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ABSTRACT

The problem of minimum-cost operation of a fleet of ships that has to carry a specific amount of cargo between two ports in a given time period for a specific, fixed contract price is studied. Detailed and realistic operating cost functions are developed. Sensitivity analyses are performed to study the effects of small or large changes of one or more cost components on the total cost. A realistic model for the annual transport capacity as a function of speed is also used, in contrast to the linear relation used in the literature.

Our problem boils down to the selection of the proper full load and ballast speeds for those ships of the fleet that we operate such that the total fleet operating costs (including lay-up costs for unused vessels) are a minimum. The algorithm developed and implemented for the solution of the above problem uses the SIMPLEX method of nonlinear optimization developed by Nelder and Meade. Sample runs of the program developed are included in the Appendices.

The above problem but with time-varying cost components has been also studied. A formulation of this problem and a solution is presented. A probabilistic analysis, for the case in which the most important cost components are random variables with known probability distribution functions, is finally presented. Examples for these algorithms are also given in the appendices.

ACKNOWLEDGEMENTS

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LIST OF SYMBOLS

Name	Description	Units
a -	Power coefficient (for the full load condition).	
a_b -	Power coefficient (for the ballast condition).	
b -	Power exponent (for the full load condition).	
b_b -	Power exponent (for the ballast condition).	
C -	Total annual operating cost.	(\$/yr)
C_a -	Annual cost of administration.	(\$/yr)
C_c -	Annual cost of capital.	(\$/yr)
C_e -	Annual cost of stores, supplies & equipment.	(\$/yr)
C_f -	Price of propulsion fuel.	(\$/lb)
C_l -	Port and route charges for fully loaded ship	(\$/RT)
C_m -	Annual cost of manning.	(\$/yr)
C_o -	Annual cost of lay-up.	(\$/yr)
C_r -	Annual cost of maintenance & repair.	(\$/yr)
C_u -	Port and route charges for ship in ballast.	(\$/RT)
D_b -	Nautical miles per round trip a vessel operates in ballast condition.	(nmiles/RT)
D_f -	Nautical miles per round trip a vessel operates in full load condition.	(nmiles/RT)
D_r -	Nautical miles per round trip a vessel operates in restricted operation.	(nmiles/RT)
DW -	Full load cargo capacity of a vessel.	(tons)
F_l -	Average amount of fuel burned per day in load port.	(lb/day)
F_r -	All-purpose fuel at restricted speed.	(lb/hp-hr)
F_u -	Average amount of fuel burned per day in unload port.	(lb/day)
G -	Tons of cargo carried per year.	(tons/yr)
g,s,d -	Coefficients for the all purpose fuel rate (loading condition).	

Name	Description	Units
g_b, s_b, d_b -	Coefficients for the all purpose fuel rate (ballast condition).	
M -	Days per year a vessel is out of service for maintenance and repair.	(days/yr)
p -	Percent of main propulsion power in full load condition.	(%)
P -	Main propulsion power in full load condition.	(hp)
p_b -	Percent of main propulsion power in ballast condition.	(%)
P_b -	Main propulsion power in ballast condition.	(hp)
P_f -	Maximum propulsion power.	(hp)
P_r -	Average power required in restricted operation.	(hp)
R_f -	All-purpose fuel rate in full load condition.	(lb/hp-hr)
RT -	Round trips per year.	(RT/yr)
V_r -	Average speed at which a vessel travels in restricted operation.	(knots)
T_u -	Average days per round trip a vessel spends in the unload port.	(days/RT)
T_l -	Average days per round trip a vessel spends in the load port.	(days/RT)
X -	Ship speed in full load condition.	(knots)
X_{max} -	Maximum full load ship speed.	(knots)
X_{min} -	Minimum acceptable full load ship speed.	(knots)
Y -	Ship Speed in ballast condition.	(knots)
Y_{max} -	Maximum ballast speed.	(knots)
Y_{min} -	Minimum ballast speed.	(knots)

I. INTRODUCTION

Fleet deployment covers a wide range of problems concerned with fleet operations, scheduling, routing, and fleet design. Many of these problems use some kind of economic criteria such as profitability, income or costs on which to base decisions. Others use noneconomic criteria such as utilization or service; these are more common in fleet deployment models used in the liner trades. Alexis [1] and Ronen [2] both give excellent reviews of various fleet deployment problems and models. An aspect of fleet deployment which is not covered extensively in the literature is slow steaming analysis.

Slow steaming is the practice of operating a ship or fleet of ships at a speed less than design or maximum operating speed, in order to take advantage of improved fuel economy and reduce operating costs. The impact of the improvement in operating economy is closely, but not exclusively, related to the price of fuel. Slow steaming is not strictly a policy to be practiced during times of high fuel costs. It can be economically beneficial to a fleet operator at any time and should be considered whenever the circumstances of a particular mission warrant its application.

Consider a fleet operator who has contracted to haul a certain amount of cargo over a known trade route in a given period of time. He has a fixed number of ships available to perform the service. If the full load carrying capacity of any combination of the available ships, operating at maximum (or design) speed, exceeds the amount of cargo available for transport, then there exists a combination of ships, operating at or below maximum loading and at or below maximum ballast speed, that will fulfill the cargo transport obligation at a minimum total operating cost to the operator. Thus a slow steaming analysis is valid whenever inequalities (A) and (B) below are satisfied:

$$\sum_{i=1}^{Z} G_i \left(X_{imax}, Y_{imax}\right) > \text{cargo} \qquad (A)$$

$$\sum_{i=1}^{Z} G_i \left(X_{imin}, Y_{imin}\right) < \text{cargo}, \qquad (B)$$

where

$G_i(X_{imax}, Y_{imax})$ is the full load cargo carrying capacity of ship i, $i=1,2,...,Z$, operating on a given trade route at maximum full load speed, X_{imax}, and at maximum ballast speed Y_{imax}

and

$G_i(X_{imin}, Y_{imin})$ is the full load cargo carrying capacity of ship i, operating on a given trade route at minimum full load speed X_{imin}, and at minimum ballast speed Y_{imin}.

Clearly, if (A) is not satisfied, the existing fleet is not sufficient in itself to carry the cargo and outside vessel(s) have to be added to the fleet. If (B) does not hold, one or more of the vessels of the fleet have to be laid-up or chartered to a third party.

Once the feasibility of a slow steaming analysis has been determined, the set of ships and their respective speeds that will minimize total operating costs may be computed. This paper describes a formulation of this problem, based on a number of simplifying assumptions. A computer program has been developed to solve the problem and to aid the fleet operator make slow steaming policy decisions. The program is based on the formulation presented in Section II. A discussion of the problem solution and a thorough sensitivity analysis are presented in sections III and IV and Appendices 1 and 2 respectively.

The sensitivity analysis provides the user with an understanding of the influence on the total fleet operating cost of its various components. For small to moderate changes of one or more cost components, the user can get a very accurate estimate of his new total operating cost without having to rerun the computer program. Some interesting conclusions are made on the basis of the sensitivity results (see Appendix 3).

For the basic fleet deployment problem but with time-varying cost components, a thorough analysis has been done. Discussion of the problem, its formulation and solution are presented in Section V. A computer program has been developed to implement the solution of this problem. More about this can be found in Appendix 5. Finally, the problem of fleet deployment when the cost coefficients are random variables with known probability density functions is examined in Section VI, where analytical expressions for the basic probabilistic quantities are presented.

II. PROBLEM FORMULATION

II.1. Objective Function and Constraints

A formulation of the basic slow steaming optimization problem discussed above will first be presented. A fleet, consisting of a given number of ships available to move a fixed amount of cargo between two ports, over a given period of time, for a fixed price. Each vessel in the fleet is assumed to have known operating cost characteristics.

The problem objective is to determine each vessel's full load and ballast speeds such that the total fleet operating cost is at a minimum and all contracted cargo is transported.

The problem will be formulated on an annual (per year) basis. The selection of a unit time period does not affect the generality of the solution. The unit time period of one year is selected since it is a typical contract interval, and it is considered to be the maximum period of time during which certain cost parameters in the formulation can be assumed constant. Units for all variables, parameters and constants are given in the List of Symbols.

Given a fleet of Z ships, each with a given full load cargo carrying capacity, and each having known operating cost characterics as a function of vessel speed; for a given trade route, two expressions will be derived for each vessel:

$F_i(X_i,Y_i)$ = Total operating cost of vessel i per ton of cargo carried, as a function of full load and ballast ship speeds, for $i=1,2,\ldots,Z$

$G_i(X_i,Y_i)$ = Total tons carried per year over a specified trade route, as a function of full load and ballast ship speeds, for $i=1,2,\ldots,Z$

Then, for each individual vessel of the fleet, the total operating cost of vessel i per year as a function of its loaded and ballast speeds is given by

$$C_i(X_i,Y_i) = F_i(X_i,Y_i)\cdot G_i(X_i,Y_i) \qquad \text{(II.1)}$$

The total annual cost of operating the fleet on the specified route is then:

$$C(X_1,Y_1,\ldots,X_Z,Y_Z) = \sum_{i=1}^{Z} C_i(X_i,Y_i)$$

Our optimization problem is:

$$\text{Minimize } C(X_1,Y_1,\ldots,X_Z,Y_Z) \qquad \text{(II.2)}$$

subject to the following constraints,

$$X_{imin} \leq X_i \leq X_{imax},\ i=1,2,.,Z \qquad \text{(II.3)}$$

$$Y_{imin} \leq Y_i \leq Y_{imax},\ i=1,2,\ldots,Z$$

$$\sum_{i=1}^{Z} G_i(X_i,Y_i) = \text{Cargo} \qquad \text{(II.4)}$$

The first constraint imposes upper and lower bounds on the vessel loaded and ballast speeds. These speed constraints are necessary to insure a feasible solution to the problem; which is, that the ship's speed is less than or equal to its maximum speed and greater than or equal to its minimum lower operating speed. The minimum operating speed may be any value greater than or equal to zero. In practice, the minimum speed is non-zero and determined by the lower end of the normal operating region of the vessel's main engine. Other practical considerations such as ship motions and bottom fouling clearly point against very low minimum speeds.

The minimum speed should be adequate for purposes of ship safety in maneuverability and control. The second constraint, equation II. 4, must be satisfied to insure all available or contracted cargo is transported.

As defined, this problem represents a 2Z dimensional, nonlinear, multivariable, constrained optimization problem, with 2Z independent variables, 4Z linear inequality constraints, and one nonlinear equality constraint. It will be shown later that the equality constraint can be used to eliminate one independent variable, thus reducing the problem to one of 2Z-1 independent variables.

II.2. Assumptions

This formulation is based on the following assumptions;

1) A vessel carries a full load of cargo from load port to unload port.
2) When the vessel is operating in restricted waters, it has a known and constant restricted speed which is usually the maximum allowable speed in the region in question, hence requiring a known, fixed power and fuel rate.
3) The number of days a vessel spends in the load port and unload port per round trip is known and constant.
4) The charges incurred at the load port and unload port per round trip are known and constant.
5) The amount of fuel burned per day in the load port and unload port is known and constant.
6) The annual costs of manning, stores, supplies, equipment, capital, administration, maintenance & repair, and make ready for sail are known and constant.
7) The power of vessel i may be expressed by $p_i = a_i \cdot X_i^{b_i}$ for the full load and by $P_{bi} = a_{bi} \cdot Y_i^{b_{bi}}$ for the ballast condition.

8) The all-purpose fuel rate for a fully loaded vessel i may be expressed by;

$$(R_f)_i = g_i \cdot p_i^2 + s_i \cdot p_i + d_i \qquad (II.5)$$

for the full load and by

$$(R_f)_{bi} = g_{bi} \cdot p_{bi}^2 + s_{bi} \cdot p_{bi} + d_{bi} \qquad (II.6)$$

for the ballast condition.

9) The total annual cost of laying up vessel i is known for all $i=1,\ldots,Z$.

10) The number of days per year vessel i is out of service for maintenance and repair is known and constant.

11) This problem formulation and solution is for a single stage, "one-shot" decision.

II.3. Round Trips per Year

This formulation does not consider the 'integer nature' of some fleet deployment problems. The number of transits from port to port is not treated as an integer variable. The effects of this simplification on the analysis results are addressed in Section III. Assuming a real number of round trips per year, the number of round trips per year made by vessel i, as a function of the vessel's full load speed, is the number of days per year a vessel is available for service, divided by the number of days per round trip it spends in each mode of operation (in port, ballast condition, restricted operation, and in the full load condition), given by;

$$RT_i(X_i,Y_i) = \frac{(365 - M_i)}{T_{li} + T_{ui} + \frac{D_r}{24(V_{ri})} + \frac{D_b}{24(Y_i)} + \frac{D_f}{24(X_i)}}$$

This expression may be rewritten as:

$$RT_i(X_i,Y_i) = \frac{TA_i' \cdot X_i \cdot Y_i}{TB_i' \cdot X_i \cdot Y_i + TC_i' \cdot Y_i + TD_i' \cdot X_i} \qquad \text{(II.7)}$$

where,

$TA_i' = (365 - M_i)$

$TB_i' = (T_{li} + T_{ui}) + D_r/24V_{ri}$

$TC_i' = D_f/24$

$TD_i' = D_b/24$

II.4. Tons Carried per Year

Having expressed the number of round trips per year as a function of ship speeds, Eq. (II.7), the expression for the number of tons carried per year as a function of ship full load and ballast speeds is determined by:

$$G_i(X_i,Y_i) = RT_i(X_i,Y_i) \times DW_i \qquad \text{(II.8)}$$

A typical plot of this function, for various ballast speeds, is shown in figure II.1. A 3-dimensional plot of the function $G_i(X_i,Y_i)$ would consist of a surface asymptotic to the level $G_o = TA' \cdot DW/TB'$.

II.5. Operating Costs

This section presents a method of deriving a general cost function using mean values of cost variables, an expression representing the vessel's speed/power relationship, and an expression representing the vessel's all-purpose fuel consumption, for the full load and ballast conditions.

Operating costs are considered to fit into one of two categories.

1) Annual operating costs which do not vary with ship speed. For example, it is assumed that manning costs per year are the same

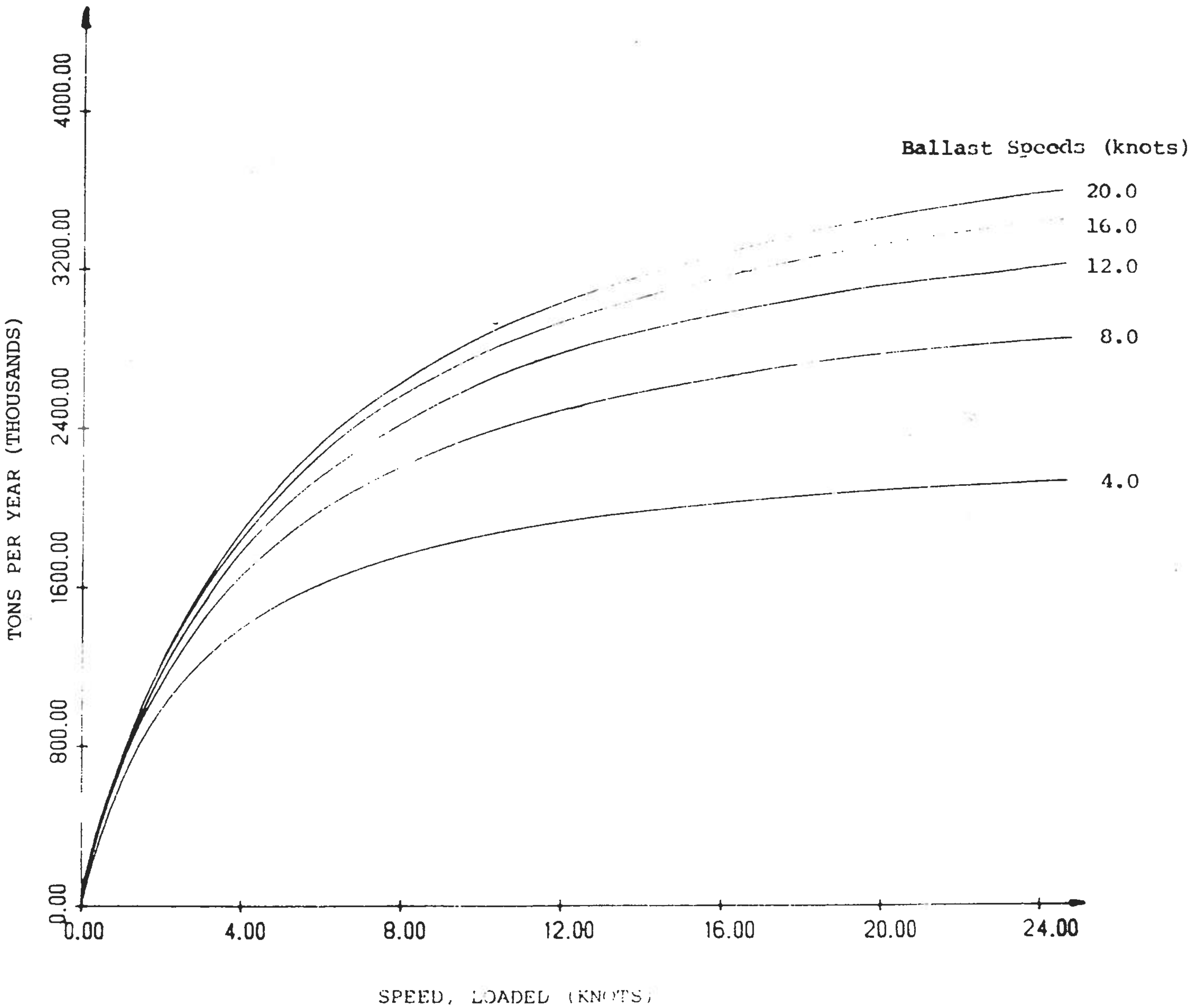

Figure II.1 Typical Plots for the Tons Carried Per Year for Various Ballast Speeds

whether the ship full load speed is 15 knots or 12 knots.

2) Annual operating costs which vary with ship speed. For example, annual fuel costs will be different if the ship travels at a full load speed of 15 knots or 12 knots.

Category 1 Costs:

The annual operating costs considered not to vary with ship speed are;

Manning costs - Annual cost of manning a vessel including wages, benefits, etc.

Stores, supplies & equipment costs - Annual cost of supplies, equipment, provisions, etc.

Capital investment costs - Annual cost of having the vessel to operate; cost of financing, chartering, leasing, etc.

Administrative costs - Annual cost of insurance, overhead, fees, etc.

Maintenance & repair costs - Annual cost of all maintenance and repair activities.

Make ready for sailing costs - Annual cost of making a vessel ready for service. This cost is zero if the vessel is ready for service.

These cost parameters are flexible and are defined with the intention that all significant fixed annual operating costs will fit into one of the above categories.

The sum of these fixed annual costs is;

$$C_i = C_{mi} + C_{ei} + C_{ci} + C_{ai} + C_{ri} + C_{si} \tag{II.9}$$

C_i is a constant parameter defined for each vessel in a fleet, and

$$CPT_i = \frac{C_i}{G_i(x_i, y_i)} \tag{II.10}$$

is an expression of cost per ton of cargo carried, and is a component of the expression $F_i(x_i,y_i)$, total cost per ton for vessel i , in Eq. (II.1).

Category 2 Costs:

The annual operating costs considered to vary with ship speed are;

- -Cost of fuel burned in the full load condition.
- -Cost of fuel burned in the ballast condition.
- -Cost of fuel burned in restricted operation.
- -Cost of fuel burned in port.
- -Port and route charges, including charges incurred in ports, canals, locks, etc.

Port and route charges vary with ship speed because they are incurred more often as ship speed increases and more round trips per year are made.

For each vessel the cost of fuel burned per ton of cargo carried while in restricted operation is;

$$C_{rest,\ fuel} = \frac{D_{ri} \cdot F_{ri} \cdot P_{ri} \cdot C_f}{V_{ri} \cdot DW_i} \qquad \text{(II.11)}$$

The cost of fuel burned per ton of cargo carried while in port is;

$$C_{port\ fuel} = \left[(F_{li} \cdot T_{li}) + (F_{ui} \cdot T_{ui})\right] \cdot \frac{C_f}{DW_i} \qquad \text{(II.12)}$$

The cost of fuel burned per ton of cargo carried while in full load operating condition can be expressed as;

$$C_{loaded\ fuel} = \frac{D_f \cdot (\text{Fuel Rate}) \cdot (\text{Power}) \quad C_f}{X_i \cdot DW_i} \qquad \text{(II.13)}$$

The fuel rates in the above expressions are considered to be an all-purpose fuel rate, reflecting the consumption rate of main propulsion fuel, and all other fuels and lubricants that are consumed on a significant basis. An all-purpose, or equivalent fuel rate, is determined by converting all significant consumption rates to an equivalent consumption rate of main propulsion fuel, based on the price difference of the fuels. The sum of all fuel rates is then an all-purpose fuel rate. See reference [5] for more details on equivalent fuel rates.

A typical plot of an all-purpose fuel rate vs. percent power will resemble the curve in figure II.2 (figure taken from ref. 5).

