materials</td>
<td>$250 000</td>
</tr>
<tr>
<td>Plan approval</td>
<td>$1 000 000</td>
</tr>
<tr>
<td>Owner's supervision</td>
<td>$1 500 000</td>
</tr>
<tr>
<td>Administration &amp; legal fees</td>
<td>$400 000</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$3 750 000</strong></td>
</tr>
</tbody>
</table>

Exercises

1) Rework the cost estimate of Table 7.1 assuming a steel weight 20 percent higher than that shown, outfitting and hull engineering weight 30 percent higher, and BHP 50 percent higher.
2) If the light ship weight is 6600 tons and the first cost is $27 million, derive the value of C used in the equation for cost based on light ship. (See page 92.)

**Answers**

1) $33.16 million, rounded to $33.2 million.

2) $12 834, rounded to $12 800.

**Independent thinking**

The equation for structural hull man-hours shows the weight of steel raised to an exponent less than one. Explain the logic.

---

**Junior level**

*Knowledge is one. Its division into subjects is a concession to human weakness.*

Sir Halford John Mackinder

### 7.6 Shipyards bidding

In the first part of this chapter we considered some simple cost estimating methods suitable for preliminary design. Absolute values there were not necessary, the aim being largely to spotlight the differences in costs between competing alternatives. When shipyard cost estimators get down to serious business, however, their aim is to predict with all possible accuracy the cost of building one specific ship. (Invitations to bid may ask for alternative bids on one or more sister ships, or perhaps on some major technical variation, but the estimator’s central aim is to predict the cost of that first basic ship.)

As mentioned in Section 7.4, most of their work consists of analyzing one at a time each of around two hundred individual components of the proposed ship. They have a lot more information on hand than was the case in preliminary design: a book of specifications and one or two dozen preliminary drawings such as the midship section, general arrangements, machinery arrangements, and major piping systems. Bidding is usually a team effort, with several estimators working up the man-hour requirements, purchasing department personnel soliciting material costs, and managers predicting labor rates, overhead costs, and attainable profits. Part of the managerial skill is directed toward trying to guess what competing shipyards are going to bid. The aim is to win the contact with a bid that is only slightly below the second lowest. The time for estimating on a big contract is generally measured in months.

Shipyards have mountains of labor cost data acquired during the construction of ships in the past. They obtain such data by keeping careful records of hours actually expended on each of those couple of hundred components. Then, when time is available, they relate the recorded hours to some key physical characteristic of the component. In the case of structural hull components, for example, they would derive man-hours per ton; or, for deck covering: man-hours per square foot. Labor costs for installing major items of equipment, such as a steering engine, might be taken simply as so many hours per unit.

Another step in refinement comes, for example, in recognizing that man-hours per ton of shell plating will be greater in the curving ends of the ship than along mid-length.
Chapter 7: Building Costs, Junior level

Estimators may further divide predicted hours between direct and indirect labor. Direct labor is that that is clearly identified with some specific part of the completed ship: shell plating, vent ducts, anchor windlass, or whatever. Indirect costs are general costs that can be associated with a given shipbuilding contract, but not with any specific part of the ship. Examples include launching, cleaning, temporary lights, and staging. Both material and labor are included. Indirect costs are not the same as overhead. The former are clearly identified with a given ship, the latter are not.

7.7 Duplicate cost savings

Some prospective ship owners ask shipyards to quote costs for building alternative numbers of identical ships. Such bidding is usually in the form of cost for one ship, or each of two, each of three, and so forth. Experience shows that unit costs go down as the number of identical units go up. Why should this be? There are two categories of reasons. The first is the matter of non-recurring costs. These are costs required to build the first ship but which need not be repeated for follow-on ships. Examples are engineering, plan approval, and preparation of numerical controls for fabrication. The second category consists primarily of labor learning, the increased efficiency workers acquire through repetitive work (and learning from their own mistakes). There are also savings in material costs because suppliers, too, may experience savings. These two factors are shown in Figure 7.1.

Labor learning tends to be most pronounced in building complex ships (Figure 7.2) or when the work force is inexperienced (Figure 7.3).

The overall effect of non-recurring costs and labor learning usually results in cumulative average costs that decrease in a log-linear fashion as shown in Figure 7.4. Be careful, these are costs for each of so many units, not the cost of each additional unit. The general equation for the cumulative average cost (abbreviated \( \bar{Y} \)) is then:

\[
\bar{Y} = \frac{a}{N^x}
\]

where

- \( a \) = cost of the first unit
- \( N \) = number of identical units

The exponent \( x \) will vary with the complexity of the ship and workers' experience. Unfortunately, I can cite no reliable numbers. A good many years ago we concluded that a value of about 0.10 was appropriate for cargo ships built in American shipyards. We also concluded that the savings would be less pronounced in shipyards overseas. That was expected because in foreign yards labor costs (where the big savings arise) formed a much smaller part of the total costs.

It is worth noting that with log-linear savings, the relative drop in cost remains the same every time you double the quantity. For example, if each of two ships costs 90 percent of the cost of one ship, then the cost for each of four ships would be 90 percent of the cost for each of two.

Exercises

1) Which of the following costs would you assign to overhead and which to miscellaneous?
   (a) Repairing a typewriter for the president's secretary
(b) Fuel for the shipyard's power house  
(c) Platform for the christening ceremony  
(d) Stationery and postage  
(e) Salary for the chief accountant  
(f) Sea trials

2) Estimators predict the cost of building one ship will be $75 million. A second ship would cost $65 million. What would be the cumulative average cost for two? For four?

3) Derive the value of $x$ in the $\bar{Y}$ equation for the cost pattern outlined above.