It is assumed that an expression of the form;

$$R_f = g(p)^2 + s(p) + d \tag{II.14}$$

will adequately describe an all-purpose fuel rate curve similar to that in figure II.2. Percent power (p) is in the interval [0,1], and g,s, and d are constants. It is further assumed that the power term in Eq. (II.13) is of the form;

$$P = a_i X_i^{b_i} \tag{II.15}$$

where a and b are constants. Percent full load power can be written as;

$$p_i = \frac{P_i}{P_{fi}} = \frac{a_i X_i^{b_i}}{P_{fi}} \tag{II.16}$$

Substitute Eq. (II.16) into Eq. (II.14);

$$R_f(x_i) = \frac{g_i (a_i X_i^{b_i})^2}{P_{fi}^2} + \frac{s_i a_i X_i^{b_i}}{P_{fi}} + d_i \tag{II.17}$$

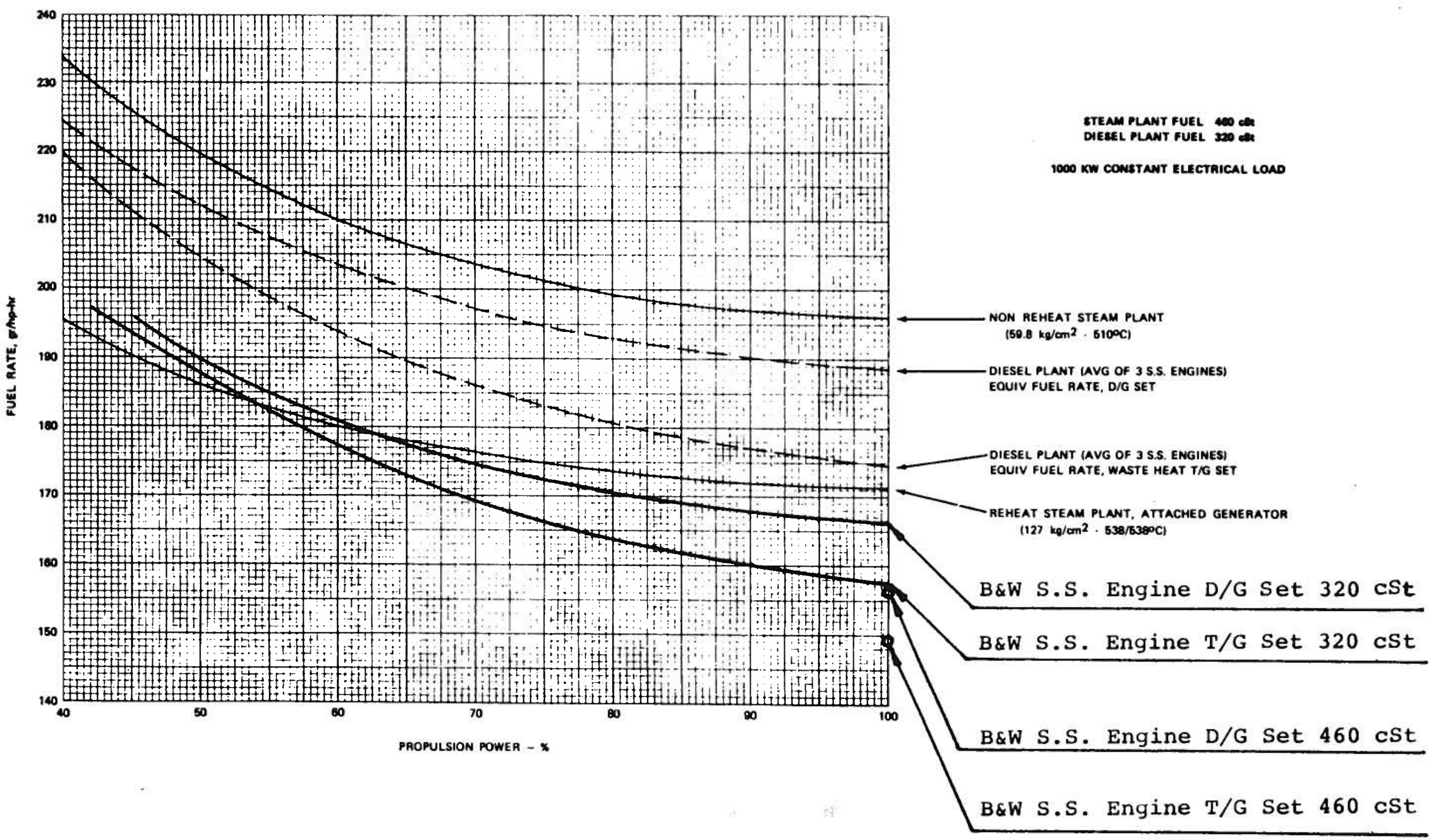

Figure II.2 (from Ref. [5])
Typical Fuel Rate vs. Propulsion Power Curves

Substituting Eq. (II.15) and Eq. (II.17) into Eq. (II.13) results in an expression for the cost of fuel burned per ton of cargo carried while operating in the full load condition, given by;

$$C_i(X_i)_{\text{loaded fuel}} = \frac{D_f \cdot C_f}{DW_i}\left[\frac{g_i a_i^3 X_i^{3bi-1}}{P_{fi}^2} + \frac{s_i a_i^2 X_i^{2bi-1}}{P_{fi}} + d_i a_i X_i^{bi-1}\right] \tag{II.18}$$

The fuel rate for the ballast condition can be written as:

$$R_{fb} = g_b \cdot (P_b)^2 + S_b \cdot (P_b) + d_b \tag{II.19}$$

(Note that a good approximation for the parameters g_b , s_b , d_b is: $g_b = g$, $s_b = s$ and $d_b = d$) . The cost of fuel burned per ton of cargo carried while each ship is in ballast condition can be expressed as:

$$C(Y_i)_{\text{bal.fuel}} = \frac{D_b \cdot (\text{Fuel Rate}) \cdot (\text{Power}) \cdot C_f}{Y_i \; DW_i} \tag{II.20}$$

For the percent power P_b we use the equation:

$$P_{bi} = \frac{P_{bi}}{P_{fi}} \tag{II.21}$$

with

$$P_{bi} = a_{bi} \cdot X_i^{b_{bi}} \tag{II.22}$$

Using equation (II.19) - (II.22) we can find the final expression for the cost of fuel burnt per ton of cargo carried while ship i operates in the ballast condition.

$$C_i(Y_i)_{\text{bal.fuel}} = \frac{D_b \cdot C_f}{DW_i}\left[\frac{g_i \cdot a_{bi}^3 \cdot Y_i^{3b_{bi}-1}}{P_{fi}^2} + \frac{s_i a_{bi}^2 \cdot Y_i^{2b_{bi}-1}}{P_{fi}} + d_i \cdot a_{bi} \cdot Y_i^{b_{bi}-1}\right] \tag{II.23}$$

Port and route charges of a constant amount are incurred per round trip. Thus, the cost per ton of cargo carried attributed to these charges may be expressed as:

$$C_{port\&route} = \frac{C_{li}+C_{ui}}{DW_i} \qquad (II.24)$$

When the shipowner is also the owner of the cargo there is a finite inventory cost that has to be added to the total operating cost. For a given interest rate r_i , a capital cost C_{ci} per ton of cargo carried and a tax rate t_i , the corresponding inventory cost is:

$$C_{inventory/ton} = \frac{r_i}{365} \cdot \frac{C_{ci}}{1-t_i} \cdot T_i \qquad (II.25)$$

where T_i is the number of days at sea while fully loaded which can be calculated by the formula:

$$T_i = \frac{D_f}{24 \cdot X_i} + \frac{D_r/2}{24 \cdot V_{ri}} \qquad (II.26)$$

Finally the inventory cost for the i_{th} ship is:

$$C_{inventory} = \frac{r_i}{365} \cdot \frac{G_i(X_i,Y_i) \cdot C_{ci}}{1-t_i} \times \left[\frac{D_f}{24 \cdot X_i} + \frac{D_r/2}{24 \cdot V_{ri}}\right] \qquad (II.27)$$

The use of the inventory cost, as a part of the total annual operating cost, produces an increase of the minimum operating cost, an increase of the full loading speeds X_i and a decrease of the ballast speeds Y_i .

We can see the effects of inventory cost of goods in transit by comparing the results for the examples with inventory cost with the corresponding examples without inventory cost (see Appendix 1).

The sum of fuel costs and port & route charges, Eqs. (II.11), (II.12), (II.18), (II.23), (II.24) is the part of a vessel's total cost per ton of cargo carried, that varies with ship speed. It is given by:

$$f_i(X_i,Y_i) = C_{restricted\ fuel} + C(X_i)_{loaded\ fuel} + C(Y_i)_{bal.fuel}$$
$$+ C_{port\ fuel} + C_{port\ \&\ route}$$

This can be as:

$$f_i(X_i,Y_i) = A_i + B_i \cdot X_i^{3bi-1} + C_i \cdot X_i^{2bi-1} + D_i \cdot X_i^{bi-1} +$$
$$B_i' \cdot Y_i^{3b_{bi}-1} + C_i' \cdot Y_i^{2b_{bi}-1} + D_i' \cdot Y_i^{b_{bi}-1} \qquad \text{(II.28)}$$

where,

$$A_i = C_{restricted\ fuel} + C_{port\ fuel} + C_{port\ \&\ route}$$

$$B_i = \frac{D_f \cdot C_f \cdot g_i \cdot a_i^3}{P_{fi}^2 \cdot DW_i}$$

$$C_i = \frac{D_f \cdot C_f \cdot s_i \cdot a_i^2}{P_{fi} \cdot DW_i}$$

$$D = \frac{D_f \cdot C_f \cdot d_i \cdot a_i}{DW_i}$$

$$B' = \frac{D_b \cdot C_f \cdot g_i \cdot a_{bi}^3}{DW_i \cdot P_{fi}^2}$$

$$C' = \frac{D_b \cdot C_f \cdot S_i \cdot a_{bi}^2}{DW_i \cdot P_{fi}}$$

$$D' = \frac{D_b \cdot C_f \cdot d_i \cdot a_{bi}}{DW_i}$$

Adding Eq. (II-10) and Eq. (II-28) results in an expression for the total operating cost per ton of cargo carried by vessel i, given by:

$$F_i(X_i,Y_i) = f_i(X_i,Y_i) + \frac{C_i}{G_i(X_i,Y_i)} \qquad (II.23)$$

Typical plots for the total (not per ton) operating costs per year for a particular ship, for various ballast speeds, are given in Fig. II.3.

A typical plot of $F(X_i,Y_i)$ is shown in Fig. II.4, as a function of the full load and ballast speed. It is seen that F is a smooth convex curve or surface with a single minimum. There is also a finite speed range in which F is not very different from its minimum value, a property which allows approximate solutions to the problem using very different speeds for individual ships to produce total fleet costs very close to one another and to the optimum cost itself. For X_i and/or Y_i going towards either 0 or ∞ , F approaches infinity. Figures II.3 and II.4 are for the same ship and for constant route data D_f, D_b, D_r .

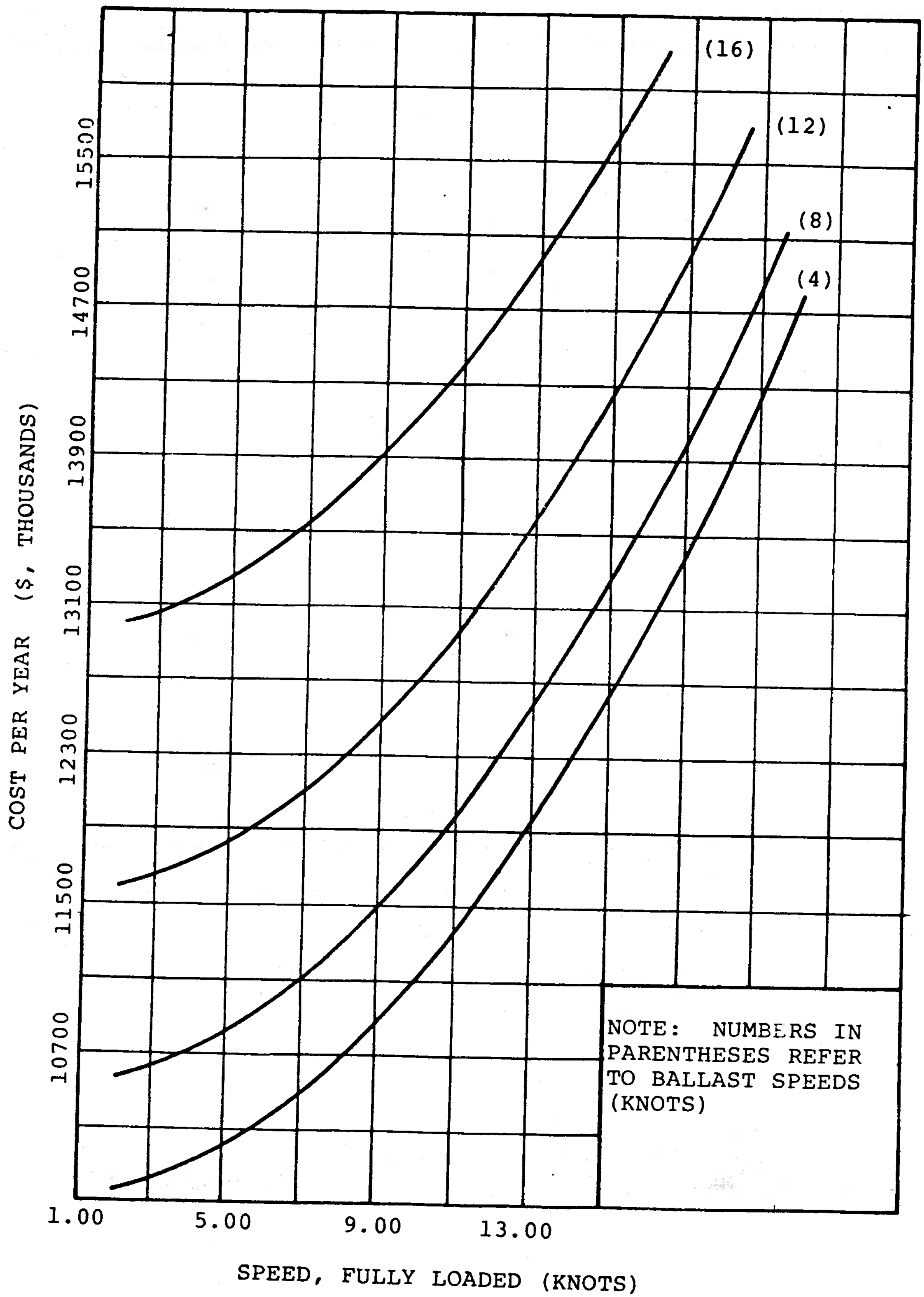

Figure 2.3. Typical Plot for the Total Operating Cost Per Year as a Function of Ship Full Load and Ballast Speeds

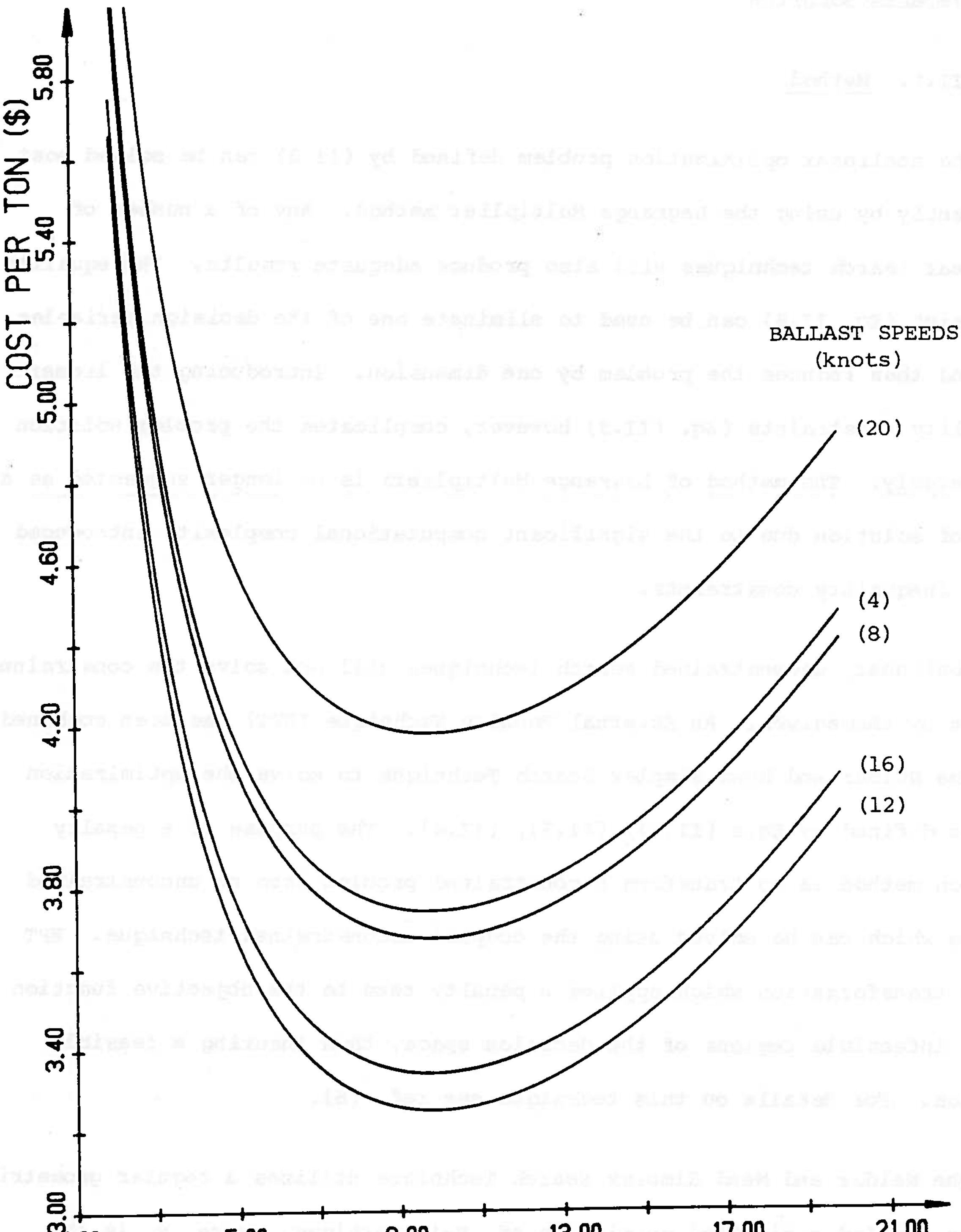

Figure II.4. Typical Plot for the Total Cost per Ton as a Function of Ship Full Load and Ballast Speeds

III. PROBLEM SOLUTION

III.1. Method

The nonlinear optimization problem defined by (II.2) can be solved most efficiently by using the Lagrange Multiplier method. Any of a number of nonlinear search techniques will also produce adequate results. The equality constraint (Eq. II.4) can be used to eliminate one of the decision variables x_i , and thus reduces the problem by one dimension. Introducing the linear inequality constraints (Eq. (II.3) however, complicates the problem solution considerably. The method of Lagrange Multipliers is no longer suggested as a means of solution due to the significant computational complexity introduced by the inequality constraints.

Nonlinear, unconstrained search techniques will not solve the constrained problem by themselves. An External Penalty Technique (EPT) has been combined with the Nelder and Mead Simplex Search Technique to solve the optimization problem defined by Eq.s (II.2), (II.3), (II.4). The purpose of a penalty function method is to transform a constrained problem into an unconstrained problem which can be solved using the coupled unconstrained technique. EPT uses a transformation which applies a penalty term to the objective function in the infeasible regions of the decision space, thus insuring a feasible solution. For details on this technique see ref. [6].

The Nelder and Mead Simplex Search Technique utilizes a regular geometric polygon (called a simplex) consisting of N+1 vertices, where N is the number of independent decision variables. The geometric simplex adapts itself to the local landscape of the objective function, using reflected, expanded and contracted points to locate the function minimum. A note of caution; if the landscape of the objective function is not convex, that is, if it is made up of a number of peaks and valleys (local maxima and minima), the routine will settle in the first valley it finds and return that value as the minimum. In that case, it is advised that the optimization routine be run from a number of different starting points (initial vessel speeds) if it is suspected that the objective function is not convex. For details on this technique refer to ref. [6].

III.2. Problem Reduction

The single equality constraint, Eq. (II.4), can be used to reduce the number of independent decision variables by one, to 2Z-1 . This is done by solving for one of the variables in terms of the others, as follows;

Eq. (II.4): $$\sum_{i=1}^{Z} G_i(X_i, Y_i) = \text{Cargo}$$

Solving for X_Z gives;

$$X_Z = \frac{\left[\text{Cargo} - \sum_{i=1}^{Z-1} \frac{TA_i' \cdot DW_i \cdot X_i \cdot Y_i}{TB_i' \cdot X_i \cdot Y_i + TC_i' \cdot Y_i + TD_i' \cdot X_i}\right] \cdot TC_Z' \cdot Y_Z}{TA_Z' \cdot DW_Z \cdot Y_Z - (TB' \cdot Y_Z + TD_Z') \cdot \left[\text{Cargo} - \sum_{i=1}^{Z-1} \frac{TA_i' \cdot DW_i \cdot X_i \cdot Y_i}{TB_i' \cdot X_i \cdot Y_i + TC_i' Y_i + TD_i' X_i}\right]} \qquad \text{(III.1)}$$

This expression for X_Z may be substituted into the objective function, Eq. (II.2), to give a new objective function with 2Z-1 independent decision variables:

$$\text{minimize} \quad C(X_i, Y_i, \ldots, X_Z, Y_Z) = \sum_{i=1}^{Z-1} C_i(X_i, Y_i) + C_Z(X_Z, Y_Z) \qquad \text{(III.2)}$$

subject to: $X_{imin} < X_i < X_{imax}$ $\quad$ for $i=1,2,\ldots,Z$ $\quad$ (III.3)

$Y_{imin} < Y_i < Y_{imax}$

III.3. Solution - Computational Issues

A computer program has been written to solve this problem using the techniques mentioned above and the formulation of section II. The solution returned consists of the ship speeds, for those vessels specified for analysis, that will minimize the total mission operating costs and fulfill the cargo transport obligation. This solution set is defined as a local optimum. As mentioned above, it is advised that the analysis be run for several sets of initial starting speeds to give a measure of assurance that a true local minimum has been found.

There are two specific parameters that influence the efficiency of the program. These are the stopping limit for the SIMPLEX search and the initial speeds.

(I) Stopping limit EPSI

The SIMPLEX search for the optimum point will stop when the following inequality has been accomplished:

$$\frac{1}{2N}\left[\sum_{k=1}^{2N}[F(X_k) - FB]^2\right]^{1/2} < EPSI$$

where $F(X_k)$ are the values of the objective function corresponding to the kth corner of the simplex with $X_k=(X'_1,Y'_1,\ldots,X'_{N-1},Y'_{N-1},Y'_N)^T$ and FB the value of the objective function at the center of the simplex.

Note that the above inequality can be valid for several combinations of the independent variables $X_1,Y_1,\ldots,X_{Z-1},Y_{Z-1},Y_Z$. That means that it defines a region of possible X_i,Y_i which increases as the number of variables is increased.

If we want greater accuracy in the calculation of X_i, Y_i we have to use very small values for EPSI. But it is not really advisable since we have a very minor improvement in the total operating cost, obtained at a much higher CPU time.

The value for EPSI is an user-specified input. Note that value at the minimum operating cost is independent of the EPSI since the objective function is very flat near the optimum point. Selected results for two examples (with 3 and 10 ships) with different values at EPSI are shown in Appendix 1, where we can see the dependence of the needed CPU time on the value of EPSI.

Generally speaking, a value for EPSI producing quite accurate results is about $(10 \div 20) \cdot Z$ where Z is the number of ships. However, it was observed that much higher values of EPSI have also produced reasonable results.

(II) Initial Speeds

The simplex search will be more efficient if the initial speeds give an initial simplex which lies in the feasible region (this feasible region is defined by the 2N inequality constraints). The user inputs an initial estimate of X_i , Y_i which satisfies these 2N inequalities. The program then checks that the equality constraint is satisfied. After the user has entered the initial full load and ballast speeds the program automatically changes these until they produce a feasible starting solution. There may also be a big difference in the number of required iterations and relatively big differences for the optimum speeds when we are starting from different initial points, but not significant differences in the total operating cost, which is the quantity of interest for the user.

III.4. Lay-Up Option

A single run of the SIMPLEX algorithm for a fleet of N ships gives the best speeds and the minimum (optimum) total operating cost for that specific fleet. However, this is just a local optimum, while the fleet operator is clearly looking for the global optimum, which is defined as the minimum of all local optima, obtained by running the SIMPLEX for all possible groups of $K<N$ ships, while laying up or chartering the other $N-K$ ships. The obvious (and most assured) method of determining the global optimum is to evaluate every feasible combination of vessels in the available fleet. A combination of vessels is considered feasible if;

$$\sum_{i=1}^{Z} G_i(X_{imax}, Y_{imax}) > \text{Cargo} > \sum_{i=1}^{Z} G_i(X_{imin}, Y_{imin})$$

Complete enumeration of all feasible ship combinations is computationally feasible if small fleets are being considered. It can be shown that the maximum number M of runs of the SIMPLEX method that will be required is $M = \binom{Z}{1} + \binom{Z}{2} + \cdots + \binom{Z}{Z-1}$, where $\binom{Z}{K}$ denotes the number of combinations of Z items taken in groups of K. Hence, for $Z = 3$, $M = 6$ runs, for $Z = 6$, $M = 62$ runs and for $Z = 10$, $M = 1792$ runs.