4) Based on the above, what would be the cumulative average cost of three ships?

**Answers**

1) a, b, d, and e would probably be charged to overhead, c and f to miscellaneous.

2) Cost for each of two = $70 million  
   Cost for each of four = $65.3 million

3) About 0.10

4) $\bar{Y} = \frac{\$75M}{3^{0.10}} = $67.2 million

**Independent thinking**

Overhead costs as a percentage of labor costs tend to be high in well-managed yards that invest in labor-saving devices. Explain why this is so.

In buying hydraulically operated folding stern ramps for new ships, the purchasing department finds duplicate cost savings that are considerably more pronounced (in relative terms) than when buying two-horsepower electric motors. Explain why this is so.
Fig. 7.1: Duplicate cost savings

Fig. 7.2: Labor learning vs complexity (plotted on log paper)

Fig. 7.3: Labor learning vs experience (Plotted on log paper)

Fig. 7.4: Cumulative average costs (plotted on log paper)

Fig. 7.5: Overhead-to-labor ratio as a function of yard activity
Senior level

*It is a great advantage for a system of philosophy to be substantially true.*

George Santayana

7.8 Estimating overhead

You, as a naval architect, will seldom be called upon to delve into detailed estimates of overhead costs to be assigned to a ship under bid. You should, nevertheless, have some understanding of the difficulties involved. That is what this section is all about.

To begin with, there are two basic kinds of overhead: (a) those that remain much the same regardless of how busy the yard may be: fixed overhead, and (b) those that vary with the level of activity within the yard: variable overhead. Figure 7.5 shows how total overhead (shown as Σ O.H.) is made up of those two components. The horizontal scale is a measure of the yard’s relative degree of busyness. At 100 percent on the scale the yard is operating full blast 24 hours per day, seven days per week, and every machine, every crane, every work station is fully occupied. At zero on the scale no productive work is underway, but the fixed overhead costs must nevertheless be paid. Also shown in the figure is an upwardly sloping line representing labor costs. If we follow the usual custom of taking overhead cost as a percentage of labor costs, we should arrive at the downward curving line that you see.

All this leads up to the rather obvious fact that the person who must predict overhead on a hoped-for contract must first gaze into the crystal ball and predict what other contracts will be on hand during that future period when the ship may be built.

7.9 Contingencies

Recognizing the impossibility of predicting future costs, careful estimators try to minimize risks by the technique of starting with optimistic estimates, which assume that all will go right during construction, no mistakes will be made, the weather will be good, all suppliers will come through on schedule, and that labor relations will be friendly. Then, to that result they add a margin for contingencies. By definition, this is a mark-up in the estimate to give equal probabilities to overruns and underruns in the actual costs. Allan Jenks, an estimator with Exxon, cites these typical contingency factors:

In early studies: 20 to 25 percent, with a 33.33 percent probability of a 10 percent overrun

When the design basis is frozen: 10 to 15 percent, with a 20 percent probability of a 10 percent overrun

When design specifications are complete: 10 percent, with a 10 percent probability of a 10 percent overrun.

7.10 Extrinsic factors

Altogether aside from a proposed ship’s technical characteristics, there is a family of influential factors that will have an impact on cost. Perhaps your yard has recently made improvements in productivity (better supervisors, incentive systems, better equipment), government regulations may change, some classification societies are more stringent than others.
Chapter 7: Building Costs, Senior level

Some external factors result in differing costs between different yards. Some yards may have considerable experience in building certain kinds of ships, some are limited in the size of ship they can build, some may be frozen in during winters and unable to deliver when needed, and so forth.

Also aside from the proposed ship's technical characteristics, before any bid is finally turned in, the yard managers should weigh the potential risks implied by the terms of the proposed contract. These may include liquidated damages, bonuses or penalties, escalation, payment schedules, and currency exchange rates.

7.11 Unconventional methods

There is, rightfully, widespread dissatisfaction with the estimating methods I have tried to explain in this chapter. The traditional division of costs into shell, framing, decks, bulkheads, and so forth is altogether too crude and leaves more to art than to science in the estimate. Some shipyards have tried breaking structural hull costs down according to the fore-and-aft location: forepeak, after peak, engine room, cargo hold within parallel mid length, cargo holds elsewhere, and erections. A somewhat related approach keeps track of costs (both structure and outfitting) in terms of the major three-dimensional assemblies in which they occur.

Every good cost estimator develops his own variations on standard methods in order to account for non-standard influences on cost. What is needed is a universal approach that will segregate out all the important influences on costs: parts of the ship, location within the ship, direct vs indirect costs, different kinds of material, work done on ship vs work done on the ground, recurring, vs non-recurring costs, fixed vs variable overhead, according to function (cargo, propulsion, habitability, etc.) and labor broken down by department. With all the modern managerial tools now available, such a level of sophistication should be within reach. Perhaps you will be the first to succeed in developing such a method and seeing it adopted.

7.12 The curse of little black books

There is throughout the marine industry a cultural benightedness that inhibits progress in cost estimating. I refer to the tendency of shipyard managers to suppress the publication of cost estimating data, and their estimators' resulting habit of keeping all essential information carefully hidden away in their little black books, to be locked in their desks overnight. (Some estimators, it is rumored, take their books home and sleep with them under their pillows.) The free exchange of other kinds of technical information evident in maritime journals and transactions is in sharp contrast to what is published with dollar signs attached. Many other branches of American industry freely publish cost information. (See, for example, the work of the American Association of Cost Engineers.) Until the marine industry learns to do the same, cost estimating will remain something of a black art.

Independent thinking

A house-building contractor is asked to quote a firm price for building a certain house. He is given a complete set of plans and specifications. The site has been selected. In arriving at a price, what are some important factors that must be considered but which are not indicated in the plans or specifications? What are some unpredictable factors that may call for some contingency margin?
Useful references


University of Michigan Department of Naval Architecture & Marine Engineering Reports


Benford, Harry, Shipbuilding Cost Estimates, Videotape lectures V2 and V3.
Chapter 8
Operating Costs

The whole is more than the sum of the parts.
Aristotle

Sophomore level

8.0 Perspective

My aim in this chapter is to give you a basic understanding of the various components that go to make up the annual costs of operating a ship (including both voyage costs and daily costs). Unfortunately, there is no practical way for me to give you a tidy handbook of actual quantitative values. Toward the end of the chapter, however, I explain some techniques I have used in the past to worm such information out of fleet managers.

My breakdown of costs represents standard accounting practice in the U. S. marine industry. Perhaps the first thing you should know about these accounting practices is that they can be misleading. As an example, the maintenance and repair category includes only money paid to outside entities (usually repair yards). Maintenance or repairs carried out by the ship's crew are charged to wages; and materials used are charged to stores and supplies.

8.1 Schedule analysis

One of your first steps should be to project the times involved in a typical round trip voyage (sometimes called a proforma voyage). You can start this imaginary voyage at any point, perhaps just after loading and ready to leave port, and then mentally follow the ship until it has completed one round trip and is once more ready to leave that same initial port and start another round trip.

Typically, such a proforma voyage would include, in sequence, estimated times for proceeding down a river, through a harbor, and out in the open sea, perhaps some time in passing through a canal, then more time in the open sea, followed by time in speed-restricted waters of a harbor, time to unload cargo, time to shift to another pier, time to load cargo, and then perhaps a mirror image of all of the foregoing sequence until a complete round trip is included and the ship is once more loaded at the first port and ready to leave. Factored into this must be some reasonable allowances for port and canal queuing delays and speed losses in fog or heavy weather. Time may also be lost in taking on bunkers or pumping out holding tanks.

The total time for the proforma voyage, when divided into the estimated operating days per year (typically 350), will give you the total number of round trips per year, which need not be a whole number.

8.2 Other applications of the voyage analysis

These scheduling calculations serve other purposes as well. In bulk ships where deadweight is critical, they are used to establish the weight of fuel that must be aboard when the ship reaches that point in its voyage where draft is most limited. (We'll discuss fuel needs in the next section.) In this phase of your work, give thought to the relative benefits of taking on bunkers for a round trip versus only enough for one leg. And don't forget some prudent margin (often 20 or 25 percent) for bad weather or other kinds of delays.
Chapter 8: Operating Costs, Junior level

The days per round trip estimate can also be used to establish the weight of other non-payload parts of total deadweight that are a function of days away from port; fresh water, victuals, and supplies. Finally, all this may lead to that critical number: the annual cargo (or passenger) transport capacity. Your estimate of actual annual transport achievement will of course be tempered by some realistic assumptions as to probable amounts available to be carried on each leg of the voyage. In the bulk trades that might amount to 100 percent use one way, and return in ballast. In the liner trades one might typically assume 85 percent full outbound, 45 percent inbound.

In more advanced studies you may need to make adjustments for minimum allowable freeboard changes brought on by geographic or seasonal requirements. Ice operations may also be a factor.

Exercise

1) Create an imaginary round trip voyage that involves loading bulk cargo at some river port, proceeding at restricted speed to the open sea, proceeding at cruising speed to a canal, passing through it at restricted speed (with time for one or more locks), then on to the unloading port, which is also some ways up a river. Assign arbitrary distances. Use your own intuitive judgement to select reasonable speeds and times for cargo handling and lockage. Derive resulting time (without margins) for a round trip.

Independent thinking

A ship owner tells you that company policy calls for a fuel margin of 20 percent. Although fuel is available at both ends of the voyage, the decision is made to take on enough bunkers for a round trip at the port where fuel costs are less. Should the margin be 20 percent of the bunkers required for the round trip?

The pursuit of an idea is as exciting as the pursuit of a whale.

Henry Norris Russell

Junior level

8.3 Voyage costs

As you may recall from Chapter 6, the voyage costs are those that are influenced primarily by the particular voyage in which the ship is engaged.

The biggest such expense is usually that for fuel. Now we should return to our proforma voyage and make a "steaming" profile: a table showing for each segment the hours consumed, the horsepower required, the fuel rate per horsepower-hour (which is usually higher at reduced powers) and the resulting amount of fuel required. Adding all those bits will yield the total fuel required for a single round trip. Multiply that by the round trips per year and you'll come up with your figure for the annual main engine fuel requirement. Multiply that number by the unit cost of the stuff and you'll have your estimated annual main engine fuel bill.

Next, repeat the steaming profile exercise to come up with the annual costs for generator fuel. This step should be kept separate from the main engine estimate because the
Chapter 8: Operating Costs, Junior level

amounts required follow different patterns and perhaps, being a higher quality fuel, may have a higher unit price.

Table 8.1 gives some convenient conversion factors for use in figuring fuel costs.

The other components of voyage costs (port and canal fees, tug service, pilotage fees) vary widely and are hard to generalize. Some port costs are on a per-use basis, others are on a per-day basis. Pier charges may be based on ship length. Pilotage may be based on draft. If I had to relate these cumulative costs to a single parameter, I should choose cubic number.