Therefore it seems that for $N>5$ it is rather too time-consuming (although always possible) to use this exhaustive enumeration scheme. However, a dynamic programming -- like sequential optimization approach significantly reduces the computational burden. In the first stage we decide which of the Z ships to eliminate to obtain the best (least total operating cost) (Z-1) - ship fleet. The process is then repeated successively and stops when inequality I.A is violated. By that time, the global optimum corresponding to one of the previously examined optimum fleets, must have already been obtained. The maximum number of fleets we will have to examine using this approach is $M_{max} = Z + (Z-1) + \ldots + 2 = Z(Z+1)/2-1$. For example, for $Z = 3$, $M_{max} = 5$ and for $Z = 10$, $M_{max} = 54$. The actual number of fleets which we will have to consider will be significantly less than M_{max}, due to the elimination of several fleets as infeasible (inequality I.A not satisfied).

The above scheme has been implemented and referred to in the following as the "exact" lay-up method. However, it seems that we should offer an approximate option for large N that selects a best (possibly suboptimal) combination of vessels in an efficient way. Such an option has been implemented and is discussed below:

One output of every SIMPLEX run is a "utilization factor" for each vessel. This factor is the ratio of the amount of cargo carried annually by the vessel at a particular speed divided by the amount of cargo it can carry annually at its maximum speed. This utilization factor will clearly be in the interval [0,1]. A ranking of vessels from high to low according to utilization factor roughly represents a ranking of the vessels relative operating economy (from good to poor). That is, a vessel that is more economical will be assigned to carry a greater proportion of its cargo carrying capacity. Applying this reasoning, the vessel(s) with the highest utilization factors should be retained for further analysis, and the vessel(s) with the lowest (nonzero) utilization factors should be eliminated from the analysis.

In the computer program we have developed, the above approximate procedure starts by running the SIMPLEX for the whole fleet (if feasible) and getting the initial utilization factors for all vessels. Then the smallest possible fleet, consisting of the K "most economical" ships is used and the minimum annual operating cost is calculated. Then we increase the number of ships by 1 by adding the most economical ship (from those not used), always checking for feasibility, and calculate the new minimum total operating cost.

Besides the above "exact" and "approximate" options, our program gives the user the option to choose himself which ships to operate and which to lay-up and run the SIMPLEX for this and any other fleets of his choice, provided they are feasible (i.e. their maximum carrying capacity is less than or equal to the cargo). For each ship in the fleet that will not be used, the program asks the user to specify the lay-up charges. These charges include all lay-up costs (including capital cost) for that vessel. In the first two options the program adds the lay-up cost directly.

Another important conclusion is that the above schemes for the lay-up problem can be also used in the case that some of the ship(s) in the fleet are chartered instead of being laid-up. This can be obtained by using negative lay-up costs for the chartered ships. These negative lay-up costs should represent the net revenue from the chartering(s).

In Section II it is noted that this model does not restrict the number of round trips per time period of a particular vessel to be an integer. This simplification generally results in a fractional number of round trips for a vessel at its optimum speed. A corresponding amount of cargo is considered transported on this fractional voyage and is included in the calculation that insures the cargo obligation is met. The more round trips a vessel makes, the less significant this restriction becomes. The usefulness of the results does not suffer if it is assumed there is some measure of flexibility in the vessel scheduling system, especially if this operation goes on for several consecutive years.

Selected results obtained by using the above (exact and approximate) lay-up methods are presented and discussed in Appendix 4.

IV. SENSITIVITY ANALYSIS

IV.1. Sensitivity Analysis for an Individual Ship

The sensitivity of the total annual operating cost for each ship with respect to a parameter p is given by the equation:

$$S_{C(X_i,Y_i)}^{p} = \frac{\partial C(X_i,Y_i)}{\partial p} \cdot \frac{p}{C(X,Y)} \qquad \text{(IV.1)}$$

where $C(X_i,Y_i)$ is the total annual operating cost for each ship as defined in eq. (II.4). The most important parameters for the sensitivity analysis are the fuel cost C_f, the ship power coefficients a and a_b and the fixed annual cost C_i (the last includes the manning cost, C_m, the cost of stores, supplies and equipment, C_e, the capital cost, C_c, the cost of maintenance and repair, C_r, the cost of administration, C_a, and the cost of changing the status of the vessel, C_s).

With respect to these parameters the sensitivity of the cost function is:

$$S_{C}^{C_f}(X_i,Y_i) = 1 - \frac{C_i + \dfrac{C_{li} + C_{ui}}{DWi} \cdot G_i(X_i,Y_i)}{C_i(X_i,Y_i)} \qquad \text{(IV.2)}$$

or approximately,

$$S_{C}^{C_f}(X_i,Y_i) = 1 - \frac{C_i}{C_i(X_i,Y_i)} \qquad \text{(IV.3)}$$

Also

$$S_{C}^{a_i}(X_i,Y_i) = \left(3B_iX_i^{3b_i-1} + 2C_iX_i^{2b_i-1} + D_iX_i^{b_i-1}\right) \frac{G_i(X_i,Y_i)}{C_i(X_i,Y_i)} \qquad \text{(IV.4)}$$

$$S_{C}^{a_{bi}}(X_i,Y_i) = \left(3B'Y_i^{3b_{bi}-1} + 2C'Y_i^{2b_{bi}-1} + D'Y_i^{b_{bi}-1}\right) \frac{G_i(X_i,Y_i)}{C_i(X_i,Y_i)} \qquad \text{(IV.5)}$$

$$S_C^{C_{mi}}(X_i,Y_i) = \frac{C_{mi}}{C_i(X_i,Y_i)} \qquad \text{(IV.6)}$$

The sensitivities of the cost function with respect to C_{ei} , C_{ci} , C_{ri} , C_{si} , are obtained from (IV.6) by replacing C_{mi} by each of these cost components respectively.

IV.2. Sensitivity Analysis for the Total Fleet Operating Cost

The sensitivity of the total operating cost $C(X_i,Y_i,\ldots,X_Z,Y_Z)$ with respect to the fuel price C_f is:

$$S_C^{C_f}(X_1,\ldots,Y_Z) = 1 - \frac{\sum_{i=1}^{Z}\left[C_i + \frac{C_{li}+C_{ui}}{DW_i} \cdot G(X_i,Y_i)\right]}{C(X_1,Y_1,\ldots,X_Z,Y_Z)} \qquad \text{(IV.7)}$$

The computer program developed calculates the above sensitivities of the cost functions $C(X_i,Y_i)$ and $C(X_1,Y_1,\ldots,X_Z,Y_Z)$ at the optimum point (sample outputs are presented in Appendix 1 and Appendix 2).

IV.3. How to Use Sensitivities

From equation (IV.1) we can obtain:

$$C_{new}(X_i,Y_i) = \left(1 + S_C^P(X_i,Y_i)\frac{\Delta P}{P}\right) \cdot C(X_i,Y_i) \qquad \text{(IV.8)}$$

which gives the new value of the cost function $C(X_i,Y_i)$ when a parameter P was changed by ΔP . Equation (IV.8) is an exact relation when the cost $C(X_i,Y_i)$ is a linear function of the parameter P . It can also be used generally in nonlinear cases, provided that we have small to moderate changes of the corresponding parameters. Note that the annual operating cost for each ship is a linear function of the fuel cost C_f and the constant costs C_m , C_e , C_c , C_a , C_z and C_s .

We can use equation (IV.6) in order to find the new operating cost of each ship for any change of the above costs, without any error. Equation (IV.8) can be written as follows:

$$C_{new}(X_i,Y_i) = \left(1 + S^{C_f}_{C(X_i,Y_i)} \cdot \frac{\Delta C_f}{C_f}\right) \cdot C(X_i,Y_i) \qquad \text{(IV.9)}$$

$$\text{with } \Delta C_f = C_{fnew} - C_f$$

Similar equations can be derived for coefficients, a , a_b and for any constant cost component.

Also for the total annual operating cost we have:

$$C_{new}(X_1,Y_1\ldots,X_z,Y_z) = \left(1 + S^{C_f}_{C}(X_i,\ldots,Y_z) \cdot \frac{\Delta C_f}{C_f}\right) \cdot C(X_1,\ldots,Y_z) \qquad \text{(IV.10)}$$

IV.4. <u>Superposition of Several Changes</u>

Recall the expression for the power:

$P = a \cdot X_i^b$ for the full load condition

and

$P_b = a_b \cdot Y_i^{b_b}$ for the ballast condition

Coefficients a and a_b increase when the resistance of each ship is increasing. Coefficient a increases between two successive hull cleanings. Coefficient a_b behaves similarly.

For concurrent changes of both a and a_b we can find the change of the annual operating cost: For each ship,

$$C_{new}(X_i,Y_i) = \left[1 + S^{a}_{C(X_i,Y_i)} \cdot \frac{\Delta a}{a} + S^{a_b}_{C(X_i,Y_i)} \cdot \frac{\Delta a_b}{a}\right] \cdot C(X_i,Y_i) \qquad \text{(IV.11)}$$

We can also find the new annual operating cost for each vessel for concurrent changes in a, a_b, C_f by using the expression:

$$C_{new}(X_i,Y_i) = \left[1 + S^{a}_{C(X_i,Y_i)} \cdot \frac{\Delta a}{a} + S^{a_b}_{C(X_i,Y_i)} \frac{\Delta a_b}{a} + S^{C_f}_{C(X_i,Y_i)} \cdot \frac{\Delta C_f}{C_f}\right] \cdot C(X_i,Y_i) \tag{IV.12}$$

We can see (Appendix 2) that the error in the calculation of the $C_{new}(X_i,Y_i)$ with the expressions (IV.14), (IV.15) is less than 5% for relatively big changes in a, a_b and C_f.

IV.5. Sensitivity of the Optimal Speeds

As it is shown in Appendix 3, the optimum operating speeds for the particular problem we are solving in this work are practically independent of the fuel cost C_f. They are also independent from the constant cost C_i. So we have that:

$$\frac{\partial X_c}{\partial C_f} = 0, \quad \frac{\partial Y_i}{\partial C_f} = 0 \qquad \text{for } i=1,\ldots,z$$

$$\frac{\partial X_i}{\partial C_i} = 0, \quad \frac{\partial Y_i}{\partial C_i} = 0$$

In other words when the fuel price or the constant cost is changed we can find the new operating cost by using the expressions (IV.9) to (IV.15) and leave X_i, Y_i unchanged. Using the above we can avoid a new run of the program when C_f or C_i are changed.

We must remember that the above result holds for a given fleet and not in cases when one or more ships are laid-up or chartered (on top of the changes in the cost components).

V. TIME-VARYING COST COMPONENTS

V.1. Introduction

Consider the initial formulation of the problem of Section II. To tackle the variable cost components problem, we will divide the one-year time horizon into a number of stages. We will represent the time-varying cost coefficients with step-wise functions. In other words, we will have constant costs in each stage, but we will find the optimum solution with respect to the same time horizon (one year) taking a single fleet deployment decision at the start of that year. We note here that the following analysis can be used for any other time horizon.

V.2. Variable Costs

The parametric costs which have been assumed constant in the intial problem formulation are described in Section II. These costs can be separated into three different kinds:

a. Fuel price C_f as defined in Section II.

b. Constant cost C_i (which is the sum of the manning, administrative, maintenance, supplies, equipment, capital investment and "make ready for sailing" costs).

c. Port and route charges C_ℓ and C_u for the loading and unloading port respectively.

V.3. Approximations for the Variable Costs

Suppose that any one of the three costs mentioned above is a given function $C_K(t)$ in a time interval equal to one year, that can be approximated by a step function. We will use step functions taking two or at most three discrete values over one year. Superposition of the changes in two or three different kinds of costs will be used in the more complicated cases.

V.4. Problem Formulation

(a) Round trips per year-tons carried per year.

Assuming a real number of round trips per time interval ΔT_j, within which the cost components will remain constant, the number of round trips in this time interval, RT_{ji}, made by vessel i as a function of the vessel's full load and ballast speeds X_{ji} and Y_{ji} is:

$$RT_{ji} = \frac{\Delta T_j - M_{ji}}{T_{li} + T_{ui} + \frac{D_r}{24V_r} + \frac{D_b}{24\ Y_{ji}} + \frac{D_f}{24\ X_{ji}}} \qquad (V.1)$$

where:

M_i = days per Δt_j the i^{th} vessel is out of service for maintenance and repair

T_{li}, T_{ui} = average days per round trip spent by the i^{th} ship in the loading and unloading ports.

The rest of the above symbols are defined in the list of symbols (pages v - vii).

The number of round trips per year, RT_i, made by the i^{th} ship can be written in the form:

$$RT_i = \sum_{j=1}^{J} RT_{ji} \qquad (V.2)$$

where J is the number of the distinct time-intervals which will be defined in the analysis.

The number of tons carried per year is:

$$G_i(X_{1i}, Y_{1i}, \ldots, X_{Ji}, Y_{Ji}) = RT_i \cdot DW_i \quad , \qquad (V.3)$$

or

$$G_i(X_{1i},Y_{1i},\ldots,X_{Ji},Y_{Ji}) = \sum_{j=1}^{J} G_{ji}(X_{ji},Y_{ji})$$

where:

DW_i is the cargo capacity (in tons) for the i^{th} ship and

G_{ji} are the tons carried by the i^{th} ship in the j^{th} time interval.

(b) Costs involved in the problem.

The expressions for the constant cost C_i, the variable cost $f_i(X_i,Y_i)$, the operating cost per ton of cargo carried $F_i(X_i,Y_i)$ for the i^{th} ship used in the previous analysis are valid now only within the time intervals in which the cost coefficients are constant.

The new equations for the above costs and for the i^{th} ship are:

-variable cost per ton:

$$f_i(X_{1i},Y_{1i},\ldots X_{Ji},Y_{Ji}) = \sum_{j=1}^{J} \Big[A_{ji} + B_{ji} \cdot X_{ji}^{3b_i-1} + C_{ji}X_{ji}^{2b_i-1} + D_{ji}X_{ji}^{b_i-1} + B'_{ji} \cdot Y_{ji}^{3b_{bi}-1} + C'_{ji} \cdot Y_{Ji}^{2b_{bi}-1} + D'_{ji} \cdot Y_{ji}^{b_{bi}-1}\Big] \quad (V.4)$$

or $$f_i(X_{1i},Y_{1i},\ldots X_{Ji},Y_{Ji}) = \sum_{j=1}^{J} f_{Ji}(X_{ji},Y_{ji}) \quad (V.5)$$

where f_{Ji} is the variable cost per ton for the i^{th} ship in the j^{th} time interval.

-constant cost (independent of ship speeds)

$$C_i = \sum_{j=1}^{J} C_{ji} \quad (V.6)$$

$$\text{or} \quad C_i = \sum_{j=1}^{J} (C_{jm}+C_{je}+C_{jc}+C_{ja}+C_{jr}+C_{js}) \quad (V.7)$$

-cost per ton cargo carried

$$F_i(X_{1i},Y_{1i},\ldots X_{Ji},Y_{Ji}) = f_i(X_{1i},\ldots,Y_{Ki}) + \sum_{j=1}^{J} \frac{C_{ji}}{G_{ji}(X_{ji},Y_{ji})} \quad (V.8)$$

- Total annual operating cost of the i^{th} ship

$$C_i(X_{1i},Y_{1i},\ldots X_{Ji},Y_{Ji}) = \sum_{j=1}^{J} \left[f_{ji}(X_{ji},Y_{ji}) \cdot G_{ji}(X_{ji},Y_{ji}) + C_{ji}\right] \quad (V.9)$$

The symbols used in the above equations were defined in the list of symbols. The only difference is in the subscript $j = 1,\ldots,J$ which describes the correponding time interval in which the cost coefficients remain constant.

V.5. Problem Solution

The total annual operating cost C of a fleet consisting of Z ships is given by the expression:

$$C = \sum_{i=1}^{Z} C_i(X_{1i},Y_{1i},\ldots X_{Ji},Y_{Ji}) \quad (V.10)$$

$$\text{or} \quad C = \sum_{i=1}^{Z} \sum_{j=1}^{J} \left[f_{ji}(X_{ji},Y_{ji}) \cdot G_{ji}(X_{ji},Y_{ji}) + C_{ji}\right] \quad (V.11)$$

The equality constraint has the form:

$$\sum_{i=1}^{Z} G_i(X_{1i},Y_{1i},\ldots X_{Ji},Y_{Ji}) = \text{Cargo}$$

$$\text{or} \quad \sum_{i=1}^{Z} \sum_{j=1}^{J} G_{ji}(X_{ji},Y_{ji}) = \text{Cargo} \quad (V.12)$$

This constraint will be used to reduce the number of independent variables by one as presented in Section III.

The inequality constraints are:

$$X_{i\min} \leq X_{ji} \leq Y_{i\max} \qquad \text{for} \quad \begin{array}{l} i=1,2,\ldots,Z \\ j=1,2,\ldots,J \end{array} \qquad (V.13)$$
$$Y_{i\min} \leq Y_{ji} \leq Y_{i\max}$$

The above non-linear optimization problem can be solved using the combination of the Nelder and Mead "Simplex" search technique and External Penalty technique, already employed in the first phase of our research. We now have an optimization problem with $2 \cdot Z \cdot J-1$ unknown speeds and $2 \cdot ZJ$ inequality constraints.

The optimal solution obtained corresponds to a non-integer number of round trips per each time interval within which the cost coefficients are constant. In fact the optimal solution should give an <u>integer</u> number of round trips lying in a small region around that non-integer optimum. This capability is especially needed in intervals with small numbers of round trips. In order to find the optimal solution with integer number of round trips per each time interval, a sequential optimization approach has been used. From the obtained real numbers of round trips RT_{ji} $(j=1,\ldots,J, i=1,\ldots,Z)$ which form a $J \times Z$ matrix, we construct all possible such matrices with elements RT'_{ji}. Each element RT'_{ji} will be the integer part of RT_{ji} or $RT_{ji}+1$. At the same time, the matrix with elements $\Delta_{ji} = RT'_{ji} - [RT_{ji}]$ where $[RT_{ji}]$ represents the integer part of the real RT_{ji}, should have as many as possible zero elements. Furthermore, each matrix (RT'_{ji}) has to comply with the constraint:

$$\text{Cargo} \leq \sum_{i=1}^{Z} \sum_{j=1}^{J} RT'_{ji} \cdot DW_i \leq \text{Cargo} + \frac{1}{Z} \cdot \sum_{i=1}^{J} DW_i \qquad (V.14)$$

In equation (V.14) the term $\frac{1}{Z} \cdot \sum_{i=1}^{Z} DW_i$ is used to avoid the stiff equality constraint, which combined with the fact that we use discrete values for the capacities, possibly will not allow any integer solution. The above term allows us to look for an integer optimum in a relatively large region around the initial non-integer optimum.

The operating cost $C_{ji}(X_{ji},Y_{ji})$ of each ship per each time interval Δt_j can be approximated by its derivatives with respect to the speeds X_{ij}, Y_{ij} as follows:

$$C_{ji}(X_{ji},Y_{ji}) = \alpha_{ji} + \beta_{ji} \cdot X_{ji} + \gamma_{ji} \cdot Y_{ji} \qquad (V.15)$$

$$\text{with } \alpha_{ji} = C_{ji}(\overline{X}_{ji},\overline{Y}_{ji}) - \frac{\partial C_{ji}(\overline{X}_{ji},\overline{Y}_{ji})}{\partial X_{ji}} \cdot \overline{X}_{ji} - \frac{\partial C_{ji}(\overline{X}_{ji},\overline{Y}_{ji})}{\partial Y_{ji}} \cdot \overline{Y}_{ji}$$

$$\beta_{ji} = \frac{\partial C(\overline{X}_{ji},\overline{Y}_{ji})}{\partial X_{ji}}$$

$$\gamma_{ji} = \frac{\partial C(\overline{X}_{ji},\overline{Y}_{ji})}{\partial Y_{ji}}$$

where $\overline{X}_{ji}$, $\overline{Y}_{ji}$ are the ship speeds which correspond to the initial real solution.