Another important cost is that of cargo handling, which may or may not be included in the contract depending on the trade. If it is to be included it would logically be treated as a voyage cost. Associated with this may be brokerage fees and cargo damage claims, hold cleaning, dunnage, rain tents, and other miscellaneous cargo-related expenses. In some studies cargo handling costs will be the same for all alternatives, in which case you can be cavalier in their treatment.

A hidden cost ignored by accountants is the inventory cost of the goods in transit. If you are going to apply the economic cost of transport criterion, you will need to calculate that cost, as explained in Section 5.13.

8.4 Daily costs

Our other major family of operating costs comprise those that continue more or less year-round regardless of the voyage. Principal among these, usually, is that of crew wages and benefits. There was a time when we began our estimate of these costs by predicting the crew complement, with deck crew proportional to ship size (perhaps cubic number), engine crew proportional to horsepower and number of screws, and steward's crew proportional to the sum of those other two. Now, however, with rational schemes for reducing personnel, crew complements are nearly independent of ship size and power. Numbers now usually vary between one and two dozen, depending on union agreements and owner's willingness to invest in automated equipment, more reliable components, and minimum-maintenance equipment (better coatings, for example).

In addition to direct daily wages you must recognize many benefits paid to seafarers. In some instances there may be crew rotation schemes so that crew members are on year-round salary, with vacation times that may amount to as much as a day ashore for every day aboard. There are sick benefits, payroll taxes, and repatriation costs (travel between home and ship when rotating on or off). These are major increments that must not be overlooked.

For general studies (not specific to any owner) you may want to set up a wage and benefit equation that recognizes that total costs are not directly proportional to numbers because automation and other crew-reduction factors tend to eliminate people at the lower end of the pay scale. The general equation may take this form:

\[ \text{Annual cost of wages, benefits, etc.} = f_1 (N_C)^{0.8} + f_2 N_C \]

where

\[ N_C = \text{number in crew} \]
\[ f_1 \text{ and } f_2 = \text{coefficients that vary with time, flag, and labor contract.} \]
Table 8.1

BUNKER C FUEL OIL CONVERSION FACTORS

Basic Relationships

One long ton of fuel oil occupies, on the average, 6.63 barrels or 278.46 gallons.
One barrel = 42 gallons = 5.615 cubic feet.
One pound of oil is assumed to contain 18,500 BTU.

Cost Conversions:

\[
\text{\$/ton} = 6.63 \left(\text{\$/bbl}\right) = 2.7846 \left(\text{\$/gal}\right)
\]

\[
\text{\$/bbl} = \left(\frac{\text{\$/ton}}{6.63}\right) = 0.42 \left(\text{\$/gal}\right)
\]

\[
\text{\$/gal} = \left(\frac{\text{\$/ton}}{2.7846}\right) = \left(\frac{\text{\$/bbl}}{0.42}\right)
\]

Other Conversions:

To convert A to B multiply A by factor shown below

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Pounds per SHP-Hr</th>
<th>Tons/Day</th>
<th>Tons/Mile</th>
<th>Bbl/Day</th>
<th>Bbl/Mile</th>
<th>Gal/Day</th>
<th>Gal/Mile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pounds per SHP-Hr</td>
<td>1</td>
<td>SHP 93.33</td>
<td>SHP 2240VX</td>
<td>SHP 14.08</td>
<td>SHP 337.9VX</td>
<td>2.98SHP</td>
<td>SHP 8.04VX</td>
<td></td>
</tr>
<tr>
<td>Tons/Day</td>
<td>93.33</td>
<td>SHP</td>
<td>(\frac{1}{24V_X})</td>
<td>6.63</td>
<td>(\frac{1}{362V_X})</td>
<td>278.5</td>
<td>(\frac{11.6}{V_X})</td>
<td></td>
</tr>
<tr>
<td>Tons/Mile</td>
<td>2240V_X</td>
<td>24V_X</td>
<td>1</td>
<td>159.1V_X</td>
<td>6.63</td>
<td>6683V_X</td>
<td>278.5</td>
<td></td>
</tr>
<tr>
<td>Bbl/Day</td>
<td>14.08</td>
<td>(\frac{1}{6.63})</td>
<td>(\frac{1}{159.1V_X})</td>
<td>1</td>
<td>(\frac{1}{24V_X})</td>
<td>42</td>
<td>(\frac{1.75}{V_X})</td>
<td></td>
</tr>
<tr>
<td>Bbl/Mile</td>
<td>(\frac{337.9V_X}{SHP})</td>
<td>3.62 V_X</td>
<td>(\frac{1}{6.63})</td>
<td>24 V_X</td>
<td>1</td>
<td>1008V_X</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>Gal/Day</td>
<td>1.298 SHP</td>
<td>(\frac{1}{278.5})</td>
<td>(\frac{1}{6683V_X})</td>
<td>(\frac{1}{42})</td>
<td>(\frac{1}{1008V_X})</td>
<td>1</td>
<td>(\frac{1}{24V_X})</td>
<td></td>
</tr>
<tr>
<td>Gal/Mile</td>
<td>(\frac{8.04V_X}{SHP})</td>
<td>(\frac{V_X}{11.6})</td>
<td>(\frac{1}{278.5})</td>
<td>(\frac{V_X}{1.75})</td>
<td>(\frac{1}{42})</td>
<td>24V_X</td>
<td>1</td>
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The cost of victuals (pronounced "vittles") is a function of numbers of people aboard and operating days per year. Compared to wages, these costs are modest. Owners are wise to resist trimming costs in this category. Tasty food in generous portions is an important factor in crew morale.

The annual cost of hull and machinery insurance is usually based on the ship's initial cost. A typical figure might be one percent of the first cost. First cost is a rather illogical basis for fixing insurance premiums, but the marine insurance business is fraught with such irrational practices.

Protection and indemnity insurance (protecting the owner against law suits) may add an annual cost of about one half a percent of the first cost. The two kinds of insurance costs are frequently lumped. Their annual cost, then, may be estimated as 1.5 percent of the first cost.

Annual costs for maintenance and repair can be estimated in two parts. Hull M&R will be roughly proportional to the cubic number raised to the two-thirds power. Machinery M&R will be roughly proportional to the horsepower also raised to the two-thirds power. A refinement on this approach is embodied in the following approximation:

$$\text{Annual cost of M&R} = f_3(LBD)^{0.685} + f_4\text{MCR} + f_5(\text{MCR})^{0.6} + K_1$$

where

- L, B, and D are length BP, beam and depth, respectively
- MCR = main engine's maximum continuous rating
- $f_3$, $f_4$, and $f_5$ = coefficients that vary with kind of ship, owner's policies, and so forth
- $K_1$ = a fixed amount regardless of hull size and engine power.

The annual cost of stores and supplies would consist of three parts. The first would be proportional to the ship's size (mooring lines for example). The second would be proportional to the horsepower (machinery replacement parts, for example), the third would be proportional to the number of crew members aboard (paint and cleaning compound, for examples).

A final daily cost category covers overhead and miscellaneous expenses. This would have to absorb a prorated share of the costs associated with maintaining one or more offices ashore. Shore staffs may number anywhere from what can be counted on one hand to bureaucracies bordering on civil service throngs.

8.5 General commentary

I mentioned earlier that the constrictions of accounting practices can be misleading and that true costs of maintenance and repairs may be considerably higher than shown in the books. Similarly, the division between voyage costs and daily costs, as defined by time charters, may also be misleading. Clearly, a voyage involving frequent round trips and lockages will increase repair costs, yet M&R is treated as a daily cost. Another example is the not inconsiderable cost of lubricating oil. That will surely be influenced by the hours of full-power operation (a function of voyages selected) and yet it is by tradition entered under stores and supplies, a daily expense. As a well-known bumboat woman once remarked, "Things are seldom what they seem."
Chapter 8: Operating Costs, Senior level

Finally, in adding up all those operating costs, it may be well to interpret Aristotle's "The whole is more than the sum of the parts" as meaning you've forgotten something.

There are no secrets better kept
than the secrets that everybody guesses.
George Bernard Shaw

Senior level

8.6 On dredging up cost information

Few students have ready sources of current, reliable operating cost data. As with shipbuilders, ship operators usually look upon their cost information as strictly confidential. For most academic projects the resulting paucity of data is not a serious weakness because no one is about to use student designs to start cutting steel. Rough cost approximations are good enough.

But where to get even rough cost figures? Some fairly old figures are given in the sources listed at the end of this chapter. With suitable adjustments for inflation, they can give you a start.

Comb through the transactions of the various maritime-related technical societies looking for papers extolling the virtues of some new technology. Some of those papers try to prove the benefits of the new by comparing predicted economic performance with that of the old. If at all complete, they will lay out the details of their cost assumptions, and these often turn out to be the most valuable part of the exposition.

After graduation you should work hard to develop professional contacts with engineers and managers in all parts of the industry. The activities of SNAME and other technical societies provide a vehicle for this. Having developed such contacts, you will be in a position to solicit occasional bits of confidential information. Your sources may be more willing to provide data if you ask, not for their own company’s cost figures, but for their estimate of industry-wide averages. You can also make it easier for your sources if you propose what you judge to be a good guess and ask if they think it too high, too low, or about right. If you plan to publish, protect your sources by finding ways to disguise their figures. Above all, use several sources of information and publish only weighted averages. (By "weighted" I mean giving greater credence to your more reliable sources.)

In one study I elicited considerable information by asking a ship operating friend for the annual cost of maintenance and repair on one of the typical ships in his fleet. Then I said, "Suppose you kept the same hull but installed an engine of twice the power. Then what would the M&R cost be? And suppose you installed an engine of half the power?" His response gave me a good idea of how M&R costs vary with horsepower. Then I asked, "Suppose you kept the original engine room but doubled the size of the hull -- or halved the size of the hull?" His response then gave me an indication of how M&R costs vary with hull size. I asked the same questions applied to crew wages and also annual costs of stores and supplies. Having elicited replies to these organized questions from half a dozen ship owners, I could draw up comprehensive guides to ship operating costs. I did this first in graphic form (as shown in Figure 8.1), and from those contours I could derive general equations like those shown in Section 8.4.
Chapter 8: Operating Costs, Senior level

Remember in all this that the only operating costs that deserve extreme care are those that will differ between the various alternative designs.

8.7 A yarn with a moral

If you will forgive me, I should like to unload on you a true tale from my early days as a ship economist. My maiden effort in this field produced a paper published in the 1957 SNAME Transactions: "Engineering Economy in Tanker Design." That was well received, so I decided to do a sequel on the economics of ocean ore carriers. In seeking data for the new paper, I approached a friend with one of the ship operating companies. He replied that his company had recently completed a comprehensive economic study of ocean ore carriers. Although he was not allowed to send me a copy of the report, I could look at it next time I came to New York. In due course I showed up and he escorted me to a room in which he invited me to sit down to examine the confidential report and, indeed, copy out all the numbers I wanted to. Well, I spent half a day furiously copying numbers, transcribing data off graphs, and so forth. As I reached the final page of the massive report I read this acknowledgement: "All weight and cost figures in this report were taken from Benford's Engineering Economy in Tanker Design."

The moral of this story is that published cost figures in the industry are so scarce that some of what you see suffers from inbreeding. Not every writer admits where he came by his data, so be careful.