Coefficients β_{ji} and γ_{ji} can be calculated by the following expressions:

$$\beta_{ji} = \Big[B_{ji} \cdot (3b_i - 1) \cdot X_{ji}^{3b_i - 2} + C_{ji}\ (2b_i - 1)\ X_{ji}^{2b_i - 2} +$$

$$D_{ji} \cdot (b_i - 1) \cdot X_{ji}^{b_i - 2}\Big] \cdot G_{ji}\ (X_{ji},\ Y_{ji}) + \qquad (V.16)$$

$$\frac{(\Delta T_{ji} - M_{ji}) \cdot DW_i}{\left(T_{\ell i} + T_{ui} + \frac{D_r}{24 \cdot V_{ri}} + \frac{D_b}{24 \cdot Y_{ji}} + \frac{D_f}{24 \cdot X_{ji}}\right)^2} \cdot \frac{D_f}{24 \cdot X_{ji}^2} \cdot f_{ji}(X_{ji},Y_{ji})$$

$$\gamma_{ji} = \left[B'_{ji}\cdot(3b_{bi}-1)\cdot Y_{ji}^{3b_{bi}-2} + C'_{ji}(2b_{bi}-1)\cdot Y_{ji}^{2b_{bi}-2} + \right.$$

$$\left. + D'_{ji}(b_{bi} - 1)\cdot Y_{ji}^{b_{bi}-2}\right]\cdot G_{ji}(X_{ji},Y_{ji}) + \qquad (V.17)$$

$$\frac{(\Delta T_{ji} - M_{ji})DW_i}{\left(T_{\ell i} + T_{ui} + \frac{D_r}{24\cdot V_{ri}} + \frac{D_b}{24\cdot Y_{ji}} + \frac{D_f}{24\cdot X_{ji}}\right)^2} \cdot \frac{D_b}{24\cdot Y^2_{ji}} \cdot f_{ji}(X_{ji},Y_{ji})$$

Note that X_{ji} and Y_{ji} in the above expressions are the optimim speeds $\overline{X}_{ji}$, $\overline{Y}_{ji}$ already obtained. If the inventory cost is included in the analysis, the new β_{ji} and γ_{ji} , say β'_{ji} and γ'_{ji} are:

$$\beta'_{ji} = \beta_{ji} - \frac{r_{ji}}{365} \cdot \frac{C_{cji}}{1-t_{ji}} \cdot \frac{D_f}{24} \cdot \frac{1}{X^2_{ji}} \; G_{ji}(X_{ji},Y_{ji})$$

$$+ \frac{r_{ij}}{365} \cdot \frac{C_{cji}}{1-t_{ji}} \cdot \left(\frac{D_f}{24\cdot X_{ji}} + \frac{Dr/2}{24\cdot V_{ri}}\right) \cdot$$

$$\frac{(\Delta T_{ji} - M_{ji})DW_i}{\left(T_{\ell i} + T_{ui} + \frac{D_r}{24\cdot V_{ri}} + \frac{D_b}{24\cdot Y_{ji}} + \frac{D_f}{24\cdot X_{ji}}\right)^2} \cdot \frac{D_f}{24\cdot X^2_{ji}} \qquad (V.18)$$

$$\gamma'_{ji} = \gamma_{ji} + \frac{r_{ji}}{365} \cdot \frac{C_{cji}}{1-t_{ji}} \cdot \left(\frac{D_f}{24\cdot X_{ji}} + \frac{Dr/2}{24\cdot V_{ri}}\right)\cdot$$

$$\cdot \frac{(\Delta T_{ji} - M_{ji})}{\left(T_{\ell i} + T_{ui} + \frac{D_r}{24\cdot V_{ri}} + \frac{D_b}{24\cdot Y_{ji}} + \frac{D_f}{24\cdot X_{ji}}\right)^2} \cdot \frac{D_b}{24\cdot Y^2_{ji}} \qquad (V.19)$$

The expression for the round trips per each time interval which we used till now can be transformed as follows:

$$X_{ji} = \frac{RT'_{ji} \cdot TC \cdot Y_{ji}}{(TA_{ji}-RT_{ji} \cdot TB_i) \cdot Y_{ji} - RT'_{ji} \cdot TD} \qquad (V.20)$$

with

$$TA_{ji} = \Delta t_j - M_{ji}$$

$$TB_i = T_{li} + T_{ui} + \frac{D_r}{24 \cdot V_{ri}}$$

$$TC = \frac{D_f}{24} \quad \text{and} \quad TD = \frac{D_b}{24}$$

Using equation (V.20) equation (V.15) gives:

$$C_{ji}(Y_{ji}) = \alpha_{ji} + \beta_{ji} \frac{RT'_{ji} \cdot TC \cdot Y_{ji}}{(TA_{ji}-RT_{ji} \cdot TB_i)Y_{ji} - TD \cdot RT'_{ji}} + \gamma_{ji} \cdot Y_{ji} \qquad (V.21)$$

From the above equation and for any value of the variable RT'_{ji} we can find the corresponding optimum ballast speed $Y_{ji}^{(0)}$ by using the equation:

$$\frac{\partial C(Y_{ji}^{(0)})}{\partial Y_{ji}} = 0 \qquad \text{or}$$

$$Y_{ji}^{(0)} = \frac{RT'_{ji}\ TD + \left(\frac{TC \cdot TD \cdot \beta_{ji}}{\gamma_{ij}}\right)^{0.5}}{TA_{ji} - RT'_{ji} \cdot TB_i} \qquad (V.22)$$

(note that always $TA_{ji} > RT_{ji} \cdot TB_i$)

Also for the optimum full-loaded speed $X^{(0)}_{ji}$ we have:

$$X^{(0)}_{ji} = \left(\frac{\gamma_{ji}}{\beta_{ji}} \cdot \frac{TC}{TD}\right)^{0.5} \cdot Y^{(0)}_{ji} \qquad (V.23)$$

Now using equations (V.21) to (V.23) for all possible matrices RT'_{ji} we calculate the corresponding values for $X^{(0)}_{ji}$, $Y^{(0)}_{ji}$, and $C_{ji}(Y^{(0)}_{ji})$. The new total operating cost C_o for each case can be found as follows:

$$C_o = \sum_{i=1}^{Z} \sum_{j=1}^{J} C_{ji}(Y^{(0)}{}_{ji}) \qquad (V.24)$$

From the values obtained for the total operating cost C_o, the minimum is selected.

Note that in the above heuristic the inequality constraints (eq. V.13) have been ignored. This is a reasonable approximation resulting from the fact that we are expecting relatively small changes in the initial ship speeds.

A computer program has been developed to solve the inital problem (with real number at round trips per time interval for each ship) using the above methods. The "integer solution" is an option of our program. The user can select this option whenever he expects that the final solution with integer RT_{ji} will be substantially different from the one initially found. It should be mentioned here that in any case only a small difference exists in the values of the total operating cost between the non-integer and integer solutions. This is clearly demonstrated in Table 5.2, even though the non-integer round trips per each time interval are relatively small numbers.

VI. PROBABILISTIC APPROACH TO THE FLEET DEPLOYMENT PROBLEM

VI.1. Introduction

The problem of fleet deployment when the cost coefficients are random variables with known probability density functions over a particular time interval will be examined in this section. We will try to give analytical expressions for the basic statistical quantities of the problem.

VI.2. Problem Formulation

The cost components which have been assumed to be known functions of time in Section V will now be assumed to be random variables with known probability density functions. These parameters are the fuel price C_f , the constant costs for each ship C_i , the port and route charges for each ship C_{pri} . We now assume that our random variables can be either fully dependent or fully independent statistically. This assumption makes sense for most or all of our cost components. To give an example, manning costs for all ships of a given flag will be very similar, and can be thought of as being statistically fully dependent random variables, whereas manning costs across different flag ships will be essentially statistically independent random variables. In this formulation, assume K groups of ships each group being statistically independent from any other, and consisting of L_K , $K=1,\ldots,K$ ships each one being perfectly correlated to any other ship in its group. For example in each group we will have the constant costs C_{Kl} , $l=1,\ldots,L_K$ which can be expressed as:

$$C_{Kl} = a_{Kl} + b_{Kl} \cdot C_{Km} \quad , \quad \text{with} \quad \begin{array}{l} K=1,\ldots,K \\ l=1,\ldots,L_m, \ (\ell \neq m) \end{array} \tag{VI.1}$$

for $m = 1,2,\ldots,L_K$.

As a second example, the port and route charges for ships working along the same route should be perfectly correlated random variables and we may also assume that they can be expressed as:

$$C_{pri} = d_{pri} + e_{pri} \cdot C_{prn} \quad , \quad i = 1,\ldots,Z (i \neq n) \qquad \text{(VI.2.)}$$

for $n = 1,2,\ldots,Z$.

With the above in mind, let us return to the formulation of our problem.

The objective function (from Section II) is:

$$C(X_1,Y_1,\ldots,X_Z;Y_Z) = \sum_{i=1}^{Z} C_i(X_i,Y_i)$$

with:

$$X_{imin} \leq X_i \leq X_{imax} \qquad \text{for } i = 1,\ldots,Z$$
$$Y_{imin} \leq Y_i \leq Y_{imax}$$

and

$$\sum_{i=1}^{Z} G_i(X_i,Y_i) = \text{Cargo}$$

(symbols were defined in Section II).

For the annual operating cost of each ship $C_i(X_i,Y_i)$ we have the expression (equation II.1):

$$C_i(X_i,Y_i) = F_i(X_i,Y_i)$$

or $\quad C(X_i,Y_i) = f_i(X_i,Y_i) \cdot G_i(X_i,Y_i) + C_i$

Using equations II.7 to II.9 and II.28 we can separate the random variables C_f , C_{pre} , C_{me} $(m = 1,2,\ldots,K)$ from the deterministic quantities X_i , Y_i , a , b , g , d , p_d , DW_i , S , D_f , D_b , D_r , V_r , M , T_l , T_u , a_{me} , b_{me} , d_{pri} , e_{pri} . The annual operating cost $G_i(X_i,Y_i)$ can be written as:

$$C_i(X_i,Y_i) = W_i(X_i,Y_i) \cdot C_f + Z_i(X_i,Y_i) \cdot C_{pri} + C_i \qquad \text{(VI.1)}$$

where

$$W_i(X_i,Y_i) = \left[\frac{D_{ri} \cdot F_{ri} \cdot P_{ri}}{V_{ri} \cdot DW_i} + \frac{F_{li} \cdot T_{li} + F_{ui} \cdot T_{ui}}{DW_i} + \right.$$

$$\frac{d_f \cdot g_i \cdot a^3}{P_{fi}^2 \cdot DW_i} \cdot X_i^{3b_i-1} + \frac{D_f \cdot s_i \cdot a_i^2}{P_{fi} \cdot DW_i} \cdot X_i^{2b_i-1} + \tag{VI.3}$$

$$\frac{D_f \cdot d_i \cdot a_i}{DW_i} \cdot X_i^{b_i-1} + \frac{D_b \cdot g_i \cdot a_{bi}^3}{P_{fi}^2 \cdot DW_i} \cdot Y_i^{3b_{bi}} +$$

$$\left. \frac{D_b \cdot s_i \cdot a^2_{bi}}{P_{fi} \cdot DW_i} \cdot Y_i^{2b_{bi}-1} + \frac{D_b \cdot d_i \cdot a_{bi}}{DW_i} \cdot Y_i^{b_{bi}-1} \right]$$

$$\cdot\, G_i(X_i,Y_i)$$

and

$$Z_i(X_i,Y_i) = \frac{G_i(X_i,Y_i)}{DW_i} \cdot C_{pri} \qquad \text{or}$$

$$= RT_i(X_i,Y_i) \cdot C_{pri} \tag{VI.4}$$

For the total annual operating cost of a fleet with Z ships we have:

$$C = \sum_{i=1}^{Z} C_i(X_i,Y_i)$$

$$C = C_f \cdot \sum_{i=1}^{Z} W_i(X_i,Y_i) + \sum_{i=1}^{Z} Z_i(X_i,Y_i) \cdot C_{pri} + \sum_{i=1}^{Z} C_i \tag{VI.5}$$

and using equations (VI.1) and (VI.2) we have:

$$C = C_f \sum_{i=1}^{Z} W_i(X_i,Y_i) + \sum_{\substack{i=1\\ i\neq n}}^{Z} Z_i(X_i,Y_i) \cdot d_{pri} +$$

$$\left[\sum_{\substack{i=1\\ (i\neq n)}}^{Z} Z_i(X_i,Y_i) \cdot e_{pri} + Z_n(X_n,Y_n) \right] \cdot C_{prn} +$$

$$\sum_{k=1}^{K} \left[\sum_{\substack{l=1 \\ l \neq m}}^{L_k} a_{kl} + \left(\sum_{\substack{l=1 \\ l \neq m}}^{L_k} b_{ki} + 1 \right) C_{km} \right] \tag{VI.6}$$

Equation (VI.6) shows that the total annual operating cost C is a linear function of the random variables C_f , C_{prn} , C_{km} $(k = 1,\ldots,K)$. Using the equality constraint $\sum_{i=1}^{Z} G_i(X_i,Y_i) = \text{Cargo}$ in order to eliminate the variable X_Z from the equation VI.4 we have:

$$C = C_f \cdot \left[\sum_{i=1}^{Z-1} W_i(X_i,Y_i) + W_Z(X_Z,Y_Z) \right] + \sum_{i=1}^{Z-1} Z_i(X_i,Y_i) \cdot d_{pri} +$$

$$Z_Z(X_Z,Y_Z) \cdot d_{prZ} + \left[\sum_{\substack{i=1 \\ (i \neq n)}}^{Z-1} Z_i(X_i,Y_i) e_{pri} + Z_Z(X_Z,Y_Z) \cdot e_{prZ} + \right. \tag{VI.7}$$

$$\left. Z_n(X_n,Y_n) \right] \cdot C_{prn} + \sum_{k=1}^{K} \left[\sum_{\substack{l=1 \\ (l \neq e)}}^{L_k} a_{kl} + \left(\sum_{\substack{l=1 \\ l \neq k}}^{L_k} b_{kl} + 1 \right) \cdot C_{km} \right]$$

which is also a linear function of the previously mentioned variables. (The expression for the dependent variable X_Z as function at X_1 , Y_1 ,...Y_Z is given by equation III.1). Our problem now is to find the probabilistic properties (density function, mean, variance) of the total annual operating cost C given the probabilistic properties of the r.v. C_f , C_{prm} , C_{km} (k = 1,...,K).

VI.3. Probabilistic Properties of the Total Annual Operating Cost

Probability Density Function (PDF)

In order to obtain the PDF of C we have to know the joint PDF f_{C_f} , C_{prm} , C_{1m} ,...C_{Km} $(C_f, C_{prm}, C_{1m},\ldots,C_{Km})$. The variables C_f , C_{prm} , C_{km} (k = 1,...,K) are independent and their joint PDF takes the form:

$$f_{C_f, C_{1m},\ldots C_{Km}, C_{prm}}(C_f,C_{prm},C_{1m},\ldots,C_{Km}) = f_{C_f}(C_f) \cdot f_{C_{prm}}(C_{prm}) \cdot$$

$$\cdot f_{C_{1m}}(C_{1m}) \ldots \cdot f_{C_{Km}}(C_{Km}) \tag{VI.8}$$

Using the auxiliary random variables:

$$S_1 = C_{1m}, \ldots, S_K = C_{Km}, S_{K+1} = C_{prn}$$

we form the following system of K+2 equations:

$$C = A \cdot C_f + B \cdot C_{prn} + \sum_{k=1}^{K} (D_k + E_k \cdot C_{km}) + F = g_1(C_f, C_{prm}, C_{1m}, \ldots C_{Km})$$

$$\begin{aligned} S_1 &= C_{1m} &&= g_2(C_{1m}) \\ &\vdots && \vdots \\ S_K &= C_{Km} &&= g_{K+1}(C_{Km}) \\ S_{K+1} &= C_{prn} &&= g_{K+2}(C_{prn}) \end{aligned} \qquad \text{(VI.9)}$$

where:

$$A = \sum_{i=1}^{Z-1} W_i(X_i, Y_i) + W_Z(X_Z, Y_Z)$$

$$B = \sum_{\substack{i=1 \\ (i \neq m)}}^{Z-1} Z_i(X_i, Y_i) e_{pri} + Z_Z(X_Z, Y_Z) e_{prZ} + Z_m(X_m, Y_m)$$

$$D_m = \sum_{\substack{l=1 \\ (l \neq m)}}^{L_k} a_{kl} \qquad \text{(VI.10)}$$

$$E_m = \sum_{\substack{l=1 \\ (l \neq n)}}^{L_k} b_{kl} + 1$$

$$F = \sum_{i=1}^{Z-1} Z_i(X_i, Y_i) d_{pri} + Z_Z(X_Z, Y_Z) \cdot d_{prZ}$$

The above system has the following properties:

(a) It can be solved uniquely for C_f, C_{prn}, C_{km} $(k = 1, \ldots, K)$ in terms of C, $S_1, \ldots, S_{K+1}$. The solution is:

$$C_f = \frac{1}{A}\left[C - B \cdot C_{prn} - \sum_{k=1}^{K} (D_k + E_k \cdot C_{km}) - F \right]$$

$$\begin{aligned} C_{1m} &= S_1 \\ &\vdots \\ C_{Km} &= S_K \\ C_{prn} &= S_{K+1} \end{aligned} \qquad \text{(VI.11)}$$

(b) The functions g_i $(i = 1,\ldots,K+2)$ have continuous partial derivatives at all points $(C_f, C_{prn}, C_{1m},\ldots,C_{Km})$ and such that the Jacobian determinant

$$J(C_f, C_{prn}, C_{1m},\ldots,C_{Km}) = \begin{vmatrix} \frac{\partial g_1}{\partial C_f} & \frac{\partial g_1}{\partial C_{1m}} & \cdots & \frac{\partial g_1}{\partial C_{Km}} & \frac{\partial g_1}{\partial C_{prn}} \\ \vdots & \vdots & & \vdots & \vdots \\ \frac{\partial g_{K+2}}{\partial C_f} & \frac{\partial g_{K+2}}{\partial C_{1m}} & \cdots & \frac{\partial g_{K+2}}{\partial C_{Km}} & \frac{\partial g_{K+2}}{\partial C_{prn}} \end{vmatrix} \qquad \text{(VI.12)}$$

is different from zero.

From VI.9 and VI.12 we have that:

$$J(C_f, C_{prn}, C_{1m},\ldots,C_{Kn}) = A$$

The PDF $f_{C,S_1,S_2,\ldots,S_{K+1}}(C,S_1,\ldots,S_{K+1})$ can be expressed as:

$$f_{C,S_1,S_2,\ldots,S_{K+1}}(C,S_1,\ldots,S_{K+1}) = \frac{1}{A} \cdot f_{C_f,C_{1m},\ldots,C_{Km},C_{pre}}(C_f,C_{1m},\ldots,C_{Km},C_{prm})$$

and using equation VI.8 we have that:

$$f_{C,S_1,\ldots,S_{K+1}}(C,S_1,\ldots,S_{K+1}) = \frac{1}{A} f_{C_f}(C_f) \cdot f_{C_{prn}}(C_{pre}) \cdot f_{C_{1m}}(C_{1m}) \cdots f_{C_{Km}}(C_{Km})$$

or

$$f_{C,C_{1m},\ldots,C_{Km},C_{prn}}(C,C_{1m},\ldots,C_{Km},C_{pn}) = \frac{1}{A} \cdot f_{C_f}(C_f)\cdot f_{C_{prn}}(C_{prn})\cdot f_{C_{1m}}(C_{1m})\ldots f_{C_{Km}}(C_{Km}) \quad \text{(VI.13)}$$

Finally the PDF for the r.v. C can be obtained as follows:

$$f_C(c) = \int_{(C_{1m})}\int_{(C_{2m})}\ldots\int_{(C_{Km})}\int_{(C_{pm})} \frac{1}{A} \cdot f_{C_f}(C_f)\cdot f_{C_{pm}}(C_{pm}) f_{C_{1m}}(C_{1m})\ldots f_{C_{Km}}(C_{Km}) \cdot dC_{prm}\cdot dC_{1m}\ldots dC_{Km} \quad \text{(VI.14)}$$

Where $(C_{1m}),\ldots(C_{Km}),(C_{prn})$ are the regions of all possible values for the r.v. $C_{1m},\ldots,C_{Km},C_{prn}$. Using equations (VI.11) the above integral yields

$$f_C(c) = \frac{1}{A}\int_{(C_{1m})}\int_{(C_{2m})}\ldots\int_{(C_{Km})}\int_{(C_{pm})} f_{C_f}\Big(\frac{1}{A}\cdot\big(C-B\cdot C_{prn}-\sum_{k=1}^{K}(D_k+E_k\cdot C_{km})-F\big)\Big)\cdot f_{C_{prn}}(C_{prn})\cdot f_{C_{1m}}(C_{1m})\ldots f_{C_{Km}}(C_{Km})\cdot dC_{prn}\cdot dC_{1m}\ldots dC_{Km} \quad \text{(VI.15)}$$

Expected Value of C

Since the total annual operating cost can be written as a linear function of C_f, C_{prn}, C_{km} $(k = 1,\ldots,K)$ its expected value $\overline{C}$ is also a linear function of the expected values $\overline{C}_f$, $\overline{C}_{prn}$, $\overline{C}_{km}$ $(k = 1,\ldots,K)$ of the above r.v. It is given by:

$$\overline{C} = A \cdot \overline{C}_f + B\cdot\overline{C}_{prn} + \sum_{k=1}^{K}(D_k+E_k\cdot C_{km}) + F \quad \text{(VI.16)}$$

Variance of C

For the variance of C we have the expression:

$$\mathrm{Var}(C) = A^2 \cdot \mathrm{Var}(C_f) + B^2\cdot\mathrm{Var}\big(C_{prn}\big) + \sum_{k=1}^{K} E_k{}^2\cdot\mathrm{Var}(C_{km}) \quad \text{(VI.17)}$$

VI.4. Formulation of the Optimization Problem

Under our probabilistic framework, the optimization process (as it was described in Sections II and III) has to be applied for the mean values of the randomly changing costs C_f, C_{prn}, C_{km} $(k = 1,\ldots,K)$. The optimal annual operating cost which will be found is also the minimum of the possible mean values of the total annual operating cost C. Then using the optimal values obtained for the speeds X_i, Y_i $(i = 1,\ldots,Z)$ and equations VI.3, VI.4, VI.10, VI.15, VI.17 we can find the PDF and the variance for the minimum total annual operating cost C.

VII. SUMMARY AND CONCLUSIONS

Various versions of the one-source, one-destination optimal fleet deployment problem have been formulated and solved. The operating cost components involved have been properly identified. Analytical expressions for the cost components that are a function of ship full load and/or ballast speeds have been presented (Section II). It was observed that, although the total operating costs/ton for a given ship or the total annual operating costs for a given fleet are almost constant in a range about the optimal speeds, the dependence of total annual operating costs on these speeds is much more pronounced when a lay-up option is available, i.e. when the fleet composition is allowed to change. This is what makes the problem interesting and its solution rewarding and worthwhile.

To get the optimal speeds and the associated minimum total annual operating costs, the SIMPLEX nonlinear optimization technique was used (Section III). Most cases studied and implemented in the IBM PC/XT personal computer did not require more than 20 minutes of computer time (or 1 to 5 seconds of mainframe computer CPU time). However, even if this method was not available, the analytical expressions and plots for the total operating costs/ton of cargo carried can be used as a guide to the decision maker.

It is interesting to note at this point that the typical "bathtub" curve (Fig. II.4) for the total operating costs/ton is much more dependent on the full load speed than on the ballast speed. However, when inventory costs are included, we have an increase in the full load speeds and a decrease in the ballast speeds (see Tables 1.7 and 1.8).

A sensitivity analysis on the above costs has also been performed

(Section IV). Fleet operators can use the results of that section to calculate how sensitive is the total annual operating cost with respect to several important cost components. Furthermore, they can predict the new total operating costs, if one or more cost components change by either small or moderate amounts, with considerable accuracy, and in almost no time at all. However, this accuracy will decrease if several cost components that are highly non-linear functions of the speed are changed simultaneously and all by a large amount. In such a (highly improbable) case, the full SIMPLEX program has to be used.

An important conclusion from the sensitivity analysis is that the optimal operating speeds of a given fleet (no lay-ups available) are independent of not only all constant costs (see Section II for definitions and details) but also of the fuel price. For the case of a minimum with zero first derivatives with respect to X_{ji} and Y_{ji} a formal proof is given in Appendix 3.

The lay-up problem has been dealt with in two different ways, one exact and one approximate (Section III). If time constraints prohibit the user from using either of the two methods, a "good practice," evident from our examples, is to lay-up ships with higher minimum operating costs/ton of cargo carried. In that case, one must pay attention that a particular lay-up does not force the speeds of the remaining ship(s) outside the "flat" region of their total operating cost/ton curves.

It is important to note here that whenever lay-up is allowed the solution of the fleet deployment problem (generally a fleet of fewer ships than the number of ships in the original fleet) will be much more sensitive to changes in the various cost components than the corresponding feasible fleets with a higher number of vessels operating. Table 4.1 presents one such example.

Considerable time has been spent by the project participants in developing and testing the computer programs that implement the proposed solution methods and algorithms. Sample runs for representative fleets with or without lay-up and their sensitivity analyses have been presented in Sections III and IV and in Appendices 1, 2, and 4.

The fleet deployment problem with time-varying cost components has been studied in Sections V and VI. A time horizon in this formulation is any interval within which cost components are constant but at least one of them is different than its value in another interval. In other words, our cost components are given "staircase" functions of time. In the case of rapidly changing costs, resulting in rather short intervals where these costs are constant, the problem of non-integer number of round trips per interval could be crucial. A heuristic approach has been developed to find the nearest "integer" solution corresponding to the non-integer solution generally provided by the SIMPLEX algorithm (see Section V). Examples using the computer program developed to solve the case of time-varying cost components may be found in Appendix 5.

The fleet deployment problem for the case when some of the cost components are random variables with known probability density functions is treated in Section VII. We note here that the minimum of the possible mean values of the total annual operating costs, C_{min} and the variance at C_{min} can be found relatively "easily." However, implementation of this approach on a computer has not yet been done and probably will not prove very useful even if it is done: The inputs to the problem, i.e. the user-supplied density functions, can have any theoretical or experimental form, thus discouraging the development of any general computer code for this problem.

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Note: A full Bibliography on the subject, consisting of over 70 references, has also been prepared but is not included in this report.

APPENDICES

APPENDIX 1: ORIGINAL PROBLEM-EXAMPLES

1. Characteristics of Vessels used in Examples

In this appendix two sample fleets consisting of three and ten ships respectively are presented. In these fleets we use three different types of tankers. The basic characteristics of each type are as follows:

Vessel A - Largest of the fleet, capable of carrying 100,000 tons of cargo

- Newest vessel in the fleet
- Most efficient ship in the fleet

Vessel B - Capable of carrying 50,000 tons of cargo

Vessel C - Capable of carrying 50,000 tons of cargo

- Not as efficient to operate as Vessels A or B

The first fleet consists of three ships, one of each type, and the second of ten ships, four of type A, three of type B and three of type C. Table 1.2 is a list of parameter values assigned to vessels A, B, C for the following problems. Explanation and units for the program variables are given in Table 1.1.

2. Results with different EPSI - different initial points

These are given in Table 1.3. We can see that the needed CPU time strongly depends upon the stopping limit EPSI. We can see also that there is not a serious improvement in the calculation of the Total Annual operating cost as EPSI is decreased, hence we may use relatively big values for the stopping limit EPSI. A conservative practice is to use EPSI = (10 to 20)$\cdot$Z$\cdot$K, where Z,K were defined in the report. In Table 1.5 sample results for the fleet of three ships and in Table 1.6 for the fleet of ten ships are presented. Results with sensitivities for the fleet of ten ships are presented in Table 1.7. In these examples we used the following route data:

3 ships	10 ships	
BALLEG = 800.	BALLEG = 3200.	(nmiles)
FULLEG = 1000.	FULLEG = 4000.	(nmiles)
RESLEG = 400.	RESLEG = 1600.	(nmiles)
CARGO = 6000000.	CARGO = 6700000.	(tons)
$FUEL = 0.11	$FUEL = 0.11	($/lb)

3. Optimization problem for one ship

In the case of one ship, an optimization problem with two unknowns, the full load and the ballast speed, also exists. For this particular case, an example is presented in Table 1.4.

4. Inventory cost

Results with inventory cost are presented in Table 1.8. The used inventory parameters are:

Cargo value per ton	CCOST = $120.
Interest rate	RIN = 0.12 (12%)
Tax rate	TR = 0.48 (48%)

Table 1.1

Program Variables

Name	Description	Units
ALPHA -	Power coefficient for the full load condition.	
ALPHAB -	Power coefficient for the ballast load condition.	
BALLEG -	Nautical miles per round trip a vessel operates in ballast condition.	(nmiles/RT)
BETA -	Power exponent for the full load condition.	
BETAB -	Power exponent for the full load condition.	
CAP -	Full load cargo capacity of a vessel.	(tons)
CARGO -	Contracted amount of cargo to be carried.	(tons)
CCOST -	Capital cost per ton of the carried load	($)
FULLEG -	Nautical miles per round trip a vessel operates in full load condition.	(nmiles/RT)
FULPWR -	Power required to operate vessel in full load condition at maximum speed.	(hp)
GAMMA -	Coefficient for the all purpose fuel rate (full load cond.).	
SIGMA -	"	
DELT -	"	
GAMMAB -	Coefficient for the all purpose fuel rate (ballast cond.).	
SIGMAB -	"	
DELTB -	"	
HBSPD -	Maximum value of ballast speed.	(knots)
HISPD -	Maximum value of full load ship speed.	(knots)
LBSPD -	Minimum value of ballast speed.	(knots)
LOWSPD -	Minimum acceptable value of full load ship speed.	(knots)
LPDAY -	Average days per round trip a vessel spends in the load port.	(days/RT)

Table 1.1 (continued)

Name	Description	Units
LPFUEL -	Average amount of fuel burned per day in load port.	(lb/day)
MRDAY -	Days per year a vessel is out of service for maintenance and repair.	(days/yr)
RIN -	Interest rate (absolute value)	
RSTFR -	All-purpose fuel rate at restricted speed.	(lb/hp-hr)
RSTLEG -	Nautical miles per round trip a vessel operates in restricted condition.	(nmiles/RT)
RSTPWR -	Average power required in restricted operation.	(hp)
RSTSPD -	Average speed at which a vessel travels in restricted operation.	(knots)
TR -	Tax rate (absolute value)	
UPDAY -	Average days per round trip a vessel spends in the unload port.	(days/RT)
UPFUEL -	Average amount of fuel burned per day in unload port.	(lb/day)
$ADMIN -	Annual cost of administration.	($/yr)
$CAPTL -	Annual cost of capital.	($/yr)
$FUEL -	Price of propulsion fuel.	($/lb)
$LAYUP -	Annual cost of lay-up.	($/yr)
$LP -	Port and route charges per round trip connected with the load port.	($/RT)
$MAINT -	Annual cost of maintenance & repair.	($/yr)
$MAN -	Annual cost of manning.	($/yr)
$SAIL -	Annual cost of changing the status of a vessel.	($/yr)
$STORE -	Annual cost of stores, supplies & equipment.	($/yr)
$UP -	Port and route charges per round trip connected with the unload port.	($/RT)

Table 1.2

Table of Input Values for Vessels A,B,C

SHIPNM = Vessel A

CAP = 100,000.0

RSTSPD = 7.0 RSTPWR = 2,000.0 RSTFR = 0.600

LPDAY = 2.0 $LP = 2,000.0 LPFUEL = 24,000.0

UPDAY = 2.0 $UP = 2,000.0 UPFUEL = 24,000.0

$MAN = 875,000.0 $STORE = 225,000.0 $CAPTL = 3,000,000.0

$ADMIN = 1,000,000.0 $MAINT = 630,000.0 $SAIL = 0.0

ALPHA = 5.09 BETA = 3.0 GAMMA = .2279 SIGMA = 0.446968 DELT = 0.635729

ALPHAB = 3. BETAB = 3. GAMMAB = .2279 SIGMAB = 0.446968 DELTB = 0.635729

$LAYUP = 3700000.0

MRDAY = 15.0 LOWSPD = 10.0 HISPD = 17.0 LBSPD = 8.0 HBSPD = 20.0

FULPWR = 25000.0

COOST = 0.0 RIN = 0.0 TR = 0.0

SHIPNM = Vessel B

CAP = 50,000.0

BALSPD = 14.0 BALPWR = 10,000.0 BALFR = 0.395

RSTSPD = 7.0 RSTPWR = 1,500.0 RSTFR = 0.534

LPDAY = 1.25 $LP = 1,000 LPFUEL = 24,000.0

UPDAY = 1.25 $UP = 1,000.0 UPFUEL = 24,000.0

$MAN = 750,000.0 $STORE = 150,000.0 $CAPTL = 1,500,000.0

$ADMIN = 400,000.0 $MAINT = 425,000.0 $SAIL = 0.0

ALPHA = 2.78 BETA = 3.1 GAMMA = 0.20514 SIGMA = 0.4 DELT = 0.57715

ALPHAB = 2.0 BETAB = 3.1 GAMMA = 0.20 SIGMAB = 0.4 DELTB = 0.57

Table 1.2 (continued)

$LAYUP = 1800000.

MRDAY = 15.0 LOWSPD = 9.0 HISPD = 16.0 LBSPD = 7.0 HBSPD = 22.0

FULPWR = 15000.0

CCOST = 0.0 RIN = 0.0 TR = 0.0

SHIPNM = Vessel C

CAP = 50,000.0

BALSPD = 14.0	BALPWR = 10,000.0	BALFR = 0.417
RSTSPD = 7.0	RSTPWR = 1,500.0	RSTFR = 0.550
LPDAY = 1.5	$LP = 1,000.0	LPFUEL = 24,000.0
UPDAY = 1.5	$UP = 1,000.0	UPFUEL = 24,000.0
$MAN = 825,000.0	$STORE = 175,000.0	$CAPTL = 1,000,000.0
$ADMIN = 360,000.0	$MAINT = 425,000.0	$SAIL = 0.0

ALPHA = 2.1	BETA = 3.2	GAMMA = 0.2165	SIGMA = 0.42462	DELT = 0.604
ALPHAB = 1.6	BETAB = 3.2	GAMMAB = 0.21	SIGMAB = 0.42	DELTB = 0.60

$LAYUP = 1300000.0

MRDAY = 15.0 LOWSPD = 9.0 HISPD = 16.0 LBSPD = 7.0 HBSPD = 22.0

FULPWR = 15000.0

CCOST = 0.0 RIN = 0.0 TR = 0.0

Table 1.3

Total Annual Operating Cost and

CPU Time* for Several EPSI

	3 SHIPS		10 SHIPS		
EPSI	TOTAL ANNUAL OPER. COST	SPENT CPU SEC.	TOTAL ANNUAL OPER. COST*	TOTAL ANNUAL OPER. COST**	SPENT CPU SEC.
5.	17.8593×10^6	0.151	64.4418×10^6	64.4304×10^6	3.591
10.	17.8593×10^6	0.123	64.4497×10^6	64.4306×10^6	3.213
20.	17.8594×10^6	0.119	64.4498×10^6	64.4315×10^6	2.552
200.	$17,8596 \times 10^6$	0.110	64.4813×10^6	64.4464×10^6	1.175
2000.	$17,9083 \times 10^6$	0.067	64.9116×10^6	64.8950×10^6	0.211

*In the Amdahl 470 mainframe computer of The University of Michigan Computing Center

**Starting from initial points different than the ones used in the previous column of this table

Table 1.4

RESULTS FOR THE ONE SHIP EXAMPLE

```
1TABLE OF INPUT VALUES
-SHIP NUMBER ----    1
 SHIP NAME ------  VESSEL A
 CARGO CAPACITY -  100000.0
 RSTSPD --    7.00          RSTPWR --     2000.00       RSTFR ---      0.6000
 LPDAY ---    2.00          $LP -----     2000.00       LPFUEL --   24000.00
 UPDAY ---    2.00          $UP -----     2000.00       UPFUEL --   24000.00
 $MAN ----     875000.00    $STORE --      225000.00    $CAPTL --    3000000.00
 $ADMIN --    1000000.00    $MAINT --      630000.00    $SAIL ---           0.0
 ALPHA ---    5.09000015    BETA ----     3.00000000    GAMMA ---    0.22793400
 SIGMA ---   -0.44696802    DELT ----     0.63572901
 ALPHAB ---     3.00000000    BETAB ----     3.00000000    GAMMAB ---      0.22799999
 SIGMAB ---    -0.44700003    DELTB ----     0.63000000
 $LAYUP --    3700000.00
 MRDAY ---    15.00         LOWSPD --     10.00         HISPD ---     17.00
                                   LBSPD --      8.00          HBSPD ---     20.00
 FULPWR --   25000.00
 CCOST ---         0.0     RIN --  0.0      TR --   .0
 ROUTE INPUT DATA
 ANNUAL CARGO AVAILABLE --      3000000.0
 BALLAST DISTANCE (NM)----      800.0
 LOADED DISTANCE (NM)-----     1000.0
 RESTRICTED DISTANCE (NM)-      400.0
 FUEL PRICE PER LB -------    0.1100
 FLEET #   1 - SUMMARY
 SHIP                        *COST       COST       ROUND      INITIAL    INITIAL     *TONS      CARGO
                             PER         PER        TRIPS      OPTIMUM    OPTIMUM     CARRIED    UTILITY
                             YEAR        TON        PER        SPEED      BAL.SPEED   (TONS)
                             ($)         ($)        YEAR       (KNOTS)    (KNOTS)
===================   ========   ========   ========   ========   ========   ========   ========
 SHIP#  1                                                 16.93      11.80
 VESSEL A                    8.7045      2.9015     30.00      13.12      15.80       3.0000     0.8999
-TOTAL ANNUAL OPERATING COSTS ($millions) =      8.7045
 Iterations for this run =        39
0* - Millions
                       SENS. FOR THE ANNUAL OPER. COST OF EACH SHIP
                       ============================================
                   $FUEL   ALPHA   ALPHAB   $MAN   $STORE   $CAPT   $ADM   $MAINT   $SAIL
                   -------------------------------------------------------------------------

SHIP#  1
 VESSEL A           .3279   .1227   0.082   .1005   0.026   0.345   .115    0.072   0.0
           SENS. FOR THE TOTAL OPER. COST FOR $FUEL : 0.328
```

Table 1.5

THREE SHIPS - EPSI = .5

1FLEET # 1 - SUMMARY

SHIP	*COST PER YEAR ($)	COST PER TON ($)	ROUND TRIPS PER YEAR	INITIAL OPTIMUM SPEED (KNOTS)	INITIAL OPTIMUM BAL.SPEED (KNOTS)	*TONS CARRIED (TONS)	CARGO UTILITY
SHIP# 1				12.20	12.20		
VESSEL A	8.6577	2.9001	29.85	12.93	15.73	2.9854	0.8955
SHIP# 2				12.20	12.20		
VESSEL B	4.9493	3.1251	31.67	11.56	12.99	1.5837	0.8145
SHIP# 3				15.48	12.20		
VESSEL C	4.2523	2.9717	28.62	10.54	11.51	1.4309	0.7768

-TOTAL ANNUAL OPERATING COSTS ($millions) = 17.8593
Iterations for this run = 241
0* - Millions

SENS. FOR THE ANNUAL OPER. COST OF EACH SHIP

	$FUEL	ALPHA	ALPHAB	$MAN	$STORE	$CAPT	$ADM	$MAINT	$SAIL
SHIP# 1 VESSEL A	.3244	.1209	0.081	.1011	0.026	0.347	.116	0.073	0.0
SHIP# 2 VESSEL B	.3356	.1204	0.087	.1515	0.030	0.303	.081	0.086	0.0
SHIP# 3 VESSEL C	.3316	.1177	0.086	.1940	0.041	0.235	.085	0.100	0.0

SENS. FOR THE TOTAL OPER. COST FOR $FUEL : 0.329

Table 1.6

1. TEN SHIPS EPSI = 10.

SHIP	*COST PER YEAR ($)	COST PER TON ($)	ROUND TRIPS PER YEAR	INITIAL OPTIMUM SPEED (KNOTS)	INITIAL OPTIMUM BAL.SPEED (KNOTS)	*TONS CARRIED (TONS)	CARGO UTILITY
SHIP# 1				12.60	12.60		
VESSEL A	9.1830	9.1965	9.99	12.69	15.89	0.9985	0.8557
SHIP# 2				12.60	12.60		
VESSEL A	9.0999	9.1926	9.90	12.39	15.91	0.9899	0.8483
SHIP# 3				12.60	12.60		
VESSEL A	9.4104	9.2173	10.21	13.54	15.78	1.0209	0.8749
SHIP# 4				12.60	12.60		
VESSEL A	9.2919	9.2071	10.09	13.42	15.26	1.0092	0.8649
SHIP# 5				12.60	12.60		
VESSEL B	4.8699	10.6023	9.19	10.90	12.36	0.4593	0.7481
SHIP# 6				12.60	12.60		
VESSEL B	4.7723	10.6125	8.99	10.68	11.81	0.4497	0.7324
SHIP# 7				12.60	12.60		
VESSEL B	5.0149	10.6018	9.46	11.27	13.09	0.4730	0.7704
SHIP# 8				12.60	12.60		
VESSEL C	4.1709	9.8465	8.47	9.88	11.18	0.4236	0.7020
SHIP# 9				12.60	12.60		
VESSEL C	4.3918	9.8736	8.90	10.59	12.03	0.4448	0.7371
SHIP# 10				15.96	12.60		
VESSEL C	4.2448	9.8495	8.62	10.25	11.28	0.4310	0.7142

-TOTAL ANNUAL OPERATING COSTS ($millions) = 64.4497
Iterations for this run = 2176
* - Millions

2. Same as above - different initial speeds

SHIP	*COST PER YEAR ($)	COST PER TON ($)	ROUND TRIPS PER YEAR	INITIAL OPTIMUM SPEED (KNOTS)	INITIAL OPTIMUM BAL.SPEED (KNOTS)	*TONS CARRIED (TONS)	CARGO UTILITY
SHIP# 1				12.90	12.90		
VESSEL A	9.4328	9.2198	10.23	13.24	16.46	1.0231	0.8768
SHIP# 2				12.90	12.90		
VESSEL A	9.2697	9.2026	10.07	13.00	15.87	1.0073	0.8632
SHIP# 3				12.90	12.90		
VESSEL A	9.1536	9.1933	9.96	12.87	15.36	0.9957	0.8533
SHIP# 4				12.90	12.90		
VESSEL A	9.2811	9.2052	10.08	13.33	15.34	1.0083	0.8641
SHIP# 5				12.90	12.90		
VESSEL B	4.8896	10.6010	9.22	11.01	12.37	0.4612	0.7512
SHIP# 6				12.90	12.90		
VESSEL B	4.8098	10.6079	9.07	10.68	12.16	0.4534	0.7384
SHIP# 7				12.90	12.90		
VESSEL B	4.9449	10.5994	9.33	11.21	12.55	0.4665	0.7598
SHIP# 8				12.90	12.90		
VESSEL C	4.2192	9.8477	8.57	10.28	11.01	0.4284	0.7100
SHIP# 9				12.90	12.90		
VESSEL C	4.1909	9.8463	8.51	10.00	11.18	0.4256	0.7054
SHIP# 10				9.21	12.90		
VESSEL C	4.2391	9.8489	8.61	10.24	11.24	0.4304	0.7133

-TOTAL ANNUAL OPERATING COSTS ($millions) = 64.4306
Iterations for this run = 2940
0* - Millions

Table 1.7

SAMPLE OUTPUT WITH SENSITIVITIES

TEN SHIPS - EPSI = 5.

SHIP	*COST PER YEAR ($)	COST PER TON ($)	ROUND TRIPS PER YEAR	INITIAL OPTIMUM SPEED (KNOTS)	INITIAL OPTIMUM BAL.SPEED (KNOTS)	*TONS CARRIED (TONS)	CARGO UTILITY
SHIP# 1				12.90	12.90		
VESSEL A	9.4365	9.2203	10.23	13.25	16.48	1.0235	0.8771
SHIP# 2				12.90	12.90		
VESSEL A	9.2757	9.2031	10.08	13.03	15.84	1.0079	0.8638
SHIP# 3				12.90	12.90		
VESSEL A	9.1446	9.1927	9.95	12.86	15.32	0.9948	0.8525
SHIP# 4				12.90	12.90		
VESSEL A	9.2656	9.2037	10.07	13.29	15.32	1.0067	0.8627
SHIP# 5				12.90	12 90		
VESSEL B	4.8933	10.6008	9.23	11.05	12.34	0.4616	0.7518
SHIP# 6				12.90	12.90		
VESSEL B	4.8211	10.6064	9.09	10.75	12.15	0.4545	0.7403
SHIP# 7				12.90	12.90		
VESSEL B	4.9411	10.5994	9.32	11.20	12.53	0.4662	0.7592
SHIP# 8				12.90	12.90		
VESSEL C	4.2216	9.8479	8.57	10.29	11.02	0.4287	0.7104
SHIP# 9				12.90	12.90		
VESSEL C	4.1964	9.8462	8.52	10.06	11.14	0.4262	0.7063
SHIP# 10				9.21	12.90		
VESSEL C	4.2346	9.8487	8.60	10.19	11.28	0.4300	0.7125

-TOTAL ANNUAL OPERATING COSTS ($millions) = 64.4304

Iterations for this run = 3073

* - Millions

SENS. FOR THE ANNUAL OPER. COST OF EACH SHIP

	$FUEL	ALPHA	ALPHAB	$MAN	$STORE	$CAPT	$ADM	$MAINT	$SAIL
SHIP# 1									
VESSEL A	.3884	.1559	0.108	.0927	0.024	0.318	.106	0.067	0.0
SHIP# 2									
VESSEL A	.3779	.1537	0.103	.0943	0.024	0.323	.108	0.068	0.0
SHIP# 3									
VESSEL A	.3691	.1518	0.100	0957	0.025	0.328	.109	0.069	0.0
SHIP# 4									
VESSEL A	.3772	.1568	0.100	.0944	0.024	0.324	.108	0.068	0.0
SHIP# 5									
VESSEL B	.3372	.1357	0.097	1533	0.031	0.307	.082	0.087	0.0
SHIP# 6									
VESSEL B	.3273	.1317	0.096	.1556	0.031	0.311	.083	0.088	0.0
SHIP# 7									
VESSEL B	.3435	.1375	0.099	.1518	0.030	0.304	.081	0.086	0.0
SHIP# 8									
VESSEL C	.3362	.1380	0.099	.1954	0.041	0.237	.085	0.101	0.0
SHIP# 9									
VESSEL C	.3323	.1341	0.100	.1966	0.042	0.238	.086	0.101	0.0
SHIP# 10									
VESSEL C	.3383	.1363	0.102	.1948	0.041	0.236	.085	0.100	0.0

SENS. FOR THE TOTAL OPER. COST FOR $FUEL : 0.360

Table 1.8

SAME CASE AS IN TABLE 1.7, INVENTORY COST INCLUDED

-SHIP	*COST PER YEAR ($)	COST PER TON ($)	ROUND TRIPS PER YEAR	INITIAL OPTIMUM SPEED (KNOTS)	INITIAL OPTIMUM BAL.SPEED (KNOTS)	*TONS CARRIED (TONS)	CARGO UTILITY
SHIP# 1 VESSEL A	9.2799	9.2597	10.02	12.90 14.52	12.90 13.43	1.0022	0.8589
SHIP# 2 VESSEL A	9.2049	9.2333	9.97	12.90 14.09	12.90 13.67	0.9969	0.8543
SHIP# 3 VESSEL A	9.3302	9.2490	10.09	12.90 14.41	12.90 13.88	1.0088	0.8645
SHIP# 4 VESSEL A	9.2445	9.2344	10.01	12.90 14.14	12.90 13.82	1.0011	0.8579
SHIP# 5 VESSEL B	4.8428	10.6659	9.08	12.90 12.05	12.90 10.51	0.4540	0.7395
SHIP# 6 VESSEL B	4.9377	10.6781	9.25	12.90 12.58	12.90 10.60	0.4624	0.7531
SHIP# 7 VESSEL B	4.8439	10.6576	9.09	12.90 11.97	12.90 10.62	0.4545	0.7402
SHIP# 8 VESSEL C	4.2152	9.9317	8.49	12.90 11.39	12.90 9.47	0.4244	0.7033
SHIP# 9 VESSEL C	4.3733	9.8994	8.84	12.90 11.51	12.90 10.58	0.4418	0.7321
SHIP# 10 VESSEL C	4.4958	9.9053	9.08	9.21 11.53	12.90 11.52	0.4539	0.7522

-TOTAL ANNUAL OPERATING COSTS ($millions) = 73.6122
Iterations for this run = 2610
O* - Millions

SENS. FOR THE ANNUAL OPER. COST OF EACH SHIP

	$FUEL	ALPHA	ALPHAB	$MAN	$STORE	$CAPT	$ADM	$MAINT	$SAIL
SHIP# 1 VESSEL A	.3782	.1756	0.088	.0943	0.024	0.323	.108	0.068	0.0
SHIP# 2 VESSEL A	.3732	.1678	0.090	.0951	0.024	0.326	.109	0.068	0.0
SHIP# 3 VESSEL A	.3815	.1735	0.091	.0938	0.024	0.322	.107	0.068	0.0
SHIP# 4 VESSEL A	.3758	.1686	0.091	.0947	0.024	0.325	.108	0.068	0.0
SHIP# 5 VESSEL B	.3303	.1469	0.081	.1549	0.031	0.310	.083	0.088	0.0
SHIP# 6 VESSEL B	.3431	.1536	0.081	.1519	0.030	0.304	.081	0.086	0.0
SHIP# 7 VESSEL B	.3305	.1461	0.082	.1548	0.031	0.310	.083	0.088	0.0
SHIP# 8 VESSEL C	.3353	.1538	0.079	.1957	0.042	0.237	.085	0.101	0.0
SHIP# 9 VESSEL C	.3591	.1560	0.093	.1886	0.040	0.229	.082	0.097	0.0
SHIP# 10 VESSEL C	.3765	.1562	0.104	.1835	0.039	0.222	.080	0.095	0.0

SENS. FOR THE TOTAL OPER. COST FOR $FUEL : 0.440

APPENDIX 2: SENSITIVITY ANALYSIS-EXAMPLES

The sensitivities of the cost functions $C_i(X_i,Y_i)$ and $C(X_i,Y_i,\ldots,X_z,Y_z)$ at the optimum point are given by the program (sample output in Appendix 1, Table 1.6).