Useful references


University of Michigan Department of Naval Architecture & Marine Engineering Reports


Figure 8.1: Annual Costs Maintenance and Repair
Appendix
Table Scraps from the Foregoing Feast

Let us be happy and live within our means,
even if we have to borrow the money to do it with.
Artemus Ward

All levels

A-1 Why this appendix?

In writing this opus I have tried to hold the quantity and quality to appropriate levels for college students in their sophomore through senior years. Those limits forced me to omit some facts that you may want to know if you intend to do serious work in ship economics. You may consider this appendix as being your introduction to master’s level work in the field. It is, however, strictly optional.

The following sections are independent of one another and are found in random order.

A-2 Where yield (or DCF) fails

In Section 5.3 I mentioned that the yield criterion has its detractors. One of its shortcomings may show up if you are faced with a cash flow pattern that shows a year-by-year mix of money coming in or out. That being the case, you may discover that there is more than one interest rate that will bring the net present value down to zero. In short, you may find yourself with more than one yield. You are then left with the unsatisfactory conclusion that you don’t know which to believe. Fortunately, in most ship economic studies we have simple cash flow patterns in which that dilemma does not arise.

A more serious flaw is that yield is fundamentally a less accurate measure of human satisfaction (which is what engineering economy is all about). Here is a case in point. Suppose that your instincts are such that you cannot make up your mind which you would rather have: $100 today or the firm promise of $120 a year from now. That establishes your private internal time value of money as being equivalent to 20 percent annual interest. Now suppose someone offers you two mutually exclusive opportunities to invest $100 today. Proposal A will return $200 a year from now. Proposal B will return $300 two years from now. Since the $100 investment is the same in both proposals you can ignore it. Then you can look a year into the future and ask yourself whether at that time you would prefer the promised $200 right then or the promise of $300 in another year. Applying the NPV criterion, the benefit of accepting Proposal B instead of A would be:

\[
NPV = (PW-20\%-1) \times 300 - 200
\]

\[
NPV = 250 - 200 = 50.
\]

Those numbers lend quantitative evidence to what you should have been able to guess: the second alternative is the more desirable. Suppose, however, you had used yield as your criterion. Then you should have these calculations:

For Proposal A: \( F/P = \frac{200}{100} = 2.00 \) and \( N = 1 \)

Corresponding yield = 100%
Appendix

For Proposal B: F/P = $300/$100 = 3.00 and N = 2
Corresponding yield can be found thus:

\[(1+i)^2 = 3\]
\[1+i = \sqrt{3} \text{ or } 30.5\]
\[i = 73.2\%\]

Here you can see that the yield criterion would favor Proposal A, which is clearly less desirable to anyone whose instinctive time-value of money amounts to 20% interest.

But you might get a little more sophisticated and ask, "Suppose I were to accept Proposal A and at the end of the year reinvest it in an equally profitable way?" That means doubling it again, so the initial $100 would grow to $400 at the end of the second year. But, wait a minute! If you assume reinvestment for A you must also assume reinvestment for B. These reinvestments might, in theory, go on forever. Now we can compare the investments by taking cumulative present worths. To find those values, divide each average annual cost by your interest rate, namely 20 percent. Again, the initial investments being identical can be dropped.

For A the cumulative present worth for $200 per year going on forever:

\[\text{PW of A} = \frac{\text{AAC}}{i} = \frac{200}{0.20} = 1000\]

For B; you would first need to convert $300 every other year to an equivalent annual amount by multiplying by the sinking fund factor for 20 percent interest and two years. Then divide that by the interest rate:

\[\text{PW of B} = \frac{(SF-20\%-2)\$300}{0.20} = 681.82\]

Now Proposal A looks better. From this we can conclude that yield may be superior to NPV if you can assume continuing reinvestments at the same level of profitability.

What can we conclude from all this? My own conclusion is that each measure of merit is as worthy as the other. As someone once observed, those who prefer NPV want to make money so as to exist. Those who prefer yield exist to make money. This reflects the different philosophies of the corporate executive and the entrepreneur.

A-3 Replacement analysis

When should you, as an owner, replace a capital asset? If you are a ship owner that is a question you should ask yourself from the moment you sign the construction contract. (During anticipated increases in demand, some speculators sign contracts with every intent of selling them to a less far-seeing owner before the ship is even built.) As your ship enters service you should at least once a year ask yourself the question, "Should I sell the ship today or wait a year and then ask myself the same question?" If you decide to keep it you will be foregoing the immediate net (after-tax, etc.) income -- a lost opportunity cost, abbreviated \(P_0\). That may be justified by the expectation of receiving a net after-tax income a year from now. That year-off income will be made up of three components:

*The after-tax cash flow from one more year of operation: A'*
Appendix

*The net income from selling the ship a year from now: L₁
*The hidden after-tax costs of inferiority: Z

Inferiority has four components:

* Deteriorated condition of the existing ship leading to:
  * Lessened income
  * Increased operating costs
* Lost opportunity costs of not owning the better ship available today:
  * Increased income potential
  * Reduced operating costs

You can visualize the cash flow pattern like this:

```
0  ↓
P₀  ↓
     ↓
     ↓
     ↓
     ↓
     ↓
     ↓
L₁  ↓
A'  ↓
Z
```

Now we are ready to decide whether keeping the ship for another year is worth doing. The measure of merit will be NPV. If it comes out positive, that would encourage us to keep the ship for at least another year; otherwise we should probably sell it. Our general equation will be:

\[ \text{NPV} = (PW-i' \cdot 1)(A' + L₁ - Z) - P₀ \]

Several other analytical techniques have been proposed, but the one I have outlined above is the simplest and, in my view, often most satisfactory. Needless to say, it involves a lot of educated guess-work about the future (as is also true of all the other systems), but that is a feature of nearly every element of engineering economics.

A-4 Predicting economic life

The preceding section talks about deciding on a year-by-year basis when to retire an existing ship. But, in designing a ship, all of our economic criteria ask us to foresee how long the ship should last. That is a more difficult task (and, fortunately, a less critical one). One method tries to look ahead to the changing patterns of the various components entering into the replacement analysis explained above. It then tries to predict at what future time the year-off cash flow will no longer be enough to offset the advantage of immediate sale.

There are other approaches. In one of them we predict future cash flows and try to find the total years of operation that will maximize the average annual benefit. In another, we use dynamic programming to analyze possible cash flows in a massive decision tree with a time base stretching over many decades.

If you want to learn more, one source of information is a 1980 paper by Brown and Benford reproduced as Report 192 in the University of Michigan Department of Naval Architecture & Marine Engineering series.
Appendix

A-5 Selecting an interest rate

Some of the measures of merit discussed in Chapter 5 require an assumption as to interest rates. In real life some business manager may dictate what that figure should be. In an academic setting, or when doing a general study for no particular customer, you may have to select the rate. So, the question arises; what is a reasonable rate? Under U.S. economic conditions, a ship operating company that wants to attract equity capital through the sale of stocks, or borrow money from a bank at minimum commercial rates, probably ought to aim for a minimum yield on total capital of ten to fifteen percent in constant-value terms. Captive fleets with secure sources of income, might favor the lower figure; common carriers might favor the higher.

Even the federal government has to recognize the time-value of money. The exact figure is hard to pin down. Some experts base it on the interest paid on government bonds, which is remarkably low when corrected for inflation.

If net present value is your criterion, you want to select a minimum acceptable interest rate. Business managers usually base this on the average cost of capital. If they raise half of their capital through selling stock (on which they hope to pay dividends of twelve percent) and half through bank loans (on which they pay eight percent interest) they might take the weighted average of those figures and thus discount future amounts at ten percent. They might also add half, or one, percent for margin.

A word of warning. Do not go overboard on trying to lower overall interest rates through extensive borrowing. The fact is that the more you borrow from a bank, the greater is the risk you are placing on both the bank and your equity holders. Both, then, have the right to insist on higher rates of return. The net effect is that overall rates should remain about the same regardless of source of capital.

Keep in mind that we normally think in terms of constant-value dollars in all this. We pretend, in effect, that inflation or deflation will not occur. As long as the ship owner is free to change freight rates to reflect changing costs, that is a reasonably safe assumption.

A-6 Misleading measures of merit

Marine literature contains many cost studies based on questionable logic. Perhaps the most common variety tries to minimize the unit cost of service. That is, someone looks for the alternative that minimizes the cost to the ship owner. This is technically called the fully distributed cost. It is something like the required freight rate, but ignores corporate income taxes and applies a rock-bottom interest rate to total capital, perhaps as low as six percent. By ignoring taxes and minimizing the time-value of money, this criterion is almost always misleading. Remember, what really counts is minimizing the cost to the customer.

A-7 Capitalized cost

Here is a measure of merit we skipped in Chapter 5. It's not much used in maritime studies, but was once popular in civil engineering circles. You may run across it in the literature. It assumes that each alternative, as it is retired, will be replaced by an exactly identical unit, and that all costs (both capital and operating) will remain forever the same. Called capitalized cost, it is simply the present value of the perpetual series of cash flows stretching into infinity. You might think that an infinite stream of money might add up to an infinite amount. And so it would were it not for the time-value of money and those discount factors we apply to the future amounts.
Appendix

With a little analytical thought on your part you can conclude, correctly, that the capitalized cost of an infinite stream is simply the average annual cost of the first unit divided by the interest rate used in finding that AAC.

A-8 Yield and NPVI: the incestuous relation

In most preliminary design studies we make the standard simplifying assumptions that lead to simple cash flow patterns like that shown below:

Given that the above pattern applies to all alternatives, then the best chosen on the basis of yield will also be the best chosen on the basis of net present value index. Let us explore why this is true.

By definition, the NPVI equals the net present value divided by the investment:

\[ \text{NPVI} = \frac{\text{NPV}}{P} \]

but \[ \text{NPV} = (\text{SPW}-i'-N)A' - P \]

so \[ \text{NPVI} = \frac{(\text{SPW}-i'-N)A'-P}{P} \]

and \[ \text{NPVI} = \frac{(\text{SPW}-i'-N)A'}{P} - 1 \]

but \[ \frac{A'}{P} = CR' \]

therefore \[ 
\text{NPVI} = (\text{SPW}-i'-N)CR' - 1 
\]

Since the interest rate \((i')\) and years of life \((N)\) should be the same for all alternatives, it follows that the series present worth factor \((\text{SPW})\) should also be the same. Thus the best alternative will hinge on which one has the highest after-tax capital recovery factor \((\text{CR}')\) -- which will automatically be the one producing the highest yield. This shows that NPVI and yield will lead to the same design decision. This explains a nice peculiarity of NPVI: it shows the same point of optimality regardless of the discount rate assigned.