With these sensitivities we can calculate the cost functions for any change in the fuel price C_f, in the constant costs and in the power coefficients α and α_b.

1. Change in the fuel price C_f

   In Table 2.1 the new minimum costs for final C_f = 0.07 \$/lb obtained by using eq. (IV.9) and (IV.13), are presented. In Table 2.2 we see the new minimum costs for Cf = 0.22 \$/lb also obtained by eq. (IV.9) and (IV.14). Comparing the above costs with these found by using the program with the new values of C_f we can see that differences in the total operating cost are less than 0.8%.

2. Change in power coefficients a and a_b.

   Using eq. (IV.19) with $\Delta a = + 0.1 \cdot a$ and $\Delta a_b = + 0.10 \cdot a_b$ for the case of three and ten ships respectively (results in Tables 2.3 and 2.4). The comparison of these results with those received from the program gives relatively small errors as it is shown in Table 2.3 and 2.4.

3. Change in the fuel price C_f and in the power coefficients a and ab.

   Using eq. (IV.20) with ΔC_f = + 0.11 \$/lb, $\Delta a = + 0.10 \cdot a$ and $\Delta a_b = 0.1 \cdot a_b$, we found the results shown in Table 2.5. The comparison of these with those received from the program (Table 2.6) gives very small errors.

4. The fact that the optimum operating speeds X_i, Y_i practically are independent of the fuel price C_f can be verified by comparing the results given in Tables 1.4 and 1.5 with these given in Tables 2.1 and 2.2.

Table 2.1

3 ships - Fuel cost C_f = 0.07 \$/lb (ΔC_f = - 37%)

i	1	2	3
$C_i(X_i,Y_i)$	7.6365×10^6	4.3453×10^6	3.7396×10^6
X_i	12.93	11.56	10.54
Y_i	15.73	12.99	11.51
TOTAL ANNUAL OPERATING COST = 15.7214×10^6			

Table 2.2

10 ships - Fuel cost C_f = 0.22 \$/lb (ΔC_f = + 100%)

i	X_1	Y_1	$C_i(X_i,Y_i)$*
1	12.80	15.68	12.5978
2	12.70	16.01	12.6378
3	13.26	15.83	12.9036
4	13.24	15.43	12.7656
5	11.11	12.40	6.5789
6	10.69	11.98	6.3395
7	11.21	13.40	6.8306
8	9.99	11.28	5.5984
9	10.19	12.02	5.8356
10	10.25	10.8	5.5773
TOTAL ANNUAL OPERATING COST*: 87.6408			

*In millions

Table 2.3

3 ships: $\Delta a = + 0.1 \cdot a$ and $\Delta a_b = + 0.1 \cdot a_b$

i	1	2	3
$C(X_i, Y_i)/10^6$	8.8325	5.0519	4.3389
ERROR*	-0.06%	+0.42%	+0.25%

TOTAL ANNUAL OPERATING COST: 18.2233×10^6 (error + 0.15%)

(*) Percent of the value received from the program

Table 2.4

10 ships - $\Delta a = + 0.1a$ and $\Delta a_b = + 0.1\ a_b$

i	$C(X_i, Y_i)/10^6$	ERROR*
1	9.4166	+0.32
2	9.4385	-0.7
3	9.5789	+0.58
4	9.5061	-0.66
5	9.0264	+1.96
6	4.8991	-2.3
7	5.1594	+4.9
8	4.2988	-0.7
9	4.425	+0.6
10	4.2875	-0.25

TOTAL ANNUAL OPERATING COST: 66.0362×10^6 \$ (error + 0.25%)

(*) Percent of the value received from the program

Table 2.5

3 ships: ΔC_f = 0.11 \$/lb, Δa = + 0.1a , Δa_b = + 0.1a_b

i	1	2	3
$C(X_i,Y_i)/10^6$	11.641	6.7129	5.749
ERROR*	-1.45%	-0.86%	-1.36%

TOTAL ANNUAL OPERATING COST: 24.1029 (error = 1.26%)

(*) Percent of the value received from the program

Table 2.6

Program Results with C_f = 0.22 \$/lb. Δa = + 1.1a, Δa_b = 1.1a_b

-SHIP	*COST PER YEAR ($)	COST PER TON ($)	ROUND TRIPS PER YEAR	INITIAL OPTIMUM SPEED (KNOTS)	INITIAL OPTIMUM BAL.SPEED (KNOTS)	*TONS CARRIED (TONS)	CARGO UTILITY
SHIP# 1				12.20	12.20		
VESSEL A	11.8120	3.9485	29.92	13.07	15.64	2.9915	0.8973
SHIP# 2				12.20	12.20		
VESSEL B	6.7711	4.2922	31.55	11.52	12.84	1.5775	0.8113
SHIP# 3				15.48	12.20		
VESSEL C	5.8287	4.0733	28.62	10.55	11.50	1.4309	0.7768

-TOTAL ANNUAL OPERATING COSTS ($millions) = 24.4118
Iterations for this run = 386
O* - Millions

SENS. FOR THE ANNUAL OPER. COST OF EACH SHIP

	$FUEL	ALPHA	ALPHAB	$MAN	$STORE	$CAPT	$ADM	$MAINT	$SAIL
SHIP# 1 VESSEL A	.5048	.1894	0.127	.0741	0.019	0.254	.085	0.053	0.0
SHIP# 2 VESSEL B	.5144	.1857	0.133	.1108	0.022	0.222	.059	0.063	0.0
SHIP# 3 VESSEL C	.5124	.1829	0.134	.1415	0.030	0.172	.062	0.073	0.0

SENS. FOR THE TOTAL OPER. COST FOR $FUEL : 0.509

APPENDIX 3: PROOF OF AN INDEPENDENCE PROPERTY OF THE OPTIMAL SPEEDS

For the symbols introduced in Section II.5 we use the following definitions:

$$A = C_{1i} \cdot C_f + C_{2i}$$
$$B = C_{3i} \cdot C_f$$
$$C = C_{4i} \cdot C_f$$
$$D = C_{5i} \cdot C_f \qquad \text{eq.(3.1)}$$
$$B' = C_{6i} \cdot C_f$$
$$C' = C_{7i} \cdot C_f$$
$$D' = C_{8i} \cdot C_f$$

with $C_2 = C_{port\ \&\ route}$

Using equations (II.1), (II.7), (II.8), (II.28), (III.2) and (3.1) the objective function takes the form:

$$C(X_1, \dots, Y_z) = \sum_1^Z \Big[\Big(C_{1i}C_f + C_{2i} + C_{3i}X_i^{3b-1} C_f + C_{4i}X_i^{2b-1} \cdot C_f +$$
$$+ C_{5i}X_i^{b-1} \cdot C_f + C_{6i}\, Y_i^{3b_b-1} \cdot C_f + C_{7i}\, Y_i^{2b_b-1} \cdot C_f +$$
$$+ C_{8i} \cdot Y_i^{b_b-1} \cdot C_f\Big) \,.\, G_i(X_i, Y_i) + C_i\Big] \qquad \text{eq.(3.2)}$$

Using the approximation:

$$C_{i2} << f_i(X_i, Y_i) \qquad \text{eq.(3.3)}$$

we can delete the term C_{2i} from eq. (3.2).

The new objective function is:

$$C(X_1, Y_1, \dots, Y_z) = \sum_1^Z \left[C_f \cdot E_i(X_i, Y_i) \cdot G_i(X_i, Y_i) + C_i\right]$$

with

$$E_i(X_i, Y_i) = C_{1i} + C_{3i}\, X_i^{3b-1} + C_{4i}X_i^{2b-1} + C_{5i}X_i^{b-1} +$$

$$+ C_{6i}Y_i^{3b_b-1} + C_{7i}Y_i^{1b_b-1} + C_{8i}Y_i^{b_b-1}$$

The optimum speeds can be found as the solution of the following system:

$$\frac{\partial C(X_i,Y_i,\ldots,X_z,Y_z)}{\partial X_i} = 0 \quad (1)$$

$$\frac{\partial C(X_i,Y_i,\ldots,X_z,Y_z)}{\partial Y_i} = 0 \quad (2)$$

for i=1,2,...,z eq.(3.4)

$$\sum_{1}^{z} G_i(X_i,Y_i) = 0 \quad (3)$$

$$X_{imin} < X_i < X_{imax} \qquad Y_{imin} < Y_i < Y_{imax} \quad (4)$$

$$\frac{\partial^2 C(X_1,\ldots,Y_z)}{\partial X_i} < 0 \quad (5)$$

$$\frac{\partial^2 C(X_i,\ldots,Y_z)}{\partial Y_i} < 0 \quad (6)$$

Equation (3.4.1) and (3.4.2) can be transformed as follows:

$$C_f \cdot \frac{\partial E_i(X_i,Y_i)}{\partial X_i} \cdot G_i(X_i,Y_i) + C_f \cdot \frac{\partial E_i(X_i,Y_i)}{\partial X_i} \cdot E_i(X_i,Y_i) = 0$$

$$C_f \cdot \frac{\partial E_i(X_i,Y_i)}{\partial Y_i} \cdot G_i(X_i,Y_i) + C_f \cdot \frac{\partial G_i(X_i,Y_i)}{\partial Y_i} \cdot E_i(X_i,Y_i) = 0$$

or

$$\frac{\partial E_i(X_i,Y_i)}{\partial X_i} \cdot G_i(X_i,Y_i) + \frac{\partial G_i(X_i,Y_i)}{\partial X_i} \cdot E_i(X_i,Y_i) = 0$$

$$\frac{\partial E_i(X_i,Y_i)}{\partial Y_i} \cdot G(X_i,Y_i) + \frac{\partial G_i(X_i,Y_i)}{\partial Y_i} \cdot E_i(X_i,Y_i) = 0 \qquad \text{eq.(3.5)}$$

Equations (3.5), (3.4.3) to (3.4.6) give a new system with respect to the unknown speeds X_i, Y_i $i=1,\ldots,Z$. We can see that the above equations do not contain either the fuel cost C_f or the constant cost C_i . Our final conclusion from the above analysis is that the optimum speeds (but not the optimal operating costs) are independent of C_f and C_i .

The above is a general conclusion for any slow steaming fleet deployment problem under the approximation of (eq. 3.3).

APPENDIX 4: LAY-UP ANALYSIS-EXAMPLES

1. Characteristics of Vessels Used in Examples

In this appendix two sample fleets consisting of four and ten ships respectively are presented. The first fleet consists of two ships of type A, and one from types B and C respectively (see Appendix 1 for the three types of ships used in the analysis). The second fleet is the ten ship fleet used in Appendix 1.

2. Sample Results

In Table 4.1 sample results for the first fleet are presented. In this example, we can see the difference between the exact and the approximate solution for the lay-up problem. The global optimum obtained using the "approximate" method is about 0.145×10^6\$ (0.74%) worse than that found with the "exact" procedure.

In Tables 4.2 and 4.3 sample results for the second fleet but for different route data are presented. We can see in these examples the strong relation which exists between the global optimum solution and the route data although the initial fleet remains the same. In Table 4.4 the total annual operating costs are presented as a function of the capacities of the fleets used in examples 4.2 and 4.3. We have to remember here that the costs mentioned previously include the lay-up costs as they were defined in Section III and given in Appendix 1. We can see that it is rather impossible to try to predict the global optimum solution before using our program. Generally, a good practice giving relatively small deviations from the global optimum is to avoid ship eliminations resulting in big increases for the remaining ship's speeds. That means that we have to operate our ships with speeds which give relatively small operating cost per ton of cargo carried for our route data (see Figure II.4).

Table 4.1

FOUR SHIPS LOCAL AND GLOBAL OPTIMAL SOLUTIONS

ROUTE INPUT DATA
ANNUAL CARGO AVAILABLE -- 6000000.0
BALLAST DISTANCE (NM)---- 800.0
LOADED DISTANCE (NM)----- 1000.0
RESTRICTED DISTANCE (NM)- 400.0
FUEL PRICE PER LB ------- 0.1100
FLEET # 1 - SUMMARY

SHIP	*COST PER YEAR ($)	COST PER TON ($)	ROUND TRIPS PER YEAR	INITIAL OPTIMUM SPEED (KNOTS)	INITIAL OPTIMUM BAL SPEED (KNOTS)	*TONS CARRIED (TONS)	CARGO UTILITY
SHIP# 1				6.50	6.50		
VESSEL A	6.5908	3.3480	19.69	6.44	6.77	1.9686	0.5954
SHIP# 2				6.50	6.50		
VESSEL A	6.6278	3.3074	20.04	6.54	7.08	2.0039	0.6061
SHIP# 3				6.50	6.50		
VESSEL B	3.7652	3.6300	20.74	6.00	6.60	1.0372	0.5424
SHIP# 4				6.39	6.50		
VESSEL C	3.3143	3.3468	19.81	6.07	6.14	0.9903	0.5462

TOTAL ANNUAL OPERATING COSTS ($millions) = 20.2980
Iterations for this run = 143
O* - Millions

SENS. FOR THE ANNUAL OPER. COST OF EACH SHIP

	$FUEL	ALPHA	ALPHAB	$MAIN	$STORE	$CAPT	$ADM	$MANT	$SAIL
SHIP# 1									
VESSEL A	.1187	.0408	0.022	.1328	0.034	0.455	152	0.096	0.0
SHIP# 2									
VESSEL A	.1234	.0425	0.024	1320	0.034	0.453	151	0.095	0.0
SHIP# 3									
VESSEL B	1325	.0388	0.027	1992	0.040	0.398	.106	0.113	0.0
SHIP# 4									
VESSEL C	1477	.0414	0.026	2489	0.053	0.302	.109	0.128	0.0

SENS. FOR THE TOTAL OPER. COST FOR $FUEL : 0.128

APPROXIMATE SOLUTION FOR THE LAY-UP PROBLEM

Number of ships which are eliminated. 2

Eliminated ship #: 3
Eliminated ship #: 4
Optimum cost (in millions): 20.5095

Number of ships which are eliminated 1

Eliminated ship #: 3
Optimum cost (in millions): 19.8484

*****GLOBAL OPTIMUM SOLUTION

1FLEET # 1 - SUMMARY

-SHIP	*COST PER YEAR ($)	COST PER TON ($)	ROUND TRIPS PER YEAR	INITIAL OPTIMUM SPEED (KNOTS)	INITIAL OPTIMUM BAL.SPEED (KNOTS)	*TONS CARRIED (TONS)	CARGO UTILITY
SHIP# 1				9.10	9.60		
VESSEL A	7.2266	2.9938	24.14	8.44	10.47	2.4138	0.7301
SHIP# 2				9.10	9.60		
VESSEL A	7.2451	2.9889	24.24	8.59	10.39	2.4240	0.7332
SHIP# 3				0.0	0.0		
VESSEL B	1.8000	0.0	0.0	0.0	0.0	0.0	0.0
SHIP# 4				6.22	10.60		
VESSEL C	3.5767	3.0776	23.24	7.46	8.15	1.1622	0.6410

TOTAL ANNUAL OPERATING COSTS ($millions) = 19.8484
Iterations for this run = 156
O* - Millions

Table 4.1

(continued)

SENS FOR THE ANNUAL OPER COST OF EACH SHIP

	$FUEL	ALPHA	ALPHAB	$MAIN	$STORE	$CAPT	$ADM	$MANT	$SAIL
SHIP# 1 VESSEL A	.1937	.0715	0 050	1211	0 031	0.415	.138	0 087	0.0
SHIP# 2 VESSEL A	.1957	0735	0 050	1208	0 031	0.414	138	0.087	0.0
SHIP# 3 VESSEL B	0	0	0 0	0	0 0	0.0	.0	0.0	0 0
SHIP# 4 VESSEL C	2084	.0668	0.049	.2307	0.049	0.280	101	0.119	0 0

SENS FOR THE TOTAL OPER COST FOR $FUEL 0 270

EXACT SOLUTION FOR THE LAY-UP PROBLEM

Number of ships which are eliminated 1
Optimum cost (in millions) 19 7034

Number of ships which are eliminated 2
Optimum cost (in millions) 20.5095

Number of ships which are eliminated . 3
Available cargo exceeds max capacity of any fleet

***** GLOBAL OPTIMUM SOLUTION

1FLEET # 1 - SUMMARY

-SHIP	*COST PER YEAR ($)	COST PER TON ($)	ROUND TRIPS PER YEAR	INITIAL OPTIMUM SPEED (KNOTS)	INITIAL OPTIMUM BAL.SPEED (KNOTS)	*TONS CARRIED (TONS)	CARGO UTILITY
SHIP# 1				9.00	9.50		
VESSEL A	7 1625	3 0108	23 79	8.30	10.06	2.3790	0 7196
SHIP# 2				9.00	9.50		
VESSEL A	7 1845	3 0048	23 91	8.51	9.92	2 3910	0 7232
SHIP# 3				6.14	10.50		
VESSEL B	4.0564	3 2977	24 60	7.64	8 56	1.2301	0 6433
SHIP# 4				0.0	0.0		
VESSEL C	1 3000	0.0	0.0	0.0	0 0	0 0	0 0

TOTAL ANNUAL OPERATING COSTS ($millions) = 19 7034
Iterations for this run = 153
O* - Millions

SENS. FOR THE ANNUAL OPER. COST OF EACH SHIP

	$FUEL	ALPHA	ALPHAB	$MAIN	$STORE	$CAPT	$ADM	$MANT	$SAIL
SHIP# 1 VESSEL A	.1867	.0693	0 047	1222	0 031	0 419	.140	0 088	0 0
SHIP# 2 VESSEL A	.1891	0721	0 046	1218	0 031	0 418	.139	0.088	0 0
SHIP# 3 VESSEL B	1928	0659	0 048	1849	0 037	0 370	099	0 105	0.0
SHIP# 4 VESSEL C	.0	0	0.0	.0	0.0	0 0	.0	0 0	0 0

SENS. FOR THE TOTAL OPER COST FOR $FUEL 0 243

Table 4.2

TEN SHIPS - INITIAL OPTIMUM/GLOBAL OPTIMUM

ROUTE INPUT DATA
ANNUAL CARGO AVAILABLE -- 4500000.0
BALLAST DISTANCE (NM)---- 3200.0
LOADED DISTANCE (NM)---- 4000.0
RESTRICTED DISTANCE (NM)- 1600.0
FUEL PRICE PER LB ------- 0.1100
FLEET # 1 - SUMMARY

SHIP	*COST PER YEAR ($)	COST PER TON ($)	ROUND TRIPS PER YEAR	INITIAL OPTIMUM SPEED (KNOTS)	INITIAL OPTIMUM BAL.SPEED (KNOTS)	*TONS CARRIED (TONS)	CARGO UTILITY
SHIP# 1				7.30	7.30		
VESSEL A	6.7956	10.2042	6.66	7.43	8.03	0.6660	0.5848
SHIP# 2				7.30	7.30		
VESSEL A	6.7976	10.1910	6.67	7.34	8.21	0.6670	0.5857
SHIP# 3				7.30	7.30		
VESSEL A	6.8103	10.1699	6.70	7.42	8.20	0.6697	0.5881
SHIP# 4				7.30	7.30		
VESSEL A	6.6815	10.4363	6.40	6.72	8.15	0.6402	0.5622
SHIP# 5				7.30	7.30		
VESSEL B	3.8194	11.8746	6.43	6.52	7.92	0.3216	0.5486
SHIP# 6				7.30	7.30		
VESSEL B	3.8156	11.8878	6.42	6.53	7.85	0.3210	0.5474
SHIP# 7				7.30	7.30		
VESSEL B	3.7751	12.0839	6.25	6.25	7.70	0.3124	0.5328
SHIP# 8				7.30	7.30		
VESSEL C	3.3230	10.8305	6.14	6.40	7.22	0.3068	0.5321
SHIP# 9				7.30	7.30		
VESSEL C	3.2552	11.1776	5.82	5.80	7.08	0.2912	0.5050
SHIP# 10				5.87	7.30		
VESSEL C	3.3104	10.8866	6.08	6.56	6.80	0.3041	0.5273

TOTAL ANNUAL OPERATING COSTS ($millions) = 48.3837
Iterations for this run = 341
O* - Millions

APPROXIMATE SOLUTION FOR THE LAY-UP PROBLEM

Number of ships which are eliminated: 6

Eliminated ship #: 5
Eliminated ship #: 6
Eliminated ship #: 7
Eliminated ship #: 8
Eliminated ship #: 9
Eliminated ship #:10
Optimum cost (in millions): 52.2170

Number of ships which are eliminated: 5

Eliminated ship #: 6
Eliminated ship #: 7
Eliminated ship #: 8
Eliminated ship #: 9
Eliminated ship #:10
Optimum cost (in millions): 49.5770

Number of ships which are eliminated: 4

Eliminated ship #: 7
Eliminated ship #: 8
Eliminated ship #: 9
Eliminated ship #:10
Optimum cost (in millions): 48.3832

Number of ships which are eliminated: 3

Eliminated ship #: 8
Eliminated ship #: 9
Eliminated ship #:10
Optimum cost (in millions): 47.7028

Table 4.2

(continued)

Number of ships which are eliminated: 2

Eliminated ship #: 9
Eliminated ship #:10
Optimum cost (in millions): 47.6450

Number of ships which are eliminated: 1

Eliminated ship #: 9
Optimum cost (in millions): 47.8890

*****GLOBAL OPTIMUM SOLUTION

1FLEET # 1 - SUMMARY

-SHIP	*COST PER YEAR ($)	COST PER TON ($)	ROUND TRIPS PER YEAR	INITIAL OPTIMUM SPEED (KNOTS)	INITIAL OPTIMUM BAL.SPEED (KNOTS)	*TONS CARRIED (TONS)	CARGO UTILITY
SHIP# 1 VESSEL A	6.9603	9.9221	7.01	8.40 7.83	8.40 8.84	0.7015	0.6160
SHIP# 2 VESSEL A	6.9760	9.8983	7 05	8.40 7.84	8.40 8.96	0.7048	0.6189
SHIP# 3 VESSEL A	6.9601	9.9189	7.02	8.40 7.75	8.40 8 98	0.7017	0.6162
SHIP# 4 VESSEL A	7.0318	9.8365	7.15	8.40 8.17	8.40 8.87	0.7149	0.6278
SHIP# 5 VESSEL B	3.9171	11.5132	6.80	7.40 7.18	7.90 8.23	0.3402	0.5803
SHIP# 6 VESSEL B	3.9215	11.4985	6.82	7.40 7.27	7.90 8.15	0.3410	0.5817
SHIP# 7 VESSEL B	3.9194	11.5073	6.81	7.40 7.40	7.90 7.92	0.3406	0.5809
SHIP# 8 VESSEL C	3.4779	10.3570	6.72	7.40 7.40	7.90 7.81	0.3358	0.5823
SHIP# 9 VESSEL C	1.3000	0.0	0.0	0.0 0.0	0.0 0.0	0.0	0.0
SHIP# 10 VESSEL C	3.3875	10.6024	6.39	5.71 6.67	7.90 7.73	0.3195	0.5541

-TOTAL ANNUAL OPERATING COSTS ($millions) = 47.8515
Iterations for this run = 257
0* - Millions

EXACT SOLUTION FOR THE LAY UP PROBLEM

Number of ships which are eliminated : 1
Optimum cost (in millions): 47.8890

Number of ships which are eliminated : 2
Optimum cost (in millions): 47.6450

Number of ships which are eliminated : 3
Optimum cost (in millions): 47.7028

Number of ships which are eliminated : 4
Optimum cost (in millions): 48.3832

Number of ships which are eliminated : 5
Optimum cost (in millions): 49.5770

Number of ships which are eliminated : 6
Optimum cost (in millions): 52.2170

Number of ships which are eliminated : 7
Available cargo exceeds max capacity of any fleet

Table 4.2

***** GLOBAL OPTIMUM SOLUTION (continued)

=======================

1FLEET # 1 - SUMMARY

-SHIP	*COST PER YEAR ($)	COST PER TON ($)	ROUND TRIPS PER YEAR	INITIAL OPTIMUM SPEED (KNOTS)	INITIAL OPTIMUM BAL.SPEED (KNOTS)	*TONS CARRIED (TONS)	CARGO UTILITY
SHIP# 1				9.40	9.40		
VESSEL A	7.3172	9.5762	7.64	9.18	9.44	0.7641	0.6710
SHIP# 2				9.40	9.40		
VESSEL A	7.3083	9.5853	7.62	9.17	9.38	0.7625	0.6695
SHIP# 3				9.40	9.40		
VESSEL A	7.2996	9.5966	7.61	9.19	9.29	0.7606	0.6680
SHIP# 4				9.40	9.40		
VESSEL A	7.3068	9.5861	7.62	9.16	9.38	0.7622	0.6694
SHIP# 5				8.40	8.90		
VESSEL B	4.0852	11.1215	7.35	8.17	8.76	0.3673	0.6265
SHIP# 6				8.40	8.90		
VESSEL B	4.0834	11.1256	7.34	8.19	8.71	0.3670	0.6260
SHIP# 7				8.40	8.90		
VESSEL B	4.0608	11.1678	7.27	8.09	8.60	0.3636	0.6202
SHIP# 8				5.35	8.90		
VESSEL C	3.5837	10.1634	7.05	7.65	8.70	0.3526	0.6115
SHIP# 9				0.0	0.0		
VESSEL C	1.3000	0.0	0.0	0.0	0.0	0.0	0.0
SHIP# 10				0.0	0.0		
VESSEL C	1.3000	0.0	0.0	0.0	0.0	0.0	0.0

TOTAL ANNUAL OPERATING COSTS ($millions) = 47.6450

Iterations for this run = 123

0* - Millions

Table 4.3

TEN SHIPS - GLOBAL & GLOBAL OPTIMAL SOLUTIONS
(Different route data)

ROUTE INPUT DATA
ANNUAL CARGO AVAILABLE -- 6200000 0
BALLAST DISTANCE (NM)---- 3200.0
LOADED DISTANCE (NM)----- 4000.0
RESTRICTED DISTANCE (NM)- 1600.0
FUEL PRICE PER LB ------- 0.1100
FLEET # 1 - SUMMARY

SHIP	*COST PER YEAR ($)	COST PER TON ($)	ROUND TRIPS PER YEAR	INITIAL OPTIMUM SPEED (KNOTS)	INITIAL OPTIMUM BAL.SPEED (KNOTS)	*TONS CARRIED (TONS)	CARGO UTILITY
SHIP# 1				11.20	11.20		
VESSEL A	8.4290	9.1895	9.17	11.43	13.26	0.9172	0.8055
SHIP# 2				11.20	11.20		
VESSEL A	8.5227	9.1865	9.28	11.79	13.25	0.9277	0.8147
SHIP# 3				11.20	11.20		
VESSEL A	8.6759	9.1789	9.45	11.93	13.99	0.9452	0.8300
SHIP# 4				11.20	11.20		
VESSEL A	8.6337	9.1791	9.41	11.70	14.13	0.9406	0.8260
SHIP# 5				11.20	11.20		
VESSEL B	4.5259	10.6836	8.47	9.75	10.93	0.4236	0.7225
SHIP# 6				11.20	11.20		
VESSEL B	4.5440	10.6760	8.51	9.77	11.08	0.4256	0.7259
SHIP# 7				11.20	11.20		
VESSEL B	4.5215	10.6884	8.46	9.95	10.58	0.4230	0.7215
SHIP# 8				11.20	11.20		
VESSEL C	3.9167	9.8858	7.92	8.96	10.23	0.3962	0.6871
SHIP# 9				11.20	11.20		
VESSEL C	3.9222	9.8826	7.94	9.02	10.18	0.3969	0.6883
SHIP# 10				14.19	11.20		
VESSEL C	3.9834	9.8634	8.08	9.36	10.25	0.4039	0.7004

TOTAL ANNUAL OPERATING COSTS ($millions) = 59.6751
Iterations for this run = 1106
O* - Millions

EXACT SOLUTION FOR THE LAY-UP PROBLEM
==================================

Number of ships which are eliminated : 1
Optimum cost (in millions): 61.0535

Number of ships which are eliminated : 2
Optimum cost (in millions): 62.7351

Number of ships which are eliminated : 3
Optimum cost (in millions): 65.6135

Number of ships which are eliminated : 4
Available cargo exceeds max capacity of any fleet

OPTIMUM FLEET IS THE INITIAL FLEET

Table 4.4

OPERATING COSTS FOR THE SAME FLEET AND DIFFERENT CARGO

TOTAL CAPACITY OF USED FLEETS/1000 (TONS)	CARGO = 45×10^5 TONS		CARGO = 62×10^5 TONS	
	ANNUAL COST ($ millions)	% DIFFERENCE FROM THE OPTIMUM SOLUTION	ANNUAL COST ($ millions)	% DIFFERENCE FROM THE OPTIMUM SOLUTION
400	52.2170	9.60	*	-
450	49.5770	4.05	*	-
500	48.3832	1.55	*	-
550	47.7028	0.12	65.6135	9.95
600	47.6450	**	62.7351	5.13
650	47.8890	0.51	61.0535	2.31
700	48.3837	1.55	59.6751	**

*Infeasible case

**Global Optimum

APPENDIX 5: TIME-VARYING COST COMPONENTS-EXAMPLES

1. How to Use the Program

The program requires a ship data base contining values for all vessel and route parameters used in the formulation of the problem. A sample input for a fleet of three ships is presented in Table 5.1. The parameter definitions and units are given in Table 1.1. Table 5.2 contains a sample output which corresponds to data contained in Table 5.1.

2. Examples

Two sample fleets consisting of three and ten ships respectively are examined. The first fleet consists of three different ships which are similar to the ones used in Appendix 1 but their cost characteristics are varying with time. Here we are using three time intervals within which costs remain constant. A list of the input values and the results obtained are presented in Table 5.2. The same table contains the approximate and the exact solution for the corresponding lay-up problem. The second fleet consists of ten ships and is similar to the one used in Appendix 1, but, here the costs are varying with the time, remaining constant within two time intervals. A list of the input values and the results for the initial fleet are presented in Table 5.3. In the same table, the approximate and the exact solution for the lay-up problem are presented.

In Table 5.4, results for the previous fleet but with different route data are presented. We can see again (compare wth results in Table 5.3) the dependence between the route data and the global optimum solution.

Table 5.1

SAMPLE INPUT FOR A FLEET OF THREE SHIPS

```
3
100
100
165
VESSEL A
100000.
7.0,2000.,0.600
2.,24000.
2.,24000.
5.09,3.0,.227934,-.446968,.635729
3.,3.,.228,-.447,.63
3700000.,1
6.,16.,5.,20.,25000.
0.,0.,0.
250000.,60000.,900000.,300000.,200000.,0.
600.,600.
4.
250000.,60000.,900000.,300000.,200000.,0.
600.,600.
4.
375000.,105000.,1200000.,400000.,230000.,0.
800.,800.
7.
VESSEL B
50000.
7.,1500.,.534
1.25,24000.
1.25,24000.
2.78,3.1,.205141,-.402271,.572156
2.,3.1,.2,-.4,.57
1800000.,1
5.,16.,5.,18.,15000
0.,0.,0.
230000.,45000.,450000.,130000.,130000.,0.
300.,300.
4.
230000.,45000.,450000.,130000.,130000.,0.
300.,300.
4.
290000.,60000.,600000.,140000.,165000.,0.
400.,400.
7.
VESSEL C
50000.
7.,1500.,.550
1.5,24000.
1.5,24000.
2.1,3.2,.216537,-.424620,.603943
1.6,3.2,.21,-.42,.6
1300000.,1
5.,16.,5.,18.,15000.
0.,0.,0.
250000.,50000.,300000.,100000.,130000.,0.
300.,300.
4.
250000.,50000.,300000.,100000.,130000.,0.
300.,300.
4.
325000.,75000.,700000.,160000.,165000.,0.
400.,400.
7.
4000000.
800
1000
400
.07
.11
.13
```

Table 5.2

SAMPLE OUTPUT FOR A FLEET OF THREE SHIPS

```
TABLE OF INPUT VALUES
=====================

 # OF TIME INTERVALS = 3
 TIME INTERVAL # 1:  100 DAYS
 TIME INTERVAL # 2:  100 DAYS
 TIME INTERVAL # 3:  165 DAYS
----------------------------------------
SHIP NUMBER ----     1

 SHIP NAME ------   VESSEL A
 CARGO CAPACITY -   100000.0
 RSTSPD --    7.00          RSTPWR --    2000.00       RSTFR ---      0.6000
 LPDAY ---    2.00          LPFUEL --   24000.00
 UPDAY ---    2.00          UPFUEL --   24000.00
 ALPHA ---    5.09000015    BETA ----    3.00000000    GAMMA ---     0.22793400
 SIGMA ---   -0.44696802    DELT ----   0.63572901
 ALPHAB ---    3.00000000    BETAB ----    3.00000000    GAMMAB ---    0.22799999
 SIGMAB ---   -0.44700003    DELTB ----    0.63000000
 $LAYUP --    3700000.00
 LOWSPD --     6.00         HISPD ---     16.00
 LBSPD --     5.00        HBSPD ---    20.00
 FULPWR --  25000.00
 CCOST ---        0.0      RIN --  0.0     TR --   .0

VARIABLE SHIP DATA
------------------
 TIME INTERVAL # 1
  $MAN ----      250000.00  $STORE --      60000.00  $CAPTL --      900000.00
  $ADMIN --      300000.00  $MAINT --     200000.00  $SAIL ---            0.0
  $LP -----      600.00  $UP -----     600.00
  MRDAY ---      4.00
 TIME INTERVAL # 2
  $MAN ----      250000.00  $STORE --      60000.00  $CAPTL --      900000.00
  $ADMIN --      300000.00  $MAINT --     200000.00  $SAIL ---            0.0
  $LP -----      600.00  $UP -----     600.00
  MRDAY ---      4.00
 TIME INTERVAL # 3
  $MAN ----      375000.00  $STORE --     105000.00  $CAPTL --     1200000.00
  $ADMIN --      400000.00  $MAINT --     230000.00  $SAIL ---            0.0
  $LP -----      800.00  $UP -----     800.00
  MRDAY ---      7.00
----------------------------------------
SHIP NUMBER ----     2

 SHIP NAME ------   VESSEL B
 CARGO CAPACITY -    50000.0
 RSTSPD --    7.00          RSTPWR --    1500.00       RSTFR ---      0.5340
 LPDAY ---    1.25          LPFUEL --   24000.00
 UPDAY ---    1.25          UPFUEL --   24000.00
 ALPHA ---    2.77999973    BETA ----    3.10000038    GAMMA ---     0.20514101
 SIGMA ---   -0.40227097    DELT ----   0.57215601
 ALPHAB ---    2.00000000    BETAB ----    3.10000038    GAMMAB ---    0.19999999
 SIGMAB ---   -0.39999998    DELTB ----    0.56999999
 $LAYUP --    1800000.00
 LOWSPD --     5.00         HISPD ---     16.00
 LBSPD --     5.00        HBSPD ---    18.00
 FULPWR --  15000.00
 CCOST ---        0.0      RIN --  0.0     TR --   .0
```

Table 5.2

(continued)

```
VARIABLE SHIP DATA
------------------
 TIME INTERVAL # 1
  $MAN ----      230000.00  $STORE --       45000.00  $CAPTL --      450000.00
  $ADMIN --      130000.00  $MAINT --      130000.00  $SAIL ---           0.0
  $LP -----      300.00  $UP -----     300.00
  MRDAY ---      4.00
 TIME INTERVAL # 2
  $MAN ----      230000.00  $STORE --       45000.00  $CAPTL --      450000.00
  $ADMIN --      130000.00  $MAINT --      130000.00  $SAIL ---           0.0
  $LP -----      300.00  $UP -----     300.00
  MRDAY ---      4.00
 TIME INTERVAL # 3
  $MAN ----      290000.00  $STORE --       60000.00  $CAPTL --      600000.00
  $ADMIN --      140000.00  $MAINT --      165000.00  $SAIL ---           0.0
  $LP -----      400.00  $UP -----     400.00
  MRDAY ---      7.00
----------------------------------------
SHIP NUMBER ----    3

 SHIP NAME ------   VESSEL C
 CARGO CAPACITY -    50000.0
 RSTSPD --     7.00         RSTPWR --     1500.00      RSTFR ---       0.5500
 LPDAY ---     1.50         LPFUEL --    24000.00
 UPDAY ---     1.50         UPFUEL --    24000.00
 ALPHA ---     2.10000038   BETA ----     3.19999981   GAMMA ---     0.21653700
 SIGMA ---    -0.42461997   DELT ----     0.60394299
 ALPHAB ---     1.60000038   BETAB ----     3.19999981   GAMMAB ---     0.20999998
 SIGMAB ---    -0.42000002   DELTB ----     0.60000002
 $LAYUP --     1300000.00
 LOWSPD --      5.00        HISPD ---     16.00
 LBSPD --     5.00        HBSPD ---     18.00
 FULPWR --    15000.00
 CCOST ---          0.0     RIN --   0.0     TR --    .0

VARIABLE SHIP DATA
------------------
 TIME INTERVAL # 1
  $MAN ----      250000.00  $STORE --       50000.00  $CAPTL --      300000.00
  $ADMIN --      100000.00  $MAINT --      130000.00  $SAIL ---           0.0
  $LP -----      300.00  $UP -----     300.00
  MRDAY ---      4.00
 TIME INTERVAL # 2
  $MAN ----      250000.00  $STORE --       50000.00  $CAPTL --      300000.00
  $ADMIN --      100000.00  $MAINT --      130000.00  $SAIL ---           0.0
  $LP -----      300.00  $UP -----     300.00
  MRDAY ---      4.00
 TIME INTERVAL # 3
  $MAN ----      325000.00  $STORE --       75000.00  $CAPTL --      700000.00
  $ADMIN --      160000.00  $MAINT --      165000.00  $SAIL ---           0.0
  $LP -----      400.00  $UP -----     400.00
  MRDAY ---      7.00
 ROUTE INPUT DATA
-----------------
 ANNUAL CARGO AVAILABLE --     4000000.0
 BALLAST DISTANCE (NM)----     800.0
```

Table 5.2

(continued)

LOADED DISTANCE (NM)----- 1000.0
RESTRICTED DISTANCE (NM)- 400.0
FUEL PRICE PER LB
TIME INTERVAL # 1: 0.0700
TIME INTERVAL # 2: 0.1100
TIME INTERVAL # 3: 0.1300

FLEET # 1 SUMMARY

SHIP	*COST PER INTERVAL ($)	COST PER TON ($)	ROUND TRIPS PER INTERVAL	INITIAL/ OPTIMUM SPEED (KNOTS)	INITIAL/ OPTIMUM BAL.SPEED (KNOTS)	*TONS CARRIED (TONS)
SHIP# 1						
VESSEL A						
TIME INTERVAL # 1	1.8937	3.2198	5.88	6.40 7.40	6.40 7.74	0.5882
TIME INTERVAL # 2	1.9302	3.5863	5.38	6.40 6.06	6.40 7.28	0.5382
TIME INTERVAL # 3	2.7407	3.0916	8.87	6.40 6.05	6.40 7.31	0.8865
SHIP# 2						
VESSEL B						
TIME INTERVAL # 1	1.0838	3.6945	5.87	5.40 6.35	5.40 6.77	0.2934
TIME INTERVAL # 2	1.0957	4.2800	5.12	5.40 5.23	5.40 5.64	0.2560
TIME INTERVAL # 3	1.4731	3.4848	8.45	5.40 5.36	5.40 5.52	0.4227
SHIP# 3						
VESSEL C						
TIME INTERVAL # 1	0.9435	3.1734	5.95	9.00 7.52	5.40 6.39	0.2973
TIME INTERVAL # 2	1.0033	3.5295	5.69	9.00 5.11	9.00 9.95	0.2843
TIME INTERVAL # 3	1.6558	4.2167	7.85	5.85 5.09	9.00 7.16	0.3927

DURING THE YEAR

	*COST/YR ($)	COST/TON ($)	R.T./YR	*TONS/YR	UTIL.FACTOR
SHIP# 1					
VESSEL A	6.565	3.261	20.129	2.013	0.613
SHIP# 2					

Table 5.2

(continued)

VESSEL B	3.653	3.758	19.441	0.972	0.519
SHIP# 3					
VESSEL C	3.603	3.698	19.485	0.974	0.548

-TOTAL ANNUAL OPERATING COSTS ($millions) = 13.8197
Iterations for this run = 495
STOPPING LIMIT EPSI = 200.00
O* - Millions

APPROXIMATE SOLUTION FOR THE LAY-UP PROBLEM

Number of ships which are eliminated: 1

Eliminated ship #: 2
Optimum cost (in millions): 13.7506

*****GLOBAL OPTIMUM SOLUTION

FLEET # 1 SUMMARY

SHIP	*COST PER INTERVAL ($)	COST PER TON ($)	ROUND TRIPS PER INTERVAL	INITIAL/ OPTIMUM SPEED (KNOTS)	INITIAL/ OPTIMUM BAL SPEED (KNOTS)	*TONS CARRIED (TONS)
SHIP# 1						
VESSEL A						
TIME INTERVAL # 1	2.0562	2.8628	7.18	9.63 9.68	12.64 12.44	0.7182
TIME INTERVAL # 2	2.2489	3.1282	7.19	9.63 9.39	12.64 13.16	0.7189
TIME INTERVAL # 3	3.2534	2.8348	11.48	9.63 8.96	12.64 12.19	1.1477
SHIP# 2						
VESSEL B						
TIME INTERVAL # 1	0.0	0.0	0.0	0.0 0.0	0.0 0.0	0.0
TIME INTERVAL # 2	0.0	0.0	0.0	0.0 0.0	0.0 0.0	0.0
TIME INTERVAL # 3	0.0	0.0	0.0	0.0 0.0	0.0 0.0	0.0
SHIP# 3						
VESSEL C						
TIME INTERVAL # 1	1.1348	2.7314	8.31	13.83 13.02	11.04 11.21	0.4155
TIME INTERVAL # 2	1.2254	3.1608	7.75	13.83 12.53	15.44 9.07	0.3877

Table 5.2

(continued)

TIME INTERVAL # 3	2.0318	3.8057	10.68	5.09 7.96	15.44 14.52	0.5339

DURING THE YEAR

	*COST/YR ($)	COST/TON ($)	R.T./YR	*TONS/YR	UTIL.FACTOR
SHIP# 1					
VESSEL A	7.559	2.924	25.848	2.585	0.787
SHIP# 2					
VESSEL B	1.800	0.0	0.0	0.0	0.0
SHIP# 3					
VESSEL C	4.392	3.285	26.740	1.337	0.752

-TOTAL ANNUAL OPERATING COSTS ($millions) = 13.7506
Iterations for this run = 303
STOPPING LIMIT EPSI = 200.00
O* - Millions

EXACT SOLUTION FOR THE LAY-UP PROBLEM

Number of ships which are eliminated : 1
Optimum cost (in millions): 13.1574

Number of ships which are eliminated : 2
Available cargo exceeds max capacity of any fleet

***** GLOBAL OPTIMUM SOLUTION

FLEET # 1 SUMMARY

SHIP	*COST PER INTERVAL ($)	COST PER TON ($)	ROUND TRIPS PER INTERVAL	INITIAL/ OPTIMUM SPEED (KNOTS)	INITIAL/ OPTIMUM BAL.SPEED (KNOTS)	*TONS CARRIED (TONS)
SHIP# 1						
VESSEL A						
TIME INTERVAL # 1	2.1323	2.8298	7.54	9.23 11.11	12.24 12.79	0.7535
TIME INTERVAL # 2	2.1653	3.1670	6.84	9.23 9.12	12.24 10.78	0.6837
TIME INTERVAL # 3	3.1309	2.8417	11.02	9.23 8.41	12.24 11.09	1.1018
SHIP# 2						
VESSEL B						
TIME INTERVAL # 1	1.3346	2.9062	9.18	13.83 13.34	10.64 13.62	0.4592
TIME INTERVAL # 2	1.3054	3.3909	7.70	13.83 9.53	15.44 10.36	0.3850
TIME INTERVAL # 3	1.7889	3.2979	10.85	5.06 7.75	15.44 13.08	0.5424
SHIP# 3						
VESSEL C						
TIME INTERVAL # 1	0.0	0.0	0.0	0.0 0.0	0.0 0.0	0.0
TIME INTERVAL # 2	0.0	0.0	0.0	0.0 0.0	0.0 0.0	0.0

Table 5.2

(continued)

				0.0	0.0	
TIME INTERVAL # 3	0.0	0.0	0.0	0.0	0.0	0.0

DURING THE YEAR

	*COST/YR ($)	COST/TON ($)	R.T./YR	*TONS/YR	UTIL.FACTOR
SHIP# 1					
VESSEL A	7.429	2.926	25.390	2.539	0.773
SHIP# 2					
VESSEL B	4.429	3.194	27.732	1.387	0.740
SHIP# 3					
VESSEL C	1.300	0.0	0.0	0.0	0.0

-TOTAL ANNUAL OPERATING COSTS ($millions) = 13.1574
Iterations for this run = 1224
STOPPING LIMIT EPSI = 10.00
0* - Millions