If you will recall what you learned in Section 2.5, SPW and CR are reciprocals. This might lead you to look at that last equation and conclude that NPVI should equal one minus one, or zero. This is not the case, however, because as here defined, CR' is derived from estimated values of A' and P), while SPW is based on an assigned interest rate, which would usually be something less than that corresponding to CR'.
Appendix

A-9 A shotgun wedding

Suppose for a given design study we used required freight rate (RFR) based on a ten percent yield as our criterion and arrived at a given ideal speed for our ship. If we then used the resulting freight rate to assess income and used the same ten percent interest rate to find NPV, we should discover that the maximum attainable NPV would be zero and would occur at the same speed as that found to be optimal using RFR. NPV and RFR, in short, would point to the same speed as being optimal.

Now, going further, suppose you were to assume a freight rate equal to the minimum RFR found in the first part. Then you would find that the maximum yield would equal your initially assumed ten percent, and would occur at the speed indicated as optimal by both of the other methods.

In real life, the amazing coincidences assumed above will seldom obtain. The analyst who uses NPV will start with an estimated freight rate from which annual cash flow will be projected. Those numbers will be discounted at a rate that would normally be appreciably lower than the target rate used in RFR. Also, since acceptable NPV values are always positive, there is an implication that the derived yield would be something greater than the discount rate used in finding NPV. In short, each criterion would normally point to a somewhat different speed (or other parameter) as being best.

A-10 The benign influence of flat laxity

By flat laxity I am referring to the characteristic shape of typical ship optimization curves. These show that one may select a design characteristic that is several percent above or below the theoretical point of optimality with only negligible loss in economic efficiency. This leads to the conclusion that intangible factors may be allowed to push the design well away from the indicated optimum without great loss in economic benefit. We can also conclude that advocates of different measures of merit should be able to agree on compromise decisions without drawing swords. The exception to this comes in cases where abrupt discontinuities are involved (such as a switch from single screw to twin screw propulsion), or in feasibility studies involving differing technologies.

A-11 Working capital

When you run across the term working capital, don't be alarmed. You have already run across the concept in practice. It is money that you have to pay out, but with the expectation of getting it back at some time. Ever paid a key deposit? That's working capital. If a shipyard has to pay for labor or materials faster than the prospective ship owner is paying in, that is working capital. The ship owner, in turn, may have to pay bills before collecting freight. In the marine industry the needs for working capital are seldom of great import and are usually ignored.

A-12 Curve plotting techniques

Ship economists have frequent need for deriving general conclusions from scanty data. The five-point plot shown in Figure 8.1 is a good example. True, if you have a plethora of reliable data there are some nice computer programs that will quickly deduce reliable general equations to fit those points. In ship cost studies, unfortunately, pluralities of data are rare, so we have to make do with a few data points and a good deal of common sense. If all you had to go on were the five data points shown in Figure 8.1, you would at once notice that the 10,000 BHP cost contour was not quite a straight line when plotted against the cubic number. Had it been a straight line, you could of course derive an equation in the form of Cost = constant + slope × CN. Since it is not a straight line, however, you can try plotting it on log paper or semi-log paper to see if either would
Figure A.1  Typical log-paper plot

The straight line drawn through the data points leads to this approximation:

Annual cost of hull maintenance & repair

$$= 20000 \left( \frac{CN}{1000} \right)^2$$
Appendix

produce a straight contour. Experience shows that most cost vs size relationships tend to be log-linear (meaning they plot as straight lines on log paper). Figure A.1 shows a typical example. The general equation for a straight line on log paper is:

\[ Y = aX^b \]

where \( a = Y \) intercept
and \( b = \) geometric slope of the line
\( X = \) basic (i.e., extreme left hand) value of the abscissa (independent variable).

You should also plot the cross curve (cost vs BHP at a given cubic number) to see if you cannot derive its equation as well. From those two sets of relationships you can with some confidence expand your findings to cover any combination of cubic number and horsepower.

Having established general trend lines, you may find it worth while to examine why some data points fail to fall right on the curve. See if you can discover what feature all the high points have in common, and the same for all the low points. Perhaps you can plot the amount of discrepancy against that suspected parameter. If those points fall into line, \textit{voila!} you have established a valuable secondary correction. Failing this, you may have to be content to draw upper and lower boundary lines, leaving it to the user's intuition to choose a value.

Common sense enters into all this. If some information is more reliable than others, give those data points more credence. Bring your curve back to zero on the independent variable scale. Does it yield a logical value of the dependent variable there? By such simple logical procedures you may be able to exceed the reliability of what your computer can produce.

Having derived a simple plot of cost vs some size parameter (say hull cost vs cubic number), some estimators then get fancy by keeping cubic number as the independent variable, but converting the dependent variable to cost divided by cubic number. There is nothing wrong with that, but don't start out plotting your original data in that form. Why not? Because the resulting curve will almost invariably resemble a hyperbola that will refuse to fall into a straight line regardless of what kind of graph paper you may select. That makes it difficult to derive its equation.

Finally, in all this, be logical in choosing your independent variables. As ways to estimate costs for example:

* Cubic number is better than displacement as a measure of hull size
* Light ship is even better than cubic number (or deadweight)
* Horsepower is better than speed as a measure of machinery size.

A-13 Closure

To those of you who have read this book through, my thanks and congratulations. I hope you have now mastered the subject, and will apply what you have learned in many aspects of your professional careers (and personal finances). I also hope you have perhaps found the subject interesting and are now ready throughout your years to develop new and better ways to tame the beast of ship economics.
Appendix

I want to acknowledge the continuing helpful advice advanced by my colleague Anastassios Perakis during the preparation of this booklet. Luella Miller deserves thanks for her careful work in typing the camera-ready copy. Additional thanks go to Carolyn Churchill for her patience in showing me how to coax my machine to draw cash flow diagrams.

And... but:

_The secret of being a bore is to tell everything._

Voltaire

There! Have I stopped in time?
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