INTEGER SOLUTION

SHIP	*COST PER INTERVAL ($)	COST PER TON ($)	ROUND TRIPS PER INTERVAL	OPTIMUM LOADED SPEED (KNOTS)	OPTIMUM BALLAST SPEED (KNOTS)	*TONS CARRIED (TONS)
SHIP# 1						
VESSEL A						
TIME INTERVAL # 1	1.9025	3.1709	6.00	7.20	8.70	0.6000
TIME INTERVAL # 2	1.8923	3.7846	5.00	5.43	6.47	0.5000
TIME INTERVAL # 3	2.7559	3.0621	9.00	6.27	7.36	0.9000
SHIP# 2						
VESSEL B						
TIME INTERVAL # 1	1.0899	3.6330	6.00	6.47	7.12	0.3000
TIME INTERVAL # 2	1.1450	3.8166	6.00	6.45	7.15	0.3000
TIME INTERVAL # 3	1.5043	3.3429	9.00	5.63	6.32	0.4500
SHIP# 3						
VESSEL C						
TIME INTERVAL # 1	0.9435	3.1451	6.00	6.80	7.42	0.3000
TIME INTERVAL # 2	0.9421	3.7685	5.00	5.18	5.77	0.2500
TIME INTERVAL # 3	1.6337	4.0842	8.00	5.01	5.50	0.4000

DURING THE YEAR

	*COST/YR ($)	COST/TON ($)	R.T./YR	*TONS/YR	UTIL.FACTOR
SHIP# 1					
VESSEL A	6.551	3.275	20.000	2.000	0.609
SHIP# 2					
VESSEL B	3.739	3.561	21.000	1.050	0.560
SHIP# 3					
VESSEL C	3.519	3.705	19.000	0.950	0.534

-TOTAL ANNUAL OPERATING COSTS ($millions) = 13.8092
-TOTAL TONS CARRIED(MILLIONS)- 4.0000
0* - Millions

Table 5.3

SAMPLE OUTPUT FOR A FLEET OF TEN SHIPS

```
TABLE OF INPUT VALUES
====================

 # OF TIME INTERVALS = 2
 TIME INTERVAL # 1: 100 DAYS
 TIME INTERVAL # 2: 265 DAYS
---------------------------------------------------
SHIP NUMBER ----   1

 SHIP NAME ------  VESSEL A
 CARGO CAPACITY -  100000.0
 RSTSPD --    7.00          RSTPWR --    2000.00      RSTFR ---      0.6000
 LPDAY ---    2.00          LPFUEL --  24000.00
 UPDAY ---    2.00          UPFUEL --  24000.00
 ALPHA ---    5.09000015   BETA ----    3.00000000   GAMMA ---    0.22793400
 SIGMA ---   -0.44696802   DELT ----    0.63572901
 ALPHAB ---    3.00000000   BETAB ----    3.00000000   GAMMAB ---    0.22799999
 SIGMAB ---   -0.44700003   DELTB ----    0.63000000
 $LAYUP --    3700000.00
 LOWSPD --     6.00          HISPD ---    16.00
 LBSPD --     5.00         HBSPD ---    20.00
 FULPWR --  25000.00
 CCOST ---         0.0     RIN --  0.0      TR --   .0

VARIABLE SHIP DATA
------------------
 TIME INTERVAL # 1
  $MAN ----      300000.00  $STORE --       80000.00  $CAPTL --    1000000.00
  $ADMIN --      300000.00  $MAINT --      200000.00  $SAIL ---            0.0
  $LP -----    800.00  $UP -----     800.00
  MRDAY ---    5.00
 TIME INTERVAL # 2
  $MAN ----      575000.00  $STORE --      145000.00  $CAPTL --    2000000.00
  $ADMIN --      700000.00  $MAINT --      430000.00  $SAIL ---            0.0
  $LP -----    1200.00  $UP -----    1200.00
  MRDAY ---    10.00
---------------------------------------------------
SHIP NUMBER ----   2
(Same data as SHIP #1)
SHIP NUMBER ----   3
(Same data as SHIP #1)
SHIP NUMBER ----   4
(Same data as SHIP #1)
---------------------------------------------------
SHIP NUMBER ----   5

 SHIP NAME ------  VESSEL B
 CARGO CAPACITY -   50000.0
 RSTSPD --    7.00          RSTPWR --    1500.00      RSTFR ---      0.5340
 LPDAY ---    1.25          LPFUEL --  24000.00
 UPDAY ---    1.25          UPFUEL --  24000.00
 ALPHA ---    2.77999973   BETA ----    3.10000038   GAMMA ---    0.20514101
 SIGMA ---   -0.40227097   DELT ----    0.57215601
 ALPHAB ---    2.00000000   BETAB ----    3.10000038   GAMMAB ---    0.19999999
 SIGMAB ---   -0.39999998   DELTB ----    0.56999999
 $LAYUP --    1800000.00
 LOWSPD --     5.00          HISPD ---    16.00
 LBSPD --     5.00         HBSPD ---    18.00
 FULPWR --  15000.00
 CCOST ---         0.0     RIN --  0.0      TR --   .0

VARIABLE SHIP DATA
------------------
 TIME INTERVAL # 1
  $MAN ----      250000.00  $STORE --       50000.00  $CAPTL --     500000.00
  $ADMIN --      150000.00  $MAINT --      125000.00  $SAIL ---            0.0
  $LP -----    300.00  $UP -----     300.00
  MRDAY ---    5.00
 TIME INTERVAL # 2
  $MAN ----      500000.00  $STORE --      100000.00  $CAPTL --    1000000.00
  $ADMIN --      250000.00  $MAINT --      300000.00  $SAIL ---            0.0
  $LP -----    700.00  $UP -----     700.00
  MRDAY ---    10.00
---------------------------------------------------
```

Table 5.3
(continued)

```
SHIP NUMBER ----    6
(Same data as SHIP #5)
SHIP NUMBER ----    7
(Same data as SHIP #5)
------------------------------------------------
SHIP NUMBER ----    8

 SHIP NAME ------  VESSEL C
 CARGO CAPACITY -    50000.0
 RSTSPD --    7.00        RSTPWR --    1500.00       RSTFR ---      0.5500
 LPDAY ---    1.50        LPFUEL --  24000.00
 UPDAY ---    1.50        UPFUEL --  24000.00
 ALPHA ---   2.10000038   BETA ----   3.19999981   GAMMA ---    0.21653700
 SIGMA ---  -0.42461997   DELT ----   0.60394299
 ALPHAB ---    1.60000038   BETAB ---    3.19999981   GAMMAB ---    0.20999998
 SIGMAB ---   -0.42000002   DELTB ----   0.60000002
 $LAYUP --    1300000.00
 LOWSPD --     5.00         HISPD ---     16.00
 LBSPD --    5.00         HBSPD ---    18.00
 FULPWR --   15000.00
 CCOST ---          0.0     RIN --  0.0     TR --   .0

VARIABLE SHIP DATA
-----------------
 TIME INTERVAL # 1
  $MAN ----      250000.00  $STORE --       50000.00  $CAPTL --     300000.00
  $ADMIN --      150000.00  $MAINT --      125000.00  $SAIL ---           0.0
  $LP -----      300.00   $UP -----      300.00
  MRDAY ---      5.00
 TIME INTERVAL # 2
  $MAN ----      575000.00  $STORE --      125000.00  $CAPTL --     700000.00
  $ADMIN --      210000.00  $MAINT --      300000.00  $SAIL ---           0.0
  $LP -----      700.00   $UP -----      700.00
  MRDAY ---     10.00
------------------------------------------------
SHIP NUMBER ----    9
(Same data as SHIP #8)
SHIP NUMBER ----   10
(Same data as SHIP #8)
ROUTE INPUT DATA
-----------------
ANNUAL CARGO AVAILABLE --       4900000.0
BALLAST DISTANCE (NM)----     3600.0
LOADED DISTANCE (NM)-----     4000.0
RESTRICTED DISTANCE (NM)-      800.0
FUEL PRICE PER LB
TIME INTERVAL # 1: 0.0800
TIME INTERVAL # 2: 0.1200
```

FLEET # 1 SUMMARY

SHIP	*COST PER INTERVAL ($)	COST PER TON ($)	ROUND TRIPS PER INTERVAL	INITIAL/ OPTIMUM SPEED (KNOTS)	INITIAL/ OPTIMUM BAL SPEED (KNOTS)	*TONS CARRIED (TONS)
SHIP# 1						
VESSEL A						
				8.70	9.70	
TIME INTERVAL # 1	2.2025	10.0397	2.19	8.70	9.75	0.2194
				7.70	8.70	
TIME INTERVAL # 2	4.8251	8.9974	5.36	7.70	8.75	0.5363
SHIP# 2						
VESSEL A						
				8.70	9.70	
TIME INTERVAL # 1	2.2025	10.0397	2.19	8.70	9.75	0.2194
				7.70	8.70	
TIME INTERVAL # 2	4.8251	8.9974	5.36	7.70	8.75	0.5363

Table 5.3

(continued)

SHIP# 3						
VESSEL A						
TIME INTERVAL # 1	2.2025	10.0397	2.19	8.70 8.70	9.70 9.75	0.2194
TIME INTERVAL # 2	4.8251	8.9974	5.36	7.70 7.70	8.70 8.75	0.5363
SHIP# 4						
VESSEL A						
TIME INTERVAL # 1	2.2025	10.0397	2.19	8.70 8.70	9.70 9.75	0.2194
TIME INTERVAL # 2	4.8251	8.9974	5.36	7.70 7.70	8.70 8.75	0.5363
SHIP# 5						
VESSEL B						
TIME INTERVAL # 1	1.1919	12.9207	1.84	6.70 6.70	7.70 7.75	0.0922
TIME INTERVAL # 2	2.4772	11.4096	4.34	5.70 5.70	6.70 6.75	0.2171
SHIP# 6						
VESSEL B						
TIME INTERVAL # 1	1.1919	12.9207	1.84	6.70 6.70	7.70 7.75	0.0922
TIME INTERVAL # 2	2.4772	11.4096	4.34	5.70 5.70	6.70 6.75	0.2171
SHIP# 7						
VESSEL B						
TIME INTERVAL # 1	1.1919	12.9207	1.84	6.70 6.70	7.70 7.75	0.0922
TIME INTERVAL # 2	2.4772	11.4096	4.34	5.70 5.70	6.70 6.75	0.2171
SHIP# 8						
VESSEL C						
TIME INTERVAL # 1	0.9926	10.8643	1.83	6.70 6.70	7.70 7.75	0.0914
TIME INTERVAL # 2	2.2379	10.3951	4.31	5.70 5.70	6.70 6.75	0.2153
SHIP# 9						
VESSEL C						
TIME INTERVAL # 1	0.9926	10.8643	1.83	6.70 6.70	7.70 7.75	0.0914
TIME INTERVAL # 2	2.2379	10.3951	4.31	5.70 5.70	6.70 6.75	0.2153
SHIP# 10						
VESSEL C						
TIME INTERVAL # 1	1.0220	10.3166	1.98	8.00 8.00	7.70 7.75	0.0991
TIME INTERVAL # 2	2.2975	10.8263	4.24	6.66 6.05	8.00 8.10	0.2122

Table 5.3

(continued)

DURING THE YEAR

	*COST/YR ($)	COST/TON ($)	R.T./YR	*TONS/YR	UTIL FACTOR
SHIP# 1					
VESSEL A	7.028	9.300	7.557	0.756	0.576
SHIP# 2					
VESSEL A	7.028	9.300	7.557	0.756	0.576
SHIP# 3					
VESSEL A	7.028	9.300	7.557	0.756	0.576
SHIP# 4					
VESSEL A	7.028	9.300	7.557	0.756	0.576
SHIP# 5					
VESSEL B	3.669	11.860	6.187	0.309	0.460
SHIP# 6					
VESSEL B	3.669	11.860	6.187	0.309	0.460
SHIP# 7					
VESSEL B	3.669	11.860	6.187	0.309	0.460
SHIP# 8					
VESSEL C	3.230	10.535	6.133	0.307	0.465
SHIP# 9					
VESSEL C	3.230	10.535	6.133	0.307	0.465
SHIP# 10					
VESSEL C	3.320	10.664	6.226	0.311	0.472

-TOTAL ANNUAL OPERATING COSTS ($millions) = 48.8983
Iterations for this run = 41
STOPPING LIMIT EPSI = 1000.00
O* - Millions

EXACT SOLUTION FOR THE LAY-UP PROBLEM

Number of ships which are eliminated : 1
Optimum cost (in millions): 48.4261

Number of ships which are eliminated : 2
Optimum cost (in millions): 48.4466

Number of ships which are eliminated : 3
Optimum cost (in millions): 48.8675

Number of ships which are eliminated : 4
Optimum cost (in millions): 49.9068

Number of ships which are eliminated : 5
Optimum cost (in millions): 51.7453

Number of ships which are eliminated : 6
Optimum cost (in millions): 54.9494

Number of ships which are eliminated : 7
Available cargo exceeds max capacity of any fleet

Table 5.3

(continued)

***** GLOBAL OPTIMUM SOLUTION

FLEET # 1 SUMMARY

SHIP	*COST PER INTERVAL ($)	COST PER TON ($)	ROUND TRIPS PER INTERVAL	INITIAL/ OPTIMUM SPEED (KNOTS)	INITIAL/ OPTIMUM BAL SPEED (KNOTS)	*TONS CARRIED (TONS)
SHIP# 1						
VESSEL A						
TIME INTERVAL # 1	2.1464	10.3812	2.07	7.61 7.61	9.76 9.81	0.2068
TIME INTERVAL # 2	4.9225	8.8697	5.55	7.61 7.61	9.76 9.81	0.5550
SHIP# 2						
VESSEL A						
TIME INTERVAL # 1	2.1464	10.3812	2.07	7.61 7.61	9.76 9.81	0.2068
TIME INTERVAL # 2	4.9225	8.8697	5.55	7.61 7.61	9.76 9.81	0.5550
SHIP# 3						
VESSEL A						
TIME INTERVAL # 1	2.1477	10.3702	2.07	7.61 7.61	9.76 9.86	0.2071
TIME INTERVAL # 2	4.9170	8.8749	5.54	7.61 7.61	9.76 9.76	0.5540
SHIP# 4						
VESSEL A						
TIME INTERVAL # 1	2.1450	10.3923	2.06	7.61 7.61	9.76 9.76	0.2064
TIME INTERVAL # 2	4.9170	8.8749	5.54	7.61 7.61	9.76 9.76	0.5540
SHIP# 5						
VESSEL B						
TIME INTERVAL # 1	1.2192	12.3005	1.98	7.24 7.24	8.50 8.50	0.0991
TIME INTERVAL # 2	2.7333	10.2733	5.32	7.24 7.24	8.50 8.50	0.2661
SHIP# 6						
VESSEL B						
TIME INTERVAL # 1	1.2192	12.3005	1.98	7.24 7.24	8.50 8.50	0.0991
TIME INTERVAL # 2	2.7333	10.2733	5.32	7.24 7.24	8.50 8.50	0.2661
SHIP# 7						
VESSEL B						
TIME INTERVAL # 1	1.2192	12.3005	1.98	7.24 7.24	8.50 8.50	0.0991
TIME INTERVAL # 2	2.7333	10.2733	5.32	7.24 7.24	8.50 8.50	0.2661
SHIP# 8						
VESSEL C						
TIME INTERVAL # 1	1.0206	10.4039	1.96	7.24 7.24	8.50 8.50	0.0981
TIME INTERVAL # 2	2.4987	9.4898	5.27	7.24 7.24	8.50 8.50	0.2633

Table 5.3

(continued)

SHIP# 9

VESSEL C

				11.94	8.50	
TIME INTERVAL # 1	1.1564	9.5831	2.41	11.94	8.50	0.1207
				6.34	13.20	
TIME INTERVAL # 2	2.5499	11.7524	4.34	6.21	13.20	0.2170

SHIP# 10

VESSEL C

				0.0	0.0	
TIME INTERVAL # 1	0.0	0.0	0.0	0.0	0.0	0.0
				0.0	0.0	
TIME INTERVAL # 2	0.0	0.0	0.0	0.0	0.0	0.0

DURING THE YEAR

	*COST/YR ($)	COST/TON ($)	R.T./YR	*TONS/YR	UTIL.FACTOR
SHIP# 1 VESSEL A	7.069	9.280	7.617	0.762	0.581
SHIP# 2 VESSEL A	7.069	9.280	7.617	0.762	0.581
SHIP# 3 VESSEL A	7.065	9.282	7.611	0.761	0.580
SHIP# 4 VESSEL A	7.062	9.287	7.604	0.760	0.580
SHIP# 5 VESSEL B	3.952	10.824	7.303	0.365	0.543
SHIP# 6 VESSEL B	3.952	10.824	7.303	0.365	0.543
SHIP# 7 VESSEL B	3.952	10.824	7.303	0.365	0.543
SHIP# 8 VESSEL C	3.519	9.738	7.228	0.361	0.548
SHIP# 9 VESSEL C	3.706	10.977	6.753	0.338	0.512
SHIP# 10 VESSEL C	1.300	0.0	0.0	0.0	0.0

-TOTAL ANNUAL OPERATING COSTS ($millions) - 48.6473

Iterations for this run = 37

STOPPING LIMIT EPSI = 1000.00

0* - Millions

Table 5.4

SAMPLE OUTPUT FOR A FLEET OF TEN SHIPS
(Different Route Data)

ROUTE INPUT DATA

ANNUAL CARGO AVAILABLE -- 5400000.0
BALLAST DISTANCE (NM)---- 3600.0
LOADED DISTANCE (NM)----- 4000.0
RESTRICTED DISTANCE (NM)- 800.0
FUEL PRICE PER LB
TIME INTERVAL # 1: 0.0600
TIME INTERVAL # 2: 0.1500

FLEET # 1 SUMMARY

SHIP	*COST PER INTERVAL ($)	COST PER TON ($)	ROUND TRIPS PER INTERVAL	INITIAL/ OPTIMUM SPEED (KNOTS)	INITIAL/ OPTIMUM BAL.SPEED (KNOTS)	*TONS CARRIED (TONS)
SHIP# 1						
VESSEL A						
TIME INTERVAL # 1	2.1727	9.3261	2.33	8.60 9.38	8.60 10.53	0.2330
TIME INTERVAL # 2	5.0077	9.4743	5.29	8.60 7.11	8.60 9.35	0.5286
SHIP# 2						
VESSEL A						
TIME INTERVAL # 1	2.4229	8.4895	2.85	8.60 11.19	8.60 15.57	0.2854
TIME INTERVAL # 2	4.9981	9.4852	5.27	8.60 7.36	8.60 8.84	0.5269
SHIP# 3						
VESSEL A						
TIME INTERVAL # 1	2.3108	8.7084	2.65	8.60 9.50	8.60 15.79	0.2654
TIME INTERVAL # 2	4.9966	9.4840	5.27	8.60 7.09	8.60 9.30	0.5268
SHIP# 4						
VESSEL A						
TIME INTERVAL # 1	2.2888	8.9351	2.56	8.60 11.06	8.60 11.31	0.2562
TIME INTERVAL # 2	5.1850	9.3480	5.55	8.60 7.60	8.60 9.81	0.5547
SHIP# 5						
VESSEL B						
TIME INTERVAL # 1	1.3507	10.0158	2.70	8.60 10.01	8.60 13.26	0.1349
TIME INTERVAL # 2	2.6154	11.4924	4.55	8.60 5.90	8.60 7.31	0.2276
SHIP# 6						
VESSEL B						
TIME INTERVAL # 1	1.2889	10.3863	2.48	8.60 8.83	8.60 12.37	0.1241
TIME INTERVAL # 2	2.8664	10.8666	5.28	8.60 6.76	8.60 9.14	0.2638
SHIP# 7						
VESSEL B						
TIME INTERVAL # 1	1.3782	9.9477	2.77	8.60 10.70	8.60 13.10	0.1385
TIME INTERVAL # 2	2.5300	11.9691	4.23	8.60 5.83	8.60 6.12	0.2114

Table 5.4

(continued)

SHIP# 8						
VESSEL C						
TIME INTERVAL # 1	1.1194	9.0021	2.49	8.60 10.75	8.60 10.04	0.1243
TIME INTERVAL # 2	2.5787	10.1674	5.07	8.60 6.49	8.60 8.91	0.2536
SHIP# 9						
VESSEL C						
TIME INTERVAL # 1	1.0670	9.1999	2.32	8.60 9.79	8.60 9.28	0.1160
TIME INTERVAL # 2	2.5114	10.2369	4.91	8.60 6.35	8.60 8.35	0.2453
SHIP# 10						
VESSEL C						
TIME INTERVAL # 1	1.1676	8.4332	2.77	9.00 11.39	8.60 12.59	0.1385
TIME INTERVAL # 2	2.4398	11.5903	4.21	6.00 6.00	9.00 9.12	0.2105

DURING THE YEAR

	*COST/YR ($)	COST/TON ($)	R.T./YR	*TONS/YR	UTIL FACTOR
SHIP# 1 VESSEL A	7.180	9.429	7.615	0.762	0.580
SHIP# 2 VESSEL A	7.421	9.135	8.123	0.812	0.619
SHIP# 3 VESSEL A	7.307	9.224	7.922	0.792	0.604
SHIP# 4 VESSEL A	7.474	9.218	8.108	0.811	0.618
SHIP# 5 VESSEL B	3.966	10.943	7.249	0.362	0.539
SHIP# 6 VESSEL B	4.155	10.713	7.757	0.388	0.577
SHIP# 7 VESSEL B	3.908	11.169	6.998	0.350	0.520
SHIP# 8 VESSEL C	3.698	9.784	7.560	0.378	0.573
SHIP# 9 VESSEL C	3.578	9.904	7.226	0.361	0.547
SHIP# 10 VESSEL C	3.607	10.338	6.979	0.349	0.529

-TOTAL ANNUAL OPERATING COSTS ($millions) = 52.2961

Iterations for this run = 6394

STOPPING LIMIT EPSI = 500.00

O* - Millions

Table 5.4

(continued)

```
              APPROXIMATE SOLUTION FOR THE LAY-UP PROBLEM
              ==========================================

Number of ships which are eliminated  5
-------------------------------------
Eliminated ship #: 5
Eliminated ship #: 7
Eliminated ship #: 8
Eliminated ship #: 9
Eliminated ship #:10
Optimum cost (in millions):   62.0815
-------------------------------------

Number of ships which are eliminated: 4
-------------------------------------
Eliminated ship #: 5
Eliminated ship #: 7
Eliminated ship #: 9
Eliminated ship #:10
Optimum cost (in millions):   57.4985
-------------------------------------

Number of ships which are eliminated: 3
-------------------------------------
Eliminated ship #: 5
Eliminated ship #: 7
Eliminated ship #:10
Optimum cost (in millions):   54.8970
-------------------------------------

Number of ships which are eliminated: 2
-------------------------------------
Eliminated ship #: 7
Eliminated ship #:10
Optimum cost (in millions):   53.3413
-------------------------------------

Number of ships which are eliminated: 1
-------------------------------------
Eliminated ship #: 7
Optimum cost (in millions):   52.5683
-------------------------------------

OPTIMUM FLEET IS THE INITIAL FLEET
```